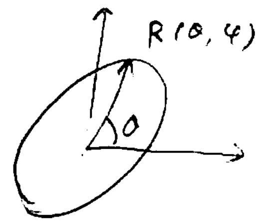
▨ 原子核の変形と回転，振動について

1. 変形パラメーター



$$R(\theta,\varphi) = R_0\left(1+\sum_{\lambda,\mu}\alpha_{\lambda\mu}\underbrace{Y_{\lambda\mu}^{*}(\hat{r})}_{=(-)^{\mu}Y_{\lambda-\mu}}\right)$$

(note) R は実数

$\Rightarrow$

$$R = R^{*} = R_0\left(1+\sum_{\lambda\mu}\alpha_{\lambda\mu}^{*}Y_{\lambda\mu}\right)$$

$$\underset{\mu\leftrightarrow-\mu}{=} R_0\left(1+\sum_{\lambda\mu}\alpha_{\lambda-\mu}^{*}Y_{\lambda-\mu}\right)$$

$$\Rightarrow \boxed{\alpha_{\lambda-\mu}^{*} = (-)^{\mu}\alpha_{\lambda\mu}}$$

$\sum_{\lambda\mu}\alpha_{\lambda\mu}Y_{\lambda\mu}^{*}$ は全体の回転に対し不変

$\longrightarrow$ $\alpha_{\lambda\mu}$ は $Y_{\lambda\mu}$ と同じ変換性を持つ

・$\lambda=2$ に限定（四重極型変形）

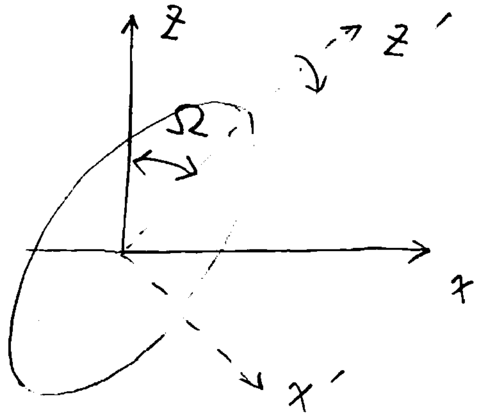
$$R(\theta,\varphi) = R_0\left(1+\sum_{\mu=-2}^{2}\alpha_{2\mu}Y_{2\mu}^{*}(\theta,\varphi)\right)$$

原子核の運動（回転，振動）$\rightarrow$ $\alpha_{2\mu}$ の時間変化によって記述される

⇝ 回転座標系で議論した方が容易
（原子核と一緒に回転する座標系）

$$Y_{20} \propto 3\cos^2\theta - 1$$
$$Y_{2\pm1} \propto \sin\theta\cos\theta\, e^{\pm i\varphi}$$
$$Y_{2\pm2} \propto \sin^2\theta\, e^{\pm 2i\varphi}$$

物体固定系への座標変換



$$Y_{\lambda\mu}(\theta',\varphi') = \mathcal{R}(\Omega)\, Y_{\lambda\mu}(\theta,\varphi)$$
$$= \sum_{\mu'} Y_{\lambda\mu'}(\theta,\varphi) \underbrace{\langle Y_{\lambda\mu'}|\mathcal{R}|Y_{\lambda\mu}\rangle}_{= D^{\lambda\,*}_{\mu'\mu}}$$

$$a_{\lambda\mu} = \sum_{\mu'} D^{\lambda\,*}_{\mu'\mu}\, \alpha_{\lambda\mu'}$$

$$R(\theta,\varphi) = R_0\left(1 + \sum_{\mu} \alpha_{2\mu} Y^*_{2\mu}(\theta,\varphi)\right)$$

を物体固定系へ座標変換

$$\longrightarrow\quad R'(\theta',\varphi') = R_0\left(1 + \sum_{\mu} a_{2\mu} Y^*_{2\mu}(\theta',\varphi')\right)$$

物体固定系から計った角度

軸の選び方:

$z'=0$ の平面に対して反転対称を要請: $R'(\theta',\varphi') = R'(\pi-\theta',\varphi')$

$\longrightarrow\quad a_{21} = a_{2-1} = 0$.

