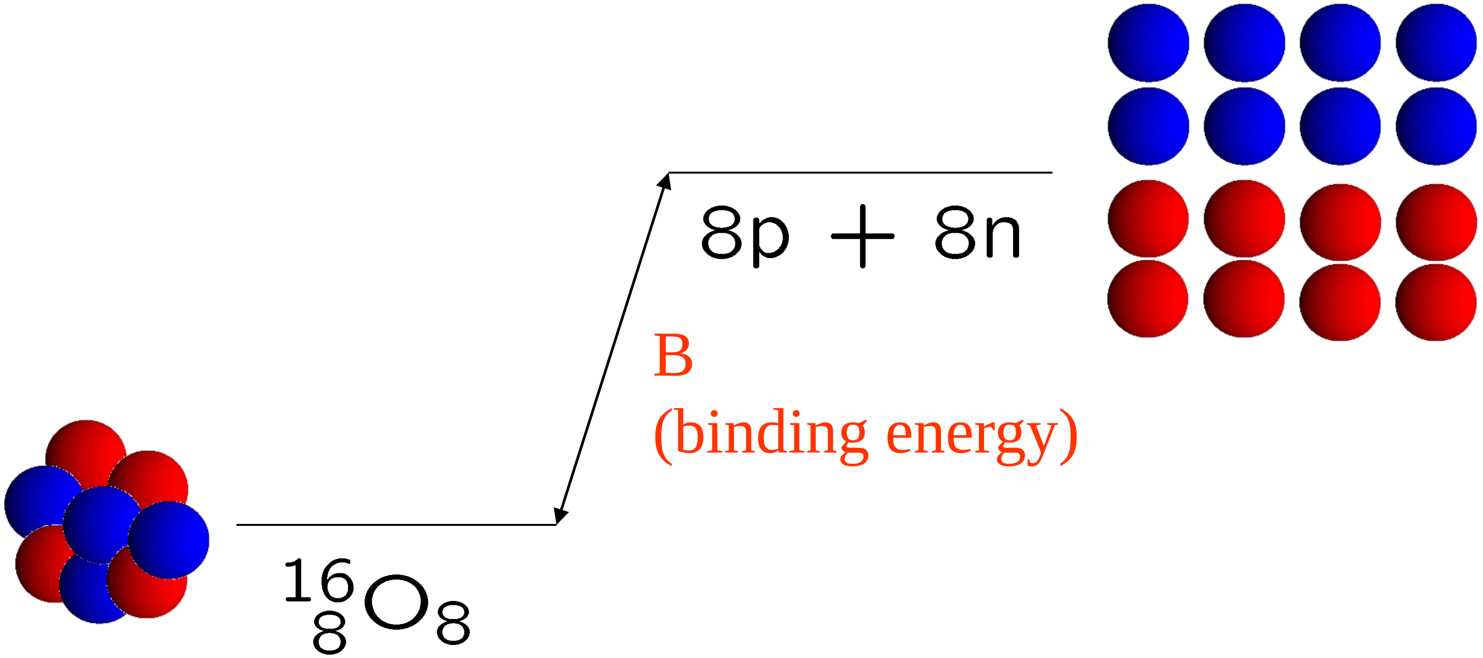
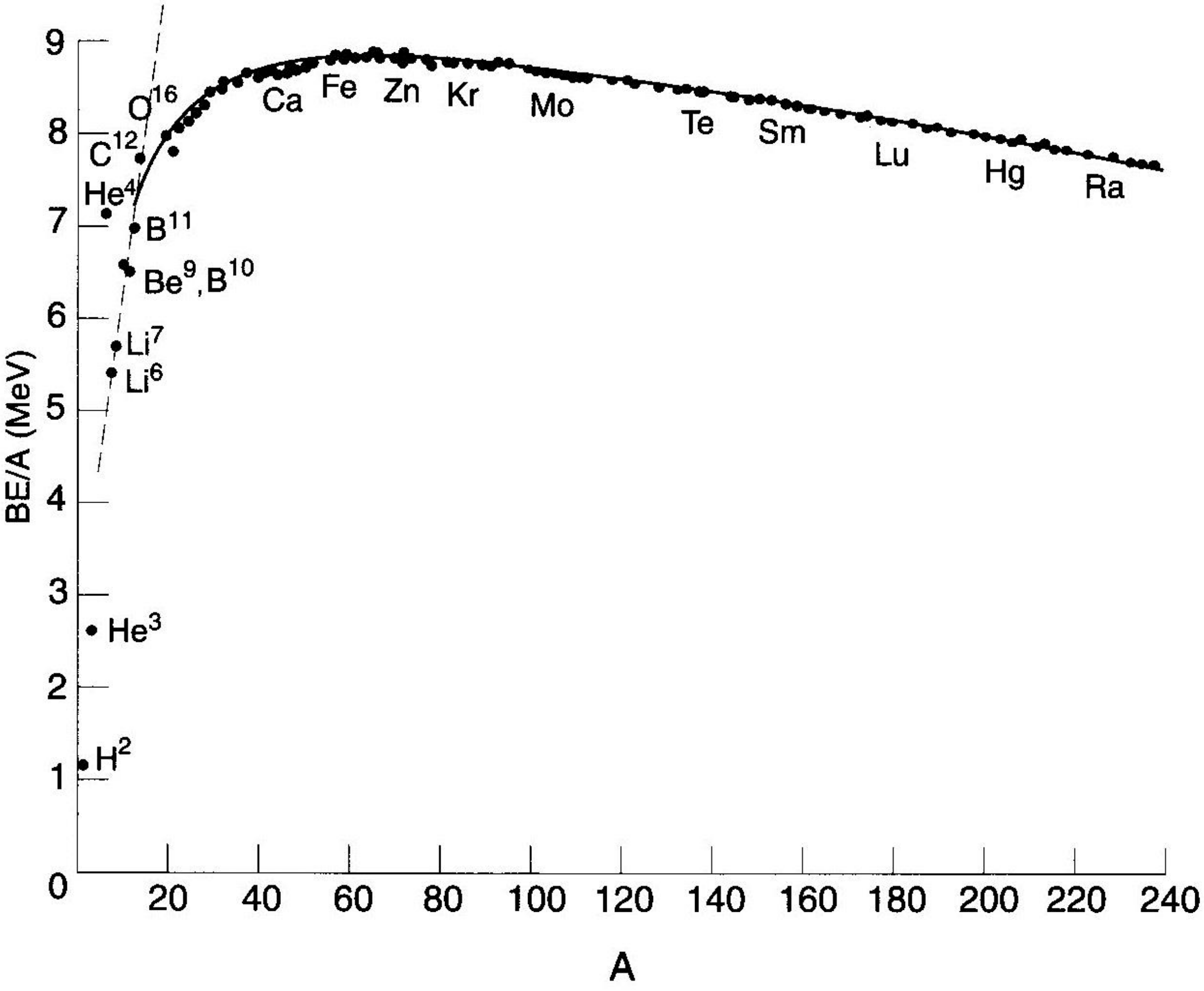
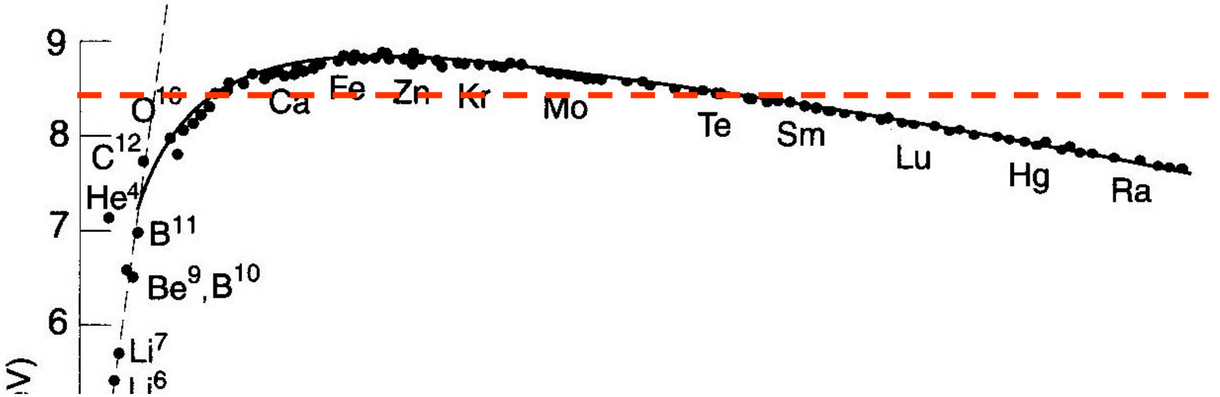
Nuclear Mass



$$m(N,Z)c^2 = Zm_pc^2 + Nm_nc^2 - B$$

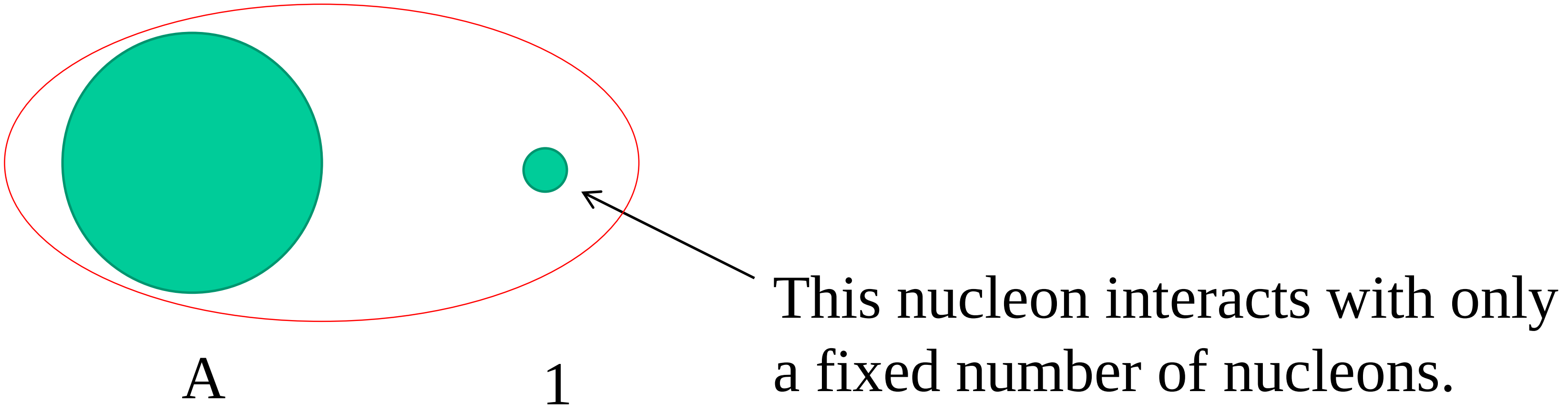


1. $B(N,Z)/A \sim 8.5$ MeV ($A > 12$) $\Longleftrightarrow$ Short range nuclear force



