Properties of one-neutron halo nuclei

- bound states
- Effects of angular momentum
- Coulomb excitations
- Deformation

weakly bound

square well potential
($l=0$ bound state)
Nuclear Physics: developed for stable nuclei (until mid 1980’s)
saturation, radii, binding energy,
magic numbers and independent particle…. 
Nuclear Physics: developed for stable nuclei (until mid 1980’s)
natural questions:
  - how many neutrons can be put into a nucleus when the number of proton is fixed?
  - what are the properties of nuclei far from the stability line?
Start of a research on unstable nuclei: interaction cross sections (1985)

\[ ^{11}\text{Li} \rightarrow \text{nuclei other than } ^{11}\text{Li} \]

target nucleus

\[ R_I(P) \quad R_I(T) \]

projectile target

if reaction takes place when two nuclei overlap with each other:

\[
\sigma_I \sim \pi [R_I(P) + R_I(T)]^2
\]

\[ \rightarrow R_I(P) \]
stable nuclei

\[ \rho(r) = \frac{\rho_0}{1 + \exp((r - R_0)/a)} \]

\[ \rho_0 \sim 0.17 \text{ (fm}^{-3}\text{)} \]

\[ R_0 \sim 1.1 \times A^{1/3} \text{ (fm)} \]

\[ a \sim 0.57 \text{ (fm)} \]
Start of a research on unstable nuclei: interaction cross sections (1985)

\[ ^{11}\text{Li} \rightarrow \text{nuclei other than } ^{11}\text{Li} \]

\[ R_I(P) \rightarrow R_I(T) \]

If reaction takes place when two nuclei overlap with each other:

\[ \sigma_I \sim \pi [R_I(P) + R_I(T)]^2 \]

\[ \rightarrow R_I(P) \]

very large radius

I. Tanihata et al., PRL55(‘85)2676
One neutron halo nuclei

A typical example: $^{11}_{4}\text{Be}_7$

One neutron separation energy

$^{10}\text{Be} + n$

$S_n = 504 +/- 6$ keV

very small

cf. $S_n = 6.81$ MeV for $^{10}\text{Be}$
One neutron halo nuclei

A typical example: $^{11}_{4}\text{Be}_7$

One neutron separation energy

$^{11}\text{Be} \rightarrow ^{10}\text{Be} + n$

$S_n = 504 \pm 6 \text{ keV}$

very small

Interpretation: a weakly bound neutron surrounding $^{10}\text{Be}$

$\psi(r) \sim \exp(-\kappa r)$

$\kappa = \sqrt{2m|\epsilon|/\hbar^2}$

weakly bound system

large spatial extension of density (halo structure)
Interpretation: a weakly bound neutron surrounding $^{10}\text{Be}$

$$\psi(r) \sim \exp(-\kappa r) \quad \kappa = \sqrt{2m|\epsilon|/\hbar^2}$$

weakly bound system

large spatial extension of density (halo structure)

Density distribution which explains the experimental reaction cross section

lunar halo
(a thin ring around moon)

M. Fukuda et al., PLB268(‘91)339
Momentum distribution

\[ S_{2n} \sim 2.1 \text{ MeV} \]
\[ S_{2n} \sim 300 \text{ keV} \]

a narrow mom. distribution when weakly bound and thus a large spatial extension

T. Kobayashi et al., PRL60 (’88) 2599
assume a 2body system with a core nucleus and a valence neutron

consider a spherical potential \( V(r) \) as a function of \( r \)

cf. mean-field potential:

\[
V(r) \sim \int v(r, r') \rho(r') \, dr'
\]

Hamiltonian for the relative motion

\[
H = -\frac{\hbar^2}{2\mu} \nabla^2 + V(r)
\]
Hamiltonian for the relative motion

\[ H = -\frac{\hbar^2}{2\mu} \nabla^2 + V(r) \]

For simplicity, let us ignore the spin-orbit interaction (the essence remains the same even if no spin-orbit interaction).

