

半経験的質量公式

(Bethe-Weizacker 質量公式: 液滴模型)

$$B(N, Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N - Z)^2}{A}$$

- 体積エネルギー: $a_v A$
- 表面エネルギー: $-a_s A^{2/3}$
- クーロン・エネルギー: $-a_C Z^2 / A^{1/3}$
- 対称エネルギー: $-a_{\text{sym}} (N - Z)^2 / A$

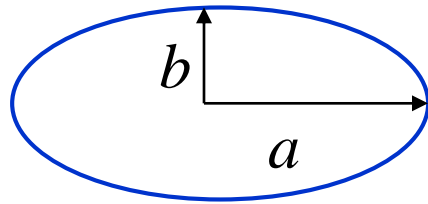
原子核が変形するとどうなるか？

$$B(N, Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N - Z)^2}{A}$$

原子核: 密度を変化させるのに大きなエネルギーが必要

→ 体積を変化させないように原子核を変形させる

回転楕円体



$$a = R \cdot (1 + \epsilon)$$

$$b = R \cdot (1 + \epsilon)^{-1/2}$$

体積項、対称項: 変化せず

表面項: 損をする(表面積が大きくなるため)

クーロン項: 得をする(平均的な陽子間距離が大きくなるため)

$$E_{\text{surf}} = E_{\text{surf}}^{(0)} (1 + 2\epsilon^2/5 + \dots)$$

$$E_C = E_C^{(0)} (1 - \epsilon^2/5 + \dots)$$

原子核の表面振動

$$E_{\text{surf}} = E_{\text{surf}}^{(0)} (1 + 2\epsilon^2/5 + \dots)$$

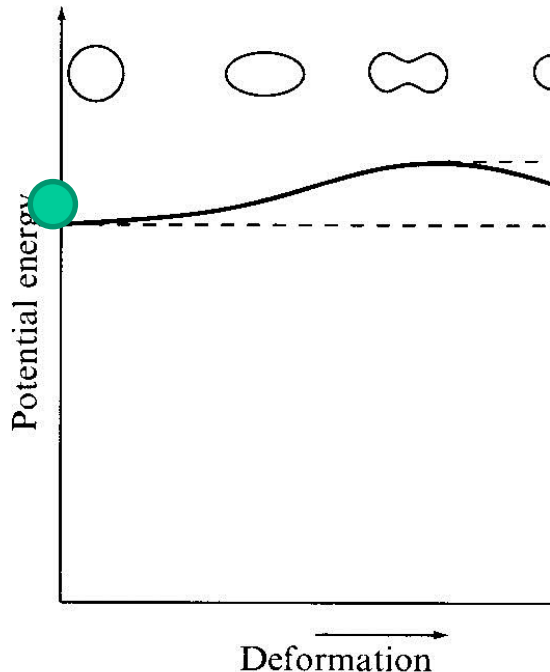
$$E_C = E_C^{(0)} (1 - \epsilon^2/5 + \dots)$$

→
$$\Delta E = E_{\text{surf}}^{(0)} \cdot \frac{2}{5} (1 - x) \epsilon^2$$

* 原子核が安定に存在するためには
 $x < 1$ が必要

$$x \equiv \frac{E_C^{(0)}}{2E_{\text{surf}}^{(0)}}$$

(fissility パラメーター)



原子核の表面振動

$$E_{\text{surf}} = E_{\text{surf}}^{(0)} (1 + 2\epsilon^2/5 + \dots)$$

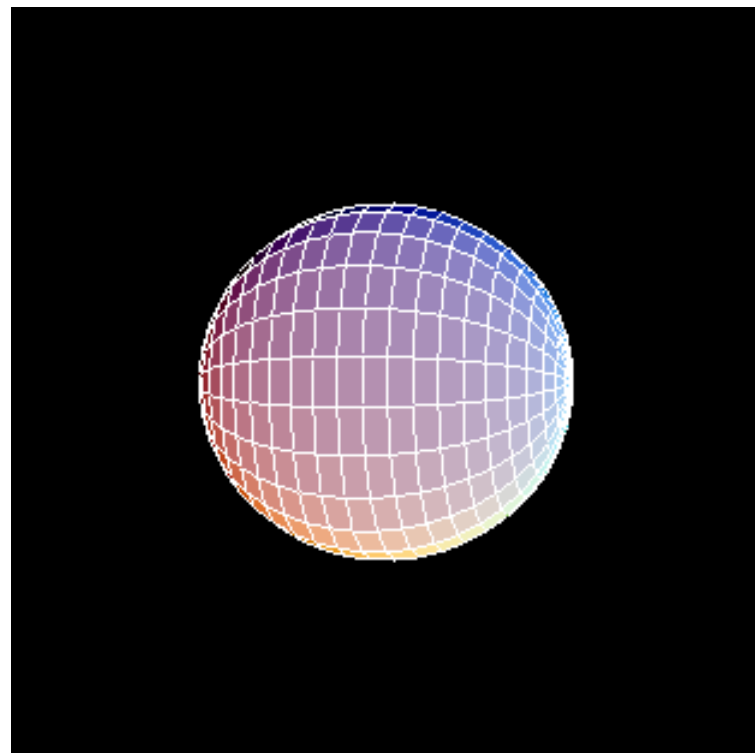
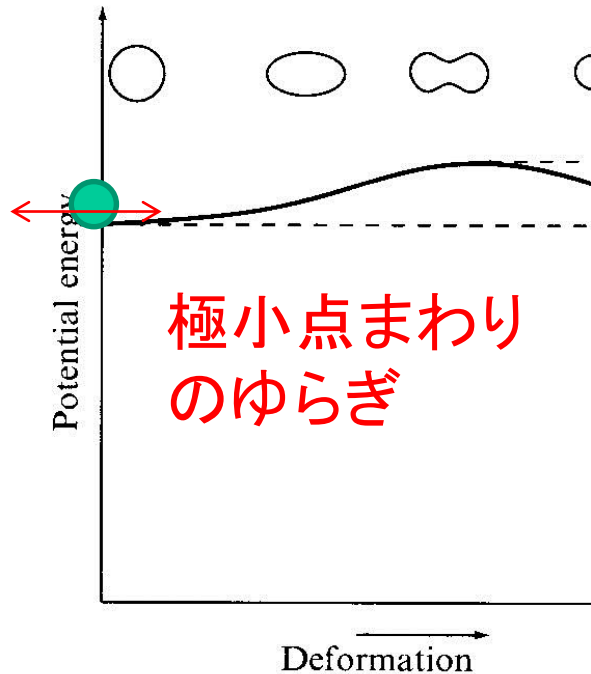
$$E_C = E_C^{(0)} (1 - \epsilon^2/5 + \dots)$$