1. B(N,Z)/A ~ 8.5 MeV (A > 12)

Binding energy: increases only by a fixed amount (~ 8.5 MeV) by adding one particle

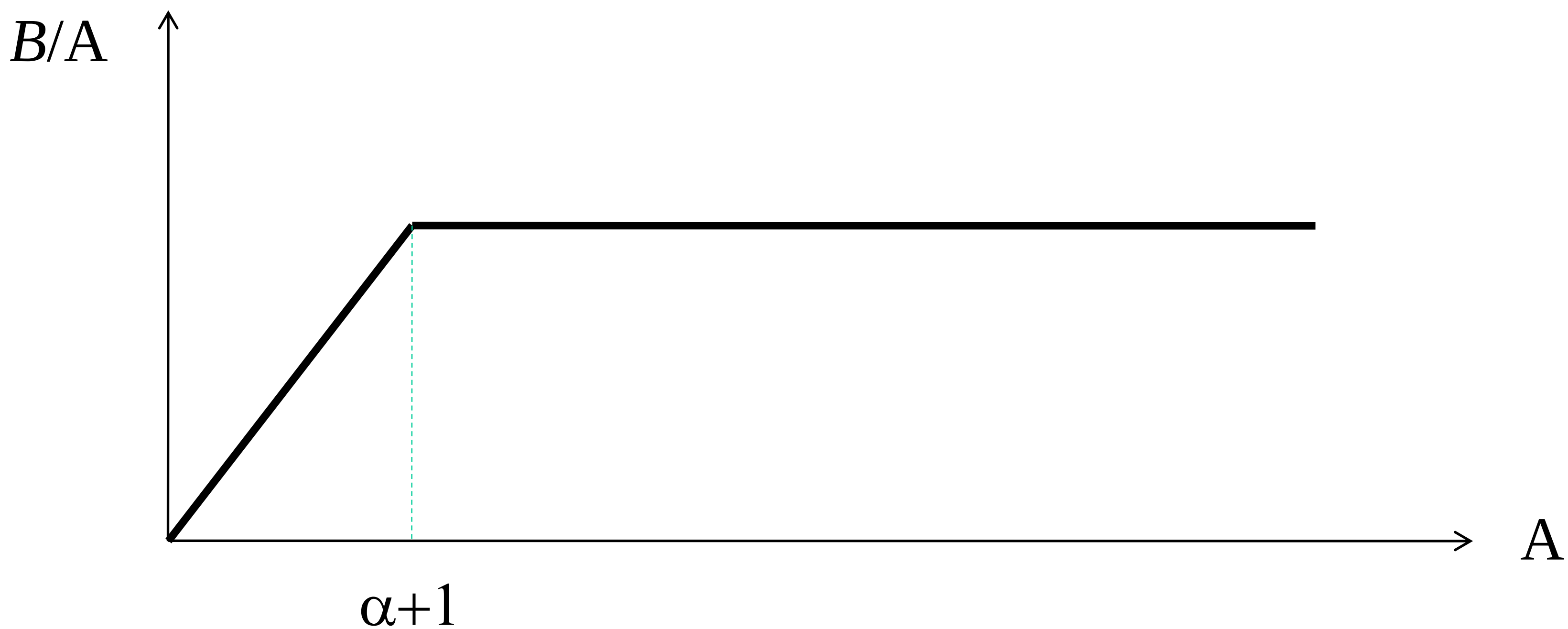


If one nucleon interacts only with surrounding α nucleons

$B \sim \alpha A/2 \longrightarrow B/A \sim \alpha/2$ (const.)

For $A < \alpha+1$, one nucleon interacts with all the other nucleons

$$\longrightarrow B/A \propto A$$

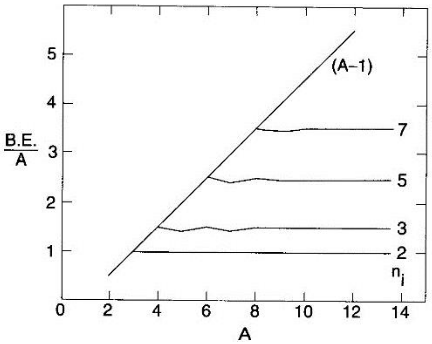


Semi-empirical mass formula

(Bethe-Weizacker formula: Liquid-drop model)

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

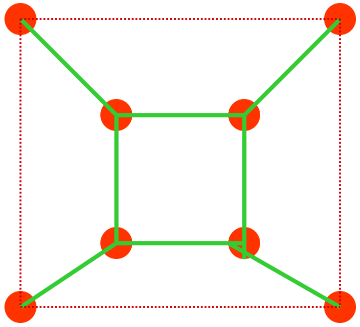
- Volume energy: $a_v A$



$$R_0 \sim 1.1 \times A^{1/3} \rightarrow V \propto A$$
$$S \propto A^{2/3}$$

- Surface energy: $-a_s A^{2/3}$

A nucleon near the surface interacts with fewer nucleons.



$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$

- Coulomb energy: $-a_C\, Z^2/A^{1/3}$

$$E_C = \frac{3}{5}\frac{Z^2 e^2}{R_C}$$ for a uniformly charged sphere

- Symmetry energy: $-a_{\mathrm{sym}}\,(N-Z)^2/A$

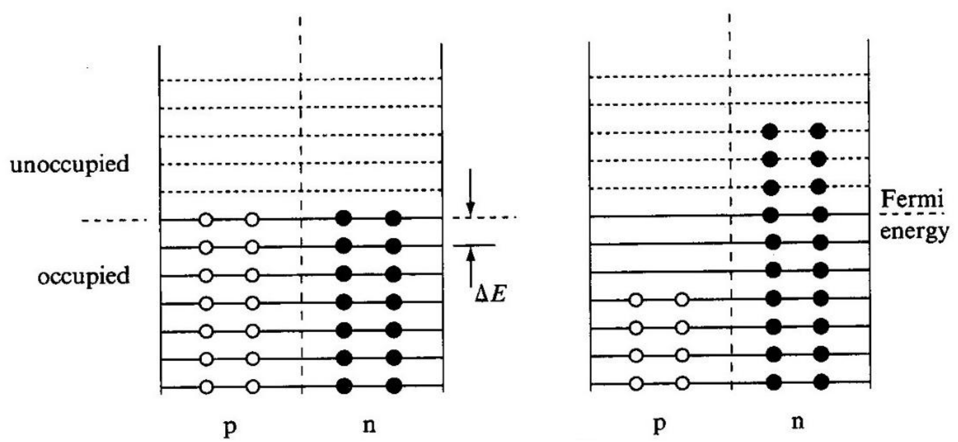
Potential energy $v_{nn} = v_{pp} = v, \quad v_{np} \sim 2v$

a nucleon interacting with nuclear matter:

$$N(v_{nn}N/A + v_{pn}Z/A) + Z(v_{pn}N/A + v_{pp}Z/A) = \frac{v}{2}(3A - (N-Z)^2/A)$$

