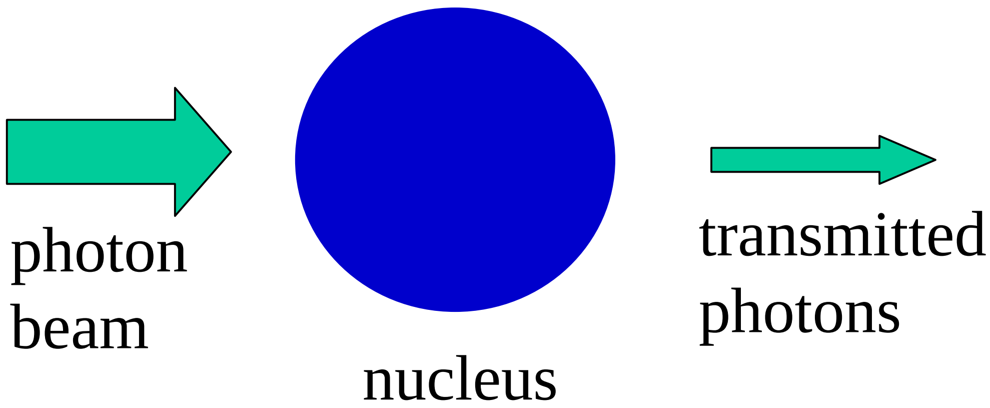
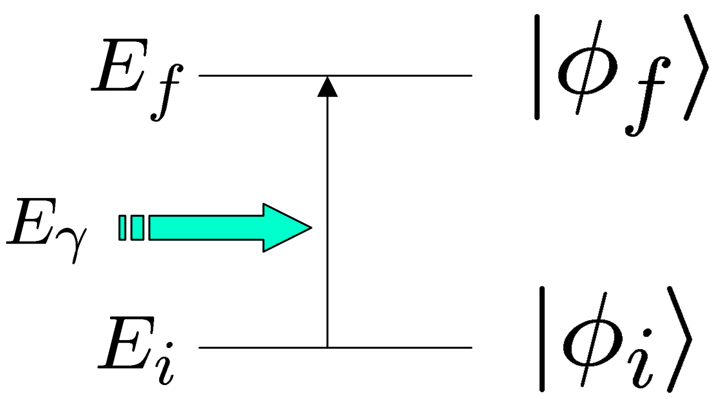
Collective Vibrations

How does a nucleus respond to an external perturbation?

i) Photo absorption cross section





The state is strongly excited when

$E_f - E_i = E_\gamma.$

Giant Dipole Resonance (GDR)

