Nuclear Reactions

Shape, interaction, and excitation structures of nuclei ⟵ scattering expt.

cf. Experiment by Rutherford (α scatt.)

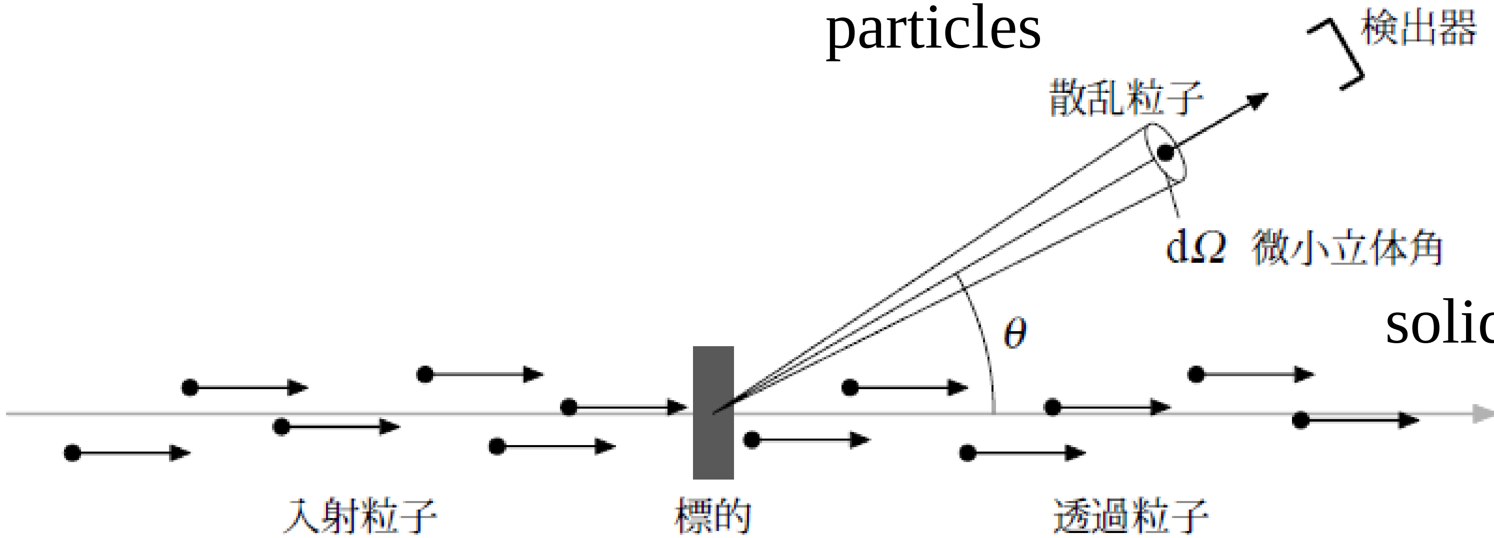
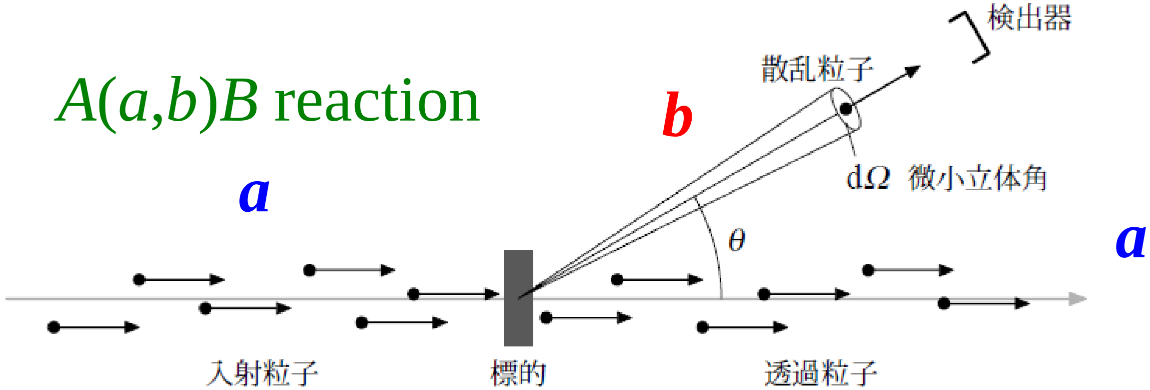


図 21.1: 散乱実験

http://www.th.phys.titech.ac.jp/~muto/lectures/QMII11/QMII11_chap21.pdf

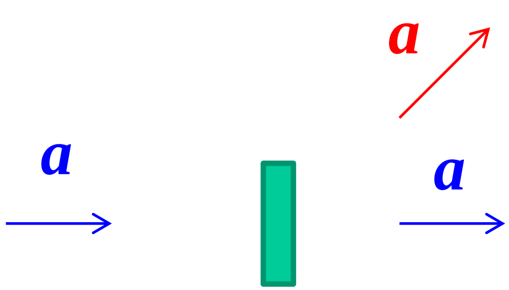
K. Muto (TIT)