$x'=0$ または $y'=0$ の平面に対して反転対称を要請:

$R'(\theta',\varphi') = R'(\theta',\pi-\varphi')$

$\longrightarrow\quad a_{22} = a_{2-2}$

5つの自由度： $\alpha_{2-2}, \cdots, \alpha_{22}$

の代わりに a_{20}, a_{22}, Ω

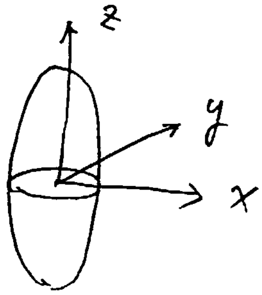
↑ 3つのオイラー角

$$a_{20} = \beta \cos\gamma$$
$$a_{22} = a_{2-2} = \frac{1}{\sqrt{2}} \beta \sin\gamma$$

↓

$$R(\theta, \varphi) = R_0 \left\{ 1 + \beta\cos\gamma \; Y_{20}(\theta) + \frac{1}{\sqrt{2}} \beta \sin\gamma \left(Y_{22}(\theta,\varphi) + Y_{2-2}(\theta,\varphi) \right) \right\}$$

$$= R_0 \left\{ 1 + \beta \sqrt{\frac{5}{16\pi}} \left(\cos\gamma (3\cos^2\theta - 1) + \sqrt{3} \sin\gamma \sin^2\theta \cos 2\varphi \right) \right\}$$



（ダッシュは省略）

$$\delta R_z = R(0,0) - R_0 = R_0 \sqrt{\frac{5}{4\pi}} \beta \cos\gamma$$

$$\delta R_x = R\left(\frac{\pi}{2}, 0\right) - R_0 = R_0 \beta \sqrt{\frac{5}{16\pi}} \left(-\cos\gamma + \sqrt{3} \sin\gamma \right)$$
$$= R_0 \beta \sqrt{\frac{5}{4\pi}} \cos\left(\gamma - \frac{2\pi}{3}\right)$$

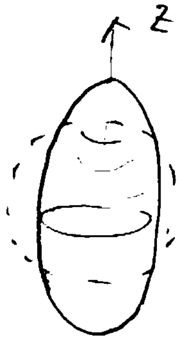
$$\delta R_y = R\left(\frac{\pi}{2}, \frac{\pi}{2}\right) - R_0 = R_0 \beta \sqrt{\frac{5}{4\pi}} \cos\left(\gamma + \frac{2\pi}{3}\right)$$

◦ $\gamma = 0$ の時：

$$\delta R_z = R_0 \sqrt{\frac{5}{4\pi}}\beta$$
$$\delta R_x = \delta R_y = -\frac{1}{2} R_0 \beta \sqrt{\frac{5}{4\pi}}$$

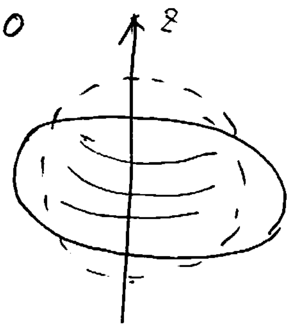
$$R(\theta) = R_0 (1+\beta Y_{20}(\theta))$$ ← φ 依存性なし
"軸対称変形"

