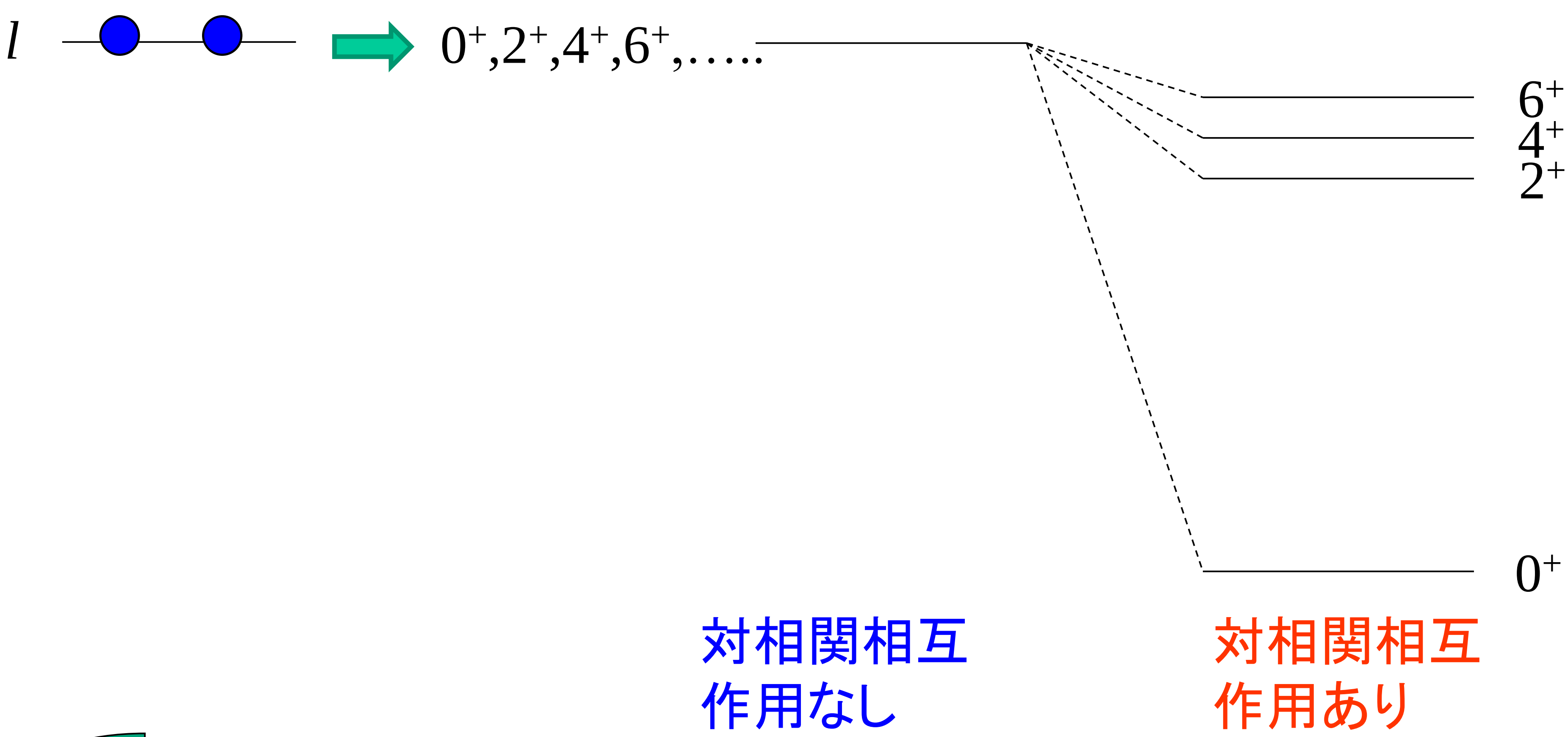
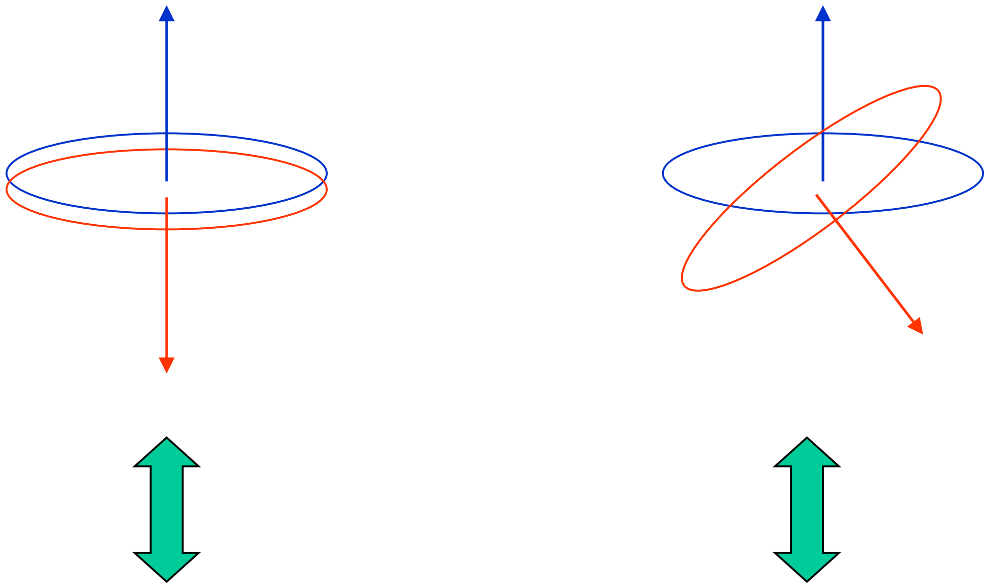
対相関(ペアリング)



原子核の基底状態のスピン

- 偶々核:例外なしに 0^+

簡単な解釈:

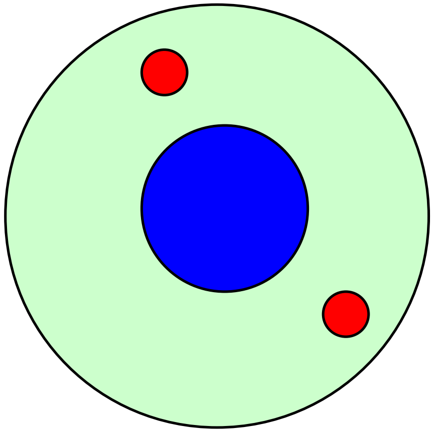


L=0 対 $\quad\quad\quad\quad$ $L \neq 0$ 対

L=0 対に対して空間的重なりが最大（エネルギー的に得）

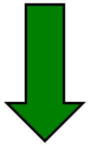
"対相関"

双中性子 (di-neutron) 相関



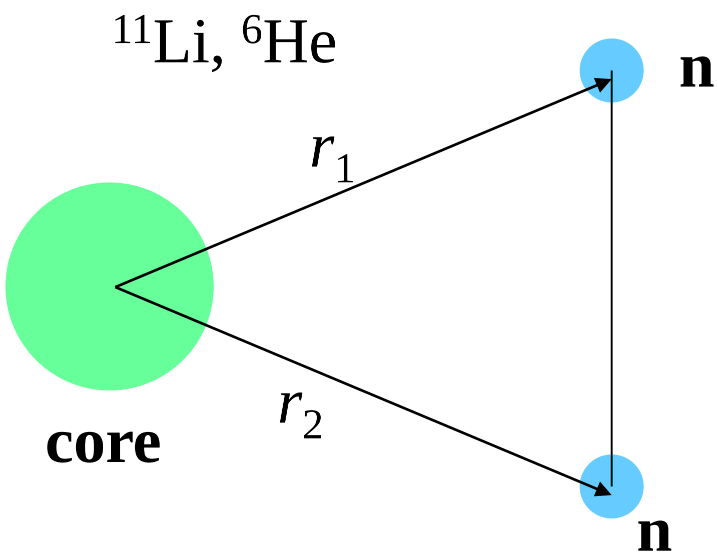
原子核中で2つの中性子は空間的に
どのように配置されているのか?

2つの中性子が独立に運動していると
すると、片方の中性子がどこにいようとも
もう片方は関知しない



対相関が働くとどうなるか?

3体模型計算: di-neutron 相関の微視的理解



$$H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn}(\boldsymbol{r}_1, \boldsymbol{r}_2) + \frac{(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2}{2A_c m}$$

（最後の項は3体系の静止系で考えた芯原子核の運動エネルギー項。）

⇨ この3体ハミルトニアンの基底状態を求め、密度分布を調べる:

（例えば）$V_{\rm nn}$ がないときの状態で展開し、展開係数を求める

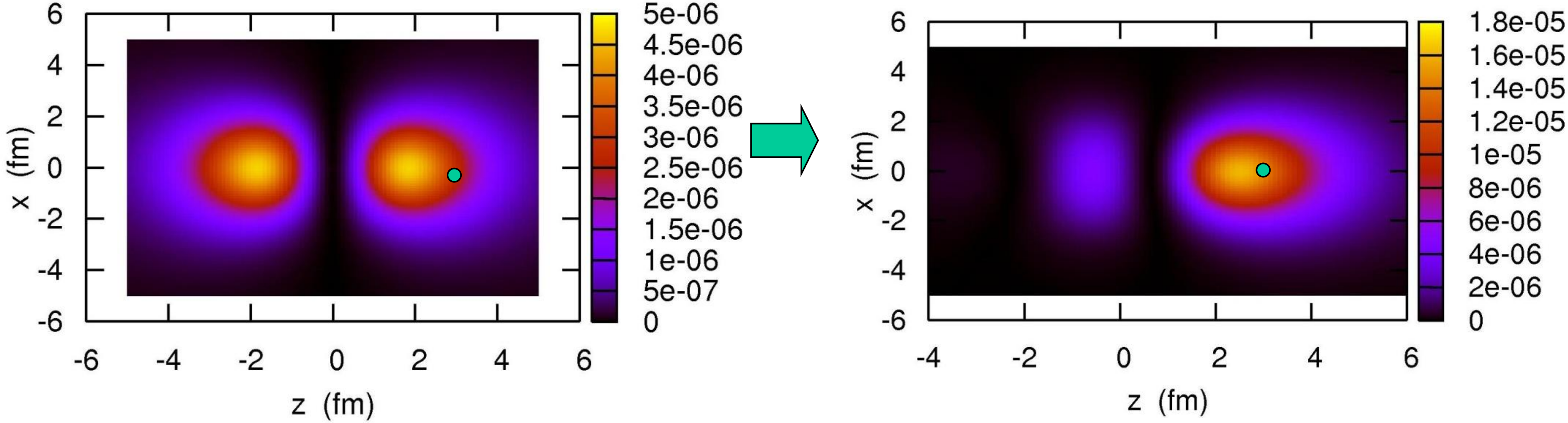
$$\Psi_{gs}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \mathcal{A} \sum_{nn'lj} \alpha_{nn'lj} \Psi^{(2)}_{nn'lj}(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

$$\Psi^{(2)}_{nn'lj}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \sum_m \langle jmj-m|00\rangle \psi_{nljm}(\boldsymbol{r}_1)\psi_{n'lj-m}(\boldsymbol{r}_2)$$

対相関力がある場合とない場合の比較

^{11}Li　1つの中性子を (z_1, x_1)=(3.4 fm, 0)に置いたときのもう一つの中性子の分布

対相関がない場合 $[1p_{1/2}]^2$　　　対相関がある場合



- 対相関がないと、z と $-z$ で対称的な分布。片方の中性子がどこにいても分布は変わらない。
- 対相関があると、2つの中性子は近くにいる。1つの中性子の場所が変わると、もう1つも変わる。

What is Di-neutron correlation?

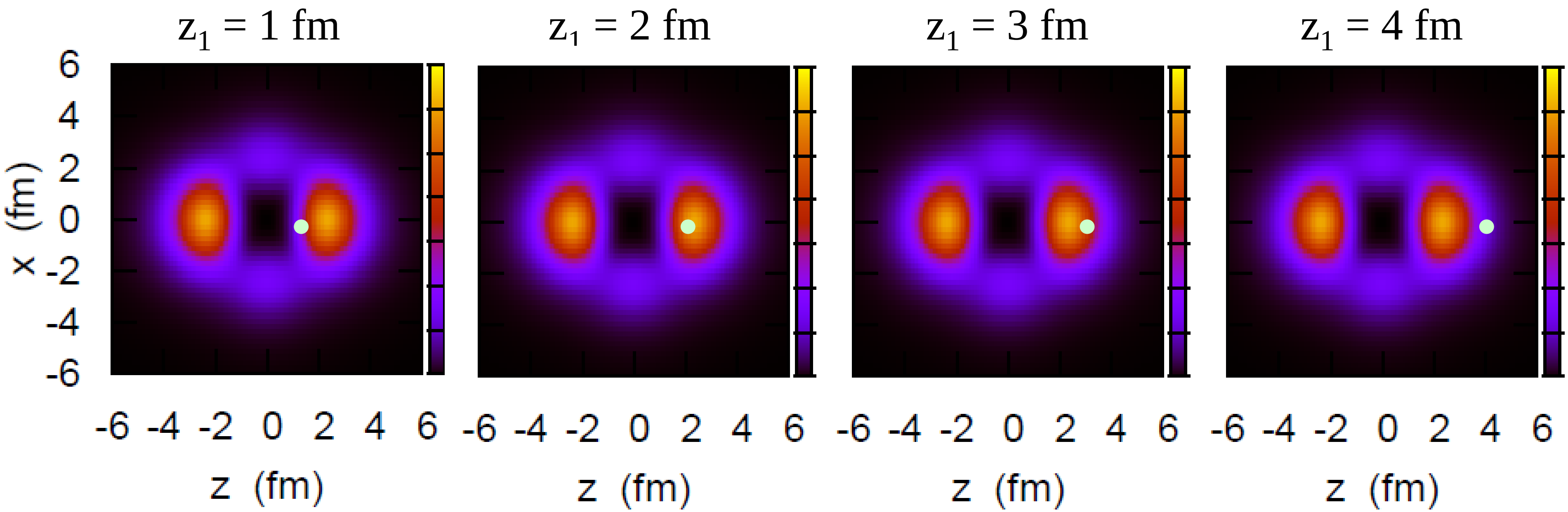
Correlation: $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

Example: ^{18}O = ^{16}O + n + n

cf. ^{16}O + n : 3 bound states ($1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$)

i) Without nn interaction: $|nn\rangle = |(1d_{5/2})^2\rangle$

Distribution of the 2nd neutron when the 1st neutron is at z_1 :



✓Two neutrons move independently

✓No influence of the 2nd neutron from the 1st neutron

$$\langle AB \rangle = \langle A \rangle \langle B \rangle$$

What is Di-neutron correlation?

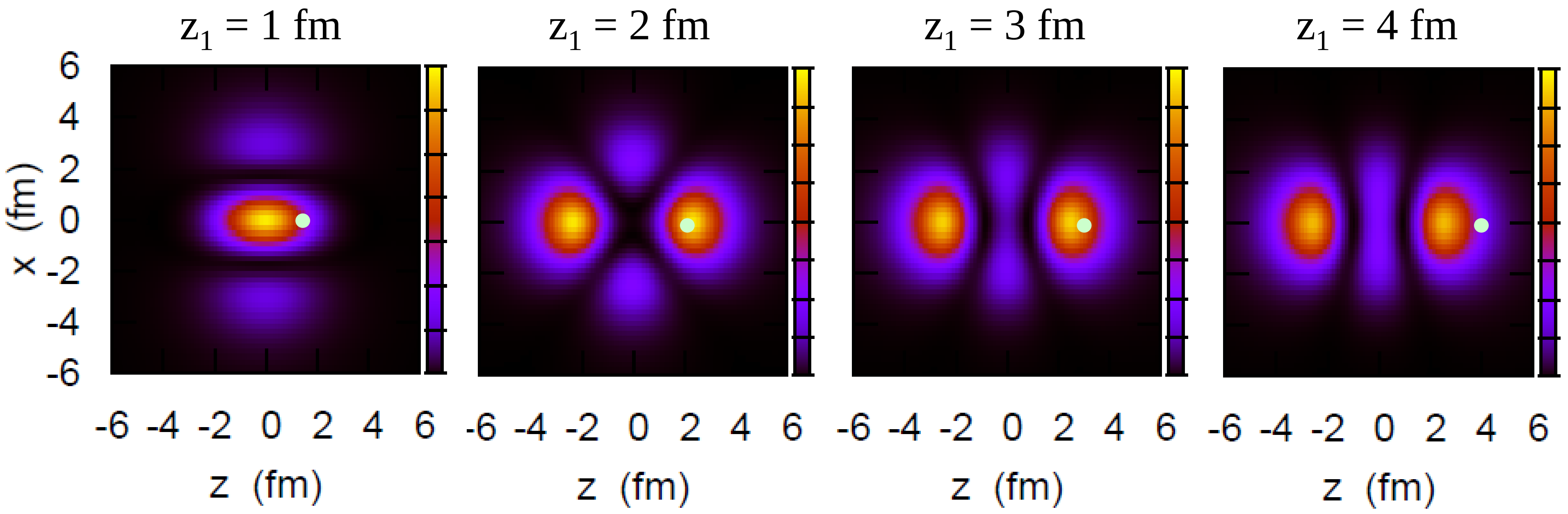
Correlation : $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

Example: $^{18}O = {}^{16}O + n + n$

cf. $^{16}O + n$: 3 bound states ($1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$)

ii) nn interaction: works only on the positive parity (bound) states

$$|nn\rangle = \alpha|(1d_{5/2})^2\rangle + \beta|(2s_{1/2})^2\rangle + \gamma|(1d_{3/2})^2\rangle$$



✓distribution changes according to the 1st neutron (nn correlation)

✓but, the distribution of the 2nd neutron has peaks both at z_1 and $-z_1$

⟶ this is NOT called the di-neutron correlation

What is Di-neutron correlation?