$A(a,b)B$ reaction



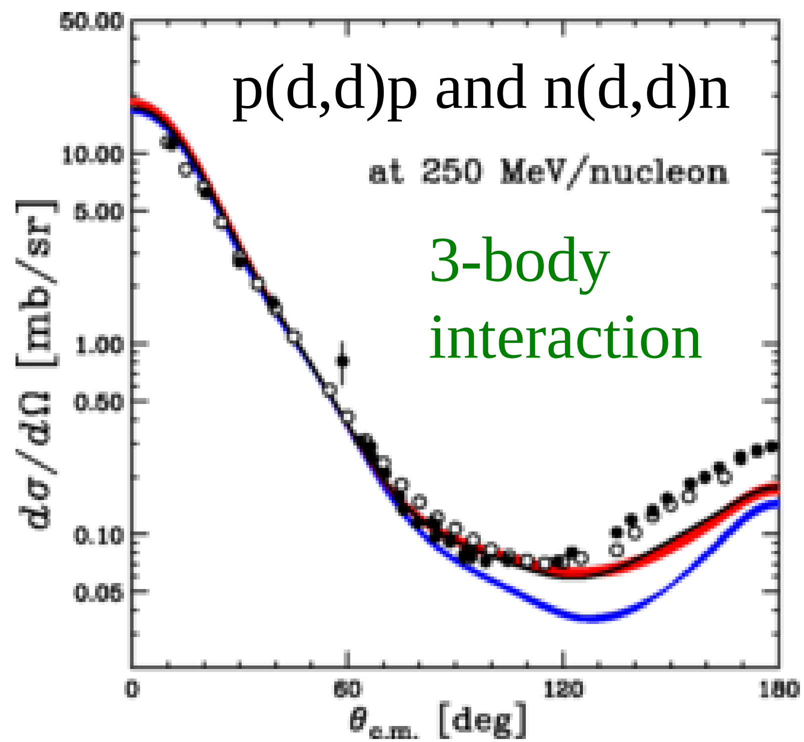
$A \rightarrow B$

✓elastic scattering



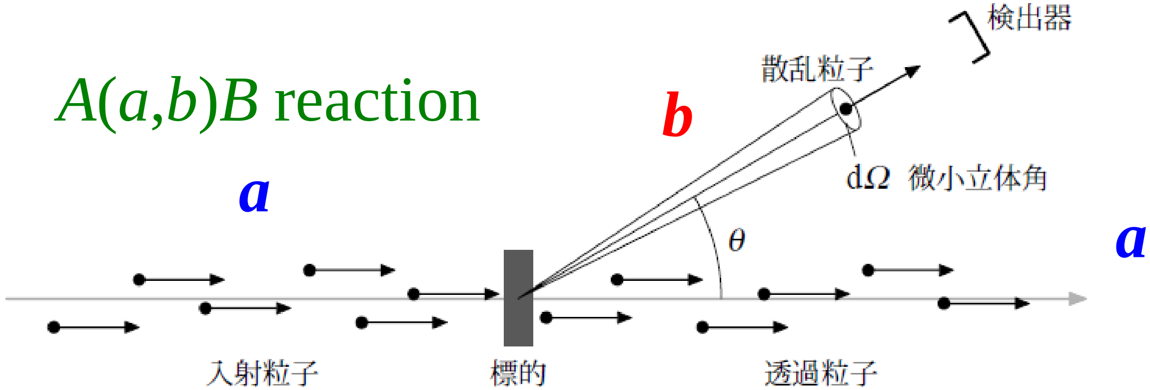
$A_{gs} \rightarrow A_{gs}$

fundamental interaction between a and A



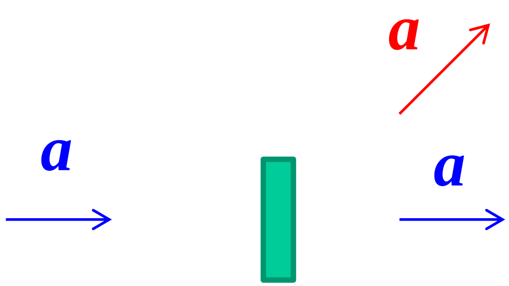
K. Sekiguchi et al., PRC89('14)064007

$A(a,b)B$ reaction



$A \rightarrow B$

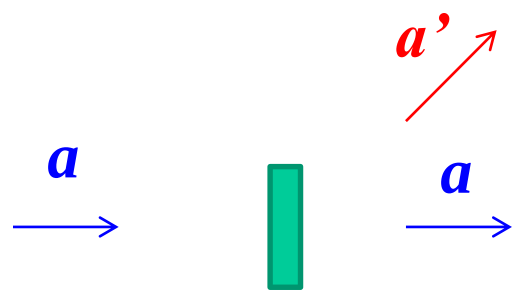
✓elastic scattering



$A_{gs} \rightarrow A_{gs}$

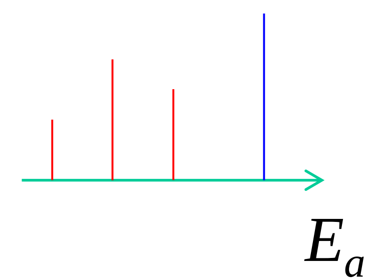
fundamental interaction between a and A

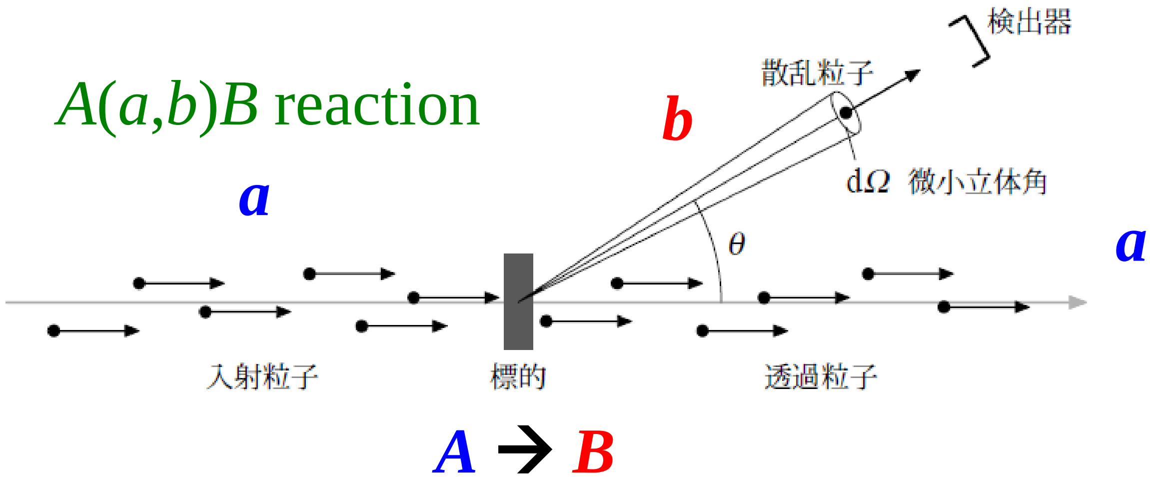
✓inelastic scattering



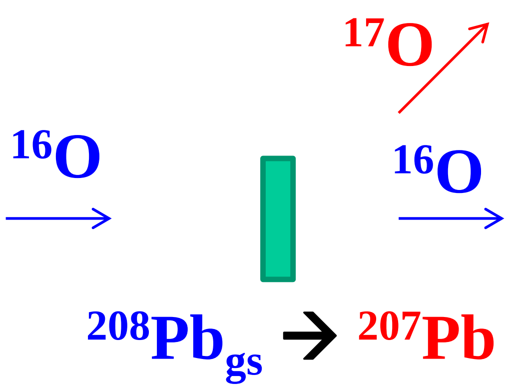
$A_{gs} \rightarrow A^*$

excitation spectrum of a nucleus A



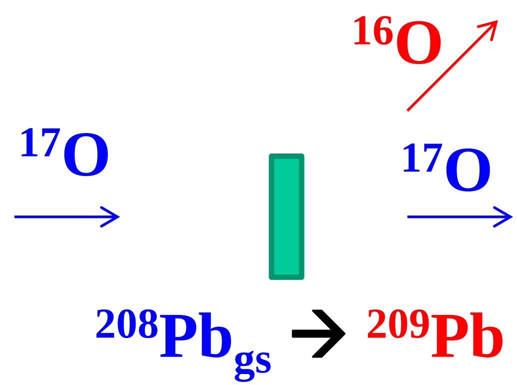


✓transfer reaction
(below: an example of pick-up reaction)



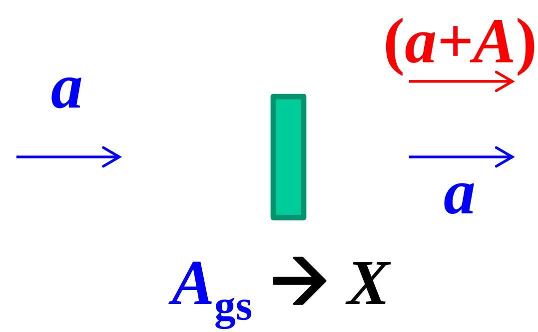
level schem of ^{207}Pb

✓transfer reaction
(below: an example of stripping reaction)



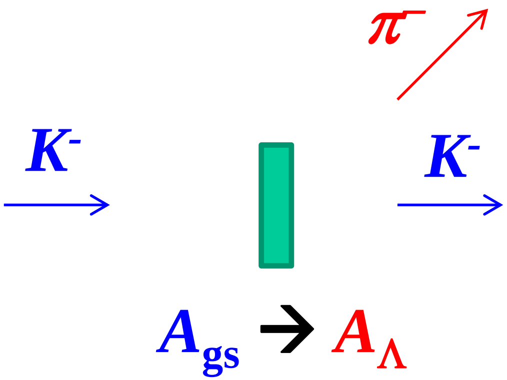
level schem of ^{209}Pb

✓fusion reaction

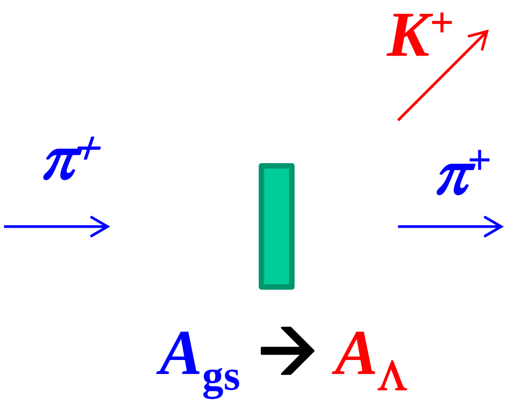


- interaction between *a* and *A*
- structure of *a* and *A*

✓(K^-,π^-) reaction

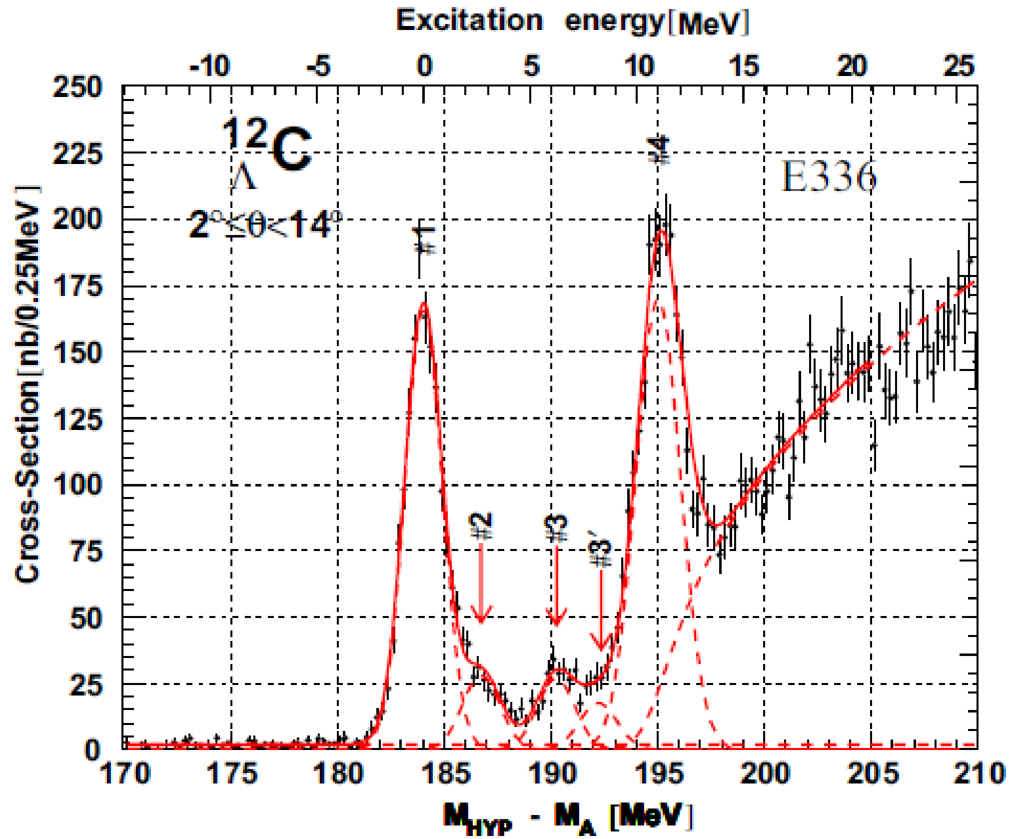


✓(π^+,K^+) reaction



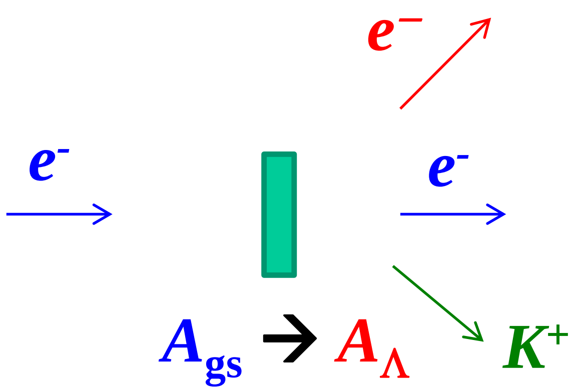
excitation spectrum
of a hypernucleus A_Λ

^{12}C (π^+,K^+) $^{12}{}_\Lambda$C reaction



O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 (‘06)564

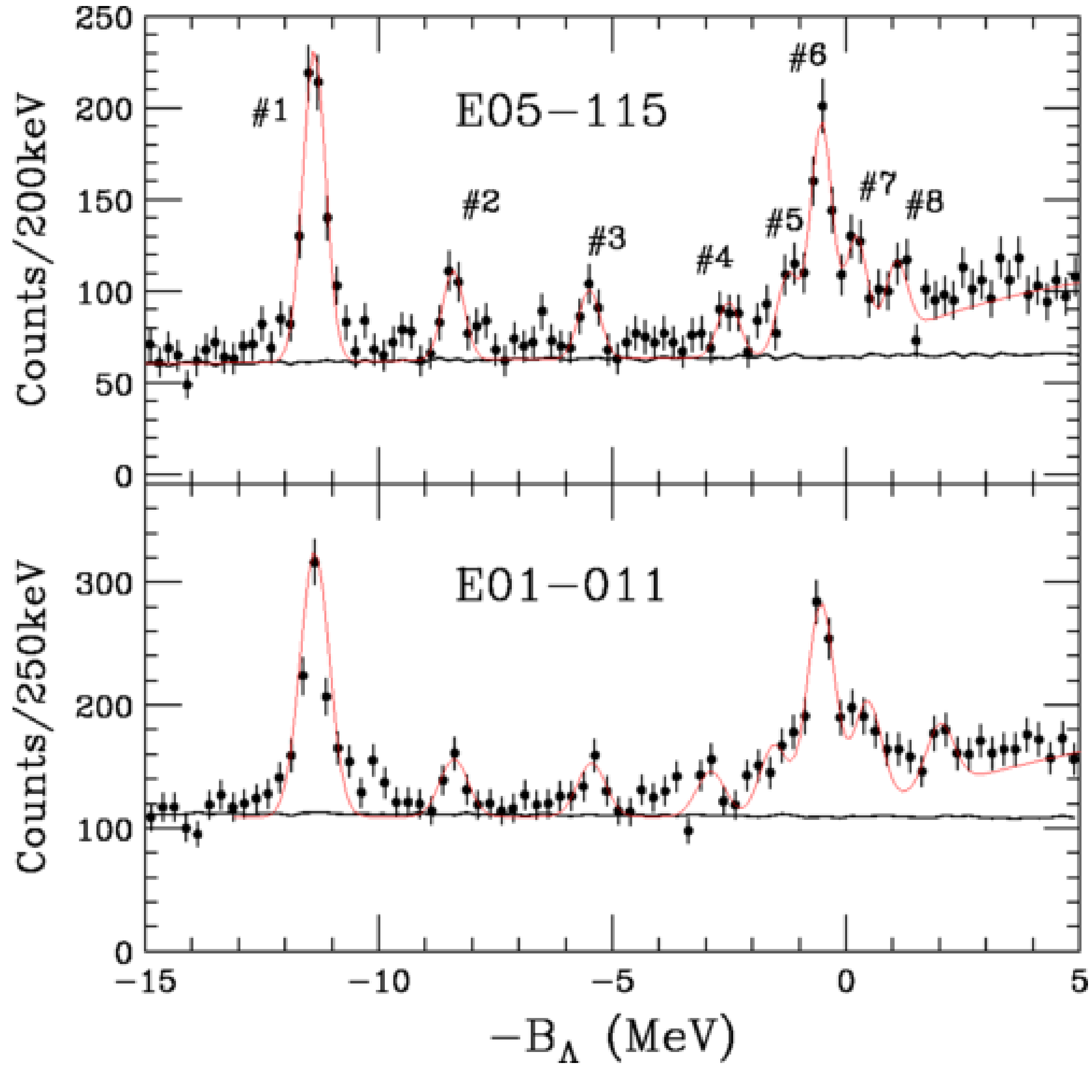
✓(e,e'K⁺) reaction



S.N. Nakamura et al., PRL110('13)012502

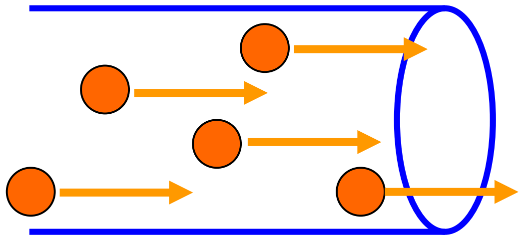
T. Gogami, Ph.D. Thesis (Tohoku U.) 2014

