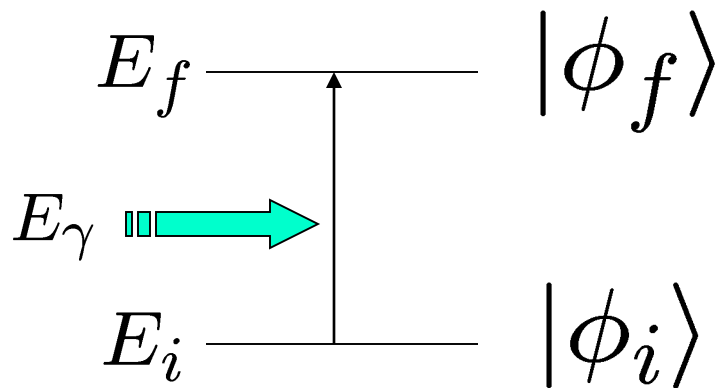
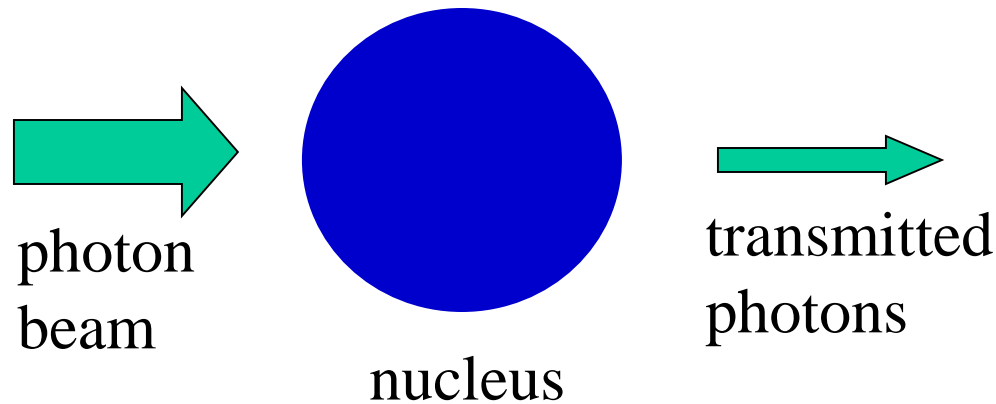


# Collective Vibrations

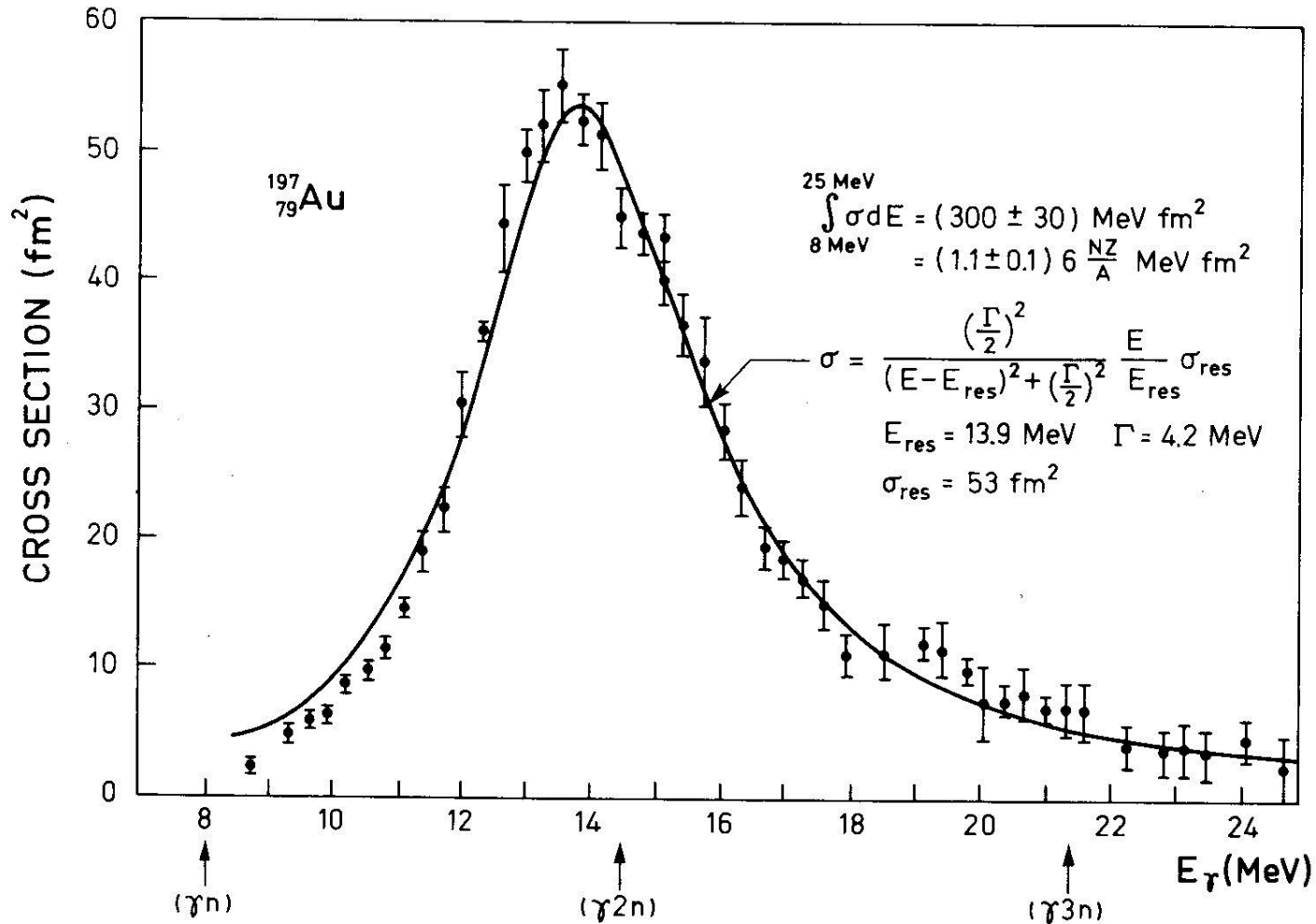
How does a nucleus respond to an external perturbation?

## i) Photo absorption cross section



The state is strongly excited when  
 $E_f - E_i = E_\gamma$ .

# Giant Dipole Resonance (GDR)



**Figure 6-18** Total photoabsorption cross section for  $^{197}\text{Au}$ . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.


## Remarks

### i) Photon interaction $\longleftrightarrow$ dipole excitation

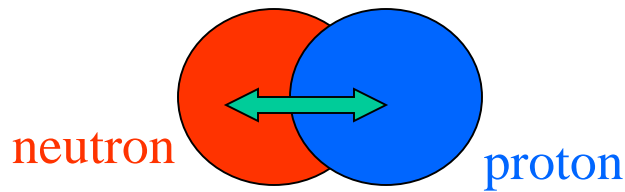
$$H_{\text{int}} = \frac{1}{2m} \frac{e}{c} (\mathbf{p} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{p})$$

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\mathbf{k}} \sum_{\alpha=1,2} \sqrt{\frac{2\pi c^2 \hbar}{\omega V}} (a_{\mathbf{k}\alpha} \boldsymbol{\epsilon}_{\alpha} e^{i\mathbf{k} \cdot \mathbf{r} - i\omega_{\mathbf{k}} t} + h.c.)$$

$$e^{i\mathbf{k} \cdot \mathbf{r}} \sim 1 \quad (\text{dipole approximation})$$

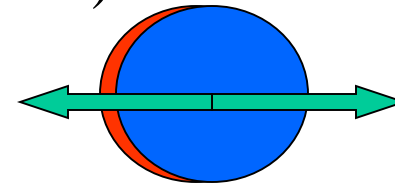

$$\sigma_{\text{abs}}(E_{\gamma}) = \frac{4\pi^2 e^2}{\hbar c} (E_f - E_i) |\langle \phi_f | \tilde{z} | \phi_i \rangle|^2 \delta(E_{\gamma} - E_f + E_i)$$

### ii) Isospin



Isovector type

(note)  $\tilde{z} = \sum_p (z_p - Z_{cm})$

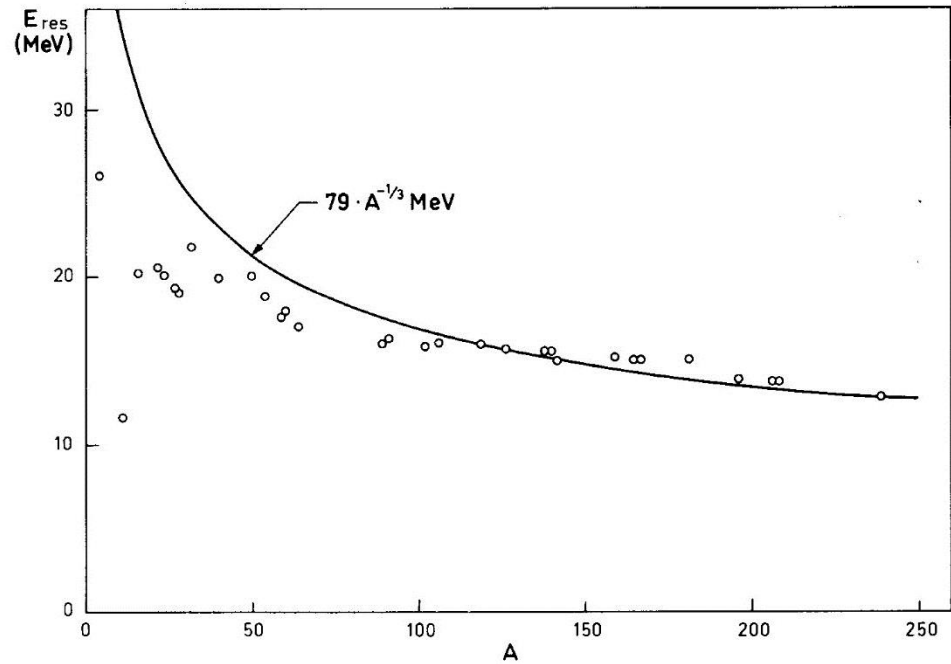
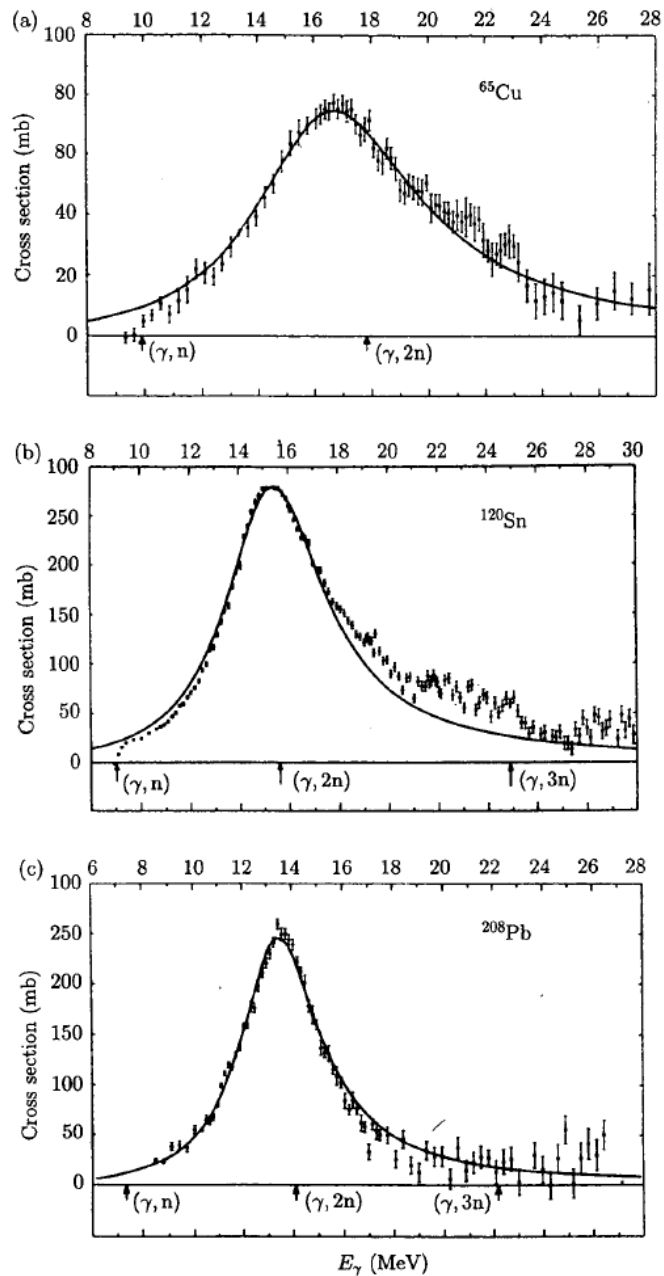