Kinetic energy

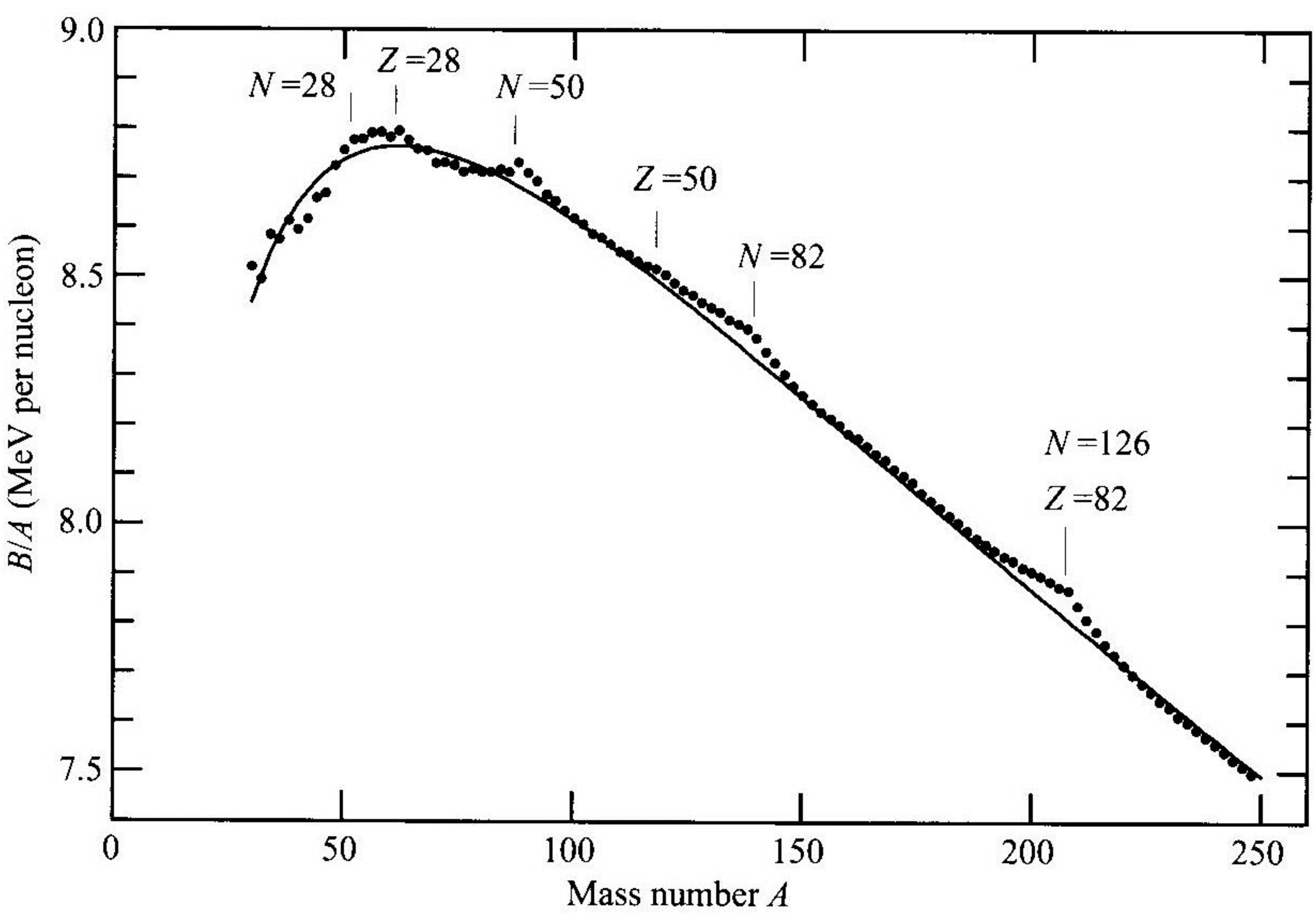
Pauli exclusion principle



When $E_k = k\,\Delta E$ and each level has two-fold degeneracy:

$$
\begin{aligned}
E &= \sum_{k=1}^{N/2} 2k\Delta E + \sum_{k=1}^{Z/2} 2k\Delta E \\
&= 2\Delta E \left(\sum_{k=1}^{N/2} k + \sum_{k=1}^{Z/2} k \right) \\
&= \frac{\Delta E}{2} \left(\frac{N^2 + Z^2}{2} + N + Z \right) \\
&= \frac{\Delta E}{2} \left(\frac{A^2}{4} + A + (N - Z)^2 \right)
\end{aligned}
$$

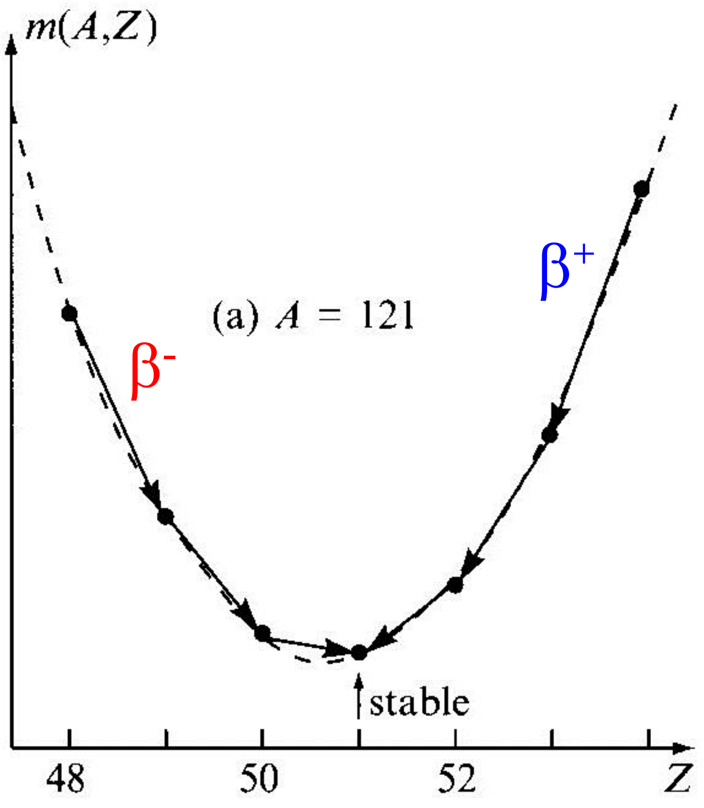
How well does the Bethe-Weizacker formula reproduce the data?



β-stability line

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

$$m(A,Z) = f(A) + a_C \frac{Z^2}{A^{1/3}} + a_{\text{sym}} \frac{(A-2Z)^2}{A}$$



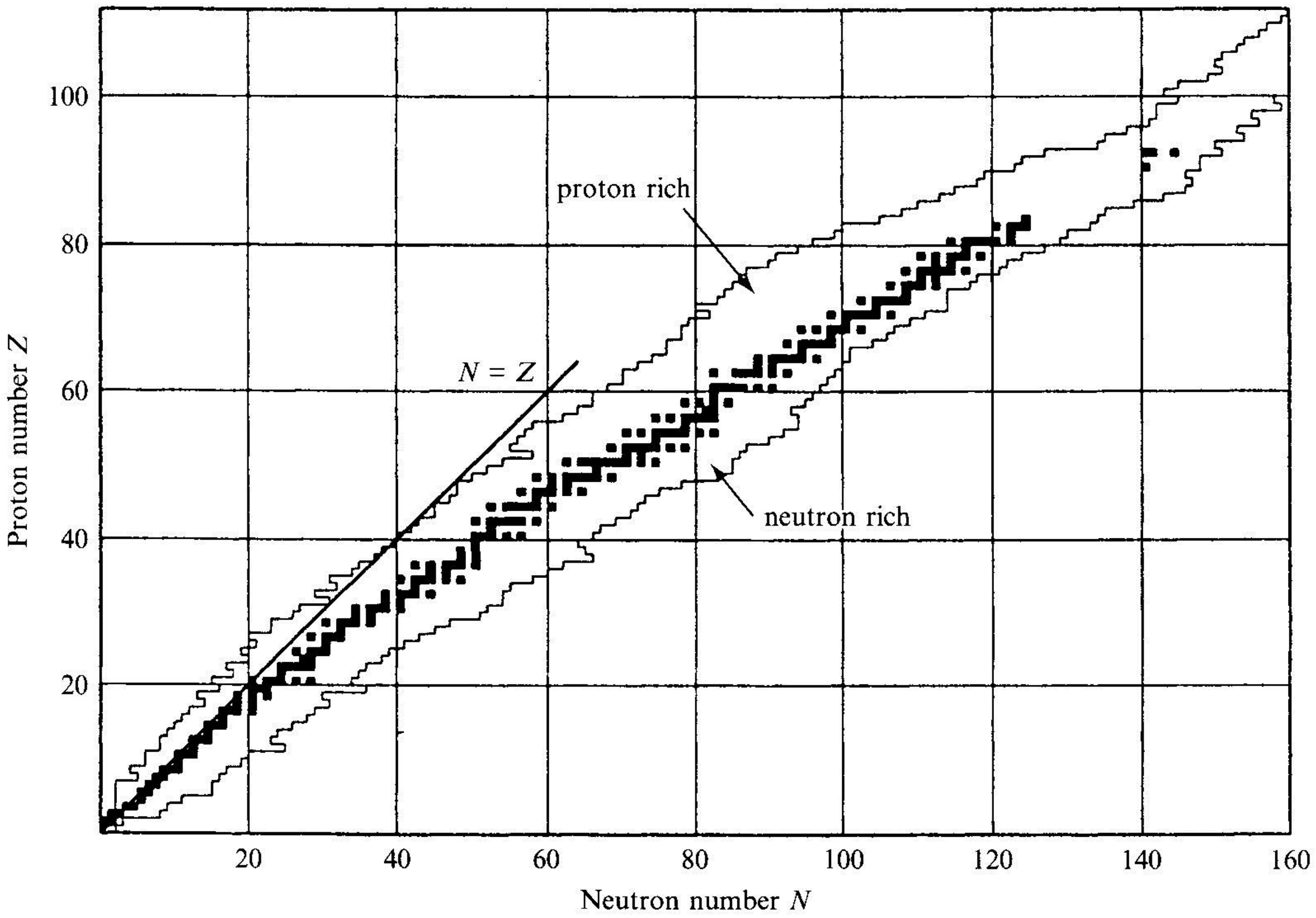
Stable nuclei (beta-stability line)

