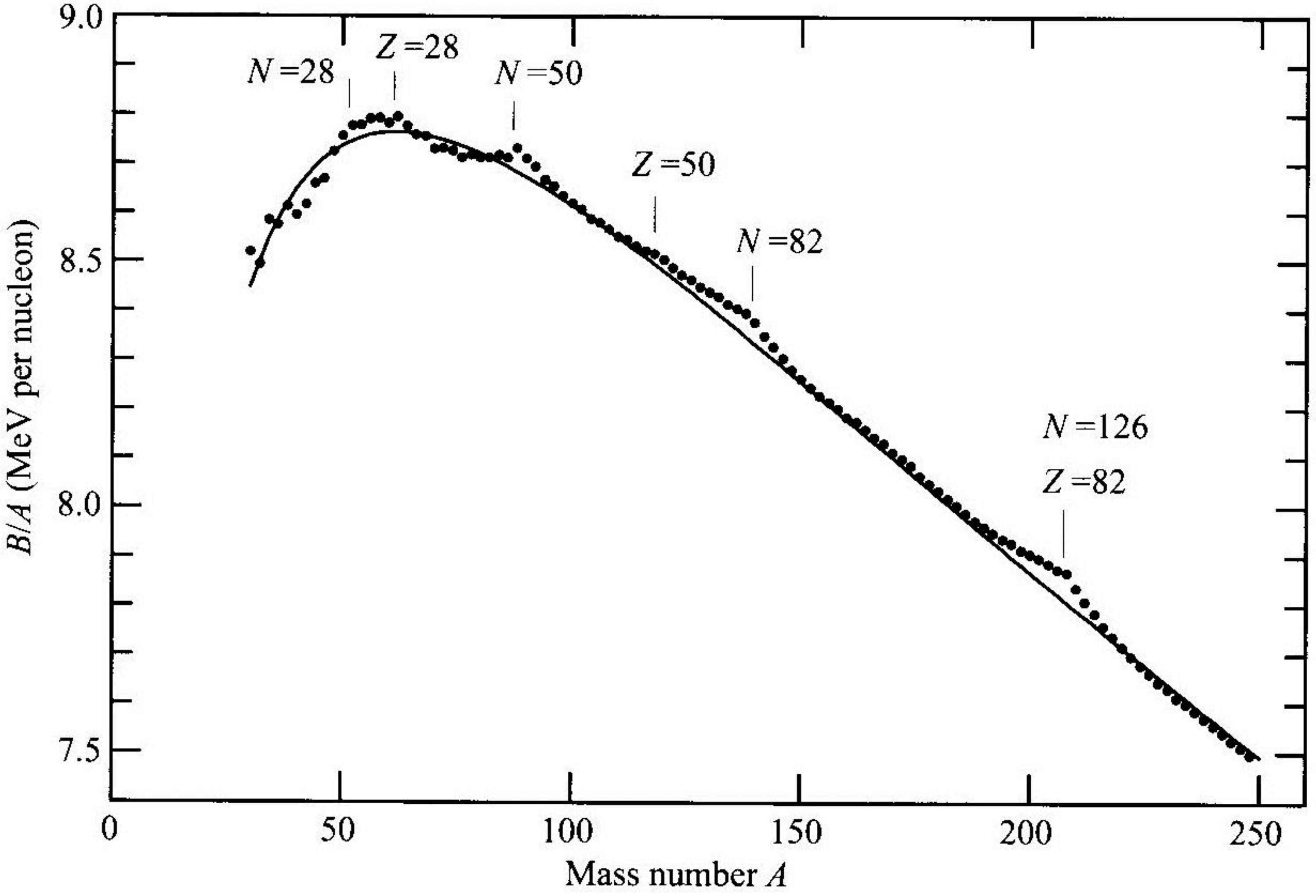
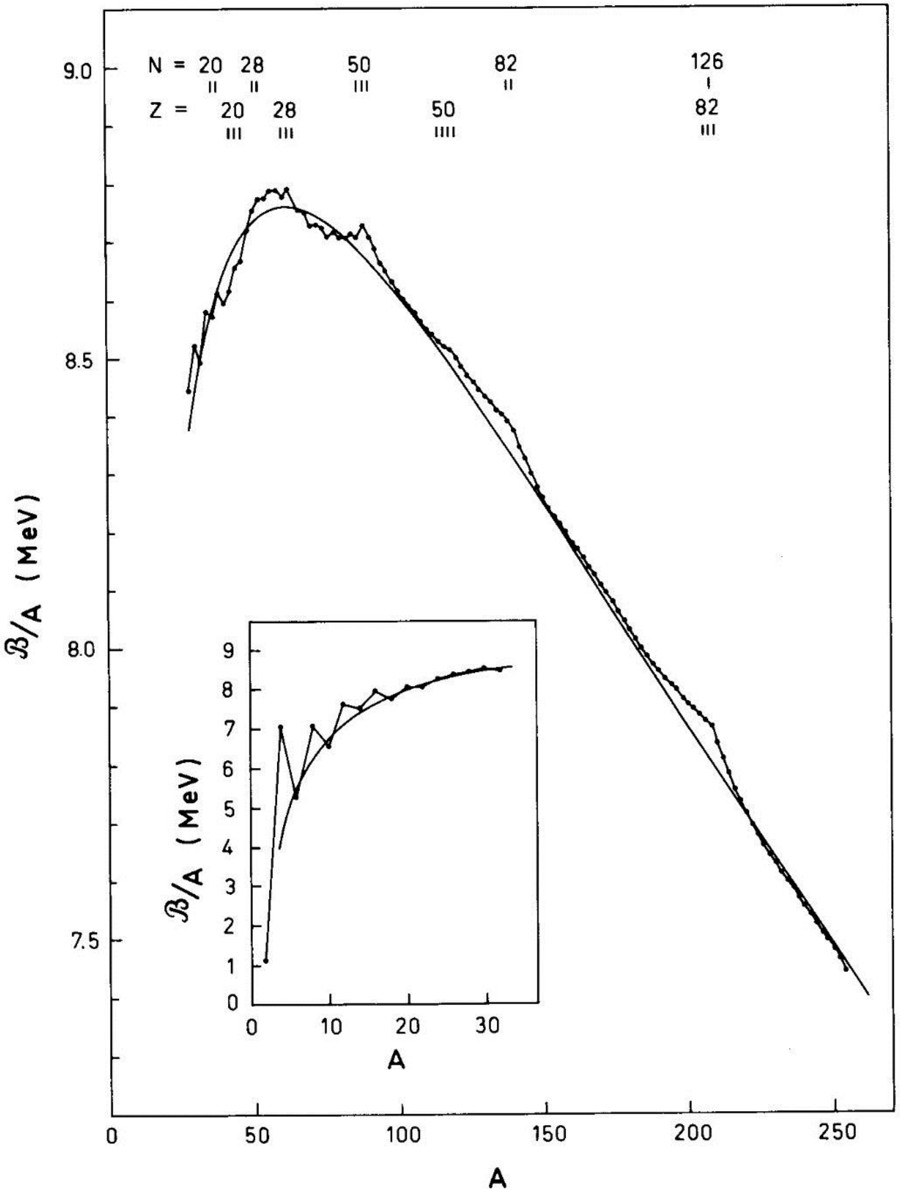


cf. N,Z = 2, 8, 20, 28, 50, 82, 126 (魔法数)に対して束縛エネルギー大

Shell Structure

$$B(N,Z) = B_{\mathsf{macro}}(N,Z) + B_{\mathsf{micro}}(N,Z)$$



- Smooth part

$$B_{\mathsf{macro}}(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathsf{sym}} \frac{(N-Z)^2}{A}$$

