Nuclear Reactions

Shape, interaction, and excitation structures of nuclei ⟵ scattering expt.

cf. Experiment by Rutherford (α scatt.)

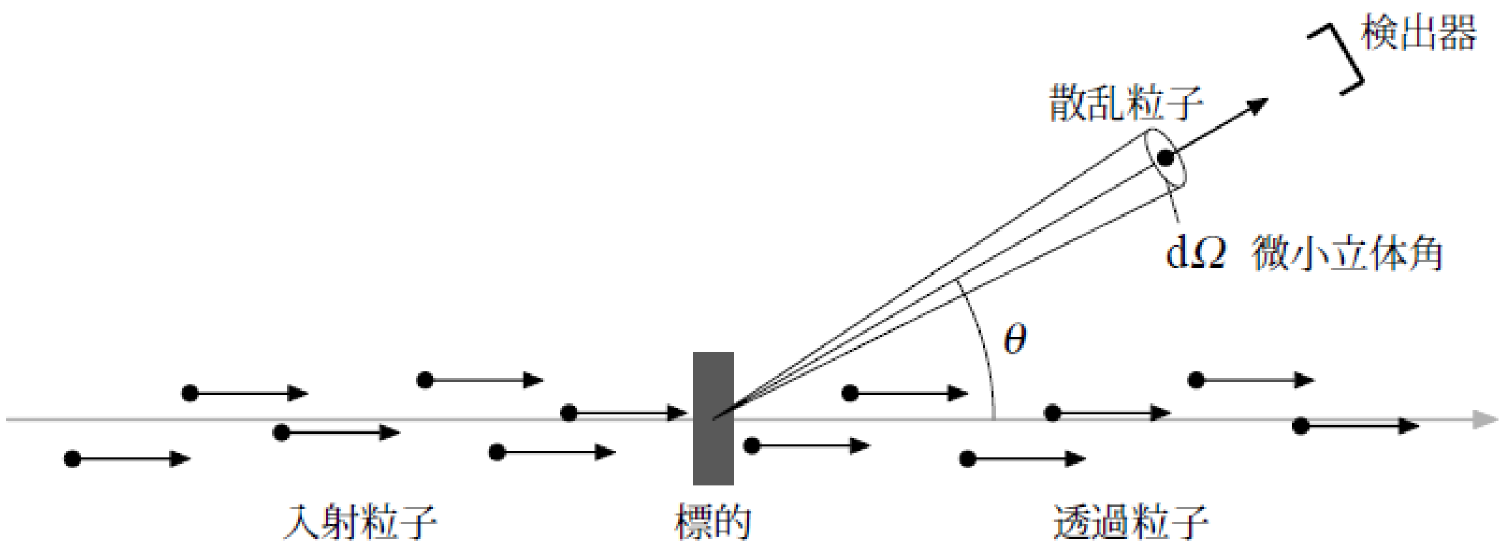
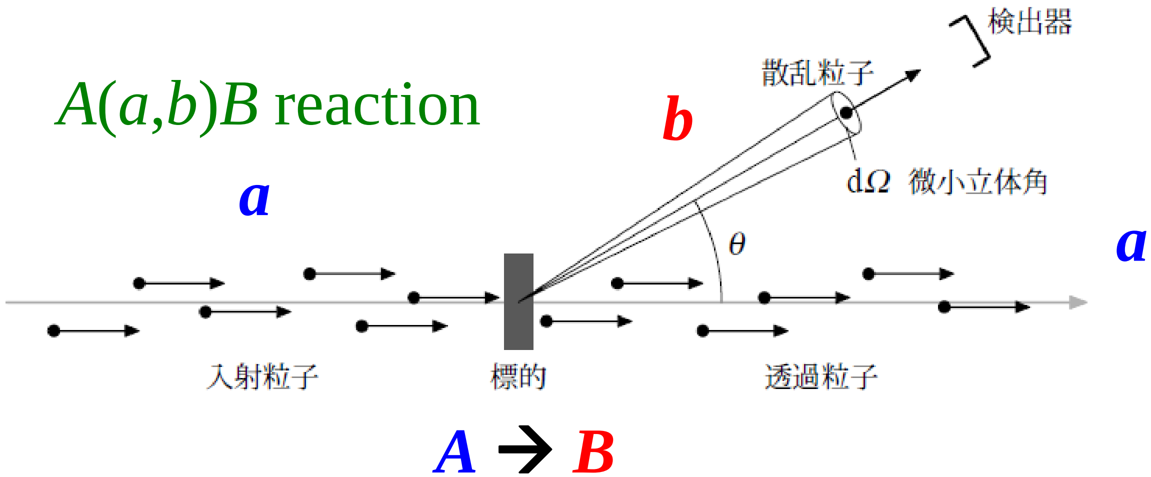