Boundary condition for bound states

\[ u_l(r) \sim r^{l+1} \quad (r \sim 0) \]
\[ \rightarrow e^{-\kappa r} \quad (r \rightarrow \infty) \]

* For a more consistent treatment, a modified spherical Bessel function has to be used
Angular momentum and halo phenomenon

\[
\left[ -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{l(l + 1)\hbar^2}{2\mu r^2} + V(r) - \varepsilon_l \right] u_l(r) = 0
\]

Centrifugal potential

Height of centrifugal barrier: 0 MeV ($l = 0$), 0.69 MeV ($l = 1$), 2.94 MeV ($l = 2$)
Wave function

Change $V_0$ for each $l$ so that $\varepsilon = -0.5$ MeV

$l = 0$ : a long tail
$l = 2$ : localization
$l = 1$ : intermediate

root-mean-square radius

$$\sqrt{\langle r^2 \rangle} = \sqrt{\int_0^\infty dr \ r^2 u_l(r)^2}$$

$7.17$ fm ($l = 0$)
$5.17$ fm ($l = 1$)
$4.15$ fm ($l = 2$)
Wave function: localized for all $l$

root-mean-square radius:

$$\sqrt{\langle r^2 \rangle} = \sqrt{\int_0^\infty dr \, r^2 |u_l(r)|^2}$$

- $3.58$ fm ($l = 0$)
- $3.05$ fm ($l = 1$)
- $3.14$ fm ($l = 2$)

For $\varepsilon = -7$ MeV

Wave function

For $\varepsilon = -7$ MeV
Wave functions

$\varepsilon = -0.5 \text{ MeV}$

$u(r)$

$|u(r)|^2$

$r \ (\text{fm})$

$2s$

$1p$

$1d$

$\varepsilon = -7 \text{ MeV}$

$u(r)$

$|u(r)|^2$

$r \ (\text{fm})$

$2s$

$1p$

$1d$
Radius: diverges for $l=0$ and 1 in the zero energy limit

Halo (a very large radius) happens only for $l=0$ or 1

\[
\langle r^2 \rangle \propto \begin{cases} 
1/|\epsilon_0| & (l = 0) \\
1/\sqrt{|\epsilon_1|} & (l = 1) \\
\text{const.} & (l = 2)
\end{cases}
\]
Other candidates for $1n$ halo nuclei

$^{19}\text{C}$: $S_n = 0.58(9)$ MeV

$^{31}\text{Ne}$: $S_n = 0.29 +/- 1.64$ MeV

Coulomb breakup of $^{19}\text{C}$

T. Nakamura et al., PRL83('99)1112

Large Coulomb breakup cross sections

T. Nakamura et al., PRL103('09)262501
Coulomb breakup of 1n halo nuclei

\[ {}^{A}Z \rightarrow {}^{A}Z^{*} \rightarrow {}^{A-1}Z + n \]

Transition from the g.s. to excited states by absorbing \( \gamma \) rays

breakup if excited to continuum states

excitations due to the Coulomb field from the target nucleus
Electromagnetic transitions

Initial state: \( |\psi_i\rangle |n_k\alpha = 1\rangle \)

Transition: \( H_{\text{int}} \) (interaction between a nucleus and EM field)

Final state: \( |\psi_f\rangle |n_k\alpha = 0\rangle \)

State of nucleus: \( \Psi_i \),
+ one photon with momentum \( k \), and polarization \( \alpha \) (\( \alpha = 1 \) or 2)
(note) time-dependent perturbation theory

\[ H_{\text{int}} = \frac{1}{Am} \cdot \frac{Ze}{c} A \cdot p \]

\[ A(r, t) = \sum_{\alpha} \int \frac{dk}{2\pi} \frac{\hbar c}{\sqrt{\hbar \omega}} \left[ a_{k\alpha} \epsilon_\alpha e^{-i\omega t} + a_{k\alpha}^{\dagger} \epsilon_\alpha e^{i\omega t} \right] = A(t) \] (dipole approximation)

transition probability per unit time due to: \( V(r, t) = F(r)e^{\pm i\omega t} \)
(for a transition to a single state)

\[ \Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle f|F|i\rangle|^2 \delta(e_f - e_i \pm \hbar \omega) \] Fermi’s Golden Rule

application to the present problem:

\[ \Gamma_{i \rightarrow f} = \frac{1}{2\pi \hbar} \left( \frac{Ze}{A + 1} \right)^2 (e_f - e_i) |\langle \psi_f|z|\psi_i\rangle|^2 \delta(e_f - e_i - \hbar \omega) \]

