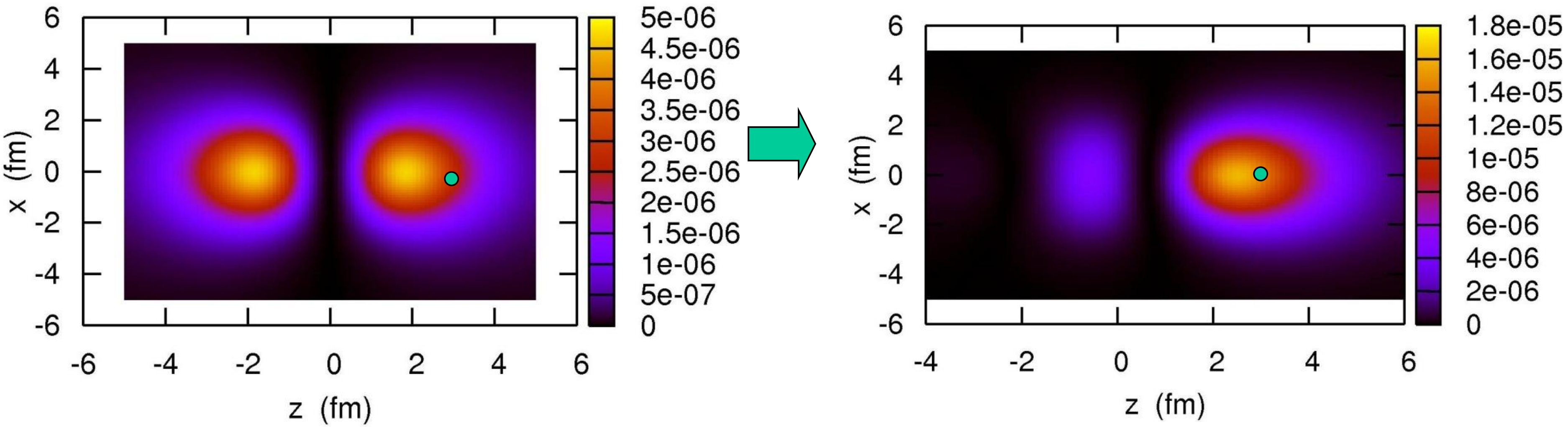
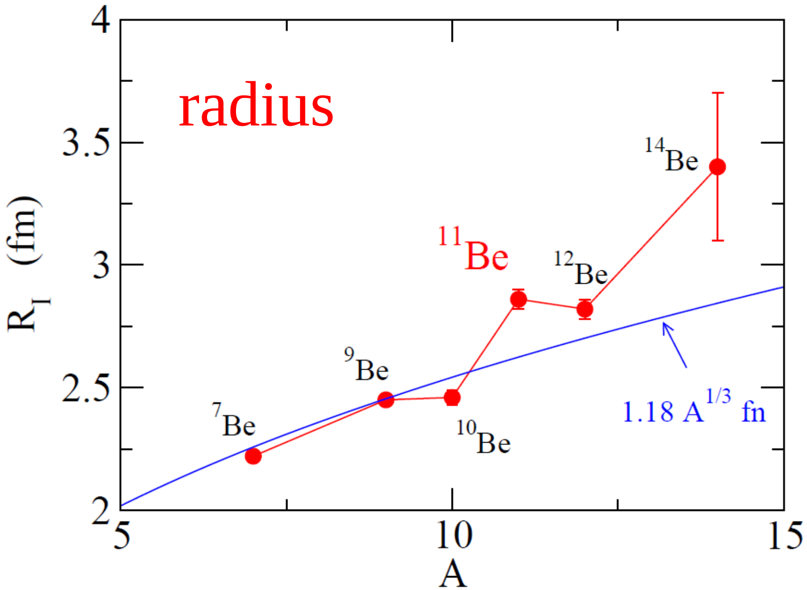
2-neutron halo nuclei and pairing correlation

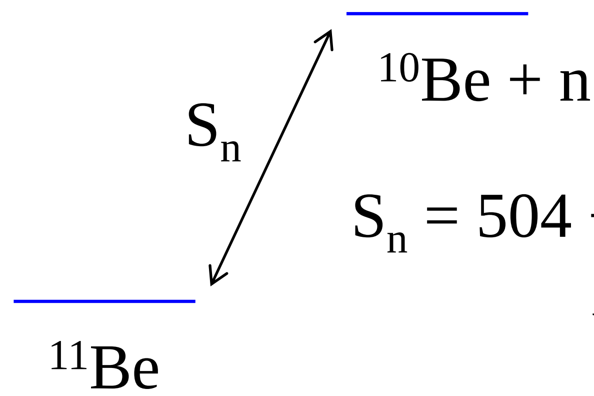


One neutron halo nuclei

A typical example: ${}^{11}{}_{4}\mathrm{Be}_{7}$



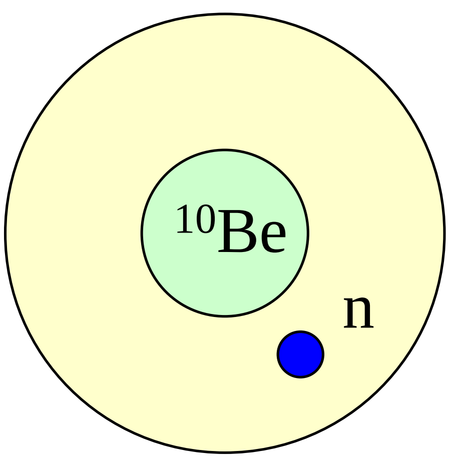
One neutron separation energy



S_n = 504 +/- 6 keV

very small

Interpretation : a weakly bound neutron surrounding ^{10}Be

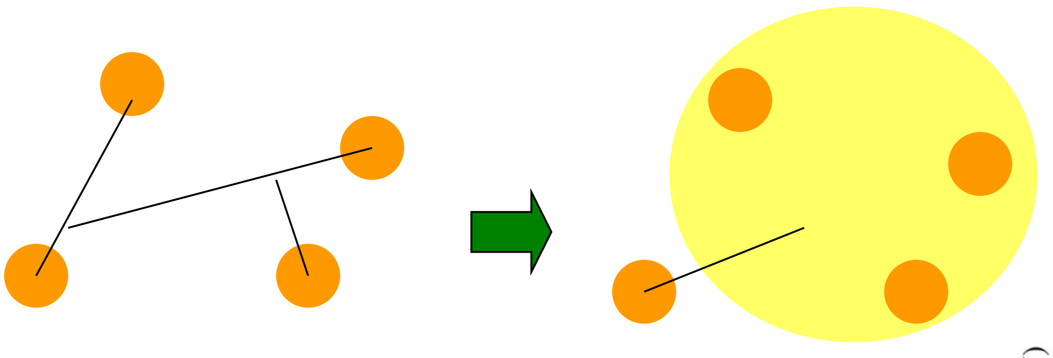


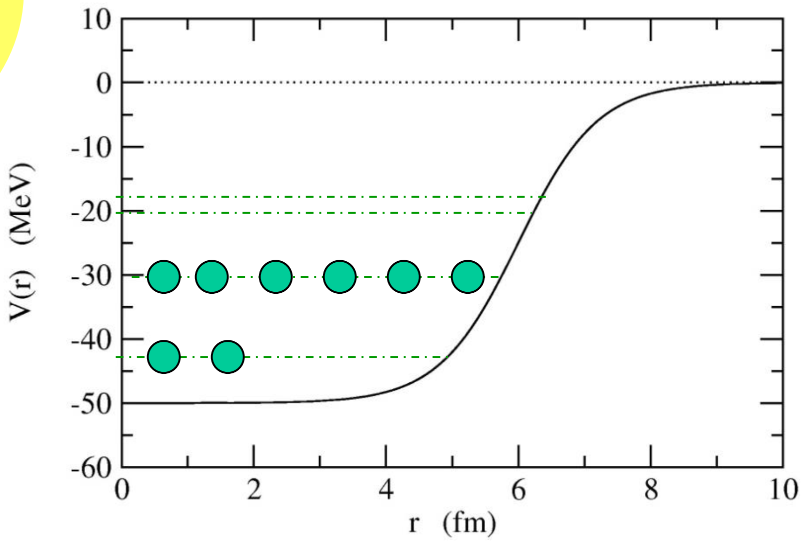
$$\psi(r) \sim \exp(-\kappa r) \qquad \kappa = \sqrt{2m|\epsilon|/\hbar^2}$$

weakly bound system

large spatial extension of density (halo structure)