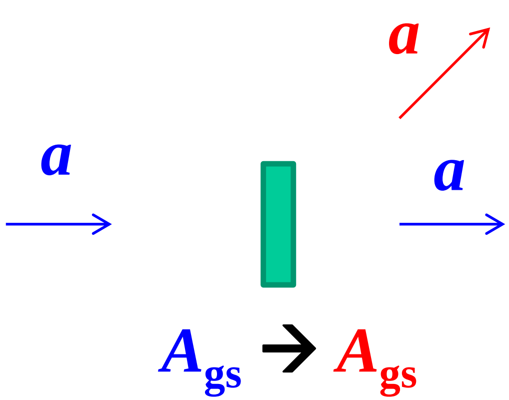
図 21.1: 散乱実験

http://www.th.phys.titech.ac.jp/~muto/lectures/QMII11/QMII11_chap21.pdf

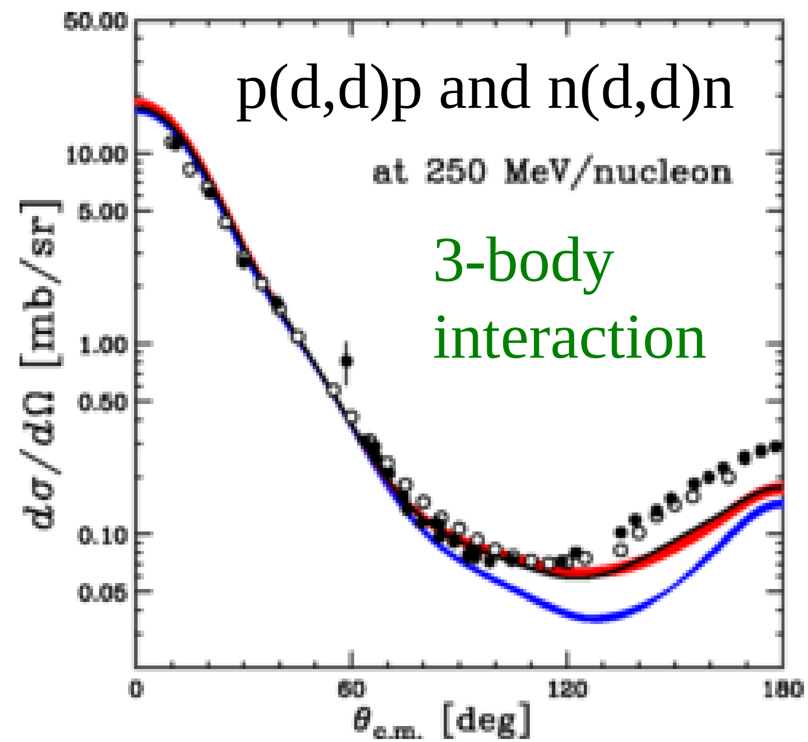
武藤一雄氏（東工大）



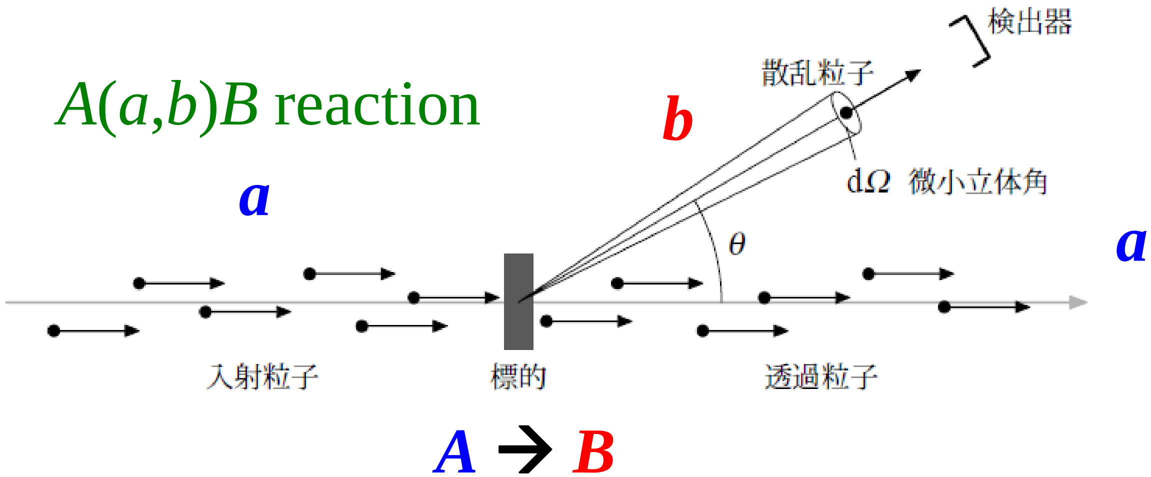
✓elastic scattering



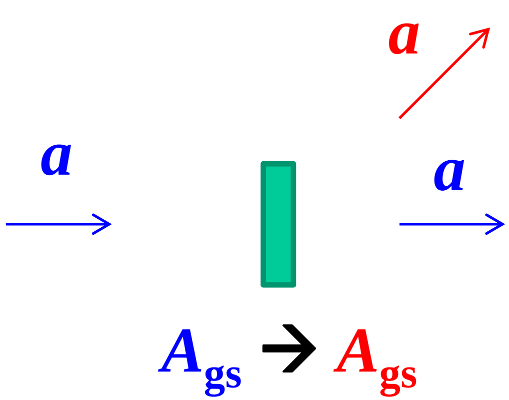
fundamental interaction
between a and A



K. Sekiguchi et al., PRC89('14)064007

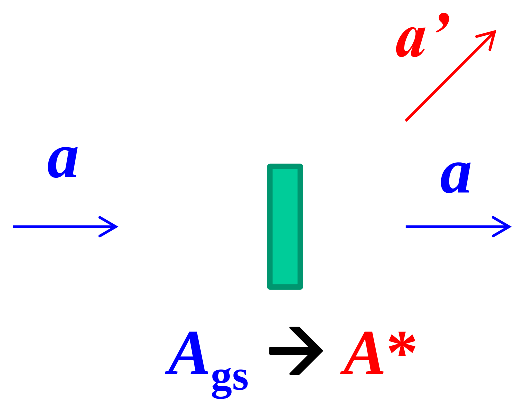


✓elastic scattering

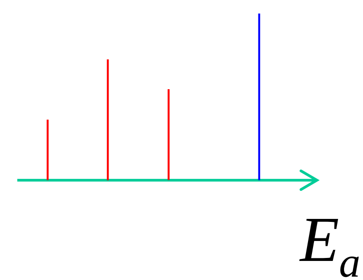


fundamental interaction between *a* and *A*

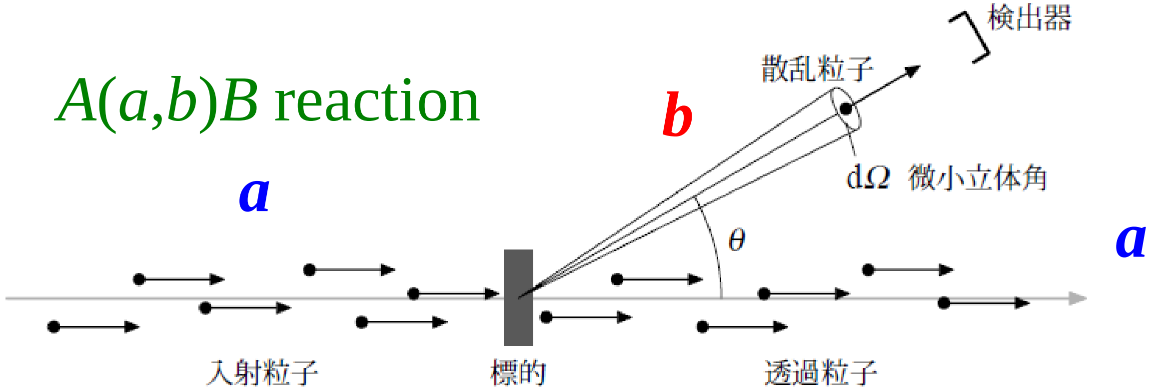
✓inelastic scattering



excitation spectrum of a nucleus *A*

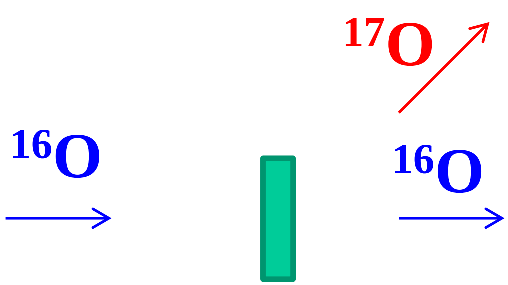


$A(a,b)B$ reaction



$A \rightarrow B$

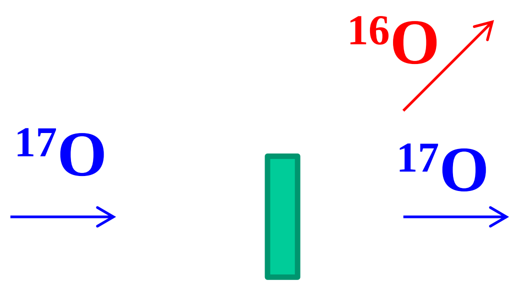
✓transfer reaction (below: an example of pick-up reaction)



$^{208}Pb_{gs} \rightarrow {}^{207}Pb$

level schem of ^{207}Pb

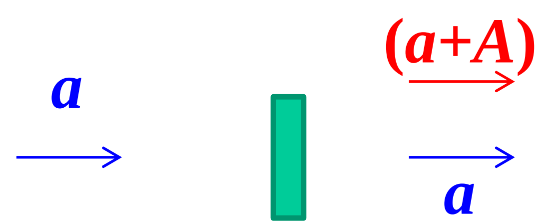
✓transfer reaction (below: an example of stripping reaction)



$^{208}Pb_{gs} \rightarrow {}^{209}Pb$

level schem of ^{209}Pb

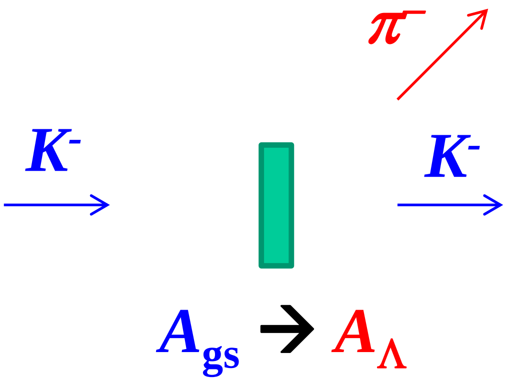
✓fusion reaction



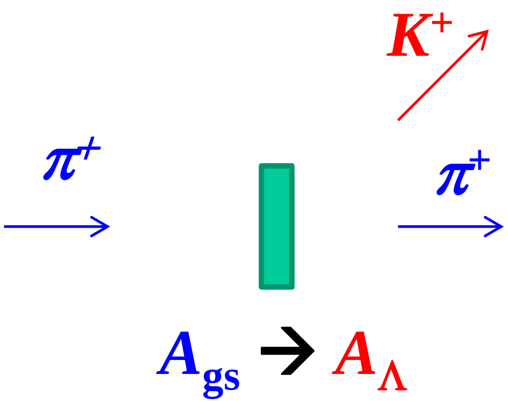
$A_{gs} \rightarrow X$

- interaction between a and A
- structure of a and A

✓(K^-,π^-) reaction

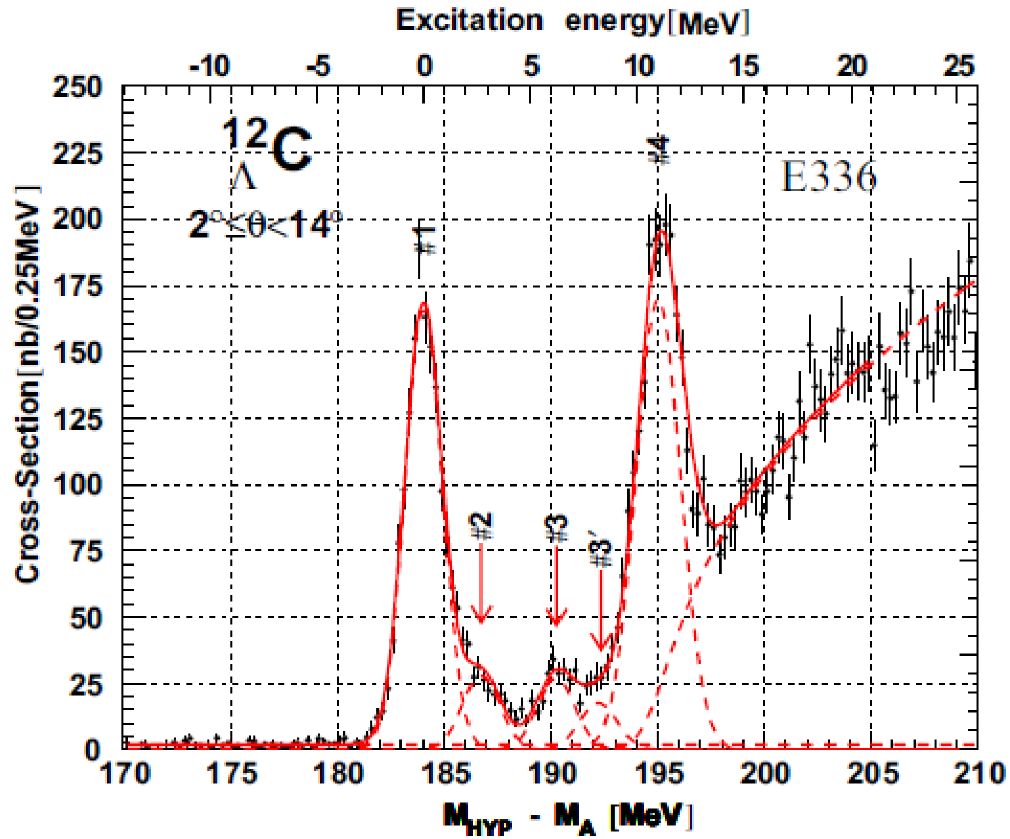


✓(π^+,K^+) reaction



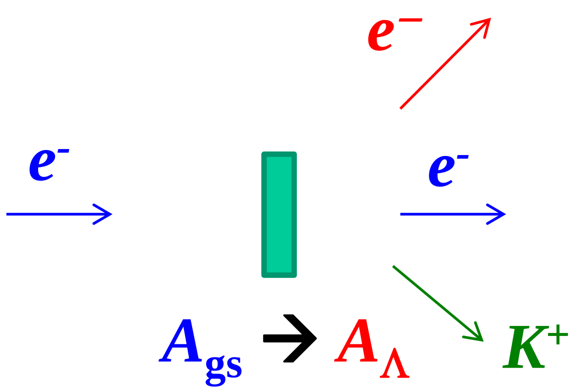
excitation spectrum
of a hypernucleus A_Λ