➔
$$\Delta E = E_{\text{surf}}^{(0)} \cdot \frac{2}{5} (1 - x) \epsilon^2$$

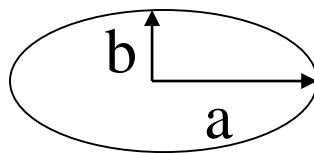
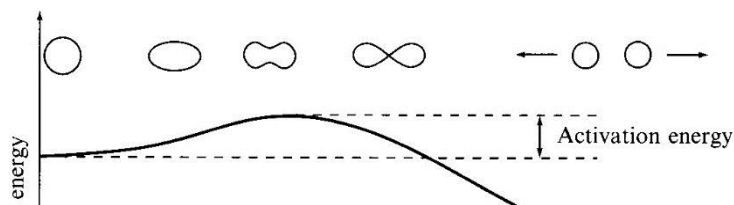
* 原子核が安定に存在するためには
 $x < 1$ が必要

$$x \equiv \frac{E_C^{(0)}}{2E_{\text{surf}}^{(0)}}$$

(fissility パラメーター)



集団振動



$$a = R \cdot (1 + \epsilon)$$

$$b = R \cdot (1 + \epsilon)^{-1/2}$$

一般的に,
$$R(\theta, \phi) = R_0 \left(1 + \sum_{\lambda, \mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi) \right)$$

(回転楕円体は $\lambda = 2, \mu = 0$ に相当)

$$V = \frac{1}{2} \sum_{\lambda, \mu} C_{\lambda} |\alpha_{\lambda\mu}|^2$$



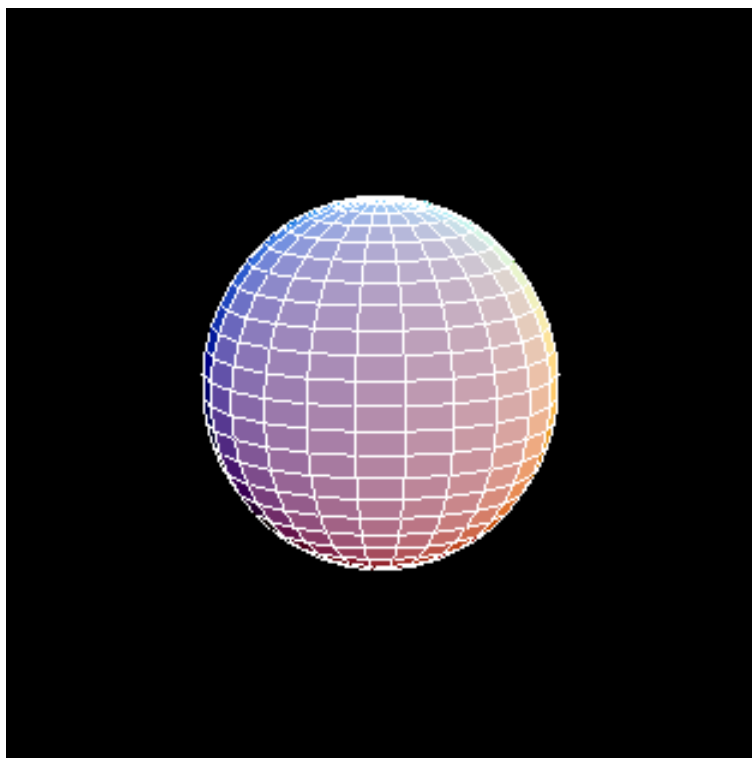
量子化: 調和振動子

(note) 慣性能率

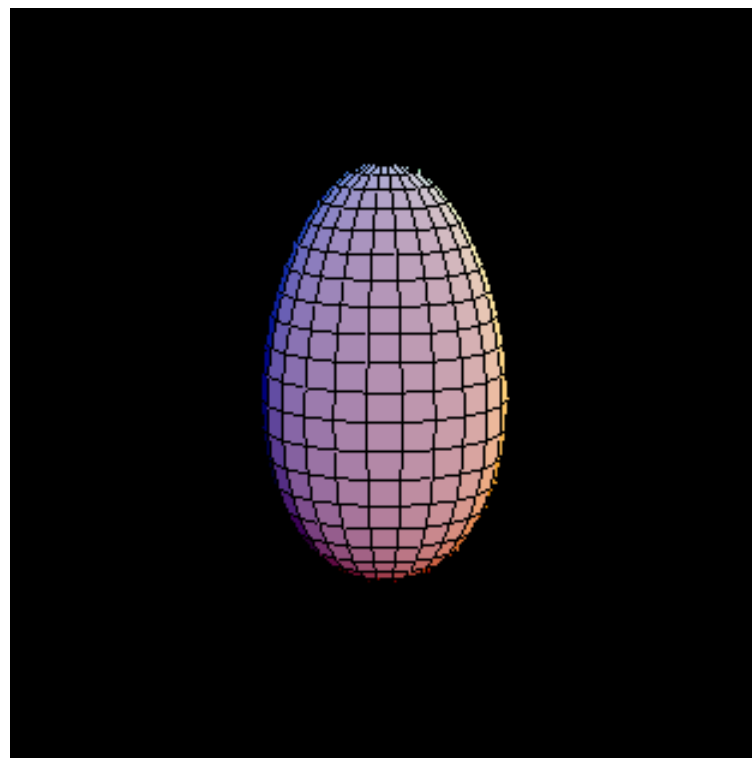
← 非圧縮性渦なし流体

$$R(\theta, \phi) = R_0 \left(1 + \sum_{\lambda, \mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi) \right)$$