$^{12}C(e,e'K^+)\ ^{12}_{\Lambda}B$



L. Tang et al., PRC90('14)034320

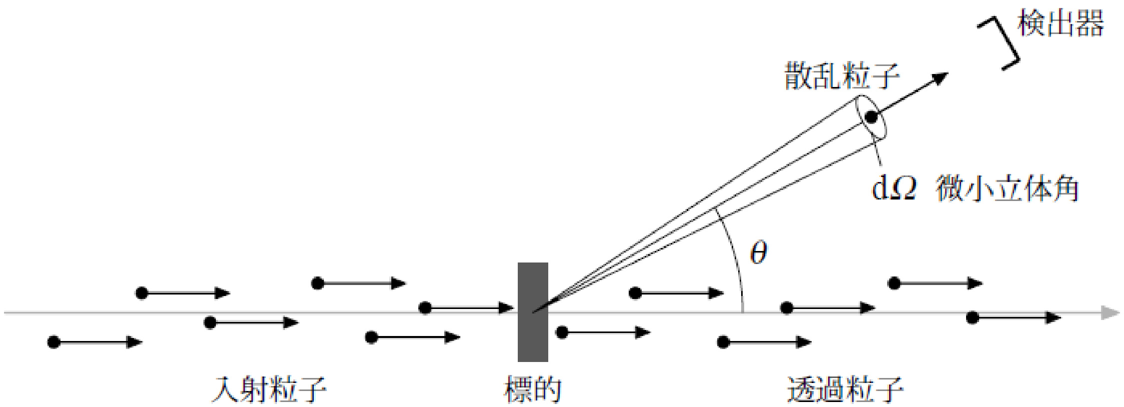
Cross sections



incident beam

flux = the number of particles crossing unit area per unit time

$$j = \rho_P \cdot v$$

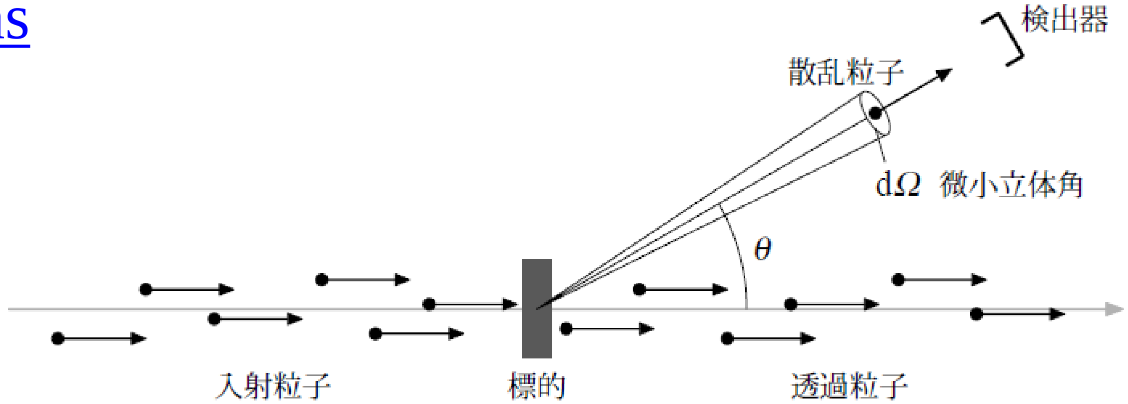


event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

$$\longrightarrow \quad R = N_T \cdot \sigma \cdot j$$

cross section

Cross sections



event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

$$\longrightarrow \quad R = N_T \cdot \sigma \cdot j$$

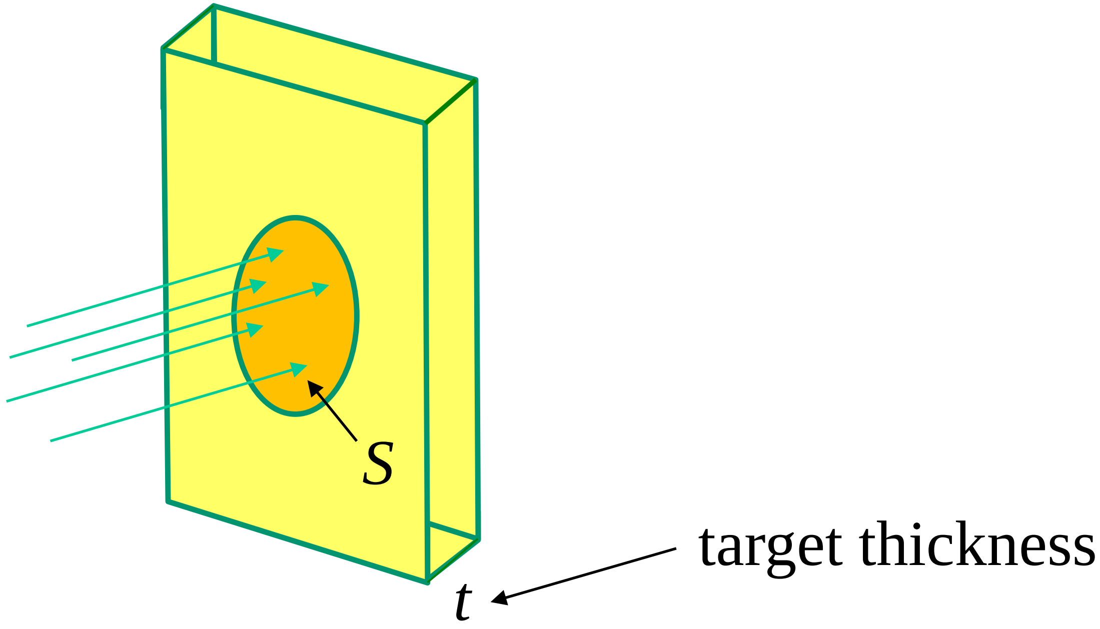
cross section

differential cross sections (angular distribution)

$$dR(\theta,\phi) = N_T \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega \qquad \sigma = \int d\Omega \frac{d\sigma}{d\Omega}$$

units: 1 barn = 10^{-24} cm^2 = 100 fm^2 (1 mb = 10^{-3} b = 0.1 fm^2)

Cross sections (experiments)



$$dR(\theta,\phi) = N_{\mathsf{T}} \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega$$

beam intensity: $I = j \cdot S$

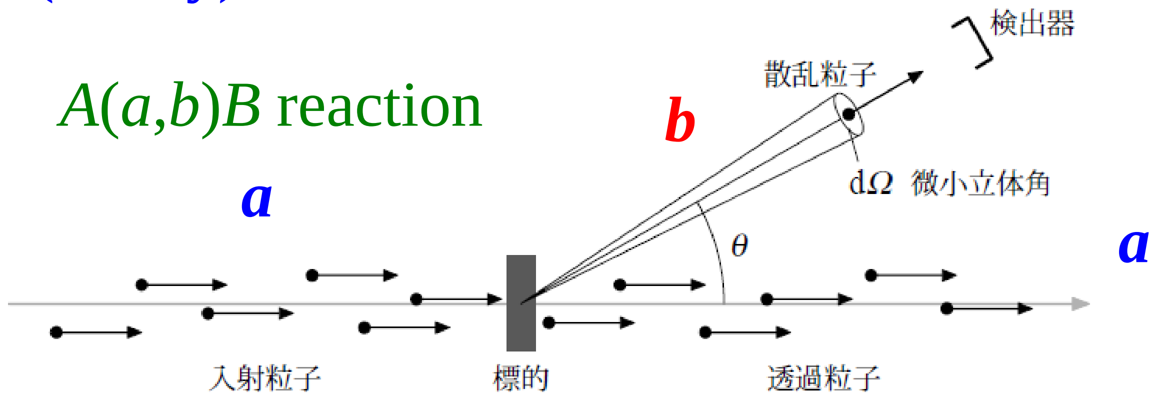
the number of target nucleus: $N_{\mathsf{T}} = S \cdot t \cdot \rho_{\mathsf{T}}$

$$dR(\theta,\phi) = I \cdot \frac{d\sigma}{d\Omega} \cdot t\rho_{\mathsf{T}} \cdot d\Omega \cdot \epsilon$$

detection efficiency

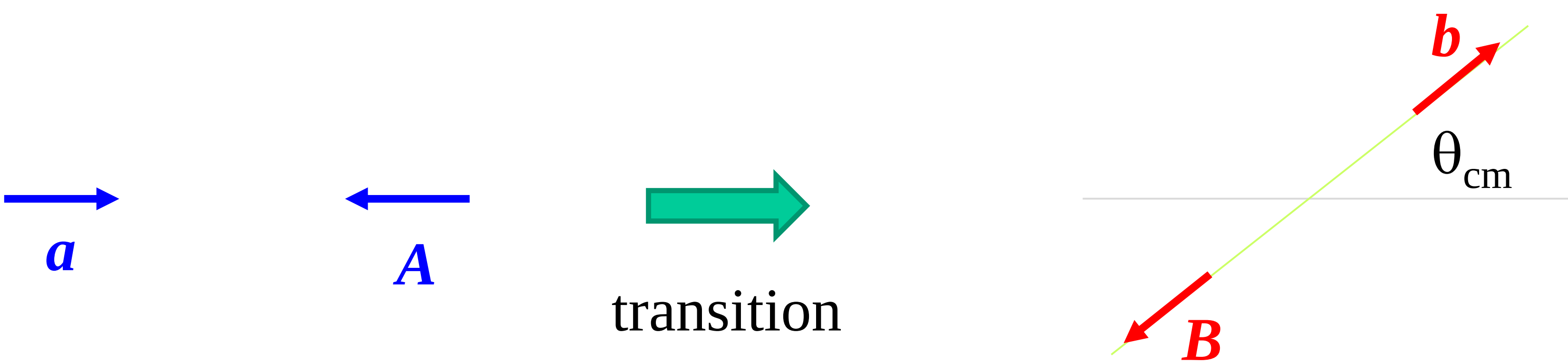
Cross sections (theory)