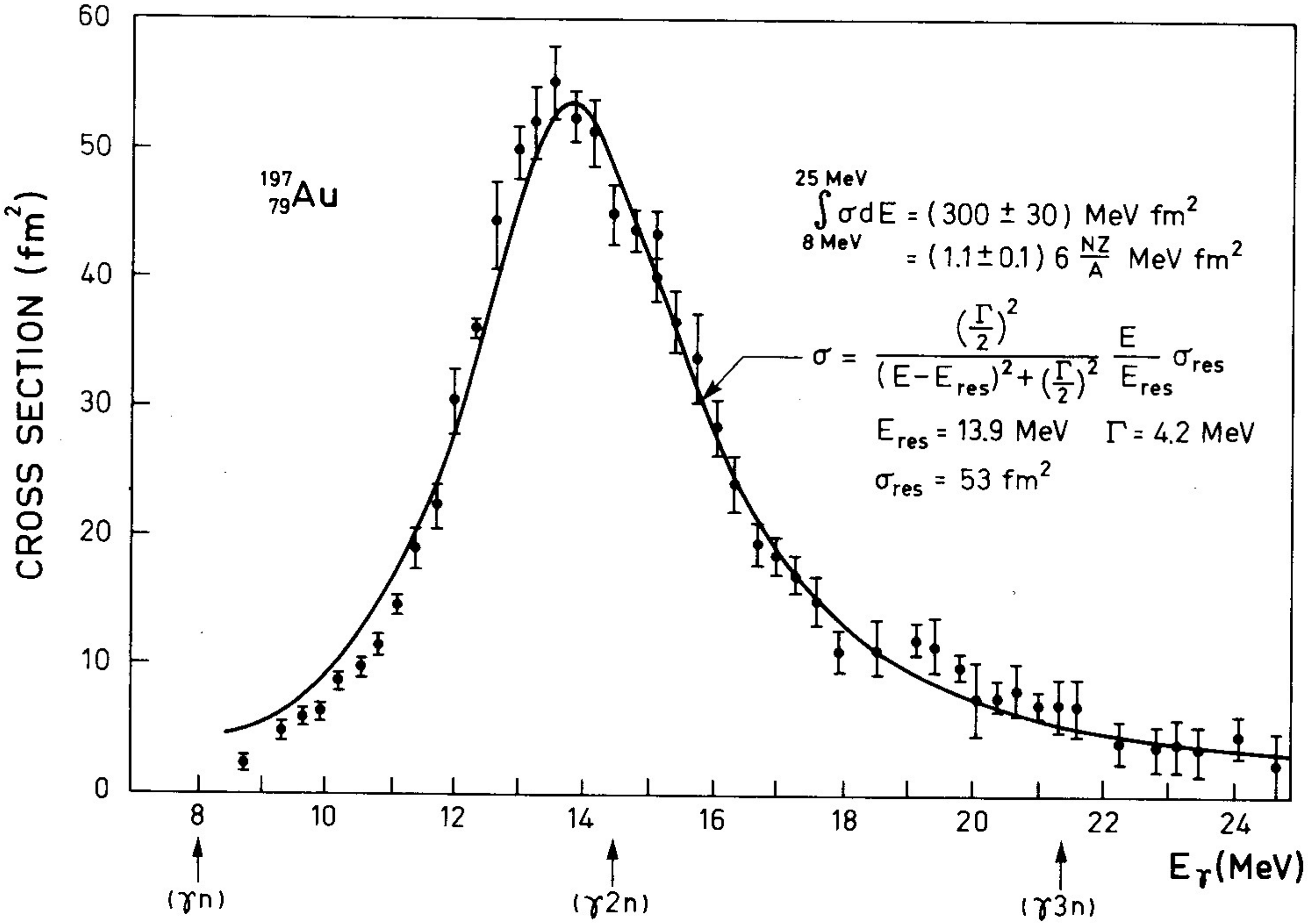


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

Remarks

i) Photon interaction $\longleftrightarrow$ dipole excitation

$$H_{\rm int} = \frac{1}{2m}\frac{e}{c}(\boldsymbol{p}\cdot\boldsymbol{A}+\boldsymbol{A}\cdot\boldsymbol{p})$$

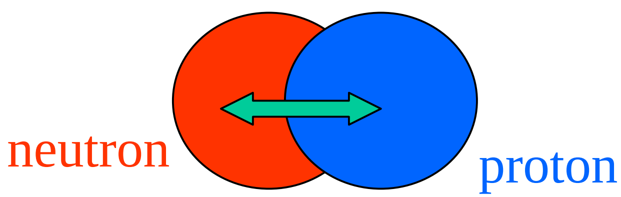
$$\boldsymbol{A}(\boldsymbol{r},t) = \sum_{\boldsymbol{k}}\sum_{\alpha=1,2}\sqrt{\frac{2\pi c^2\hbar}{\omega V}}(a_{\boldsymbol{k}\alpha}\,\boldsymbol{\epsilon}_\alpha\,e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega_k t}+h.c.)$$

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\sim 1$$ (dipole approximation)

$$\boxed{\sigma_{\rm abs}(E_\gamma)=\frac{4\pi^2e^2}{\hbar c}(E_f-E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma-E_f+E_i)}$$

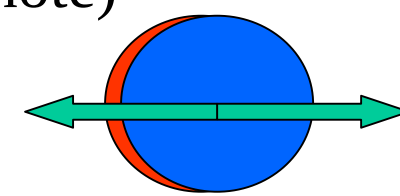
ii) Isospin

$$\tilde{z}=\sum_p(z_p-Z_{cm})$$



Isovector type

(note)



Isoscalar dipole motion

$\longleftrightarrow$ c.m. motion (to the first order)

iii) Collective motion

Motion of the whole nucleus rather than a single-particle motion

Sum Rule

Strength function:

$$\boxed{S(E) = \sum_\nu |\langle \nu | F | 0 \rangle|^2 \, \delta(E_\nu - E)}$$

Energy weighted sum rule:

$$\boxed{\begin{aligned} S_1 &= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2 \\ &= \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle \end{aligned}}$$