$$V = \frac{1}{2} \sum_{\lambda, \mu} C_\lambda |\alpha_{\lambda\mu}|^2$$

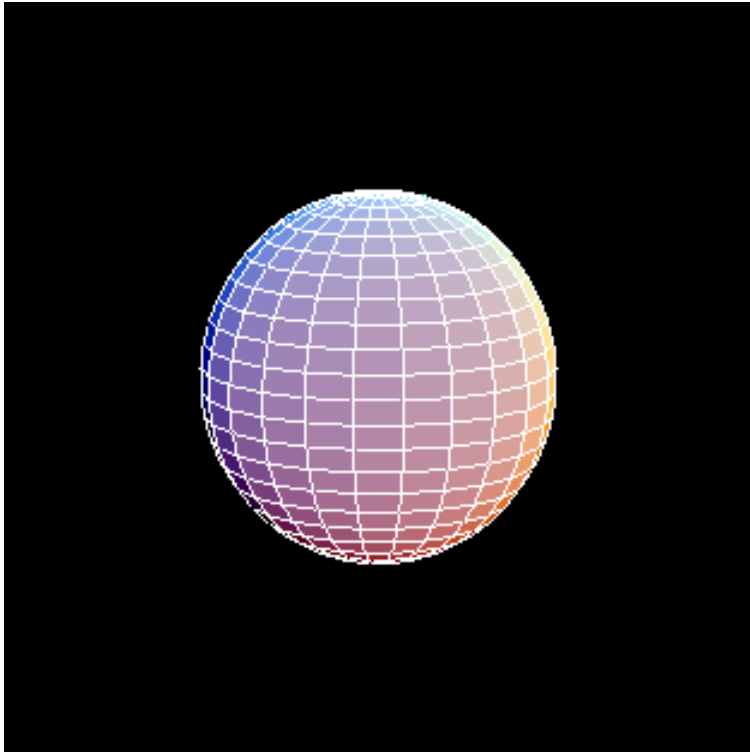


$\lambda=2$: 四重極型振動



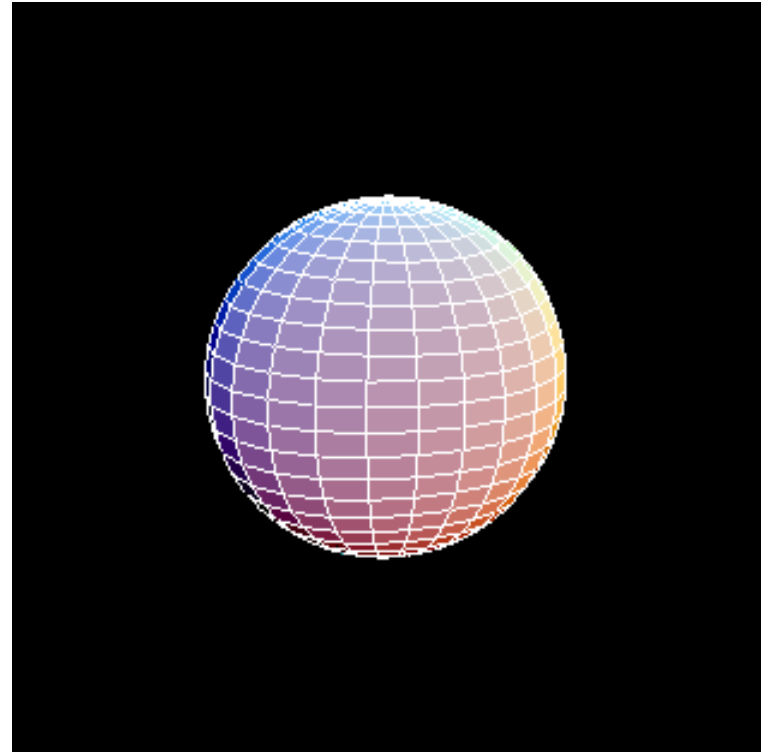
$\lambda=3$: 八重極型振動

$$R(\theta, \phi) = R_0 \left(1 + \sum_{\lambda, \mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi) \right) \quad V = \frac{1}{2} \sum_{\lambda, \mu} C_{\lambda} |\alpha_{\lambda\mu}|^2$$



Y_{20} 型振動

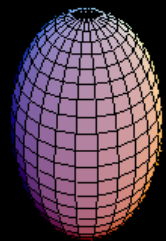
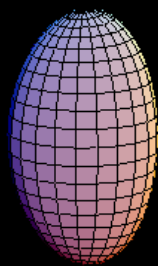
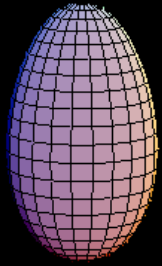
$$\lambda=2, \mu=0$$



Y_{22} 型振動

$$\lambda=2, \mu = +/- 2$$

$$R(\theta, \phi) = R_0 \left(1 + \sum_{\lambda, \mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi) \right) \quad V = \frac{1}{2} \sum_{\lambda, \mu} C_{\lambda} |\alpha_{\lambda\mu}|^2$$



Y_{30} 型振動

Y_{31} 型振動

Y_{32} 型振動

Y_{33} 型振動

$\lambda=3, \mu=0$

$\lambda=3, \mu = +/- 1$

$\lambda=3, \mu = +/- 2$

$\lambda=3, \mu = +/- 3$

どのくらいのエネルギーを与えれば原子核は振動しはじめるのか？

↔ 振動の励起エネルギー

ムービー: 在田謙一郎氏 (名古屋工大)