A(*a*,*b*)*B* reaction



A → ***B***

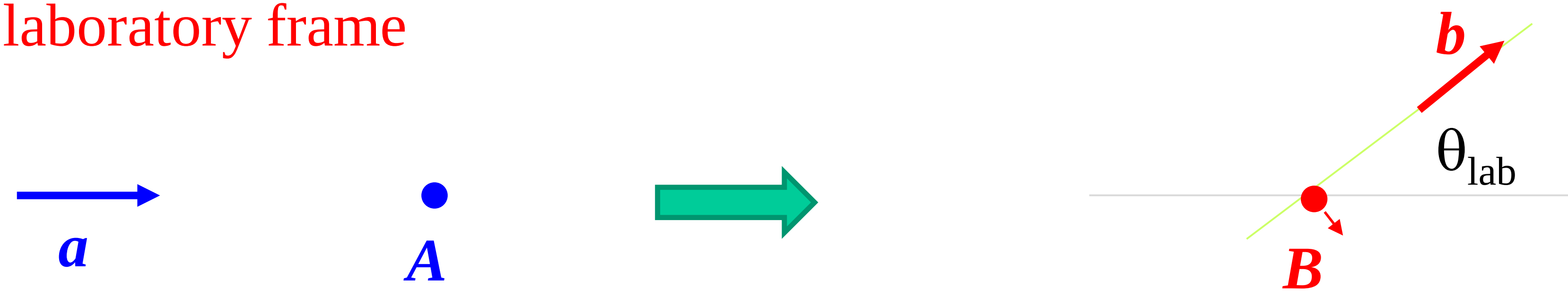
center of mass frame



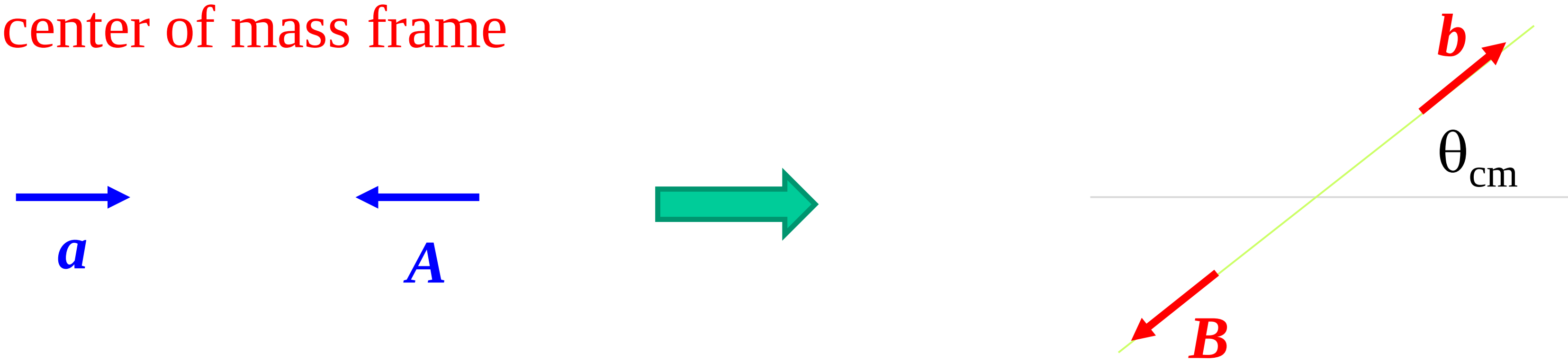
$$\frac{d\sigma}{d\Omega} = \frac{R}{j_{\text{in}}}$$

Cross sections

✓ laboratory frame



✓ center of mass frame



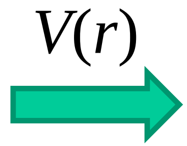
□ transformation ⟵ energy and momentum conservations

$$\tan\theta_{\text{lab}} = \frac{\sin\theta_{\text{cm}}}{\gamma + \cos\theta_{\text{cm}}}, \qquad d\Omega_{\text{lab}} = \frac{|1+\gamma\cos\theta_{\text{cm}}|}{(1+\gamma^2+2\gamma\cos\theta_{\text{cm}})^{3/2}}\, d\Omega_{\text{cm}}$$

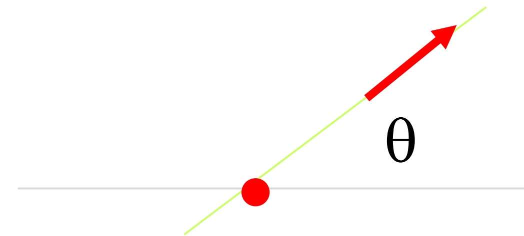
$$E_{\text{cm}} = \frac{M_A}{M_a + M_A} E_{\text{lab}}, \qquad \gamma = \sqrt{\frac{M_a M_b}{M_A M_B}\frac{E_{\text{cm}}}{E_{\text{cm}} + Q}}$$

Born approximation

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$



$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$



$$\left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

transition rate for elastic scattering:

$$\begin{aligned} W_{fi} &= \frac{2\pi}{\hbar}\int \frac{d\boldsymbol{p}_f}{(2\pi\hbar)^3}\,|\langle\psi_f|V|\psi_i\rangle|^2\delta(E_f - E_i) \\ &= \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega\left|\widetilde{V}(\boldsymbol{q})\right|^2 \end{aligned}$$

$$\widetilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r}e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar}V(r) \equiv \int d\boldsymbol{r}e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(r)$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

$$W_{fi} = \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega\left|\tilde{V}(\boldsymbol{q})\right|^2$$

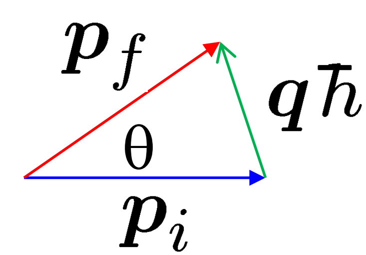
momentum transfer

$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r} e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar}V(r) \equiv \int d\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(r)$$

incident flux: $j_{\mathrm{inc}} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\mathrm{inc}}} = \int d\Omega \boxed{\frac{\mu^2}{4\pi^2\hbar^4}\left|\tilde{V}(\boldsymbol{q})\right|^2}$$

$$= \frac{d\sigma}{d\Omega}$$



$$q\hbar = 2p_i\sin\frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E \sin^2\theta/2)^2} |F(\boldsymbol{q})|^2$$

$$= \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega}\right) |F(\boldsymbol{q})|^2$$

Form factor

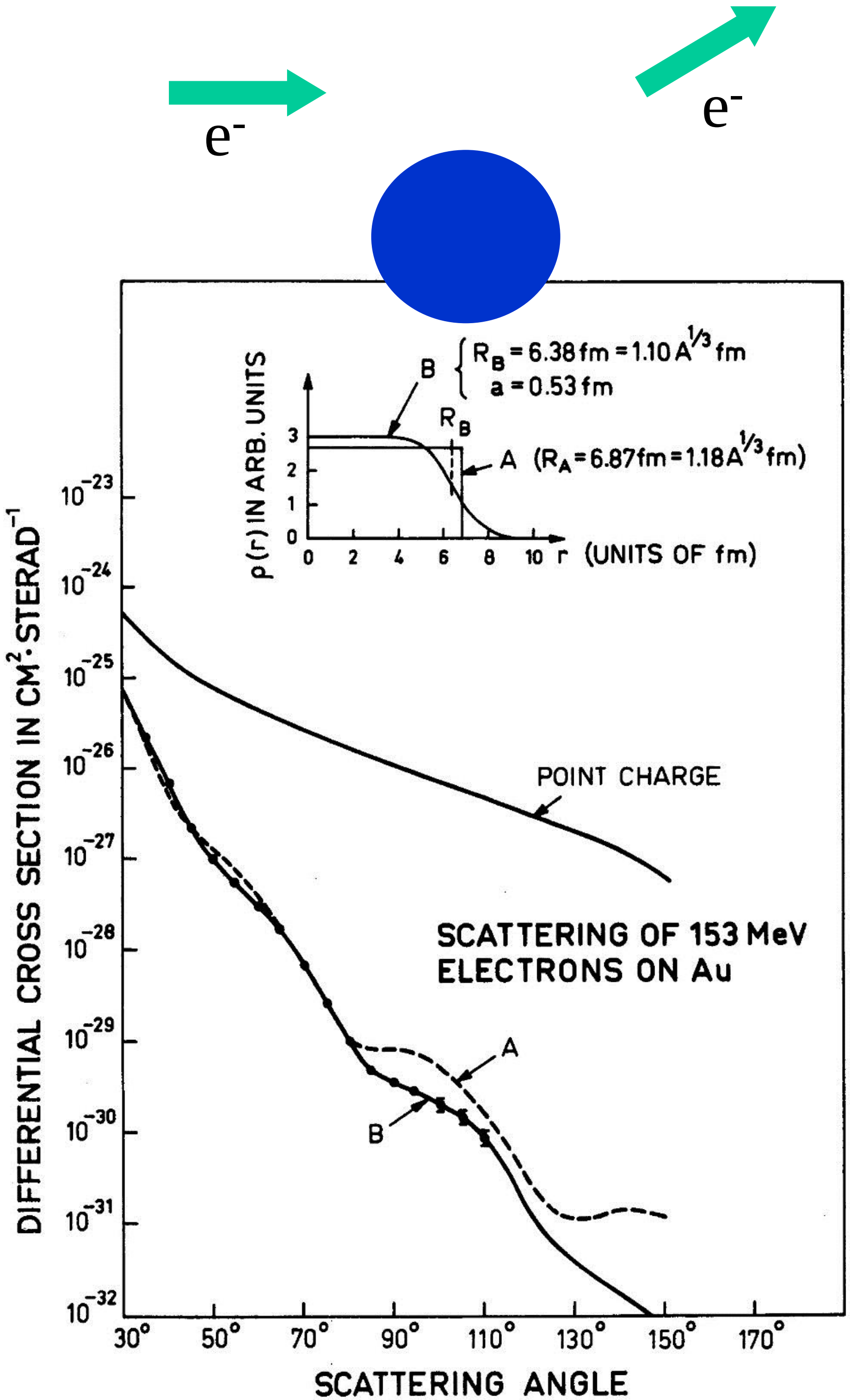
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})\, d\boldsymbol{r}$$

* relativistic correction:

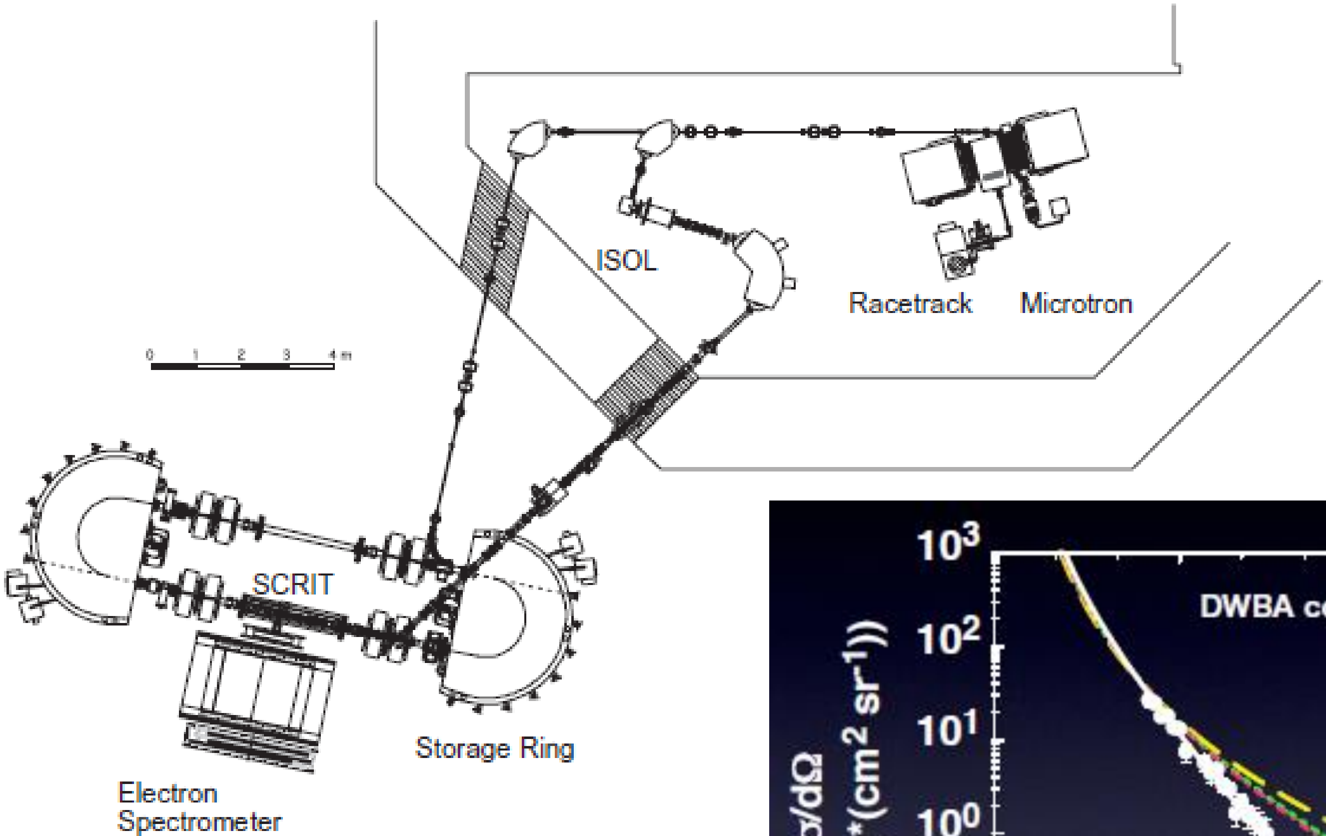
$$\frac{d\sigma_{\text{Ruth}}}{d\Omega} \rightarrow \frac{d\sigma_{\text{Mott}}}{d\Omega}$$

$$= \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \left(1 - \frac{v^2}{c^2} \sin^2 \frac{\theta}{2}\right)$$

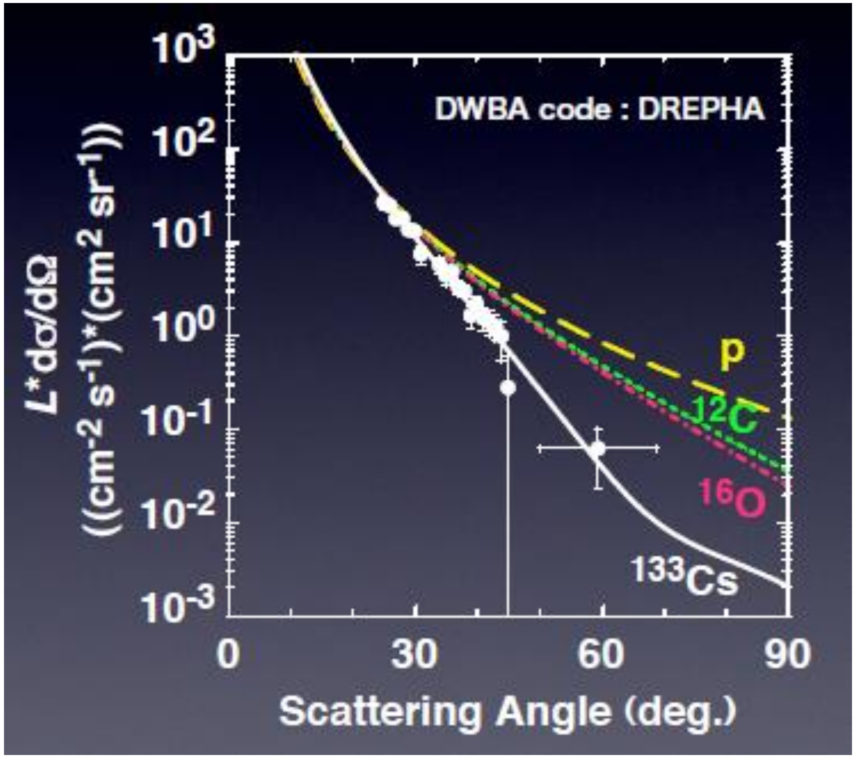
$$\sim \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \cos^2 \frac{\theta}{2} \qquad (v \rightarrow c)$$



cf. electron scattering off unstable nuclei (SCRIT)



T. Suda et al.,
PTEP 2012, 03C008 (2012)
PRL102, 102501 (2009)



Distorted Wave Born approximation (DWBA)

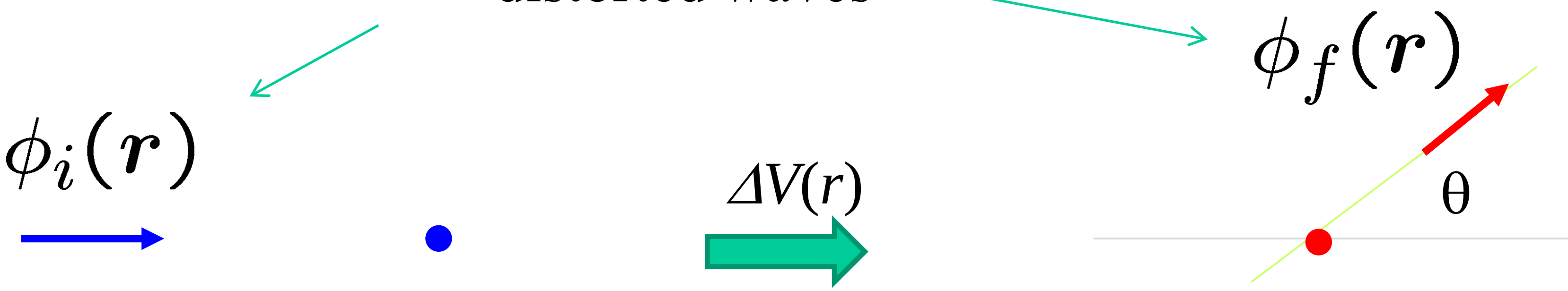
$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

$$\Rightarrow \quad \left(-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \underline{V(r) - V_0(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

“distorted waves”



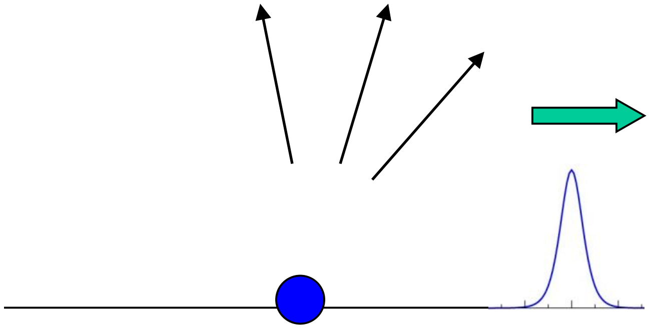
✓inelastic scattering

✓transfer reactions

Optical model

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux (absorption)

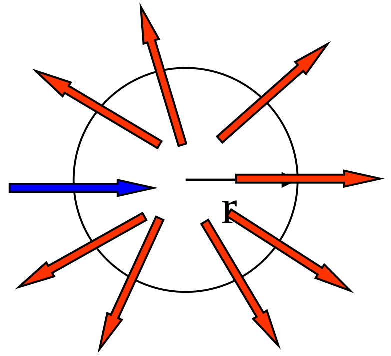
Optical potential

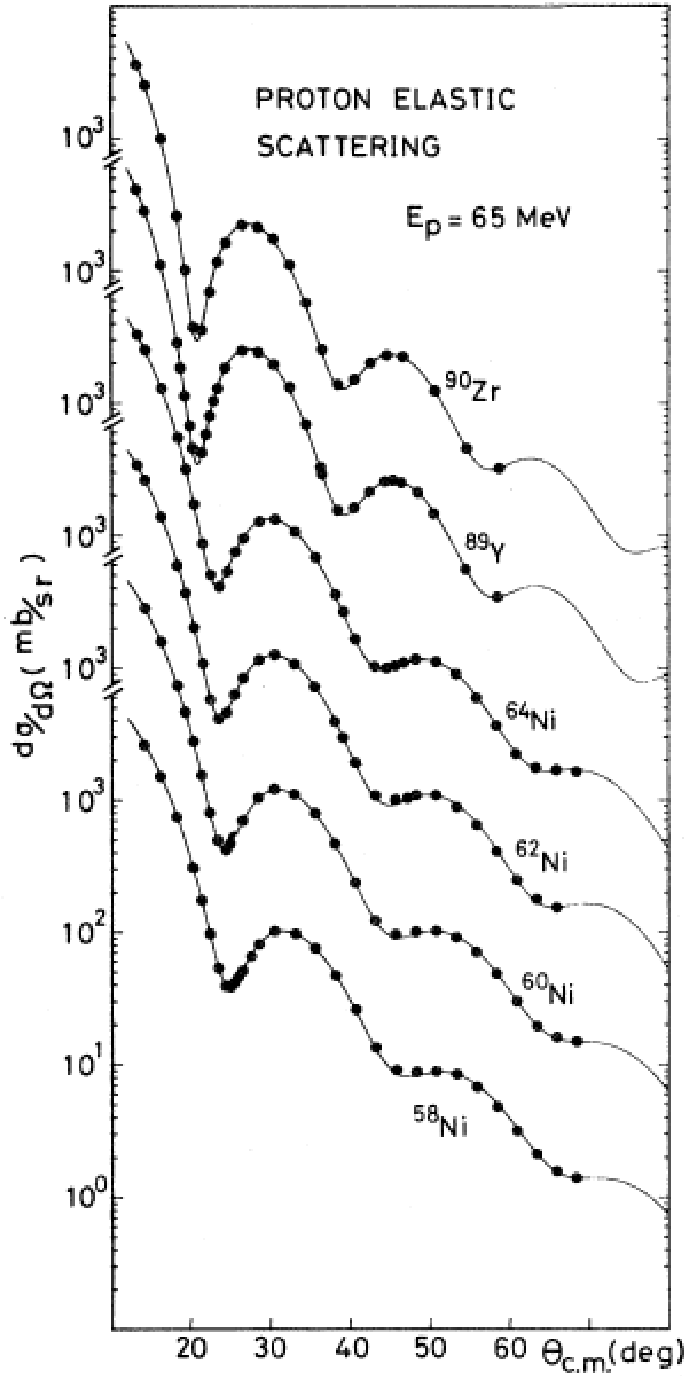
$$V_{\rm opt}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)$$

$$\Longrightarrow \quad \boldsymbol{\nabla} \cdot \boldsymbol{j} = \cdots = -\frac{2}{\hbar} W |\psi|^2$$