$\beta > 0$



プロレート型

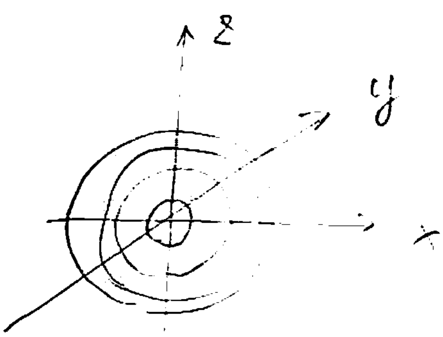
$\beta < 0$

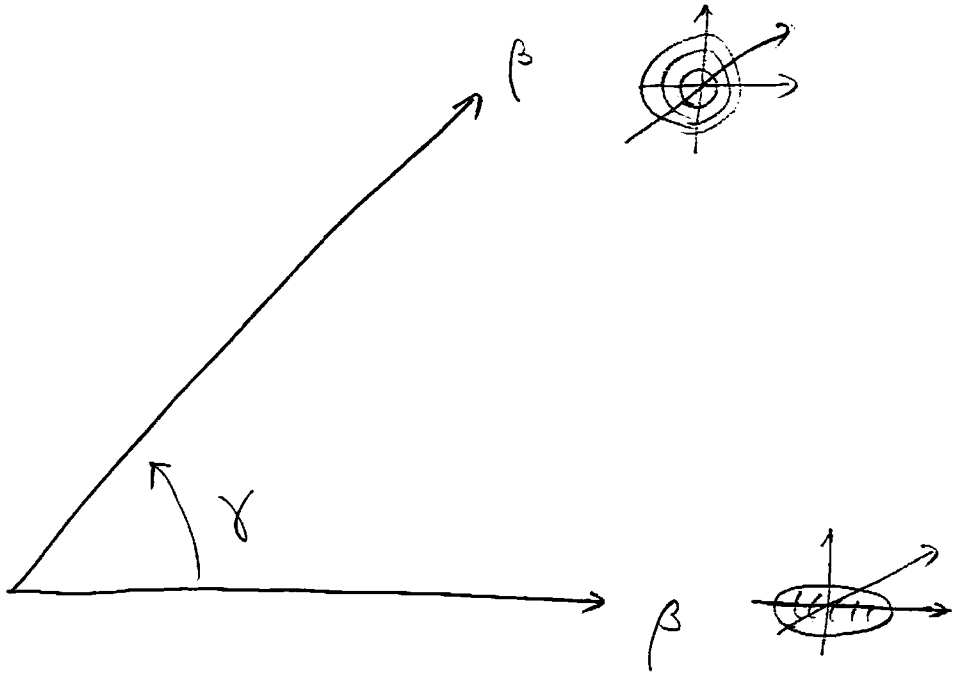


オブレート型

◦ $\beta > 0$, $\gamma = \frac{\pi}{3}$ の時：

$$\delta R_y = -R_0 \beta \sqrt{\frac{5}{4\pi}}$$
$$\delta R_x = \delta R_z = \frac{1}{2} R_0 \beta \sqrt{\frac{5}{4\pi}}$$ ← y軸まわりの軸対称変形



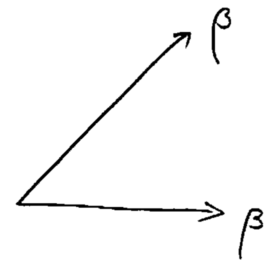


β
γ
β

2, 回転と慣性モーメント

$$H = \frac{1}{2} \sum_{\mu} \left\{ B \, |\dot{\alpha}_{2\mu}|^2 + C \, |\alpha_{2\mu}|^2 \right\}$$

- ・物体固定系へ変換
- ・平衡点周りの微小振動



$$H = \cdots = T_{rot} + \underbrace{\boxed{T_{vib} + \frac{1}{2} C_{20} \left(a_{20}(\beta,\gamma) - a_{20}^{(0)} \right)^2 + C_{22} \left(a_{22}(\beta,\gamma) - a_{22}^{(0)} \right)^2}}_{H_{vib}},$$

$$T_{rot} = \frac{1}{2} \sum_{k=1}^{3} \mathcal{J}_k \, \omega_k^2$$

$$\mathcal{J}_k = 4 B \beta^2 \sin^2 \left(\gamma - \frac{2\pi}{3} k \right)$$

$$T_{vib} = \frac{1}{2} B \left(\dot{\beta}^2 + \beta^2 \dot{\gamma}^2 \right)$$

・rotational part

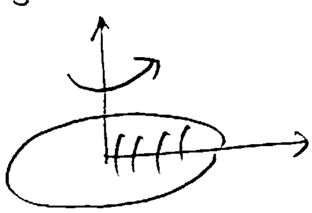
$$T_{rot} = \frac{1}{2}\sum_{k=1}^{3} \mathcal{J}_k \omega_k^2$$

$$\xrightarrow[\text{量子化}]{} \sum_k \frac{\hat{I}_k^2}{2\mathcal{J}_k}$$

渦ナシ流体 → $B = \frac{3}{8\pi} A \cdot m R_0^2$

↓ $\mathcal{J}_k^{irr} = \frac{3}{2\pi} m A R_0^2 \beta^2 \sin^2\left(\gamma - \frac{2\pi}{3}k\right)$