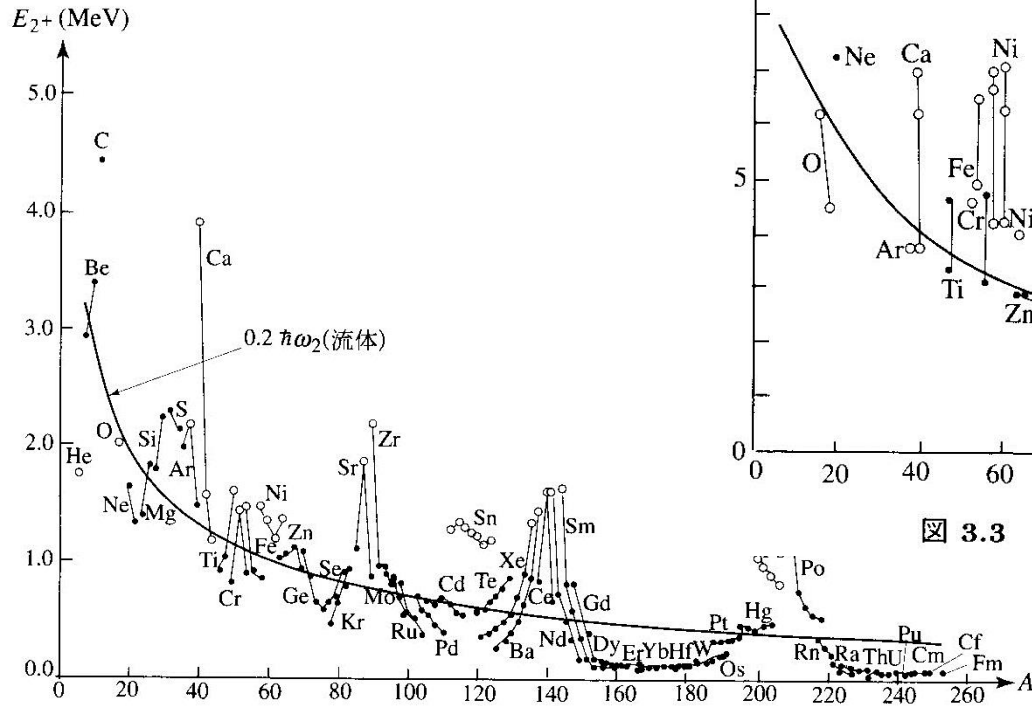


図 3.2 偶々核の第 1 励起 2^+ 状態の励起エネルギー

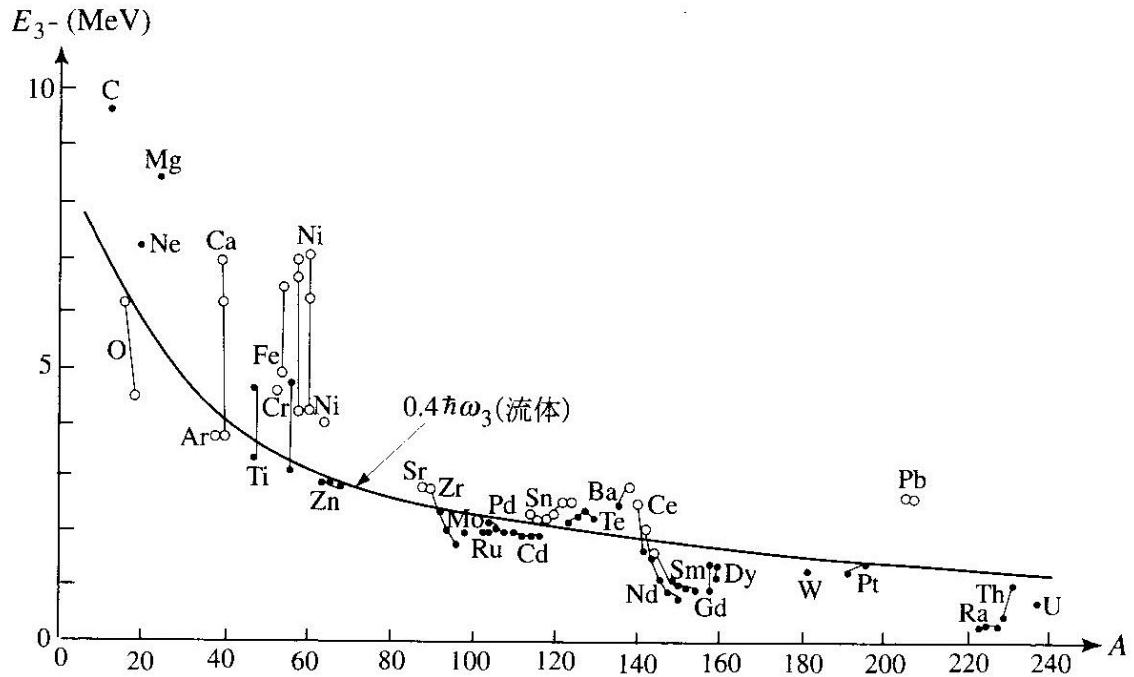


図 3.3 偶々核の第 1 励起 3^- 状態の励起エネルギー

2重フォノン状態

4^+ ————— 1.282 MeV
 2^+ ————— 1.208 MeV
 0^+ ————— 1.133 MeV

2^+ ————— 0.558 MeV

0^+ —————

^{114}Cd

(復習) 1次元調和振動子

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

$$a = \sqrt{\frac{m\omega}{2\hbar}} \left(x + \frac{ip}{m\omega} \right)$$
$$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \left(x - \frac{ip}{m\omega} \right)$$

を用いてハミルトニアン
を書き直すと

$$H = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right) \quad [a, a^\dagger] = 1$$

基底状態: $|0\rangle \quad E_0 = \frac{1}{2}\hbar\omega \quad \text{cf. } a|0\rangle = 0$

励起状態: $|n\rangle = \frac{1}{\sqrt{n!}} (a^\dagger)^n |0\rangle \quad E_n = \left(n + \frac{1}{2} \right) \hbar\omega$

1次元調和振動子

$$H = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right)$$

$$|n\rangle = \frac{1}{\sqrt{n!}} (a^\dagger)^n |0\rangle \quad E_n = \left(n + \frac{1}{2} \right) \hbar\omega$$

原子核の表面振動

$$H = \frac{1}{2} \sum_{\lambda, \mu} \left\{ B_\lambda |\dot{\alpha}_{\lambda\mu}|^2 + C_\lambda |\alpha_{\lambda\mu}|^2 \right\}$$

$$= \sum_{\lambda, \mu} \hbar\omega_\lambda \left(b_{\lambda\mu}^\dagger b_{\lambda\mu} + \frac{1}{2} \right)$$

$$[b_{\lambda\mu}, b_{\lambda'\mu'}^\dagger] = \delta_{\lambda, \lambda'} \delta_{\mu, \mu'}$$

$$\frac{1}{\sqrt{2}} [b_{20}^\dagger b_{20}^\dagger]^{(IM)} |0\rangle$$

$$\begin{array}{l} 4^+ \text{-----} 1.282 \text{ MeV} \\ 2^+ \text{-----} 1.208 \text{ MeV} \\ 0^+ \text{-----} 1.133 \text{ MeV} \end{array}$$

$$b_{2\mu}^\dagger |0\rangle$$

$$2^+ \text{-----} 0.558 \text{ MeV}$$

$$|0\rangle$$

$$0^+ \text{-----}$$

¹¹⁴Cd

(復習) 角運動量の合成

2つの角運動量を持つ系 l_1, l_2

状態の分類の仕方に2通り:

①それぞれの角運動量の固有状態(の直積)

$$|l_1 m_1 l_2 m_2\rangle = |l_1 m_1\rangle |l_2 m_2\rangle$$

②合成角運動量 $L = l_1 + l_2$ とその z 成分

$$|LM\rangle$$

(note)

L の最大値は $L_{\max} = l_1 + l_2$, 最小値は $L_{\min} = |l_1 - l_2|$

どちらも完全系を張る:

$$\begin{aligned} 1 &= \sum_{m_1, m_2} |l_1 m_1 l_2 m_2\rangle \langle l_1 m_1 l_2 m_2| \\ &= \sum_{L, M} |LM\rangle \langle LM| \end{aligned}$$

$$\begin{aligned}
 1 &= \sum_{m_1, m_2} |l_1 m_1 l_2 m_2\rangle \langle l_1 m_1 l_2 m_2| \\
 &= \sum_{L, M} |LM\rangle \langle LM|
 \end{aligned}$$



左から $|LM\rangle$ をかけると、

$$|LM\rangle = \sum_{m_1, m_2} \underbrace{\langle l_1 m_1 l_2 m_2 | LM \rangle}_{\text{クレブシュ・ゴルダン係数}} |l_1 m_1 l_2 m_2\rangle$$

同様に左から $|l_1 m_1 l_2 m_2\rangle$ をかけると、

$$|l_1 m_1 l_2 m_2\rangle = \sum_{L, M} \langle l_1 m_1 l_2 m_2 | LM \rangle |LM\rangle$$