(note)

$$\begin{aligned} \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle &= \frac{1}{2} \langle F(HF - FH) - (HF - FH)F \rangle \\ &= \langle FHF - E_0 F^2 \rangle \\ &= \sum_\nu E_\nu |\langle 0 | F | \nu \rangle|^2 - E_0 \langle 0 | F^2 | 0 \rangle \\ &= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2 \end{aligned}$$

Energy weighted sum rule:

$$\boxed{\begin{aligned} S_1 &= \sum_\nu (E_\nu - E_0)|\langle \nu|F|0\rangle|^2 \\ &= \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle \end{aligned}}$$

For $F = F(\boldsymbol{r})$ (local operator)

$$\begin{aligned} [H,F] &= \left[-\frac{\hbar^2}{2m}\nabla^2, F\right] \\ &= -\frac{\hbar^2}{2m}(\nabla^2 F + 2\nabla F\cdot\nabla) \end{aligned}$$

$$[F,[H,F]] = \frac{\hbar^2}{m}(\nabla F)^2$$

$$\boxed{S_1 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2}$$

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2$$

For *F=z*

$$\boxed{S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|z|0\rangle|^2 = \frac{\hbar^2 N_{sys}}{2m}}$$

[TRK (Thomas-Reiche-Kuhn) Sum Rule]

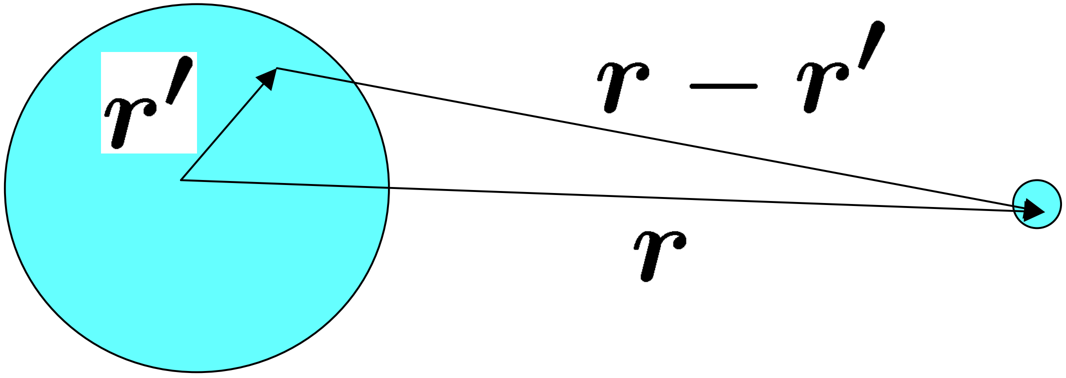
⬆

Model independent

For $F = r^\lambda Y_{\lambda\mu}(\hat{\boldsymbol{r}})$

$$\boxed{S_1 = \frac{\lambda(2\lambda+1)\hbar^2}{8\pi m} A\,\langle r^{2\lambda-2}\rangle}$$

(note)



$$V_C(\boldsymbol{r}) = e^2 \int \frac{\rho_C(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|} d\boldsymbol{r}'$$

$$\frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|} = \sum_{\lambda,\mu} \frac{4\pi}{2\lambda + 1} \frac{Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})}{r^{\lambda+1}} \cdot \underline{\underline{r'^{\lambda} Y_{\lambda\mu}(\hat{\boldsymbol{r}}')}}$$

Photo absorption cross section:

$$\sigma_{\mathsf{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c}(E_f - E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma - E_f + E_i)$$

$$\begin{aligned}\tilde{z} = \sum_p (z_p - Z_{cm}) &= \sum_p \left\{ z_p - \frac{1}{A}\left(\sum_{p'} z_{p'} + \sum_n z_n\right)\right\} \\ &= \frac{NZ}{A}\left(\frac{1}{Z}\sum_p z_p - \frac{1}{N}\sum_n z_n\right)\end{aligned}$$

$$\boxed{\begin{aligned}\int \sigma_{\mathsf{abs}}(E_\gamma)dE_\gamma &= \frac{4\pi^2 e^2}{\hbar c}\cdot\frac{\hbar^2}{2m}\cdot\frac{NZ}{A} \\ &= \frac{2\pi^2 e^2 \hbar}{mc}\cdot\frac{NZ}{A}\end{aligned}}$$

