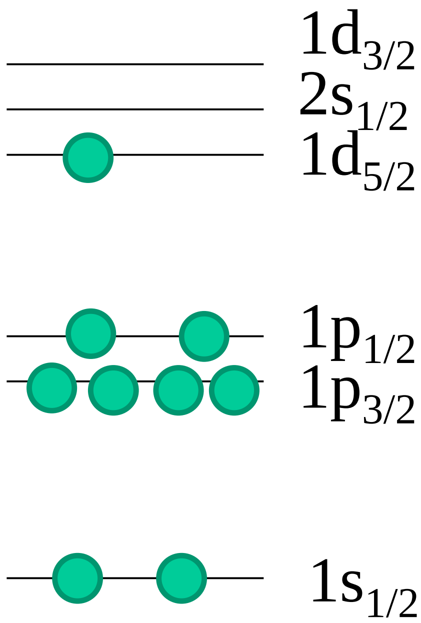
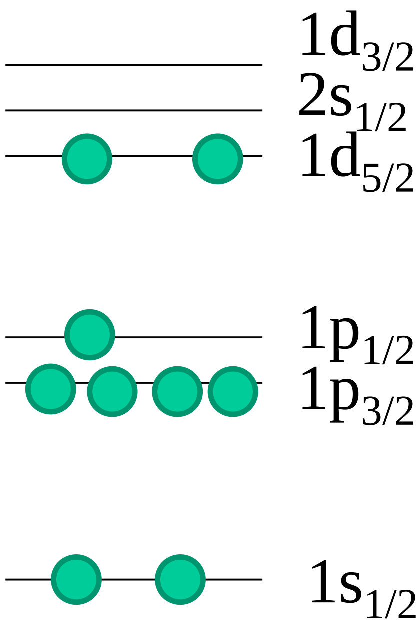
# Nuclear Shell Model

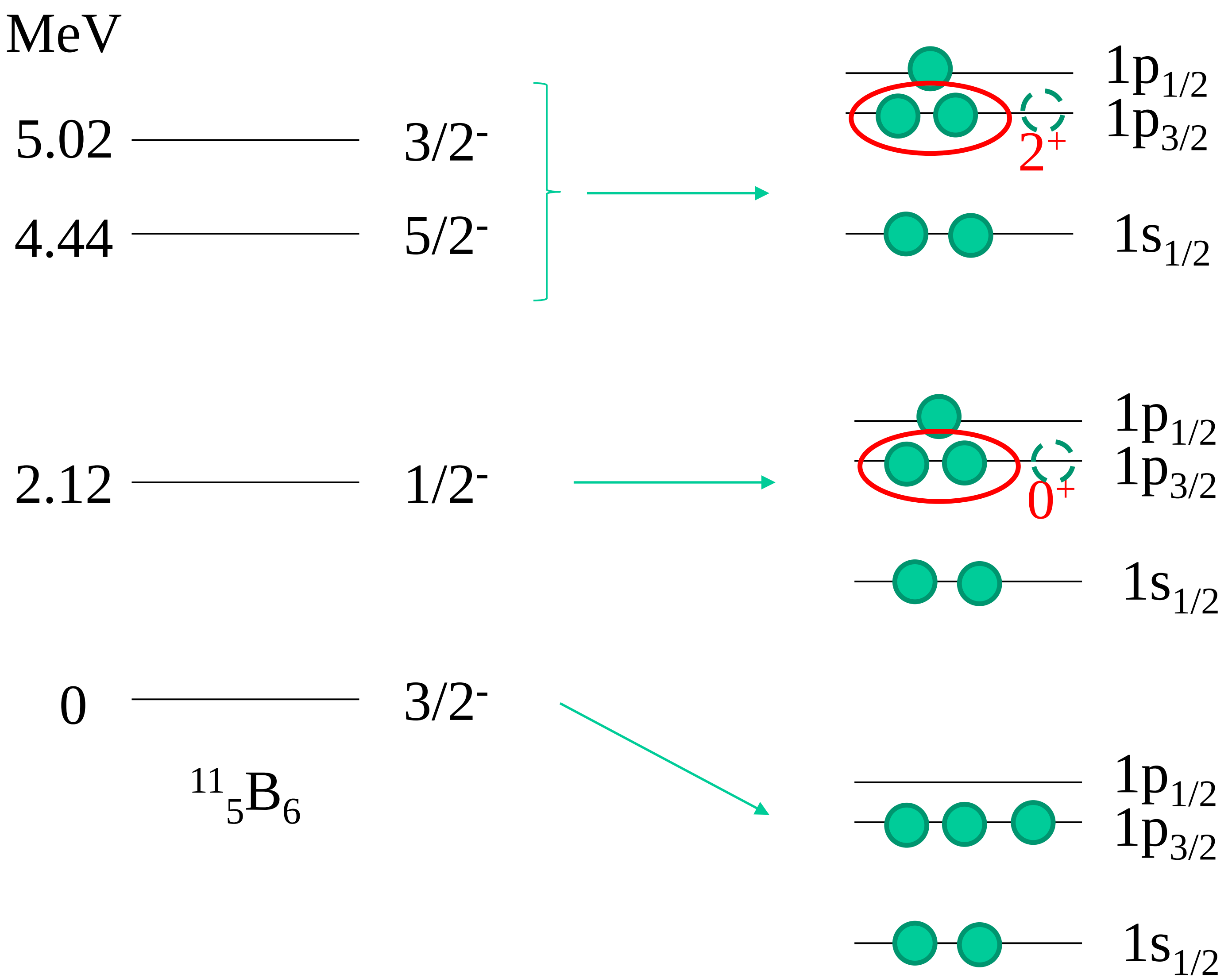



configuration 1



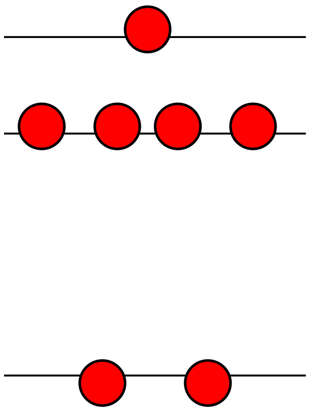

configuration 2

# Nuclear Shell Model



## Level scheme of $^{11}_{4}Be_7$

With a spherical potential :




The g.s. of $^{11}$Be : $I^{\pi} = 1/2^{-}$

very artificial

In reality…..

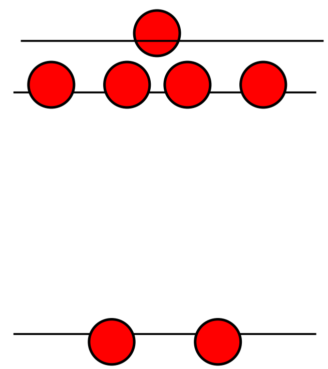
0.32 MeV

$1/2^{-}$

$1/2^{+}$

$^{11}$Be

?



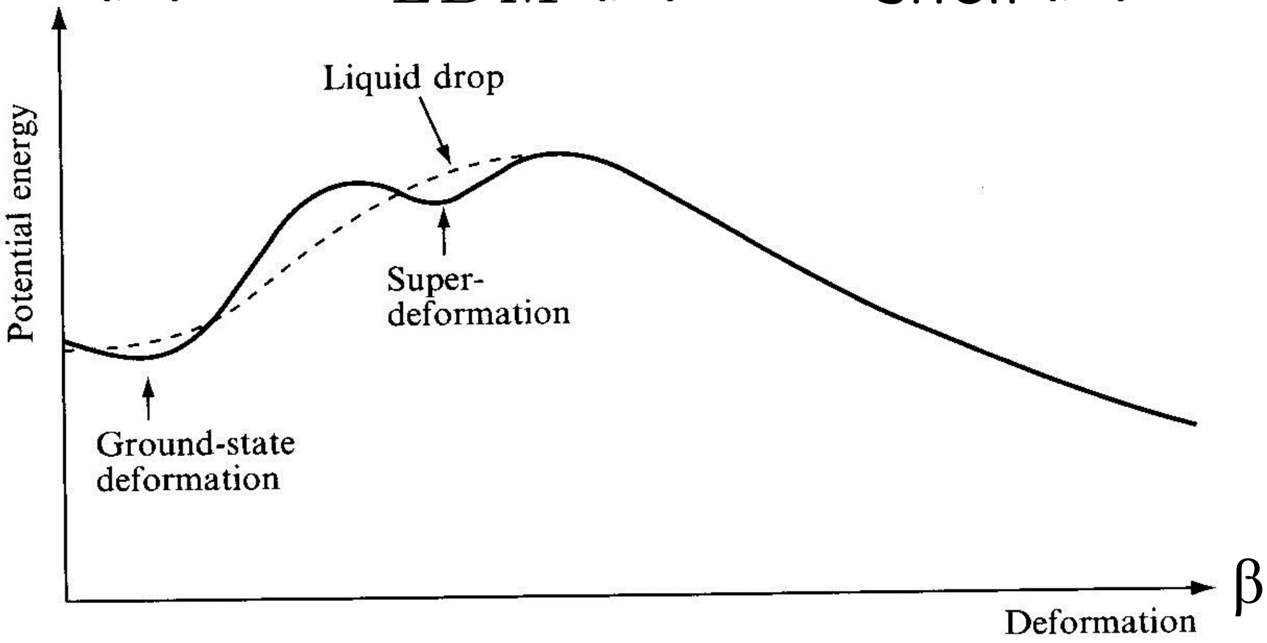

"parity inversion"

What happens if $^{11}$Be is deformed?

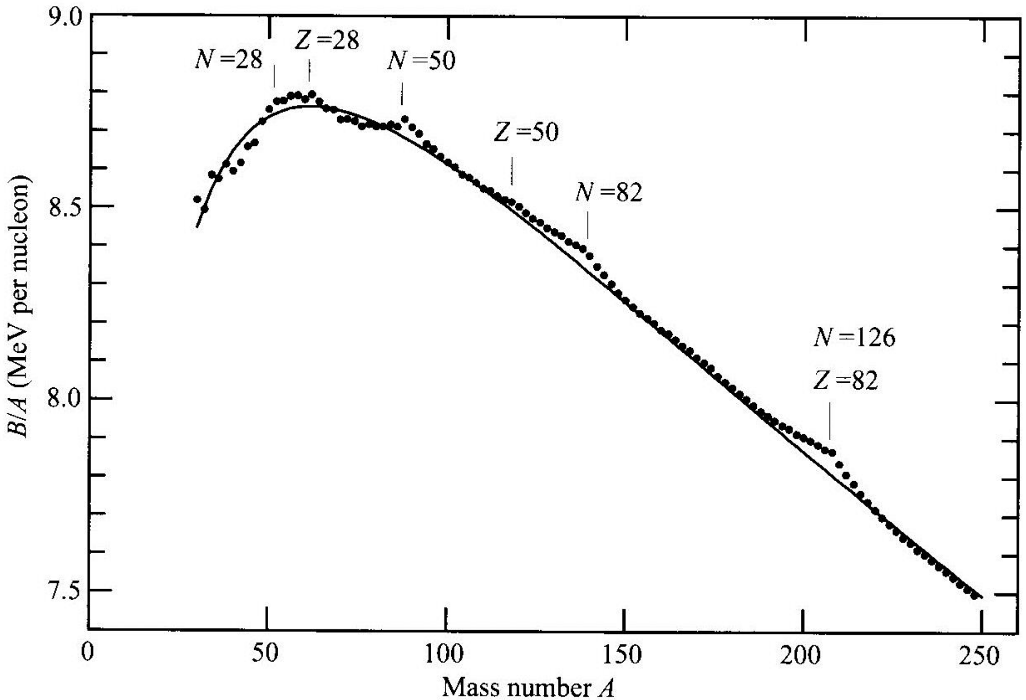
# Nuclear Deformation

Deformed energy surface for a given nucleus

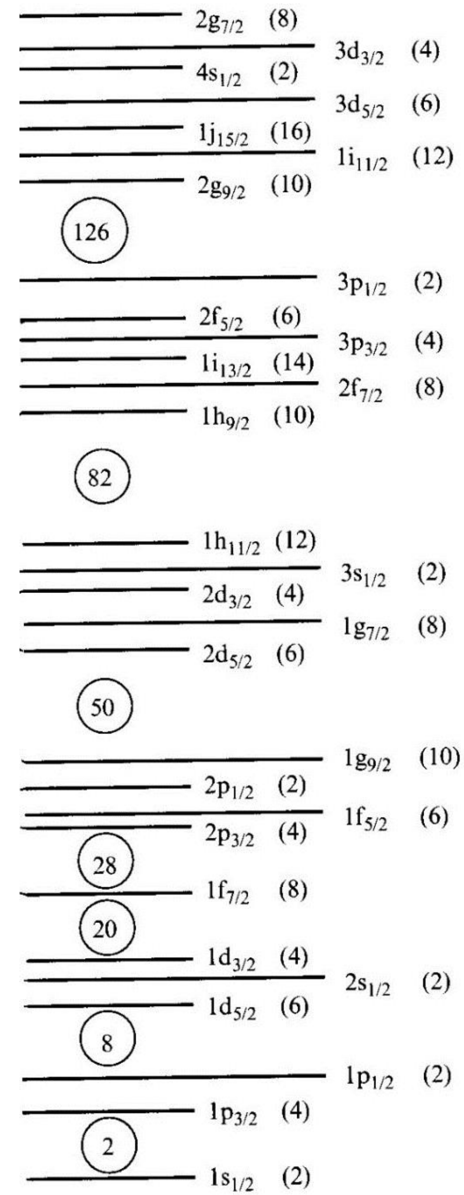
$$E(\beta) = E_{LDM}(\beta) + E_{\mathsf{shell}}(\beta)$$