(note) クレブシュ・ゴルダン係数の性質

$$\langle l_1 m_1 l_2 m_2 | LM \rangle = (-1)^{l_1 + l_2 - L} \langle l_2 m_2 l_1 m_1 | LM \rangle$$

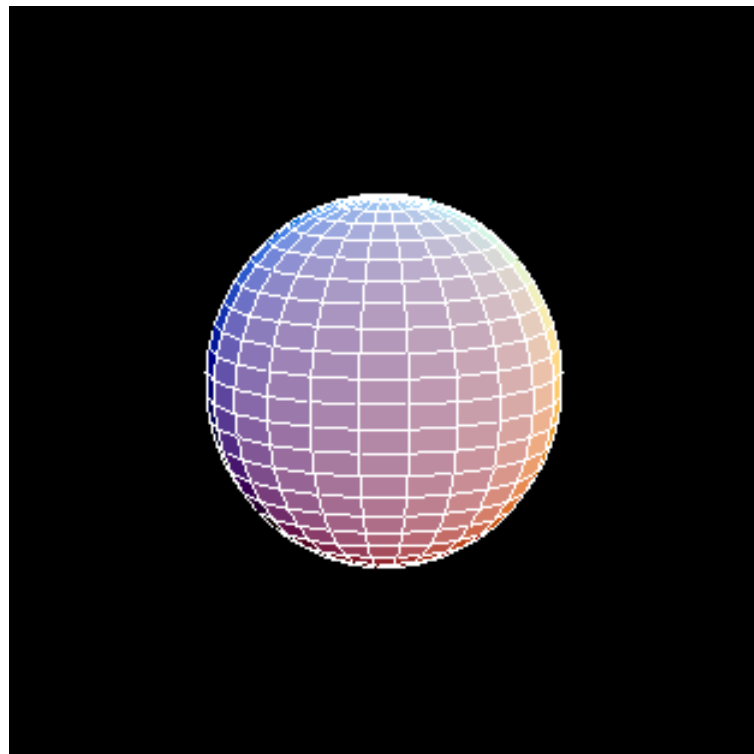
2重フォノン状態

$$\begin{array}{l}
 4^+ \text{-----} 1.282 \text{ MeV} \\
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 0^+ \text{-----} 1.133 \text{ MeV}
 \end{array}$$

$$2^+ \text{-----} 0.558 \text{ MeV}$$

$$0^+ \text{-----}$$

^{114}Cd



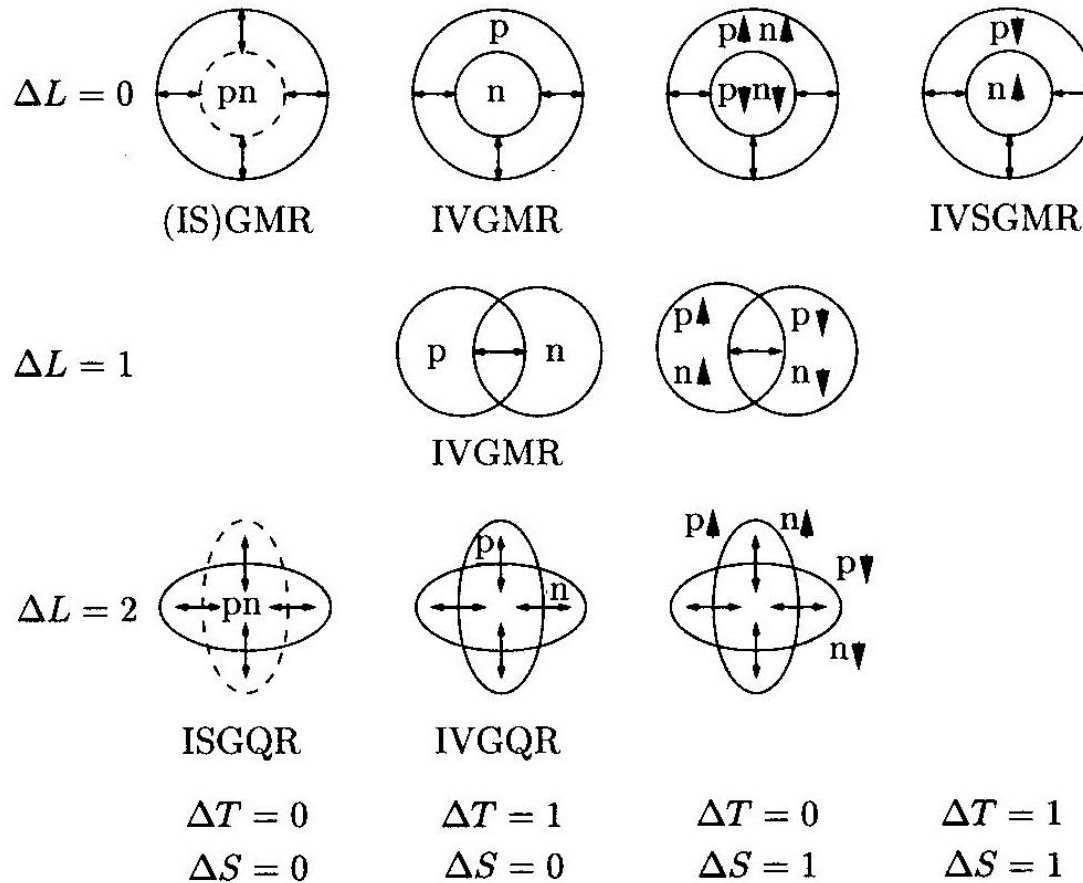
$$\begin{aligned}
 [b_2^\dagger b_2^\dagger]^{(IM)} &= \sum_{\mu, \mu'} \langle 2\mu 2\mu' | IM \rangle b_{2\mu}^\dagger b_{2\mu'}^\dagger \\
 &= \sum_{\mu, \mu'} \langle 2\mu' 2\mu | IM \rangle b_{2\mu'}^\dagger b_{2\mu}^\dagger = b_{2\mu}^\dagger b_{2\mu'}^\dagger \\
 &= (-1)^I \langle 2\mu 2\mu' | IM \rangle \quad (\text{ボソン}) \\
 &= (-1)^I \sum_{\mu, \mu'} \langle 2\mu 2\mu' | IM \rangle b_{2\mu}^\dagger b_{2\mu'}^\dagger
 \end{aligned}$$