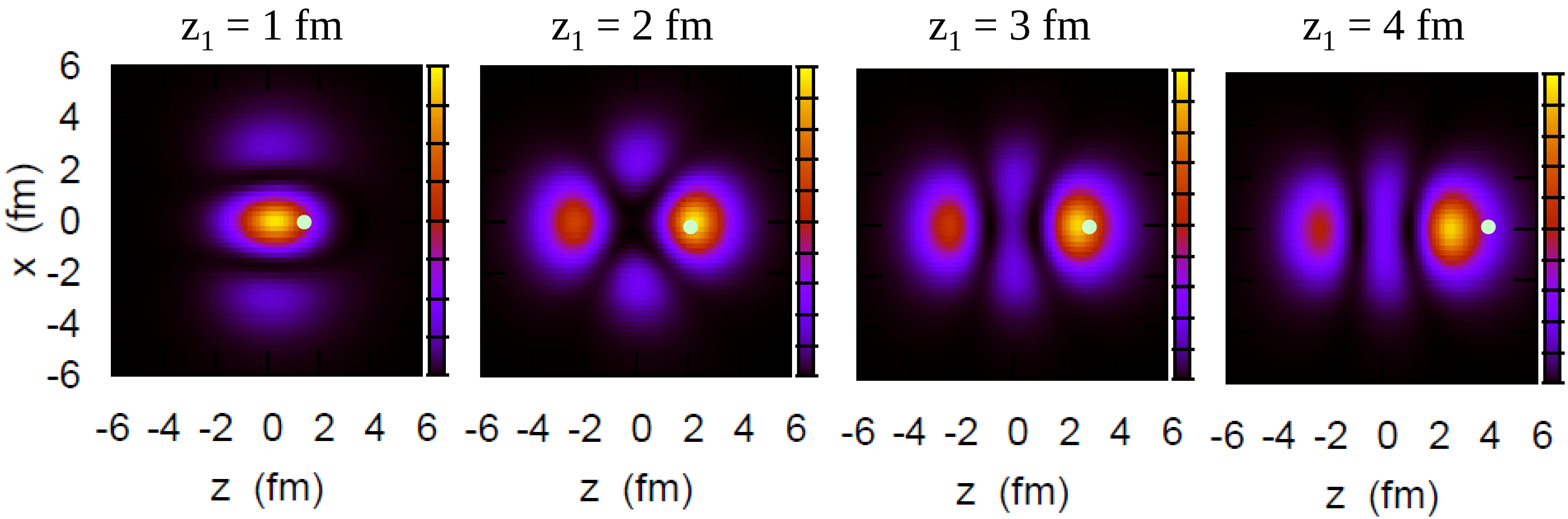
Correlation: $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

Example: ^{18}O = ^{16}O + n + n

cf. ^{16}O + n : 3 bound states ($1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$)

iii) nn interaction: works also on the continuum states

$$|nn\rangle = \sum_{n,n',j,l} C_{nn'jl} |(nn'jl)^2\rangle$$

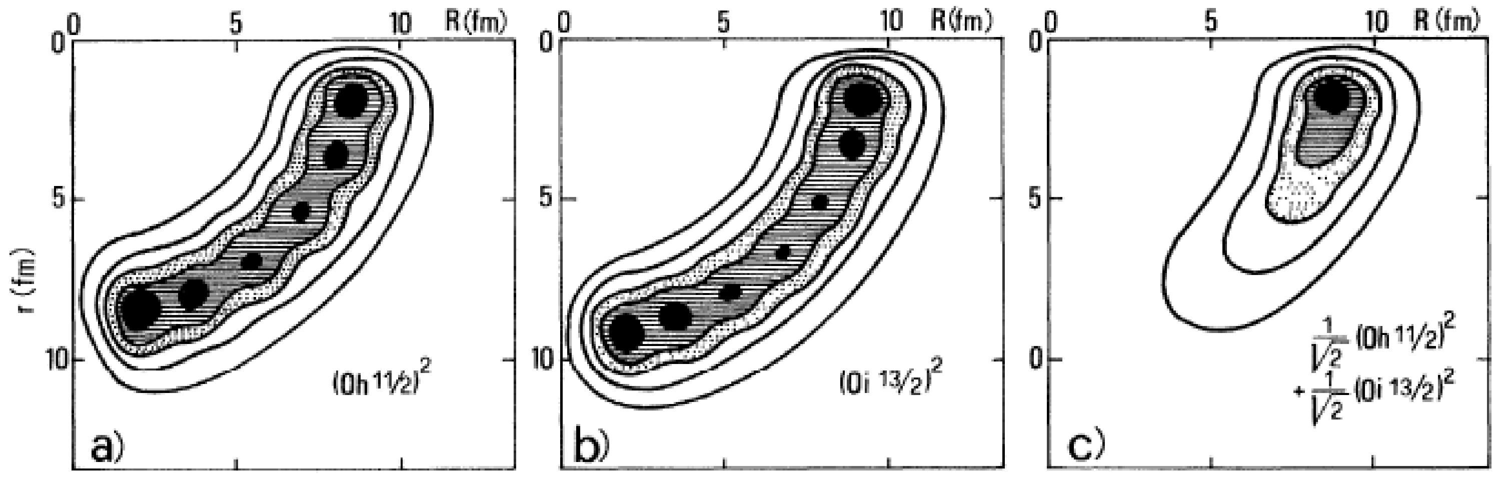


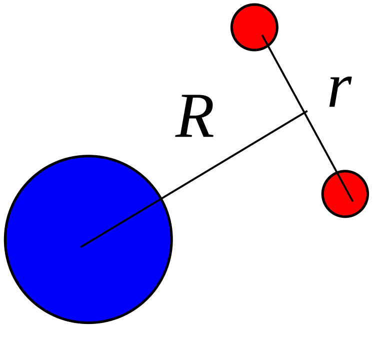
✓spatial correlation: the density of the 2nd neutron localized close to the 1st neutron (dineutron correlation)

✓parity mixing: essential role

cf. F. Catara et al., PRC29('84)1091

dineutron correlation: caused by the admixture of different parity states





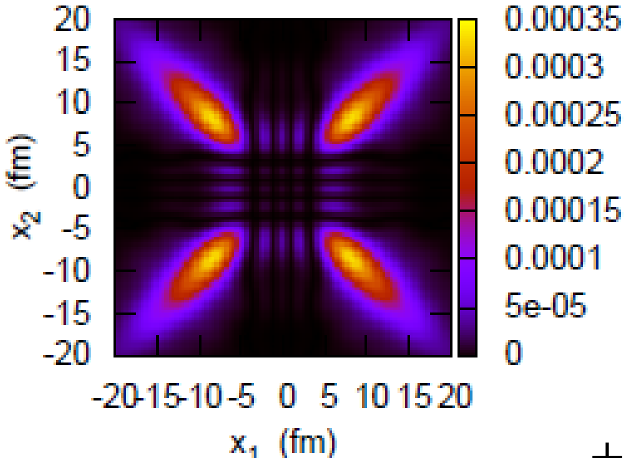
F. Catara, A. Insolia, E. Maglione, and A. Vitturi, PRC29('84)1091

何故、異なるパリティが混ざると dineutron 相関が生じるのか?

$$\Psi_{gs}(x_1,x_2) = \Psi_{ee}(x_1,x_2) + \Psi_{oo}(x_1,x_2)$$

$$\longrightarrow \rho_2(x_1,x_2) = |\Psi_{ee}(x_1,x_2)|^2 + |\Psi_{oo}(x_1,x_2)|^2 + 2\Psi_{ee}(x_1,x_2)\Psi_{oo}(x_1,x_2)$$

$|\Psi_{ee}(x_1,x_2)|^2$

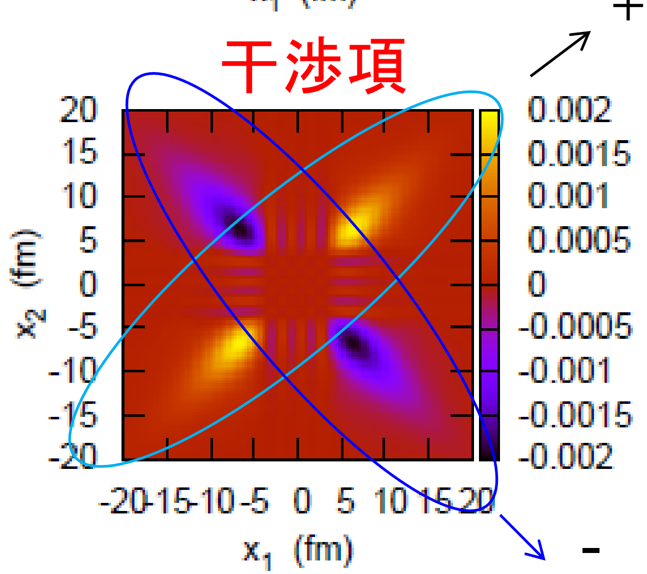


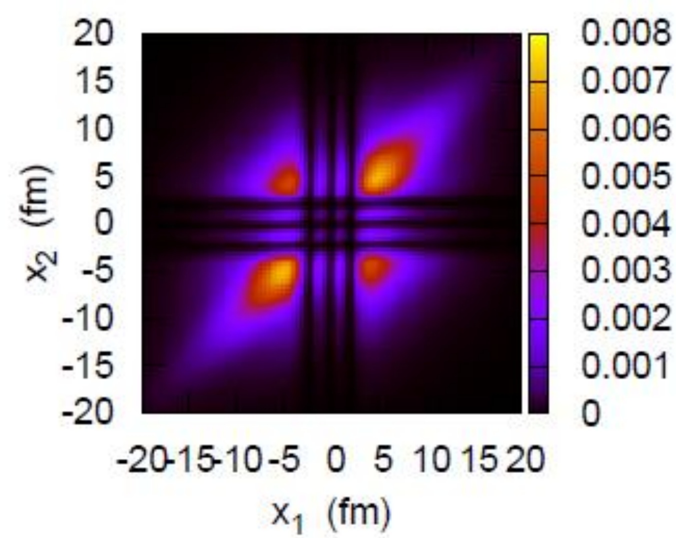
$$\Psi_{ee}(-x_1,x_2) = \Psi_{ee}(x_1,x_2)$$

$$\Psi_{oo}(-x_1,x_2) = -\Psi_{oo}(x_1,x_2)$$

$$\rho_2(-x_1,x_2) = |\Psi_{ee}(x_1,x_2)|^2 + |\Psi_{oo}(x_1,x_2)|^2 - 2\Psi_{ee}(x_1,x_2)\Psi_{oo}(x_1,x_2)$$