- Fluctuation part

$$B_{\mathsf{micro}} = B_{\mathsf{pair}} + B_{\mathsf{shell}}$$

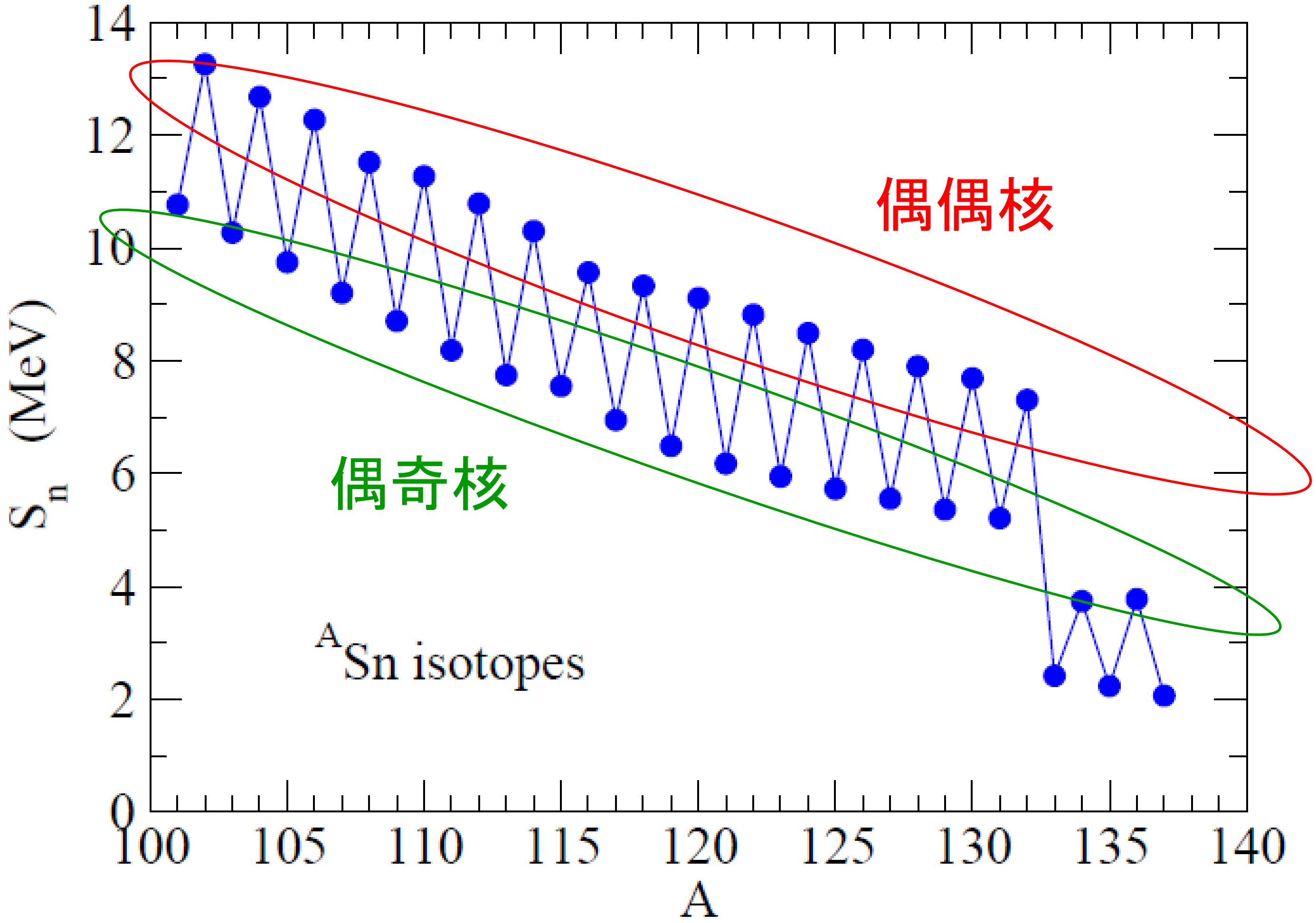
Liquid drop model:

$$B_{\mathsf{LDM}} = B_{\mathsf{macro}} + B_{\mathsf{pair}}$$

対相関エネルギー

偶数個の中性子から1つ中性子を取る方が奇数個から取るより大きなエネルギーが必要：対相関

even-odd staggering



1n separation energy: $S_n(A,Z) = B(A,Z) - B(A-1,Z)$

Pairing Energy

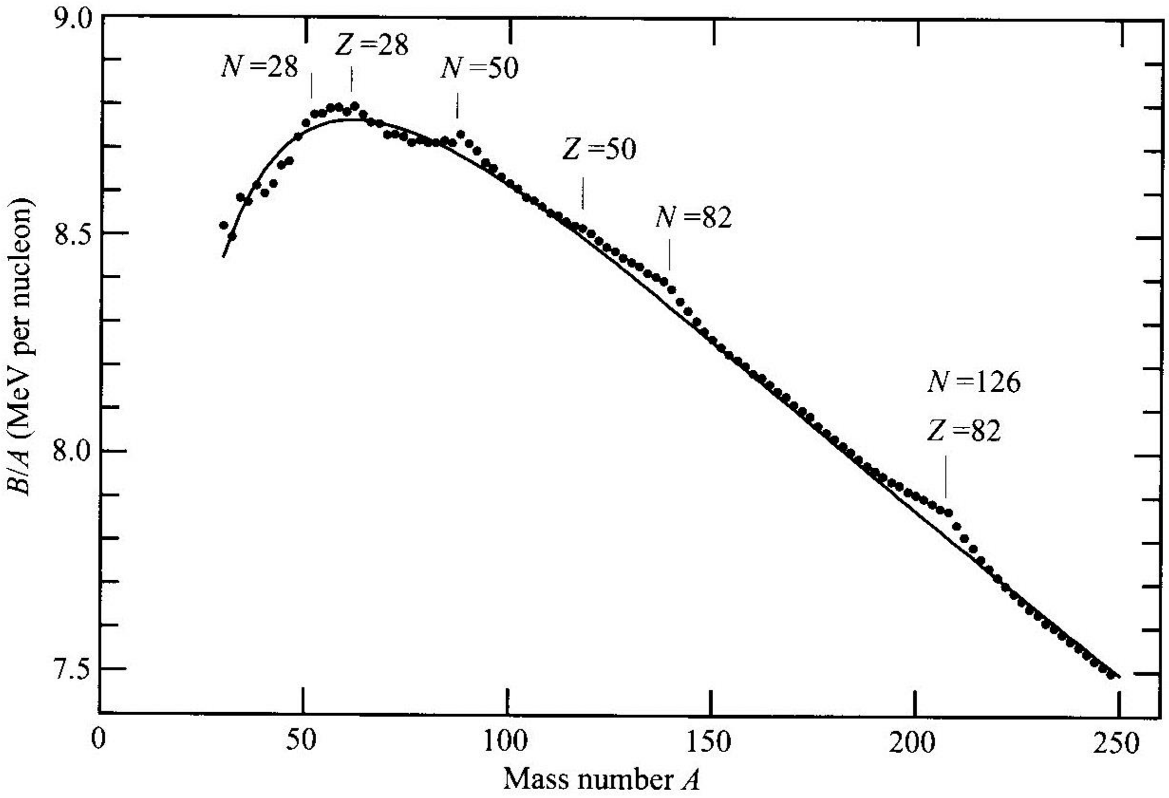
Extra binding when like nucleons form a spin-zero pair

Example:

	Binding energy (MeV)
${}^{210}_{82}\mathrm{Pb}_{128} = {}^{208}_{82}\mathrm{Pb}_{126} + 2n$	1646.6
${}^{210}_{83}\mathrm{Bi}_{127} = {}^{208}_{82}\mathrm{Pb}_{126} + n + p$	1644.8
${}^{209}_{82}\mathrm{Pb}_{127} = {}^{208}_{82}\mathrm{Pb}_{126} + n$	1640.4
${}^{209}_{83}\mathrm{Bi}_{126} = {}^{208}_{82}\mathrm{Pb}_{126} + p$	1640.2

$$\boxed{\begin{aligned} B_{\mathrm{pair}} &= \Delta && (\text{for even} - \text{even}) \\ &= 0 && (\text{for even} - \text{odd}) \\ &= -\Delta && (\text{for odd} - \text{odd}) \end{aligned}}$$

Shell Energy



Extra binding for N,Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

⇨ Very stable

$^{4}{}_{2}He_{2}, ^{16}{}_{8}O_{8}, ^{40}{}_{20}Ca_{20}, ^{48}{}_{20}Ca_{28}, ^{208}{}_{82}Pb_{126}$