(note) Gauss's law

$$\int_S \boldsymbol{j} \cdot \boldsymbol{n}\, dS = \int_V \boldsymbol{\nabla} \cdot \boldsymbol{j}\, dV$$





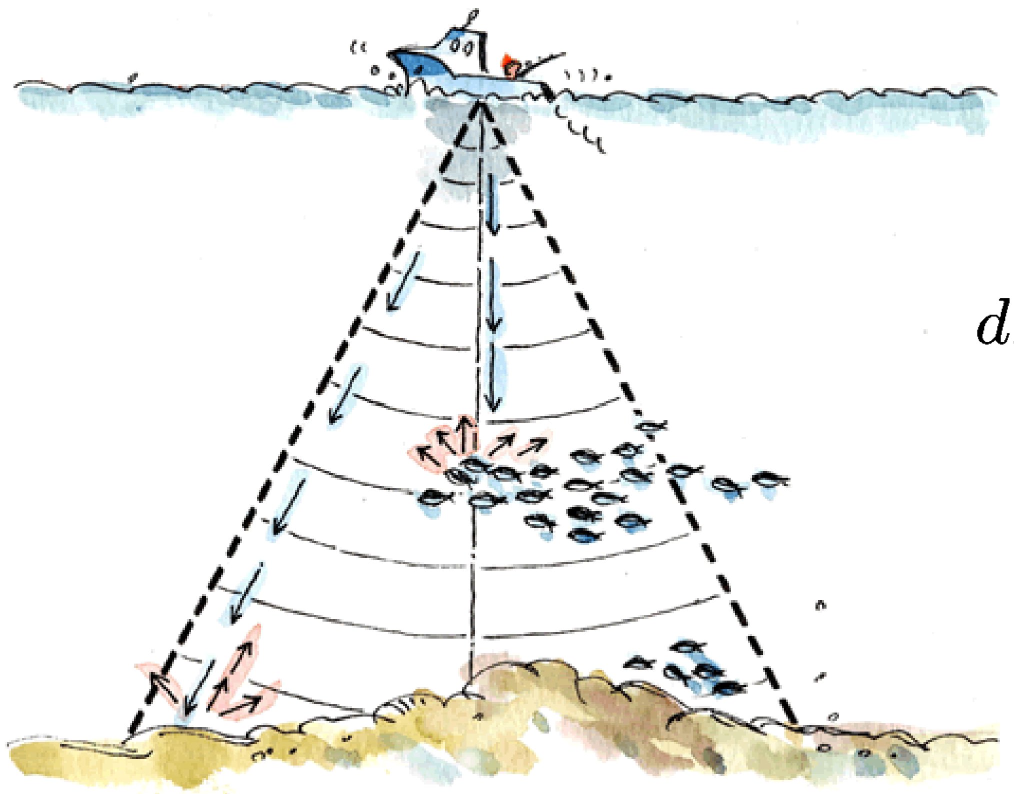
$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \frac{Z_P Z_T e^2}{r} + V_{\mathrm{opt}}(r) - E\right)\psi(\boldsymbol{r}) = 0$$

Woods-Saxon + volume &surface imaginary parts

H. Sakaguchi et al.,
PRC26 (1982) 944

Appendix: DWBA in ocean acoustics

Fishfinder



https://www.furuno.co.jp/technology/about/fishfinder1.html

(backward) scattering of (ultra-)sonic waves due to fish etc.

$$dR(\theta,\phi) = N_{\mathsf{T}} \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega$$

$$\downarrow$$

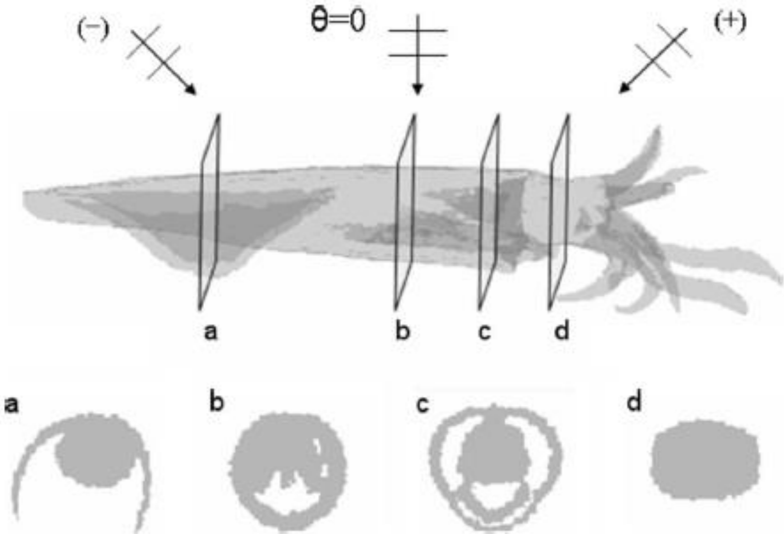
$$N_{\mathsf{T}} = \frac{\frac{dR}{d\Omega}}{j \cdot \frac{d\sigma}{d\Omega}}$$

one can know the number of fish N_{T} if one knows the differential cross sections

Use of the distorted wave Born approximation to predict scattering by inhomogeneous objects: Application to squid

Benjamin A. Jones,[a] Andone C. Lavery, and Timothy K. Stanton
Department of Applied Ocean Physics and Engineering, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts 02543-1053