名前の
つけかえ

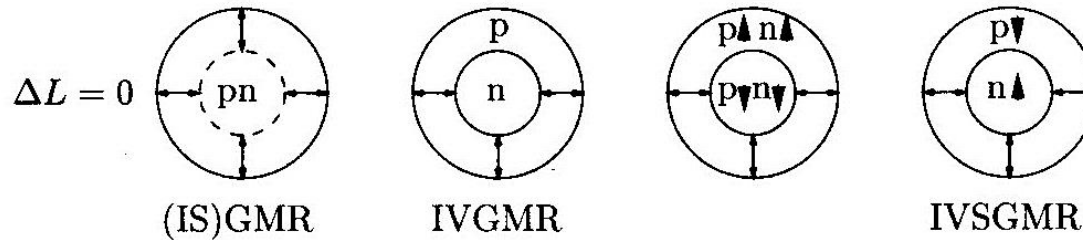
原子核:陽子と中性子の2種類の粒子

どのように動くかで2種類の振動

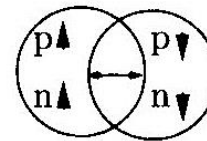
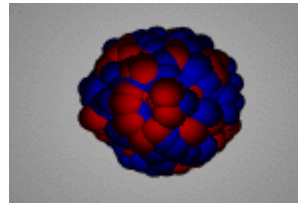
- 陽子と中性子が一緒に動く(アイソ・スカラー型)
- 陽子と中性子が反対方向に動く(アイソ・ベクトル型)



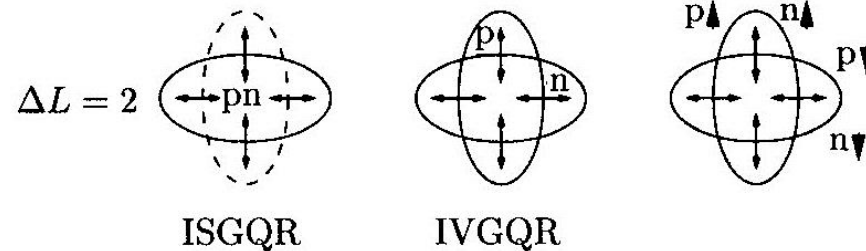
- 陽子と中性子が一緒に動く(アイソ・スカラー型)
- 陽子と中性子が反対方向に動く(アイソ・ベクトル型)



$\Delta L = 1$



巨大双極子共鳴
(GDR)



$$\Delta T = 0$$

$$\Delta S = 0$$

$$\Delta T = 1$$

$$\Delta S = 0$$

$$\Delta T = 0$$

$$\Delta S = 1$$

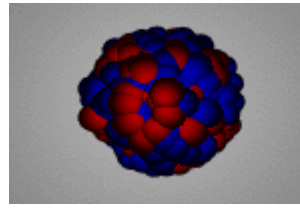
$$\Delta T = 1$$

$$\Delta S = 1$$

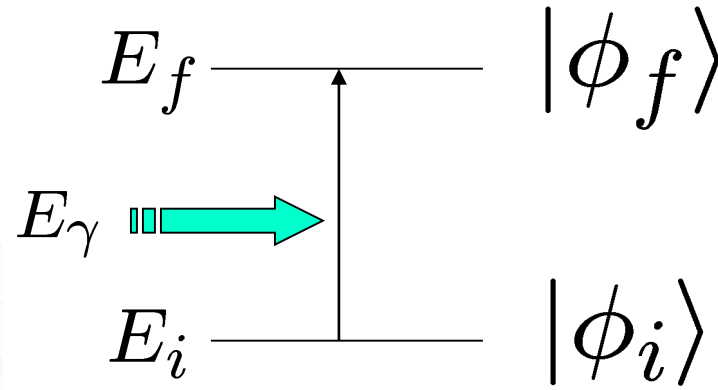
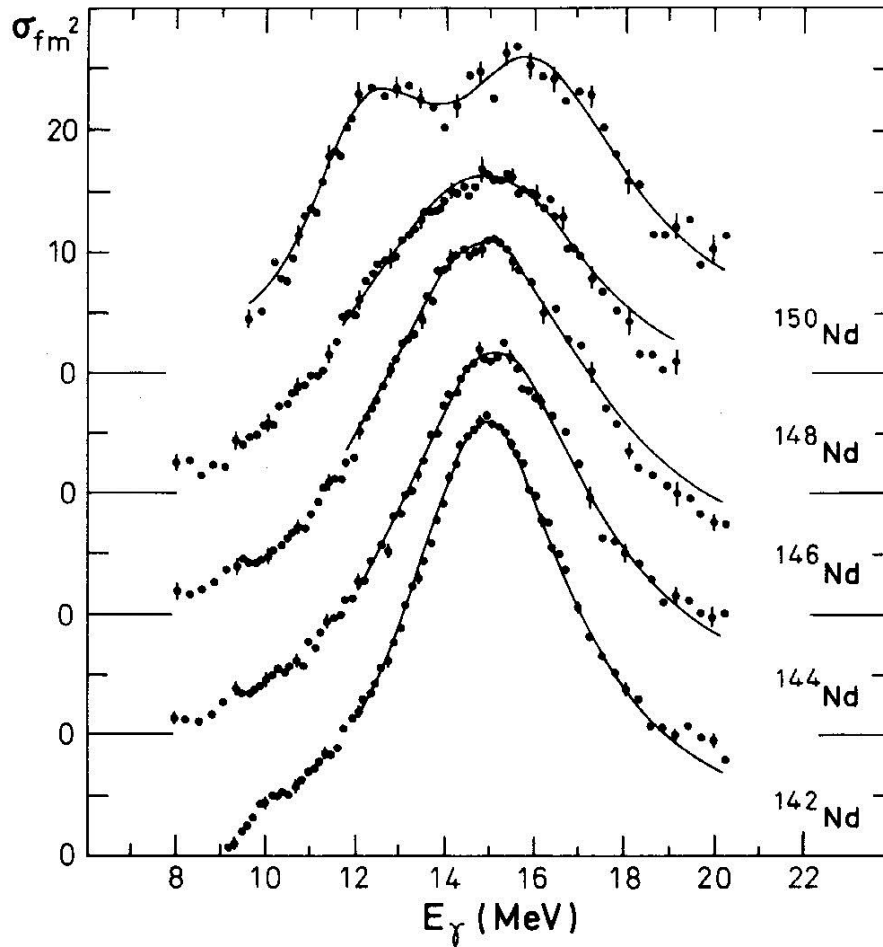
movies: H.-J. Wollersheim,