✓ asymmetric fission

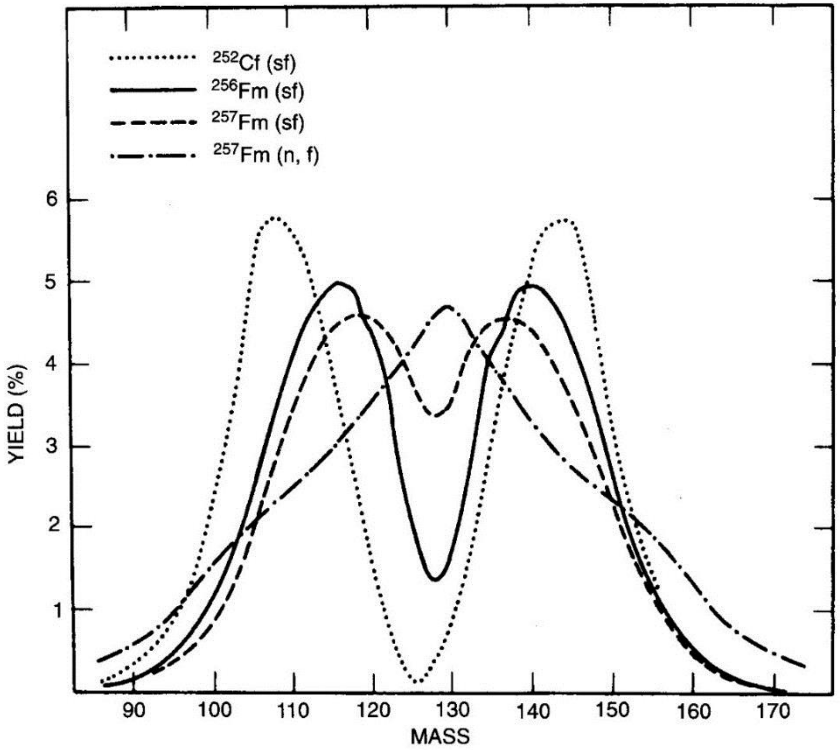


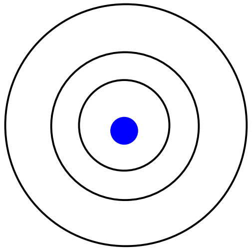
Fig. 4.1. Mass distributions in terms of the fission fragment masses for spontaneous fission of $^{252}_{98}$Cf, $^{256}_{100}$Fm and $^{257}_{100}$Fm and for neutron-induced fission of ^{257}Fm. Note the trend toward symmetric fission with increasing mass and in addition the larger number of symmetric events for neutron-induced than for spontaneous fission (from R. Vandenbosch and J.R. Huizenga, *Nuclear Fission* (Academic Press, New York and London, 1973)).

cf. $^{120}_{50}$Sn

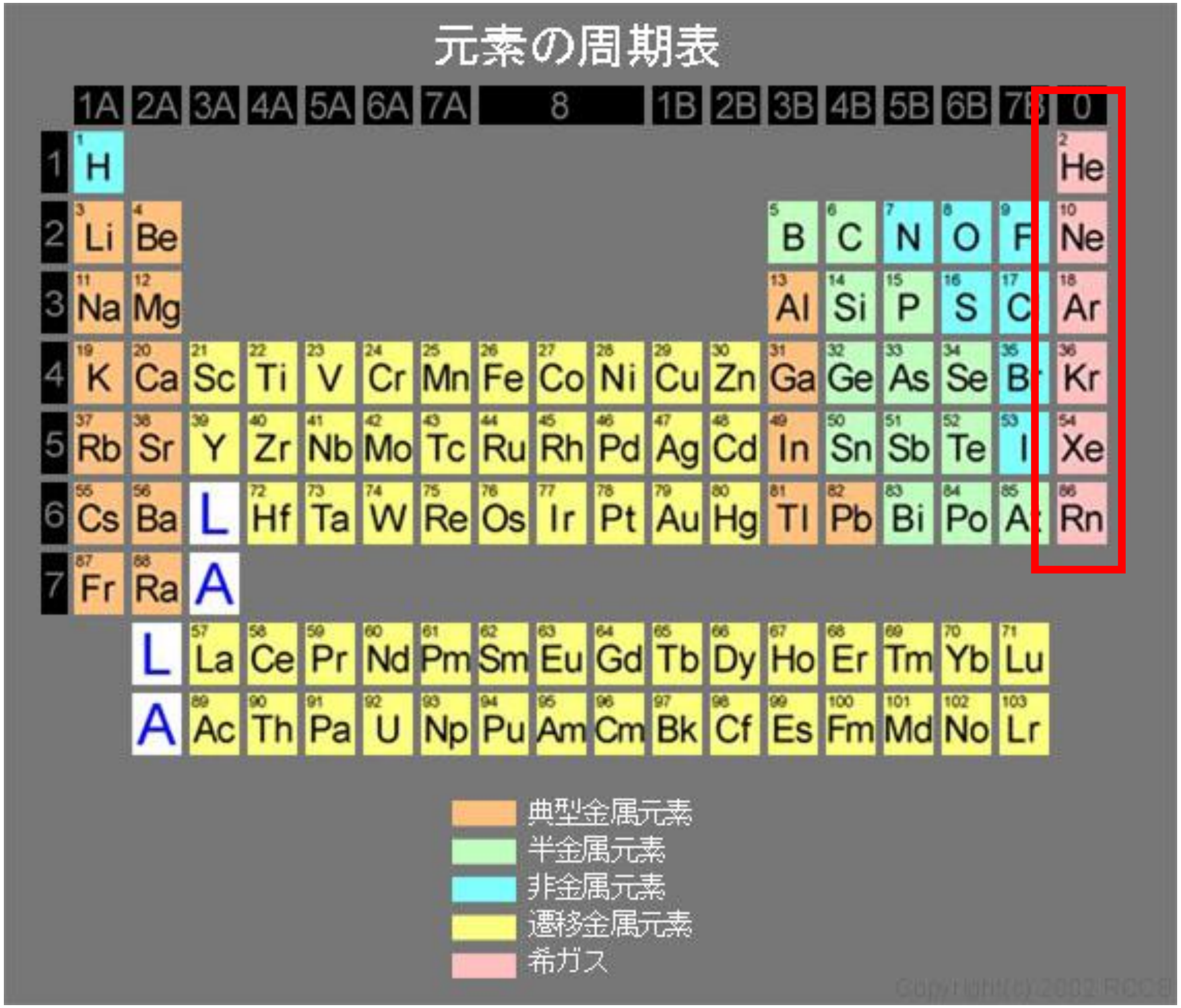
✓ stability of superheavy elements

(note) 原子の魔法数 (貴ガス)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)



殻構造



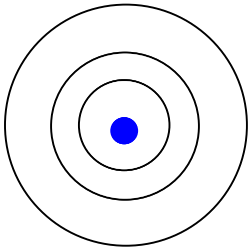
元素の周期表

	1A	2A	3A	4A	5A	6A	7A	8			1B	2B	3B	4B	5B	6B	7B	0
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	L	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	A															
		L	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
		A	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

🟧 典型金属元素
🟩 半金属元素
🟦 非金属元素
🟨 遷移金属元素
🟥 希ガス

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)

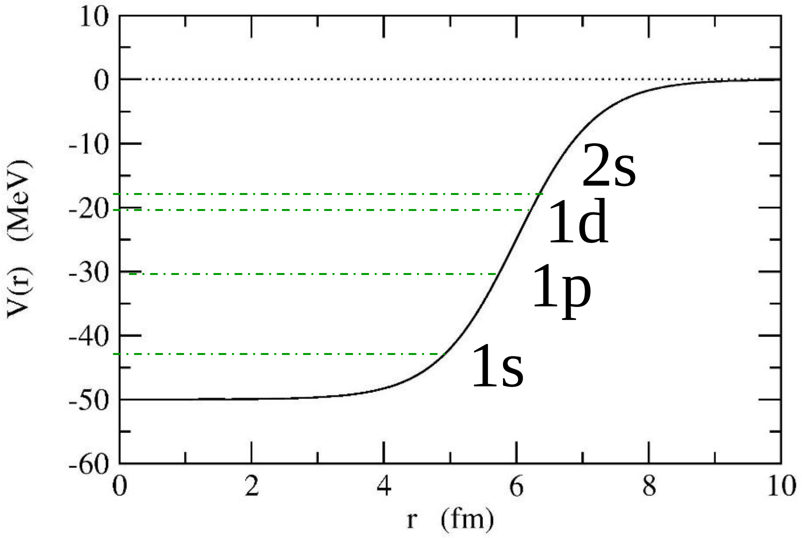


Shell structure

Similar attempt in nuclear physics: independent particle motion in a potential well