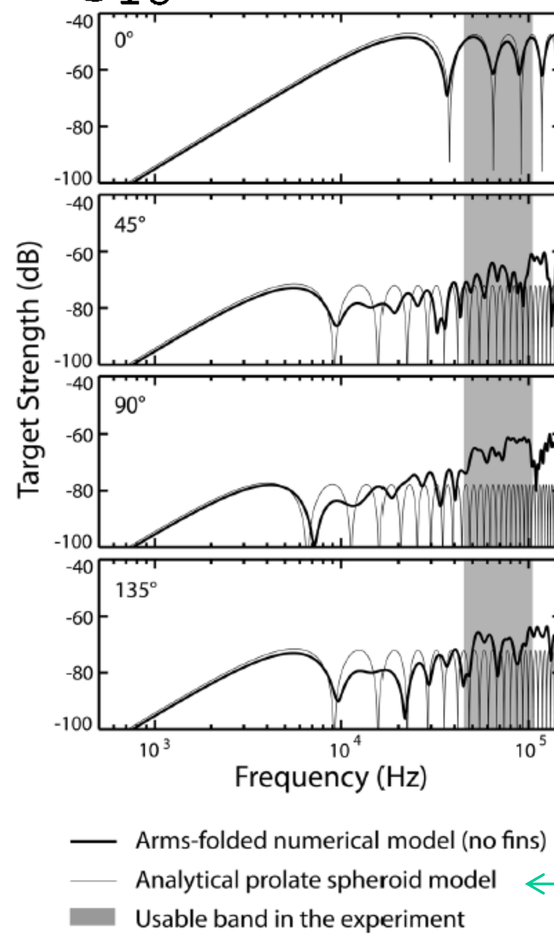
J. Accoust. Soc. Am. 125 ('09) 73



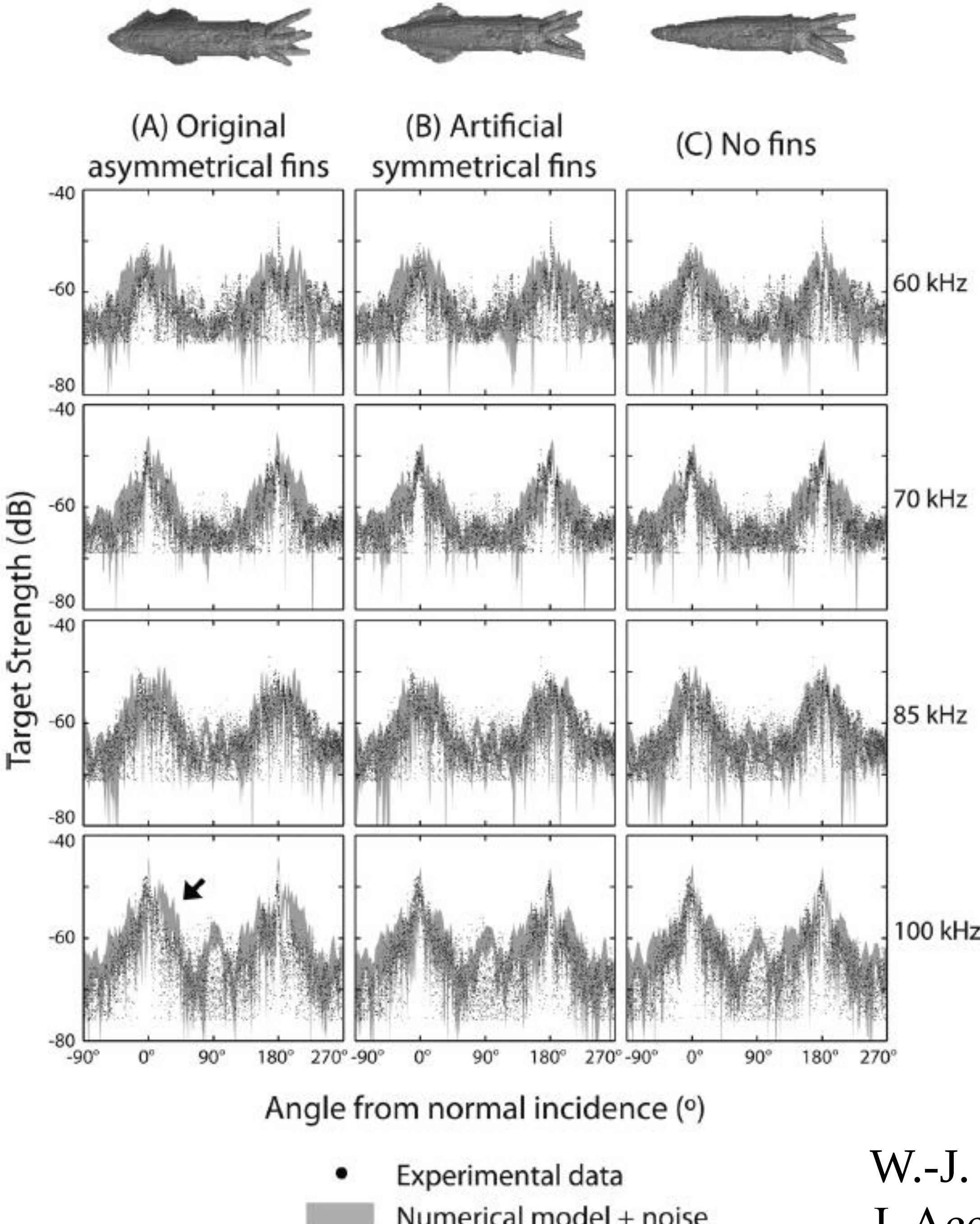
Modeling of squid

DWBA: local wave number inside a squid

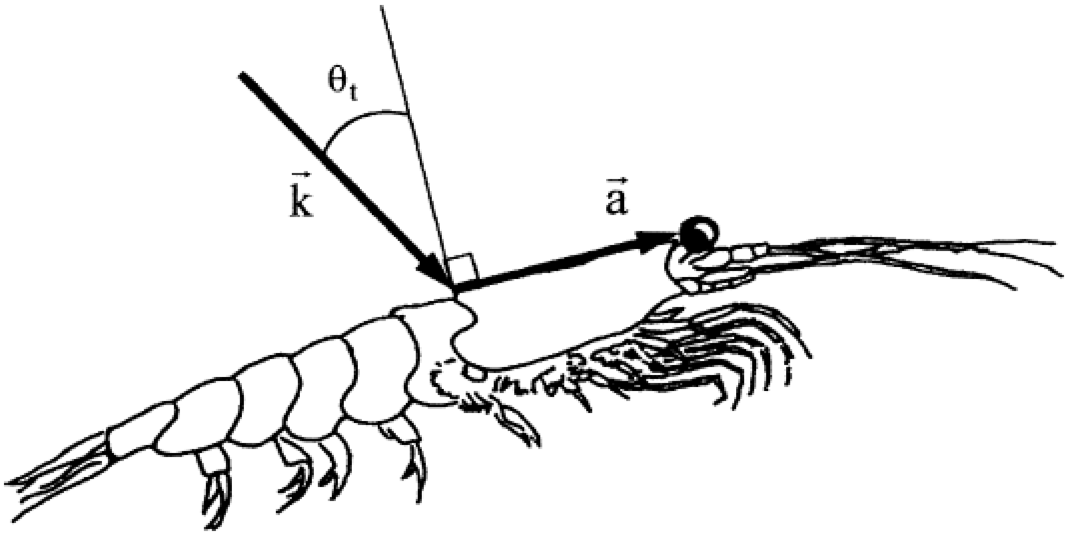
$10\log_{10}\sigma$



← !



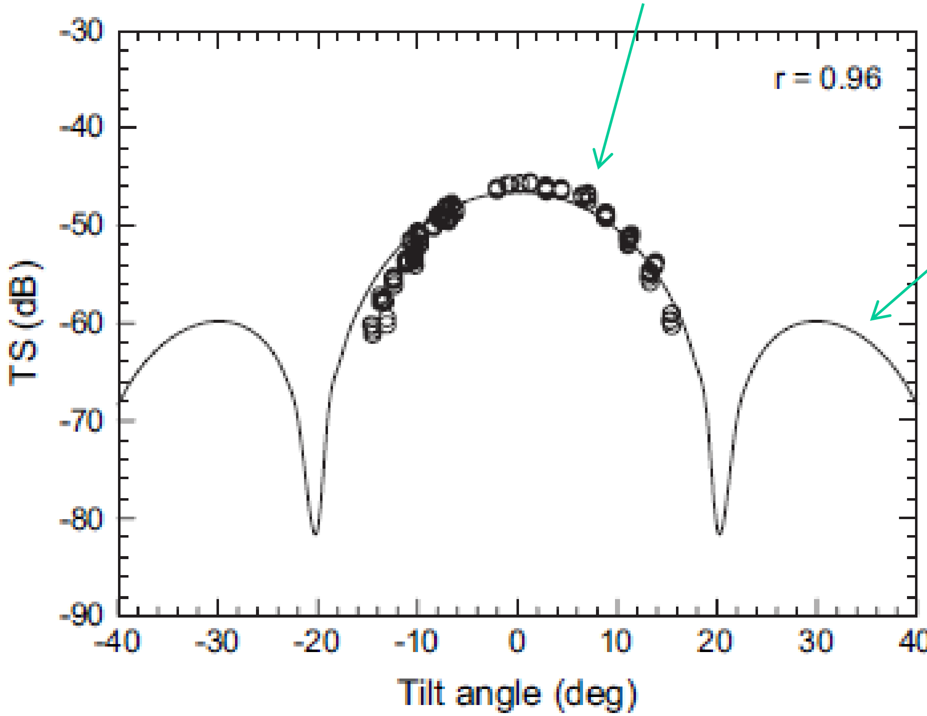
W.-J. Lee, A.C. Lavery, T. Stanton,
J. Accoust. Soc. Am. 131 (‘12) 4461



Krill (オキアミ)

measurement

DWBA

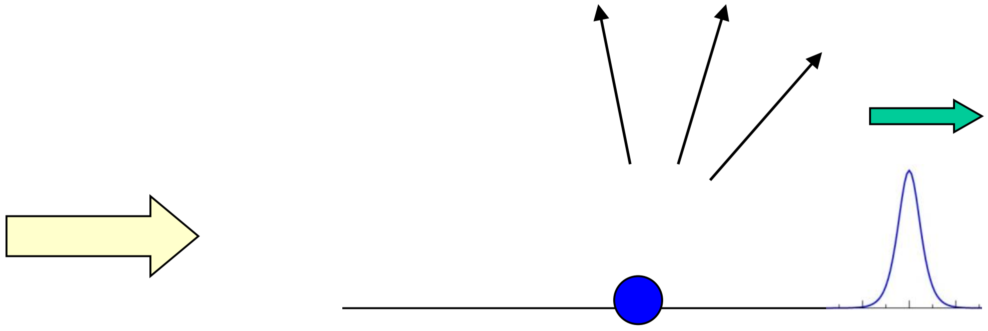


K. Akamatsu and M. Furusawa,
ICES J. of Marine Science 63 ('06) 36

Absorption cross sections

Reaction processes

- ➢Elastic scatt.
- ➢Inelastic scatt.
- ➢Transfer reaction
- ➢Compound nucleus formation (fusion)



Loss of incident flux (absorption)

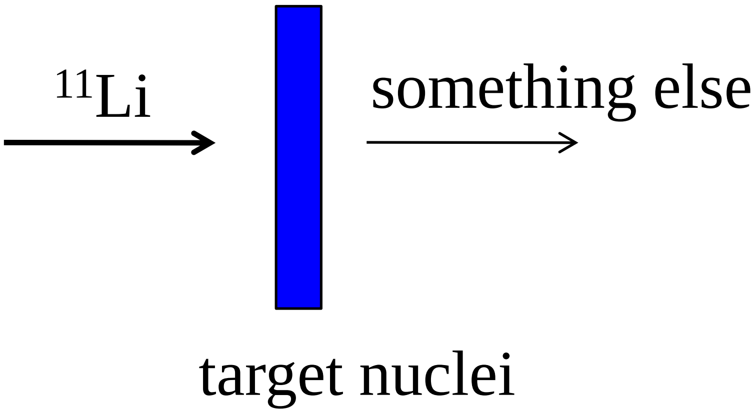
reaction cross sections

total scattering cross section - elastic cross section

$$\sigma_R = \sigma_{\mathrm{tot}} - \sigma_{\mathrm{el}}$$

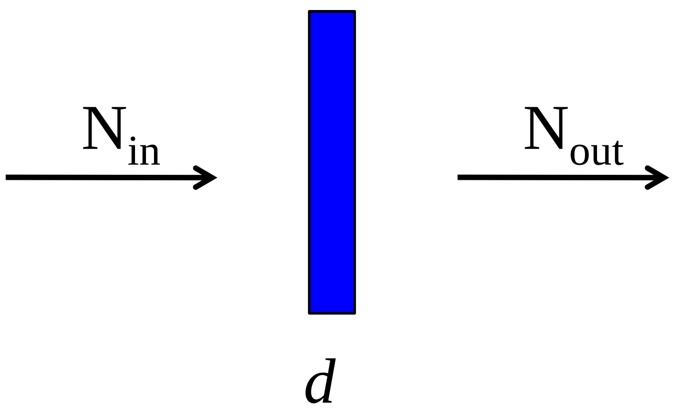
- fusion
- inelastic
- transfer

Interaction cross sections and halo nuclei



interaction cross section σ_I
= cross section for the change
of Z a/o N in the incident nucleus

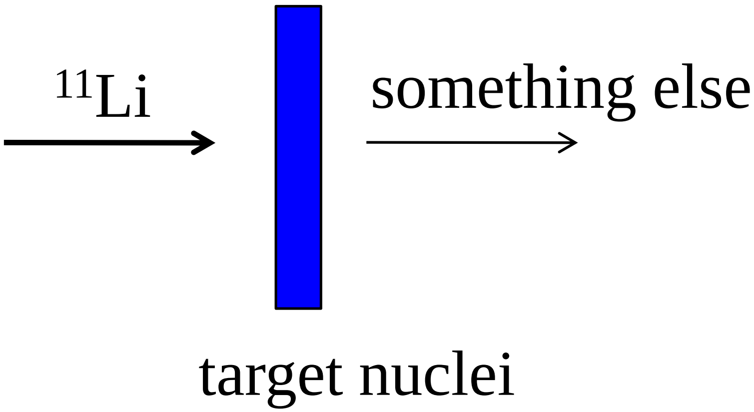
transmission method



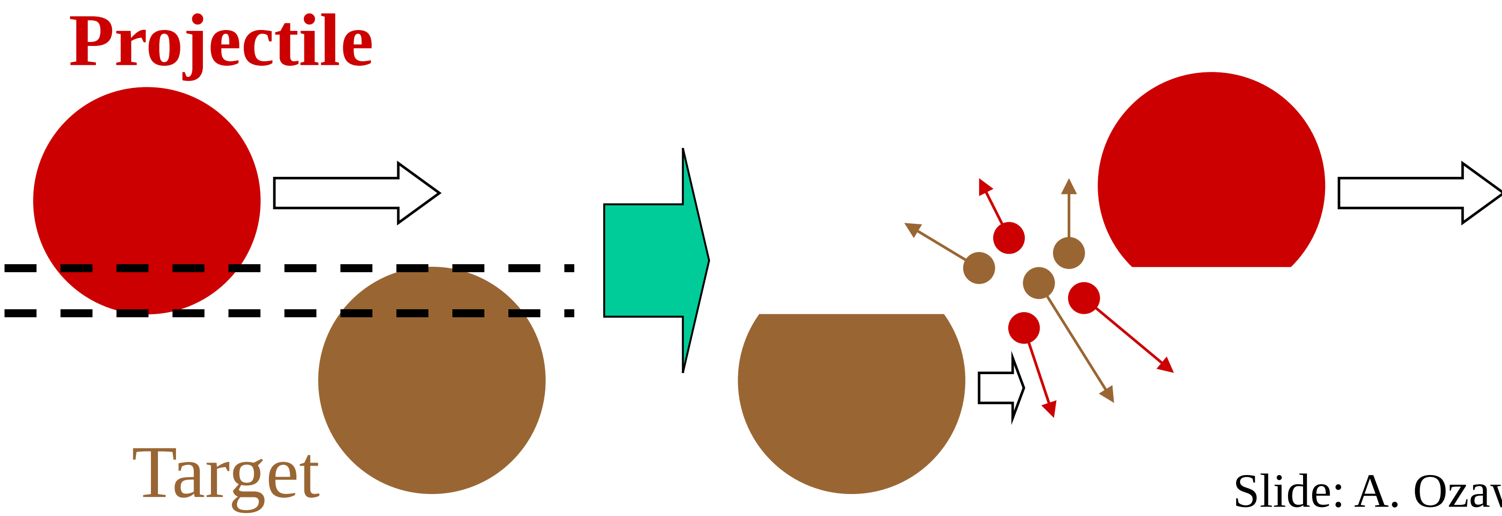
$$\sigma_R = -\frac{1}{t}\ln\left(\frac{N_{\rm out}}{N_{\rm in}}\right)$$