LDM only → always spherical ground state

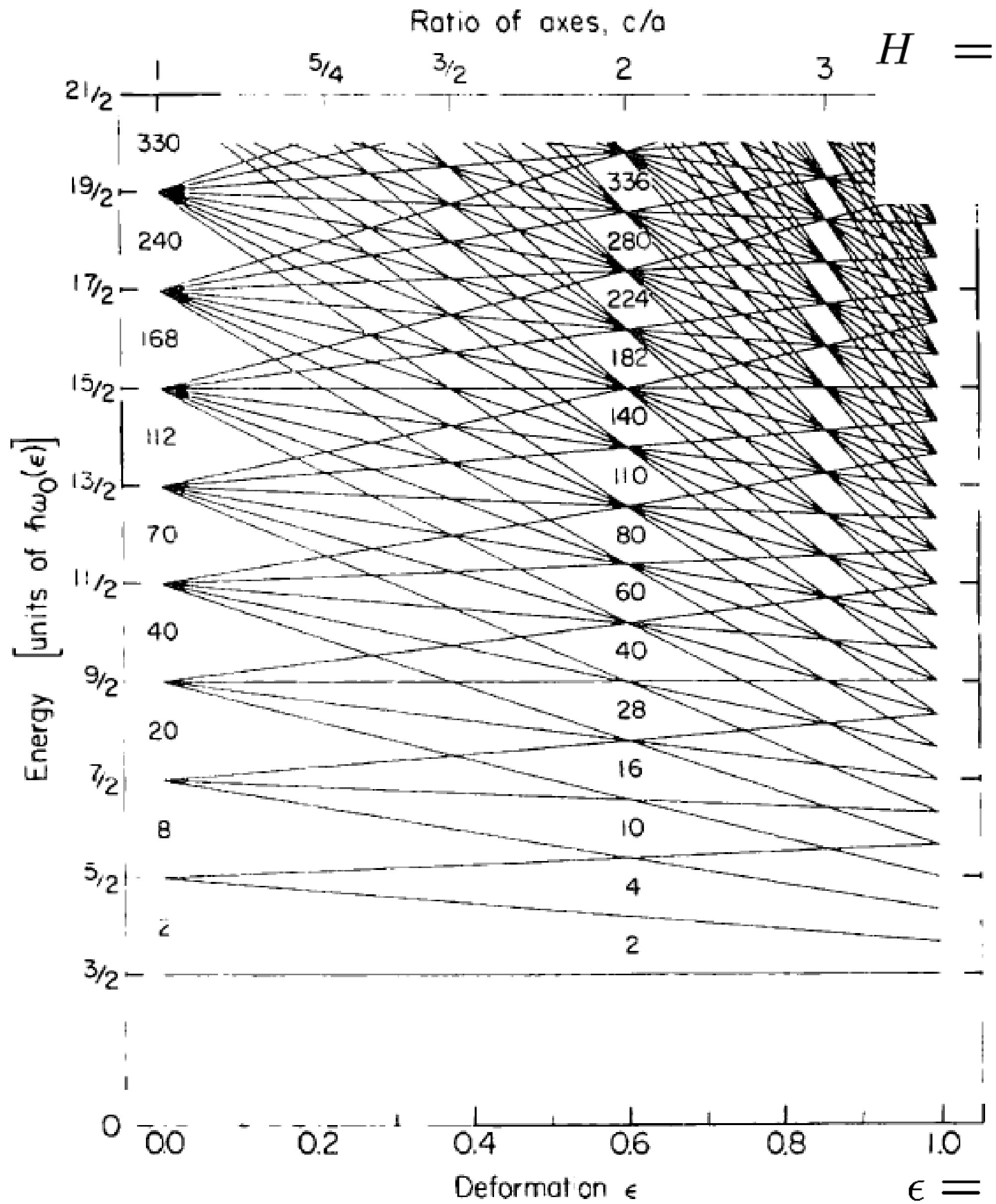



$$B = B_{\mathrm{LDM}} + B_{\mathrm{sh}}$$




Voods-Saxon plus
pin-orbit coupling

$$
\begin{aligned}
H &= -\frac{\hbar^2}{2m}\nabla^2 \\
&\quad +\frac{1}{2}m\omega_z^2\, z^2 + \frac{1}{2}m\omega\, \omega_\perp^2 (x^2+y^2) \\
&\qquad \omega_\perp^2 \omega_z = \omega_0^3
\end{aligned}
$$




$$
\begin{aligned}
E &= \hbar\omega_z(n_z + 1/2) \\
&\quad + \hbar\omega_\perp(n_\perp + 1)
\end{aligned}
$$

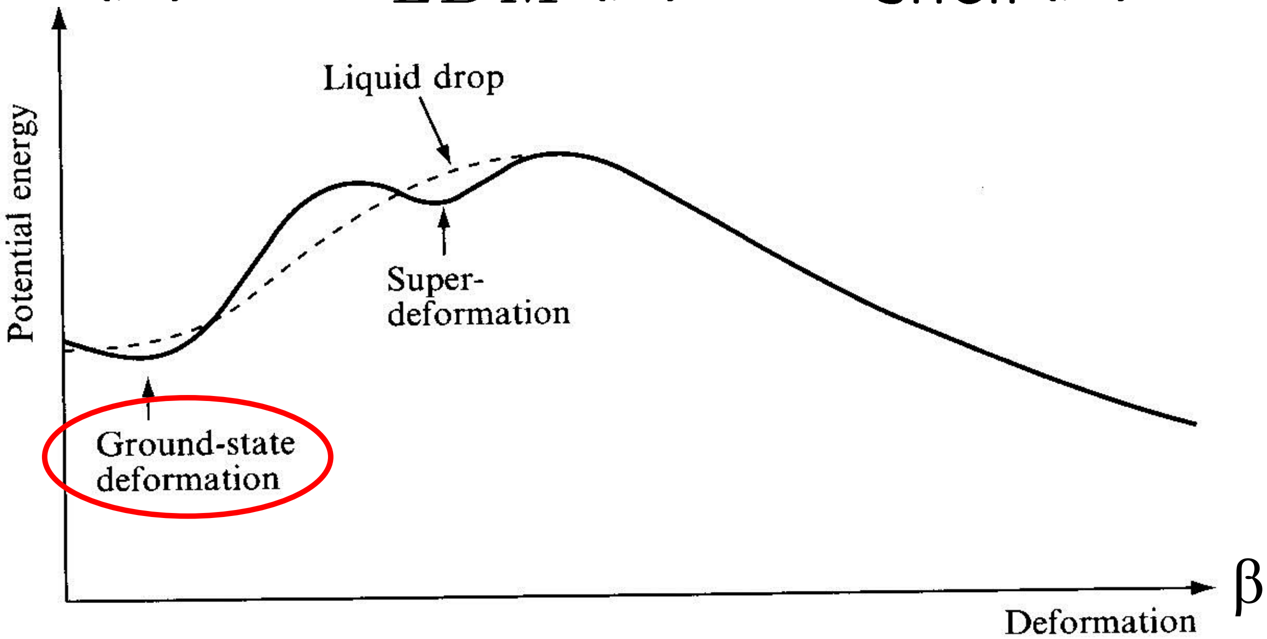
$$\epsilon = (\omega_\perp - \omega_z)/\omega_0$$

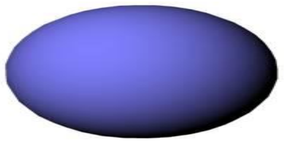
**Figure 2.25.** Energy levels of an harmonic-oscillator potential for prolate spheroidal deformations $\epsilon$. (From [MN 73].)

# Nuclear Deformation

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{\text{shell}}(\beta)$$



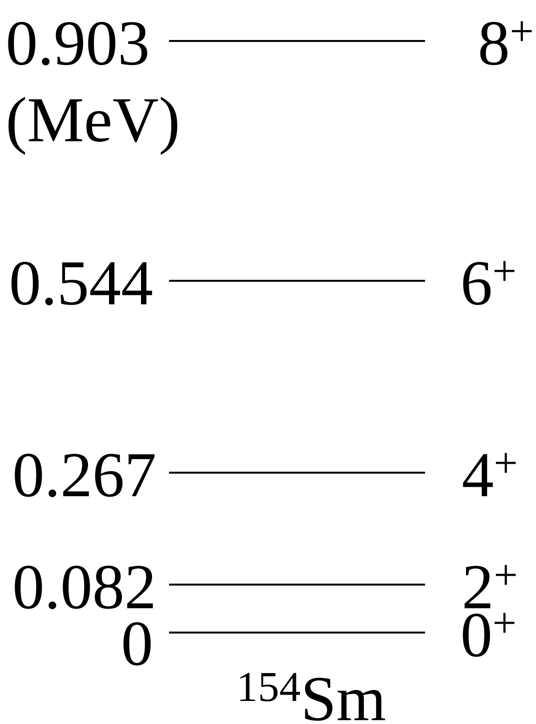
LDM only → always spherical ground state

Shell correction → may lead to a deformed g.s.