$^{12}C\ (\pi^+,K^+)\ ^{12}{}_{\Lambda}C$ reaction



O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 (‘06)564

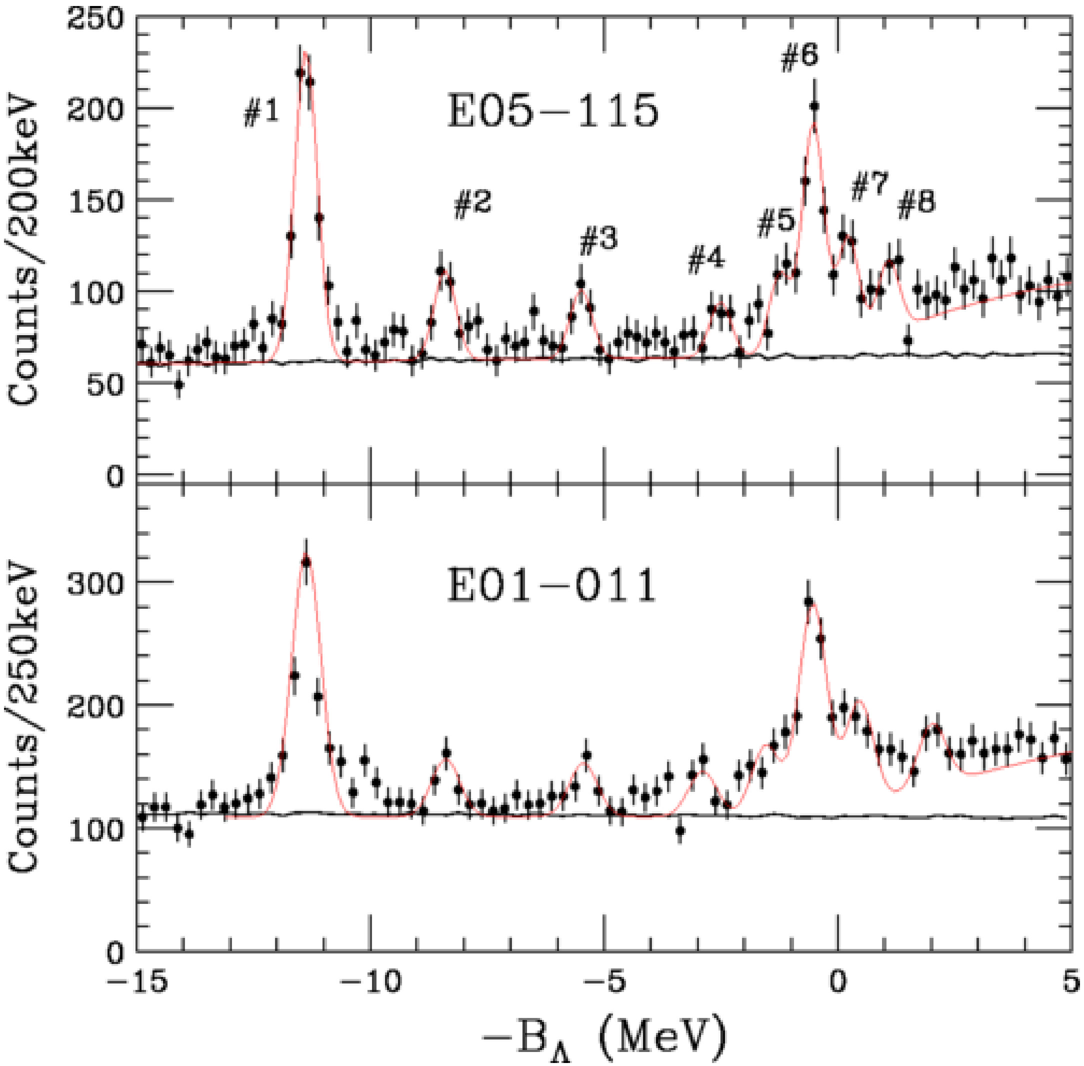
✓(e,e'K^+) reaction



S.N. Nakamura et al.,
PRL110('13)012502

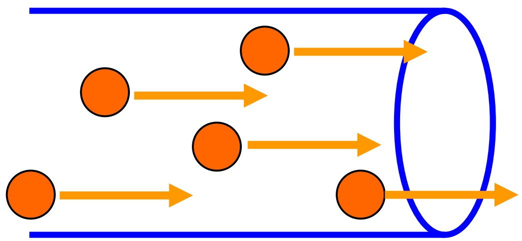
T. Gogami,
Ph.D. Thesis (Tohoku U.)
2014

$^{12}C(e,e'K^+)\ ^{12}_{\Lambda}B$



L. Tang et al., PRC90('14)034320

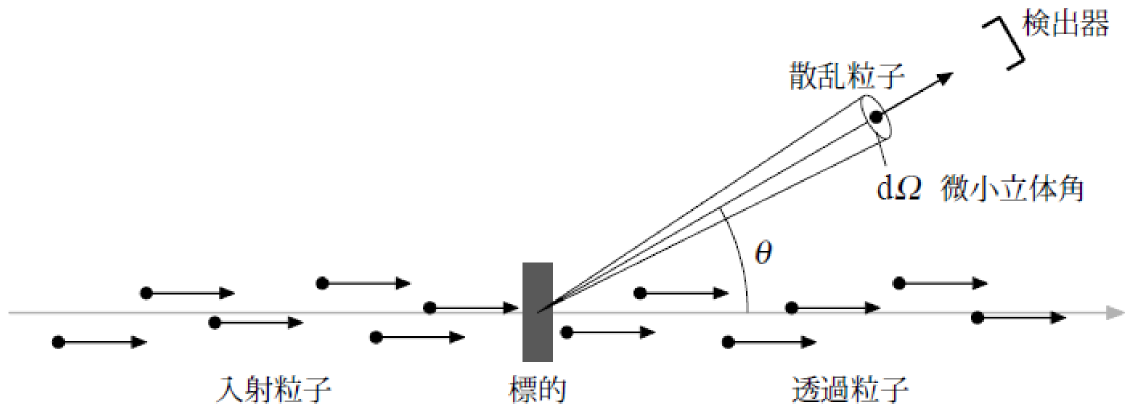
Cross sections



incident beam

flux = the number of particles
crossing unit area
per unit time

$$j = \rho_{\mathrm{P}} \cdot v$$

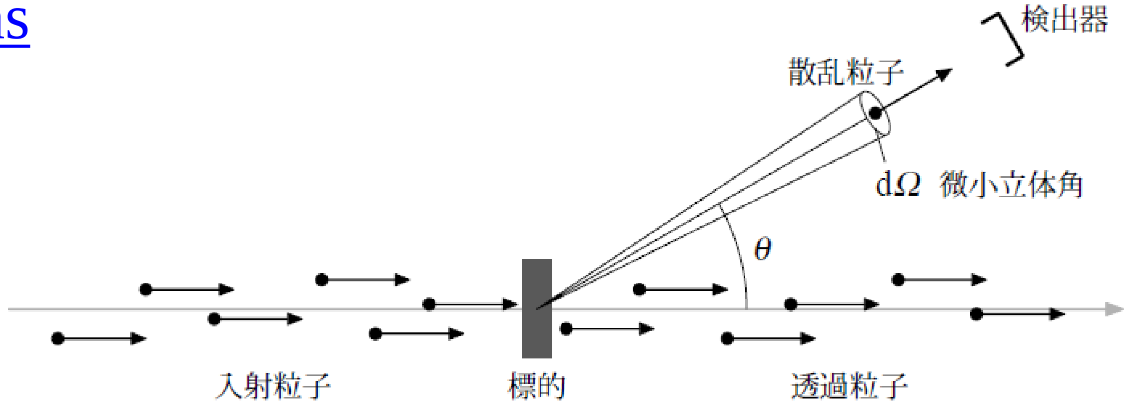


event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

$$\longrightarrow \quad R = N_{\mathrm{T}} \cdot \sigma \cdot j$$

cross section

Cross sections



event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

$$R = N_{\mathrm{T}} \cdot \sigma \cdot j$$

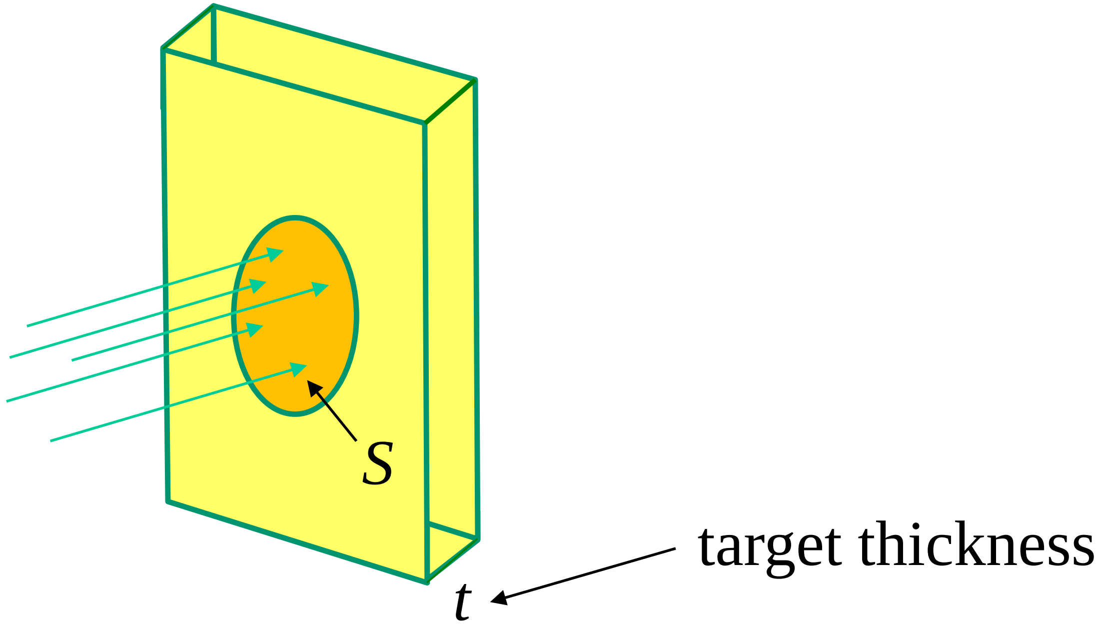
cross section

differential cross sections (angular distribution)

$$dR(\theta, \phi) = N_{\mathrm{T}} \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega \qquad \sigma = \int d\Omega \frac{d\sigma}{d\Omega}$$

units: 1 barn = 10^{-24} cm^2 = 100 fm^2 (1 mb = 10^{-3} b = 0.1 fm^2)