(note) photo-absorption cross section if \( \Gamma \) is divided by the photon flux \( c/(2\pi)^3 \):

\[ \sigma_\gamma = \frac{4\pi^2}{\hbar c} \left( \frac{Ze}{A + 1} \right)^2 (e_f - e_i) |\langle \psi_f|z|\psi_i\rangle|^2 \delta(e_f - e_i - \hbar \omega) \]
Application to the present problem:

\[ \Gamma_{i \rightarrow f} = \frac{1}{2\pi \hbar} \left( \frac{Z e}{A + 1} \right)^2 (e_f - e_i) \left| \langle \psi_f | z | \psi_i \rangle \right|^2 \delta(e_f - e_i - \hbar \omega) \]

\[ P_{i \rightarrow f} \sim \left| \langle \psi_f | z | \psi_i \rangle \right|^2 \]

\[ \sum_f P_{i \rightarrow f} = \]
Application to the present problem:

\[ \Gamma_{i \rightarrow f} = \frac{1}{2\pi\hbar} \left( \frac{Ze}{A + 1} \right)^2 (e_f - e_i) \left| \langle \psi_f | z | \psi_i \rangle \right|^2 \delta(e_f - e_i - \hbar \omega) \]

\[ P_{i \rightarrow f} \sim \left| \langle \psi_f | z | \psi_i \rangle \right|^2 \]

\[ \sum_f P_{i \rightarrow f} = \sum_f \langle \psi_i | z | \psi_f \rangle \langle \psi_f | z | \psi_i \rangle \]

\[ = \langle \psi_i | z^2 | \psi_i \rangle \]

large transition probability if the spatial extentation in \( z \) is large.
Wigner-Eckart theorem and reduced transition probability

\[
\sigma_\gamma = \frac{16\pi^3}{3\hbar c} \left( \frac{Ze}{A+1} \right)^2 E_\gamma \left| \langle \psi_f | rY_{10} | \psi_i \rangle \right|^2 \delta(e_f - e_i - E_\gamma)
\]

\[
E_\gamma = e_f - e_i = \hbar \omega
\]

\[
\left| \langle \psi_f | rY_{10} | \psi_i \rangle \right|^2 \rightarrow \frac{1}{2l + 1} \sum_{m,m'} \left| \langle \psi_{l'm'} | rY_{10} | \psi_{lm} \rangle \right|^2
\]

\[
= \frac{1}{3} \cdot \frac{1}{2l + 1} \left| \langle \psi_{l'} | rY_1 | \psi_i \rangle \right|^2
\]

\[
\sigma_\gamma = \frac{16\pi^3}{9\hbar c} E_\gamma \cdot \frac{1}{2l + 1} \left| \langle \psi_f | eE_1 rY_1 | \psi_i \rangle \right|^2 \delta(e_f - e_i - E_\gamma)
\]

\[
= \frac{16\pi^3}{9\hbar c} \frac{dB(E1)}{dE_\gamma}
\]

Reduced transition probability

\[
\frac{dB(E1)}{dE_\gamma} = \frac{1}{2l + 1} \left| \langle \psi_f | e_{E1} rY_1 | \psi_i \rangle \right|^2 \delta(e_f - e_i - E_\gamma)
\]
E1 effective charge

\[ \sigma_\gamma = \frac{16\pi^3}{3\hbar c} \left( \frac{Ze}{A+1} \right)^2 (e_f - e_i) \left| \langle \psi_f | r Y_{10} | \psi_i \rangle \right|^2 \delta(e_f - e_i - \hbar \omega) \]

dipole operator: \[ \hat{D}_\mu = e_{E1} \cdot r Y_{1\mu}(\theta, \phi) \]

\[ e_{E1} = \frac{Z}{A+1} e \]

Distribution of charges measured from the center of mass

\[ Z_1 (r_1 - R) + Z_2 (r_2 - R) \]

\[ R = \frac{A_1 r_1 + A_2 r_2}{A_1 + A_2} \]

\[ = \ldots \]

\[ = \frac{Z_1 A_2 - Z_2 A_1}{A_1 + A_2} (r_1 - r_2) \]

\[ = \frac{Z_1 A_2 - Z_2 A_1}{A_1 + A_2} r \]

\[ e_{E1} = \frac{Z_1 A_2 - Z_2 A_1}{A_1 + A_2} e \]

(a general formula for 2-body)
Coulomb breakup cross sections

\[ \sigma_\gamma = \frac{16\pi^3}{9\hbar c} E_\gamma \cdot \frac{d\gamma}{dE_\gamma} \]

\[ \frac{d\sigma_\gamma}{dE_\gamma} \sim \frac{16\pi^3}{9\hbar c} \cdot \frac{d\gamma}{dE_\gamma} \]

In actual nuclear reactions, absorption of virtual photons rather than real photons

\[ \frac{d\sigma}{dE_{ex}} \sim \frac{16\pi^3}{9\hbar c} \cdot N_{E1}(E_{ex}) \cdot \frac{dB(E1)}{dE_{ex}} \]