* Spontaneous Symmetry Breaking

# Nuclear Deformation

Excitation spectra of $^{154}$Sm

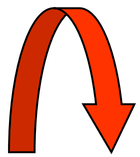



$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

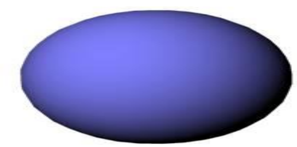
cf. Rotational energy of a rigid body
(Classical mechanics)

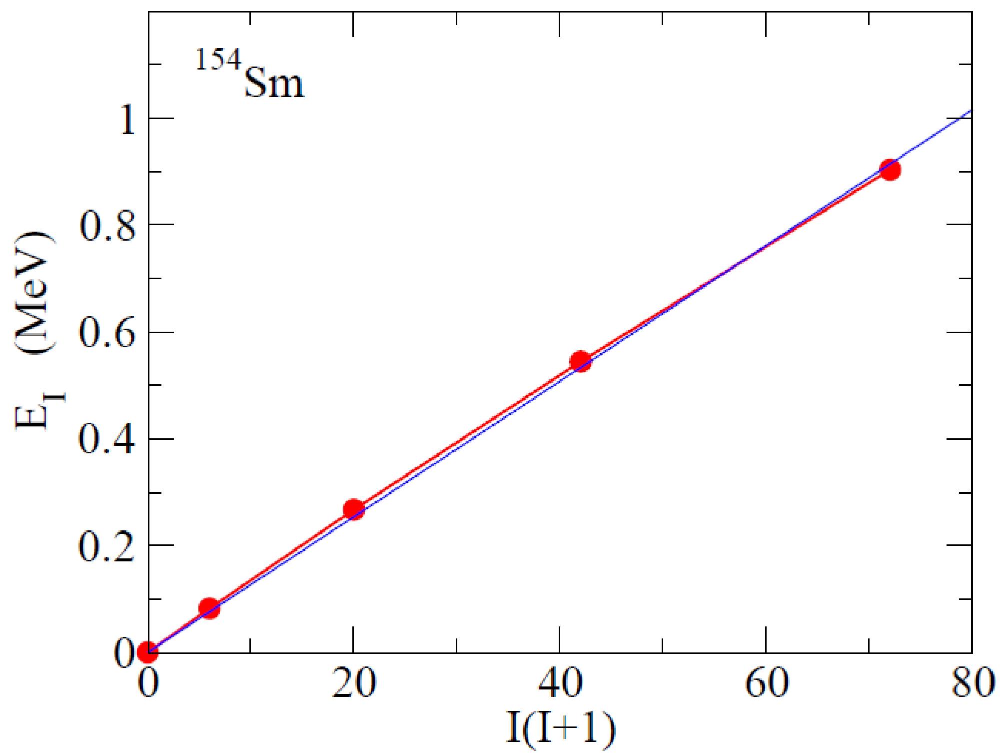
$$E = \frac{1}{2}\mathcal{J}\omega^2 = \frac{I^2}{2\mathcal{J}}$$

$$(I = \mathcal{J}\omega, \ \ \omega = \dot{\theta})$$



$^{154}$Sm is deformed

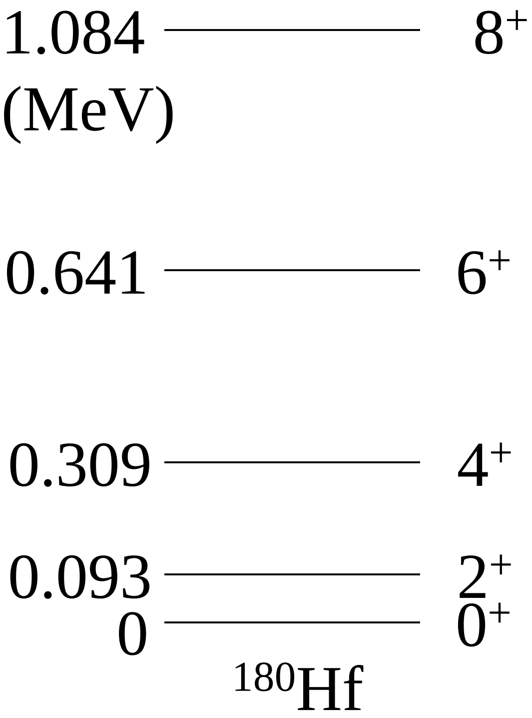




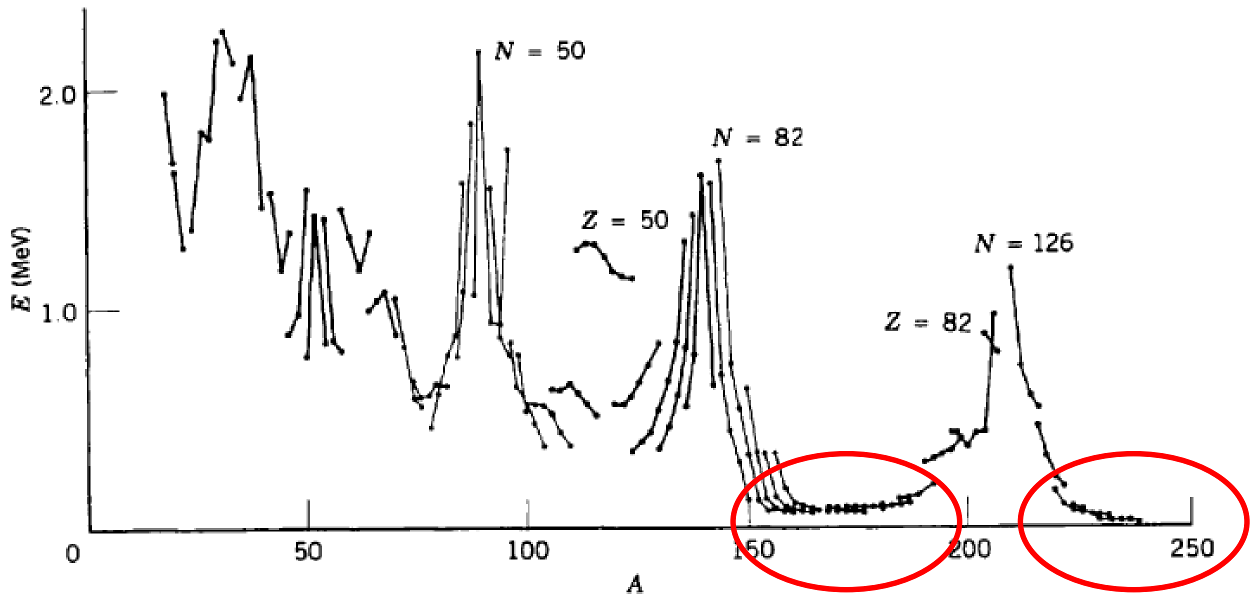
# Evidences for nuclear deformation

- existence of rotational band

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$



# The energy of the first $2^+$ state in even-even nuclei

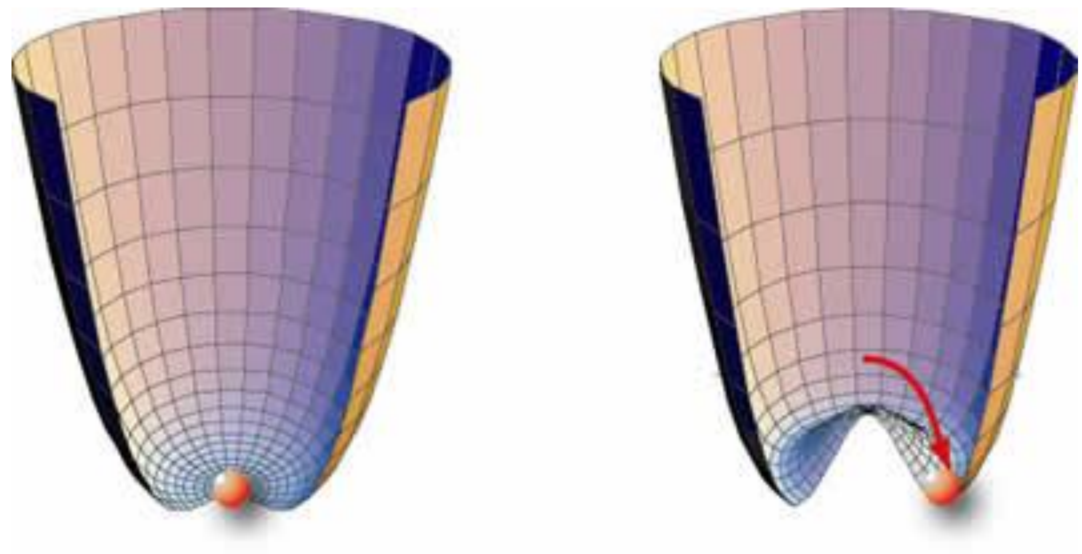



deformed nuclei

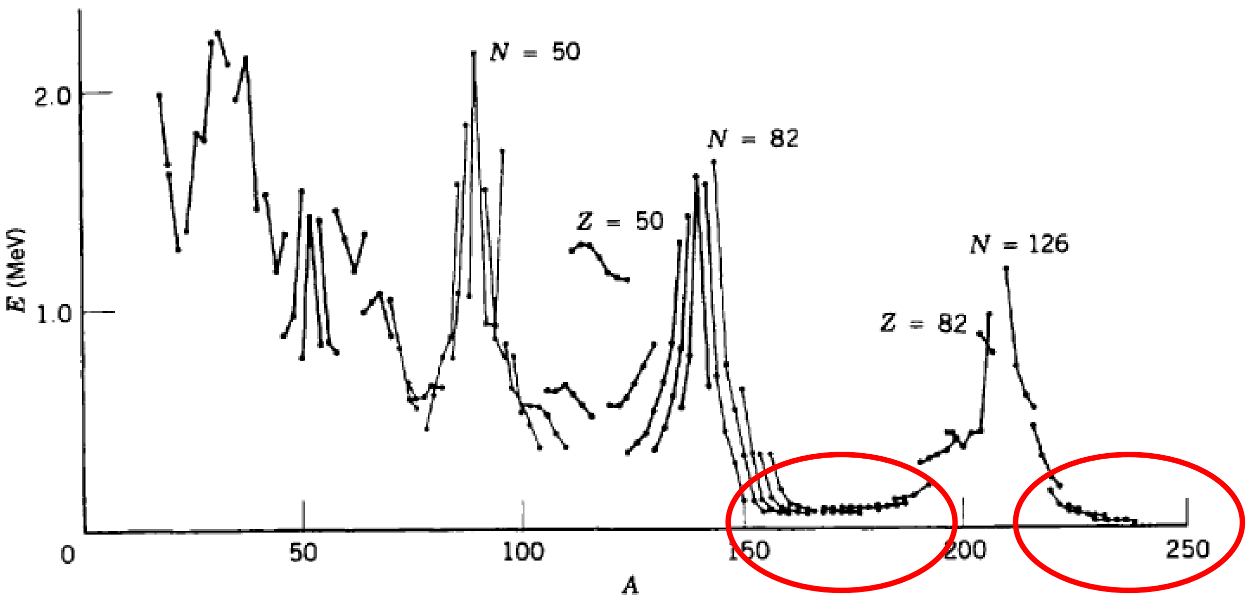
K.S. Krane, “Introductory Nuclear Physics”

## Spontaneous symmetry breaking

The vacuum state does not have (i.e, the vacuum state violates) the symmetry which the Hamiltonian has.

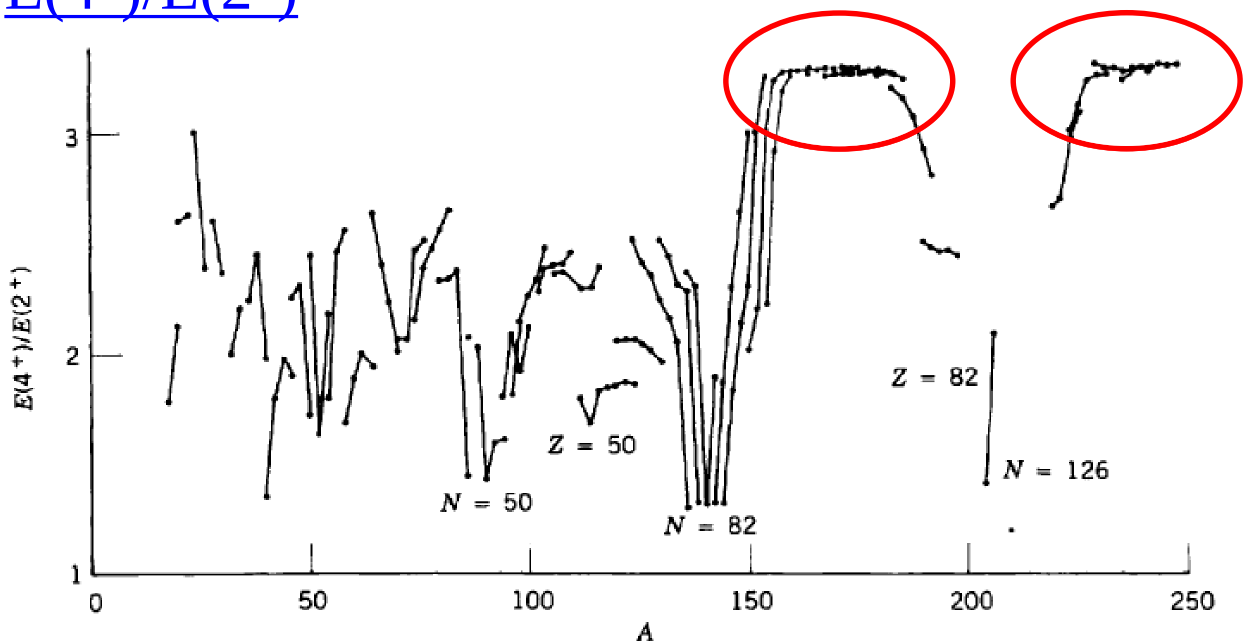


(A Nambu-Goldstone mode (zero-energy mode) appears in order to restore the symmetry.)