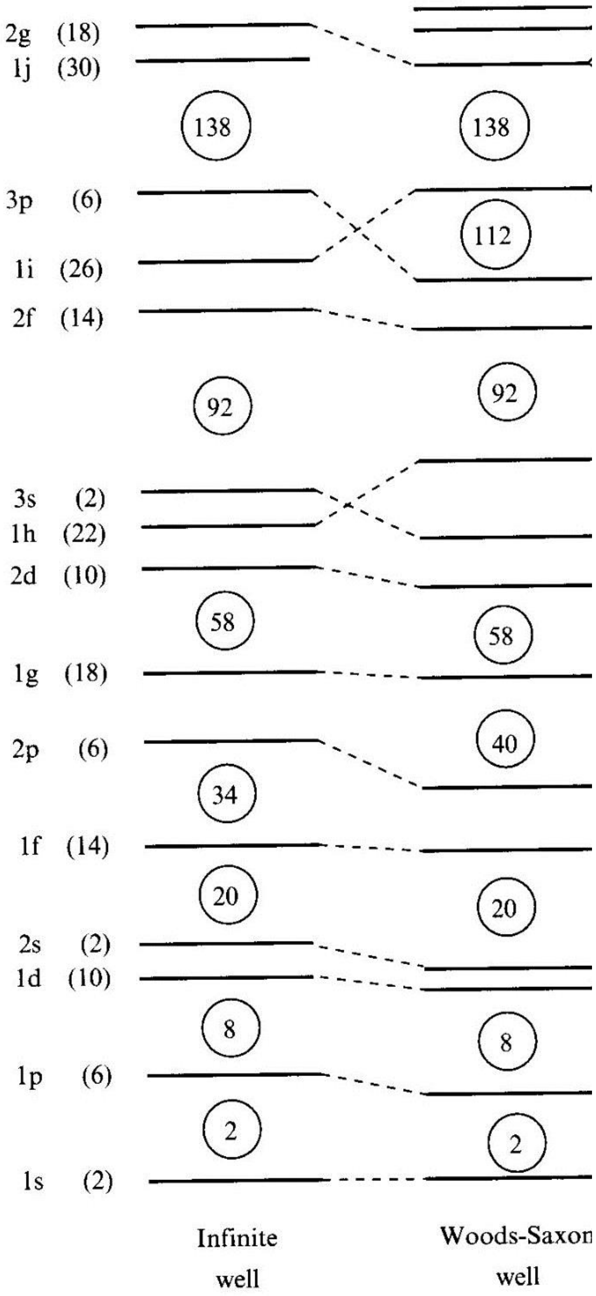
Woods-Saxon potential

$$V(r) = -V_0/[1+\exp((r-R_0)/a]$$



$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$\psi(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$



Woods-Saxon itself does not provide the correct magic numbers (2,8,20,28, 50,82,126).

Meyer and Jensen (1949):

Strong spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(r) = 0$$

$$V_{ls}(r) \sim -\lambda\frac{1}{r}\frac{dV}{dr} \qquad (\lambda \; > \; 0)$$

jj coupling shell model

$$\left[-\frac{\hbar^2}{2m}\nabla^2+V(r)-\epsilon\right]\psi(\boldsymbol{r})=0 \quad\Longrightarrow\quad \psi_{lmm_s}(\boldsymbol{r})=\frac{u_l(r)}{r}Y_{lm}(\hat{\boldsymbol{r}})\cdot\chi_{m_s}$$

Spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2+V(r)+V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s}-\epsilon\right]\psi(\boldsymbol{r})=0$$

(note) $\boldsymbol{j}=\boldsymbol{l}+\boldsymbol{s} \quad\Longrightarrow\quad \boldsymbol{l}\cdot\boldsymbol{s}=(\boldsymbol{j}^2-\boldsymbol{l}^2-\boldsymbol{s}^2)/2$

$$\boxed{\begin{aligned}\psi_{jlm}(\boldsymbol{r}) &= \frac{u_{jl}(r)}{r}\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}})\\ \mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) &= \sum_{m_l,m_s}\langle l\ m_l\ 1/2\ m_s|j\ m\rangle Y_{lm_l}(\hat{\boldsymbol{r}})\chi_{m_s}\end{aligned}}$$

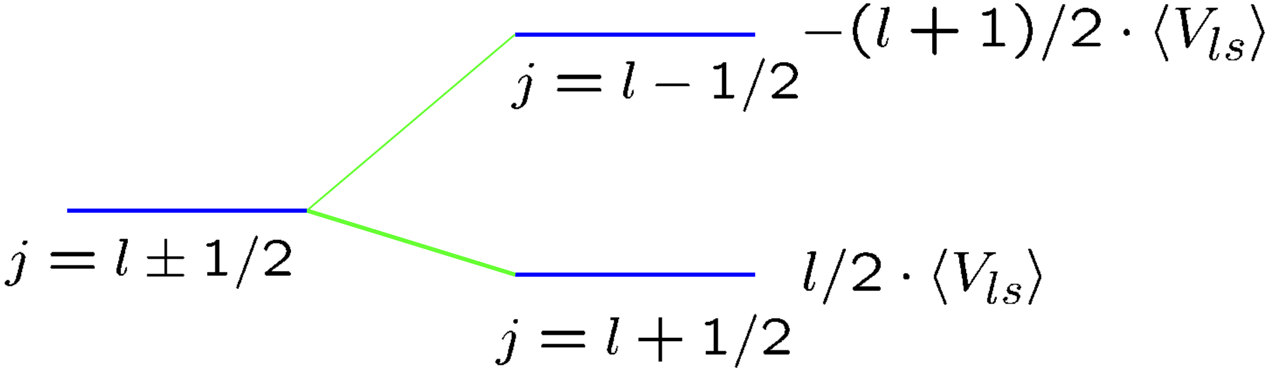
jj coupling shell model

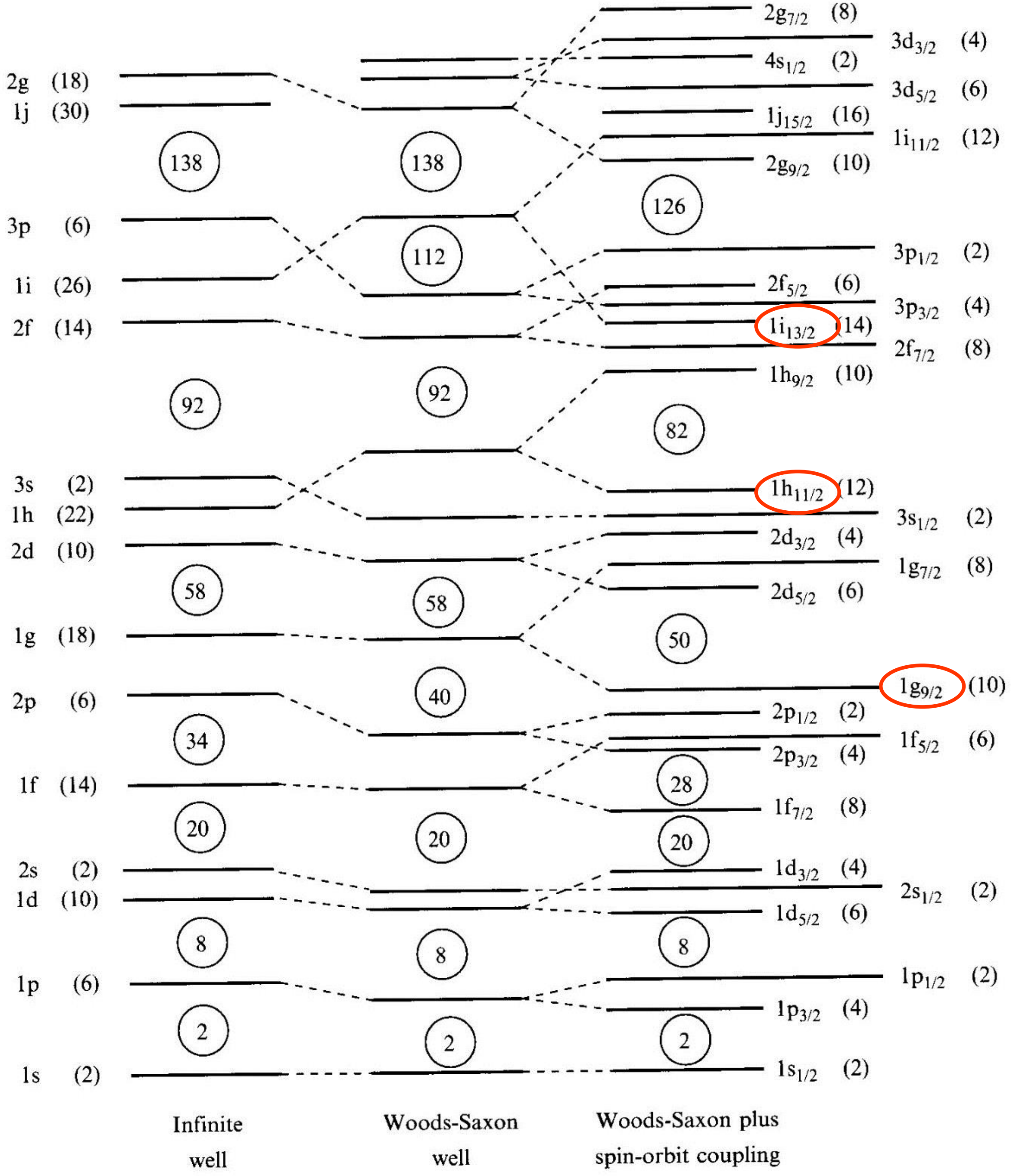
$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