干渉項



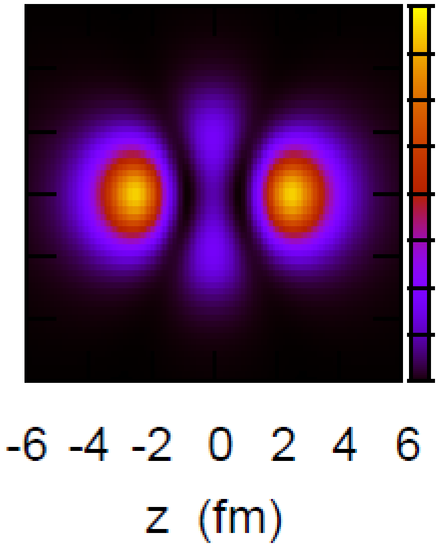


spatial localization of two neutrons
(dineutron correlation)

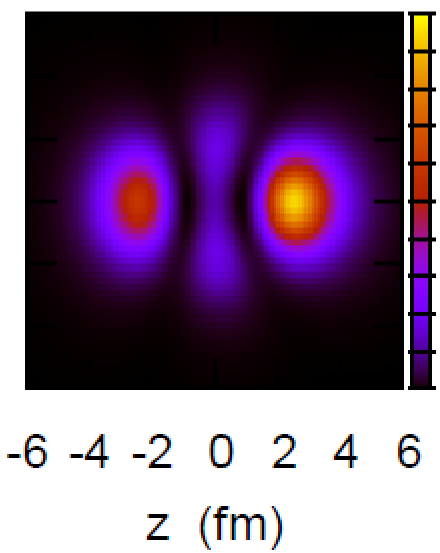
cf. Migdal, Soviet J. of Nucl. Phys. 16 ('73) 238
Bertsch, Broglia, Riedel, NPA91('67)123

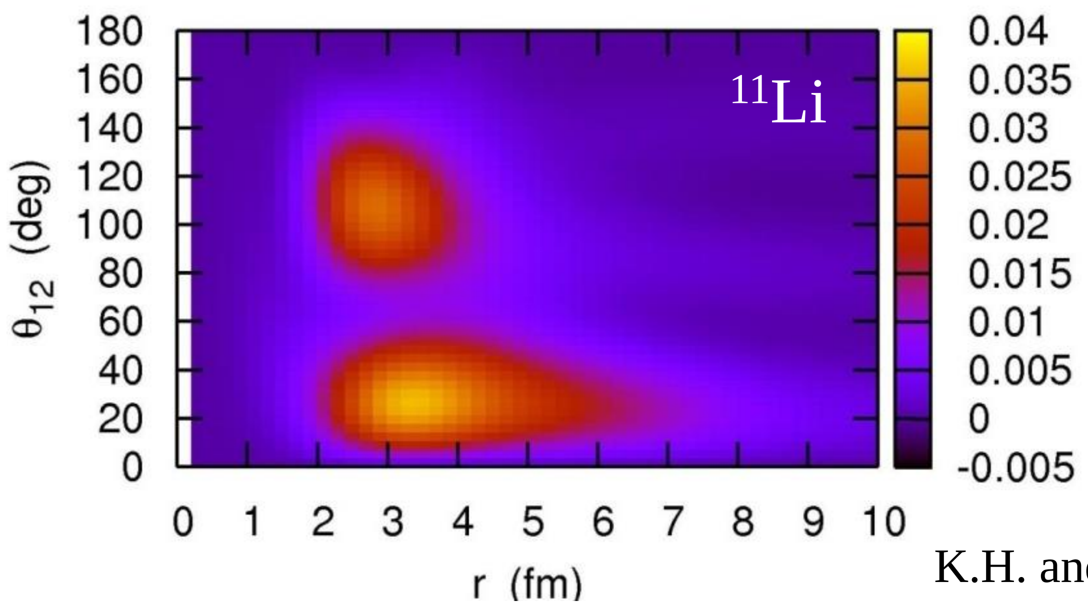
weakly bound systems

→easy to mix different parity states due to the continuum couplings
+ enhancement of pairing on the surface



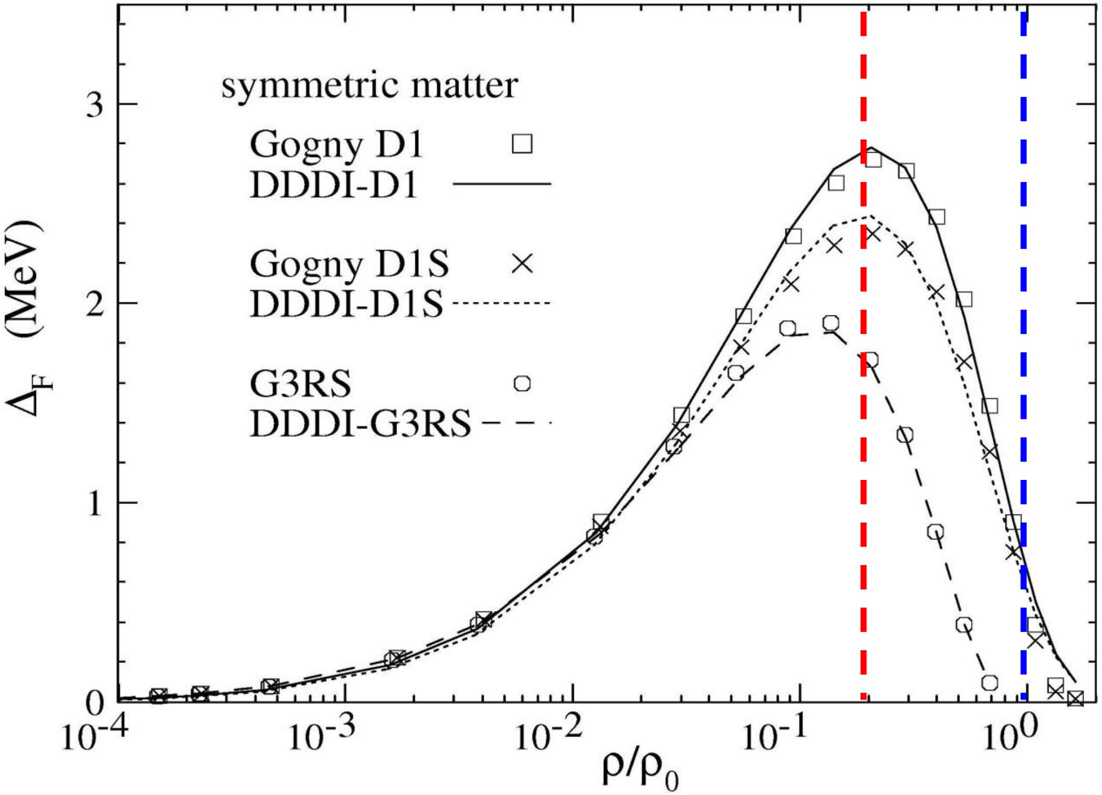
parity mixing





K.H. and H. Sagawa,
PRC72('05)044321

pairing gap in infinite nuclear matter

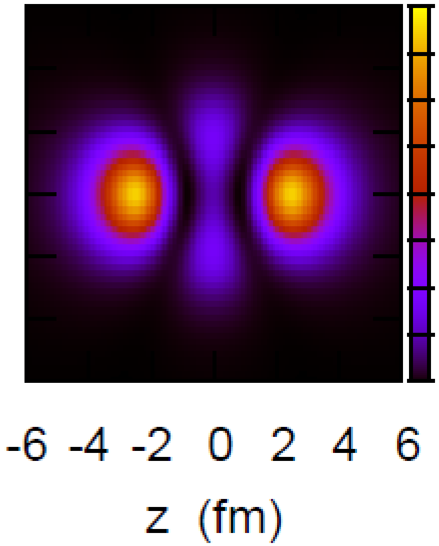


M. Matsuo, PRC73('06)044309

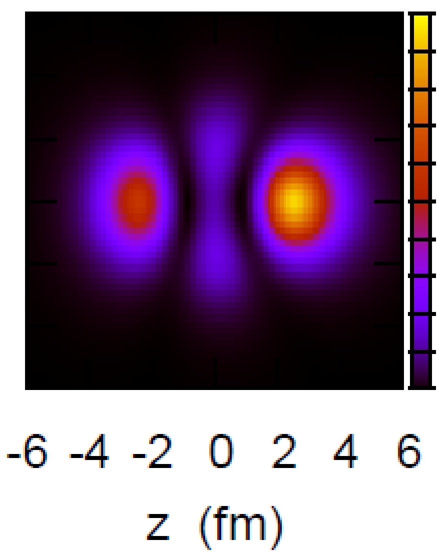
spatial localization of two neutrons

(dineutron correlation)

cf. Migdal, Soviet J. of Nucl. Phys. 16 ('73) 238
Bertsch, Broglia, Riedel, NPA91('67)123



parity mixing



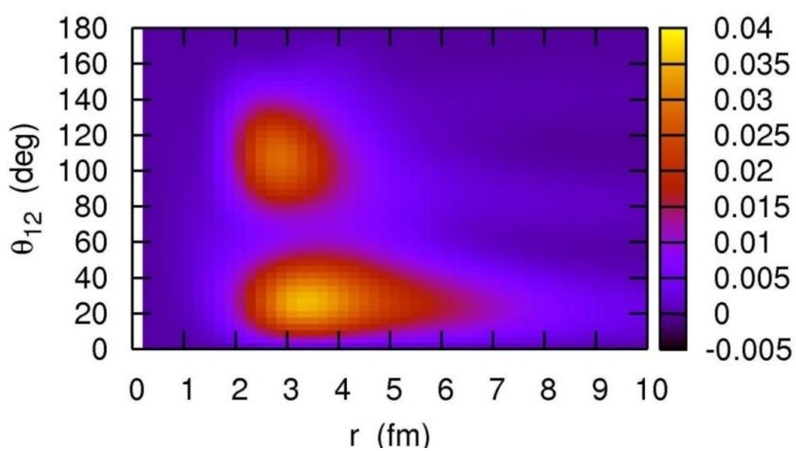
weakly bound systems

→easy to mix different parity states due to the continuum couplings

+ enhancement of pairing on the surface

⟶ dineutron correlation: enhanced

cf. - Bertsch, Esbensen, Ann. of Phys. 209('91)327
- M. Matsuo, K. Mizuyama, Y. Serizawa, PRC71('05)064326