Isoscalar dipole motion

$\longleftrightarrow$  c.m. motion (to the first order)

### iii) Collective motion

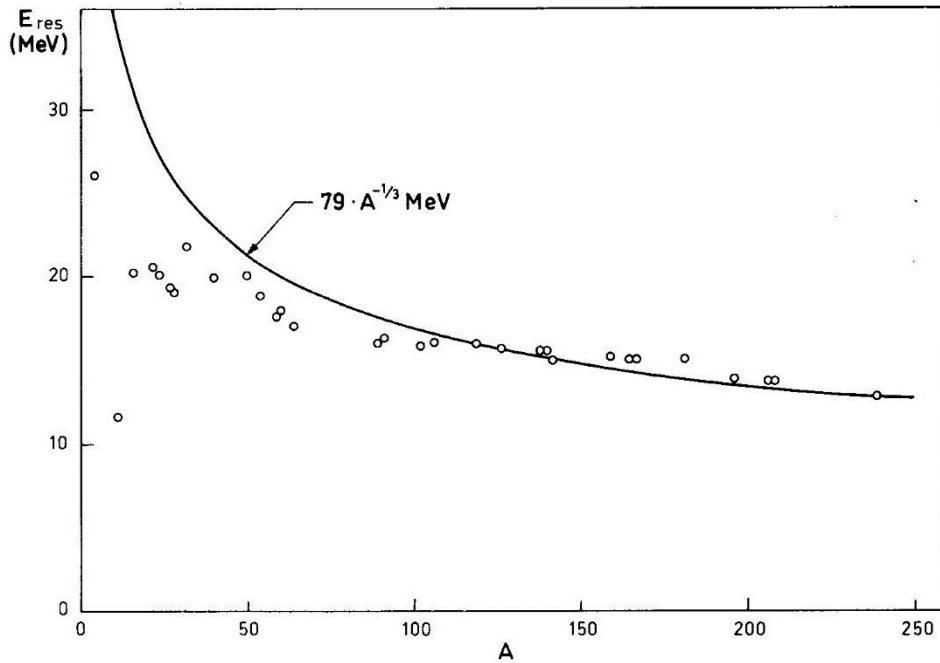
Motion of the whole nucleus rather than a single-particle motion



Bohr-Mottelson  
 “Nuclear Structure vol. II”

M.N. Harakeh and A. van der Woude,  
 “Giant Resonances”

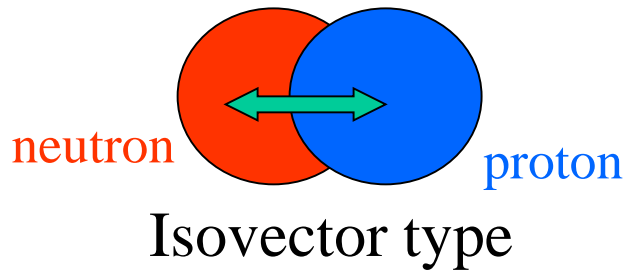
FIG. 1.2. The photo-neutron cross section  $\sigma(\gamma, n)$  as a function of the photon energy for the three nuclei  $^{208}\text{Pb}$ ,  $^{120}\text{Sn}$  and  $^{65}\text{Cu}$ . Note that for these nuclei  $\sigma(\gamma, n) \approx \sigma_{\text{abs}}(\gamma)$ . From reference (BER75).

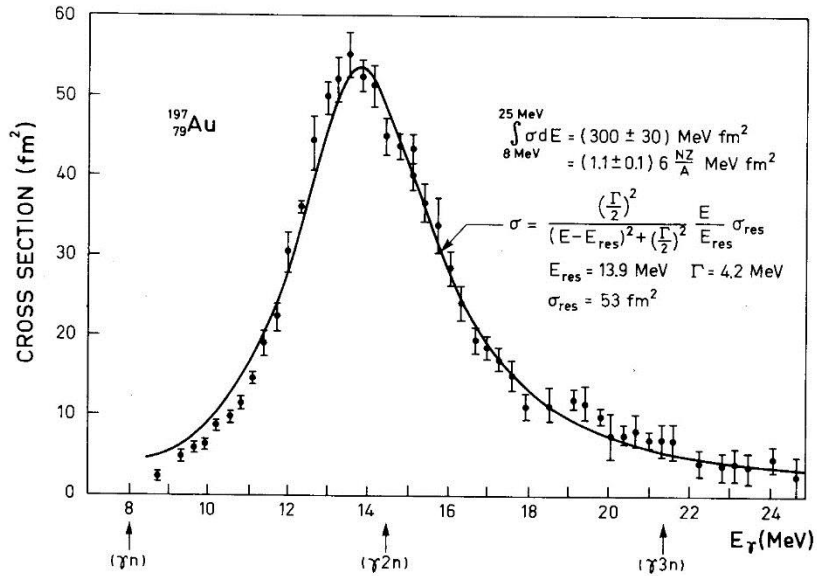


$$E_{GDR} \propto A^{-1/3}$$

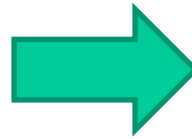
$$\propto 1/R$$

Bohr-Mottelson  
 “Nuclear Structure vol. II”

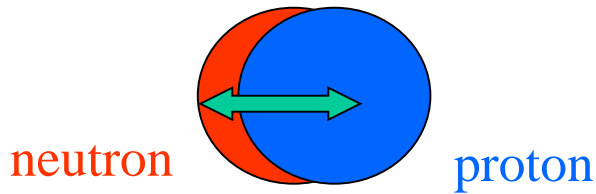




**Figure 6-18** Total photoabsorption cross section for  $^{197}\text{Au}$ . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

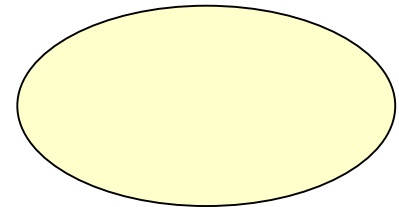


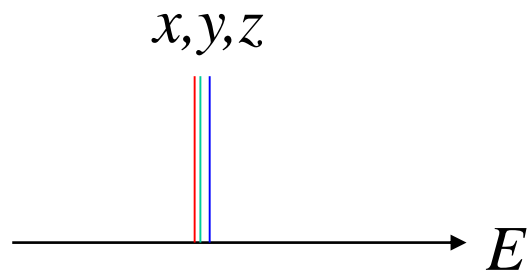
?