$$t = \rho_T \cdot d \cdot \epsilon$$

Interaction cross sections and halo nuclei



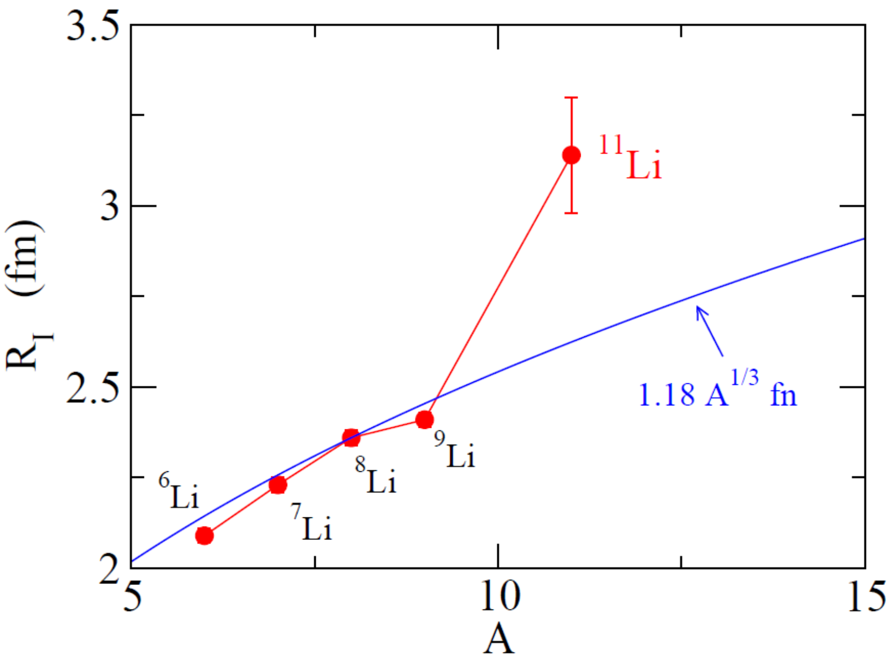
interaction cross section σ_I
= cross section for the change
of Z a/o N in the incident nucleus

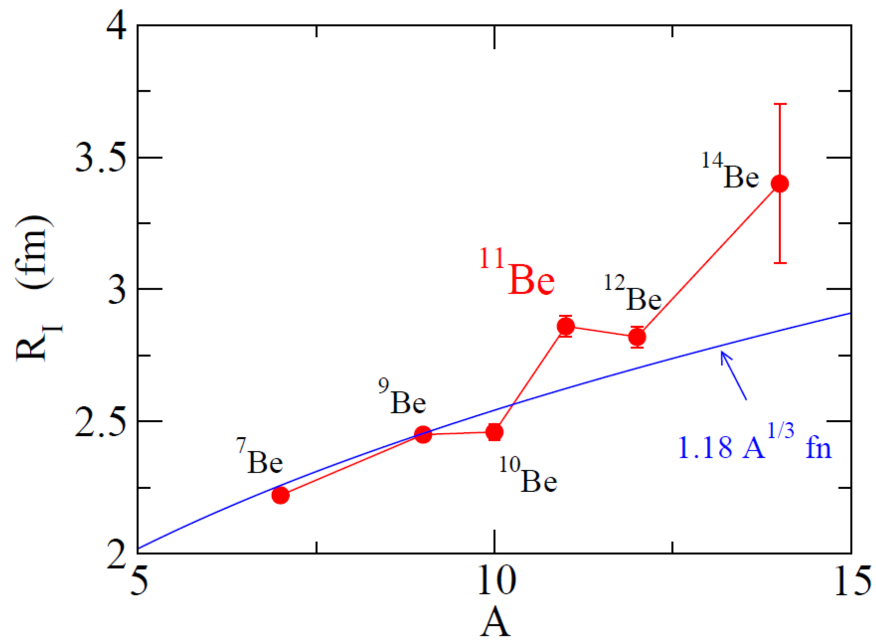


Slide: A. Ozawa

$$\sigma_I \sim \pi[R_I(P) + R_I(T)]^2 \longrightarrow R_I(P)$$

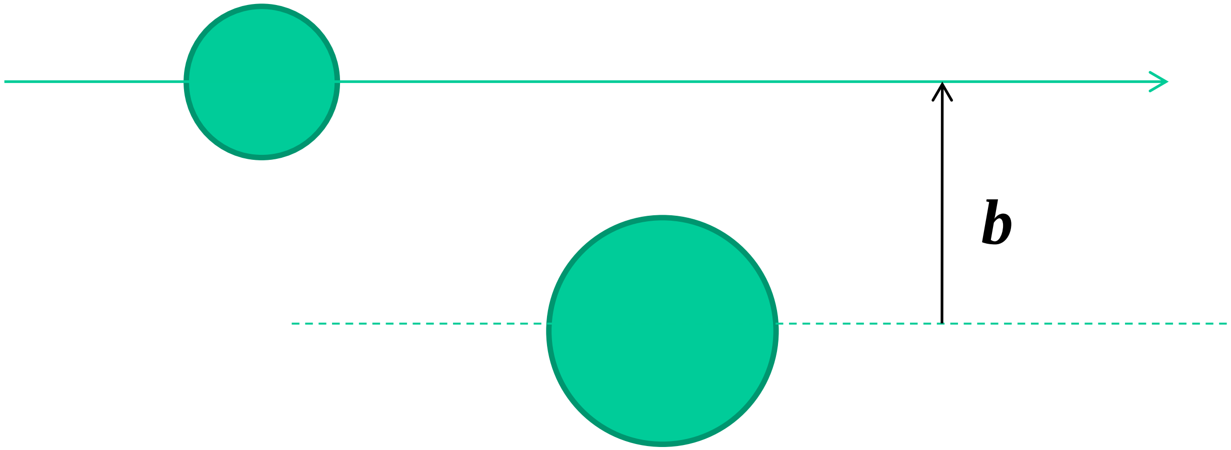
Discovery of **halo nuclei**





I. Tanihata, T. Kobayashi, O. Hashimoto et al., PRL55('85)2676; PLB206('88)592

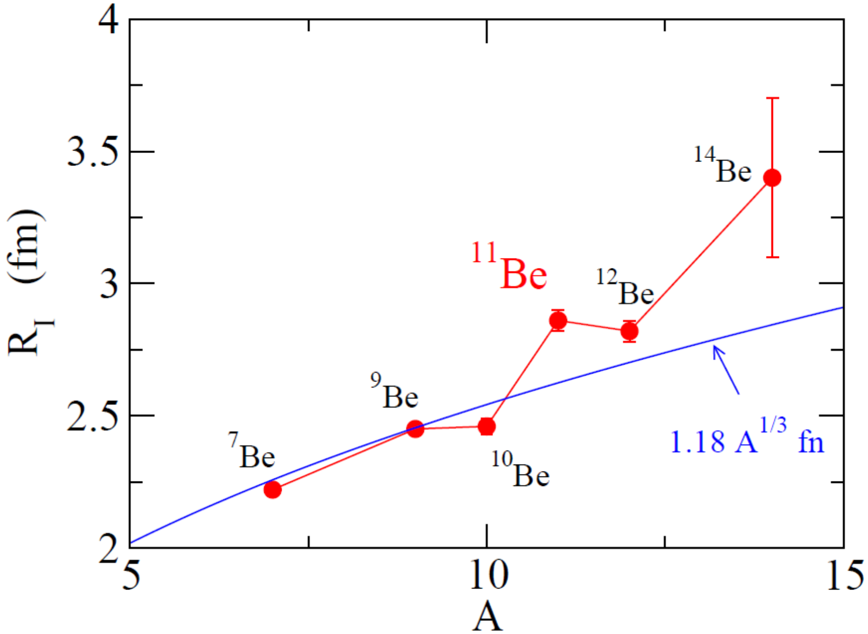
Reaction cross sections



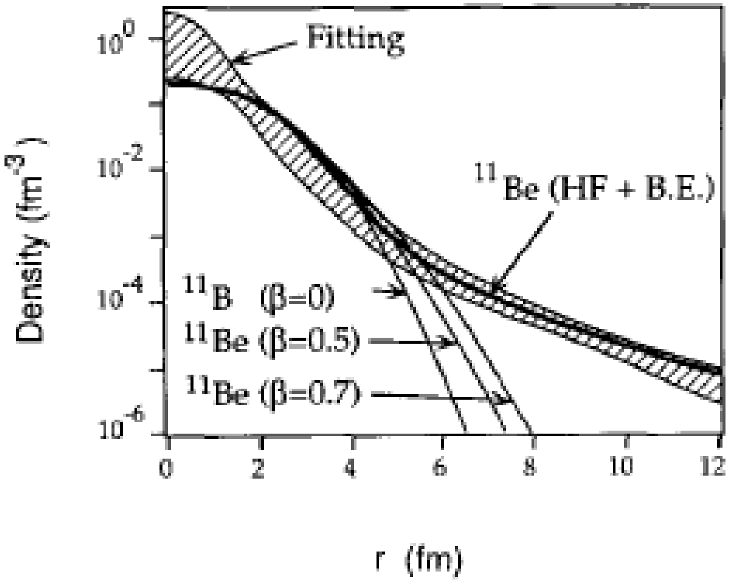
Glauber theory (optical limit approximation : OLA)

$$\sigma_R \sim 2\pi \int_0^\infty b db \left[1 - \exp\left(-\sigma_{NN} \int d^2 s \rho_P^{(z)}(\boldsymbol{s}) \rho_T^{(z)}(\boldsymbol{s} - \boldsymbol{b})\right)\right]$$

- straight-line trajectory (high energy scattering)
- adiabatic approximation
- simplified treatment for multiple scattering: $(1-x)^N \to e^{-Nx}$



Density distribution which explains the experimental σ_R



M. Fukuda et al., PLB268('91)339

$$\sigma_R \sim 2\pi \int_0^\infty bdb \left[1 - \exp\left(-\sigma_{NN} \int d^2 s \rho_P^{(z)}(\boldsymbol{s}) \rho_T^{(z)}(\boldsymbol{s}-\boldsymbol{b})\right)\right]$$