(note) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} \quad \Longrightarrow \quad \boldsymbol{l}\cdot\boldsymbol{s} = (\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2)/2$

$$\begin{aligned} \psi_{jlm}(\boldsymbol{r}) &= \frac{u_{jl}(r)}{r}\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) \\ \mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) &= \sum_{m_l,m_s}\langle l\ m_l\ 1/2\ m_s | j\ m\rangle Y_{lm_l}(\hat{\boldsymbol{r}})\chi_{m_s} \end{aligned}$$

$$\boldsymbol{l}\cdot\boldsymbol{s} = l/2\ (j = l + 1/2), \quad -(l+1)/2\ (j = l - 1/2)$$





intruder states
unique parity states

Single particle spectra

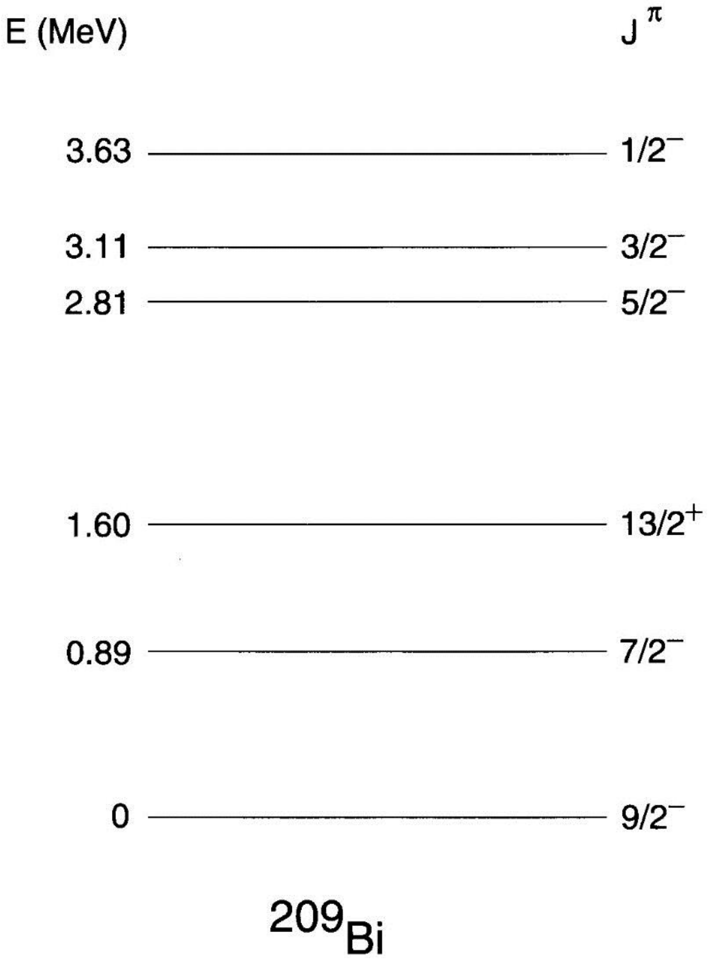
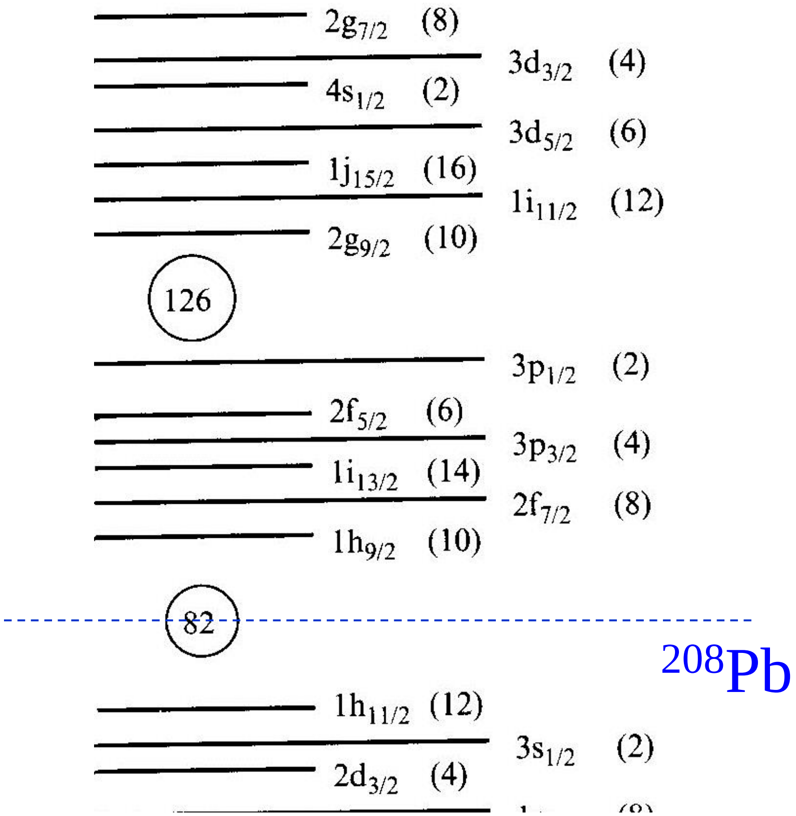


FIG. 3.6. Low-lying single-particle levels of ^{209}Bi.

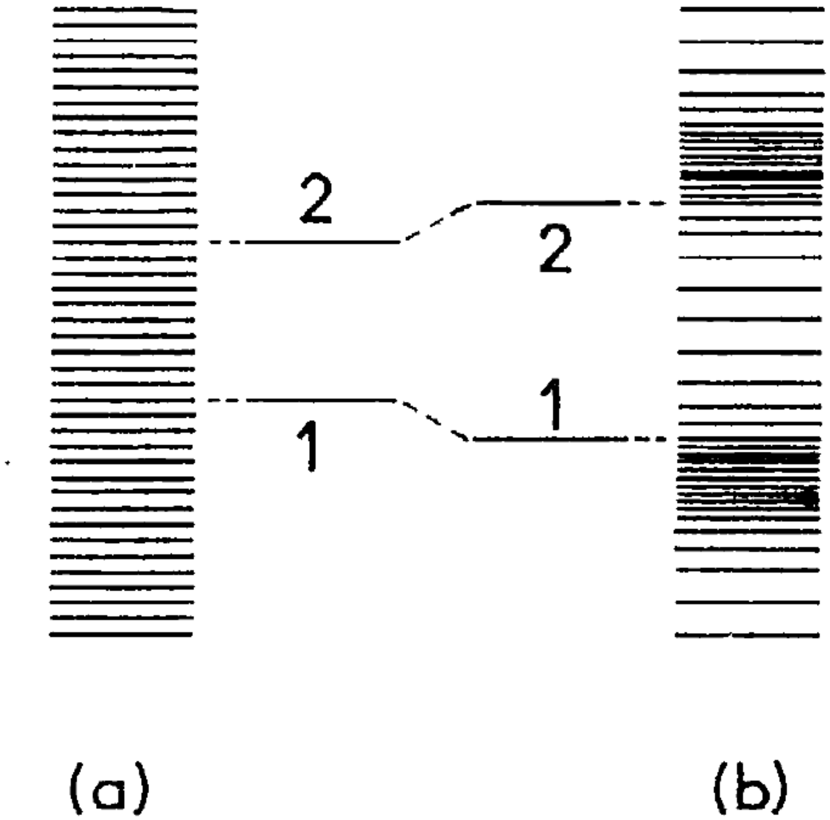


- How to construct V(r) microscopically?
- Does the independent particle picture really hold?