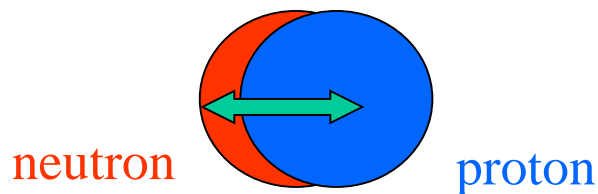
$$E_{\text{GDR}} \propto 1/R$$

deformed nucleus

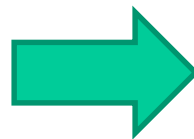




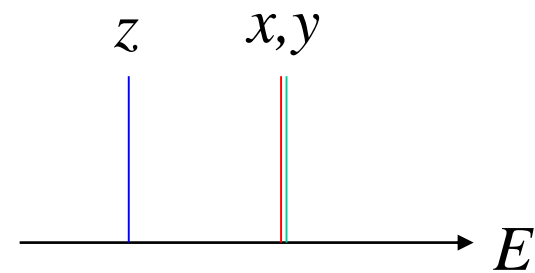
spherical nucleus



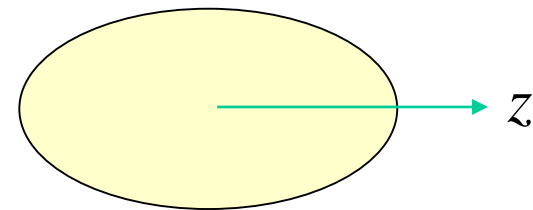
$$E_{\text{GDR}} \propto 1/R$$

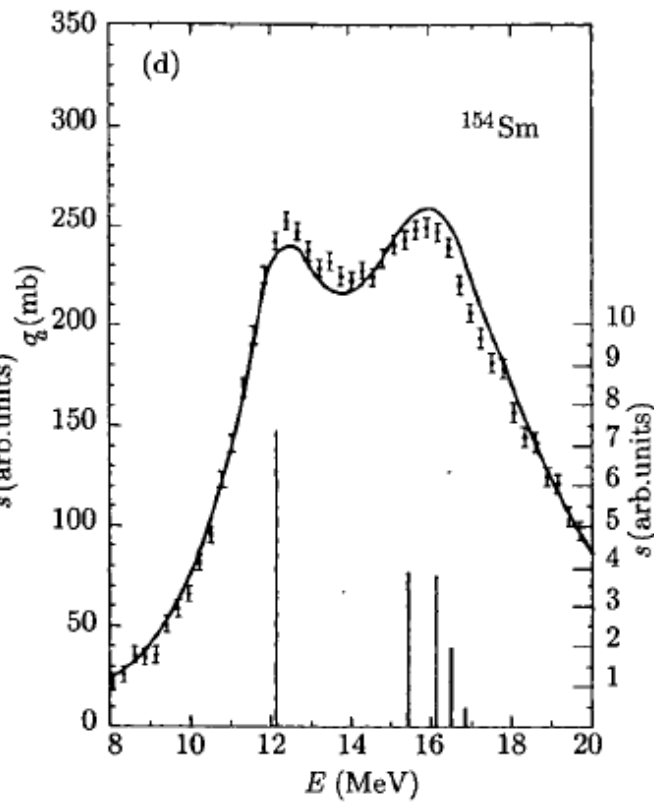
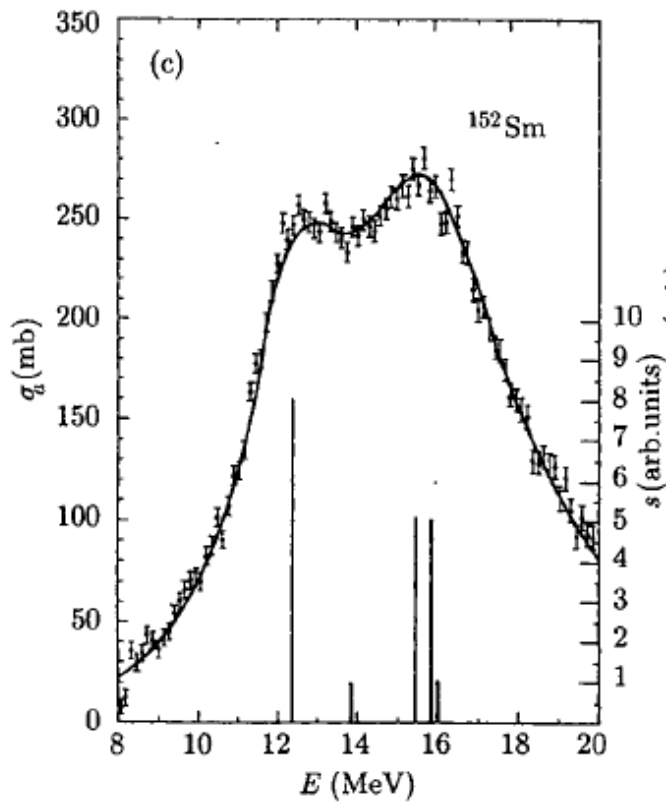


(prolate deformation)



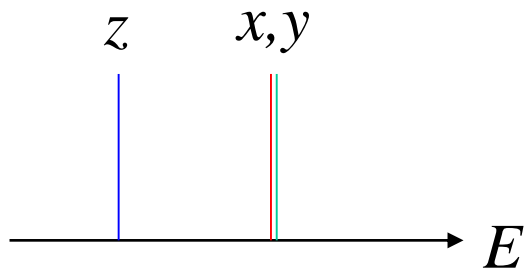
deformed nucleus



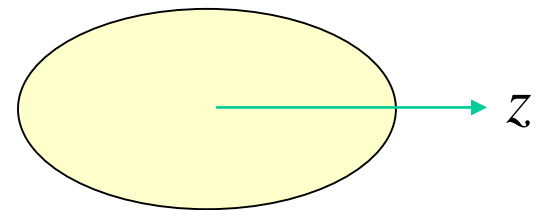


M.N. Harakeh and  
A. van der Woude,  
“Giant Resonances”

(prolate deformation)



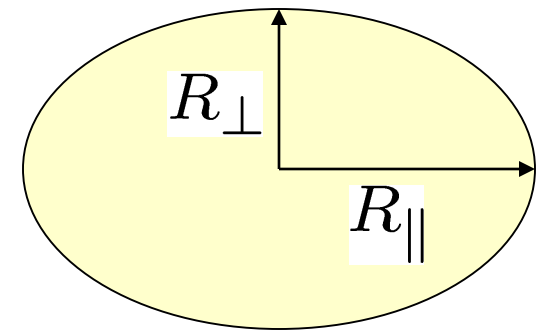
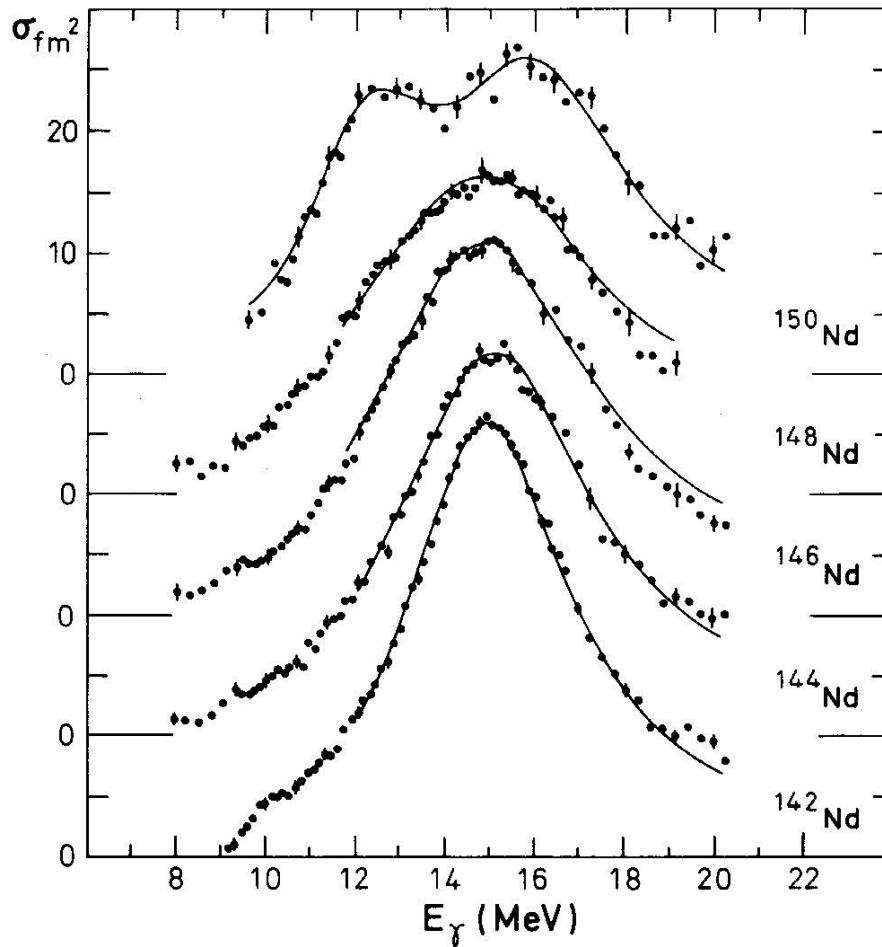
deformed nucleus





## Deformation effect

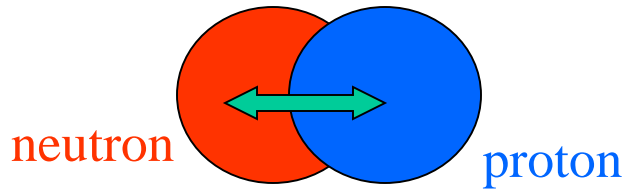
$$\hbar\omega \sim A^{-1/3} \sim 1/R$$



**Figure 6-21** Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssière, *Nuclear Phys. A172*, 437 (1971). The solid curves represent Lorentzian fits with the parameters given in Table 6-6.

# Giant Dipole Resonances

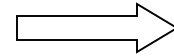
## • Goldhaber-Teller type



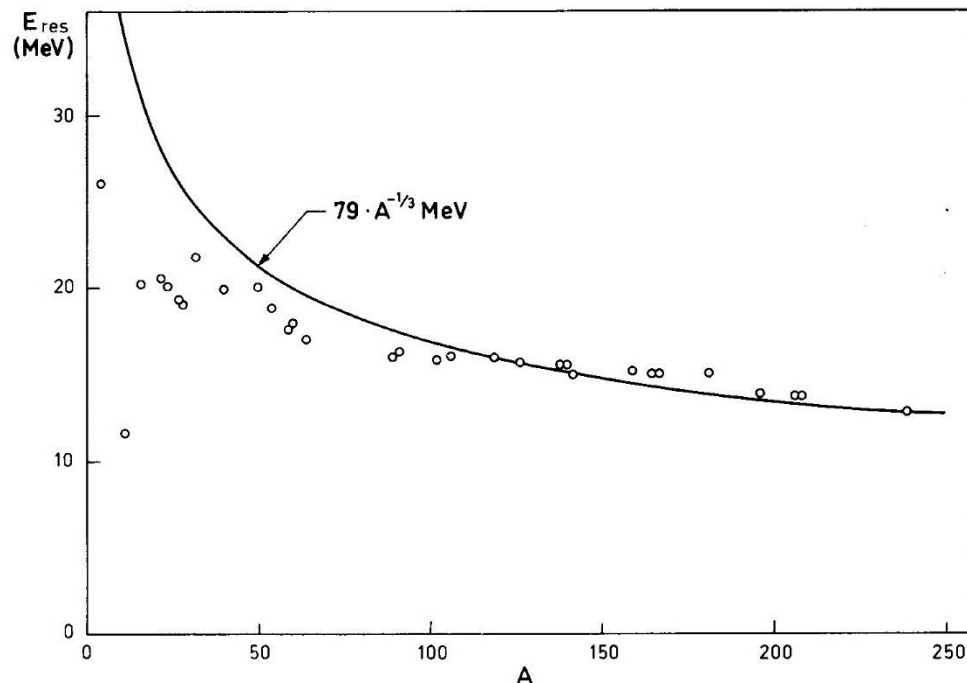
$$\hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z$$



$$\hbar\omega \sim A^{-1/6}$$

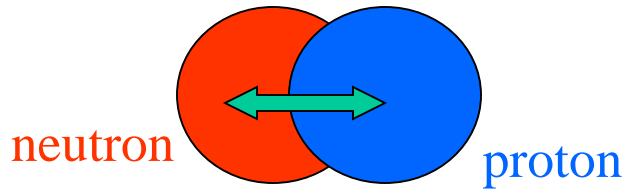


Inconsistent with expt.  
(except for light nuclei)