(note) $\gamma = 0$ のとき
$\mathcal{J}_3^{irr} = 0$ ↔ 量子力学的には対称軸周りの回転はない

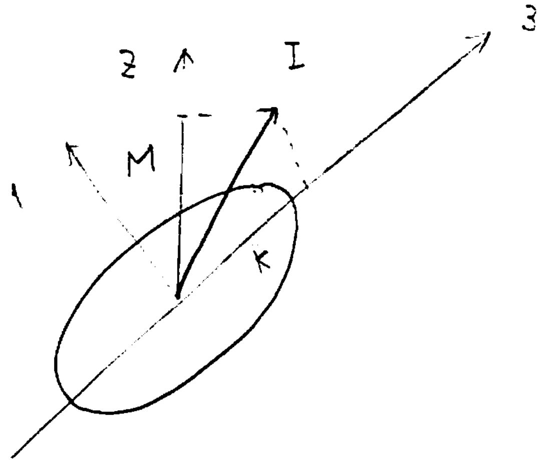


剛体では $\mathcal{J}_k^{rig} = \frac{2}{5} m A R_0^2 \left(1 - \sqrt{\frac{5}{4\pi}}\,\beta\cos\left(\gamma - \frac{2\pi}{3}k\right)\right)$

実験: $\mathcal{J}^{irr} < \mathcal{J}^{exp} < \mathcal{J}^{rig}$

・回転体の波動関数

$$H = \sum_k \frac{\hat{I}_k^{2}}{2\mathcal{J}_k} + H_{vib}$$



$\vec{I}^2, I_z, I_3$ の同時固有状態

$$|IMK\rangle = \sqrt{\frac{2I+1}{8\pi^2}}\, D^{I*}_{MK}(\Omega)$$

$$\vec{I}^2 |IMK\rangle = I(I+1)|IMK\rangle$$
$$I_z |IMK\rangle = M |IMK\rangle$$
$$I_3 |IMK\rangle = K |IMK\rangle$$

Hの固有状態：一般に $|\Psi_{IM}\rangle = \sum_K \underbrace{g_K(\beta,\gamma)}_{\text{振動}} \underbrace{|IMK\rangle}_{\text{回転}}$

1軸回りのπ回転で波動関数が不変

$$\longrightarrow g_K(\beta,\gamma) = (-)^I g_{-K}(\beta,\gamma)$$

2軸回りのπ回転で波動関数が不変

$$\longrightarrow g_K(\beta,\gamma) = (-)^{I+K} g_{-K}(\beta,\gamma)$$

↓ K：偶数

$$|\Psi_{IM}\rangle = \sum_K g_K(\beta,\gamma)\left\{ |IMK\rangle + (-)^I |IM-K\rangle \right\}$$

$$\downarrow \begin{cases} I = 0, 2, 4, \cdots & (K=0) \\ I = K, K+1, \cdots & (K\neq 0) \end{cases}$$

・軸対称変形の場合 ($\gamma=0$)

$$\mathcal{J}_1=\mathcal{J}_2\equiv\mathcal{J}_0, \quad \mathcal{J}_3=0$$

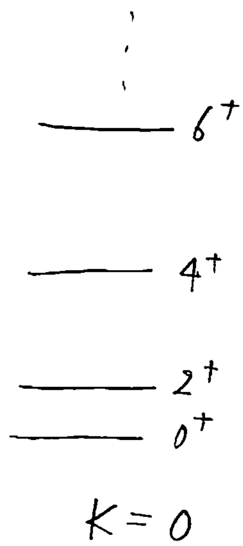
↓

$$\hat{T}_{rot}=\frac{I_1^2+I_2^2}{2\mathcal{J}_0}=\frac{\vec{I}^2-I_3^2}{2\mathcal{J}_0}$$

