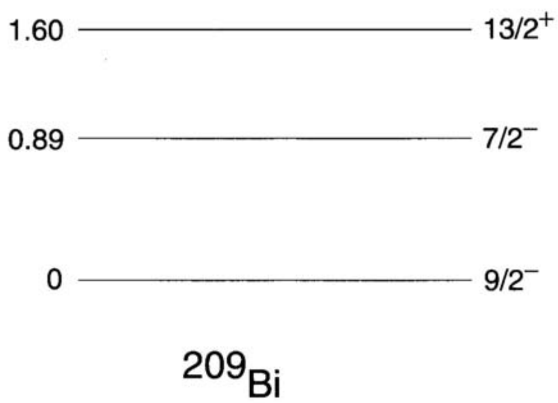
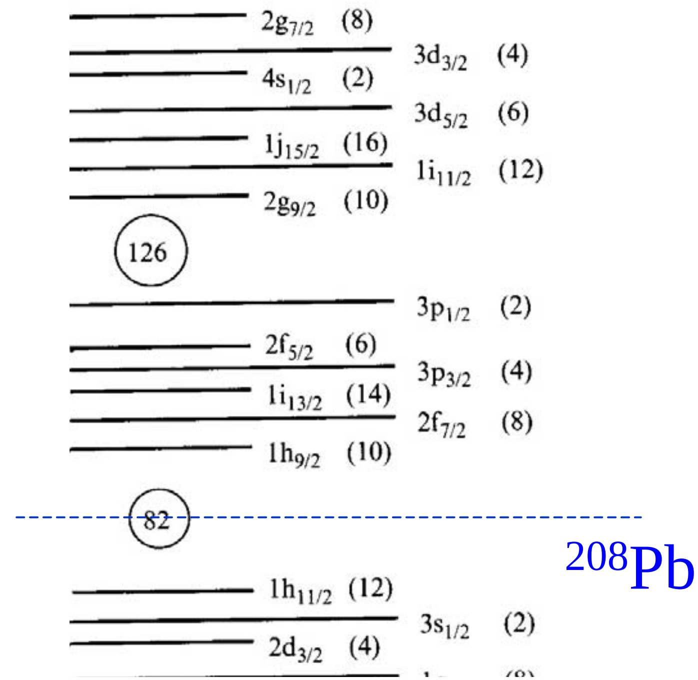
Pairing Correlations

$^{209}_{83}Bi_{126} = {}^{208}_{82}Pb_{126} + p$



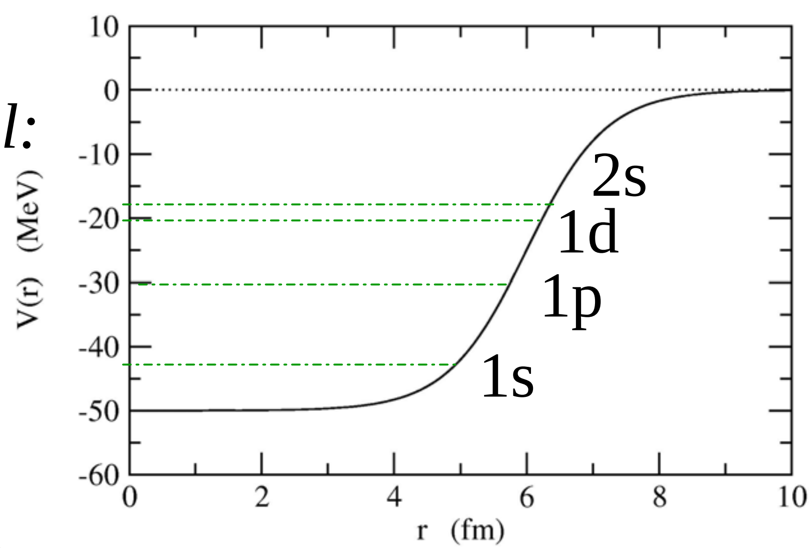


$^{210}_{84}Po_{126} = {}^{208}_{82}Pb_{126} + 2p$

expectation of the indep. particle model:

E=0: $[h_{9/2} \otimes h_{9/2}]^I$ $(I=0,2,4,6,8)$

E=0.89 MeV: $[h_{9/2} \otimes f_{7/2}]^I$ $(I=1,2,3,4,5,6,7,8)$



# of states below 1 MeV: 13

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observed spectra:

1.20 MeV —— 4^+

0.81 MeV —— 2^+

0 —— 0^+

^{210}Po

↔ Effects of the residual interaction

$$H = \sum_{i=1}^{A} \left(-\frac{\hbar^2}{2m} \boldsymbol{\nabla}_i^2 + V_{\mathrm{HF}}(i) \right) + \underline{\underline{\frac{1}{2} \sum_{i,j}^{A} v(\boldsymbol{r}_i, \boldsymbol{r}_j) - \sum_i V_{\mathrm{HF}}(i)}}$$

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$$\sim -g\,\delta(\boldsymbol{r} - \boldsymbol{r}') \qquad \text{(short range force)}$$

$$= -g\frac{\delta(r - r')}{rr'}\sum_{\lambda\mu} Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})Y_{\lambda\mu}(\hat{\boldsymbol{r}}')$$

$$\Delta E_I \sim \langle [j \bigotimes j]^I | -g\delta(\boldsymbol{r} - \boldsymbol{r}') | [j \bigotimes j]^I \rangle$$

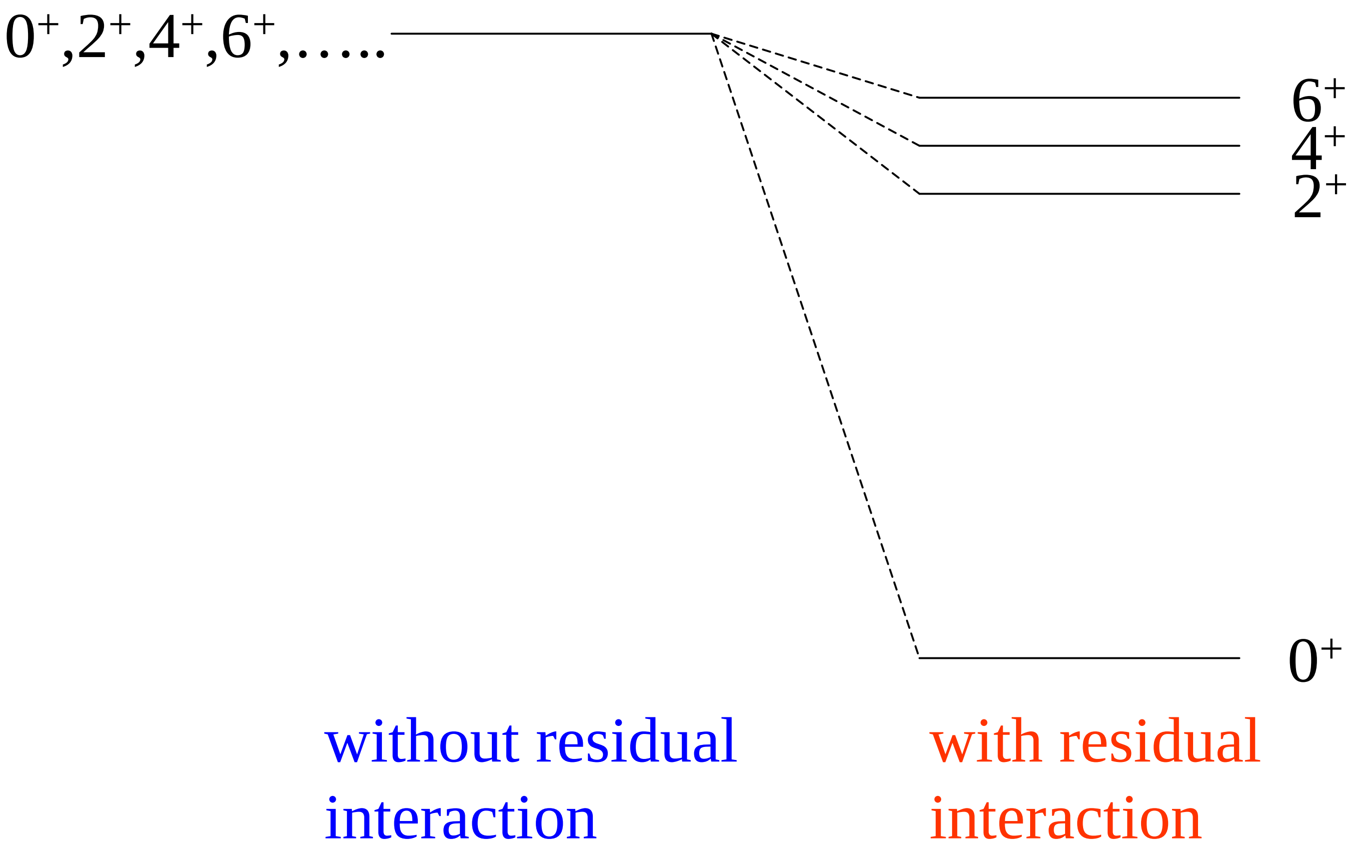
$$= -g\,F_r\,\frac{(2j+1)^2}{2}\begin{pmatrix} j & j & I \\ 1/2 & -1/2 & 0 \end{pmatrix}^2$$