# Giant Dipole Resonances

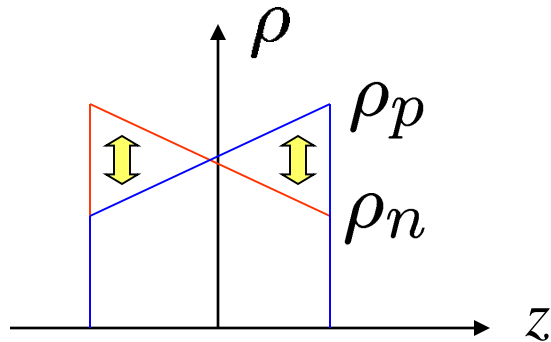
## • Goldhaber-Teller type



$$\hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z$$

$$\longrightarrow \hbar\omega \sim A^{-1/6}$$

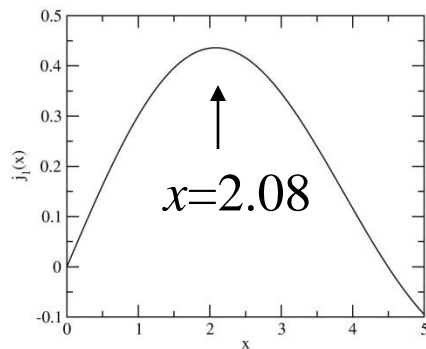
## • Steinwedel-Jensen type



$$\hat{Q} = j_1(kr) Y_{1\mu}(\hat{r}) \tau_z$$

$$\longrightarrow \hbar\omega \sim A^{-1/3}$$

$$kR = 2.08$$



$$j_1(x) = (\sin x - x \cos x) / x^2$$

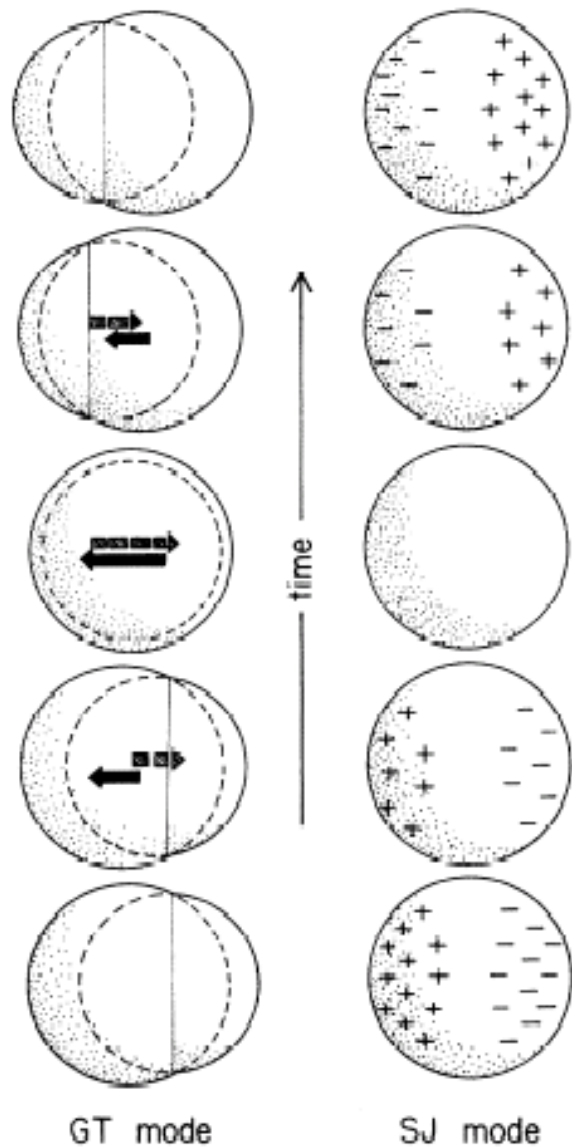
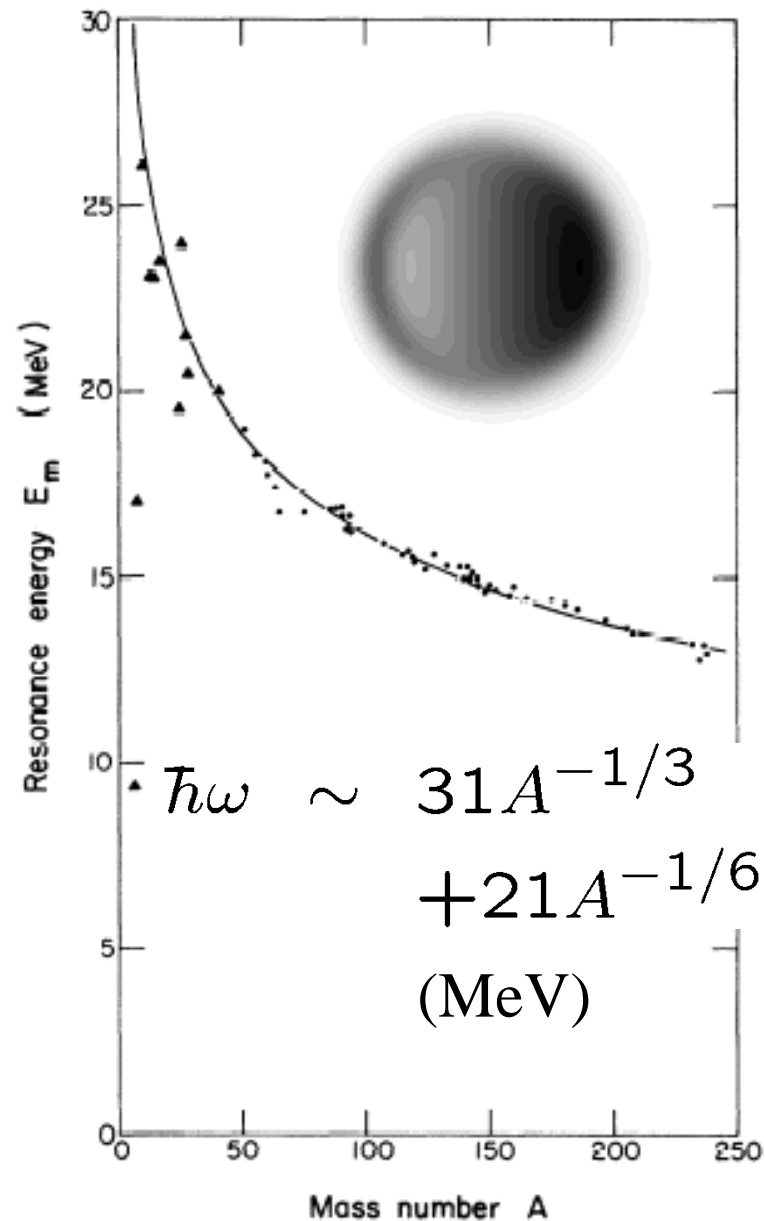
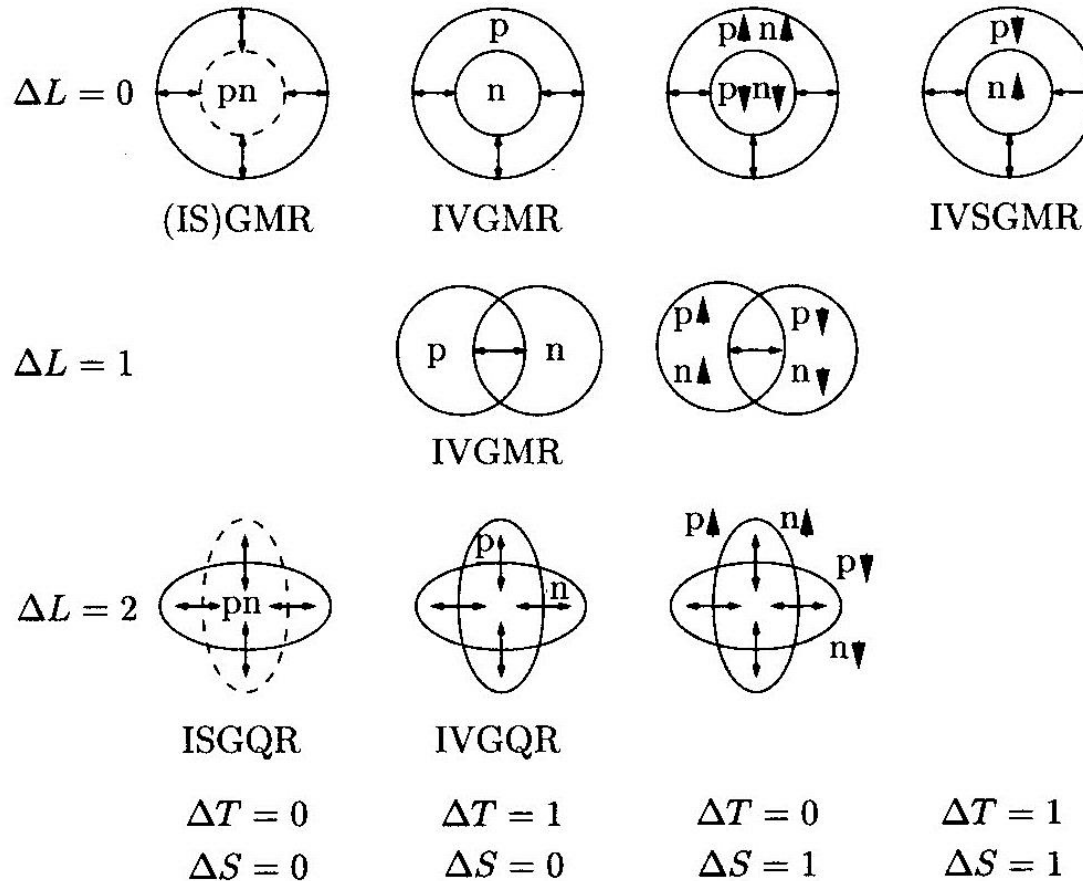


FIG. 1. Schematic drawings that serve to illustrate the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.