$$\left.\frac{\partial m}{\partial Z}\right|_{A=const.} = 0$$

$$Z = \frac{4a_{\text{sym}}}{2a_C/A^{1/3} + 8a_{\text{sym}}/A}$$

⇨ Z < A/2

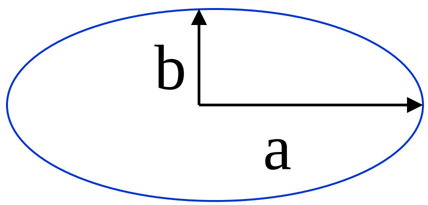
Nuclear Chart



Stable nuclei: $N \geq Z$

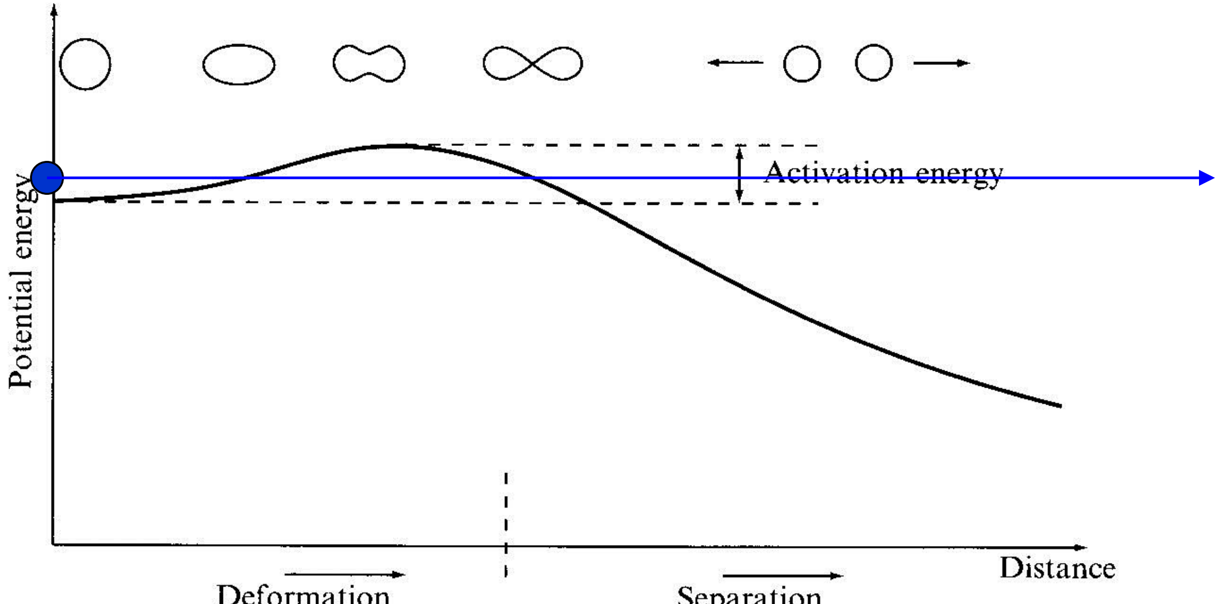
Nuclear Fission

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$



$$a = R \cdot (1+\epsilon)$$
$$b = R \cdot (1+\epsilon)^{-1/2}$$

$$E_{\mathrm{surf}} = E_{\mathrm{surf}}^{(0)} (1 + 2\epsilon^2/5 + \cdots)$$
$$E_C = E_C^{(0)} (1 - \epsilon^2/5 + \cdots)$$



Quantum tunneling

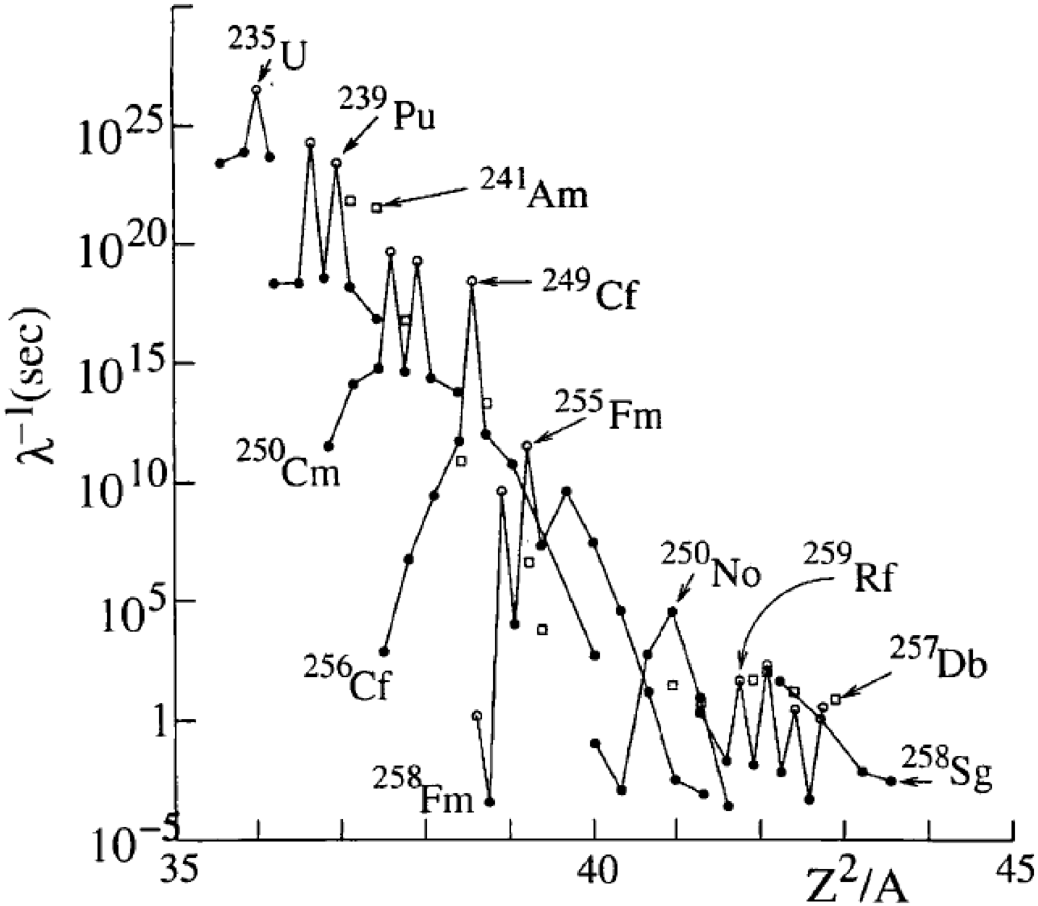


Fig. 6.4. Spontaneous fission lifetimes as a function of the fission parameter Z^2/A for selected nuclei. Circles are for even-Z nuclei. filled circles for even-even nuclei and open circles for even-odd nuclei. Squares are for odd-Z nuclei.

Life times for spontaneous fission:

large Z^2/A ⟶ low fission barrier

⟶ short half-life

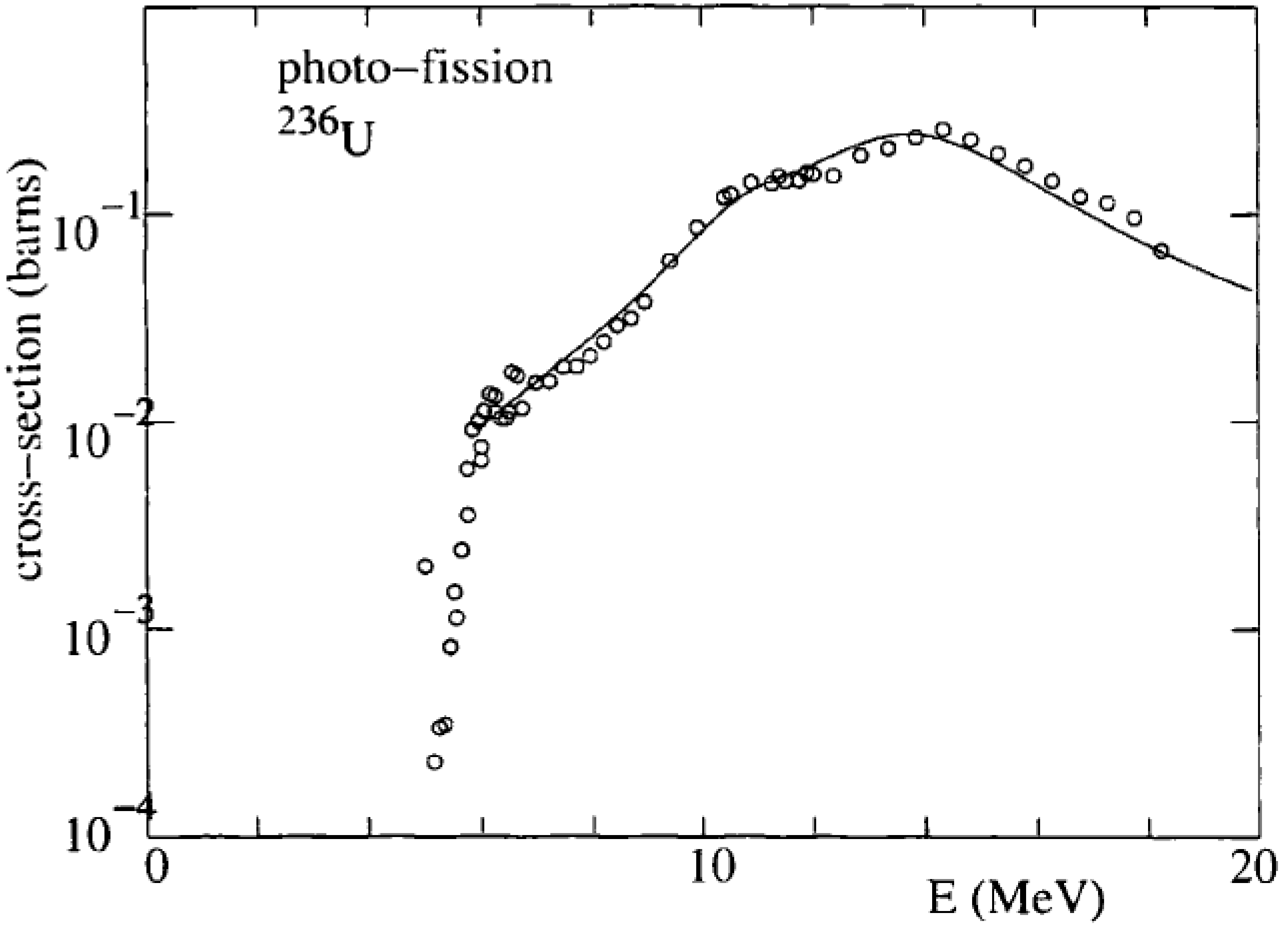
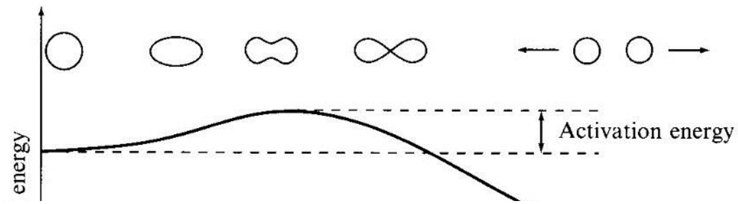