↓

I_3 も いい量子数 $(=K)$

基底状態バンド



3. 振動運動

（軸対称変形の場合）

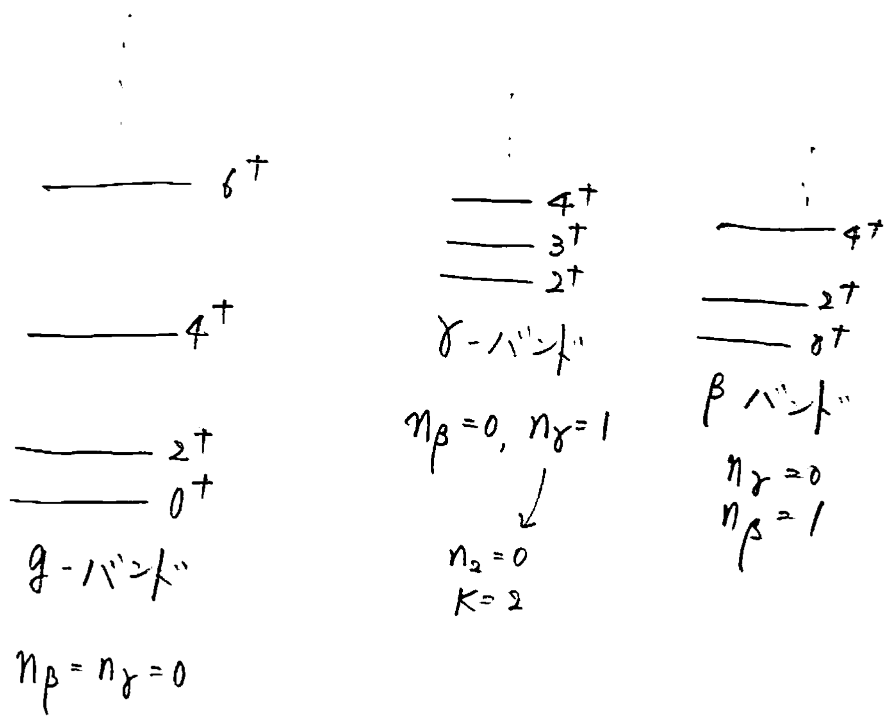
$$H_{vib} = \frac{1}{2} B \left(\dot{\beta}^2 + \beta_0^2 \dot{\gamma}^2 \right) + \frac{1}{2} C_\beta \left(\beta - \beta_0 \right)^2 + \frac{1}{2} C_\gamma \beta_0^2 \gamma^2$$

→ 量子化

→ 固有値：

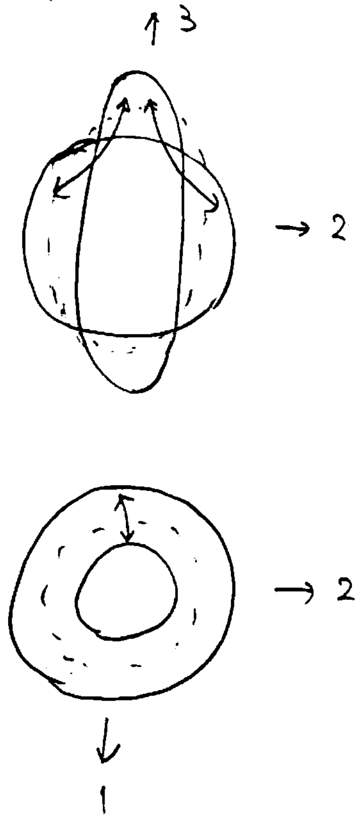
$$E = \left(n_\beta + \frac{1}{2} \right) \hbar \omega_\beta + \left(n_\gamma + 1 \right) \hbar \omega_\gamma$$

$$n_\gamma = 2 n_2 + \frac{1}{2} K$$



β 振動

(β → 振動, γ=0)



γ 振動

(β = const. , γ → 振動)