Cross sections (experiments)



$$dR(\theta, \phi) = N_T \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega$$

beam intensity: $I = j \cdot S$

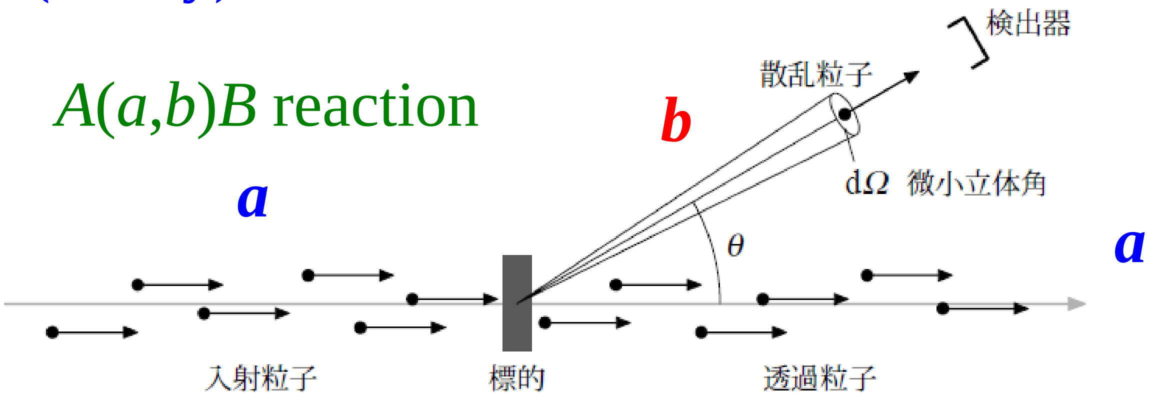
the number of target nucleus: $N_T = S \cdot t \cdot \rho_T$

$$dR(\theta, \phi) = I \cdot \frac{d\sigma}{d\Omega} \cdot t\rho_T \cdot d\Omega \cdot \epsilon$$

detection efficiency

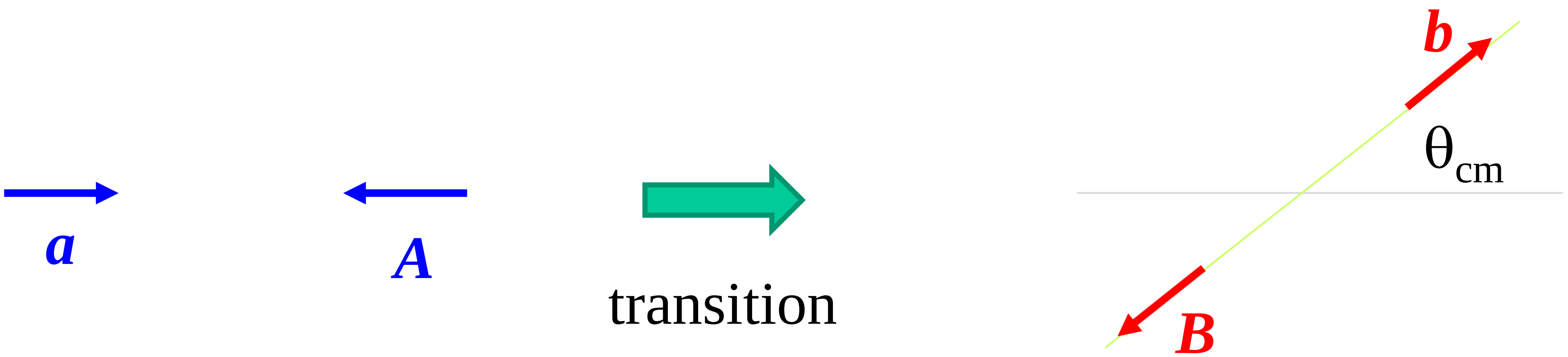
Cross sections (theory)

$A(a,b)B$ reaction



$A \rightarrow B$

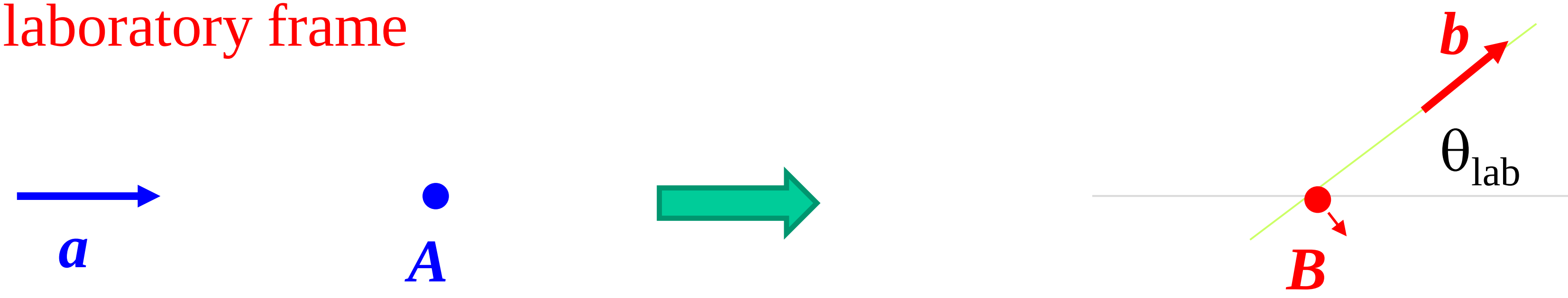
center of mass frame



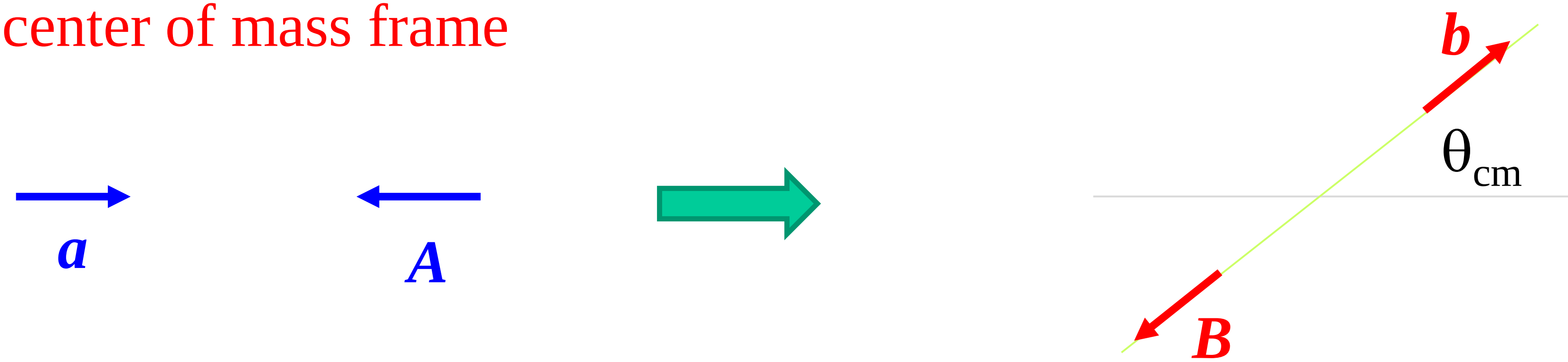
$$\frac{d\sigma}{d\Omega} = \frac{R}{j_{\rm in}}$$

Cross sections

✓ laboratory frame



✓ center of mass frame



□ transformation ⟵ energy and momentum conservations

$$\tan\theta_{\rm lab} = \frac{\sin\theta_{\rm cm}}{\gamma+\cos\theta_{\rm cm}}, \qquad d\Omega_{\rm lab} = \frac{|1+\gamma\cos\theta_{\rm cm}|}{(1+\gamma^2+2\gamma\cos\theta_{\rm cm})^{3/2}}d\Omega_{\rm cm}$$

$$E_{\rm cm} = \frac{M_A}{M_a+M_A}E_{\rm lab}, \qquad \gamma = \sqrt{\frac{M_aM_b}{M_AM_B}\frac{E_{\rm cm}}{E_{\rm cm}+Q}}$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

transition rate for elastic scattering:

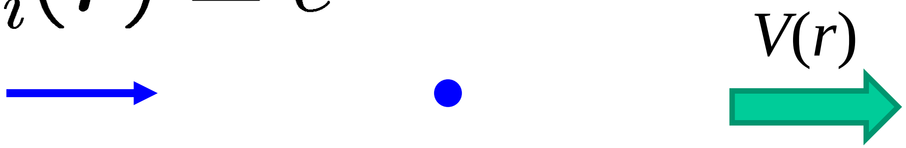
$$\begin{aligned} W_{fi} &= \frac{2\pi}{\hbar}\int\frac{d\boldsymbol{p}_f}{(2\pi\hbar)^3}\,|\langle\psi_f|V|\psi_i\rangle|^2\delta(E_f - E_i) \\ &= \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega\left|\widetilde{V}(\boldsymbol{q})\right|^2 \end{aligned}$$