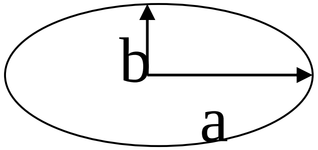
Fig. 6.5. Cross-section for $\gamma\,^{236}U \rightarrow$ fission [30].

photo-fission cross sections: threshold at ~ 5.7 MeV

(fission barrier height: ~ 5.7 MeV)

Collective Vibrations





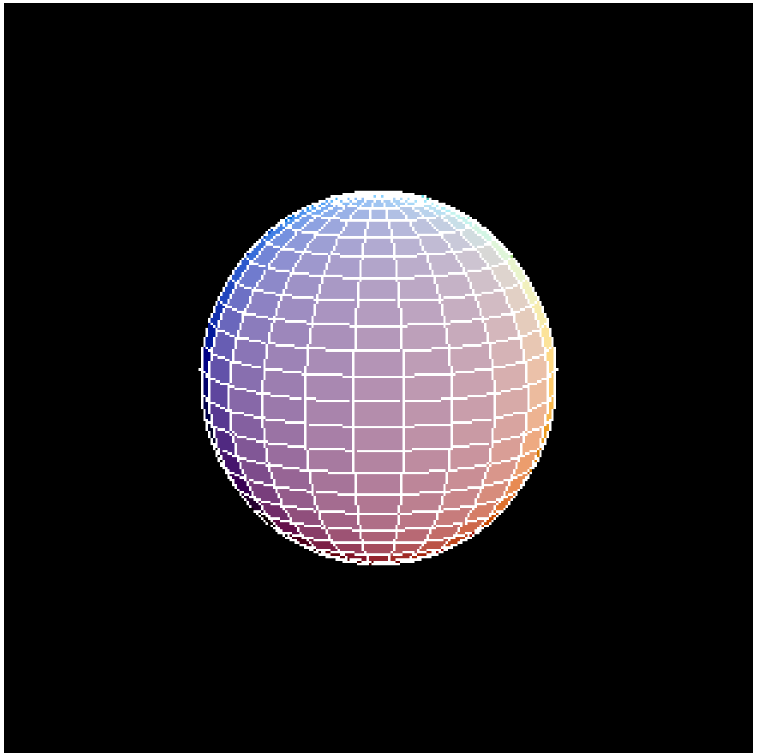
$$\begin{aligned} a &= R\cdot(1+\epsilon) \\ b &= R\cdot(1+\epsilon)^{-1/2} \end{aligned}$$

In general, $R(\theta,\phi) = R_0\left(1+\sum_{\lambda,\mu}\alpha_{\lambda\mu}Y^*_{\lambda\mu}\right)$

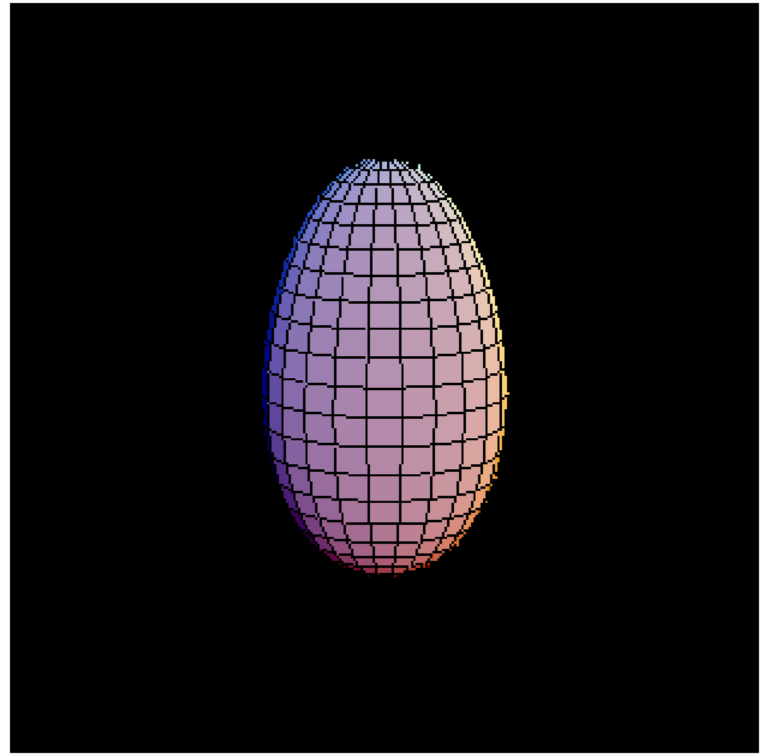
$$V = \frac{1}{2}\sum_{\lambda,\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$

Quantization: Harmonic Vibrations

$$R(\theta,\phi) = R_0\left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}\right) \qquad V = \frac{1}{2}\sum_{\lambda,\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$



λ=2: Quadrupole vibration



λ=3: Octupole vibration

Double phonon states

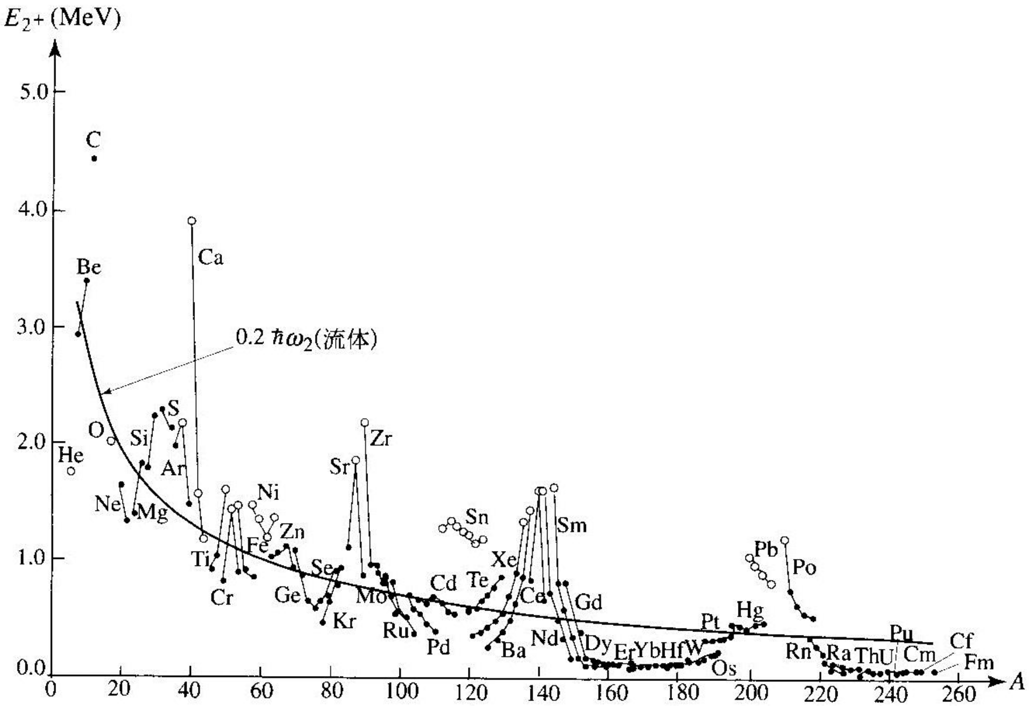
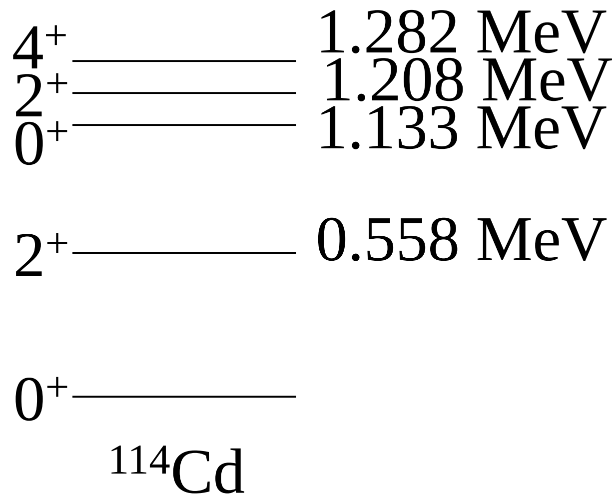