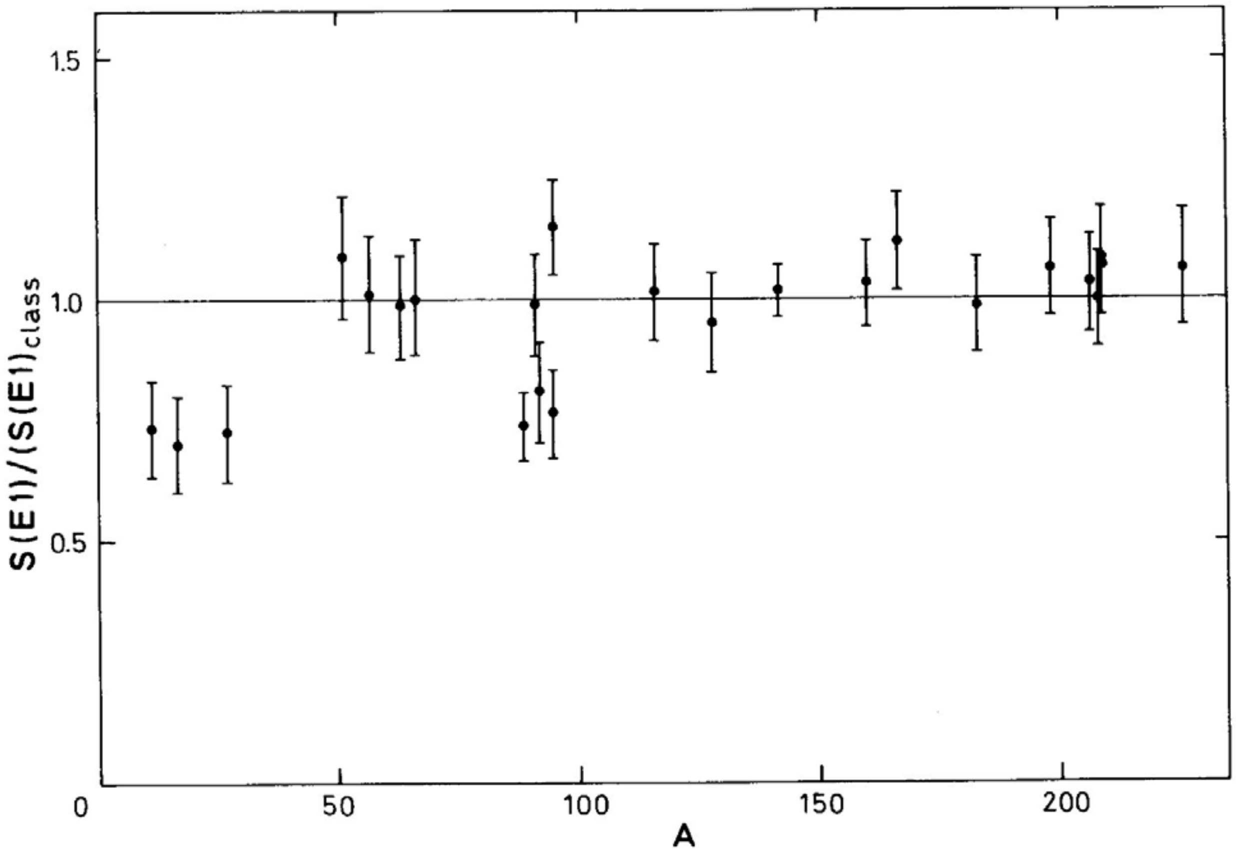


Figure 6-20 Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with $A > 50$, the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic γ rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proc. Intern. Nuclear Physics Conference*, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of (γp) processes must be included and the data are from: ^{12}C and ^{27}Al (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, *Phys. Rev.* **143**, 790, 1966); ^{16}O (Dolbilkin *et al.*, *loc.cit.*, Fig. 6-26). For the heavy nuclei ($A > 50$), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssière *et al.*, 1970).