deformed nuclei

E($4^+$)/E($2^+$)




deformed nuclei:
E($4^+$)/E($2^+$) ~ 3.3

spherical nuclei:
E($4^+$)/E($2^+$) ~ 2

K.S. Krane, "Introductory Nuclear Physics"

# Nuclear deformation

Energy change due to nuclear deformation:

$$E(\beta) = E_{LDM}(\beta) + E_{\mathsf{shell}}(\beta)$$

deformation in nuclei

$\longrightarrow$ deformation in a potential which nucleons feel

$\longrightarrow$ deformation dependent shell correction energy

# One-particle motion in a deformed potential

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' \sim -g\rho(\boldsymbol{r}) \qquad \text{if} \quad v(\boldsymbol{r},\boldsymbol{r}') = -g\delta(\boldsymbol{r}-\boldsymbol{r}')$$

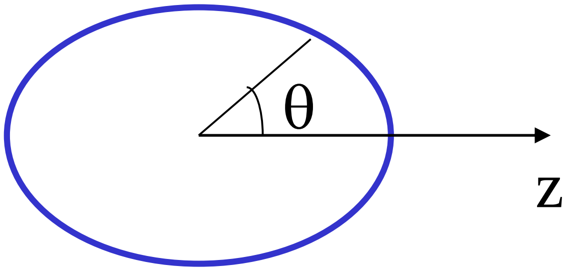
if the density is deformed, so is the mean-field potential

(note) radius of ellipsoid (axial symm.): $R(\theta) = R_0(1+\beta_2 Y_{20}(\theta))$

Woods-Saxon potential

$$V(r) = -V_0/[1+\exp((r-R_0)/a)]$$

$$R_0 \rightarrow R(\theta)$$




Deformed Woods-Saxon potential

$$\boxed{\begin{aligned} V(r,\theta) &= -V_0/[1+\exp((r-R_0-R_0\beta_2 Y_{20}(\theta))/a] \\ &\sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots \end{aligned}}$$

# One-particle motion in a deformed potential

## Deformed Woods-Saxon potential

$$\boxed{\begin{aligned} V(r,\theta) &= -V_0/[1+\exp((r-R_0-R_0\beta_2 Y_{20}(\theta))/a] \\ &\sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots \end{aligned}}$$

breaking of rotational symmetry

$\longrightarrow$ angular momentum: is not a good quantum number (non-conservation)

■ Let us discuss the effect of $Y_{20}$ term using the first order perturbation theory (note) the first order perturbation theory

$$H = H_0 + H_1$$

Suppose we know all the eigen-values and eigen-functions of $H_0$ :

$$H_0|\phi_n^{(0)}\rangle = E_n^{(0)}|\phi_n^{(0)}\rangle$$

The eigen-values and the eigen-functions are modified by $H_1$ as:

$$\begin{aligned} E_n &= E_n^{(0)} + \langle\phi_n^{(0)}|H_1|\phi_n^{(0)}\rangle + \cdots \\ |\phi_n\rangle &= |\phi_n^{(0)}\rangle + \sum_{m\neq n} \frac{\langle\phi_m^{(0)}|H_1|\phi_n^{(0)}\rangle}{E_n^{(0)} - E_m^{(0)}}|\phi_m\rangle + \cdots \end{aligned}$$

# One-particle motion in a deformed potential

## Deformed Woods-Saxon potential

$$\boxed{V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots}$$

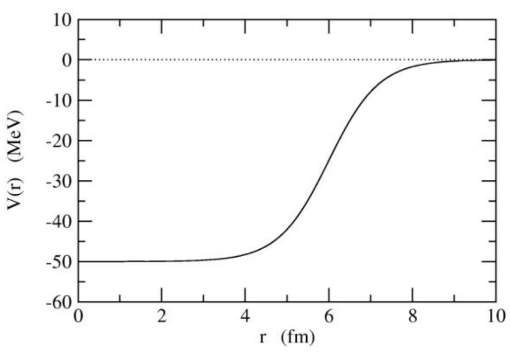
■the effect of $Y_{20}$ term $\longleftarrow$ the first order perturbation theory

Eigen-functions for $\beta_2=0$ (spherical pot.) : $\psi_{nlK}(\boldsymbol{r}) = R_{nl}(r)Y_{lK}(\hat{\boldsymbol{r}})$

eigen-values : $E_{nl}$ (no dependence on $K$)

The energy change :

$$\begin{aligned} E_{nl} &\rightarrow E_{nl} + \langle \psi_{nlK} | \Delta V | \psi_{nlK} \rangle \\ &= E_{nl} - \beta_2 R_0 \underbrace{\left[\int_0^\infty r^2 dr\, \frac{dV_0}{dr} (R_{nl}(r))^2\right]}_{\text{positive quantity}} \cdot \underbrace{\langle Y_{lK} | Y_{20} | Y_{lK} \rangle}_{\propto\, -(3K^2 - l(l+1))} \end{aligned}$$



# One-particle motion in a deformed potential

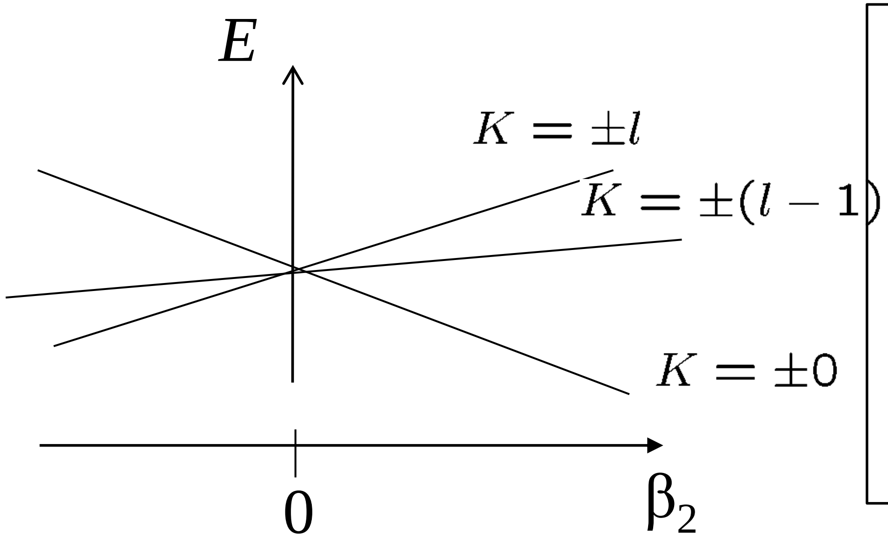
## Deformed Woods-Saxon potential

$$\boxed{V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots}$$

■ the effect of $Y_{20}$ term $\longleftarrow$ the first order perturbation theory

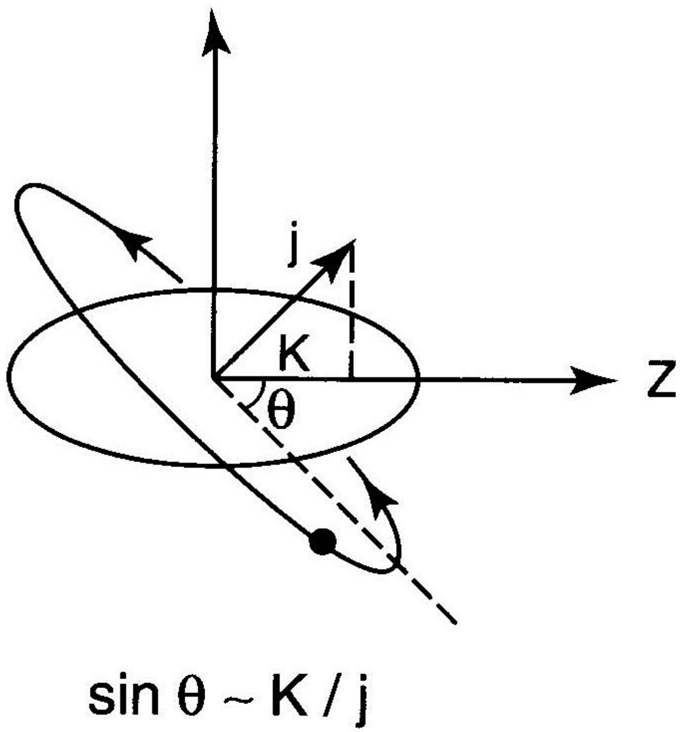
The energy change :

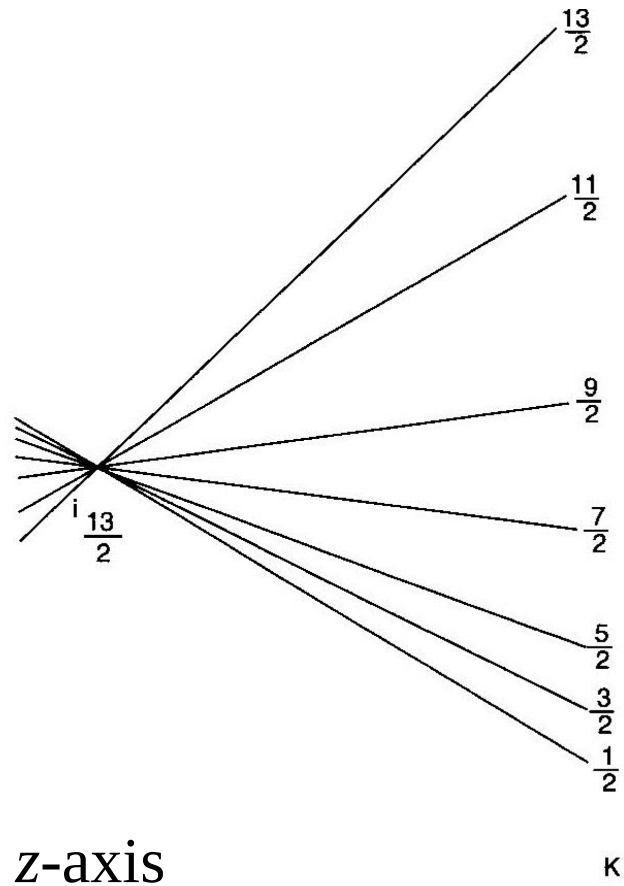
$$E_{nl} \;\to\; E_{nl} + \alpha_{nl}\,\beta_2\,(3K^2 - l(l+1)) \qquad (\alpha_{nl} > 0)$$



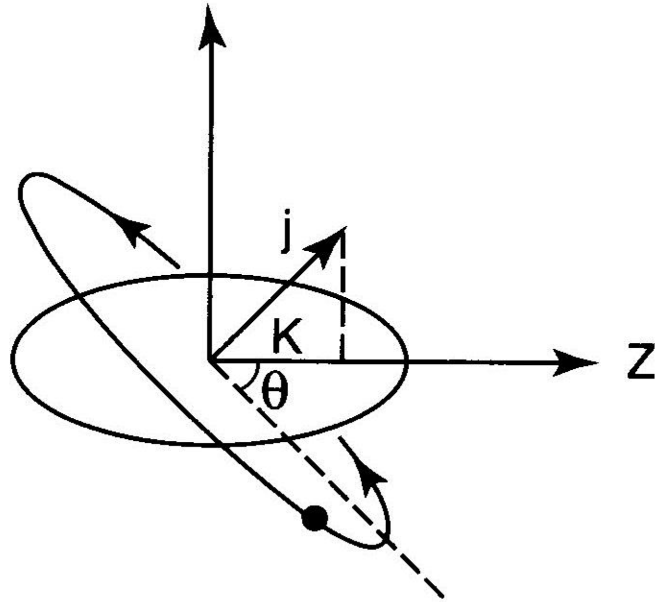

- ✓ different energy changes for $K$ (→ non-degenerate)
- ✓ For $\beta_2 > 0$, the energy is lower for smaller K
- ✓ Opposite when $\beta_2 < 0$
- ✓ $K$ and $-K$ : degenerate