(for even *j*)

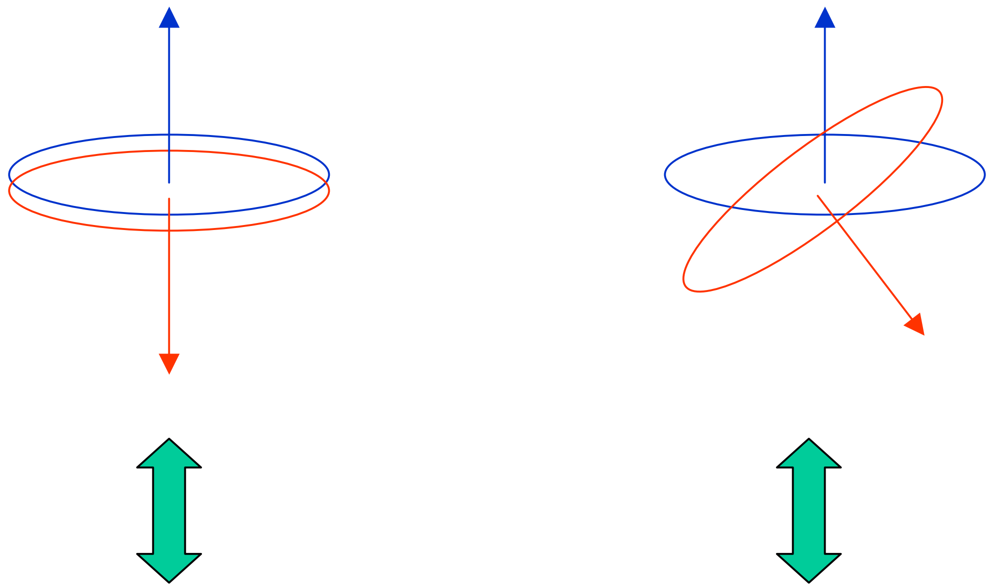
$$F_r = \int dr \frac{u^4_{jl}(r)}{4\pi r^2} \qquad \text{(radial integral)}$$

$$\Delta E_I \sim -g\,F_r\,\frac{(2j+1)^2}{2}\begin{pmatrix} j & j & I \\ 1/2 & -1/2 & 0 \end{pmatrix}^2 \equiv -g\,F_r\,A(jj;I)$$

A(jj;I)	*I*=0	*I*=2	*I*=4	*I*=6
j=5/2	3.00	0.685	0.286	---
j=7/2	4.00	0.95	0.467	0.233



Simple interpretation:



I=0 pair $\qquad$ $I \neq 0$ pair

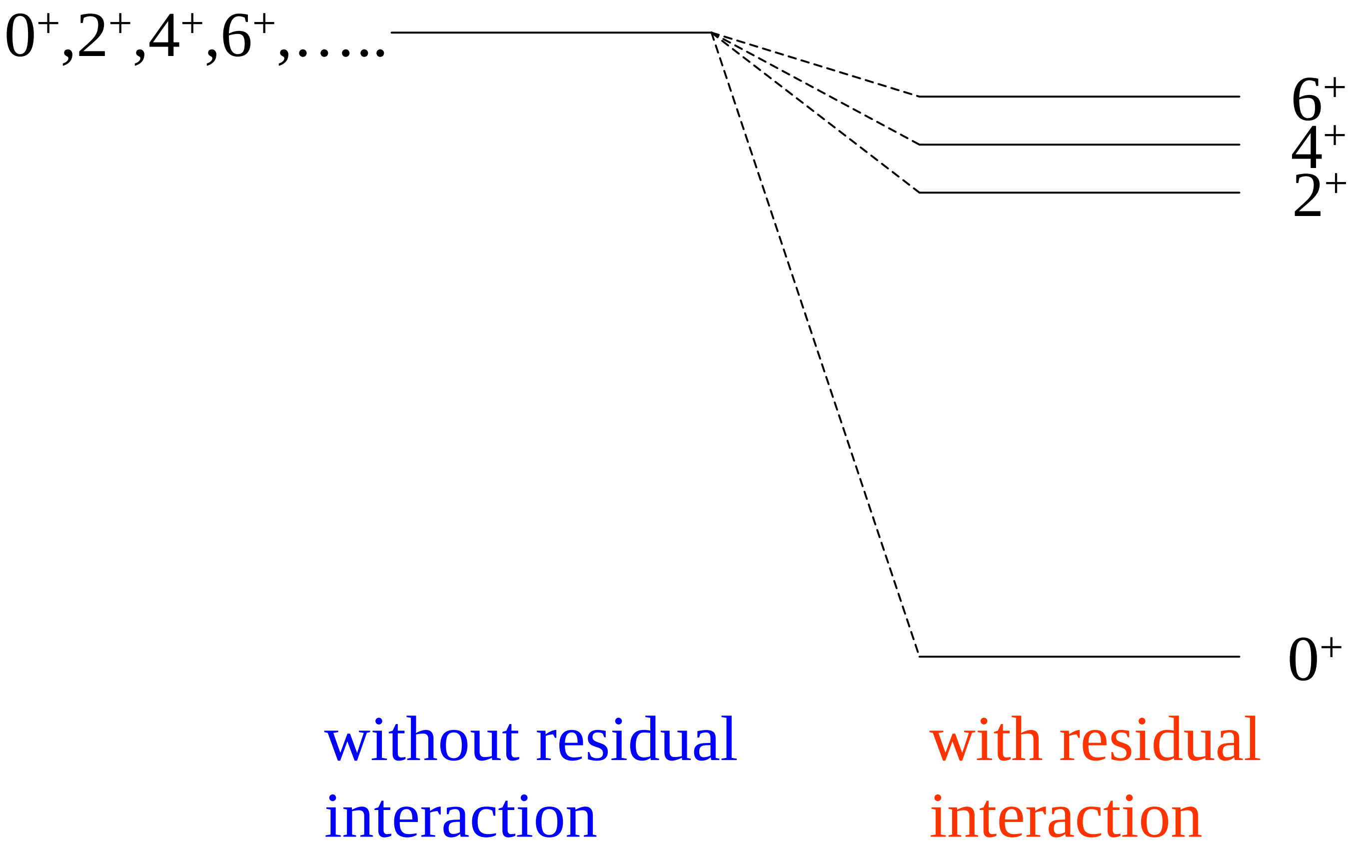
The spatial overlap is the largest for the I=0 pair.

"Pairing Correlation"

(note) The I=$2j$ pair is unfavoured due to the Pauli principle.

(note)

$$\psi(l^2; L=0) = \sum_{\mu} \langle l\mu l-\mu | L=0,0\rangle \, Y_{l\mu}(\hat{\boldsymbol{r}}_1) Y_{l-\mu}(\hat{\boldsymbol{r}}_2) = Y_{l0}(\theta_{12})/\sqrt{4\pi}$$



<u>The ground state spin of nuclei</u>

- Even-even nuclei: 0^+
- Even-odd nuclei: the spin of the valence particle

Mass Formula (Even-odd mass difference)

Extra binding when like nucleons form a spin-zero pair

Example:

	Binding energy (MeV)
${}^{210}_{82}Pb_{128} = {}^{208}_{82}Pb_{126}+2n$	1646.6
${}^{210}_{83}Bi_{127} = {}^{208}_{82}Pb_{126}+n+p$	1644.8
${}^{209}_{82}Pb_{127} = {}^{208}_{82}Pb_{126}+n$	1640.4
${}^{209}_{83}Bi_{126} = {}^{208}_{82}Pb_{126}+p$	1640.2

$$\begin{aligned} B_{\text{pair}} &= \Delta && (\text{for even} - \text{even}) \\ &= 0 && (\text{for even} - \text{odd}) \\ &= -\Delta && (\text{for odd} - \text{odd}) \end{aligned}$$