Hartree-Fock Method

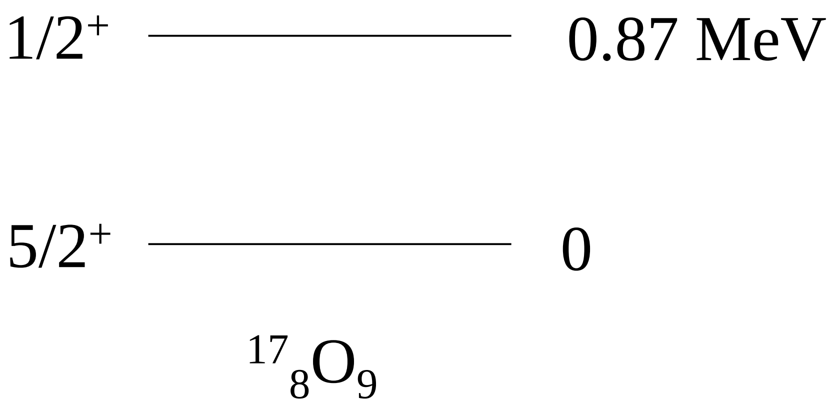


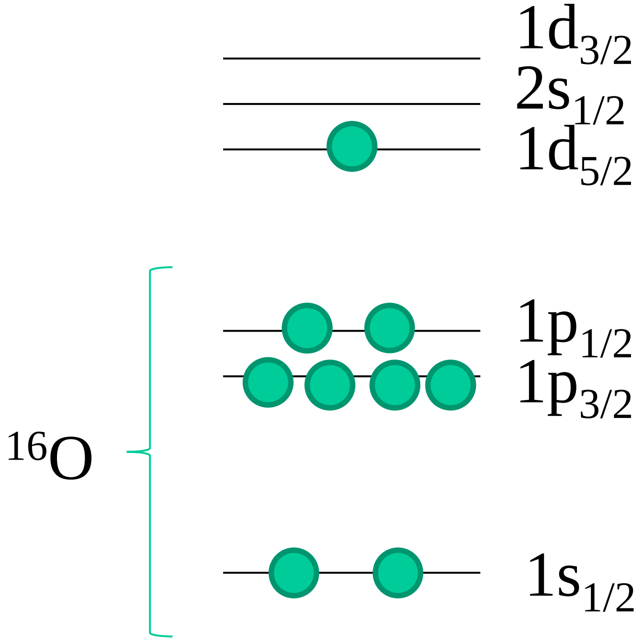


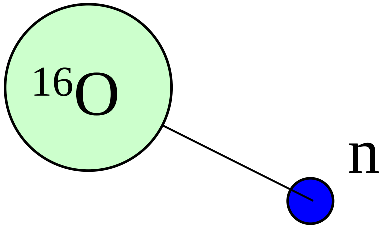
independent particle motion
in a potential well

$$\Psi(1,2,\cdots,A) = \mathcal{A}[\psi_1(1)\psi_2(2)\cdots\psi_A(A)]$$
$$= \frac{1}{\sqrt{A!}}\begin{vmatrix} \psi_1(1) & \psi_2(1) & \cdots & \psi_A(1) \\ \psi_1(2) & \psi_2(2) & \cdots & \psi_A(2) \\ \vdots & & & \\ \psi_1(A) & \psi_2(A) & \cdots & \psi_A(A) \end{vmatrix}$$

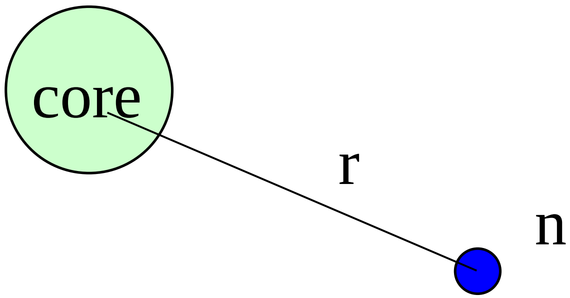
Slater determinant: antisymmetrization due to the Pauli principle


1/2+
0.87 MeV
5/2+
0
$^{17}_{8}O_9$

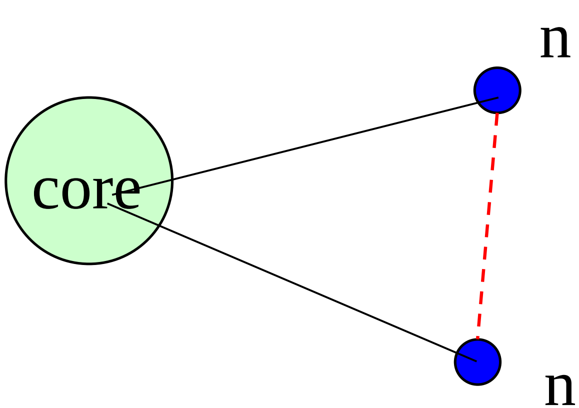

$1d_{3/2}$
$2s_{1/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
^{16}O
$1s_{1/2}$


^{16}O
n

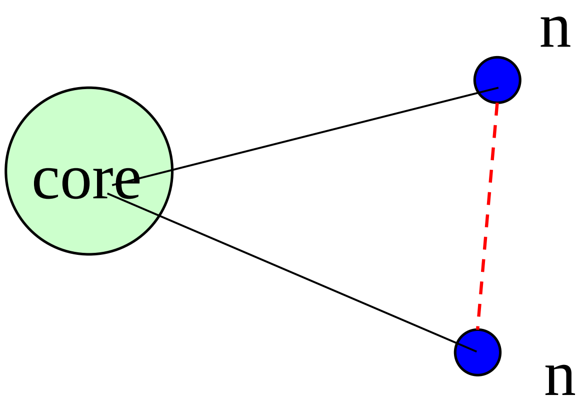
Pairing correlation



What happens if there are two neutrons outside the core nucleus?

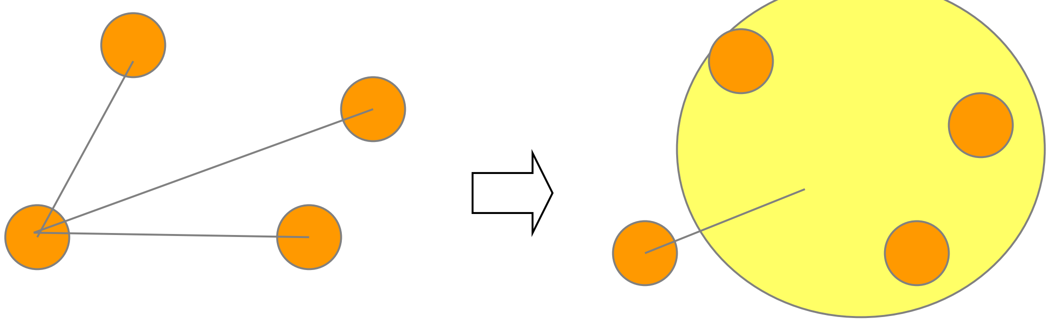


What is the influence of the interaction between the two neutrons?



What is the influence of the interaction between the two neutrons?

Mean-field theory



treat the interaction among particles only on average

the pure mean-field picture

→ the interaction between the two neutrons :

only through the mean-field potential,

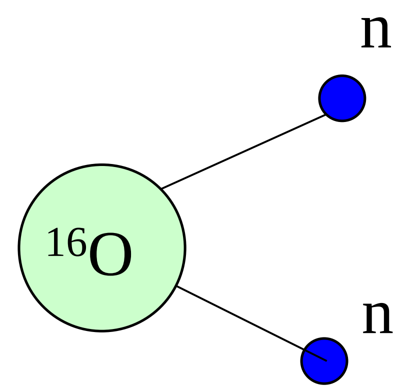
(the two neutrons: uncorrelated).

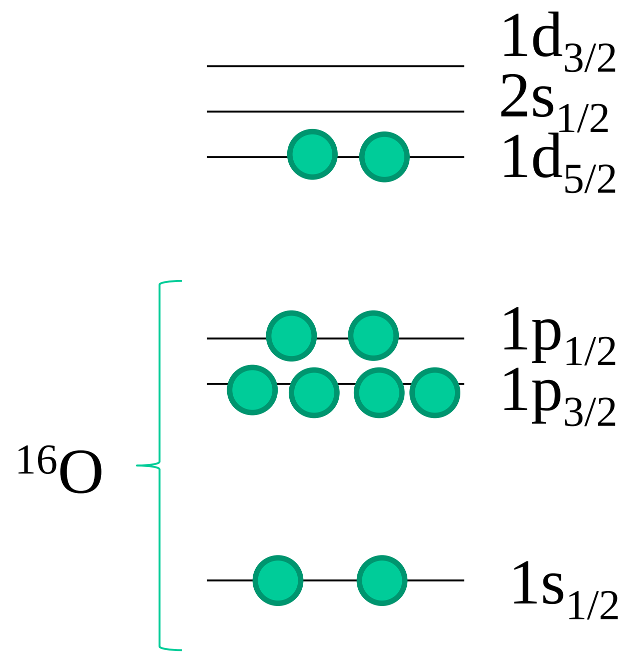
$$H = \sum_i T_i + \sum_{i<j} v_{ij} \rightarrow H = \sum_i (T_i + V_i) + \sum_{i<j} v_{ij} - \sum_i V_i$$

deviation from the average
(residual interaction)

Can the residual interaction be neglected completely?