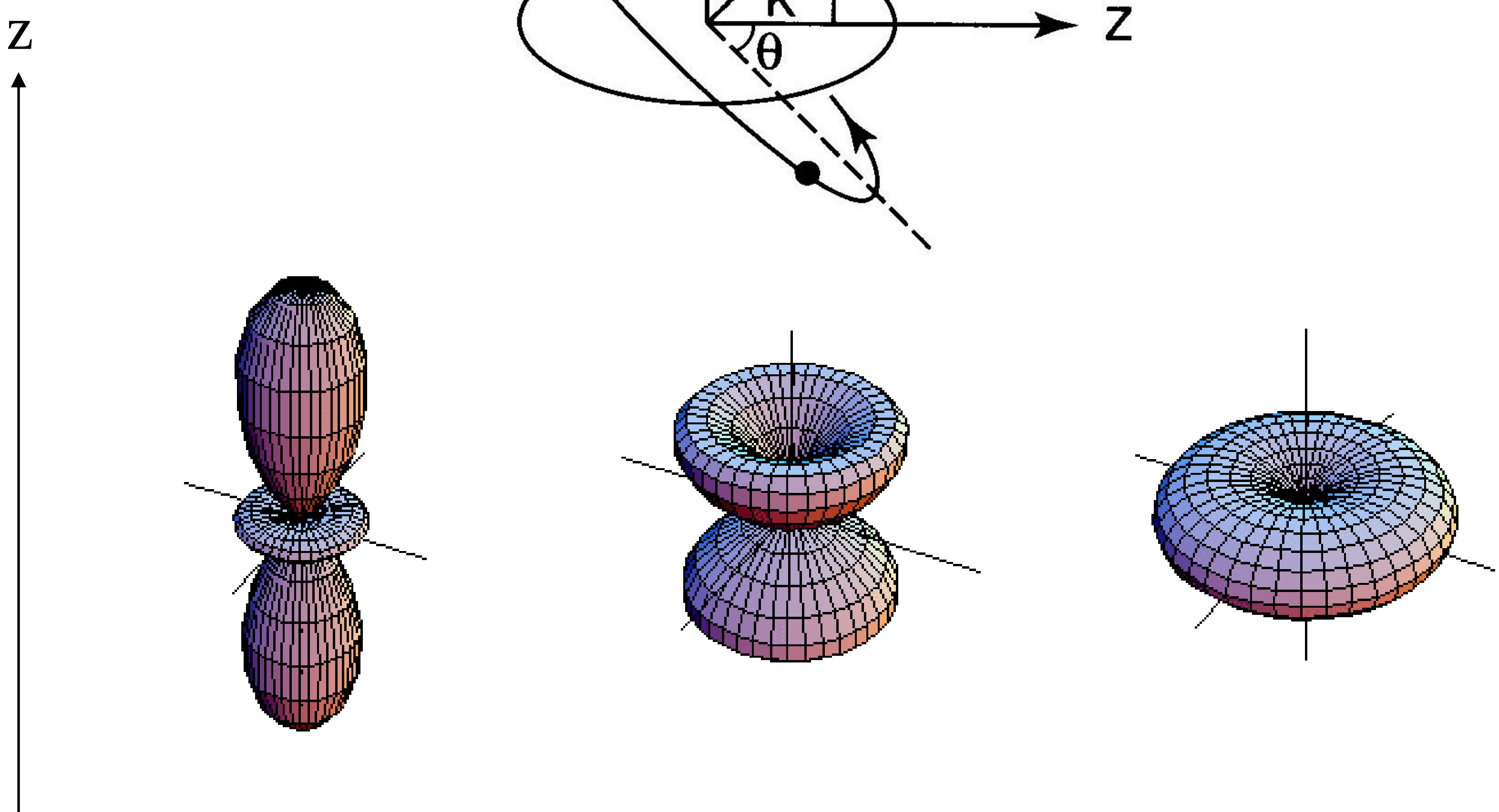
# Geometrical interpretation




- *K*: projection of angular momentum onto *z*-axis
- nucleon motion: in a plane perpendicular to the ang. mom. vector
- for prolate deformation, a motion with small *K* is along the longer axis
- therefore, the energy is lowered
- a motion with large *K* is along the shorter axis, and loses the energy




$r = Y_{20}$ (K=0) $\quad$ $r = Y_{21}$ (K=1) $\quad$ $r = Y_{22}$ (K=2)

# One-particle motion in a deformed potential

$$\boxed{V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots}$$

■the effect of $Y_{20}$ term $\longleftarrow$ the first order perturbation theory

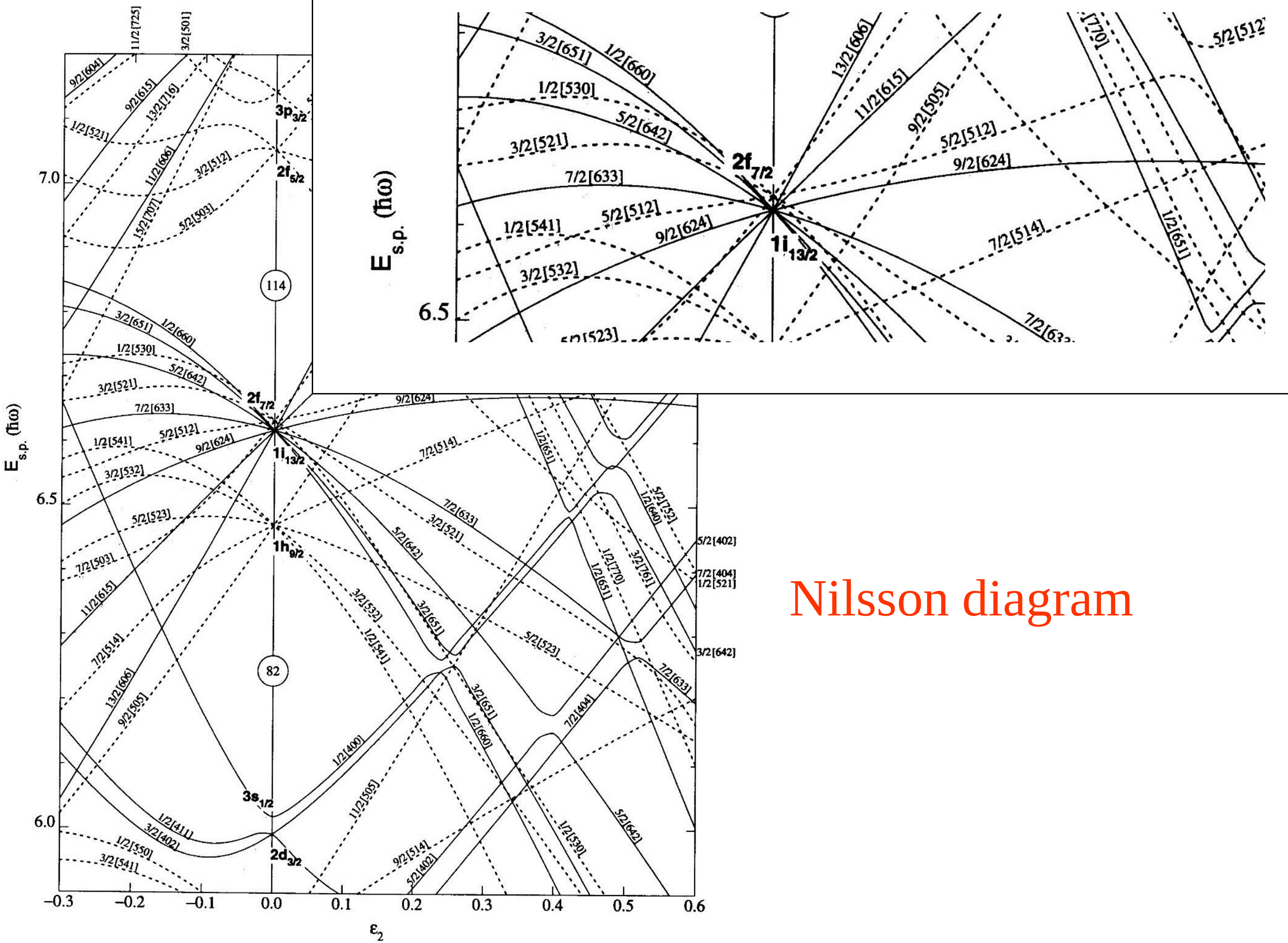
Next, a change in wf: $$|\phi_n\rangle = |\phi_n^{(0)}\rangle + \sum_{m\neq n} \frac{\langle\phi_m^{(0)}|H_1|\phi_n^{(0)}\rangle}{E_n^{(0)} - E_m^{(0)}} |\phi_m\rangle + \cdots$$

Eigen-functions for $\beta_2$=0 (spherical pot.) : $\psi_{nlK}(\boldsymbol{r}) = R_{nl}(r) Y_{lK}(\hat{\boldsymbol{r}})$

$$\boxed{\psi_{nlK} \to \psi_{nlK} + \sum_{n'l'K'} \frac{\langle\psi_{n'l'K'}|\Delta V|\psi_{nlK}\rangle}{E_{nl} - E_{n'l'}} \psi_{n'l'K'}}$$

mixing of states which are connected by $\langle Y_{l'K'}|Y_{20}|Y_{lK}\rangle$

- *l* does not conserve, and the wf includes several *l* components
- For axial symmetry ($Y_{20}$), *K* does not change ($K' = K$), therefore *K* is a good quantum number
- $Y_{20}$ does not change parity. The parity is thus also conserved.

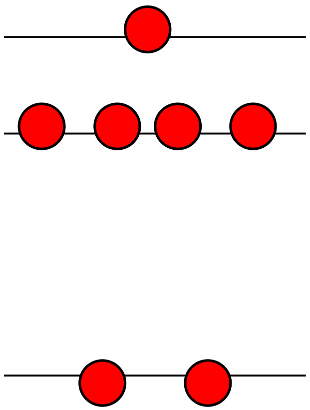


# Nilsson diagram

**Figure 13. Nilsson diagram for protons, Z ≥ 82 ($\varepsilon_4 = \varepsilon_2^2/6$).**

## Level scheme of $^{11}_{4}Be_7$

With a spherical potential :



$1p_{1/2}$ [2]

$1p_{3/2}$ [4]

$1s_{1/2}$ [2]

The g.s. of $^{11}$Be : $I^{\pi} = 1/2^-$

very artificial

In reality…..

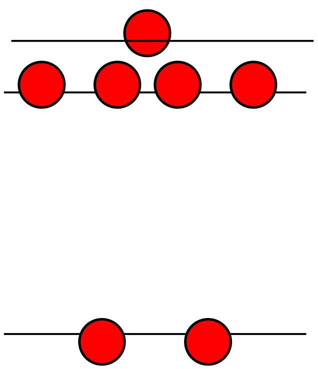
0.32 MeV

$1/2^-$

$1/2^+$

$^{11}$Be

?