⇒ Later in this lecture

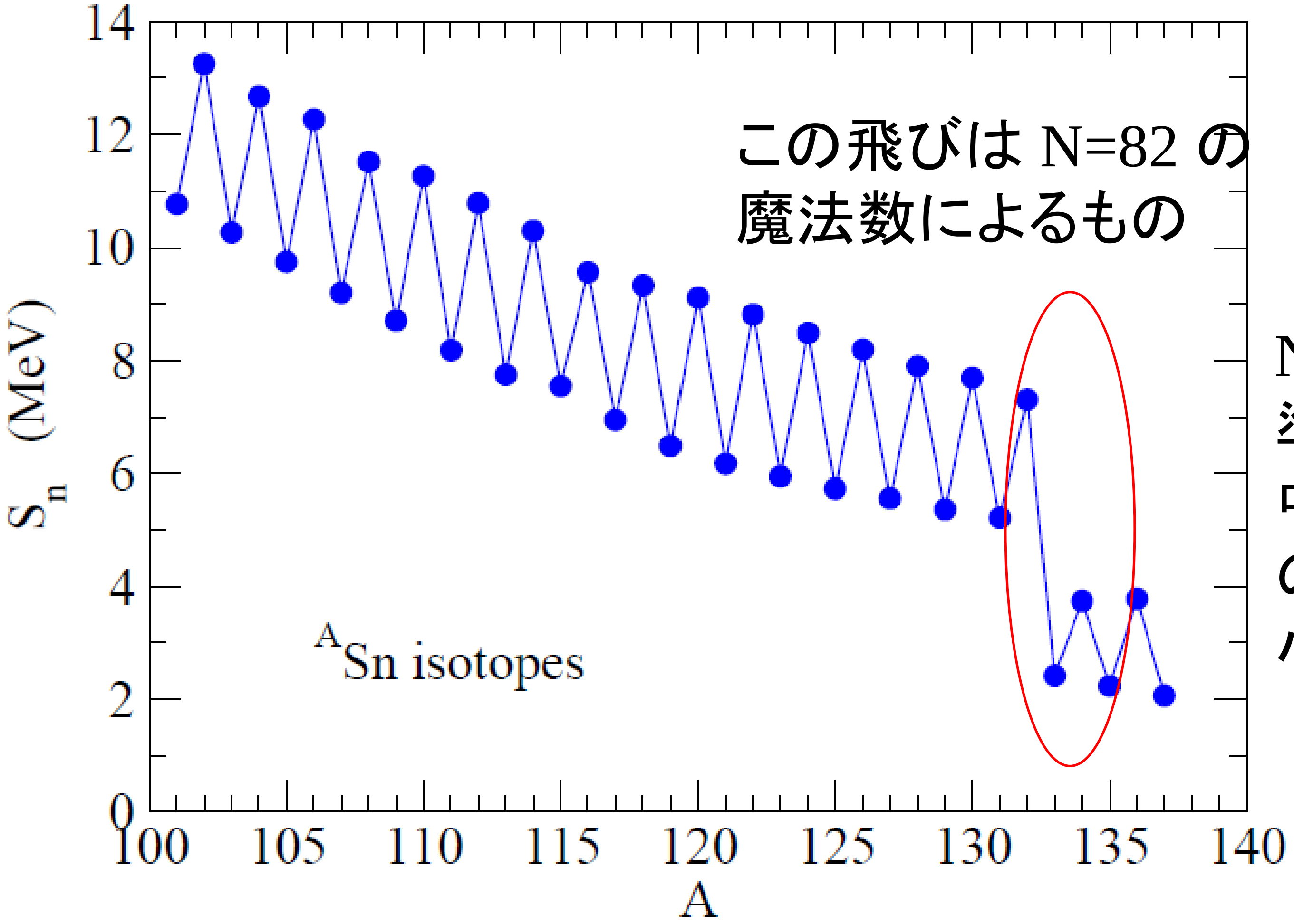
何故、閉殻の原子核は安定になるのか?

準位密度



均一の場合　　濃淡がある場合

準位密度に濃淡があれば、下から数えて濃淡の終わりまで準位がつまると(図の1の場合)、均一の場合に比べてエネルギーが小さい



この飛びは N=82 の
魔法数によるもの

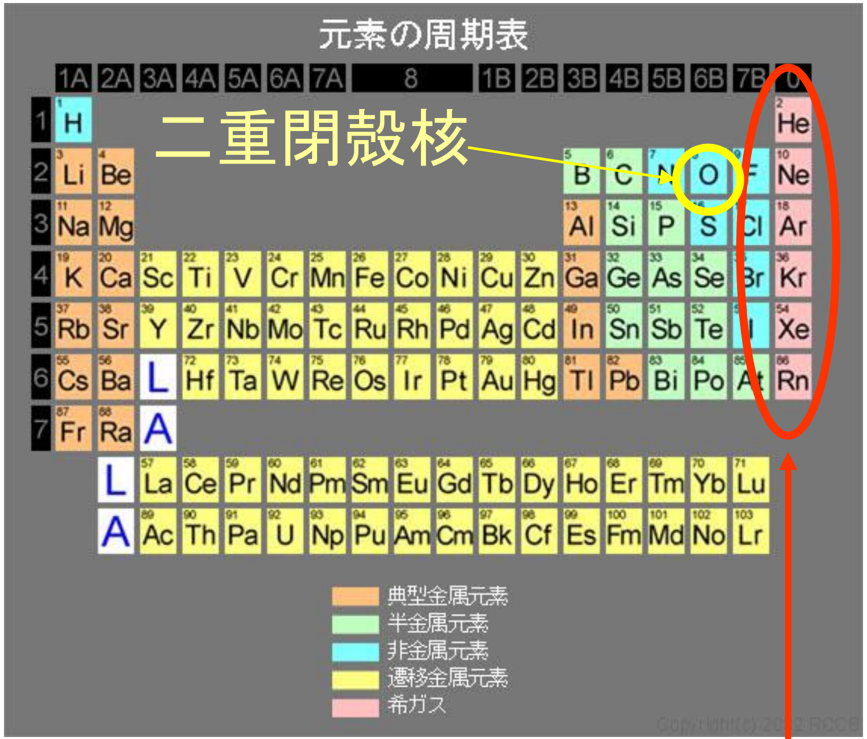
N=83から上の
準位がつまるため
中性子をとりのぞく
のにエネルギーが
小さくてすむ

1n separation energy: $S_n\ (A,Z) = B(A,Z) – B(A-1,Z)$

生命誕生のための幸運な偶然

原子の魔法数

電子の数が 2, 10, 18, 36, 54, 86



元素の周期表

	1A	2A	3A	4A	5A	6A	7A	8			1B	2B	3B	4B	5B	6B	7B	0
1	H																	He
2	Li	Be											B	C	N	O	F	Ne
3	Na	Mg											Al	Si	P	S	Cl	Ar
4	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
5	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
6	Cs	Ba	L	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
7	Fr	Ra	A															
		L	La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu	
		A	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