K.H. and H. Sagawa, PRC72('05)044321

The BCS theory

Many-particles in non-degenerate levels
~ mean-field approx. for the pairing channel ~

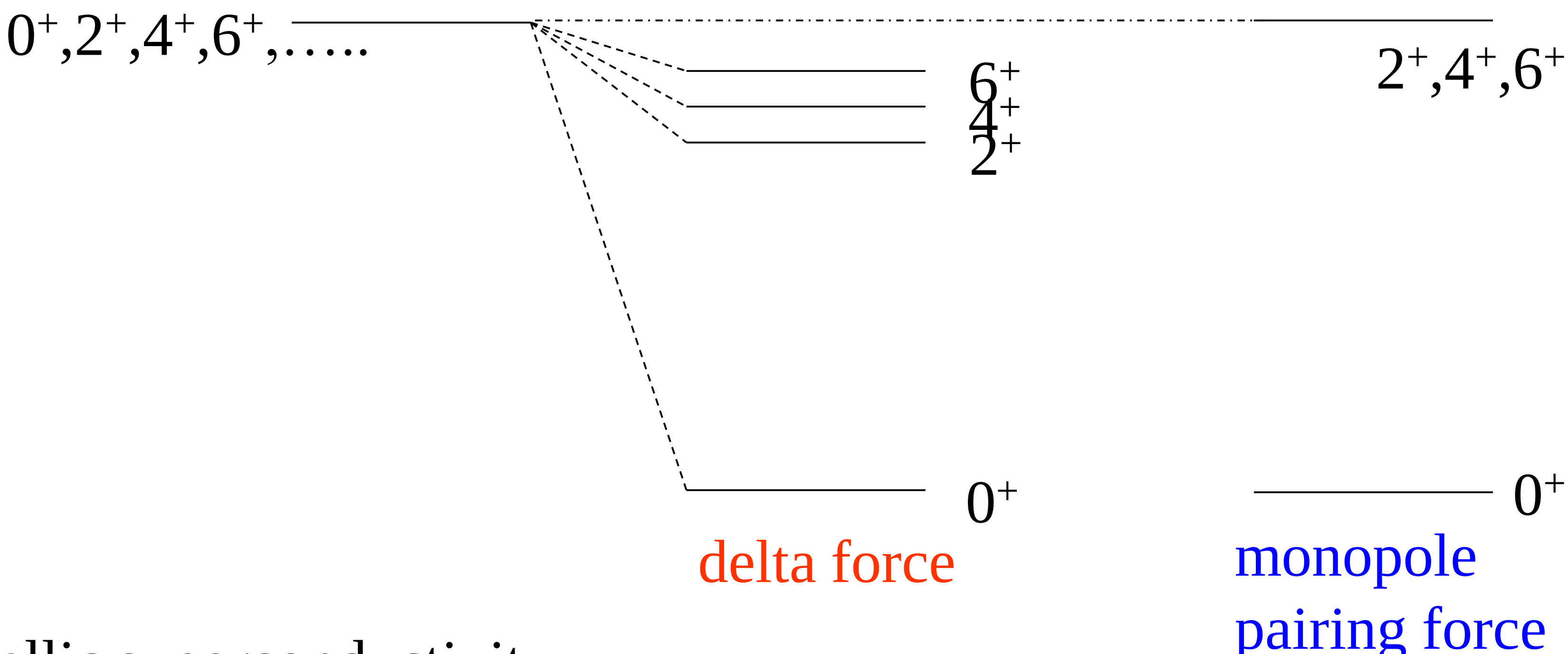
Simplified pairing interaction

$$V = -G\,P^\dagger P; \qquad P^\dagger = \sum_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger$$

$\bar{\nu}$: the time reversed state of ν

e.g.,

$$|\nu\rangle = |njlm\rangle, \quad |\bar{\nu}\rangle = |njl-m\rangle$$



Cf. Metallic superconductivity

Solve the pairing Hamiltonian

$$H = \sum_\nu \epsilon_\nu (a_\nu^\dagger a_\nu + a_{\bar{\nu}}^\dagger a_{\bar{\nu}}) - G \left(\sum_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger \right) \left(\sum_{\nu>0} a_{\bar{\nu}} a_\nu \right)$$

in the mean-field approximation

- Mean-field approximation:

$$V = -G\, P^\dagger P \to -G \left(\langle P^\dagger \rangle P + P^\dagger \langle P \rangle \right) = -\Delta (P^\dagger + P)$$

Cf. HF potential

$$V_H(\boldsymbol{r}) = \int v(\boldsymbol{r}, \boldsymbol{r}') \rho_{\mathrm{HF}}(\boldsymbol{r}') d\boldsymbol{r}$$

⟺ particle number violation

we consider $H' = H - \lambda \hat{N}$ instead of H :

$$H' \to \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k)$$

$$H' \to \sum_{k>0}(\epsilon_k - \lambda)(a^\dagger_k a_k + a^\dagger_{\bar{k}} a_{\bar{k}}) - \Delta \sum_{k>0}(a^\dagger_k a^\dagger_{\bar{k}} + a_{\bar{k}} a_k)$$

Bogoliubov transformation

$$\alpha^\dagger_\nu = u_\nu a^\dagger_\nu - v_\nu a_{\bar{\nu}}, \qquad \alpha^\dagger_{\bar{\nu}} = u_\nu a^\dagger_{\bar{\nu}} + v_\nu a_\nu$$

(Quasi-particle operator)

- Transform *H'* in a form of

$$H' = const. + \sum_{k>0} E_k(\alpha^\dagger_k \alpha_k + \alpha^\dagger_{\bar{k}} \alpha_{\bar{k}})$$

⟹ g.s.: $\alpha_k|BCS\rangle = 0$

1st excited state: $|1_k\rangle = \alpha^\dagger_k|BCS\rangle$ at E_k

…. and so on.

Ground state wave function: $\alpha_k|BCS\rangle = 0$

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu\, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|\,0\Big\rangle$$

(note) $\left\langle BCS|a^\dagger_\nu a_\nu|BCS\right\rangle = |v_\nu|^2$: occupation probability

(note) $E'_{\rm BCS} = \langle BCS|H'|BCS\rangle \sim 2\sum_{\nu>0}(\epsilon_\nu - \lambda)v_\nu^2 - \dfrac{\Delta^2}{G}$

$$H' = const. + \sum_{k>0} E_k(\alpha^\dagger_k\alpha_k + \alpha^\dagger_{\bar{k}}\alpha_{\bar{k}})$$

$$u_\nu^2 = \frac{1}{2}\left(1 + \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}}\right)$$

$$v_\nu^2 = \frac{1}{2}\left(1 - \frac{\epsilon_\nu - \lambda}{\sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2}}\right)$$

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$$

Self-consistency condition:

$$\Delta = G\langle BCS|\hat{P}|BCS\rangle = G\sum_{\nu>0} u_\nu v_\nu$$

$$= \frac{G}{2}\sum_{\nu>0}\frac{\Delta}{E_\nu}$$

Gap equation

i) Trivial solution: always exists

$$\Delta = 0$$

$$v_\nu^2 = 1 \quad (\epsilon_\nu \leq \lambda)$$
$$\quad = 0 \quad (\epsilon_\nu > \lambda)$$

$$|\Psi\rangle = \prod_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger |0\rangle$$

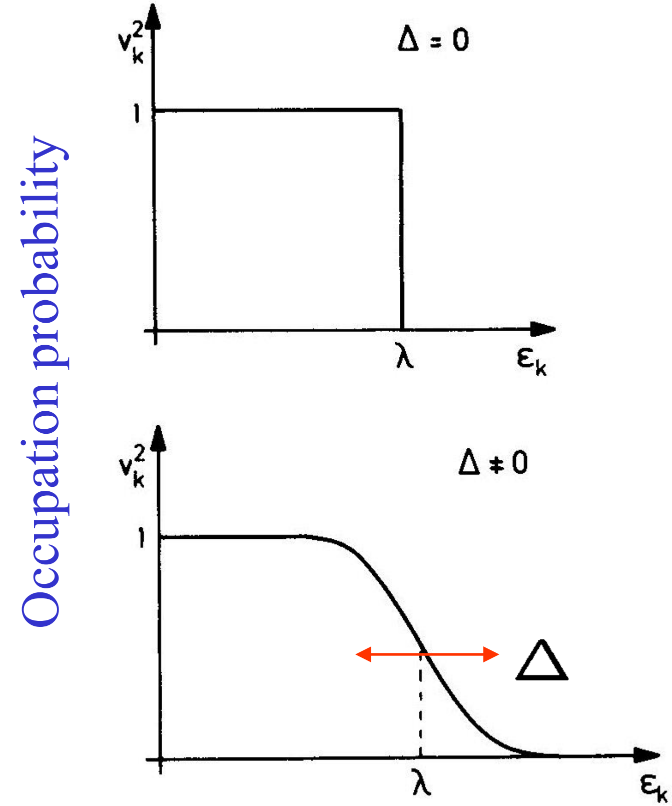
G a/o $N \longrightarrow$ large

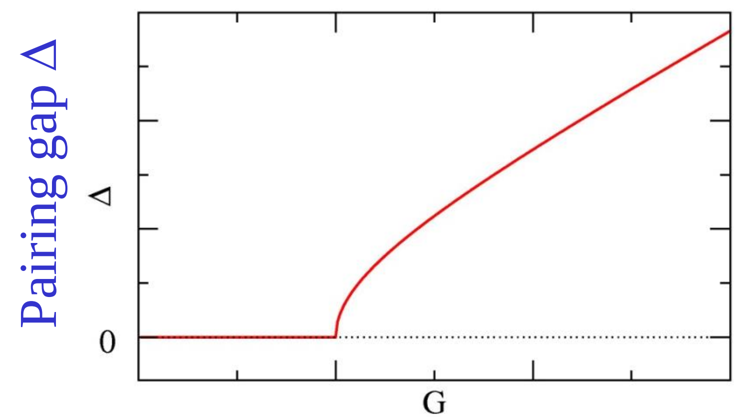
ii) Superfluid solution

$$\Delta \neq 0$$

$$v_\nu^2 < 1$$

$$|BCS\rangle = \prod_{\nu>0} \left(u_\nu + v_\nu\, a_\nu^\dagger a_{\bar{\nu}}^\dagger\right) \Big| 0 \Big\rangle$$