$1p_{1/2}$

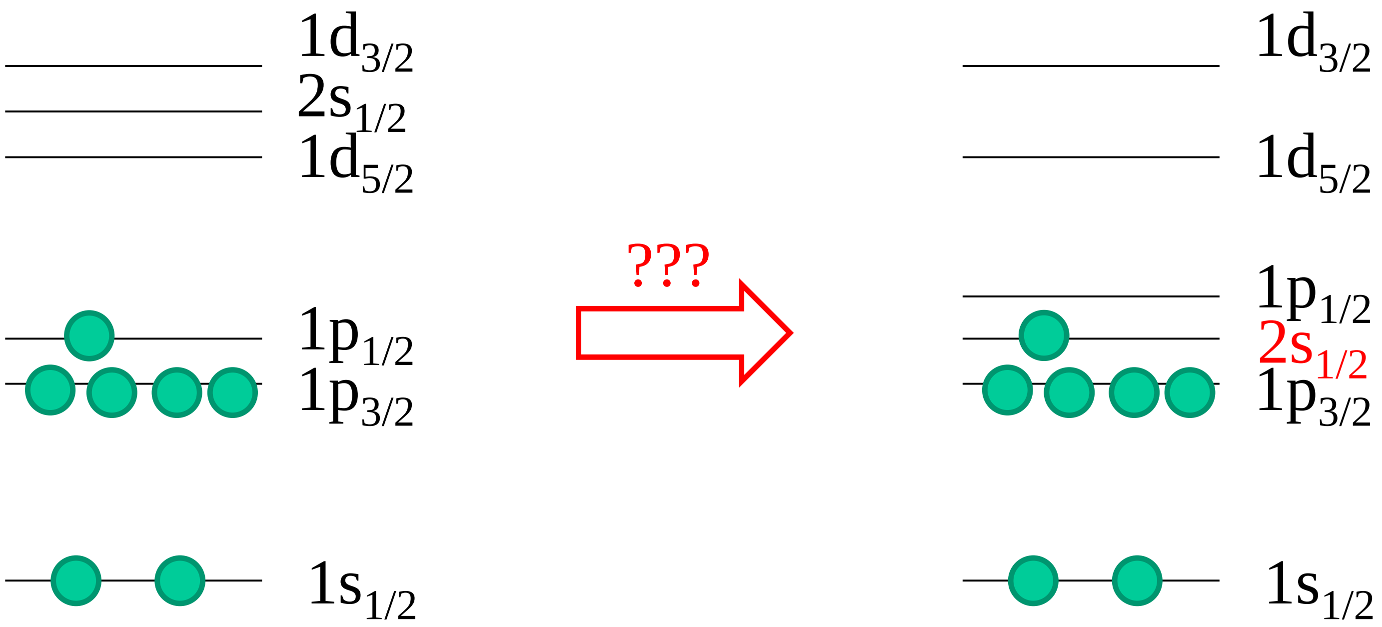
$2s_{1/2}$

$1p_{3/2}$

$1s_{1/2}$

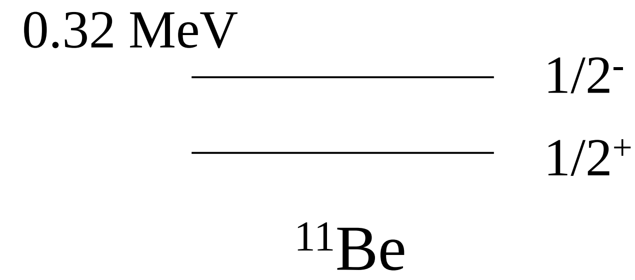
"parity inversion"

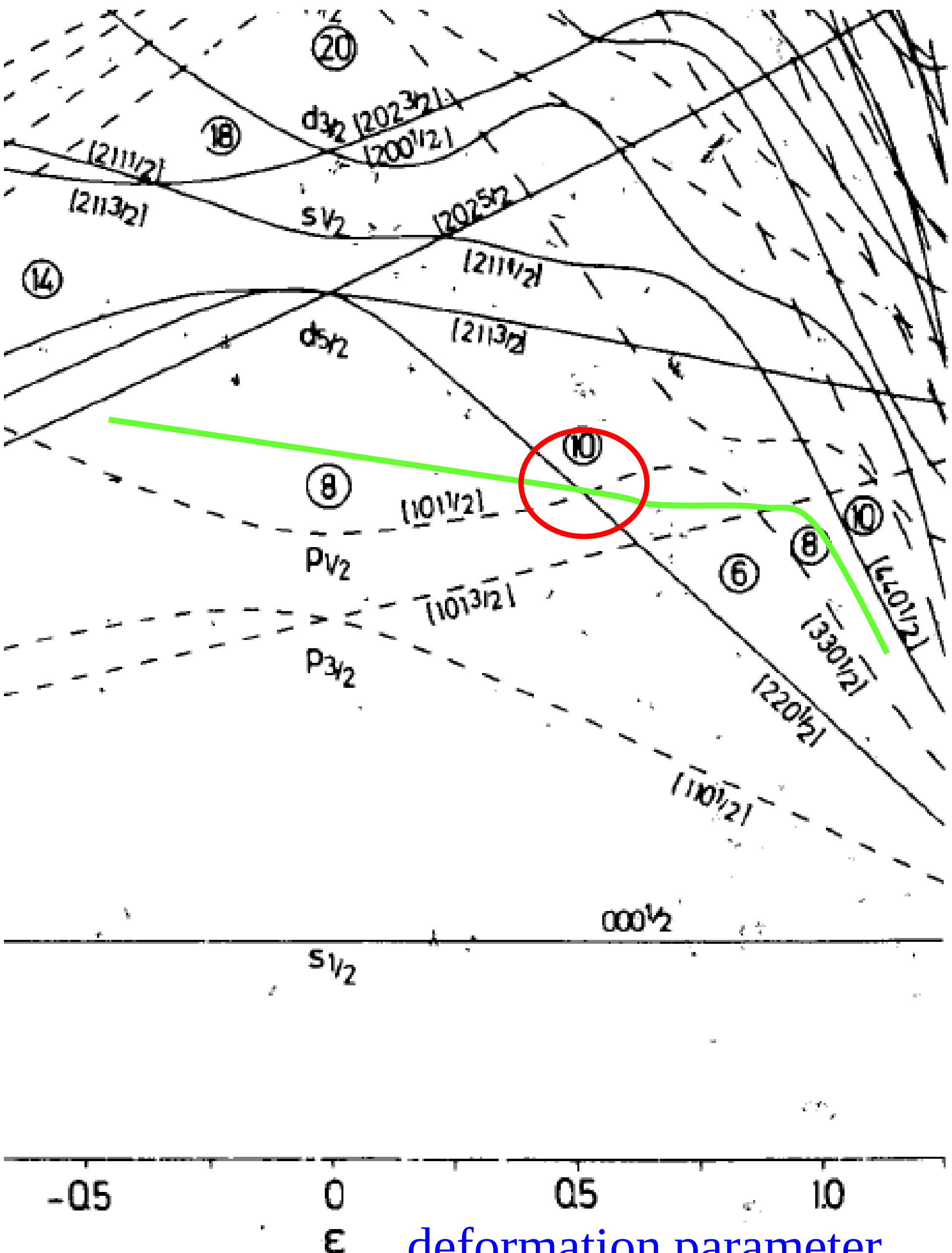
What happens if $^{11}$Be is deformed?




Very unnatural.
The $2s_{1/2}$ state is more naturally explained if one considers a deformation of $^{11}$Be.

$^{11}_{4}\mathrm{Be}_{7}$



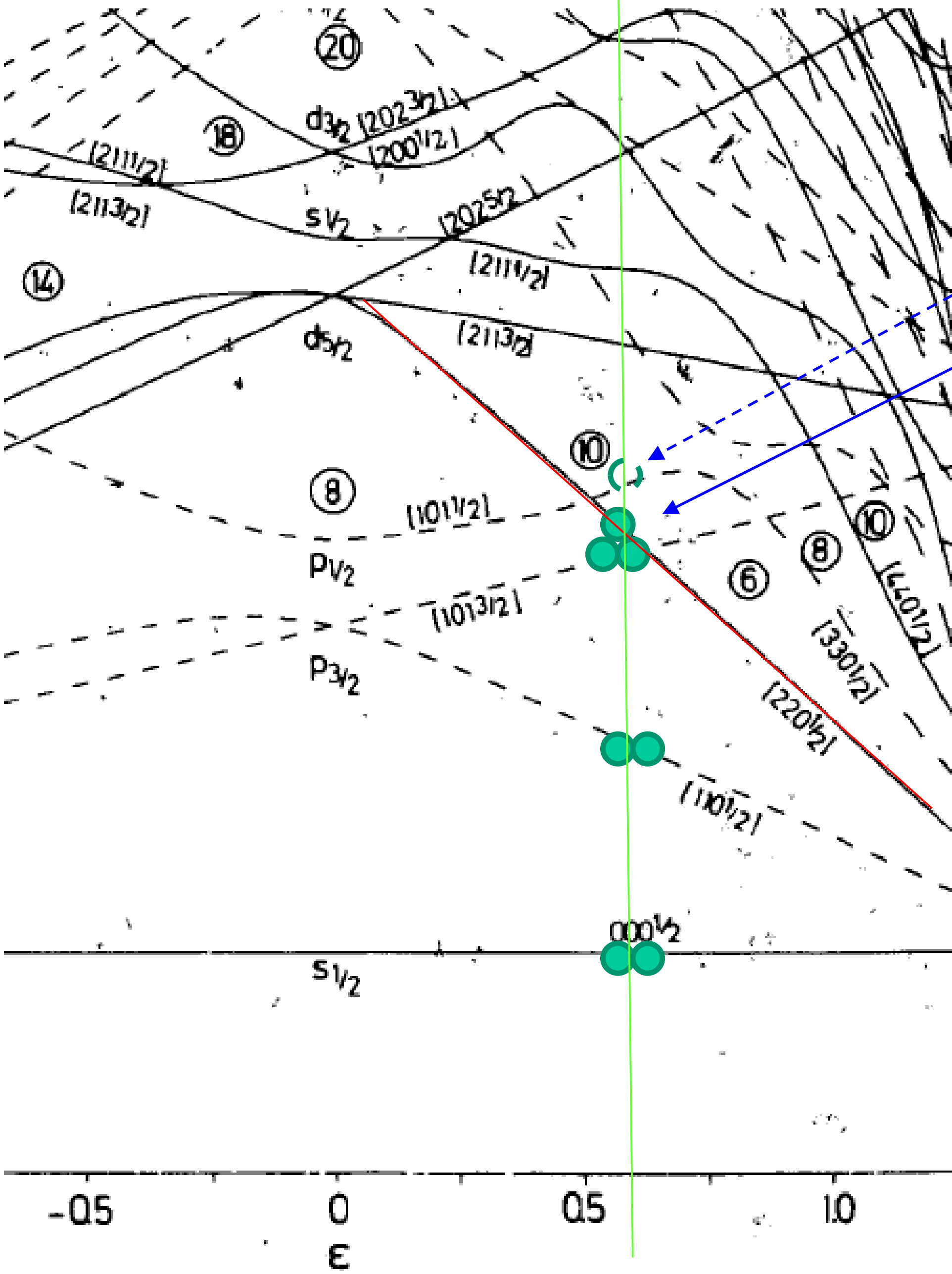



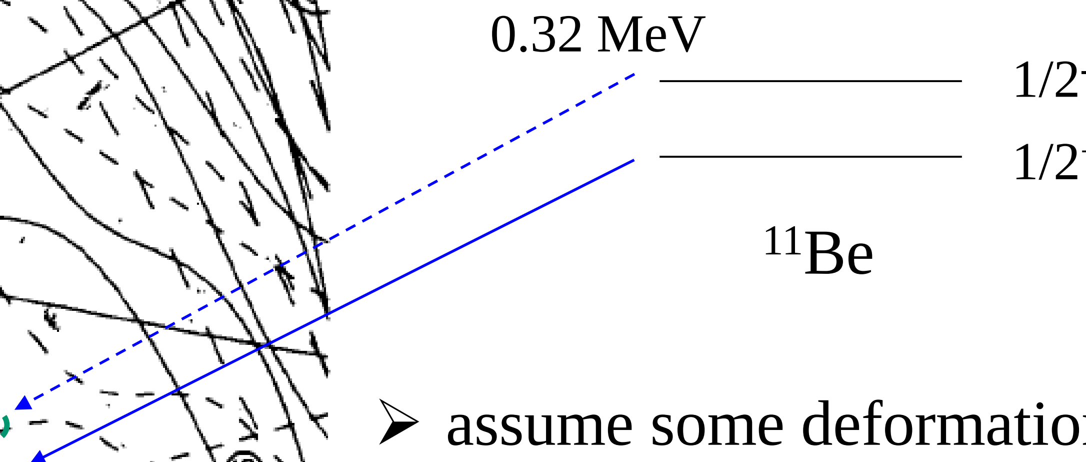

deformation parameter

- assume some deformation, and put 2 nucleons in each level from the bottom (degeneracy of $+K$ and $-K$)
- Look for the level which is occupied by the valence nucleon（the 7th level for $^{11}$Be)
- Identify the value of $K^{\pi}$ for that level with the spin and parity of the whole nucleus.