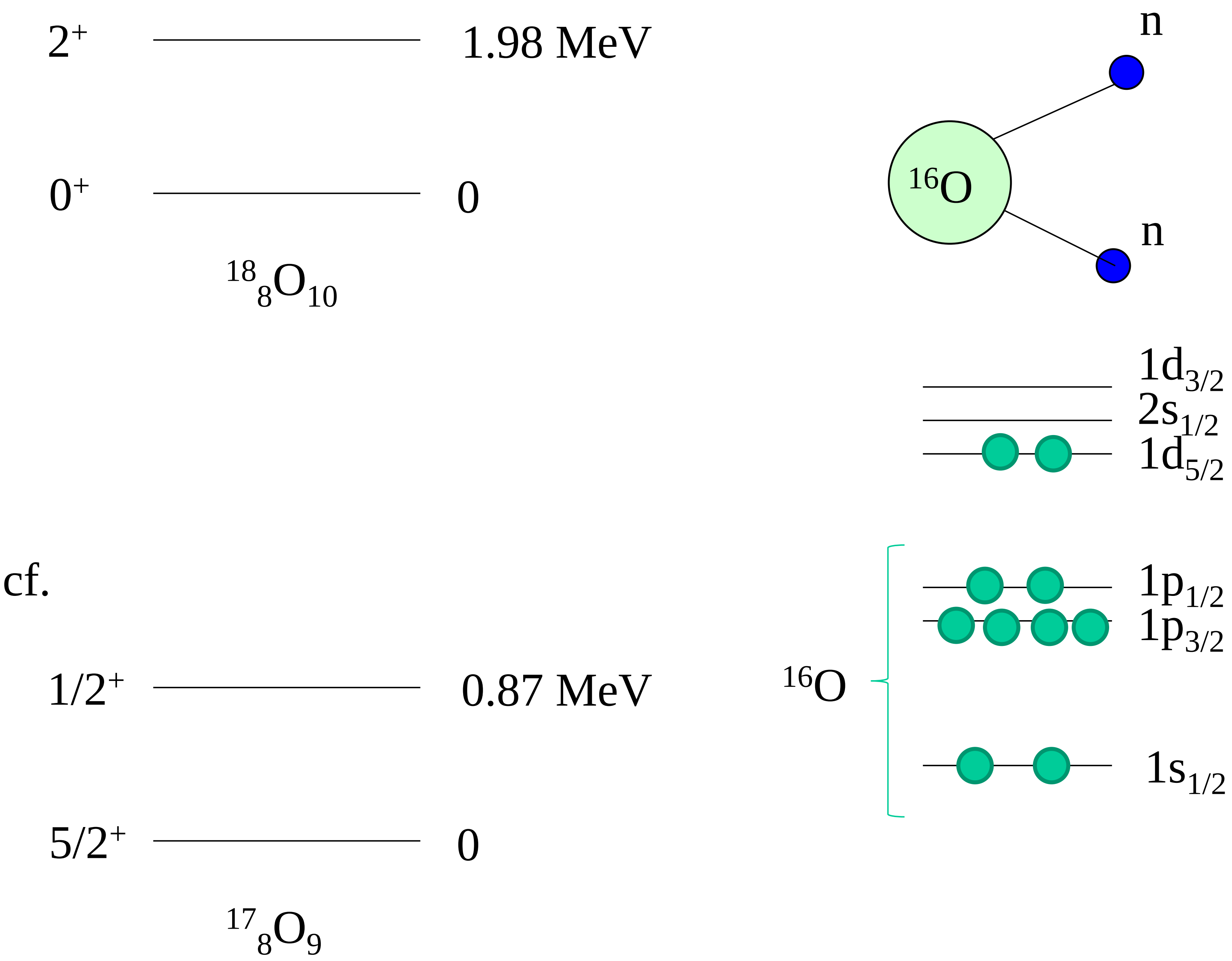
$\longrightarrow$ it has been known that it plays an important role in open-shell nuclei (pairing correlation)





cf.

$^{17}_{8}O_9$



2+
1.98 MeV
0+
0
18 8 O 10
n
16O
n
1d3/2
2s1/2
1d5/2
cf.
1p1/2
1p3/2
16O
1/2+
0.87 MeV
1s1/2
5/2+
0
17 8 O 9

$$H = \sum_i T_i + \sum_{i<j} v_{ij} \rightarrow H = \sum_i (T_i + V_i) + \underbrace{\sum_{i<j} v_{ij} - \sum_i V_i}$$

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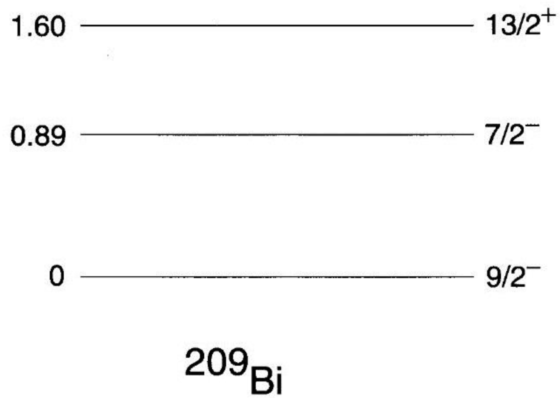
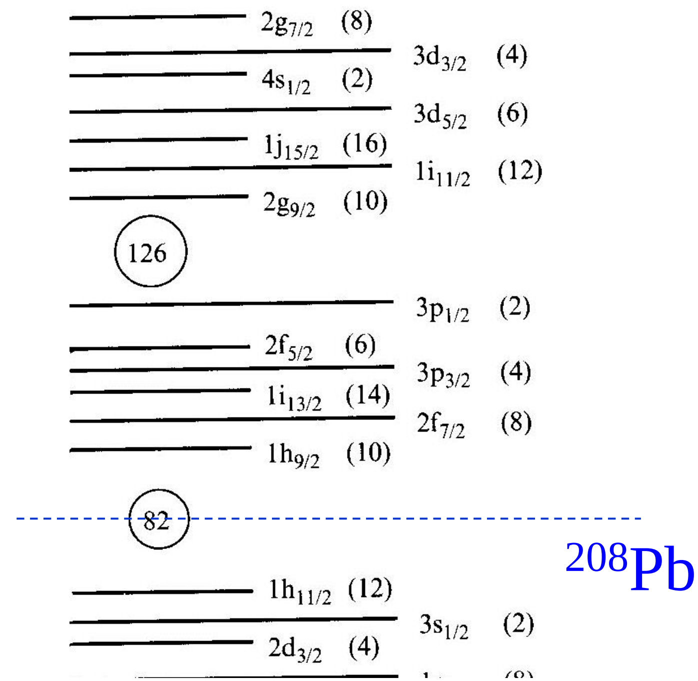
Pairing Correlations

$^{209}_{83}Bi_{126} = {}^{208}_{82}Pb_{126} + p$





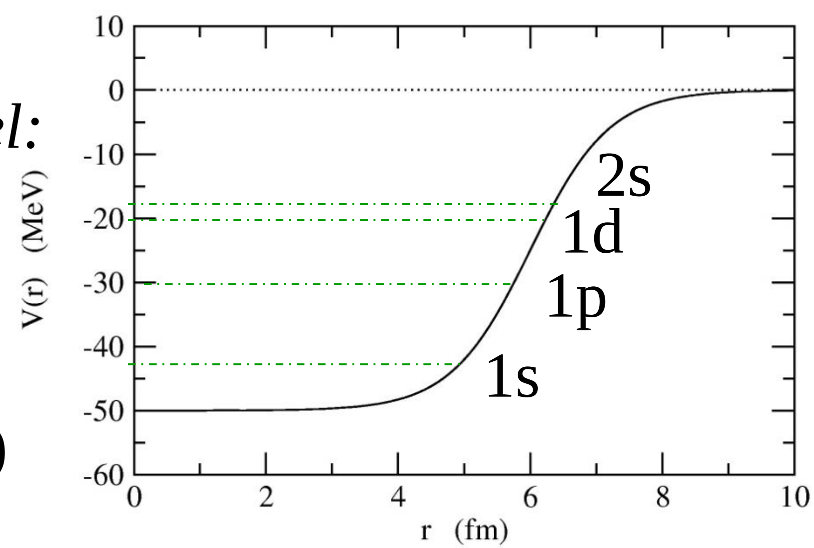
$^{210}_{84}Po_{126} = {}^{208}_{82}Pb_{126} + 2p$

expectation of the indep. particle model:

E=0: $[h_{9/2} \otimes h_{9/2}]^I$ $(I=0,2,4,6,8)$

E=0.89 MeV: $[h_{9/2} \otimes f_{7/2}]^I$

$(I=1,2,3,4,5,6,7,8)$



⇒ # of states below 1 MeV: 13

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⇨ # of states below 1 MeV: 13

observed spectra:

1.20 MeV ———— 4^+

0.81 MeV ———— 2^+

0 ———— 0^+

^{210}Po

⟺ Effects of the residual interaction

$$H = \sum_{i=1}^{A} \left(-\frac{\hbar^2}{2m} \nabla_i^2 + V_{\mathrm{HF}}(i) \right) + \underline{\frac{1}{2} \sum_{i,j}^{A} v(\boldsymbol{r}_i, \boldsymbol{r}_j) - \sum_i V_{\mathrm{HF}}(i)}$$

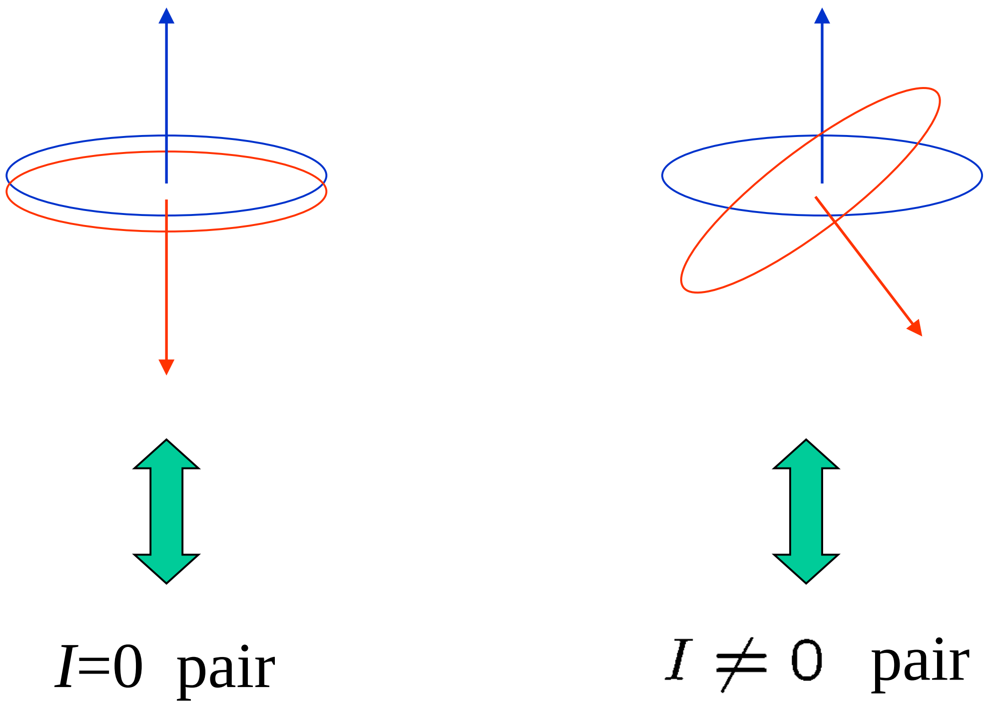
Pairing correlation

$$H = \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + V_{\rm HF}(i)\right) + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i, \boldsymbol{r}_j) - \sum_i V_{\rm HF}(i)$$

A delta function interaction for a residual interaction:
(an extremely short range interaction)

$$v_{\rm res}(\boldsymbol{r}, \boldsymbol{r}') \sim -g\,\delta(\boldsymbol{r} - \boldsymbol{r}')$$
$$= -g\frac{\delta(r - r')}{rr'}\sum_{\lambda\mu} Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})Y_{\lambda\mu}(\hat{\boldsymbol{r}}')$$

Simple interpretation:



The spatial overlap is the largest for the I=0 pair.

"Pairing Correlation"

(note) The I=$2j$ pair is unfavoured due to the Pauli principle.

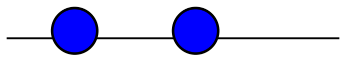
(note)

$$\psi(l^2; L=0) = \sum_{\mu} \langle l\mu l - \mu | L=0,0\rangle \, Y_{l\mu}(\hat{\boldsymbol{r}}_1) Y_{l-\mu}(\hat{\boldsymbol{r}}_2) = Y_{l0}(\theta_{12})/\sqrt{4\pi}$$

Pairing correlations

$$v_{\rm res}(\boldsymbol{r},\boldsymbol{r}') \sim -g\,\delta(\boldsymbol{r}-\boldsymbol{r}')$$

$$= -g\frac{\delta(r-r')}{rr'}\sum_{\lambda\mu}Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})Y_{\lambda\mu}(\hat{\boldsymbol{r}}')$$

l 

$$|(ll)LM\rangle = \sum_{m,m'}\langle lmlm'|LM\rangle\psi_{lm}(\boldsymbol{r})\psi_{lm'}(\boldsymbol{r}')$$

⇩

The energy change due to the residual interaction:

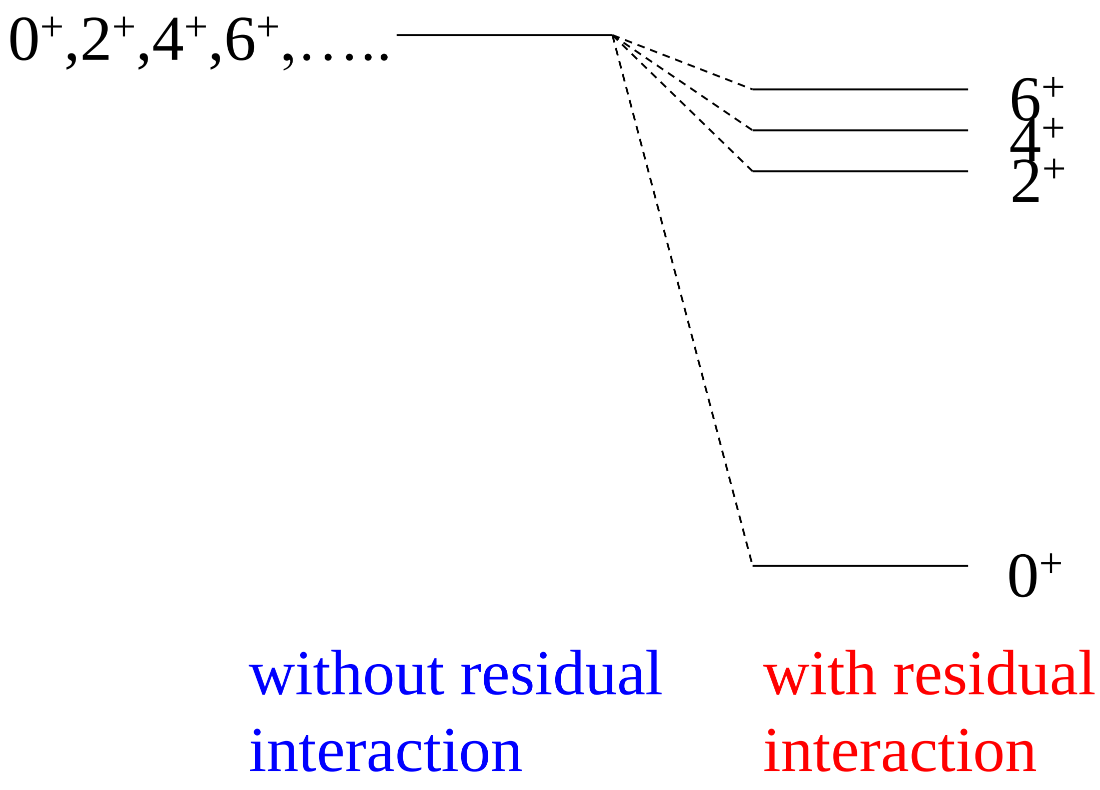
$$\Delta E_L = \langle (ll)LM|v_{\rm res}|(ll)LM\rangle$$

$$= -g\,I_r^{(l)}\,\frac{(2l+1)^2}{4\pi}\begin{pmatrix} l & l & L \\ 0 & 0 & 0 \end{pmatrix}^2$$