Number fluctuation

Normal-Superfulid phase transition

Quasi-particle excitations

$$H \sim E_{BCS} + \sum_\nu E_\nu \, \alpha^\dagger_\nu \alpha_\nu$$

- g.s. of even-even nuclei: $|BCS\rangle$
- One quasi-particle states:

$$|\nu_1\rangle = \alpha^\dagger_{\nu_1}|BCS\rangle = a^\dagger_{\nu_1} \prod_{\nu \neq \nu_1} \left(u_\nu + v_\nu \, a^\dagger_\nu a^\dagger_{\bar{\nu}}\right)\Big|0\Big\rangle$$

Wave function for odd-mass nuclei

$$\langle \nu_1|H|\nu_1\rangle = \langle H\rangle + E_{\nu_1}$$

- Two quasi-particle states:

$$|\nu_1\nu_2\rangle = \alpha^\dagger_{\nu_1}\alpha^\dagger_{\nu_2}|BCS\rangle$$

Excited state of the even-even nuclei

$$\langle \nu_1\nu_2|H|\nu_1\nu_2\rangle - \langle H\rangle \;=\; E_{\nu_1} + E_{\nu_2}$$
$$\geq \; 2\Delta$$

← Energy gap

(note) no pairing limit:

$$\alpha^\dagger_p \alpha^\dagger_h \to a^\dagger_p a_h, \qquad E_p + E_h \to (\epsilon_p - \lambda) + (\lambda - \epsilon_h)$$

(particle-hole excitation)

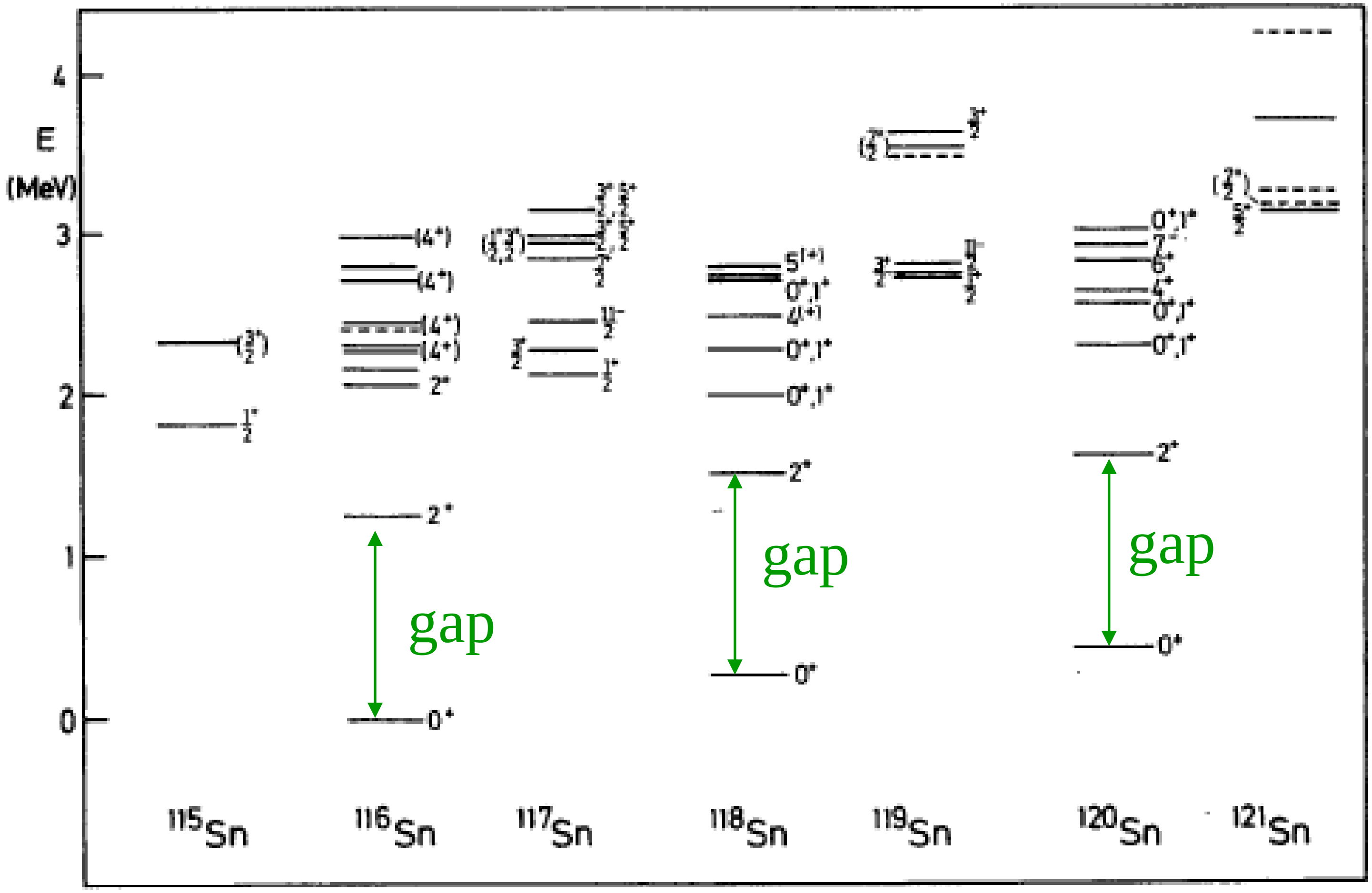


Figure 6.1. Excitation spectra of the $_{50}$Sn isotopes.

Ring-Schuck

Effects of pairing on moment of inertia

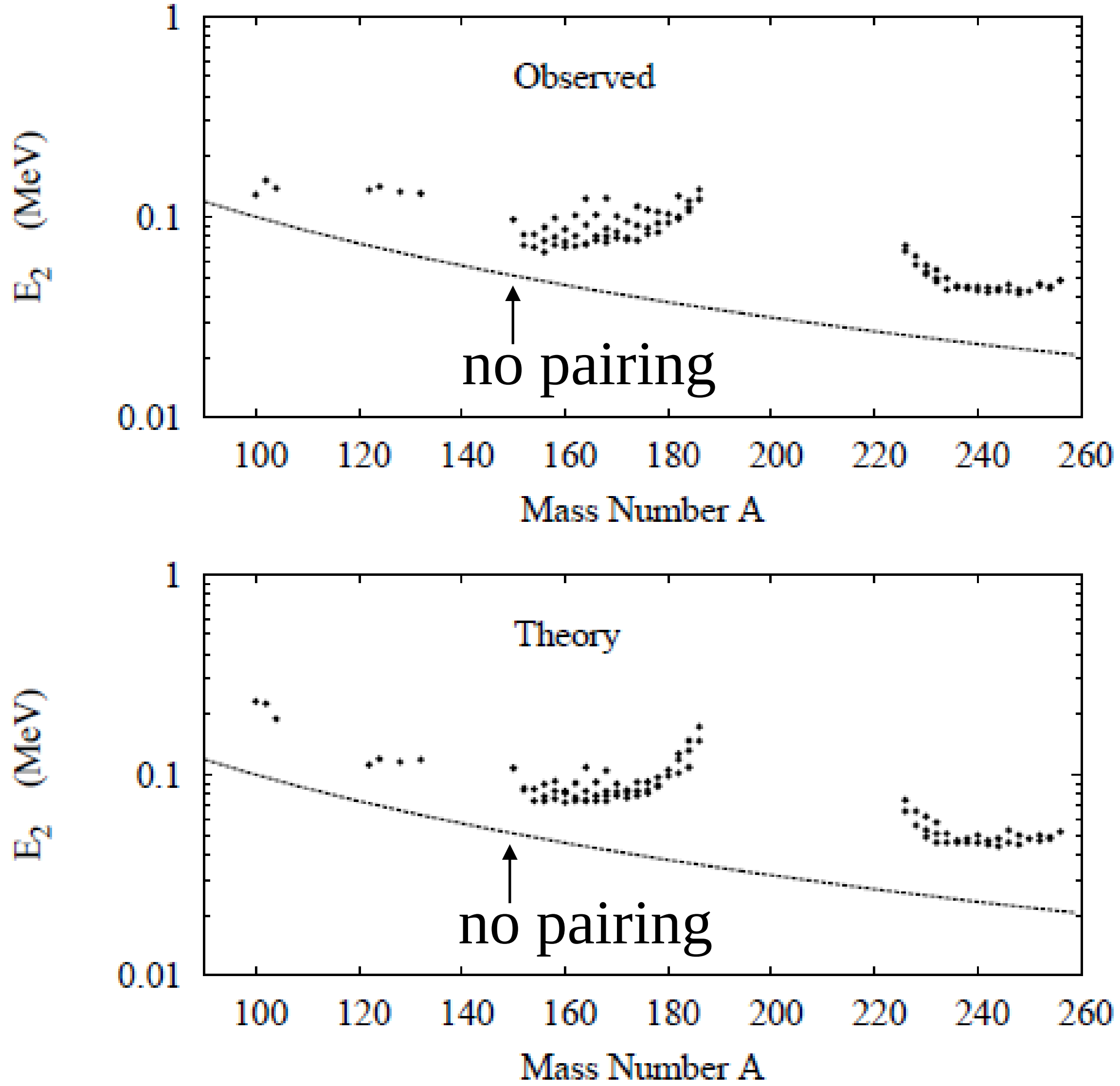


Fig. 9. Excitation energy of the first 2^+ state in deformed nuclei. The line shows the prediction assuming a rigid rotor.