cf. particle-rotor model

$^{11}_{4}Be_{7}$

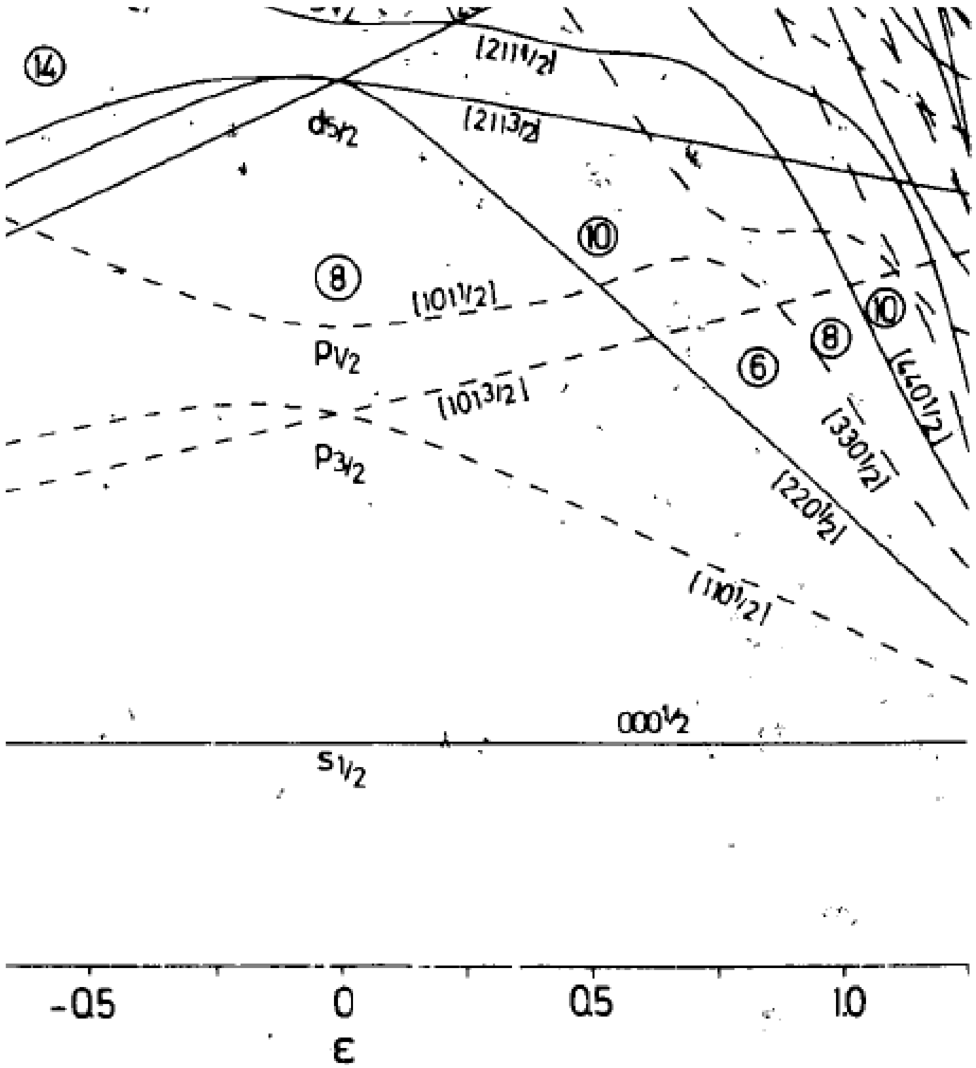


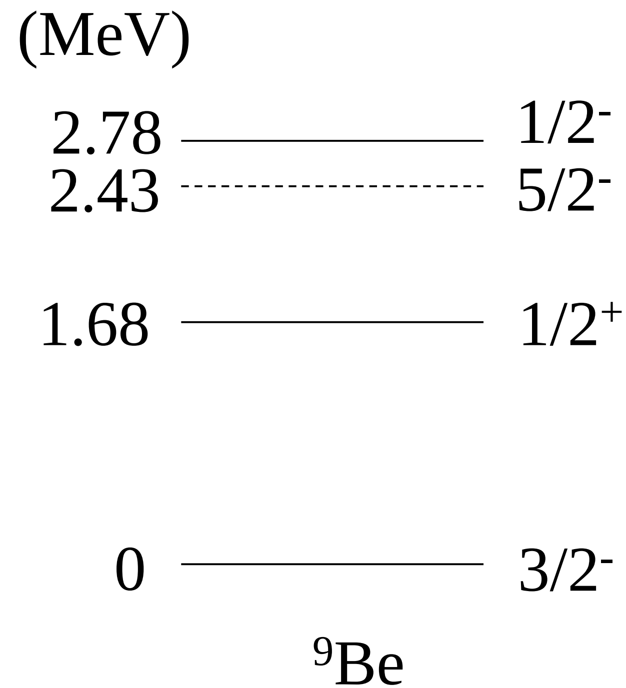




- assume some deformation, and put 2 nucleons in each level from the bottom
- Look for the level which is occupied by the valence nucleon（the 7th level for $^{11}Be$)
- Identify the value of $K^{\pi}$ for that level with the spin and parity of the whole nucleus.

# Can the level scheme of $^{9}_{4}Be_5$ be explained in a similar way?

cf. $^{10}B(e,e'K^+)^{10}{}_{\Lambda}Be$ (= $^{9}Be+\Lambda$)

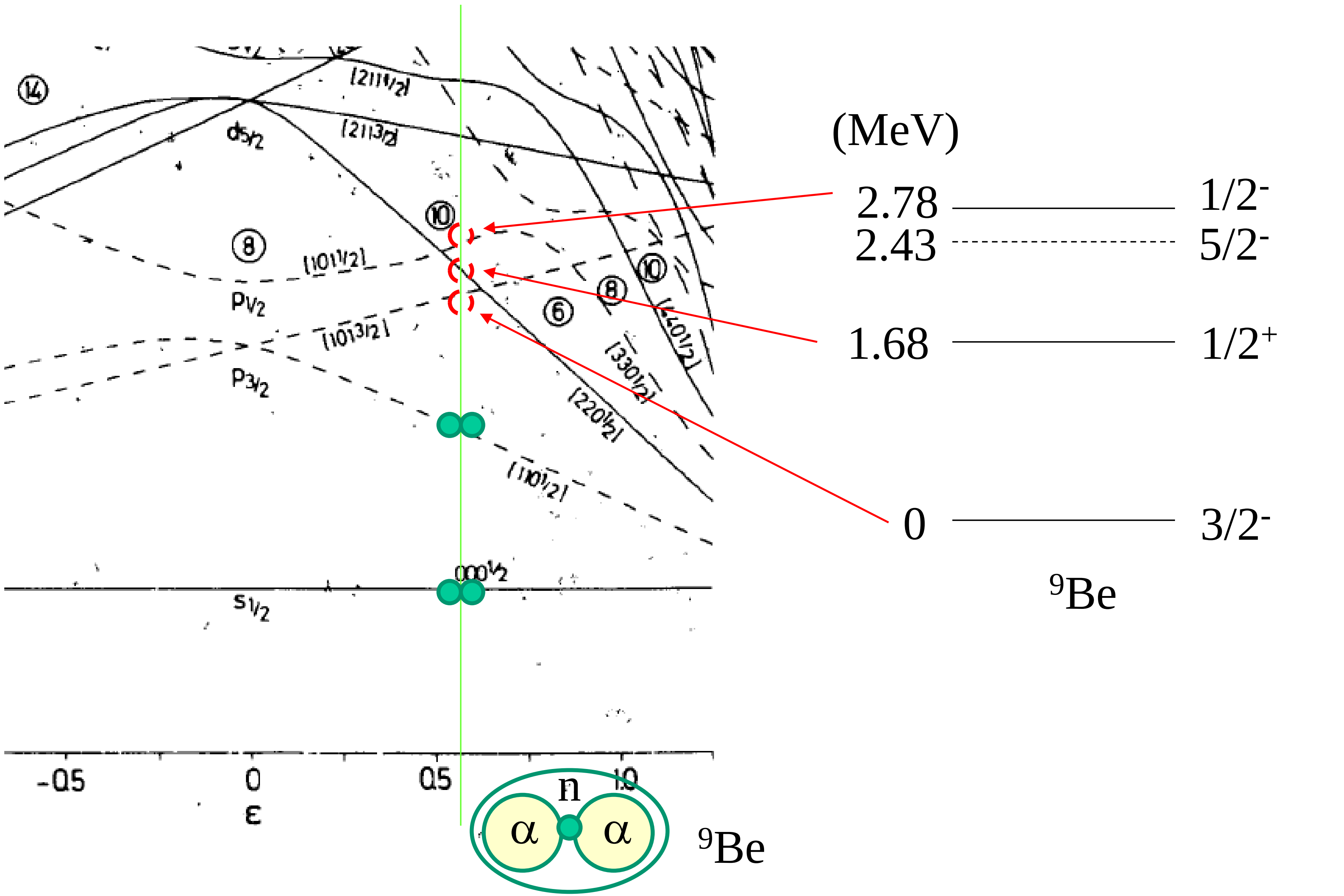



The $5/2^-$ state at 2.43 MeV: rotational state with the same configuration as the g.s. state (not considered here)

# Can the level scheme of ${}^{9}_{4}Be_5$ be explained in a similar way?

cf. ${}^{10}B(e,e'K^+){}^{10}_{\Lambda}Be$ (= ${}^{9}Be+\Lambda$)



## Nobel prize in physics (2008)

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

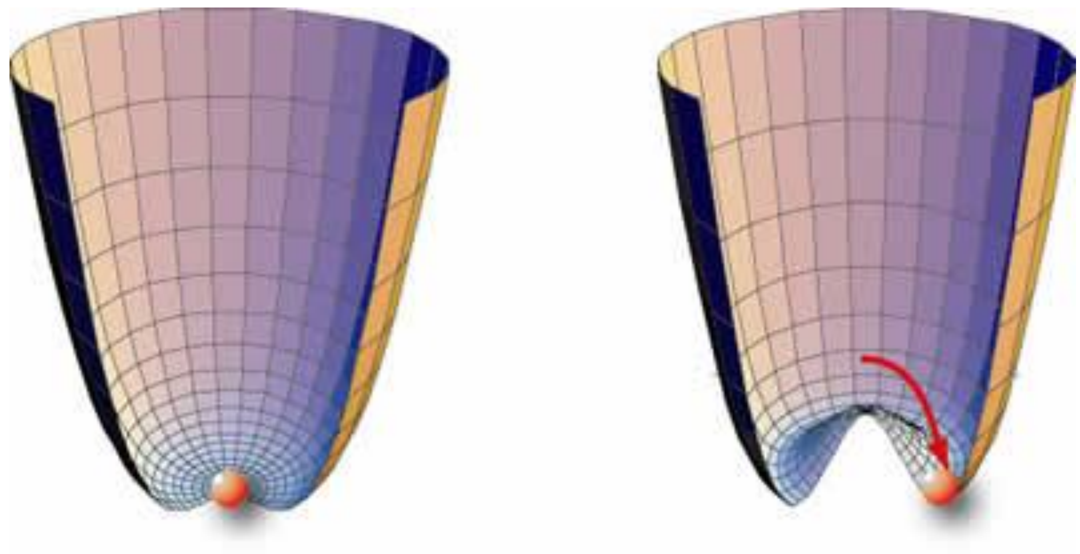
Prof. Y. Nambu

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

Kobayashi and Maskawa

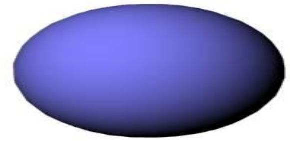
# Spontaneous symmetry breaking

The vacuum state does not have (i.e, the vacuum state violates) the symmetry which the Hamiltonian has.