🟧 典型金属元素
🟩 半金属元素
🟦 非金属元素
🟨 遷移金属元素
🟥 希ガス

不活性ガス：He, Ne, Ar, Kr, Xe, Rn

原子核の魔法数

陽子または中性子の数が
2, 8, 20, 28, 50, 82, 126 の時安定

➡ 例えば $^{16}_{8}\mathrm{O}_{8}$（二重閉殻核）

➡ 酸素元素は元素合成の過程で数多く生成された

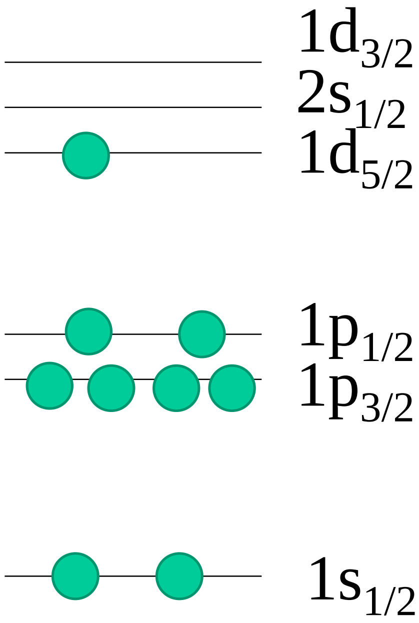
➡ しかし、酸素は化学的には「活性」

➡ 化学反応により様々な複雑な物質をつくり生命に至った

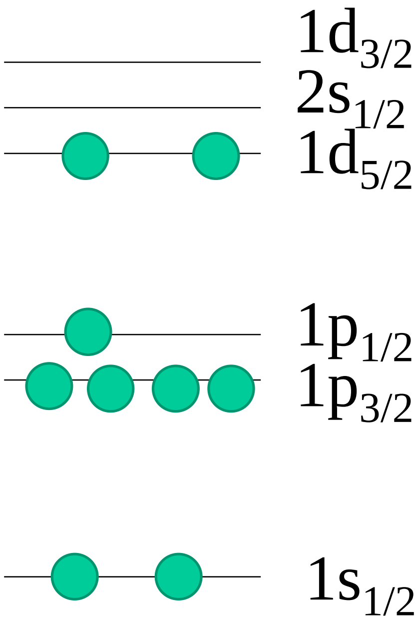
参考：望月優子　ビデオ「元素誕生の謎にせまる」　http://rarfaxp.riken.go.jp/~motizuki/contents/genso.html

single-j model

shell model



configuration 1



configuration 2

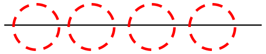
…… several others

angular momentum (spin) and parity for each configuration?

→ let us first investigate a single-j case single-j level: one level with an angular momentum j

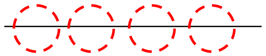
j

example: j = $p_{3/2}$



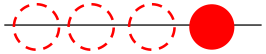
$p_{3/2}$

can accommodate 4 nucleons
(j_z= +3/2, +1/2, -1/2, -3/2)



$p_{3/2}$ can accommodate 4 nucleons (j_z= +3/2, +1/2, -1/2, -3/2)

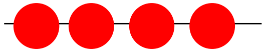
i) 1 nucleon



$p_{3/2}$ → $I^\pi = 3/2^-$

(there are 4 ways to occupy this level)

ii) 4 nucleons



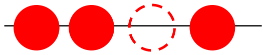
$p_{3/2}$ → $I^\pi = 0^+$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1

iii) 3 nucleons



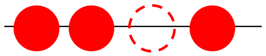
$p_{3/2}$ → $I^\pi = 3/2^-$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

iii) 3 nucleons



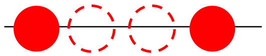
$p_{3/2}$ $\Rightarrow$ $I^{\pi} = 3/2^{-}$

$$I = j_1 + j_2 + j_3$$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

iv) 2 nucleons



$p_{3/2}$

$$I = j_1 + j_2$$

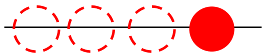
there are 4 x 3/2=6 ways to occupy this level with 2 nucleons.

$\Rightarrow$ $I^{\pi} = 0^{+}$ or 2^{+}

$$3/2 + 3/2 \longrightarrow I = 0, \cancel{1}, 2, \cancel{3}$$

anti-symmetrization

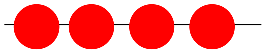
i) 1 nucleon



$p_{3/2}$ → $I^\pi = 3/2^-$

(there are 4 ways to occupy this level)