## ii) Inelastic scattering

(e,e'), (p,p'), ( $\alpha,\alpha'$ ), Heavy-ion  $\longrightarrow$  Higher multipolarities

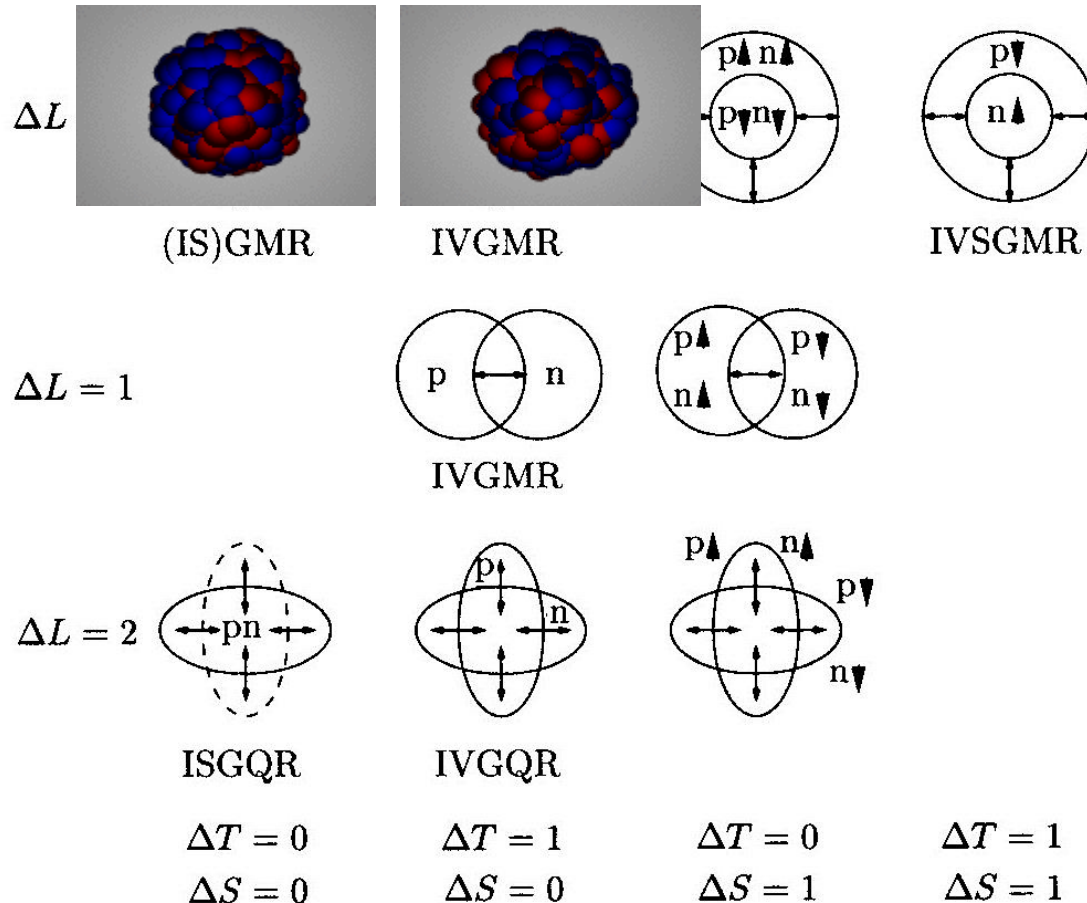


(note)  $\Delta L = 2 \longrightarrow \Delta N = 2$  Giant Resonance (GQR)

$\Delta N = 0$  Low-lying state

## ii) Inelastic scattering

$(e,e')$ ,  $(p,p')$ ,  $(\alpha,\alpha')$ , Heavy-ion  $\longrightarrow$  Higher multipolarities

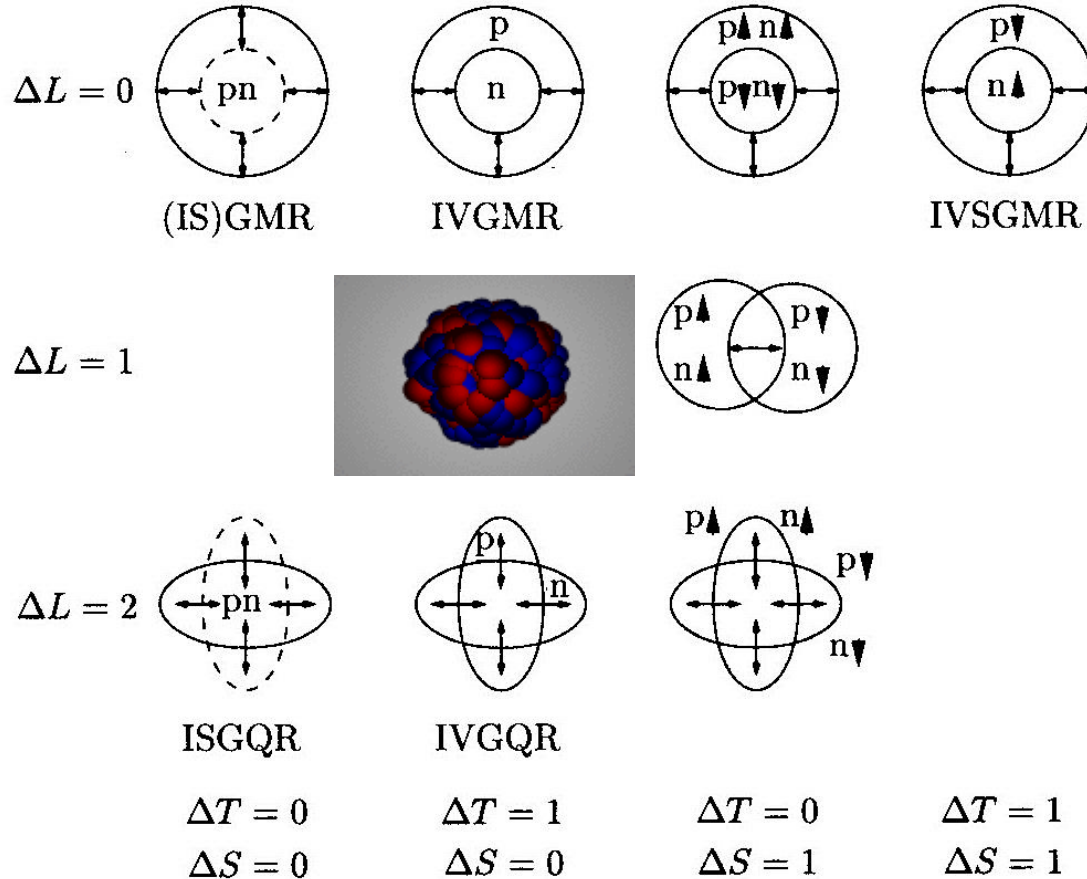


movies: H.-J. Wollersheim,

<https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html>

## ii) Inelastic scattering

(e,e'), (p,p'), ( $\alpha,\alpha'$ ), Heavy-ion  $\longrightarrow$  Higher multipolarities

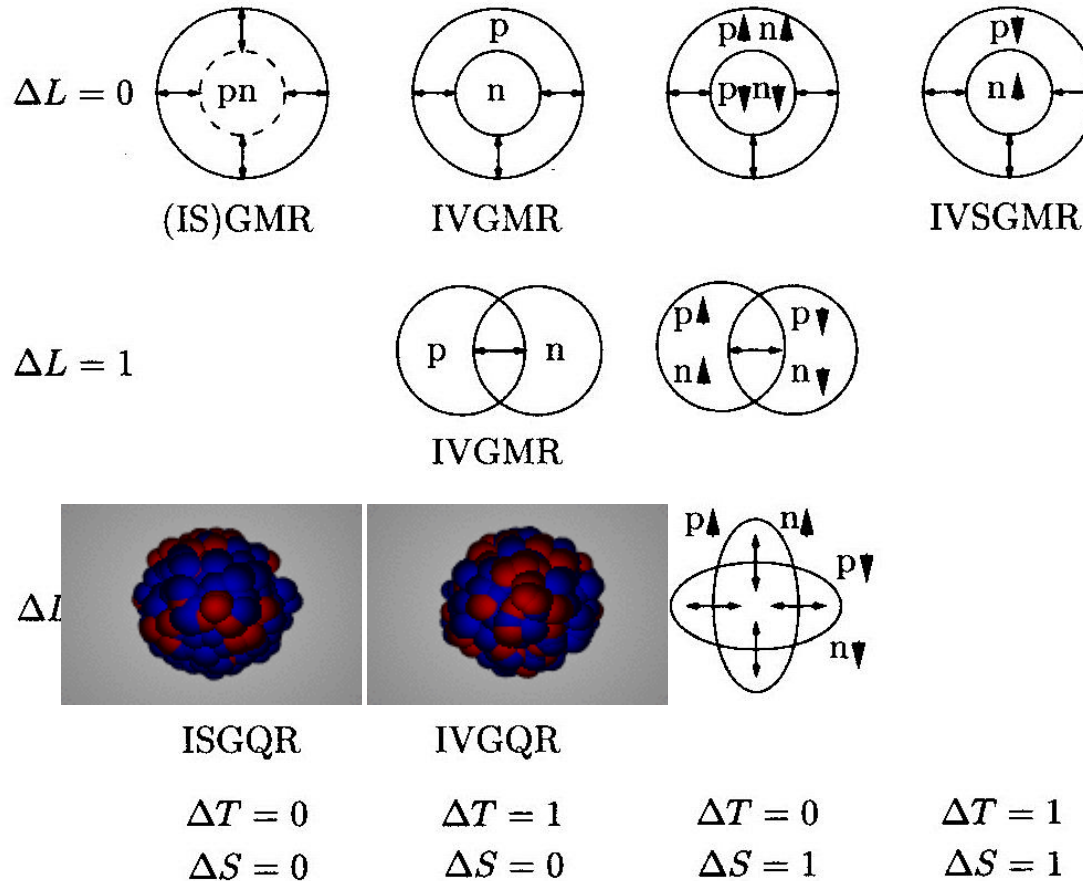


movies: H.-J. Wollersheim,

<https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html>

## ii) Inelastic scattering

(e,e'), (p,p'), ( $\alpha,\alpha'$ ), Heavy-ion  $\longrightarrow$  Higher multipolarities



movies: H.-J. Wollersheim,

<https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html>



# Discovery of Giant Quadrupole Resonance (GQR)

VOLUME 29, NUMBER 16

PHYSICAL REVIEW LETTERS

16 OCTOBER 1972

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## Giant Multipole Resonances in $^{90}\text{Zr}$ Observed by Inelastic Electron Scattering

S. Fukuda and Y. Torizuka

*Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan*

(Received 24 August 1972)

Inelastic electron scattering from the giant dipole resonance region in  $^{90}\text{Zr}$  was measured. In addition to the usual dipole resonance we have found new resonances at 14.0 MeV and around 28 MeV. The spins and parities and transition strengths of these states are discussed.