(A Nambu-Goldstone mode (zero-energy mode) appears in order to restore the symmetry.)

# Nuclear Deformation



## Excitation spectra of $^{154}$Sm

| | |
|---|---|
| 0.903 (MeV) | $8^+$ |
| 0.544 | $6^+$ |
| 0.267 | $4^+$ |
| 0.082 | $2^+$ |
| 0 | $0^+$ |

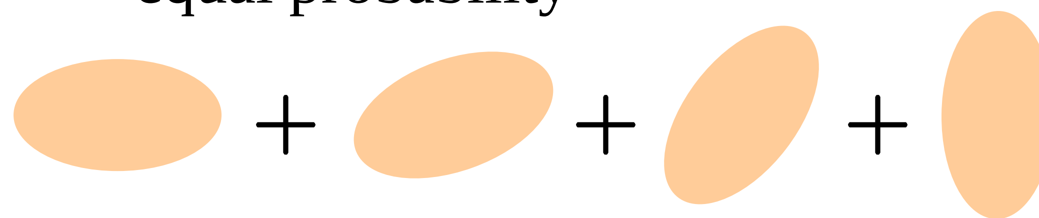
$^{154}$Sm

$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

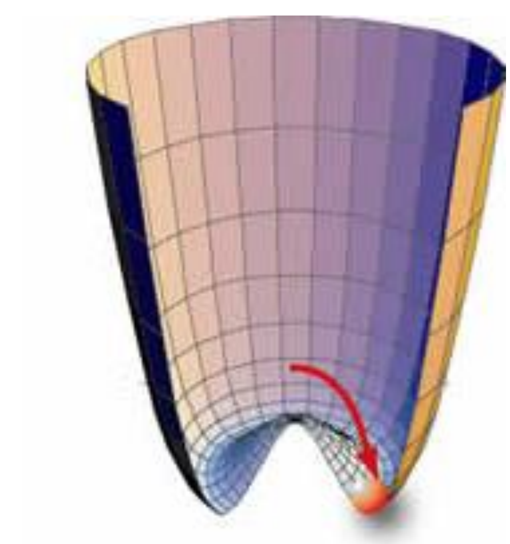
## What is $0^+$ state (Quantum Mechanics)?

$0^+$: no preference of direction (spherical)

→ Mixing of all orientations with an equal probability



c.f. HF + Angular Momentum Projection



## Quiz

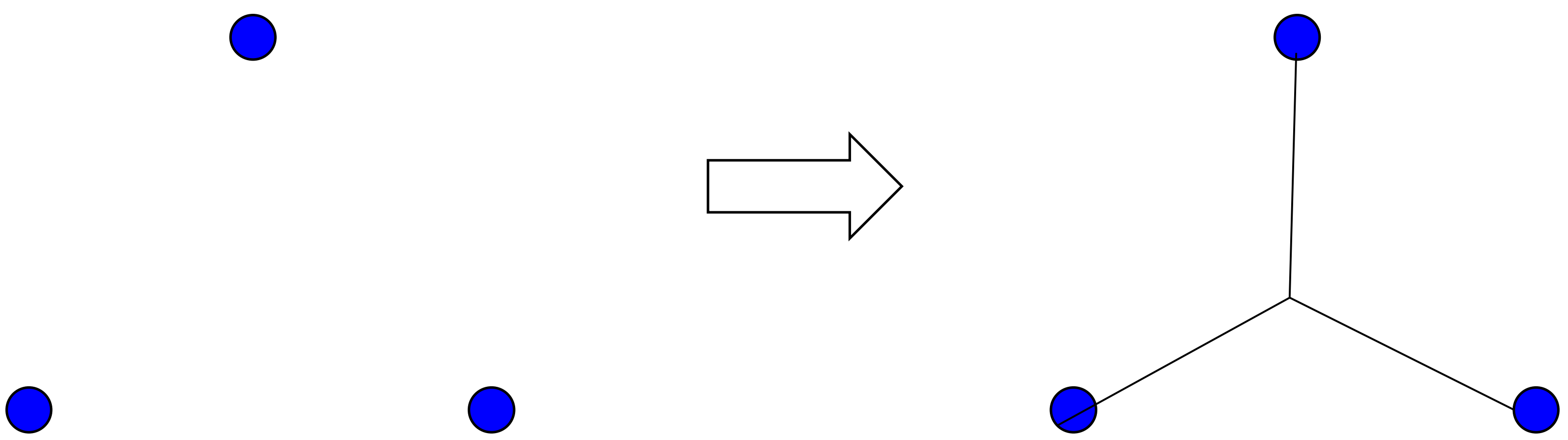
There are a few dots.

- Connect the dots.
- The number of lines is not limited.
- Two lines can cross.
- Connect the dots so that one can go from one dot to all the other dots.

How do you connect the lines if you want to make the total length of lines the shortest?

e.g.) Equilateral triangle

Connect symmetrically



[Quiz](#)

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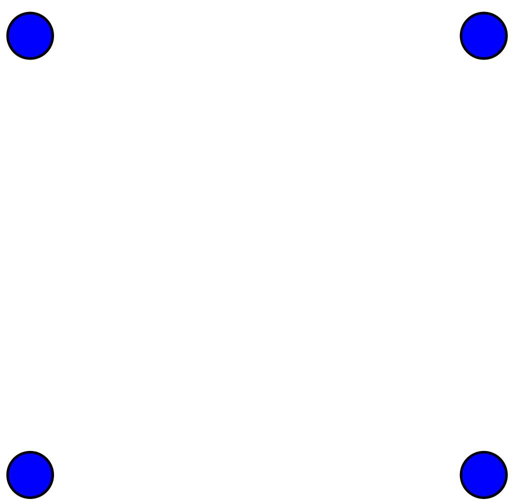
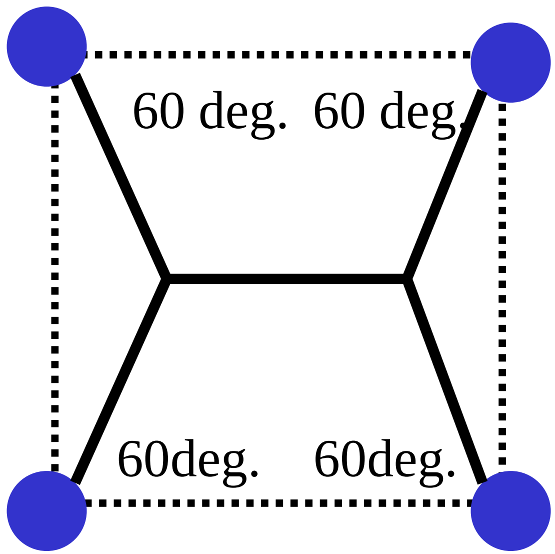
- Two lines can cross.
- Connect the dots so that one can go from one dot to all the other dots.

How do you connect the lines if you want to make the total length of lines the shortest?

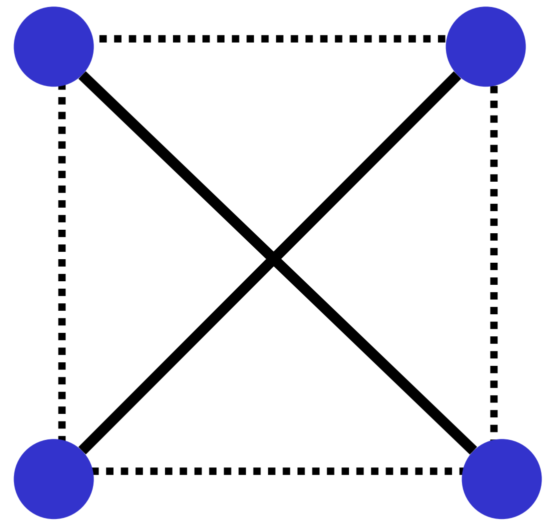
(question) how about the case for a square?



(answer)




cf.



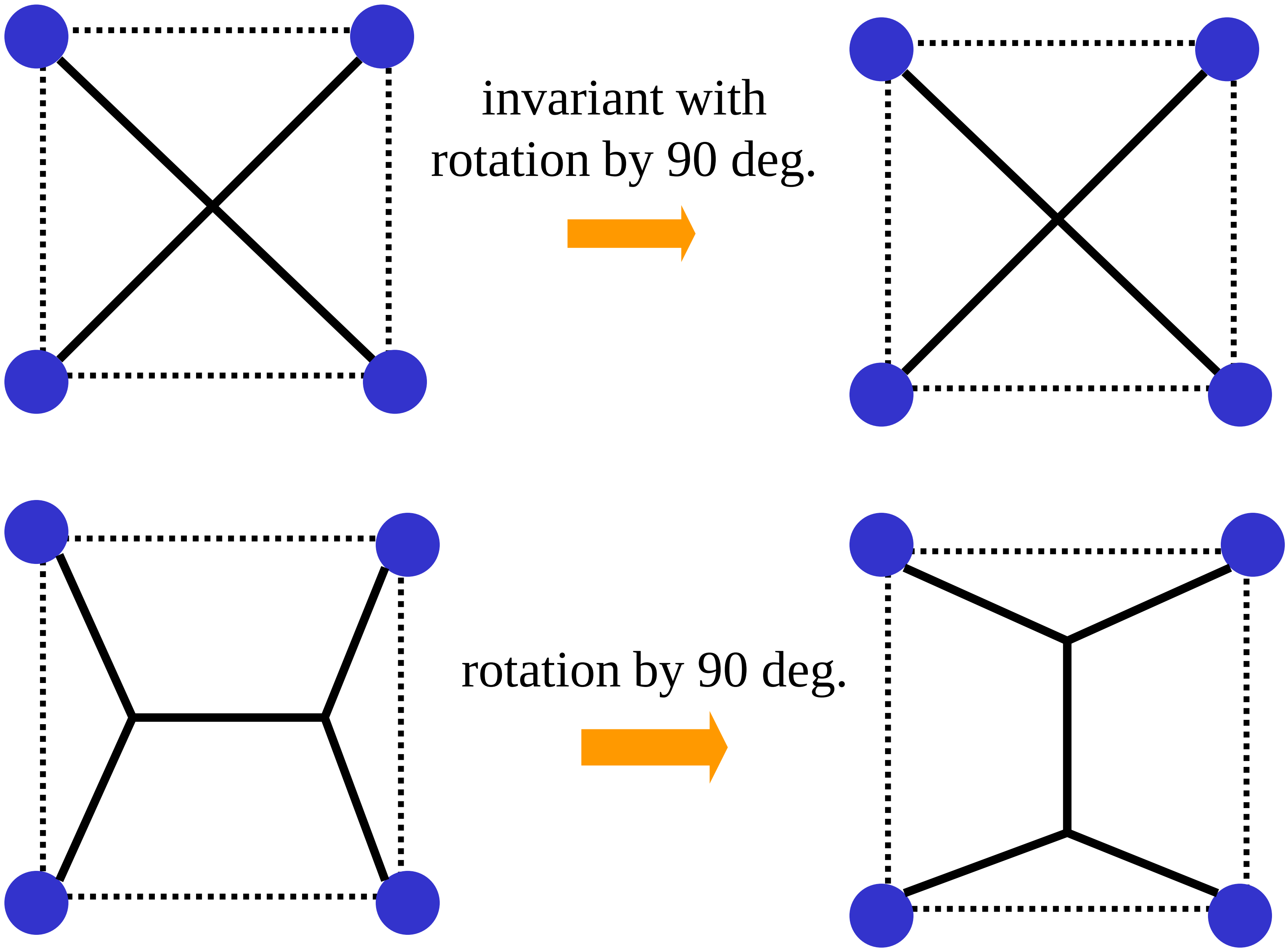
Length

$$4 \times \frac{1}{\sqrt{3}} + \left(1 - 2 \times \frac{1}{2\sqrt{3}}\right)$$
$$= 1 + \sqrt{3}$$
$$= 2.732 \cdots$$

Length

$$2 \times \sqrt{2} = 2.828 \cdots$$

Ref. Takeshi Koike,
"Genshikaku Kenkyu" Vol. 52 No. 2, p. 14




a good example of spontaneous symm. breaking

Courtesy: Takeshi Koike