# of virtual photon

See: C.A. Bertulani and P. Danielwicz, “Introduction to Nuclear Reactions” for more details
Simple estimate of E1 strength distribution (analytic model)

Transition from an \( l = 0 \) to an \( l = 1 \) states:

\[
\begin{align*}
&\text{WF for the initial state:} \quad \Psi_i(r) = \sqrt{2\kappa} \frac{e^{-\kappa r}}{r} Y_{00}(\hat{r}) \quad \kappa = \sqrt{\frac{2\mu |E_b|}{\hbar^2}} \\
&\text{WF for the final state:} \quad \Psi_f(r) = \sqrt{\frac{2\mu k}{\pi \hbar^2}} j_1(k r) Y_{1m}(\hat{r}) \quad j_1(k r) : \text{spherical Bessel function} \\
&k = \sqrt{\frac{2\mu E_c}{\hbar^2}}
\end{align*}
\]

\[
\frac{d B(E1)}{dE} = \frac{3}{4\pi} e^{2}_{E1} \left| \int_{0}^{\infty} r^2 dr \cdot \frac{\sqrt{2\kappa e^{-\kappa r}}}{r} \cdot \sqrt{\frac{2\mu k}{\pi \hbar^2}} j_1(k r) \right|^2
\]

The integral can be performed analytically

\[
\frac{d B(E1)}{dE} = \frac{3\hbar^2}{\pi^2 \mu} e^{2}_{E1} \sqrt{|E_b|} E_{c}^{3/2} \frac{E_c^{3/2}}{(|E_b| + E_c)^4}
\]

Refs. (for more general \( l_i \) and \( l_f \))
- S. Typel and G. Baur, NPA759(‘05)247
\[
\frac{dB(E1)}{dE} = \frac{3\hbar^2}{\pi^2 \mu} \frac{e^2}{E_1} \sqrt{|E_b|} \frac{E_c^{3/2}}{(|E_b| + E_c)^4}
\]

peak position:

\[E_c = \frac{3}{5} |E_b|\]

\[(E_x = E_c - E_b = \frac{8}{5} |E_b|)\]

peak height:

\[\propto \frac{1}{|E_b|^2}\]

Total transition probability:

\[B(E1) = S_0 = \frac{3\hbar^2 e^2 E_1}{16\pi^2 \mu |E_b|}\]

➢ a high and sharp peak as the bound state energy, \(|E_b|\), becomes small

➢ As the bound state energy, \(|E_b|\), gets small, the peak appears at a low energy

\[E_{\text{peak}} = 0.28 \text{ MeV} \quad (E_b = -0.5 \text{ MeV})\]

cf. \[\frac{3}{5} |E_b| = 0.3 \quad \text{MeV}\]
\( ^{11}\text{Be} = ^{10}\text{Be} + n \)

transition from the \(2s_{1/2}\) state (bound) to the p-wave (\(l = 1\)) state

Comparison between a weakly-bound case and a strongly-bound case

Actual numerical calculations with a Woods-Saxon potential
\[ 11\text{Be} = ^{10}\text{Be} + n \quad 2s_{1/2} \rightarrow \text{p state} \]

- A high and sharp peak as the bound state energy, \(|E_b|\), becomes small.

\[
S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c} \\
= 1.53 \text{ e}^2\text{fm}^2 (E_b = -0.5 \text{ MeV}) \\
0.32 \text{ e}^2\text{fm}^2 (E_b = -7 \text{ MeV})
\]

- As the bound state energy, \(|E_b|\), gets small, the peak appears at a low energy.

\[
E_{\text{peak}} = 0.28 \text{ MeV} (E_b = -0.5 \text{ MeV}) \\
0.96 \text{ MeV} (E_b = -7 \text{ MeV})
\]

- Weak \(E_b\) dependence when the transition strength is multiplied by \((E_c - E_b)\).

\[
S_1 = \int_0^\infty dE_c (E_c - E_b) \frac{dB(E1)}{dE_c} \\
= 2.79 \text{ e}^2\text{fm}^2 \text{ MeV} (E_b = -0.5 \text{ MeV}) \\
3.18 \text{ e}^2\text{fm}^2 \text{ MeV} (E_b = -7 \text{ MeV})
\]
Sum Rule

\[ S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c} = \frac{3}{4\pi} e_1^2 \langle r^2 \rangle_i \]