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VOLUME 30, NUMBER 21

PHYSICAL REVIEW LETTERS

21 MAY 1973

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## Electroexcitation of Giant Resonances in $^{208}\text{Pb}$

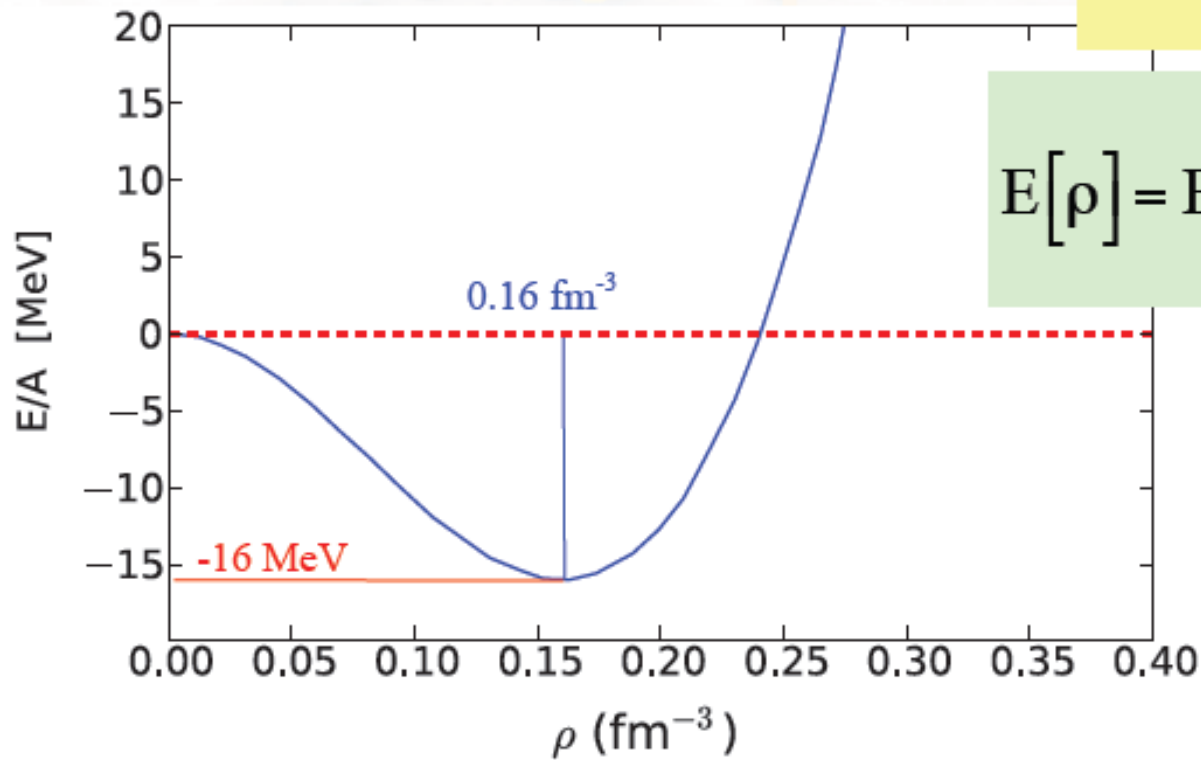
M. Nagao and Y. Torizuka

*Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan*

(Received 27 February 1973)

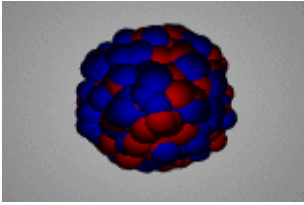
The giant-resonance region in  $^{208}\text{Pb}$  was observed by inelastic electron scattering. We present evidence for the existences of a  $2^+$  (or  $0^+$ ) state at  $\sim 22$  MeV and a  $3^-$  state at  $\sim 19$  MeV with giant-resonance character. The resonance states between 8.6 and 11.6 MeV are confirmed to be  $2^+$  (or  $0^+$ ) and the sum of their strengths exhausts about 50% of the  $E2$  sum rule or 100% of  $E0$ .

# EOS of infinite nuclear matter



$$K_{\infty} = 9\rho^2 \left. \frac{d^2[E(\rho)/\rho]}{d\rho^2} \right|_{\rho_0}$$

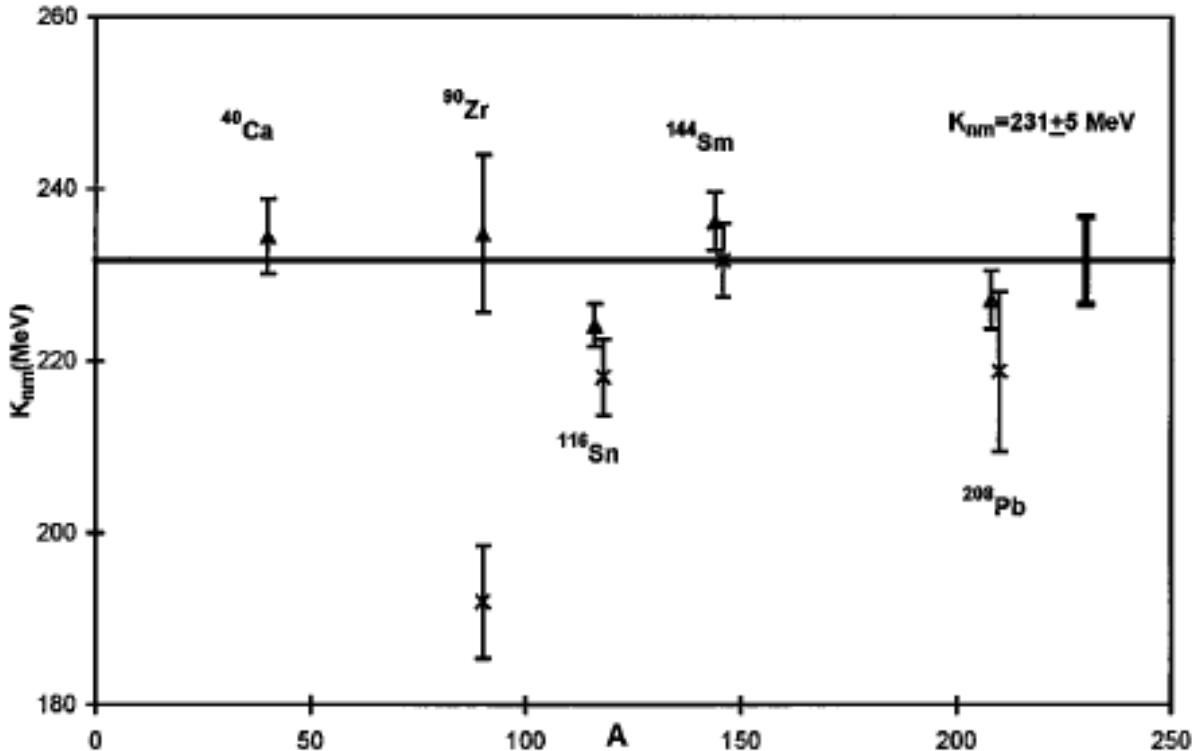
$$E[\rho] = E[\rho_0] + \frac{1}{18} K_{\infty} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2$$



# Isoscalar giant monopole resonances (breathing mode)

$$E_{\text{ISGMR}} \sim \sqrt{\frac{\hbar^2 K}{m \langle r^2 \rangle}}$$

J.P. Blaizot,  
Phys. Rep. 64 ('80) 171



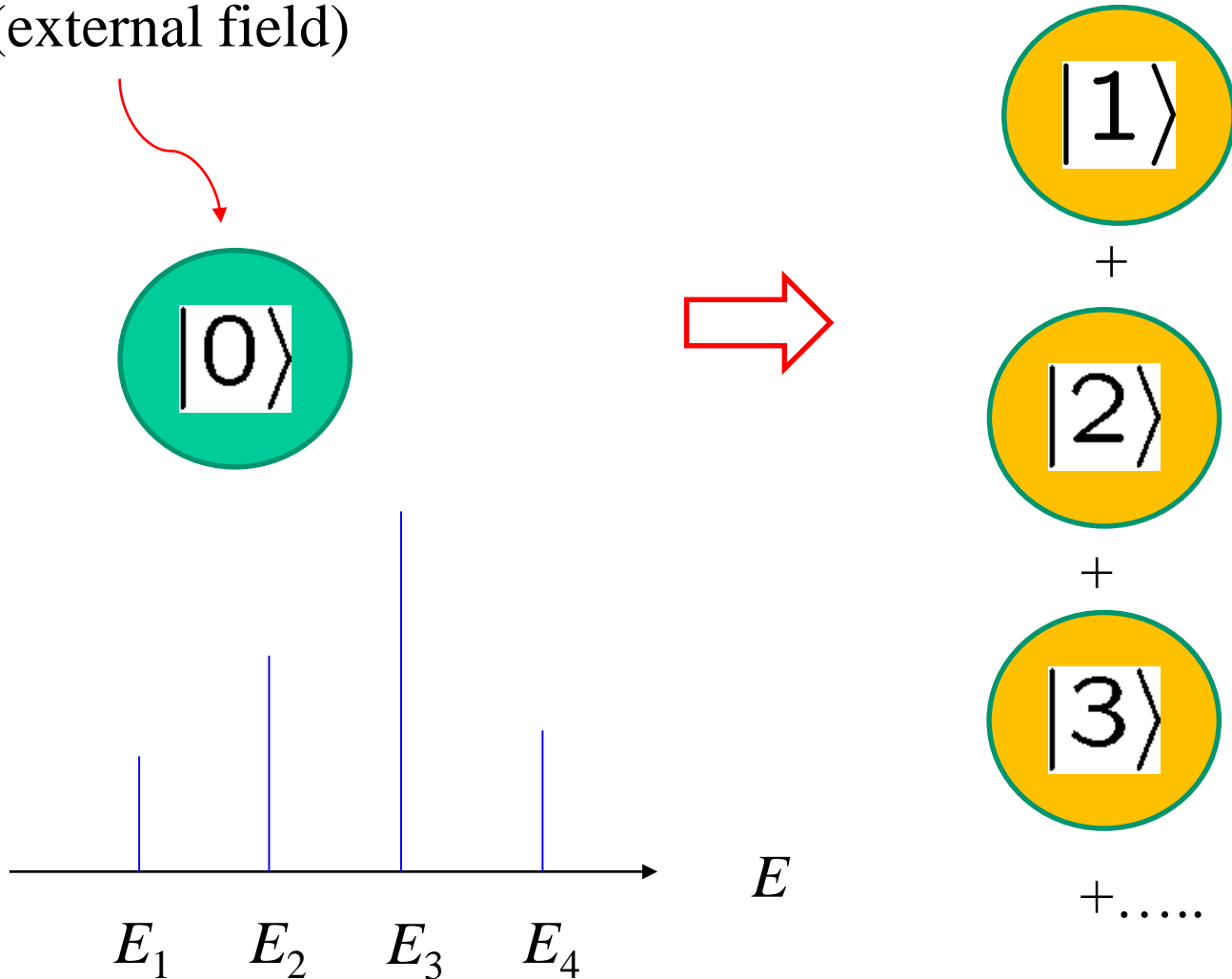
$K \sim 231 \pm 5 \text{ MeV}$

# Sum Rule

Strength function:

$$S(E) = \sum_{\nu} |\langle \nu | F | 0 \rangle|^2 \delta(E_{\nu} - E_0 - E)$$

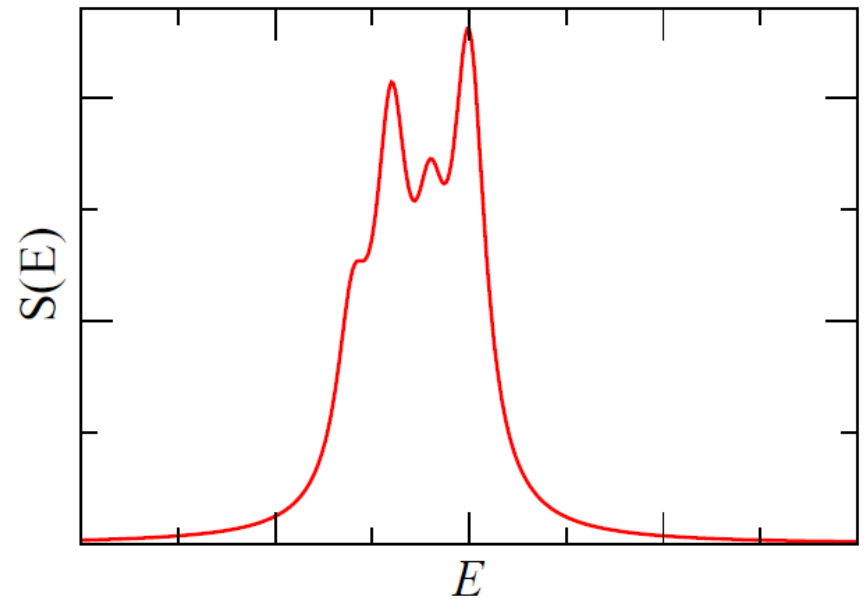
$F$  (external field)