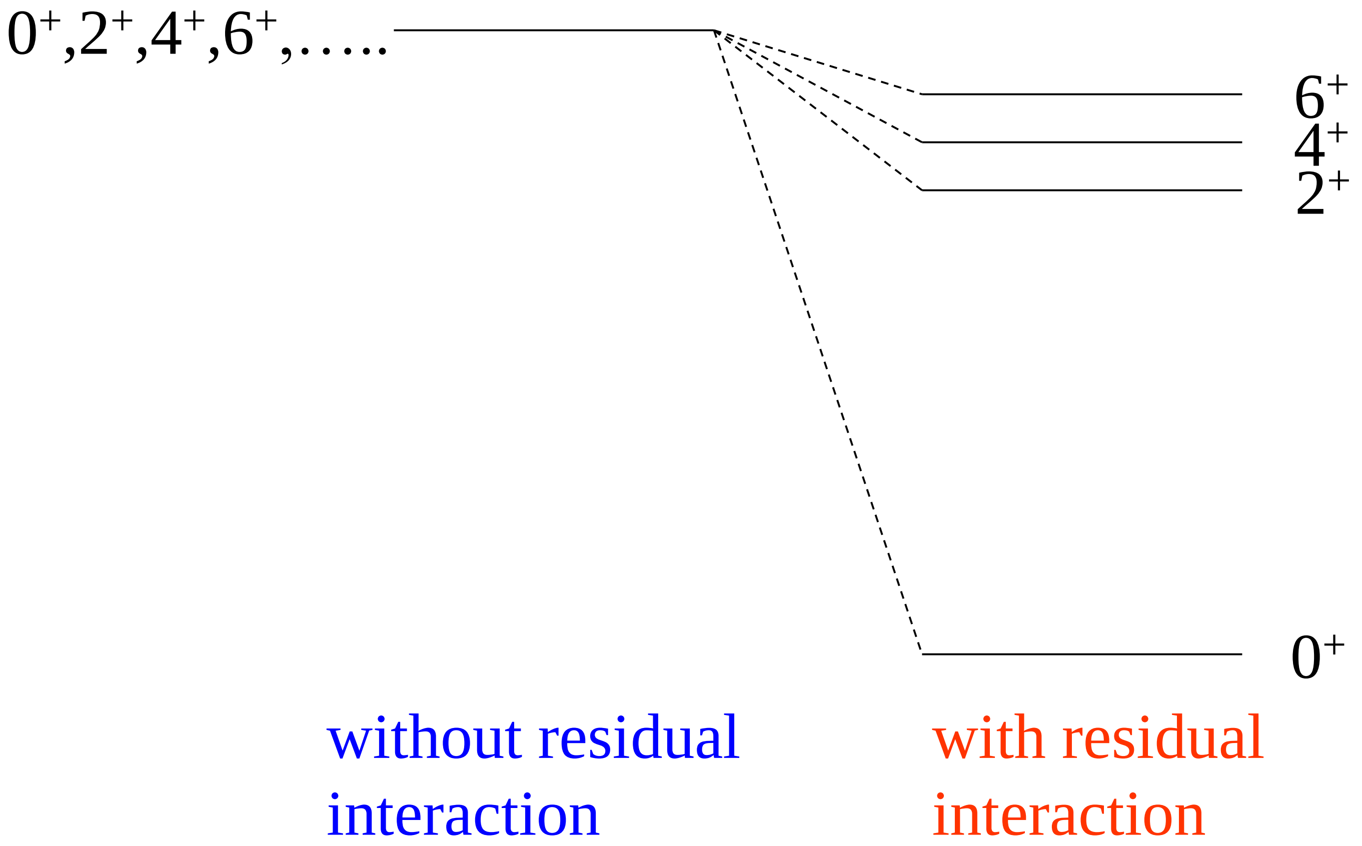
$$I_r^{(l)} = \int_0^\infty r^2dr\,(R_l(r))^4$$

$$\Delta E_L = -g\, I_r^{(l)} \frac{(2l+1)^2}{4\pi} \begin{pmatrix} l & l & L \\ 0 & 0 & 0 \end{pmatrix}^2 \equiv -g\, I_r^{(l)} \frac{A(ll;L)}{4\pi}$$

$A(ll;L)$	L=0	L=2	L=4	L=6	L=8
l = 2	5.00	1.43	1.43	---	---
l = 3	7.00	1.87	1.27	1.63	---
l = 4	9.00	2.34	1.46	1.26	1.81



“Pairing Correlation”



<u>The ground state spin of nuclei</u>

- Even-even nuclei: 0^+
- Even-odd nuclei: the spin of the valence particle

Binding energy

Extra binding when like nucleons form a spin-zero pair

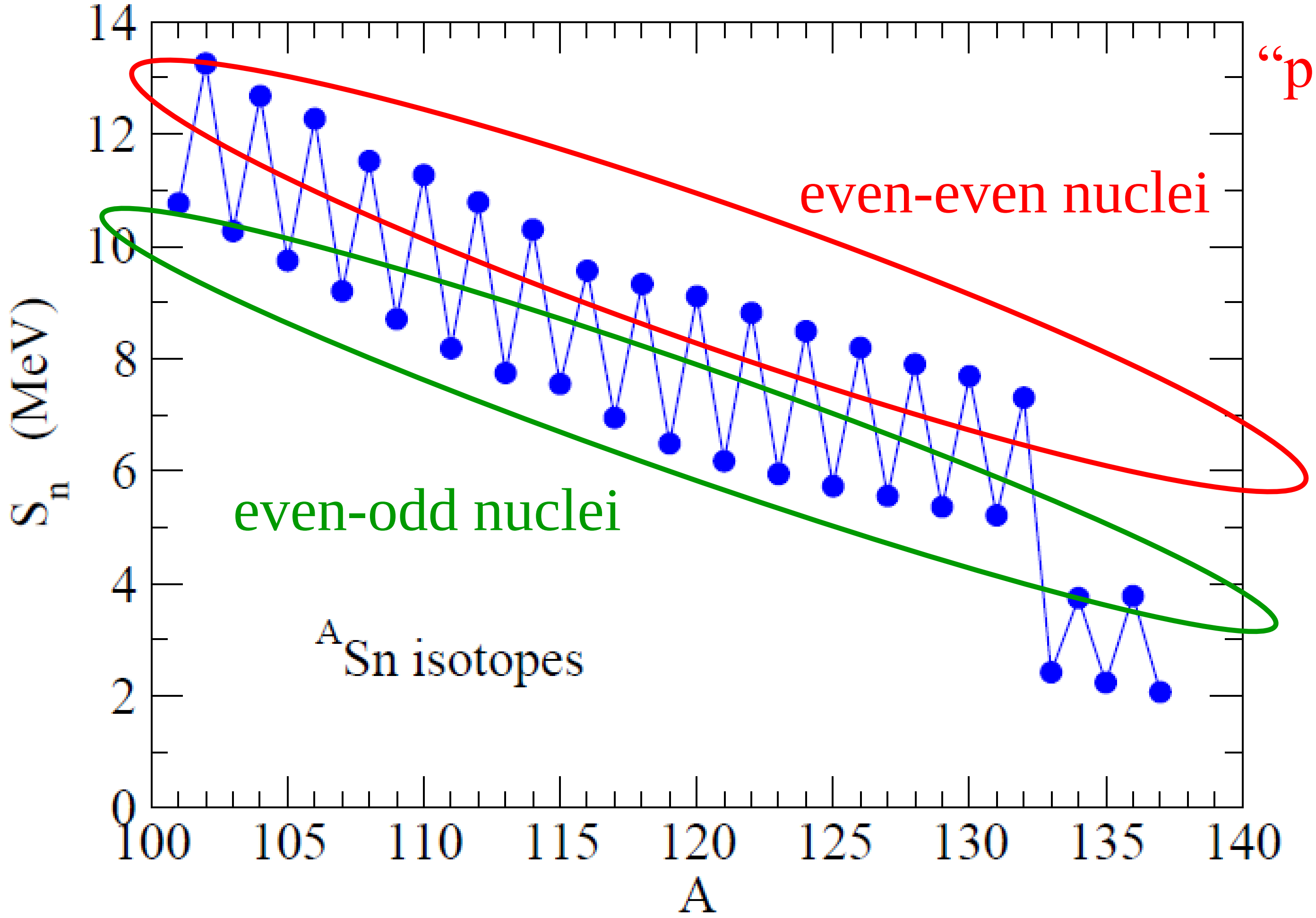
Example:

	Binding energy (MeV)
${}^{210}_{82}Pb_{128} = {}^{208}_{82}Pb_{126}+2n$	1646.6
${}^{210}_{83}Bi_{127} = {}^{208}_{82}Pb_{126}+n+p$	1644.8
${}^{209}_{82}Pb_{127} = {}^{208}_{82}Pb_{126}+n$	1640.4
${}^{209}_{83}Bi_{126} = {}^{208}_{82}Pb_{126}+p$	1640.2

Pairing energy

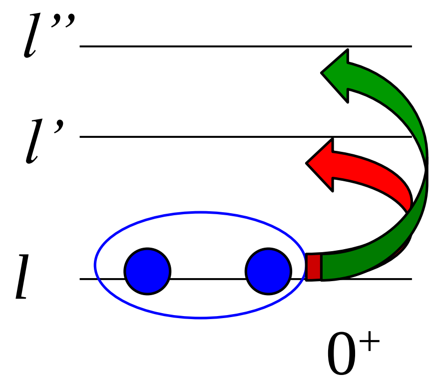
A larger energy required to remove one neutron from even number than from odd number

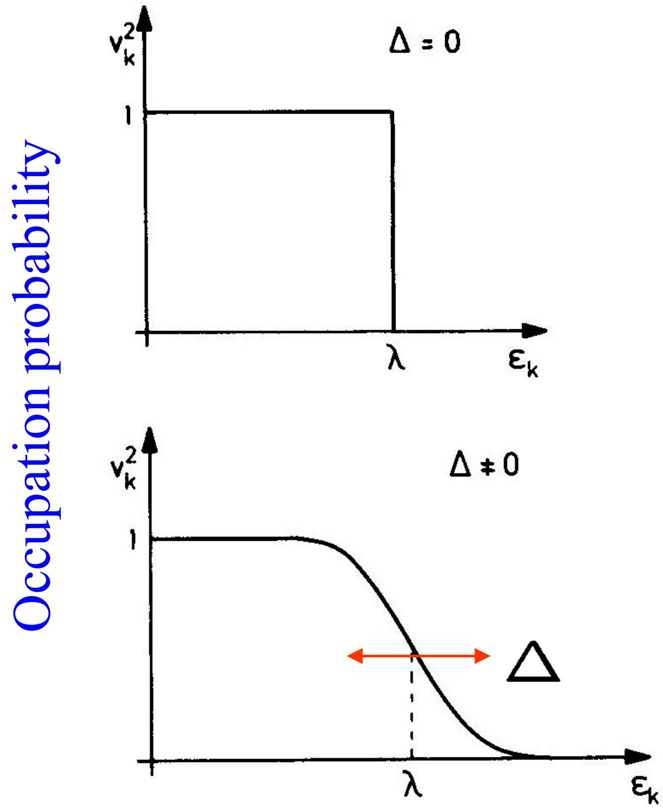
even-odd staggering



1n separation energy: S_n (A,Z) = B(A,Z) – B(A-1,Z)

Wave function:





$$|\Psi_{0+}\rangle = |(ll)L=0\rangle + \sum_{l'} \frac{\langle (l'l')L=0|v_{\rm res}|(ll)L=0\rangle}{2\epsilon_l - 2\epsilon_{l'}} |(l'l')L=0\rangle + \cdots$$

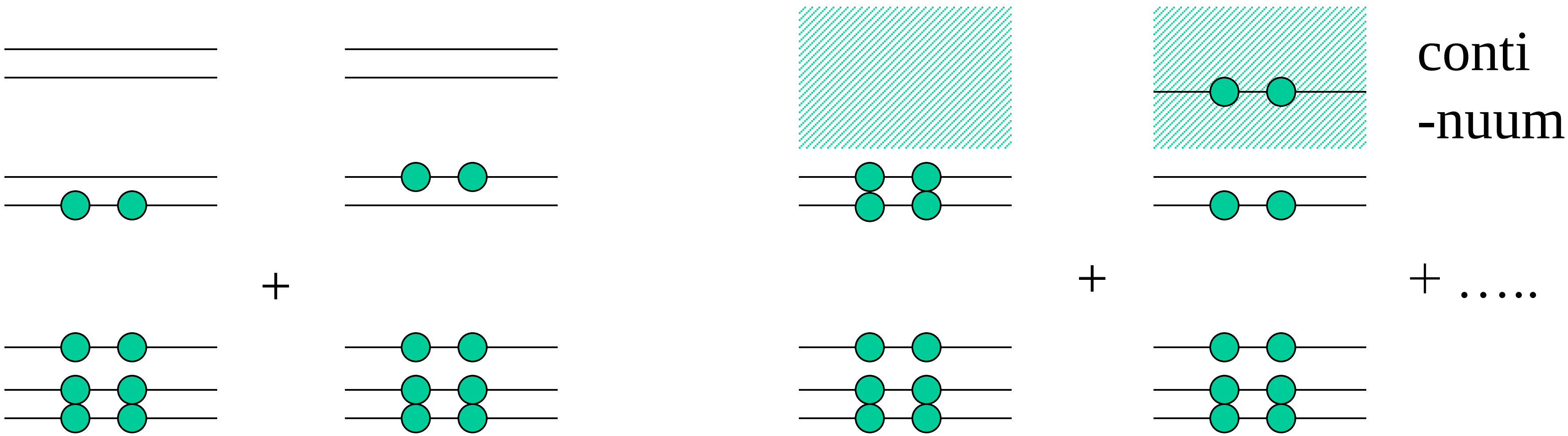
Each orbit is occupied only partially.

cf. BCS theory

Role of redidual interaction

$$H = \sum_i T_i + \sum_{i<j} v_{ij} \rightarrow H = \sum_i (T_i + V_i) + \underbrace{\sum_{i<j} v_{ij} - \sum_i V_i}$$

residual interaction
(pairing)



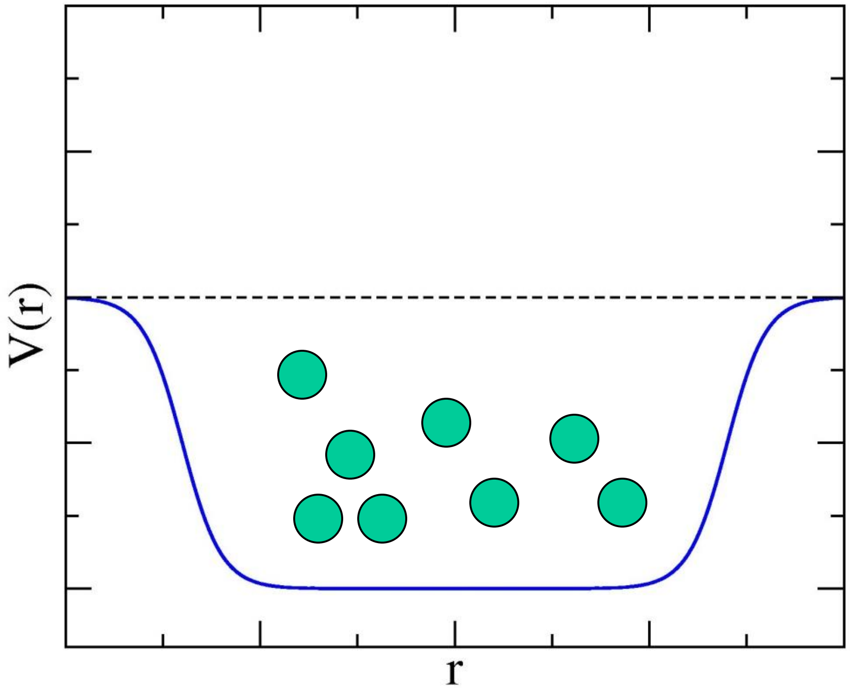
open shell nuclei
→ superfluidity

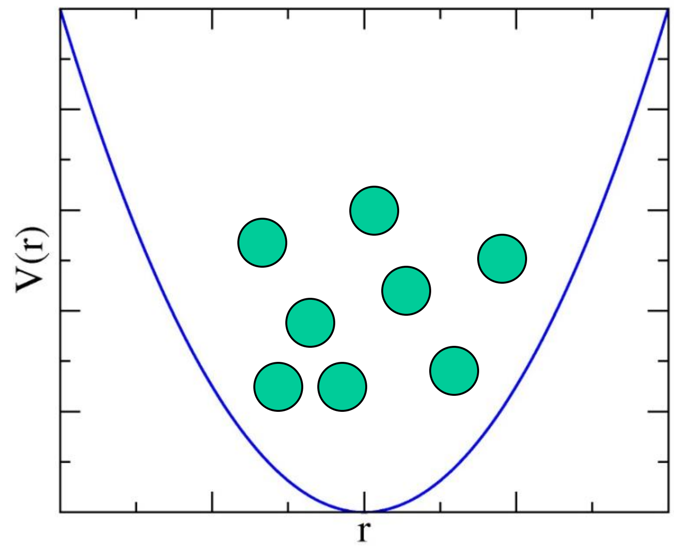
weakly bound nuclei

Neutron-rich nuclei:

- weakly bound systems: low neutron density
- residual interaction (pairing interaction)
- many-body correlations

interacting many-particles in a confining potential





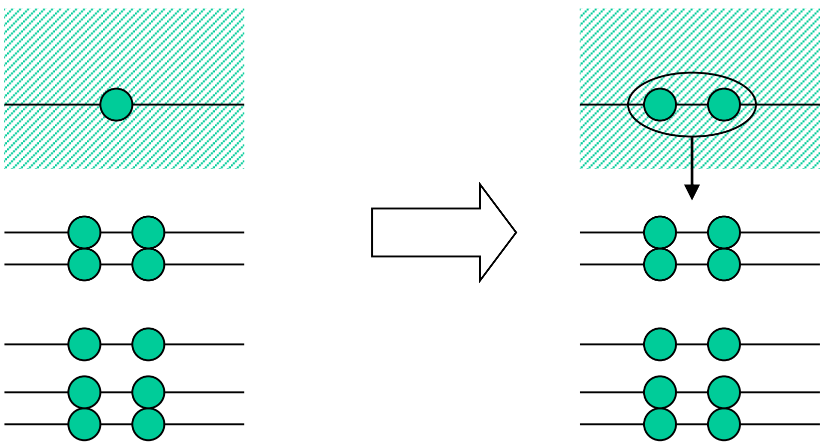
cf. a harmonic trap

- finite-well confining potential
- self-consistent potential

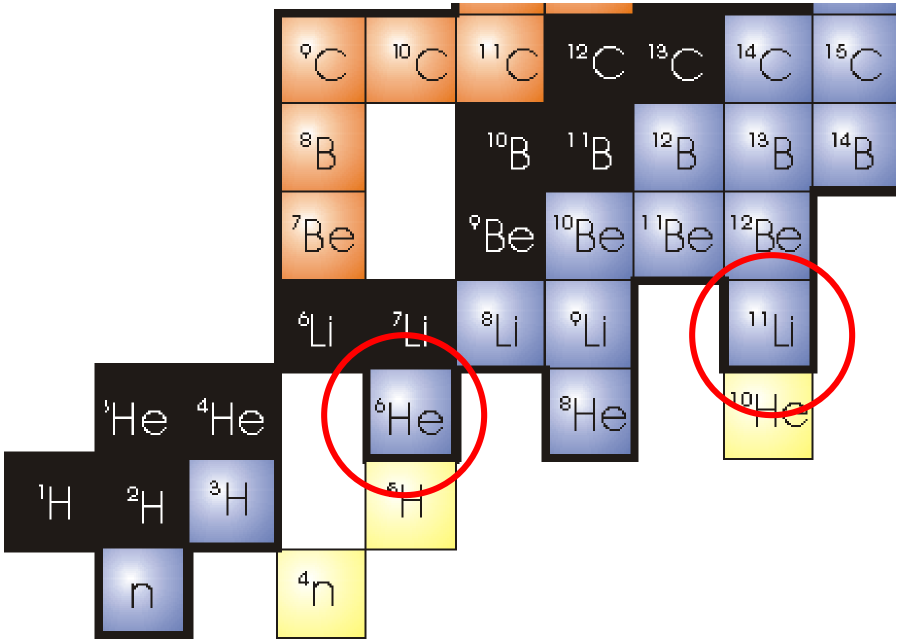
➡ a challenging problem

Borromean nucleus

residual interaction → attractive



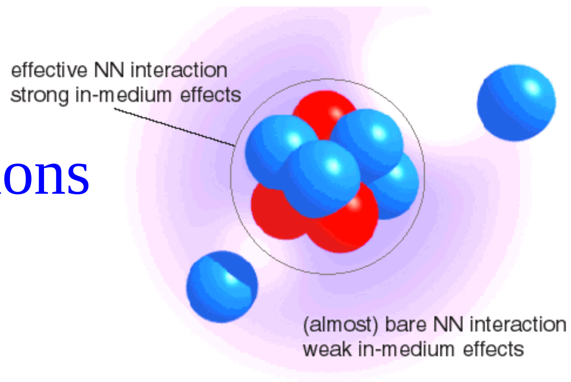
particle unstable   particle stable



"Borromean nuclei"

Structure of Borromean nuclei

- ✓non-trivial due to many-body correlations
- ✓has attracted lots of attention



What is “Borromean” ?

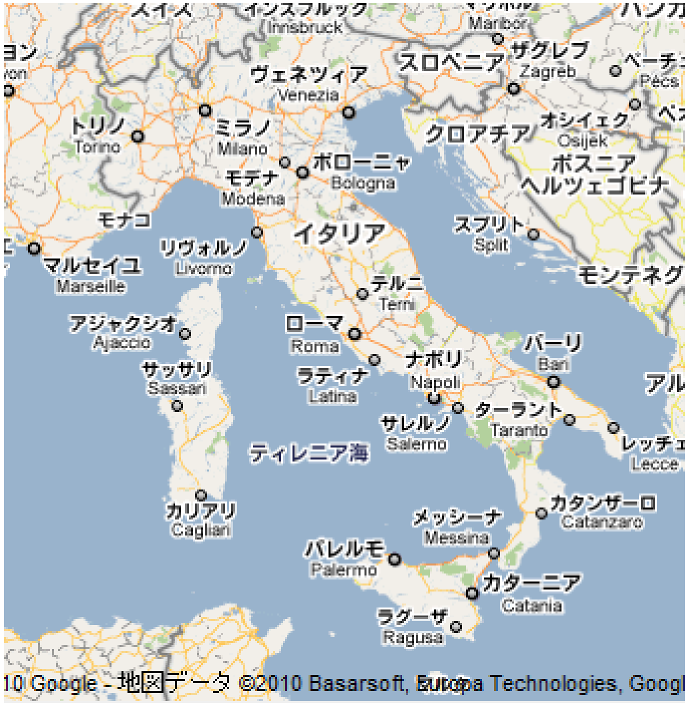
Even though three rings are tied together,
two rings can be separated once any of three is removed.

“Borromean rings”

What is “Borromean” ?

Borromean islands
(northen Italy,
in Lake Maggiore)
near Milano

Crest of Borromeo Family
(13th century)



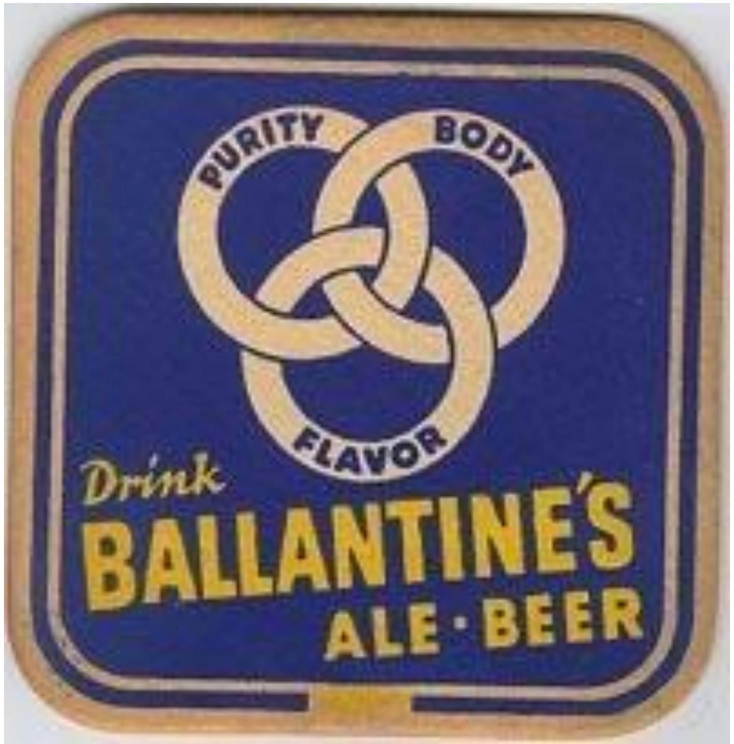
Borromean islands

What is “Borromean” ?