Total E1 transition probability: proportional to the g.s. expectation value of \( r^2 \)

\[ S_0 = \int_0^\infty dE_c \frac{dB(E1)}{dE_c} \]

\[ = 1.53 \text{ e}^2 \text{fm}^2 \ (E_b = -0.5 \text{ MeV}) \]
\[ = 0.32 \text{ e}^2 \text{fm}^2 \ (E_b = -7 \text{ MeV}) \]

\[ = 1.62 \text{ e}^2 \text{fm}^2 \ (E_b = -0.5 \text{ MeV}) \]
\[ = 0.41 \text{ e}^2 \text{fm}^2 \ (E_b = -7 \text{ MeV}) \]

* almost coincide with each other. Small difference due to Pauli forbidden transitions (the transition from 2s to 1p)
**Sum Rule**

\[
S_0 = \int_0^\infty dE_c \frac{d\mathcal{B}(E1)}{dE_c} = \frac{3}{4\pi} e_{E1}^2 \langle r^2 \rangle_i
\]

- **Total E1 transition probability**: proportional to the g.s. expectation value of \(r^2\)

  - If the initial state is \(l=0\) or \(l=1\), the radius increases for weakly bound

    ![Graph showing rms radius vs. energy](image)

    - **Enhancement of total E1 prob.**
    - Inversely, if a large E1 prob. (or a large Coul. b.u. cross sections) are observed, this indicates \(l=0\) or \(l=1\) → halo structure
Other candidates for $\text{1n halo nuclei}$

$^{19}\text{C}$: $S_n = 0.58(9)$ MeV

$^{31}\text{Ne}$: $S_n = 0.29 \pm 1.64$ MeV

Large Coulomb breakup cross sections

T. Nakamura et al., PRL83(‘99)1112

T. Nakamura et al., PRL103(‘09)262501
With a spherical potential:

\[ 1s_{1/2} [2] \]
\[ 1p_{1/2} [2] \]
\[ 1p_{3/2} [4] \]

The g.s. of $^{11}$Be: $I^\pi = 1/2^-$

In reality.....

0.32 MeV

$^{11}$Be

\[ \text{very artificial} \]

What happens if $^{11}$Be is deformed?
s.p. motion in a deformed potential

halo : only for $l = 0$ or $1$

$\Rightarrow$ however, a possibility is enlarged for a deformed nucleus

deformed potential $V(r, \theta)$ $\Rightarrow$ mixture of angular momenta

e.g.,

\begin{align*}
|d_{5/2}\rangle & \rightarrow |d_{5/2}\rangle + |s_{1/2}\rangle + |g_{7/2}\rangle + \cdots \\
|f_{7/2}\rangle & \rightarrow |f_{7/2}\rangle + |p_{3/2}\rangle + |p_{1/2}\rangle + \cdots
\end{align*}

(note) $s_{1/2}$: $\Omega^\pi = 1/2^+$ only

$p_{1/2}$: $\Omega^\pi = 1/2^-$ only

$p_{3/2}$: $\Omega^\pi = 3/2^-$ and $1/2^-$ only

$\Rightarrow$ possibility of halo only for s.p. states with $\Omega^\pi = 1/2^+, 1/2^-, 3/2^-$
s.p. motion in a deformed potential

\[ |d_{5/2}\rangle \rightarrow |d_{5/2}\rangle + |s_{1/2}\rangle + |g_{7/2}\rangle + \cdots \]
\[ \rightarrow |s_{1/2}\rangle \quad (|\epsilon| \rightarrow 0) \]

T. Misu, W. Nazarewicz, and S. Aberg, NPA614('97)44
(deformed square well)

When weakly bound, the \(l=0\) terms becomes dominant even for a very small deformation
(in the zero binding limit, 100\% of \(l=0\) component)

I. Hamamoto, PRC69('04)041306(R)
(deformed Woods-Saxon)
s-wave dominance phenomenon

I. Hamamoto, PRC69(‘04)041306(R)

$l = 0$ component is also enhanced when weakly bound (but, always less than 100%)

a possibility of deformed halo nucleus: $^{31}\text{Ne}$
Nilsson model analysis [I. Hamamoto, PRC81(‘10)021304(R)]

21st neutron

non-halo
($\Omega^\pi = 3/2^+$)

(a) Probabilities of major components of the $[330 \, 1/2]$ level
$R = 3.946$ fm (A = 30) $\beta = 0.3$

(b) Probabilities of major components of the $[321 \, 3/2]$ level
$R = 3.946$ fm (A = 30) $\beta = 0.5$
Large Coulomb breakup cross section

T. Nakamura et al.,
PRL103(‘09)262501

Y. Urata, K.Hagino, and H. Sagawa,
PRC83(‘11)041303(R)
Another example: $^{37}\text{Mg}$