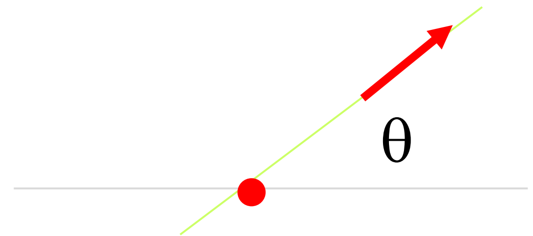
$$\widetilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r}e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar}V(r) \equiv \int d\boldsymbol{r}e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(r)$$

Born approximation

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$





$$W_{fi} = \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega \left|\tilde{V}(\boldsymbol{q})\right|^2$$

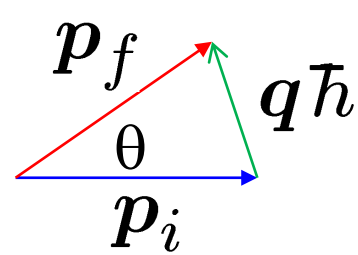
momentum transfer

$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r} e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar} V(r) \equiv \int d\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} V(r)$$

incident flux: $j_{\mathrm{inc}} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\mathrm{inc}}} = \int d\Omega \boxed{\frac{\mu^2}{4\pi^2\hbar^4}\left|\tilde{V}(\boldsymbol{q})\right|^2}$$

$$= \frac{d\sigma}{d\Omega}$$



$$q\hbar = 2p_i \sin\frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2}|F(\boldsymbol{q})|^2$$

$$= \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega}\right)|F(\boldsymbol{q})|^2$$

Form factor

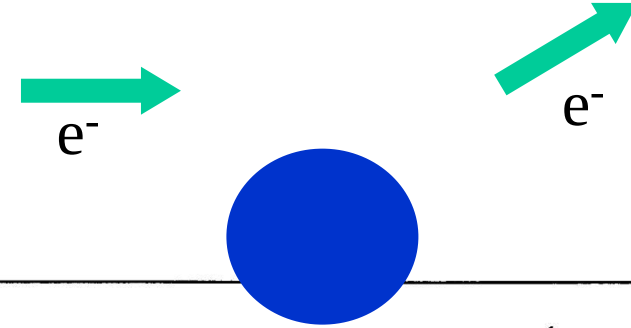
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho(\boldsymbol{r})\,d\boldsymbol{r}$$

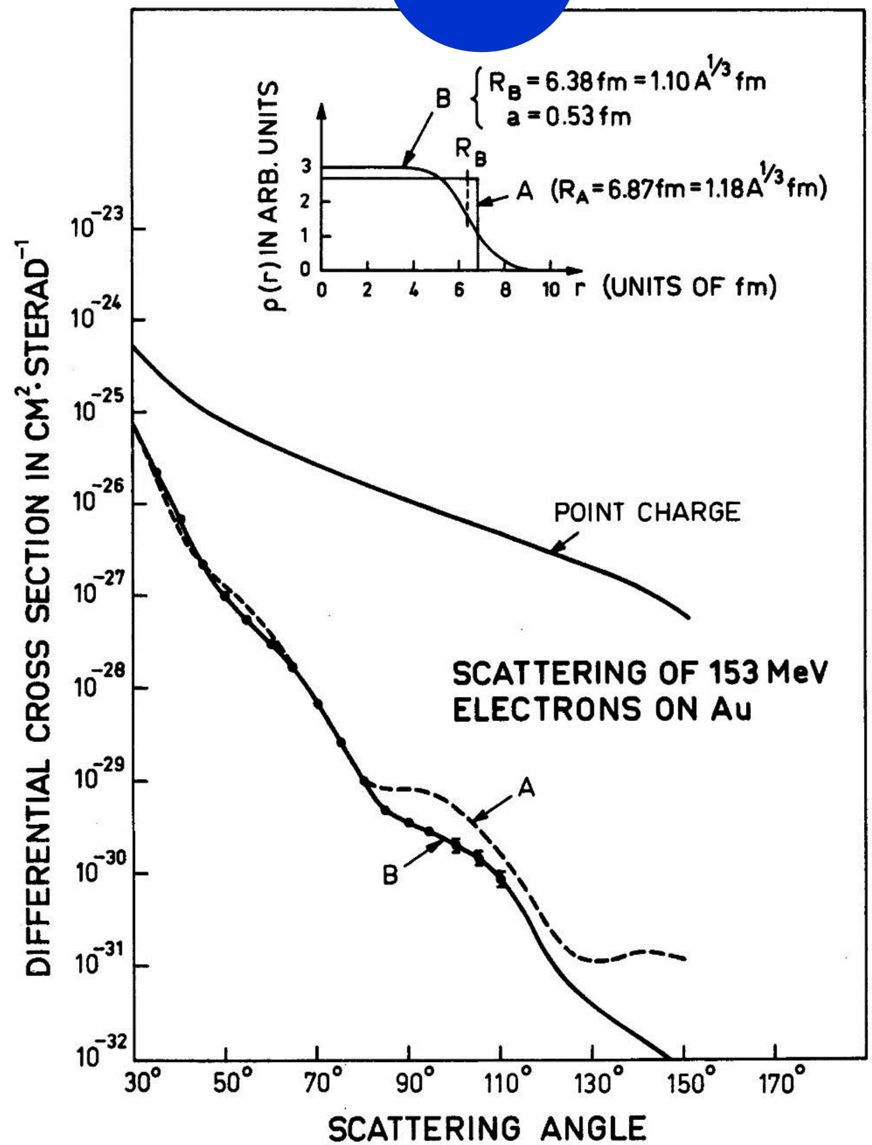
* relativistic correction:

$$\frac{d\sigma_{\text{Ruth}}}{d\Omega} \rightarrow \frac{d\sigma_{\text{Mott}}}{d\Omega}$$

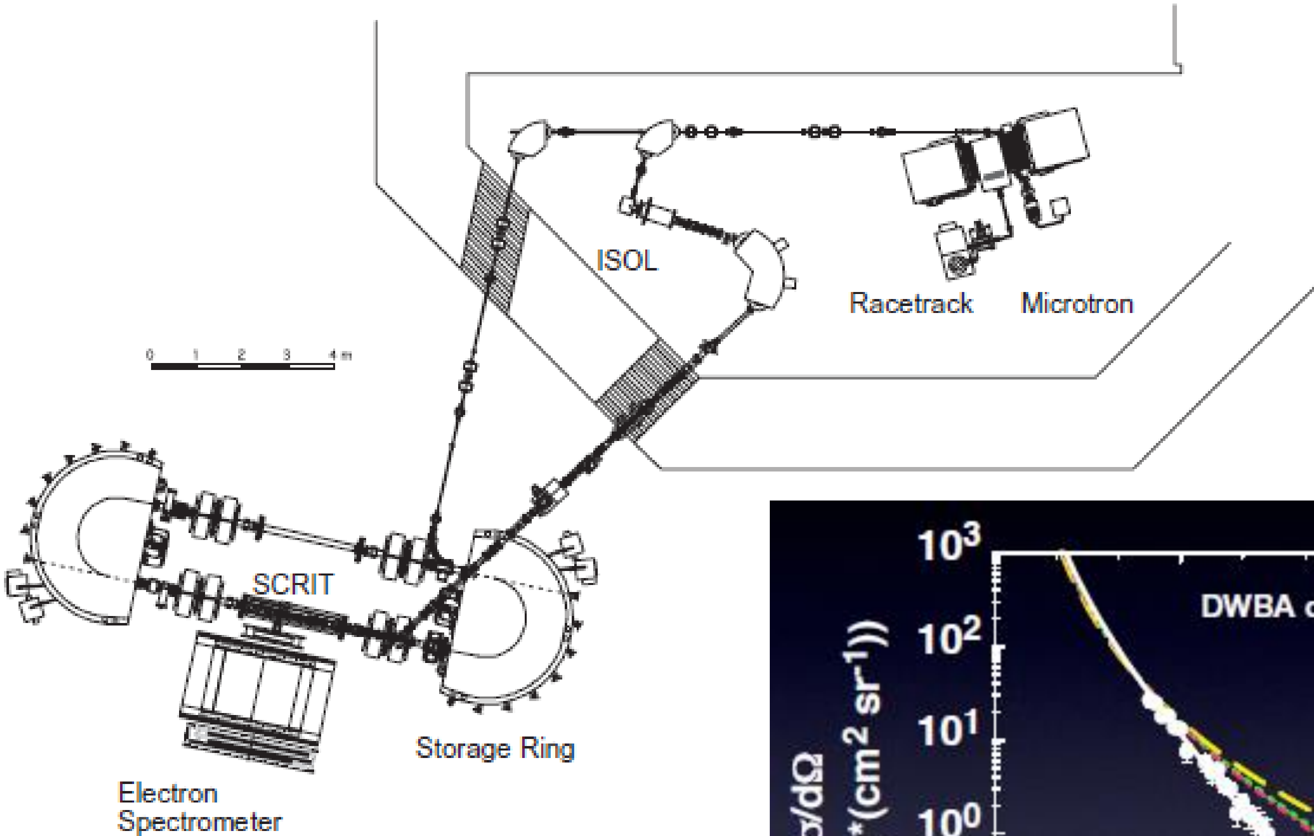
$$= \frac{d\sigma_{\text{Ruth}}}{d\Omega}\cdot\left(1-\frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right)$$

$$\sim \frac{d\sigma_{\text{Ruth}}}{d\Omega}\cdot\cos^2\frac{\theta}{2} \qquad (v\rightarrow c)$$