# Sum Rule

Strength function:

$$S(E) = \sum_{\nu} |\langle \nu | F | 0 \rangle|^2 \times \delta(E_{\nu} - E_0 - E)$$



✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E) dE = \sum_{\nu} |\langle \nu | F | 0 \rangle|^2$$

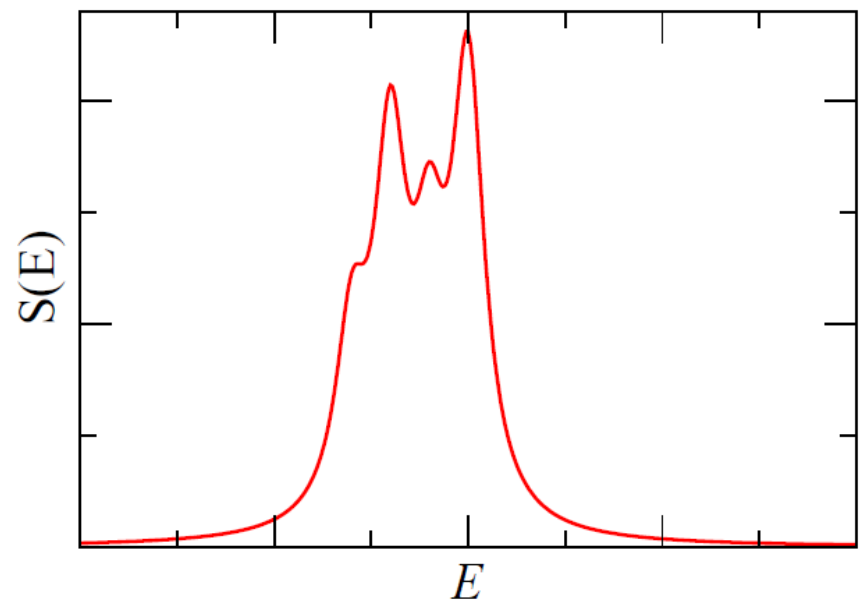
✓ energy weighted sum rule

$$S_1 \equiv \int E S(E) dE = \sum_{\nu} (E_{\nu} - E_0) |\langle \nu | F | 0 \rangle|^2$$

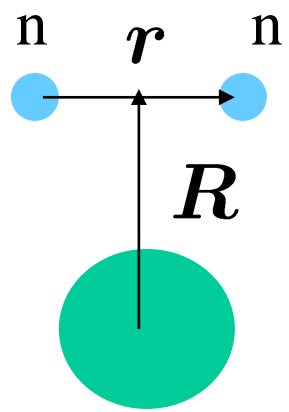
✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E) dE = \sum_{\nu} |\langle \nu | F | 0 \rangle|^2$$

$$= \langle 0 | F^2 | 0 \rangle$$



cf. geometry of Borromean nuclei



$$B(E1) = \sum_i B(E1; gs \rightarrow i)$$

$$= \frac{3}{\pi} \left( \frac{Ze}{A} \right)^2 \langle R^2 \rangle$$

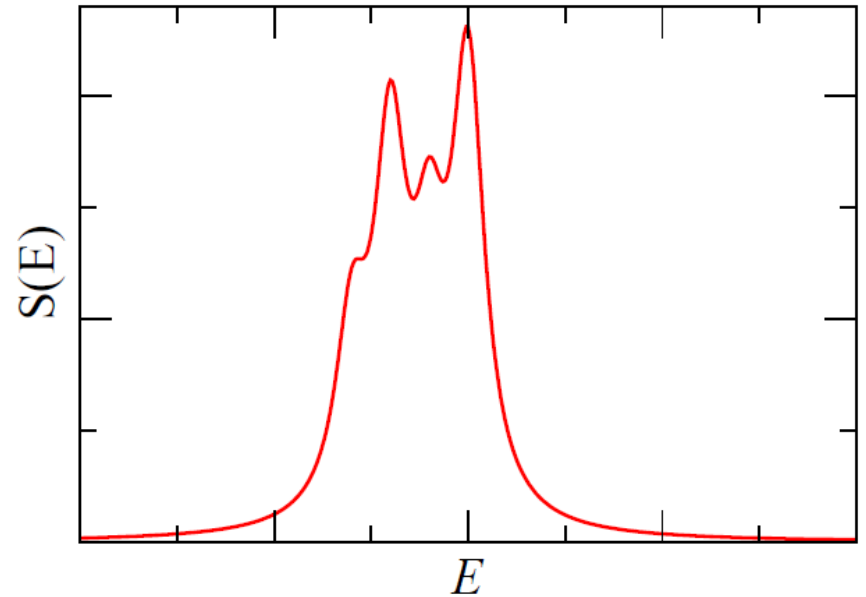
$$S(E) = \sum_{\nu} |\langle \nu | F | 0 \rangle|^2 \times \delta(E_{\nu} - E_0 - E)$$

⇒  $\langle \theta_{nn} \rangle = 65.2_{-13.0}^{+11.4} \quad ({}^{11}\text{Li})$

$= 74.5_{-13.1}^{+11.2} \quad ({}^6\text{He})$

✓ energy weighted sum rule

$$\begin{aligned} S_1 &\equiv \int E S(E) dE \\ &= \sum_{\nu} (E_{\nu} - E_0) |\langle \nu | F | 0 \rangle|^2 \\ &= \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle \end{aligned}$$



$$S(E) = \sum_{\nu} |\langle \nu | F | 0 \rangle|^2 \times \delta(E_{\nu} - E_0 - E)$$

$$\begin{aligned} \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle &= \frac{1}{2} \langle F(HF - FH) - (HF - FH)H \rangle \\ &= \langle FHF - E_0 F^2 \rangle \\ &= \sum_{\nu} E_{\nu} |\langle 0 | F | \nu \rangle|^2 - E_0 \langle 0 | F^2 | 0 \rangle \\ &= \sum_{\nu} (E_{\nu} - E_0) |\langle \nu | F | 0 \rangle|^2 \end{aligned}$$

Energy weighted sum rule:

$$\begin{aligned} S_1 &= \sum_{\nu} (E_{\nu} - E_0) |\langle \nu | F | 0 \rangle|^2 \\ &= \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle \end{aligned}$$

For  $F = F(\mathbf{r})$  (local operator)

$$\begin{aligned} [H, F] &= \left[ -\frac{\hbar^2}{2m} \nabla^2, F \right] \\ &= -\frac{\hbar^2}{2m} (\nabla^2 F + 2\nabla F \cdot \nabla) \end{aligned}$$



$$[F, [H, F]] = \frac{\hbar^2}{m} (\nabla F)^2$$



$$S_1 = \frac{\hbar^2}{2m} \int d\mathbf{r} \rho(\mathbf{r}) \cdot (\nabla F)^2$$

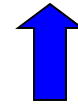


$$S_1 = \sum_{\nu} (E_{\nu} - E_0) |\langle \nu | F | 0 \rangle|^2 = \frac{\hbar^2}{2m} \int d\mathbf{r} \rho(\mathbf{r}) \cdot (\nabla F)^2$$

For  $F=z$

$$S_1 = \sum_{\nu} (E_{\nu} - E_0) |\langle \nu | z | 0 \rangle|^2 = \frac{\hbar^2 N_{sys}}{2m}$$

[TRK (Thomas-Reiche-Kuhn) Sum Rule]



Model independent

For  $F = r^{\lambda} Y_{\lambda\mu}(\hat{\mathbf{r}})$

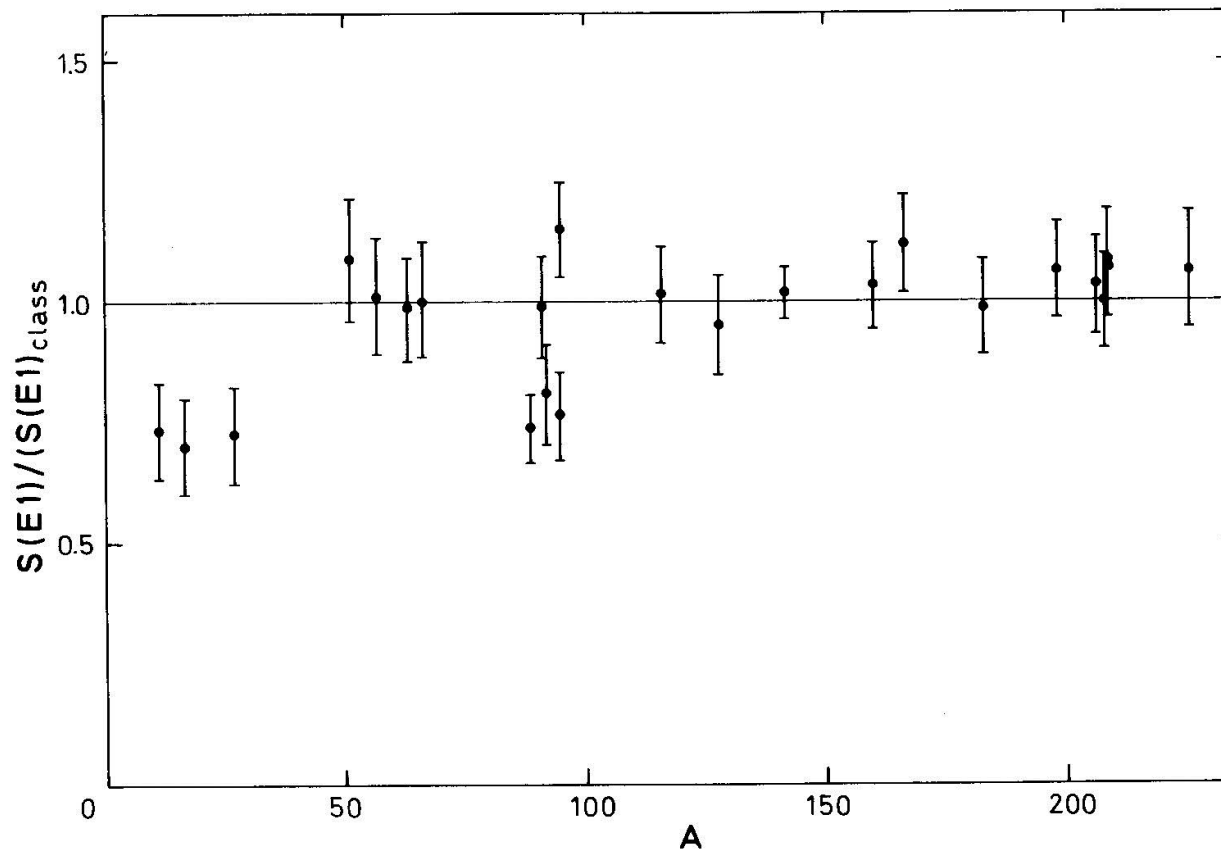
$$S_1 = \frac{\lambda(2\lambda + 1)\hbar^2}{8\pi m} A \langle r^{2\lambda-2} \rangle$$