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

G.F. Bertsch, in “Fifty years of nuclear BCS”

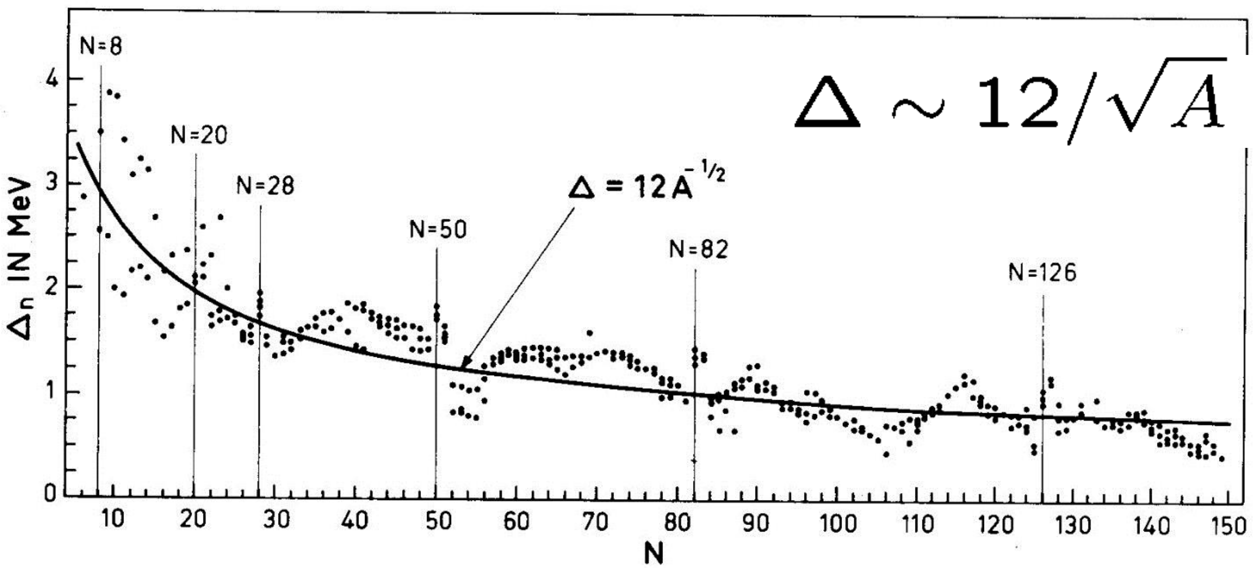
Even-odd mass difference and pairing gap

$$\begin{aligned} B_{\text{pair}} &= \Delta \quad &(\text{for even} - \text{even}) \\ &= 0 \quad &(\text{for even} - \text{odd}) \\ &= -\Delta \quad &(\text{for odd} - \text{odd}) \end{aligned}$$

$$\begin{aligned} E(N+2,Z) &= E(N,Z) + 2\lambda \\ E(N+1,Z) &= E(N,Z) + \lambda + \Delta \end{aligned}$$

$$-\Delta_n \sim [E(N+2,Z) - 2E(N+1,Z) + E(N,Z)]/2$$

$$\Delta \sim 12/\sqrt{A} \quad (\text{MeV})$$



Bohr-Mottelson ('69)

Hartree-Fock-Bogoliubov (HFB) Theory

HF+BCS method: first solve HF, and then solve the gap equation

s.p. wave functions, occupation probabilities,
chemical potential, pairing gaps

$$\psi_k(\boldsymbol{r}), u_k, v_k$$

→ Hartree-Fock-Bogoliubov (HFB) theory:

both wave functions and occupation probabilities
at the same time

$$U_k(\boldsymbol{r}), V_k(\boldsymbol{r})$$

cf. weakly bound systems

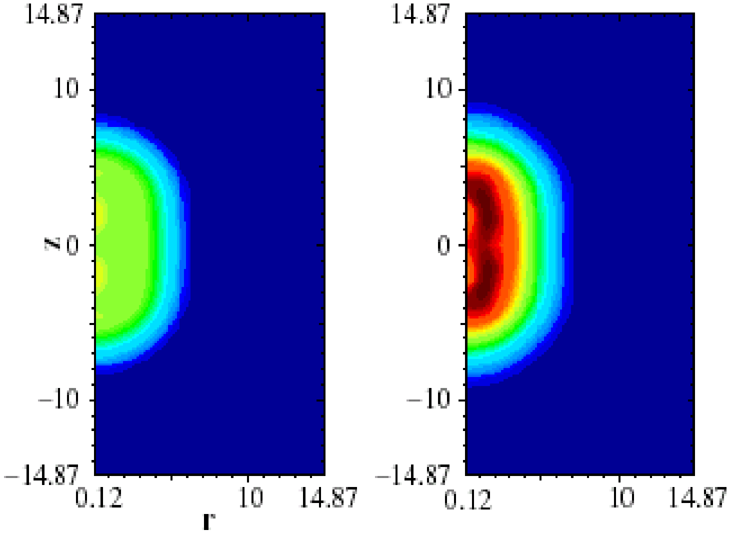
$$\begin{pmatrix} \hat{h}(\boldsymbol{r}) - \lambda & \tilde{\Delta}(\boldsymbol{r}) \\ \tilde{\Delta}(\boldsymbol{r})^* & -\hat{h}(\boldsymbol{r}) + \lambda \end{pmatrix} \begin{pmatrix} U_k(\boldsymbol{r}) \\ V_k(\boldsymbol{r}) \end{pmatrix} = E_k \begin{pmatrix} U_k(\boldsymbol{r}) \\ V_k(\boldsymbol{r}) \end{pmatrix}$$

$$\begin{aligned} \hat{h}(\boldsymbol{r}) &= -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathsf{HF}}(\boldsymbol{r}) \\ \rho(\boldsymbol{r}) &= \sum_k |V_k(\boldsymbol{r})|^2 \end{aligned}$$

u,*v* factors → *u*, *v* functions

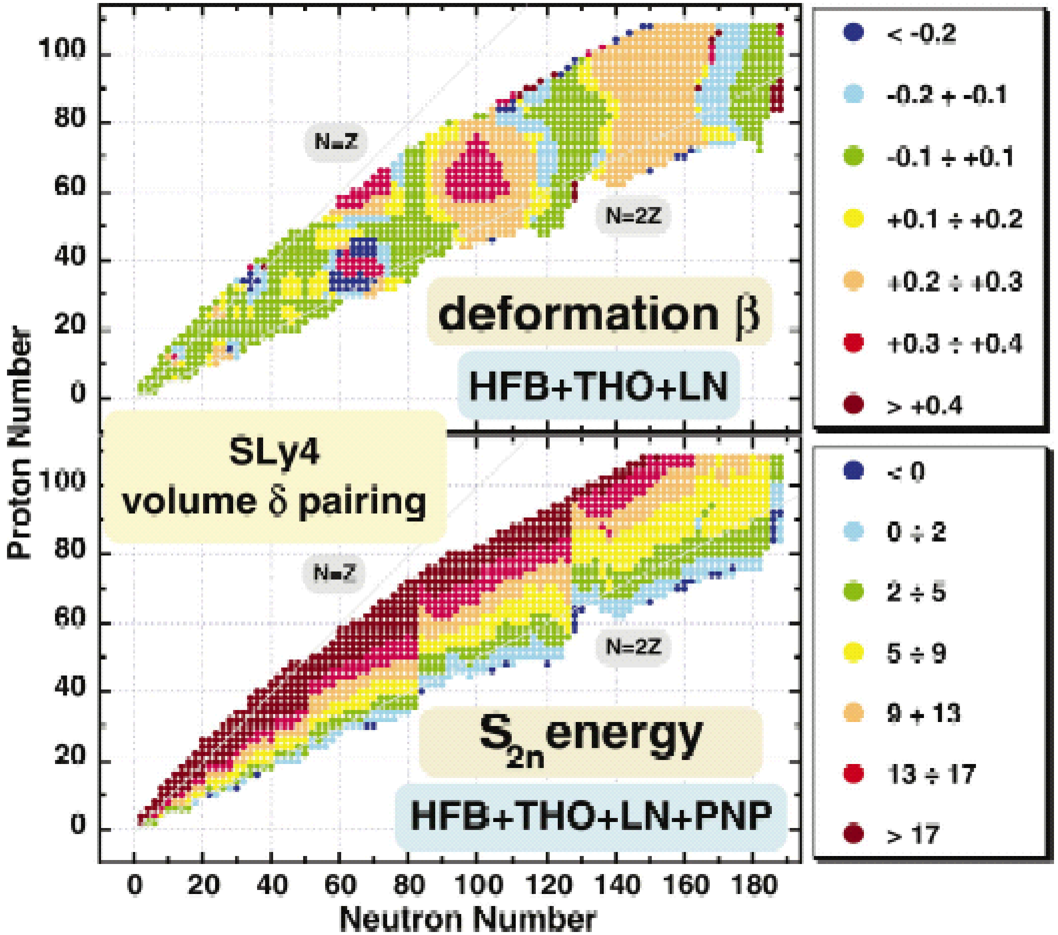
Application of the HFB method

Density of ^{110}Zr (SHFB-SLy4)