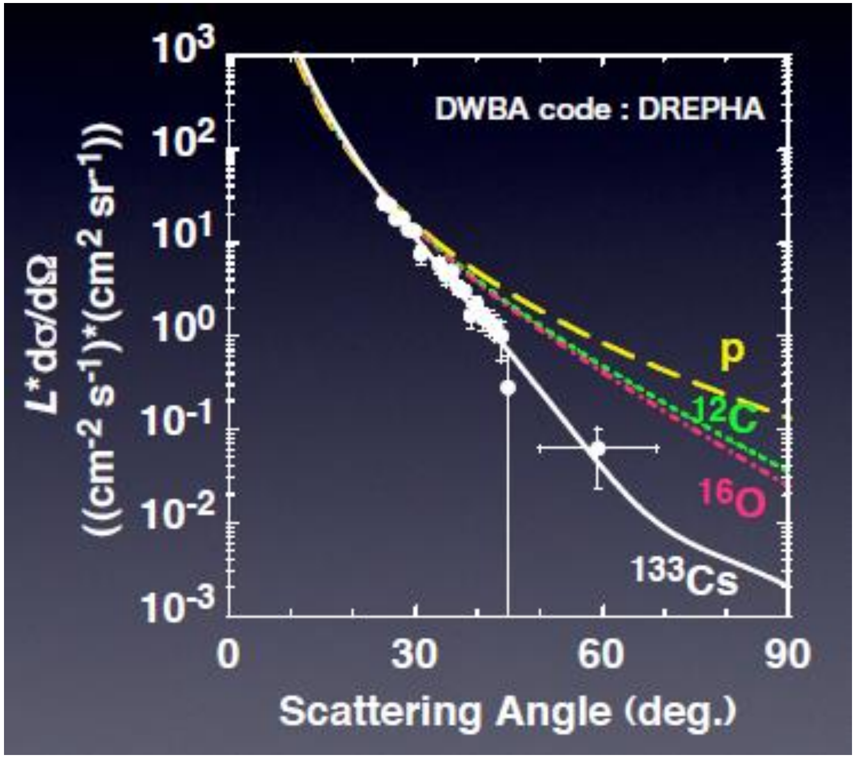




cf. electron scattering off unstable nuclei (SCRIT)



T. Suda et al.,
PTEP 2012, 03C008 (2012)
PRL102, 102501 (2009)

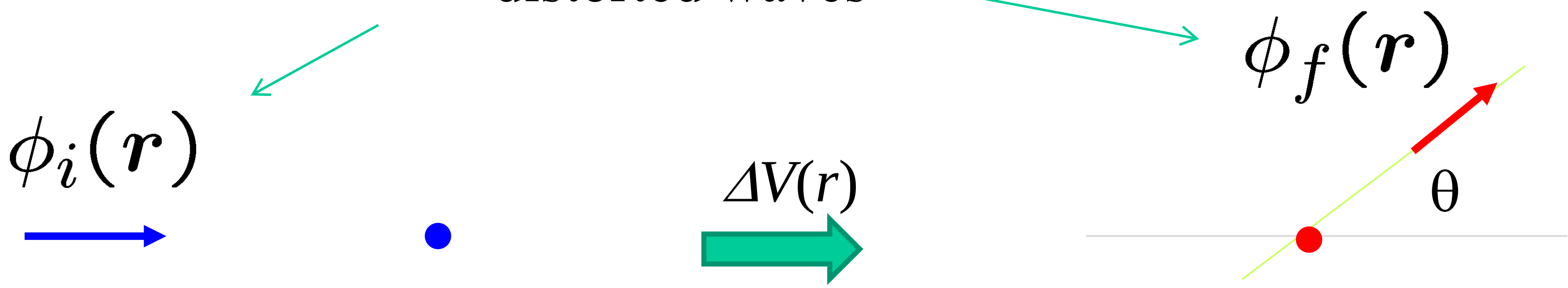


Distorted Wave Born approximation (DWBA)

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \underbrace{V(r)}_{\text{perturbation}} - E\right)\psi(\boldsymbol{r}) = 0$$

$$\Rightarrow \left(-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \underbrace{V(r) - V_0(r)}_{\text{perturbation}} - E\right)\psi(\boldsymbol{r}) = 0$$

"distorted waves"



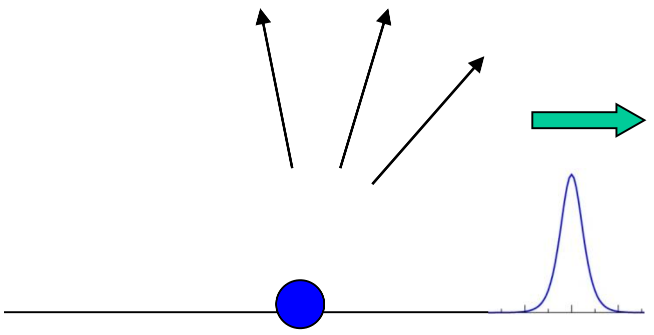
✓inelastic scattering

✓transfer reactions

Optical model

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux (absorption)

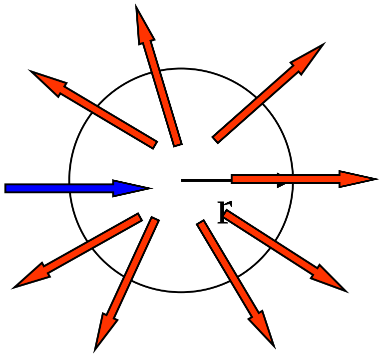
Optical potential

$$V_{\text{opt}}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)$$

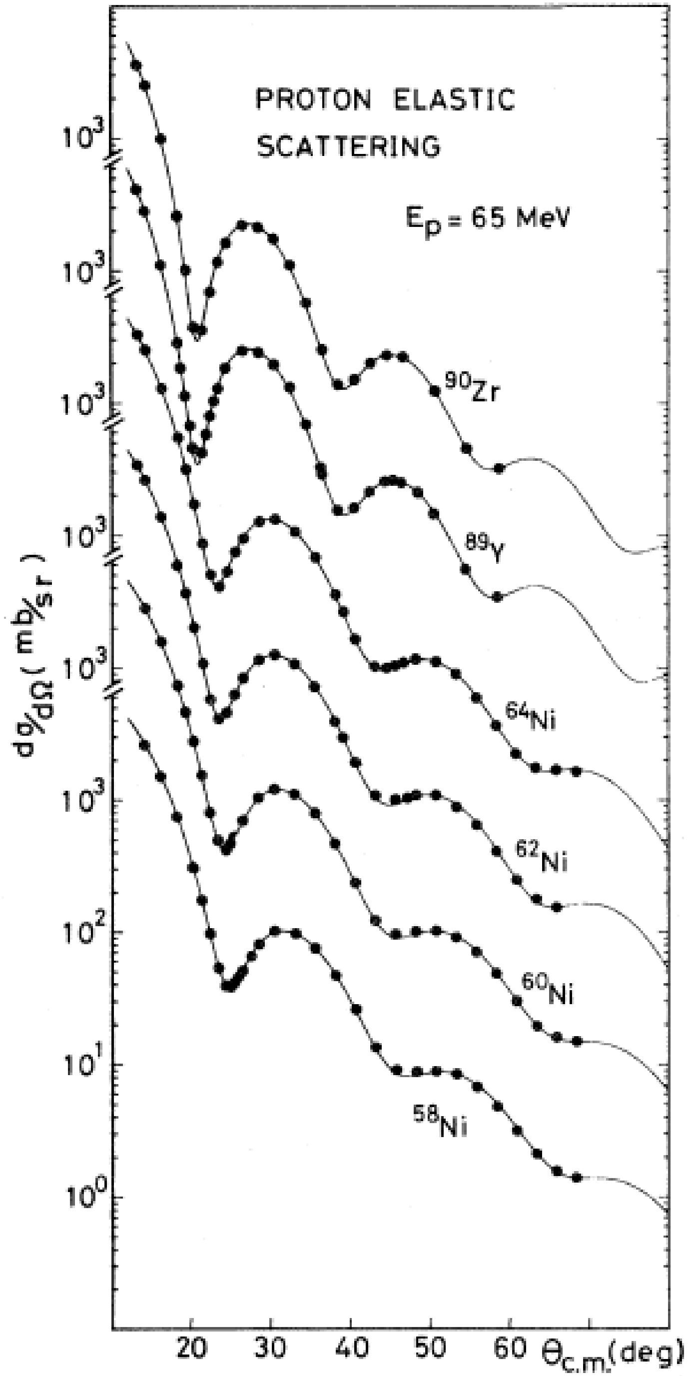
$$\longrightarrow \quad \boldsymbol{\nabla} \cdot \boldsymbol{j} = \cdots = -\frac{2}{\hbar} W |\psi|^2$$

(note) Gauss's law

$$\int_S \boldsymbol{j} \cdot \boldsymbol{n} \, dS = \int_V \boldsymbol{\nabla} \cdot \boldsymbol{j} \, dV$$



$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \frac{Z_P Z_T e^2}{r} + V_{\rm opt}(r) - E\right)\psi(\boldsymbol{r}) = 0$$