<https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html>

変形の効果



光吸収 断面積



フォトンのエネルギーと励起エネルギーが一致すると光子が原子核に吸収される

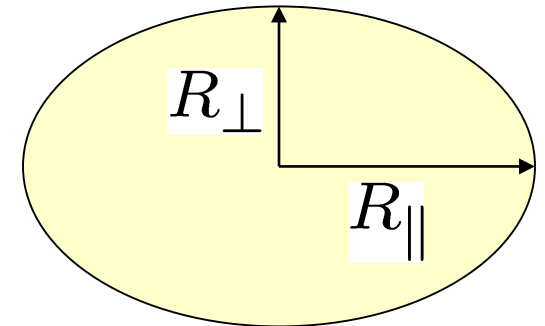
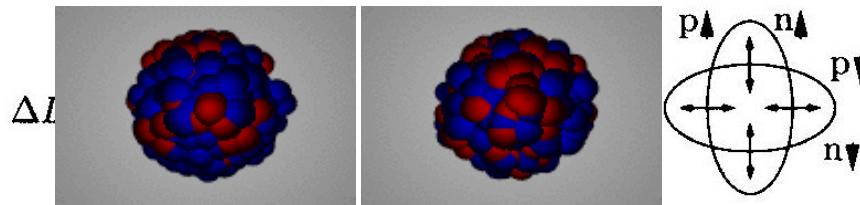
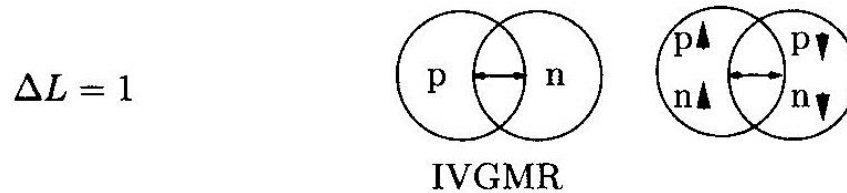
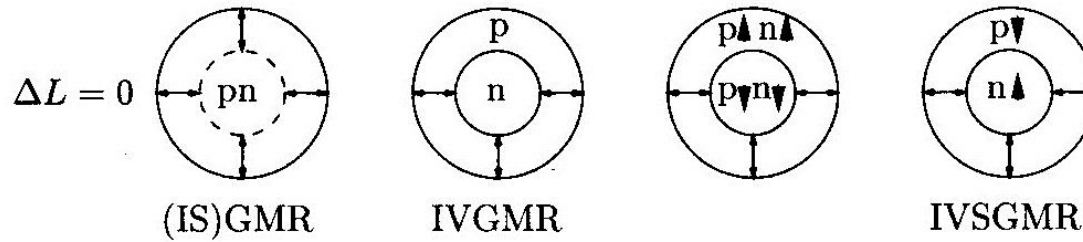


Figure 6-21 Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssièrre, *Nuclear Phys. A*172, 437 (1971). The solid curves represent Lorentzian fits with the parameters given in Table 6-6.

- 陽子と中性子が一緒に動く(アイソ・スカラー型)
- 陽子と中性子が反対方向に動く(アイソ・ベクトル型)



ISGQR

IVGQR

$$\Delta T = 0$$

$$\Delta T = 1$$

$$\Delta T = 0$$

$$\Delta T = 1$$

$$\Delta S = 0$$

$$\Delta S = 0$$

$$\Delta S = 1$$

$$\Delta S = 1$$

巨大四重極子振動

movies: H.-J. Wollersheim,

<https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html>

Giant Multipole Resonances in ^{90}Zr Observed by Inelastic Electron Scattering

S. Fukuda and Y. Torizuka

Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan

(Received 24 August 1972)

Inelastic electron scattering from the giant dipole resonance region in ^{90}Zr was measured. In addition to the usual dipole resonance we have found new resonances at 14.0 MeV and around 28 MeV. The spins and parities and transition strengths of these states are discussed.

Electroexcitation of Giant Resonances in ^{208}Pb

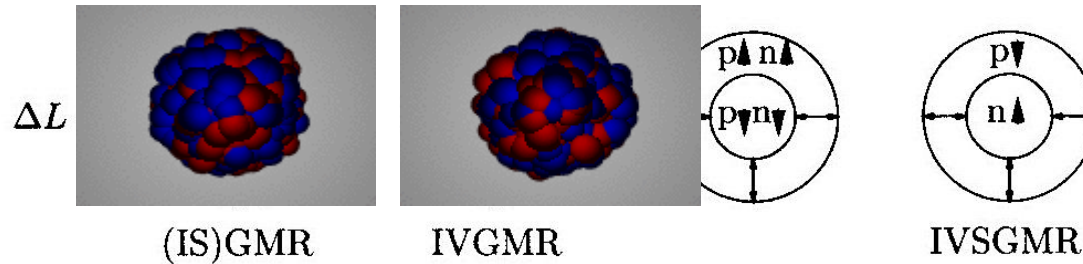
M. Nagao and Y. Torizuka

Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan

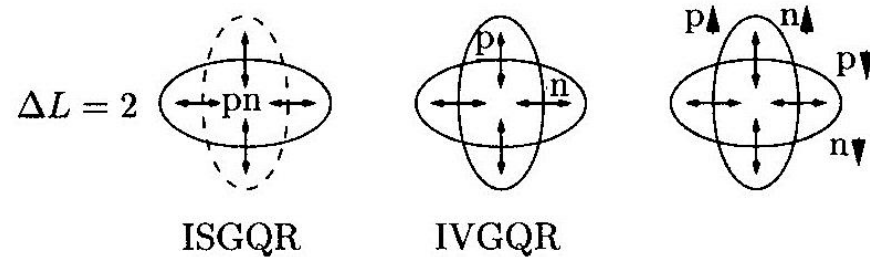
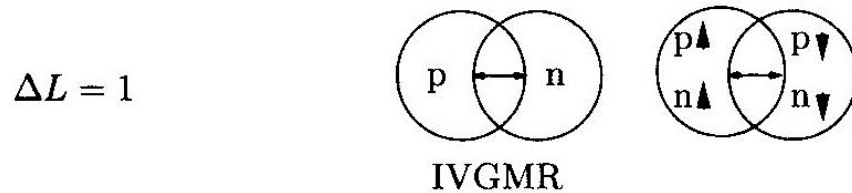
(Received 27 February 1973)

The giant-resonance region in ^{208}Pb was observed by inelastic electron scattering. We present evidence for the existences of a 2^+ (or 0^+) state at ~ 22 MeV and a 3^- state at ~ 19 MeV with giant-resonance character. The resonance states between 8.6 and 11.6 MeV are confirmed to be 2^+ (or 0^+) and the sum of their strengths exhausts about 50% of the $E2$ sum rule or 100% of $E0$.