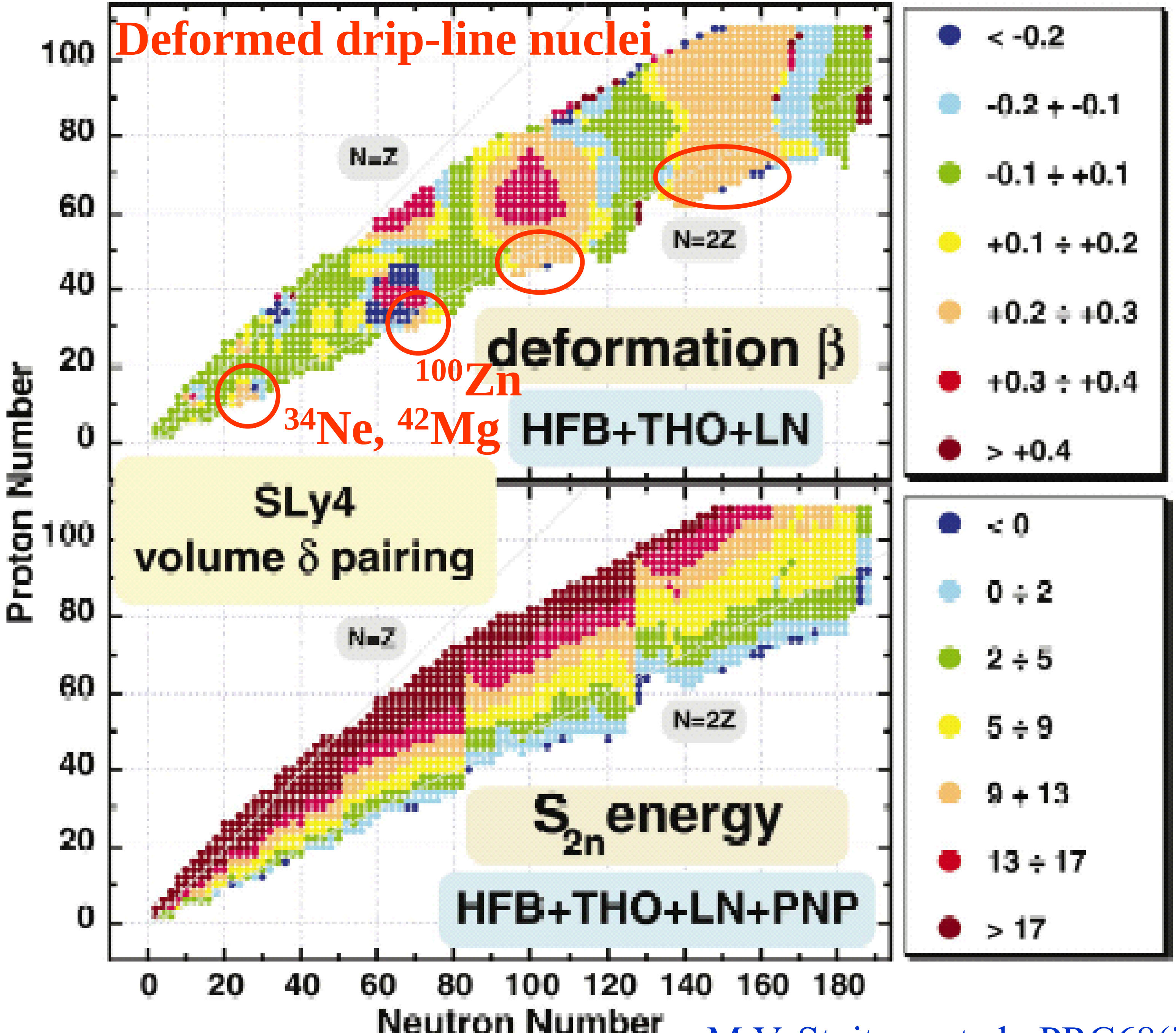


A. Blazkiewicz et al., PRC71('05)054231

Systematics of β_2 and S_{2n}

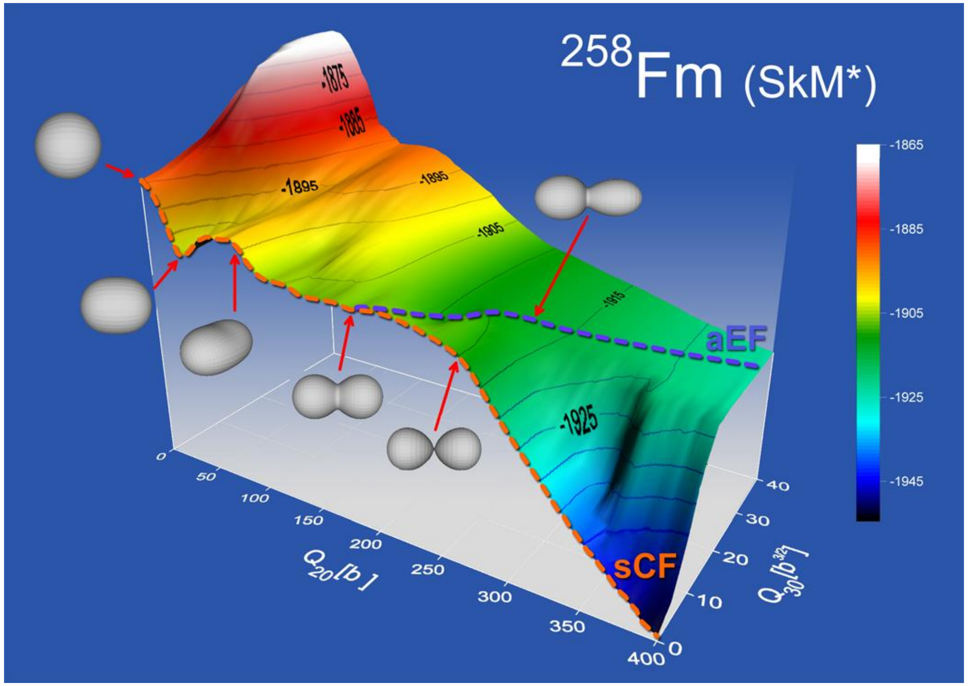


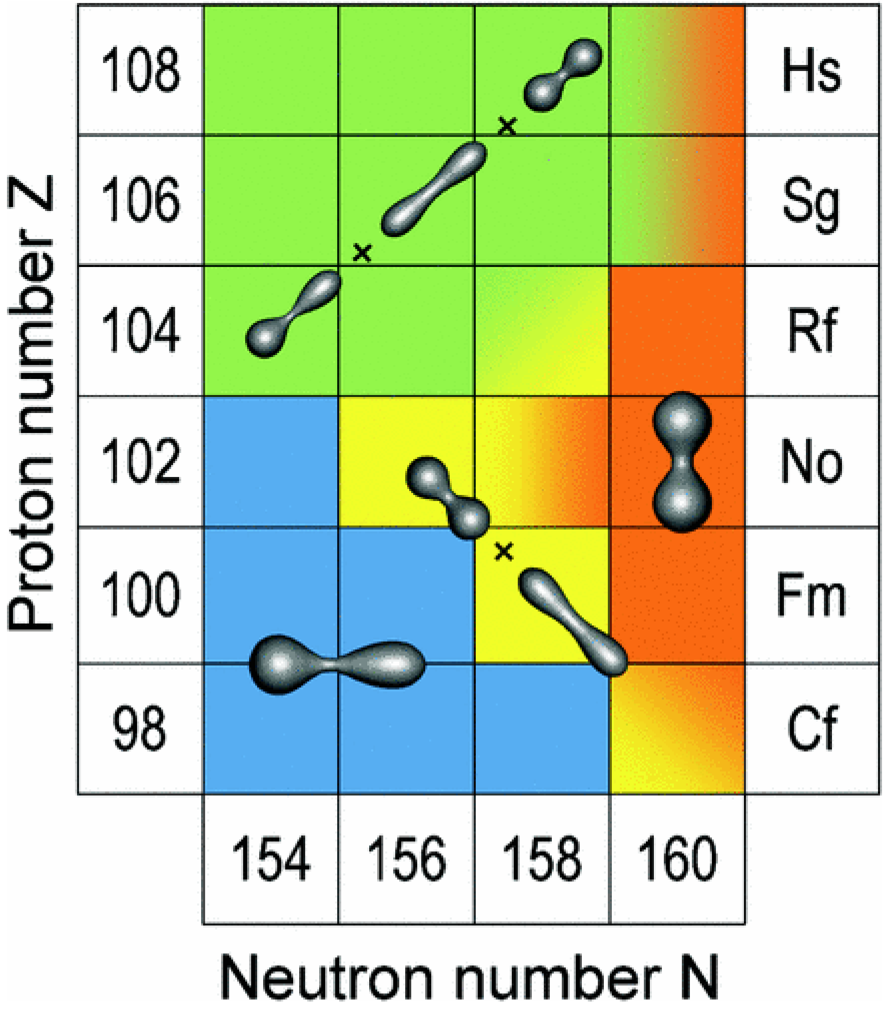
M.V. Stoitsov et al., PRC68('03)054312



M.V. Stoitsov et al., PRC68('03)054312

potential energy surface for fission process





A. Staszczak, A. Baran, J. Dobaczewski, and W. Nazarewicz, PRC80 (‘09) 014309