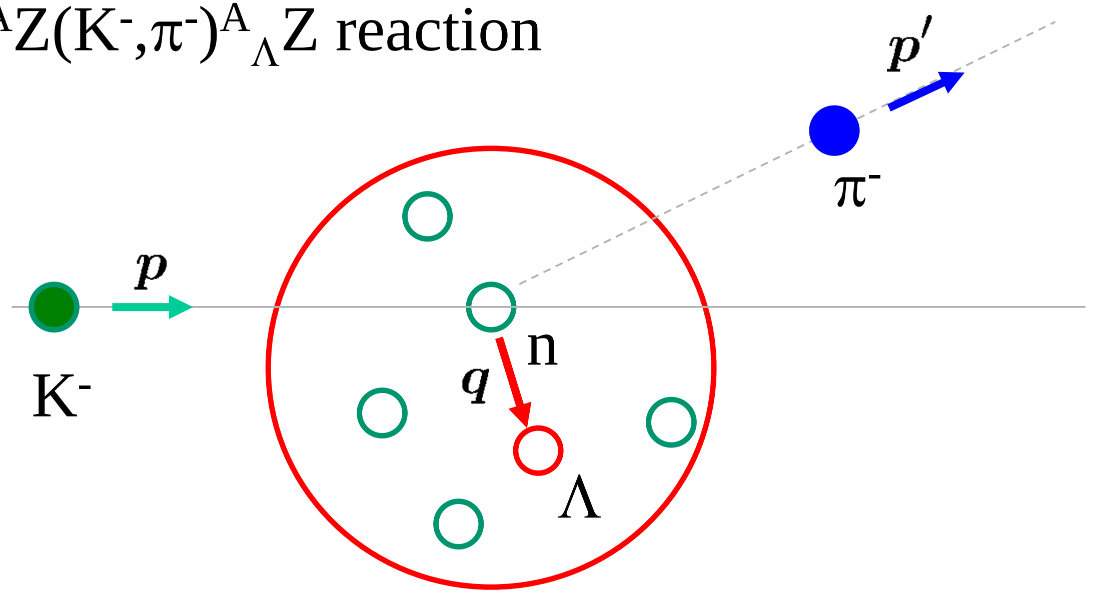


Woods-Saxon + volume &surface imaginary parts

H. Sakaguchi et al., PRC26 (1982) 944

Impulse approximation

example: ${}^{A}Z(K^{-},\pi^{-}){}^{A}{}_{\Lambda}Z$ reaction



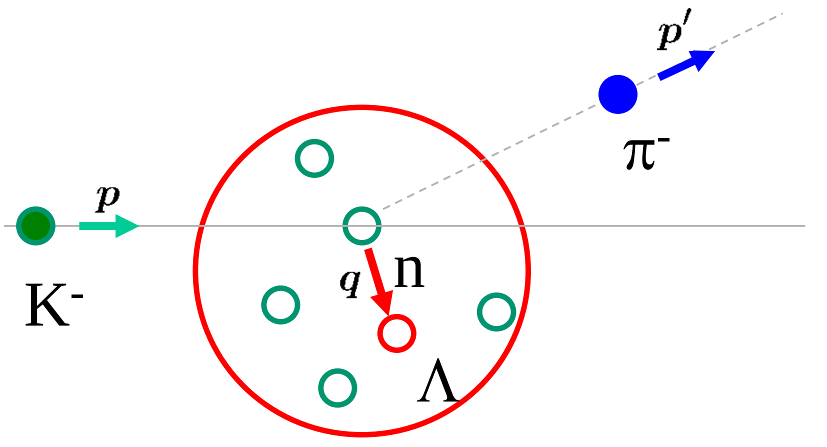
- ✓ high energy
- ✓ single scattering approximation
- ✓ (other nucleons: spectator)

$$T_{fi} \sim \left\langle \psi_{\pi^-} \middle| \left\langle \Psi_{{}^{A}_{\Lambda}Z} \middle| \sum_j v_{eff}(j) \middle| \Psi_{{}^{A}Z} \right\rangle \middle| \psi_{K^-} \right\rangle$$

effective K-n interaction
(including multiple scattering)

Impulse approximation

example: ${}^{A}Z(K^{-},\pi^{-}){}^{A}_{\Lambda}Z$ reaction



✓ high energy

✓ single scattering approximation

$$T_{fi} \sim \left\langle \psi_{\pi^-} \left| \left\langle \Psi_{{}^A_\Lambda Z} \left| \sum_j v_{eff}(j) \right| \Psi_{{}^A Z} \right\rangle \right| \psi_{K^-} \right\rangle$$

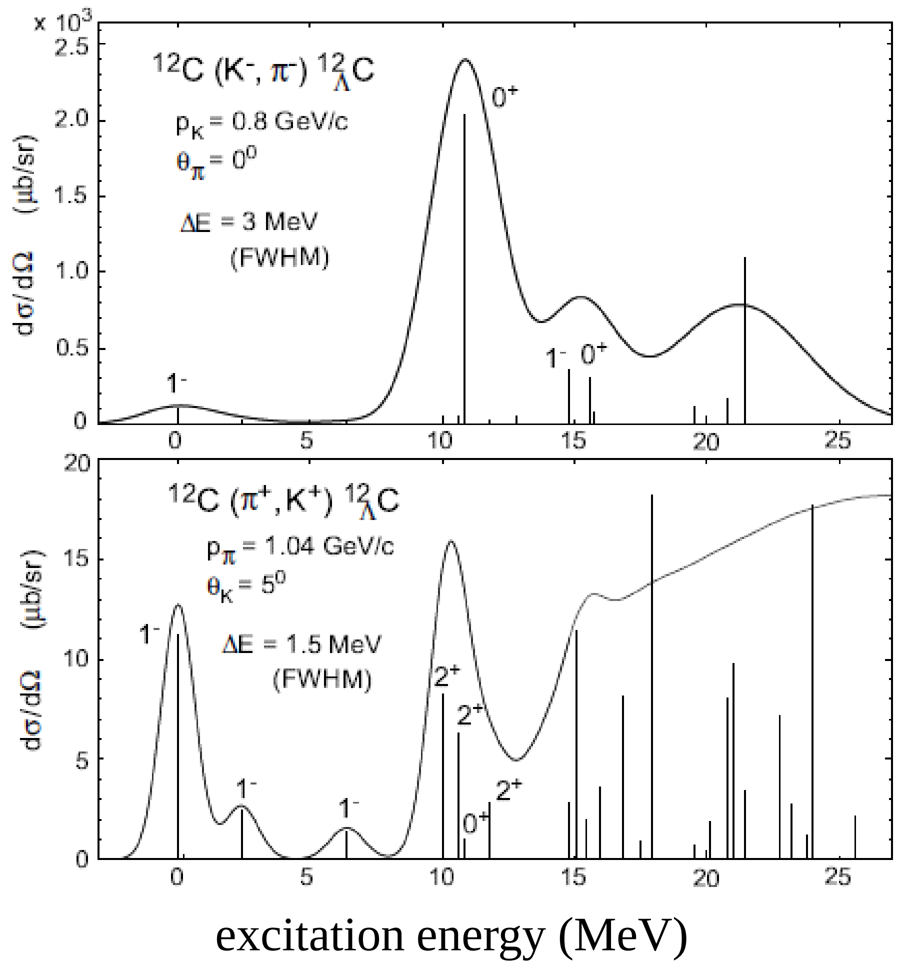
$$\frac{d\sigma}{d\Omega} \sim \alpha_{\mathsf{kin}} \left(\frac{d\sigma}{d\Omega} \right)_{K^- n \to \pi^- \Lambda} N_{\mathsf{eff}}(\theta; i \to f)$$

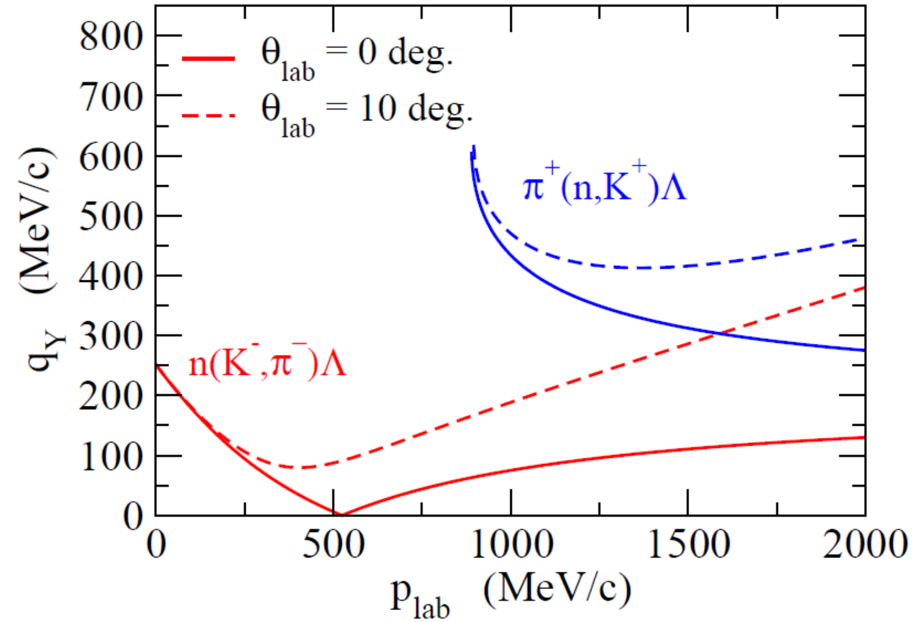
kinematical factor

elementary process

$$N_{\mathsf{eff}}(\theta; i \to f) \sim \left| \int d\boldsymbol{r}\, \psi^*_{\pi^-}(\boldsymbol{r})\, \varphi^{(\Lambda)*}_{j_\Lambda l_\Lambda m_\Lambda}(\boldsymbol{r}) \varphi^{(n)}_{j_n l_n m_n}(\boldsymbol{r})\, \psi_{K^-}(\boldsymbol{r}) \right|^2$$

- Plane wave impulse approximation (PWIA)
- Distorted wave impulse approximation (DWIA)