- 陽子と中性子が一緒に動く(アイソ・スカラー型)
- 陽子と中性子が反対方向に動く(アイソ・ベクトル型)



巨大単極子振動
(呼吸モード)



$$\Delta T = 0$$

$$\Delta S = 0$$

$$\Delta T = 1$$

$$\Delta S = 0$$

$$\Delta T = 0$$

$$\Delta S = 1$$

$$\Delta T = 1$$

$$\Delta S = 1$$

movies: H.-J. Wollersheim,

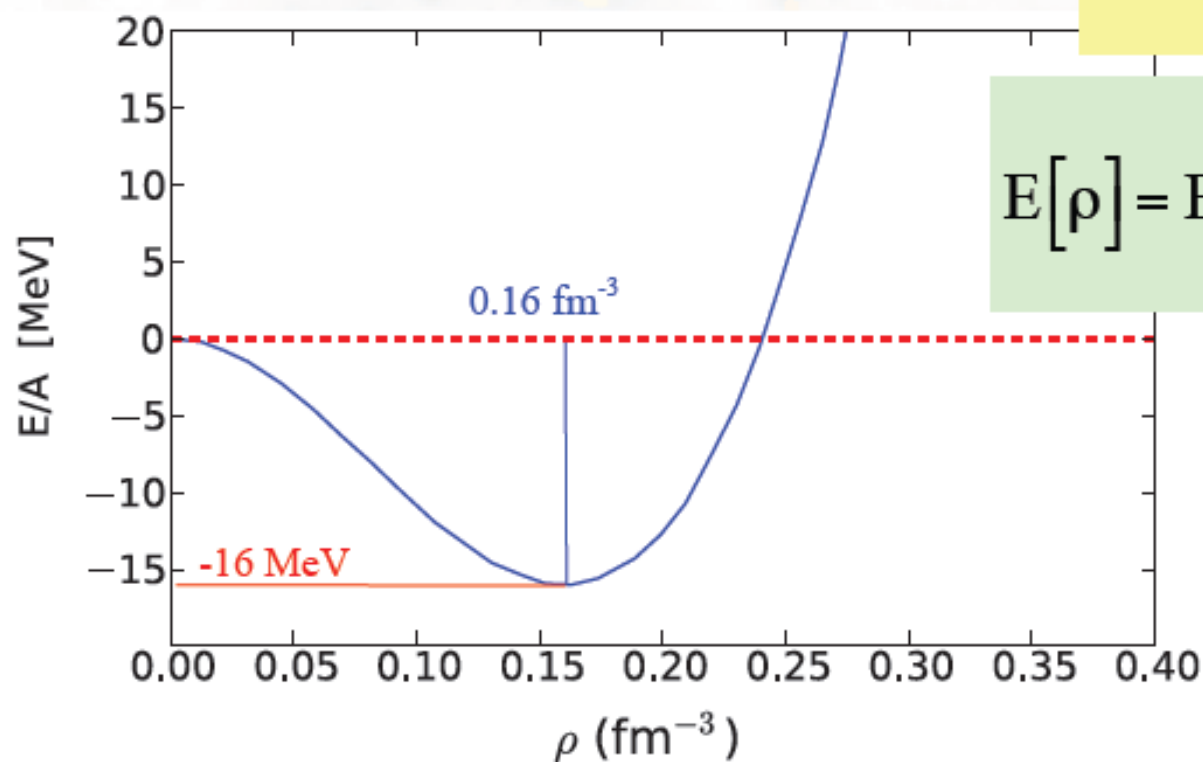
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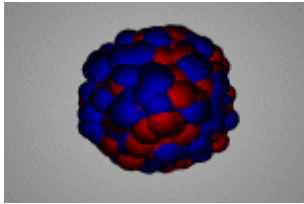
核物質の状態方程式

非圧縮度

$$K_{\infty} = 9\rho^2 \left. \frac{d^2[E(\rho)/\rho]}{d\rho^2} \right|_{\rho_0}$$

$$E[\rho] = E[\rho_0] + \frac{1}{18} K_{\infty} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2$$

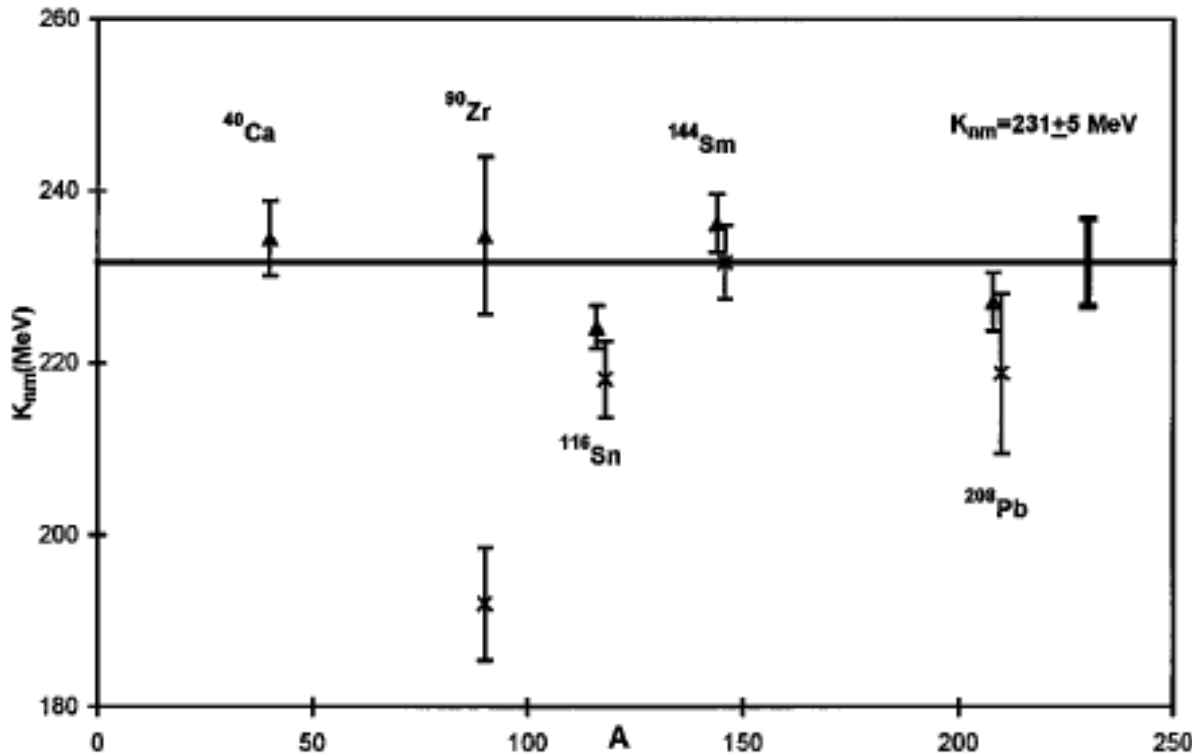




アイソ・スカラー型巨大単極子モード (呼吸モード)

$$E_{\text{ISGMR}} \sim \sqrt{\frac{\hbar^2 K}{m \langle r^2 \rangle}}$$

J.P. Blaizot,
Phys. Rep. 64 ('80) 171



$K \sim 231 \pm 5 \text{ MeV}$