Incidentally, in Japan too….

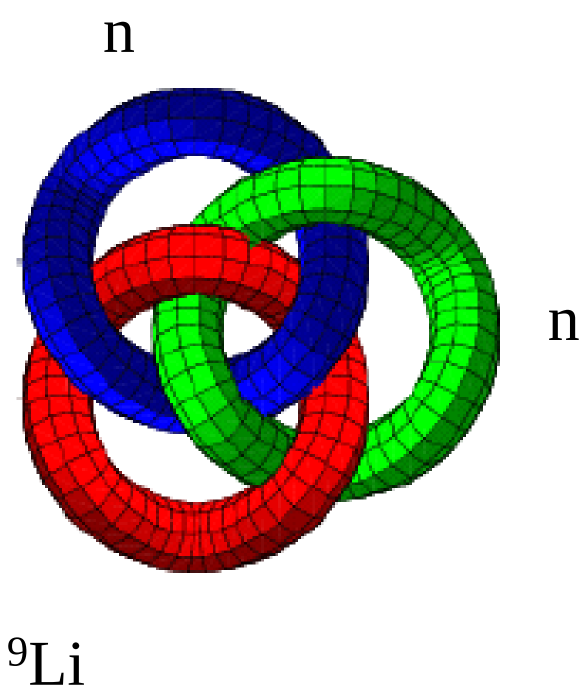
Crest of Kaneda Family

Omiwa shrine
(Sakurai city, Nara)



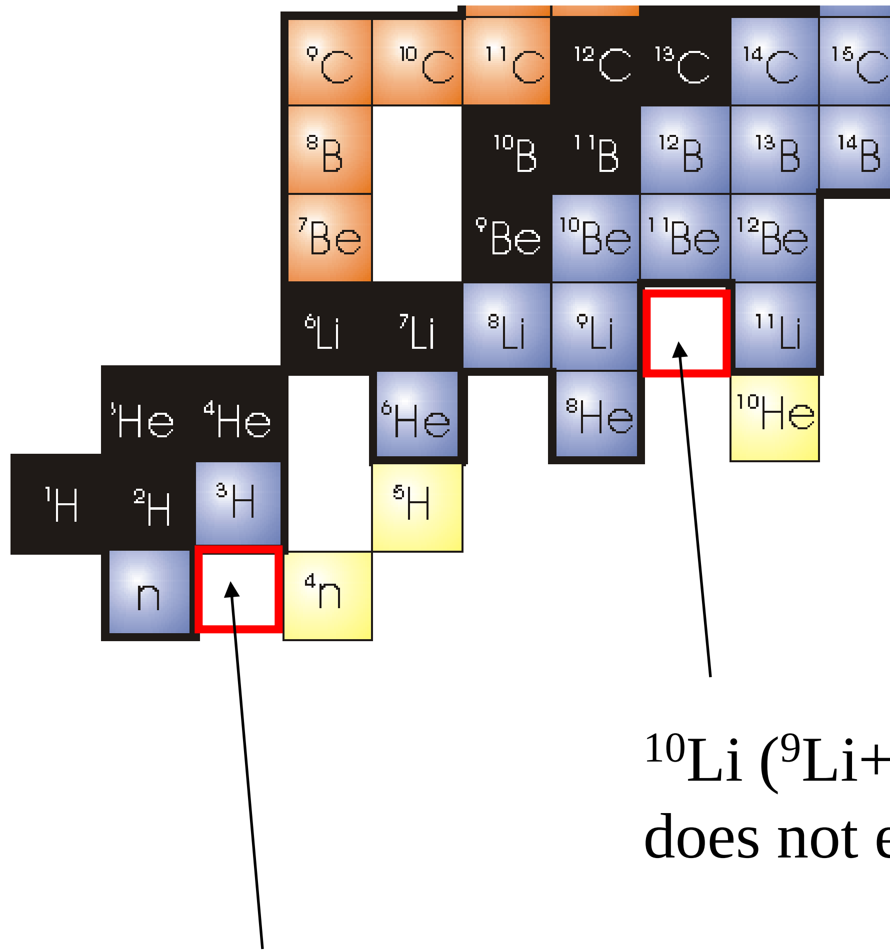
Ballantine's ale (American beer)

Borromean nuclei



Borromean nuclei

Anoter typical example: ^{6}He



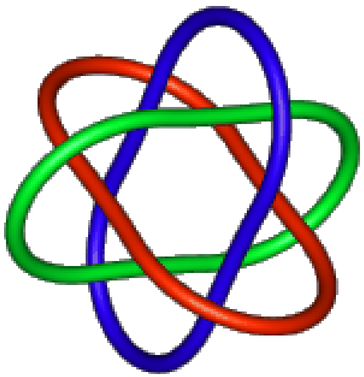
^{10}Li (^{9}Li+n) does not exist

^{2}n (n+n) does not exist

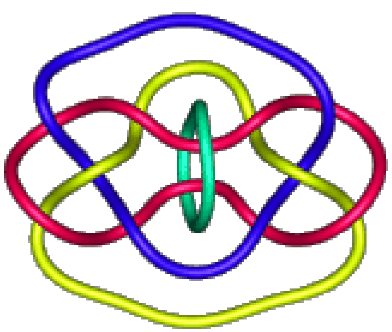
(note) Brunnian link: generalized Borromean rings

knot theory: a field in topology (mathematics)

n=3: Borromean



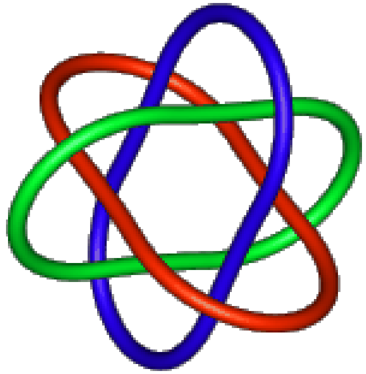
n=4



n=6

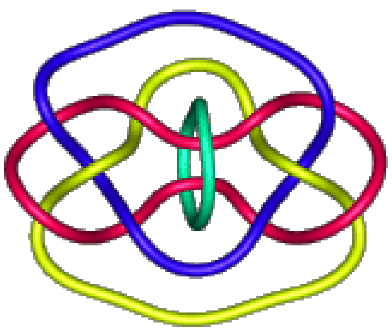
(note) Burunnian nucleus

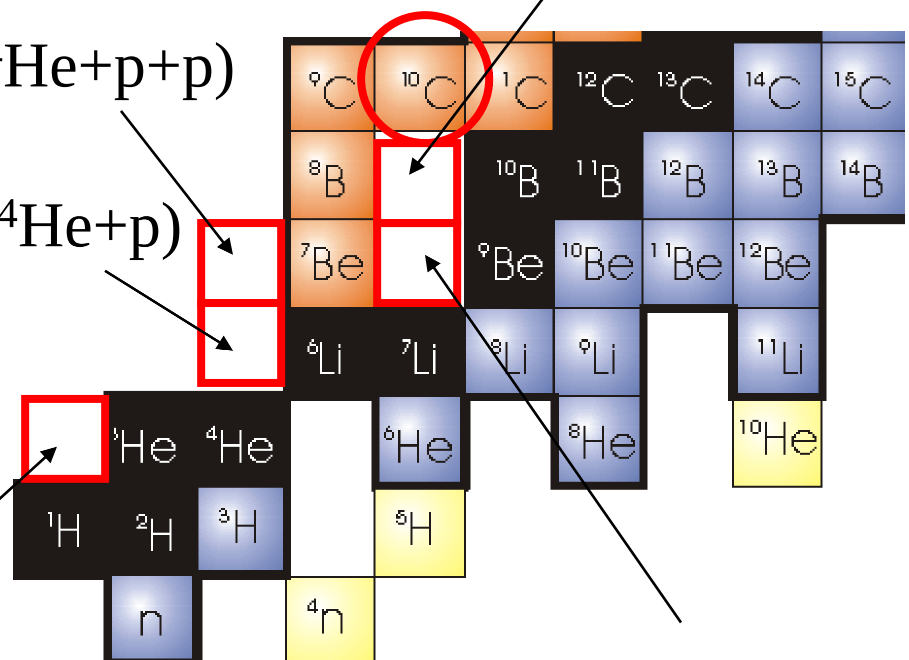
n=3: Borromean



^{11}Li, ^{6}He, etc.

n=4: $^{10}C = {}^{4}He + {}^{4}He + p + p$





cf. N. Curtis et al., PRC77('08)021301(R)