$m_n + m_K$ = 1432 MeV

$m_\pi + m_\Lambda$ = 1255.3 MeV

Q > 0

$m_\pi + m_n$ = 1079.2 MeV

$m_K + m_\Lambda$ = 1609.4 MeV

Q < 0

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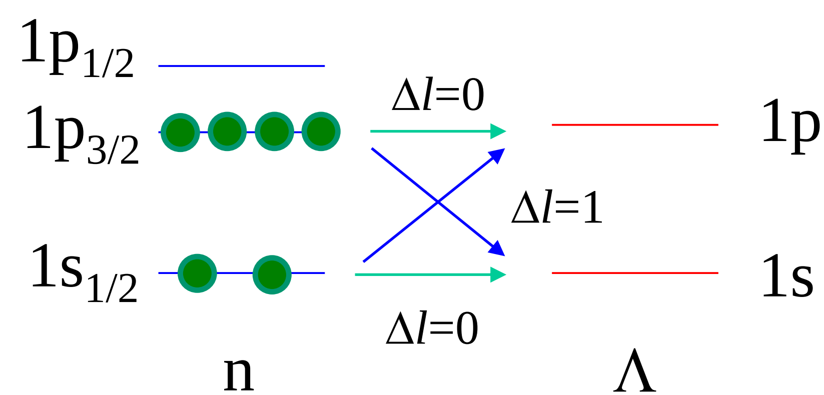
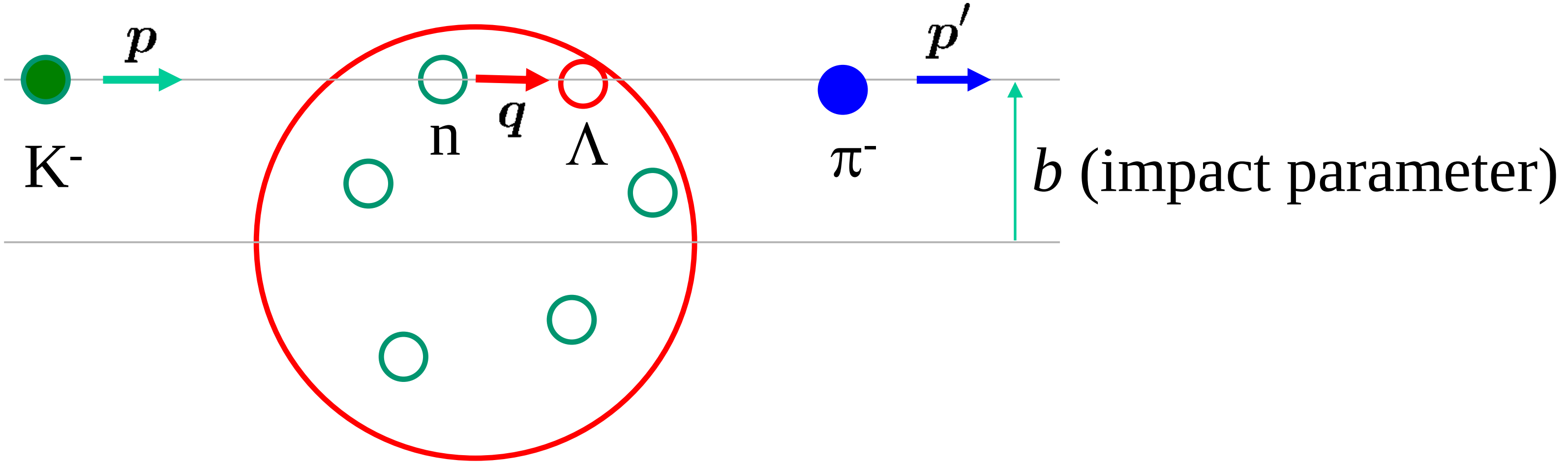
T. Motoba et al., PRC38('88)1322

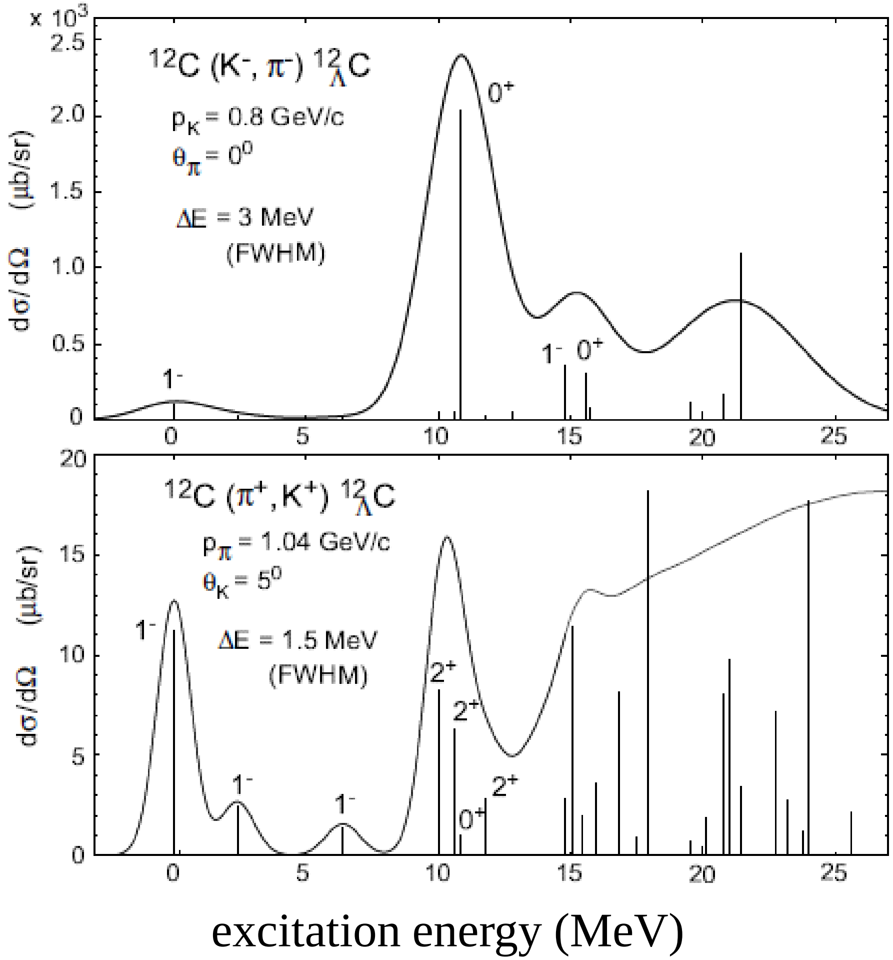


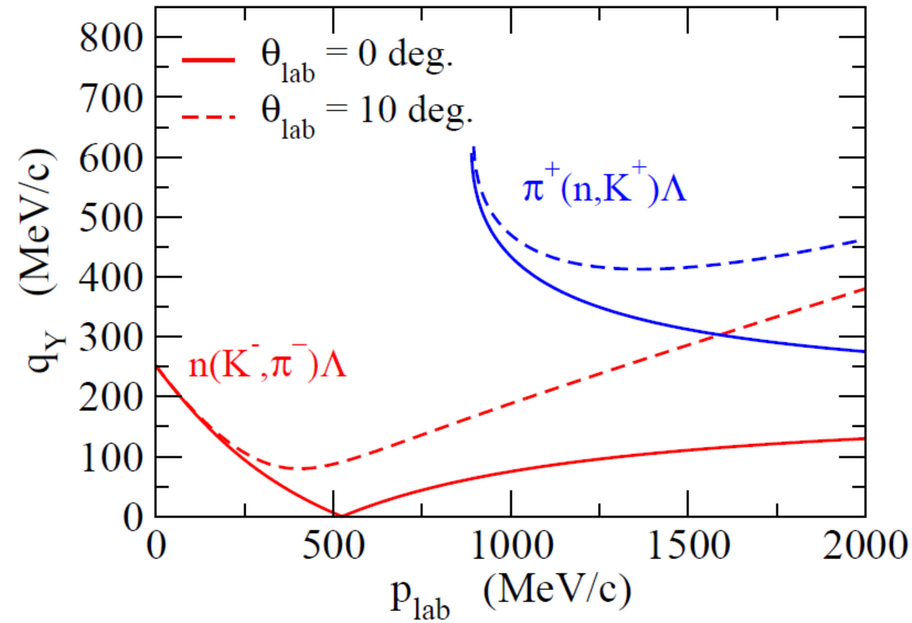
relation between q and Δl

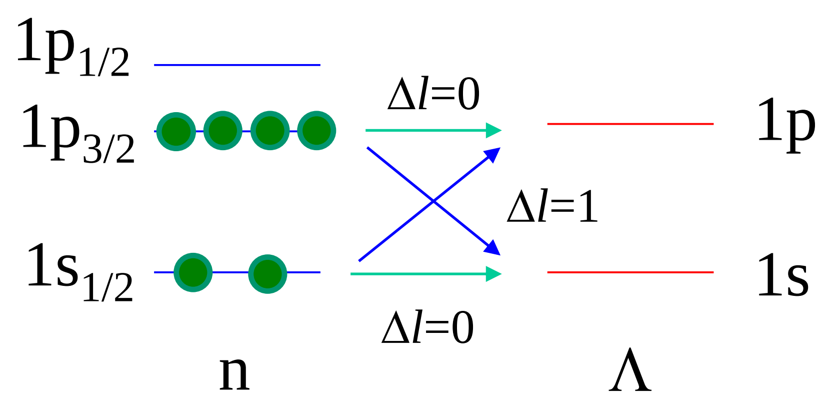


$l \sim kb$ (classically)

$\Delta l \sim b(p' - p) = bq$







O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 (‘06)564

T. Motoba et al., PRC38(‘88)1322

$$\Delta l \sim b(p' - p) = bq$$