ii) 4 nucleons

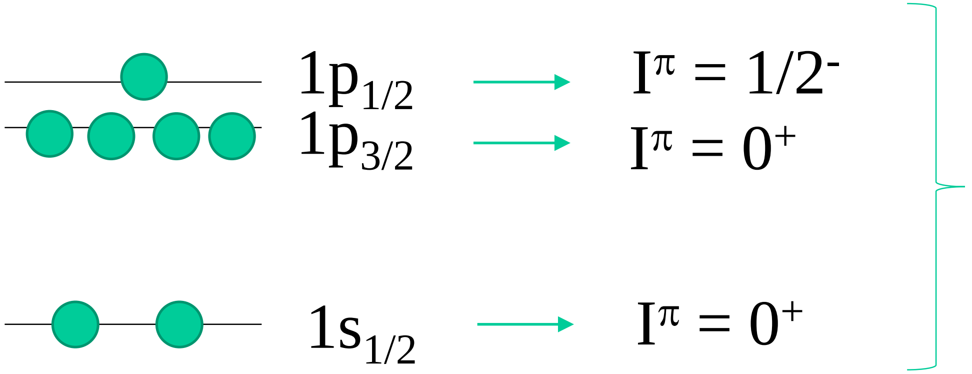


$p_{3/2}$ → $I^\pi = 0^+$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1



$1p_{1/2}$ → $I^\pi = 1/2^-$

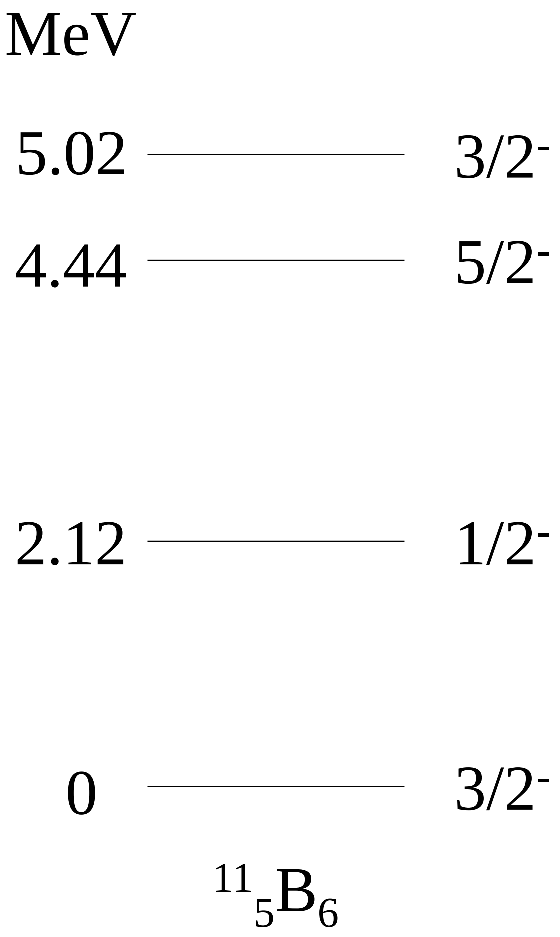
$1p_{3/2}$ → $I^\pi = 0^+$

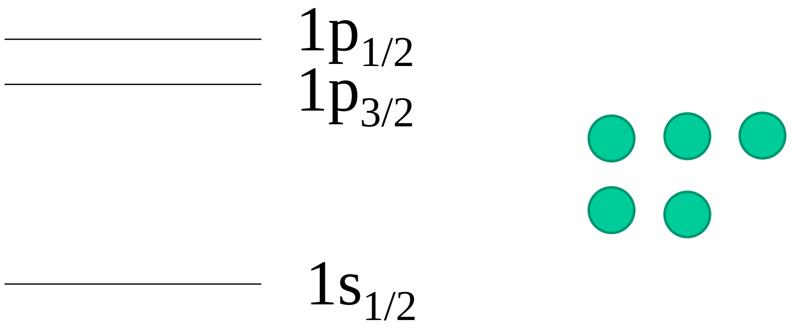
$1s_{1/2}$ → $I^\pi = 0^+$

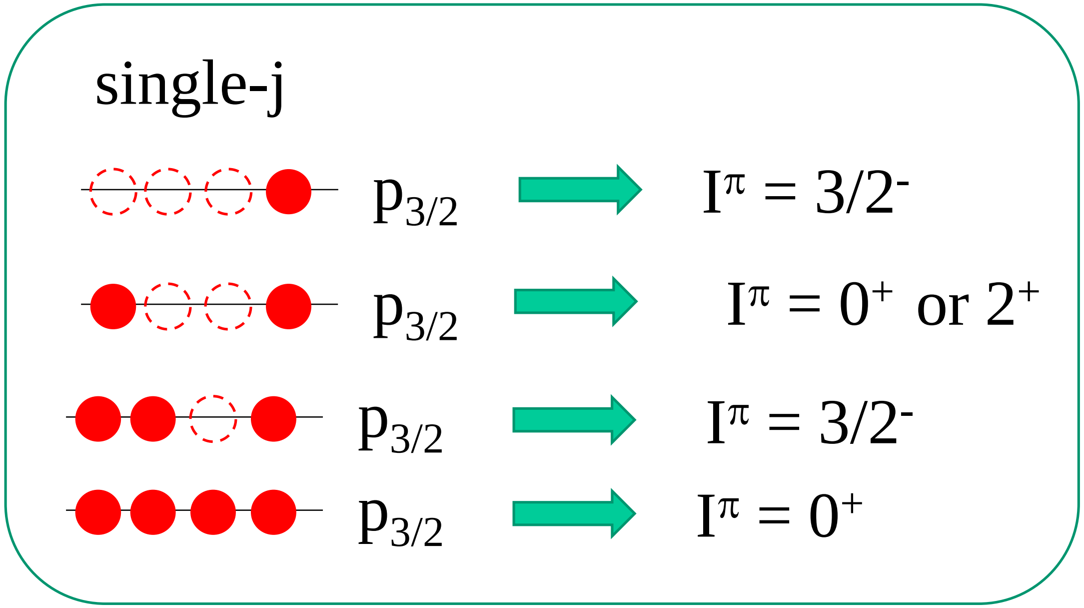
in total,
$I^\pi = 1/2^-$

example: (main) shell model configurations for ^{11}B

cf. ^{12}C(e,e'K$^+$)$^{12}{}_\Lambda$B (=^{11}B+Λ)






single-j

$p_{3/2}$ → $I^\pi = 3/2^-$

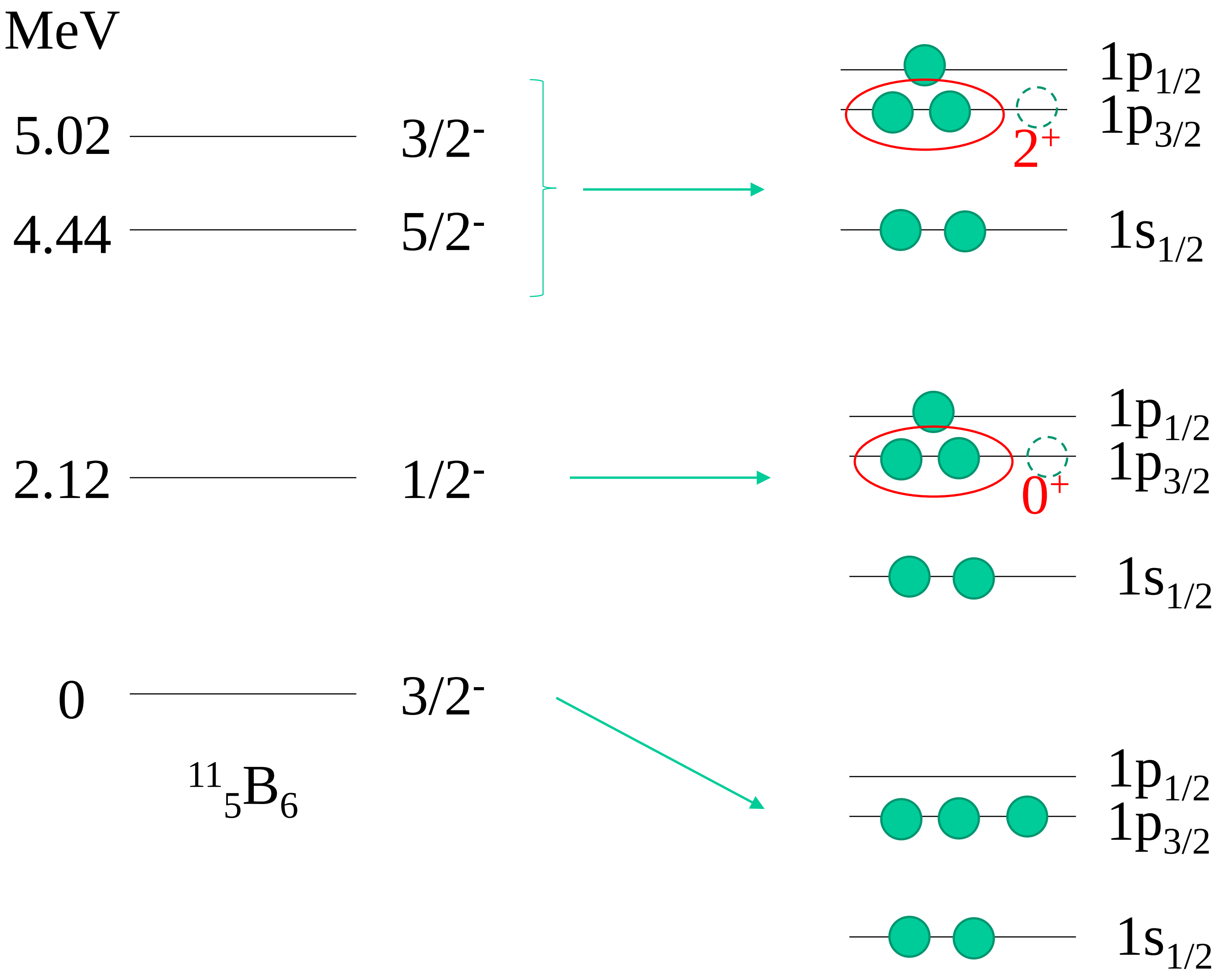
$p_{3/2}$ → $I^\pi = 0^+$ or 2^+

$p_{3/2}$ → $I^\pi = 3/2^-$

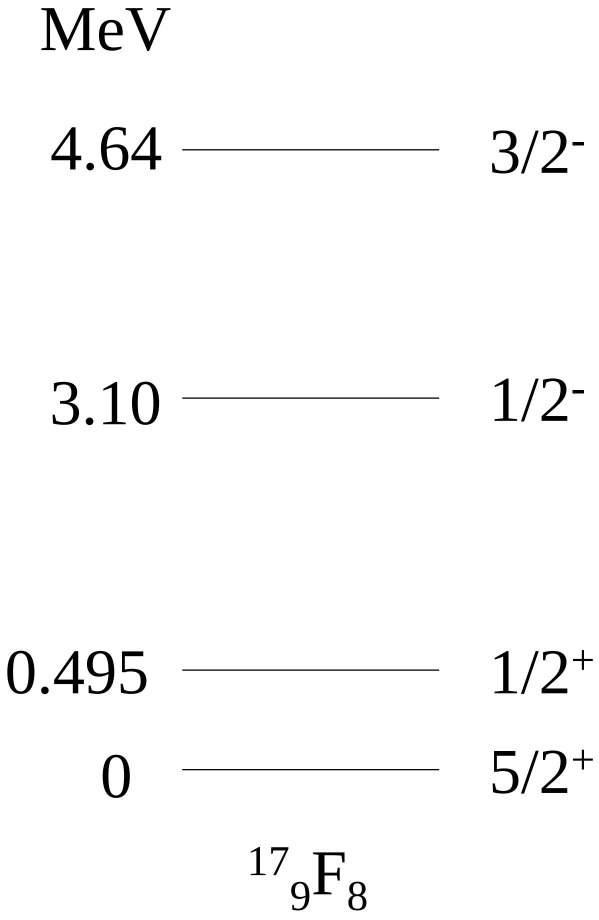
$p_{3/2}$ → $I^\pi = 0^+$

example: (main) shell model configurations for ^{11}B

cf. $^{12}\mathrm{C}(e,e'K^{+})^{12}{}_{\Lambda}\mathrm{B}$ $(={}^{11}\mathrm{B}+\Lambda)$



another example: (main) shell model configurations for ^{17}F



MeV

4.64	$3/2^-$
3.10	$1/2^-$
0.495	$1/2^+$
0	$5/2^+$

$^{17}_{9}\mathrm{F}_8$

another example: (main) shell model configurations for ^{17}F