図 3.2 偶々核の第 1 励起 2^+ 状態の励起エネルギー

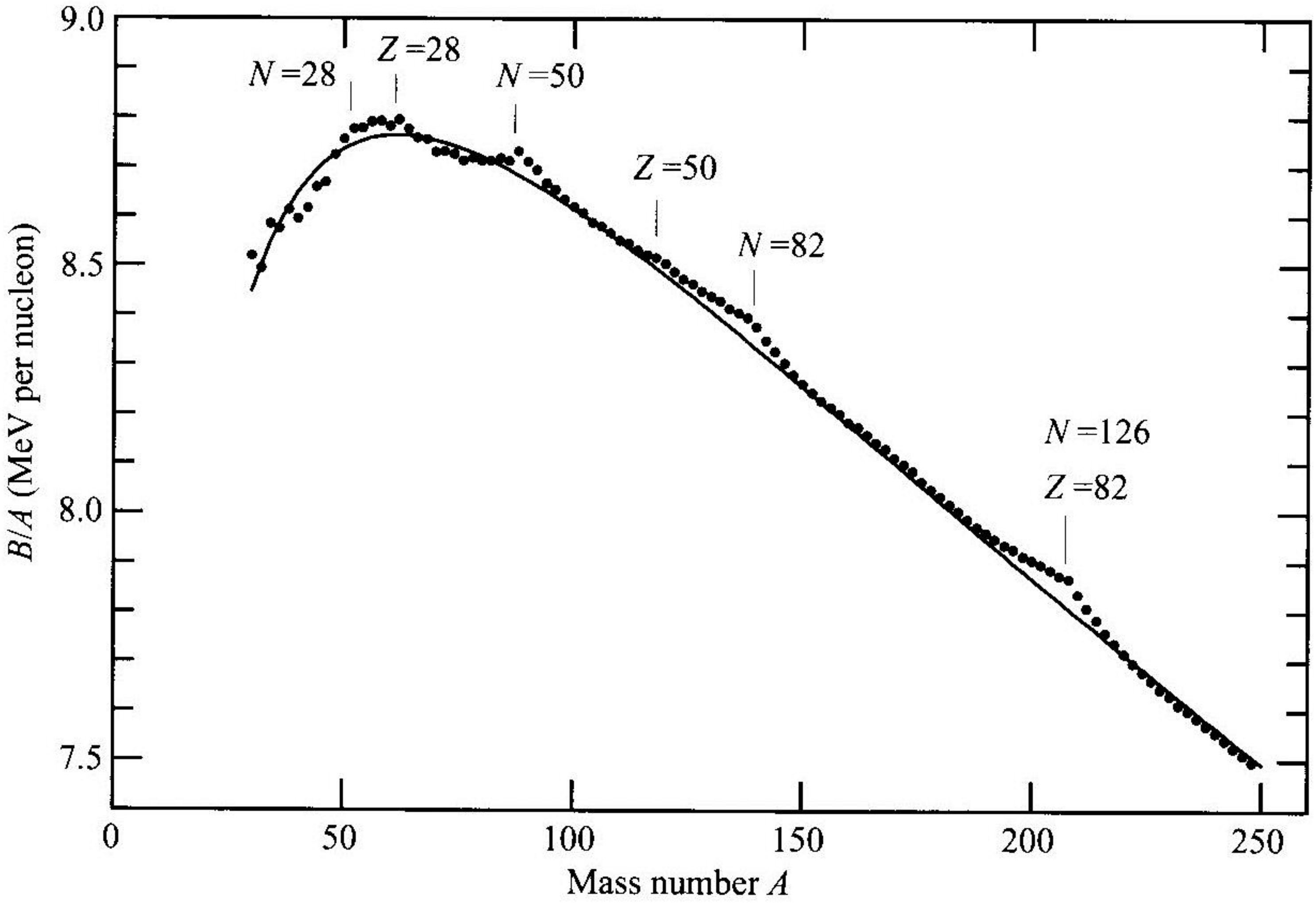


Microscopic description

⇨ Random phase approximation (RPA)

[later in this lecture]

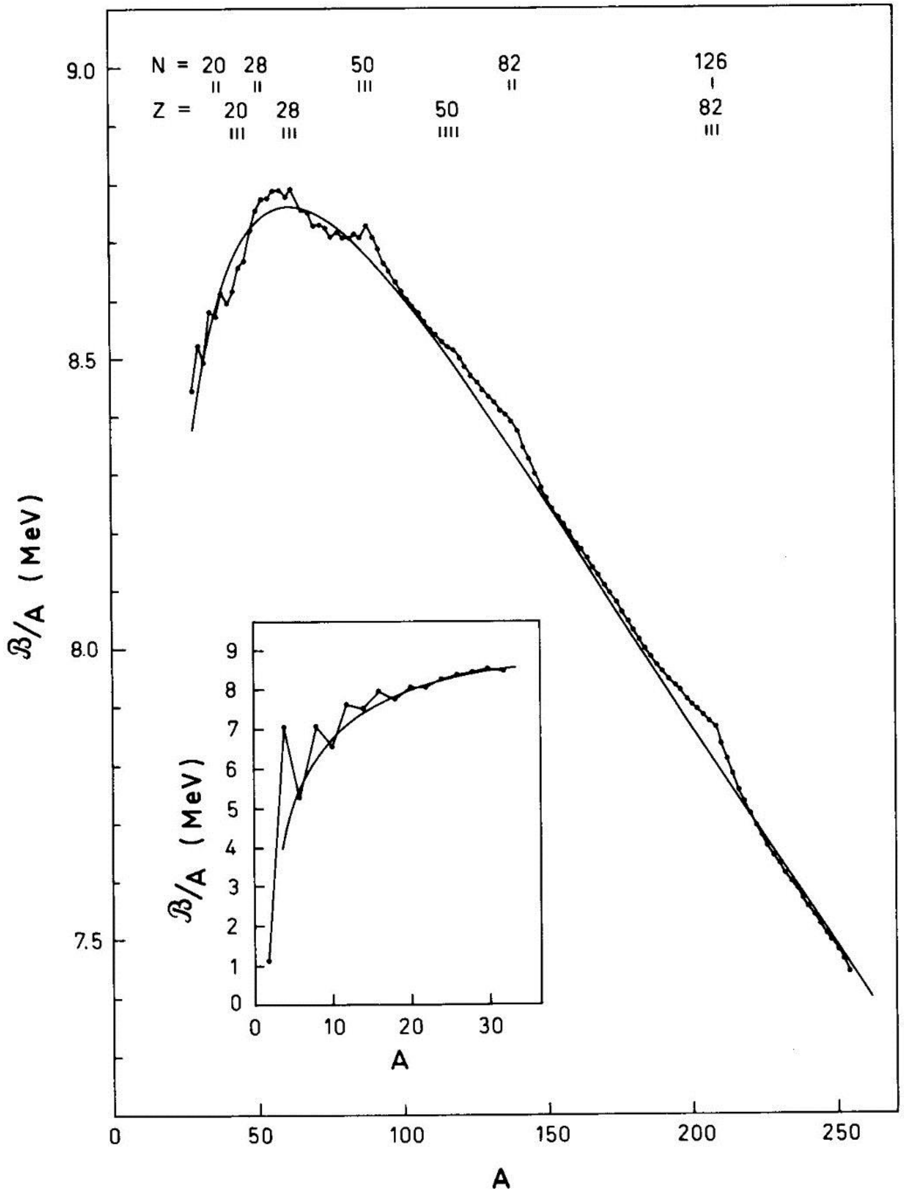
How well does the Bethe-Weizacker formula reproduce the data?



cf. N,Z = 2, 8, 20, 28, 50, 82, 126: large binding energy
"magic numbers"

Shell Structure

$$B(N,Z) = B_{\mathrm{macro}}(N,Z) + B_{\mathrm{micro}}(N,Z)$$



- Smooth part

$$B_{\mathrm{macro}}(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$

- Fluctuation part

$$B_{\mathrm{micro}} = B_{\mathrm{pair}} + B_{\mathrm{shell}}$$

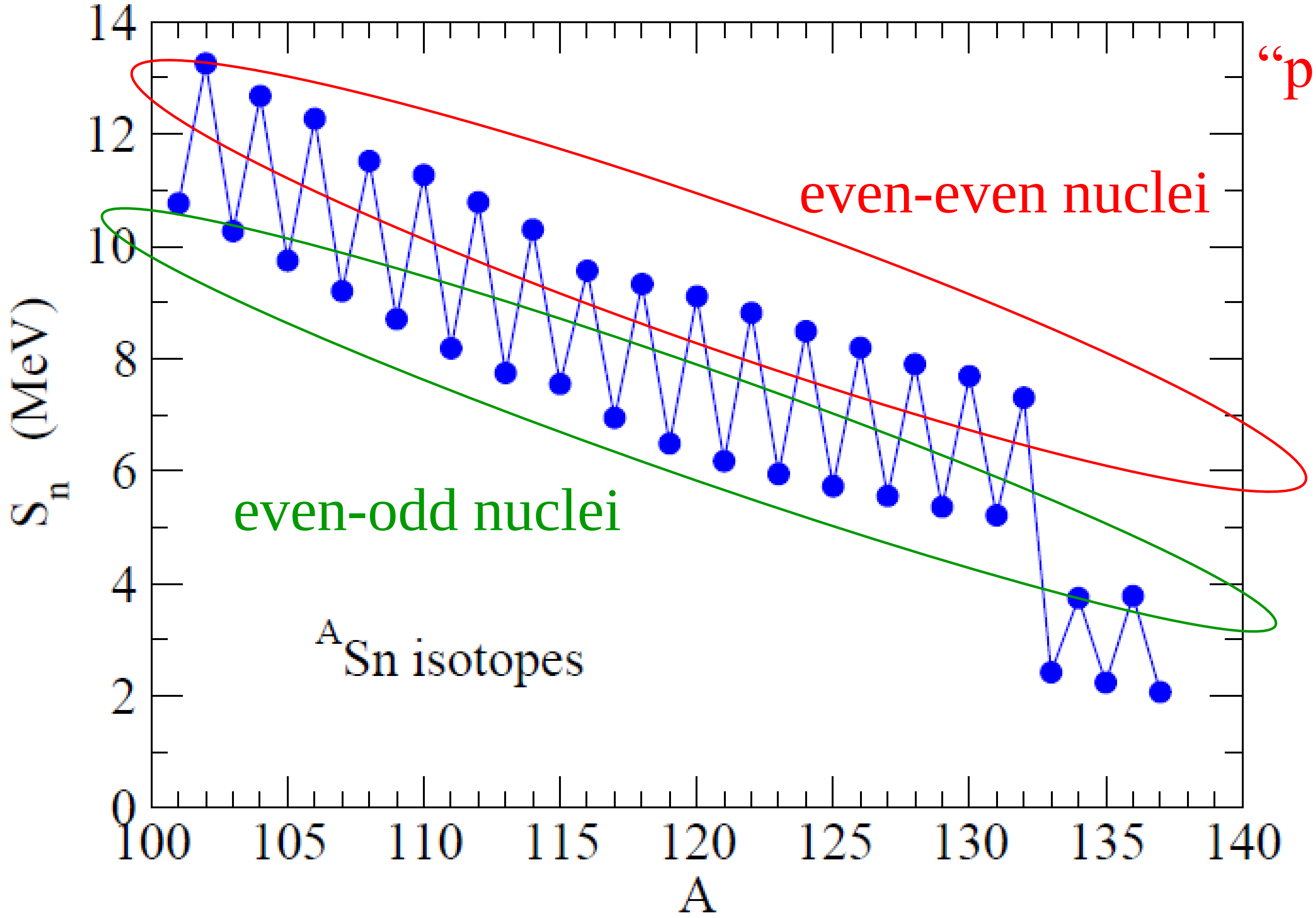
Liquid drop model:

$$B_{\mathrm{LDM}} = B_{\mathrm{macro}} + B_{\mathrm{pair}}$$

Pairing energy

even-odd staggering

A larger energy required to remove one neutron from even number than from odd number



1n separation energy: $S_n\,(A,Z) = B(A,Z) - B(A-1,Z)$

Pairing Energy

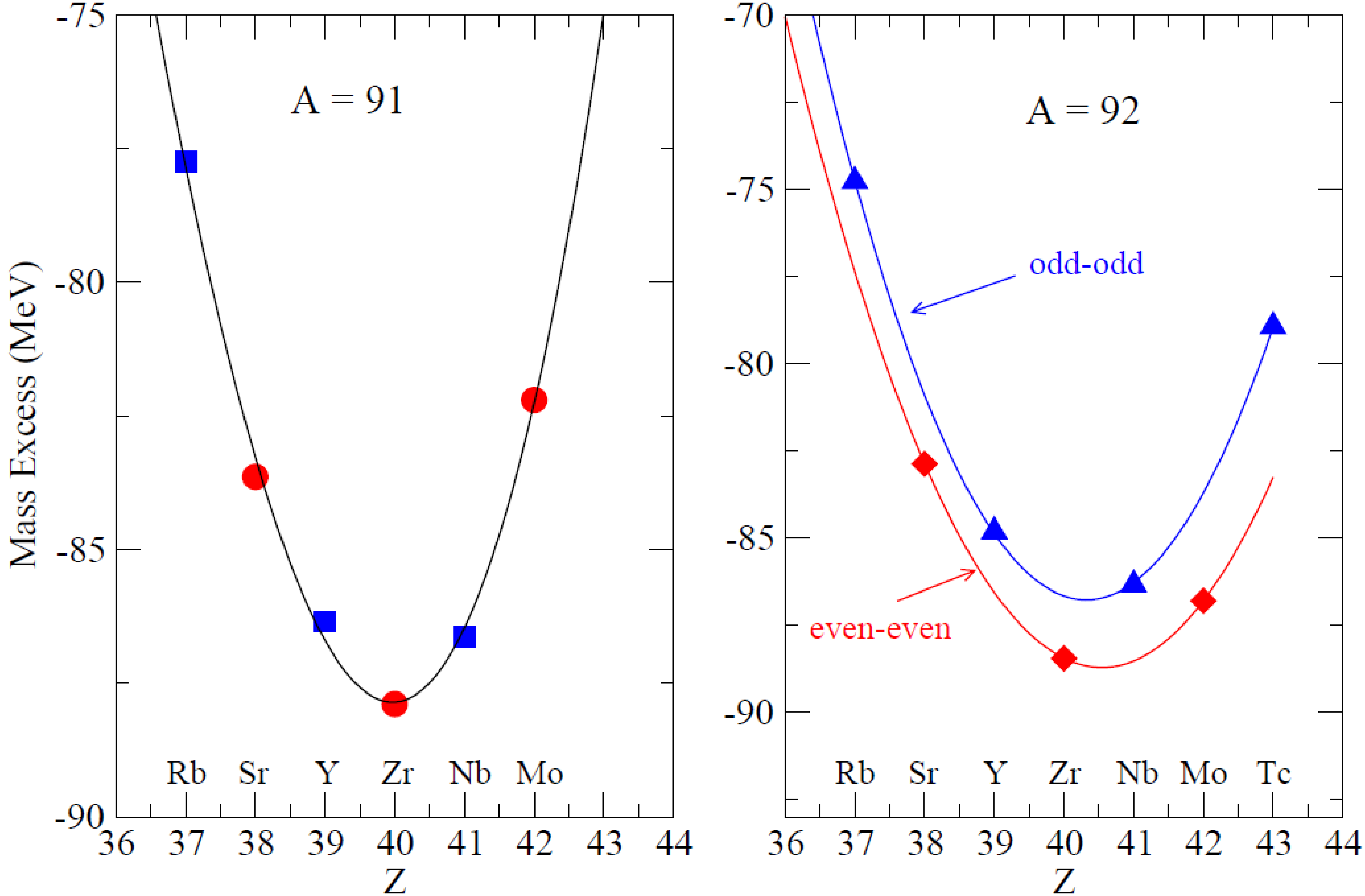
Extra binding when like nucleons form a spin-zero pair

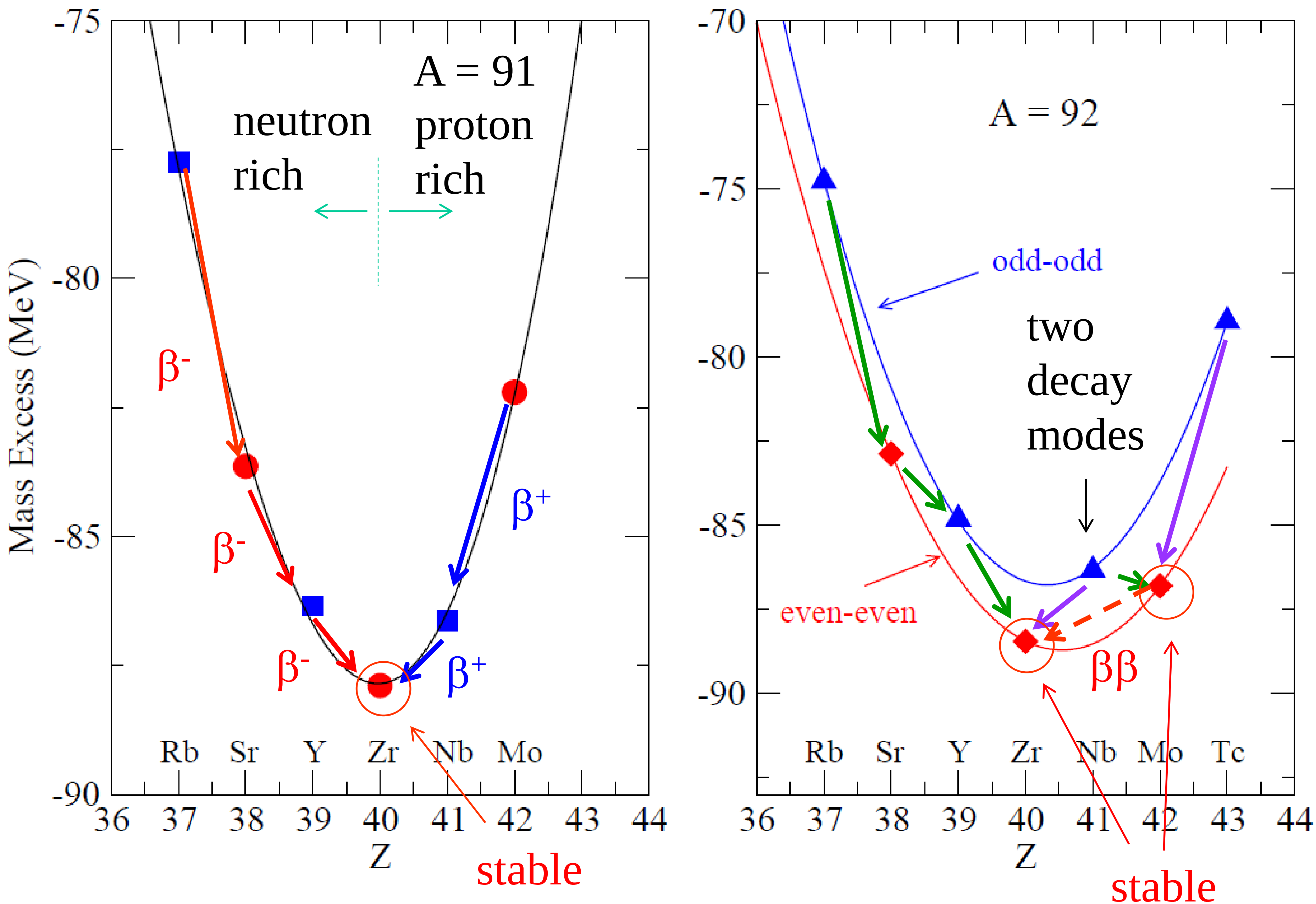
Example:

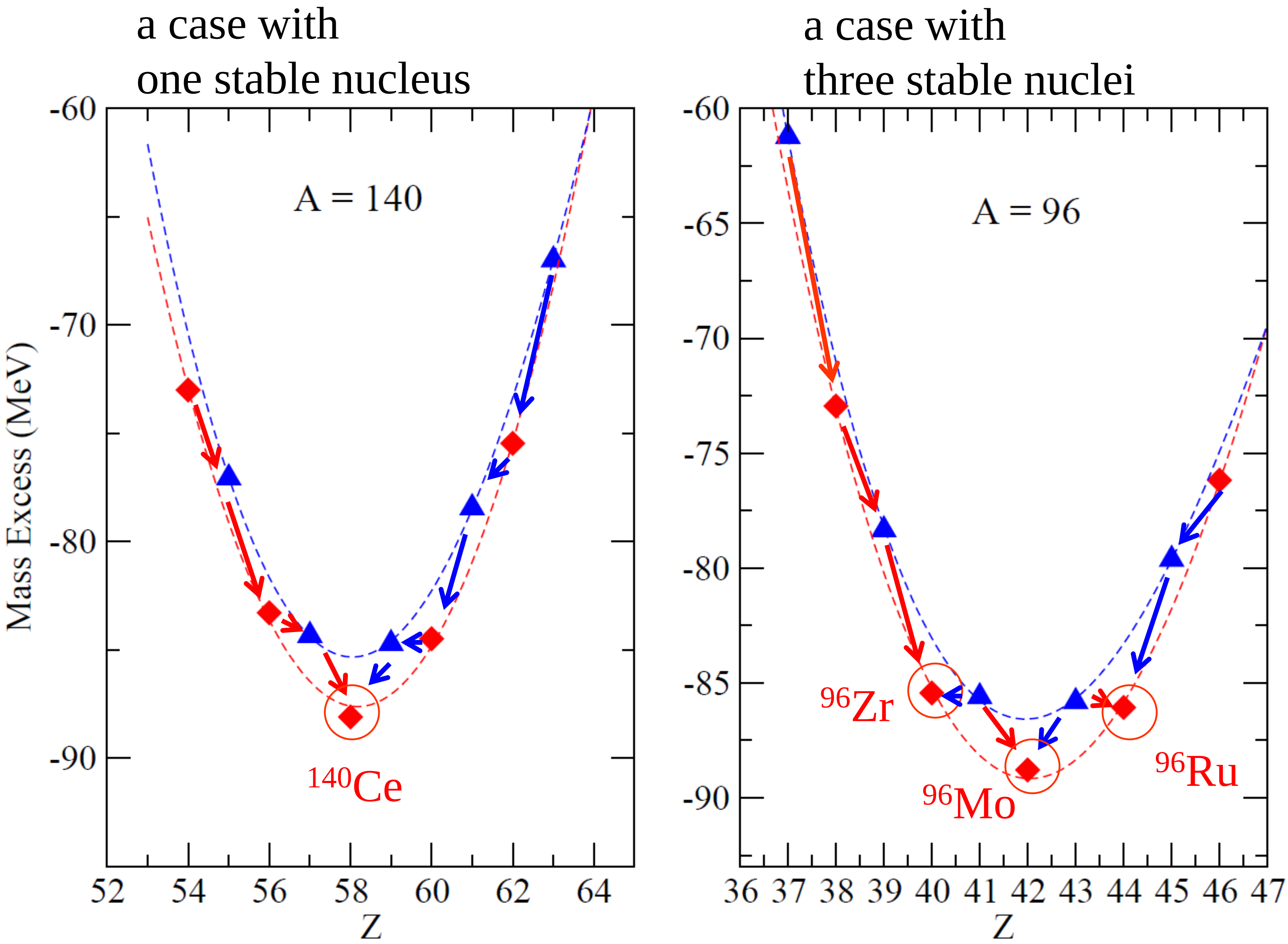
	Binding energy (MeV)
${}^{210}_{82}Pb_{128} = {}^{208}_{82}Pb_{126}+2n$	1646.6
${}^{210}_{83}Bi_{127} = {}^{208}_{82}Pb_{126}+n+p$	1644.8
${}^{209}_{82}Pb_{127} = {}^{208}_{82}Pb_{126}+n$	1640.4
${}^{209}_{83}Bi_{126} = {}^{208}_{82}Pb_{126}+p$	1640.2

$$\begin{aligned} B_{\mathrm{pair}} &= \Delta \quad &(\text{for even} - \text{even}) \\ &= 0 \quad &(\text{for even} - \text{odd}) \\ &= -\Delta \quad &(\text{for odd} - \text{odd}) \end{aligned}$$

$$
\begin{aligned}
B_{\text{pair}} &= \Delta & \text{(for even}-\text{even)} \\
&= 0 & \text{(for even}-\text{odd)} \\
&= -\Delta & \text{(for odd}-\text{odd)}
\end{aligned}
$$




Mass Excess (MeV)
-75
-80
-85
-90
A = 91
neutron
rich
proton
rich
β-
β-
β-
β+
β+
Rb
Sr
Y
Zr
Nb
Mo
36
37
38
39
40
41
42
43
44
Z
stable
-70
-75
-80
-85
-90
A = 92
odd-odd
two
decay
modes
even-even
ββ
Rb
Sr
Y
Zr
Nb
Mo
Tc
36
37
38
39
40
41
42
43
44
Z
stable



a case with
one stable nucleus
a case with
three stable nuclei
A = 140
A = 96
Mass Excess (MeV)
-60
-65
-70
-75
-80
-85
-90
Z
52 54 56 58 60 62 64
36 37 38 39 40 41 42 43 44 45 46 47
140Ce
96Zr
96Mo
96Ru

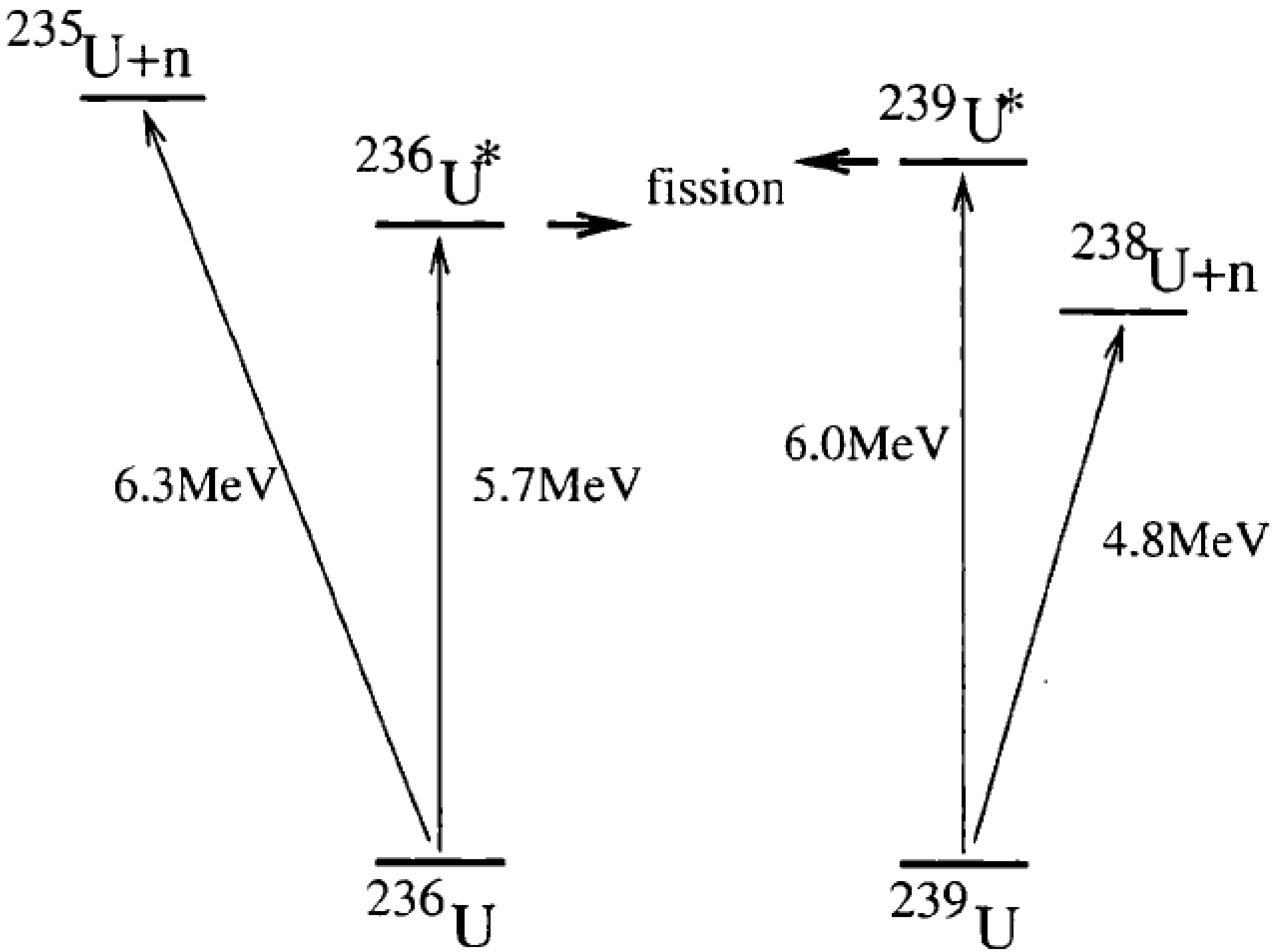


Fig. 6.6. Levels of the systems $A = 236$ and $A = 239$ involved in the fission of ^{236}U and ^{239}U. The addition of a motionless (or thermal) neutron to ^{235}U can lead to the fission of ^{236}U. On the other hand, fission of ^{239}U requires the addition of a neutron of kinetic energy $T_n = 6.0 - 4.8 = 1.2$ MeV.

Relation between fission barrier height and 1n separation energy