## Photo absorption cross section:

$$\sigma_{\text{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c} (E_f - E_i) |\langle \phi_f | \tilde{z} | \phi_i \rangle|^2 \delta(E_\gamma - E_f + E_i)$$

$$\begin{aligned} \tilde{z} = \sum_p (z_p - Z_{cm}) &= \sum_p \left\{ z_p - \frac{1}{A} \left( \sum_{p'} z_{p'} + \sum_n z_n \right) \right\} \\ &= \frac{NZ}{A} \left( \frac{1}{Z} \sum_p z_p - \frac{1}{N} \sum_n z_n \right) \end{aligned}$$

$$\begin{aligned} \int \sigma_{\text{abs}}(E_\gamma) dE_\gamma &= \frac{4\pi^2 e^2}{\hbar c} \cdot \frac{\hbar^2}{2m} \cdot \frac{NZ}{A} \\ &= \frac{2\pi^2 e^2 \hbar}{mc} \cdot \frac{NZ}{A} \end{aligned}$$

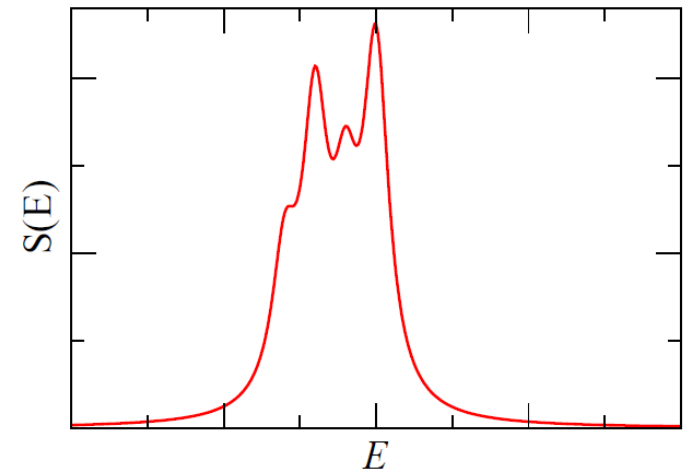


**Figure 6-20** Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with  $A > 50$ , the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic  $\gamma$  rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proc. Intern. Nuclear Physics Conference*, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of  $(\gamma p)$  processes must be included and the data are from:  $^{12}\text{C}$  and  $^{27}\text{Al}$  (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, *Phys. Rev.* **143**, 790, 1966);  $^{16}\text{O}$  (Dolbilkin *et al.*, *loc.cit.*, Fig. 6-26). For the heavy nuclei ( $A > 50$ ), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssi re *et al.*, 1970).

## Nice features of sum rule

$$S_0 = \langle 0 | F^2 | 0 \rangle$$

$$S_1 = \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle$$



Sum rule:

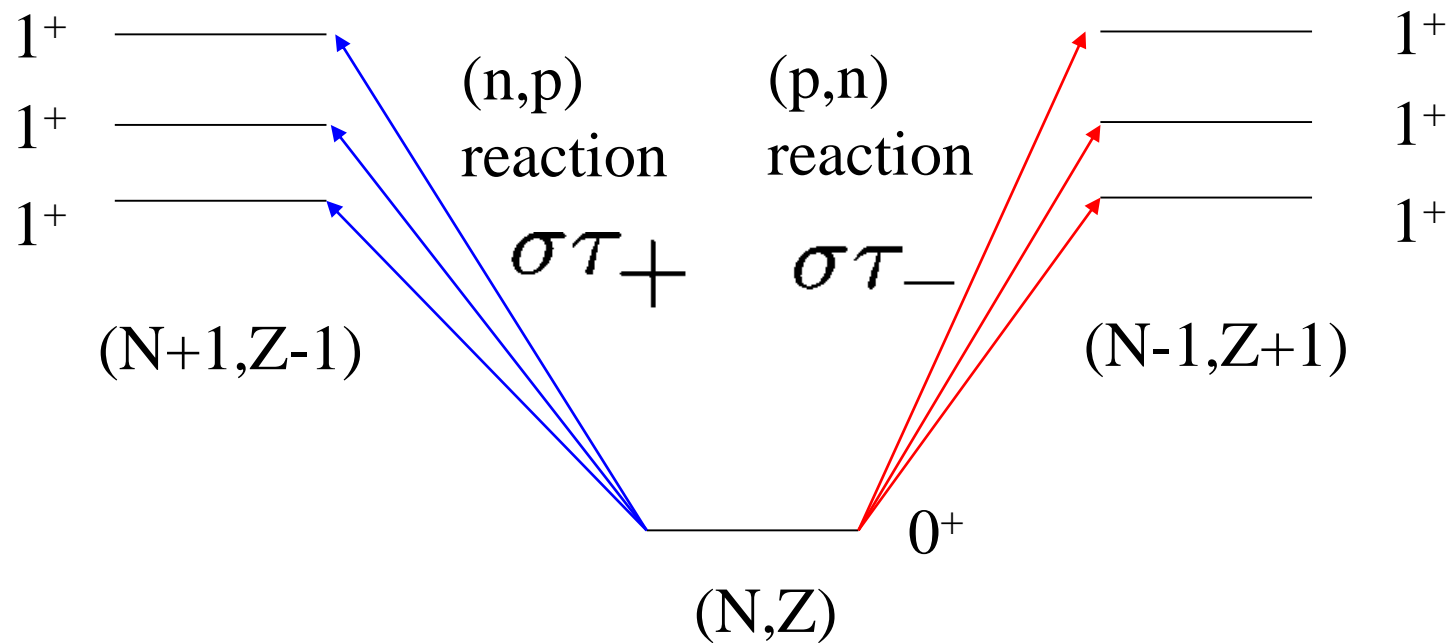
(Some) information on excited states

← only with the properties of ground state

- check a missing strength in the region outside the measurement
- Extraction of radii etc. by measuring a strength distribution
- checks of experimental data and numerical calculations (something is strange if the sum rule value exceeds considerably)
- the validity of approximation.
  - cf. RPA satisfies the TRK sum rule,  
but Tamm-Dancoff approximation does not

## Ikeda sum rule

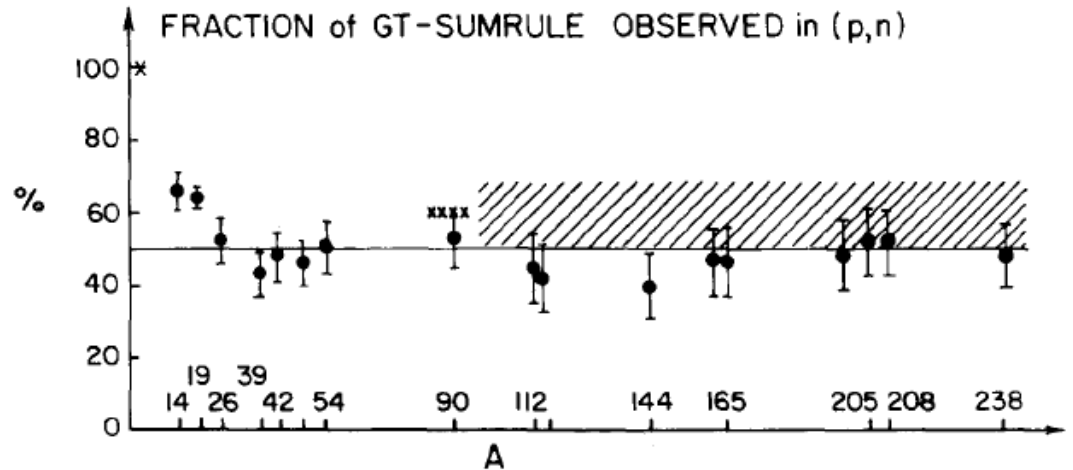
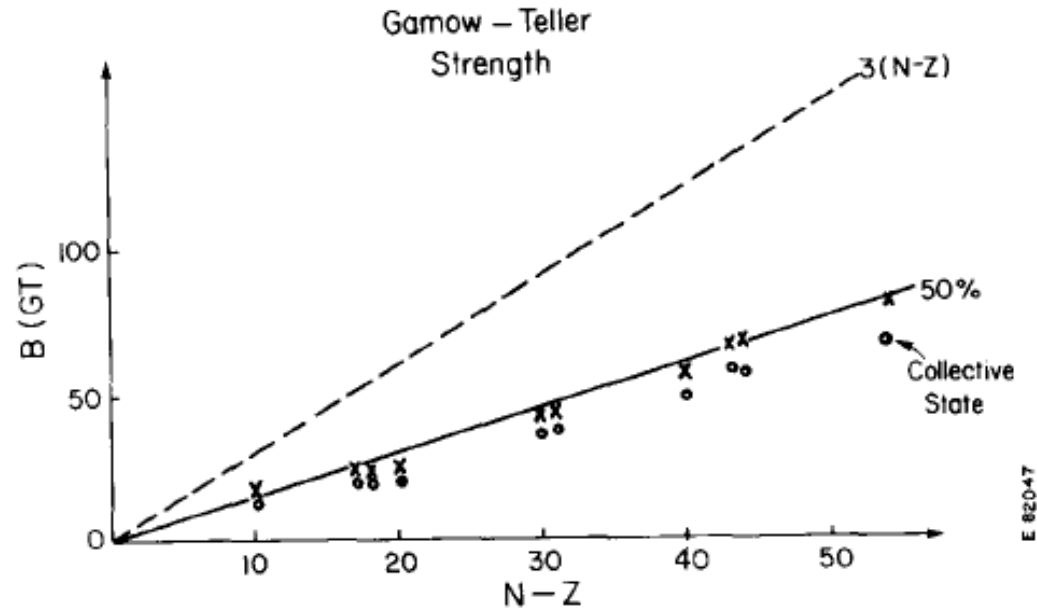
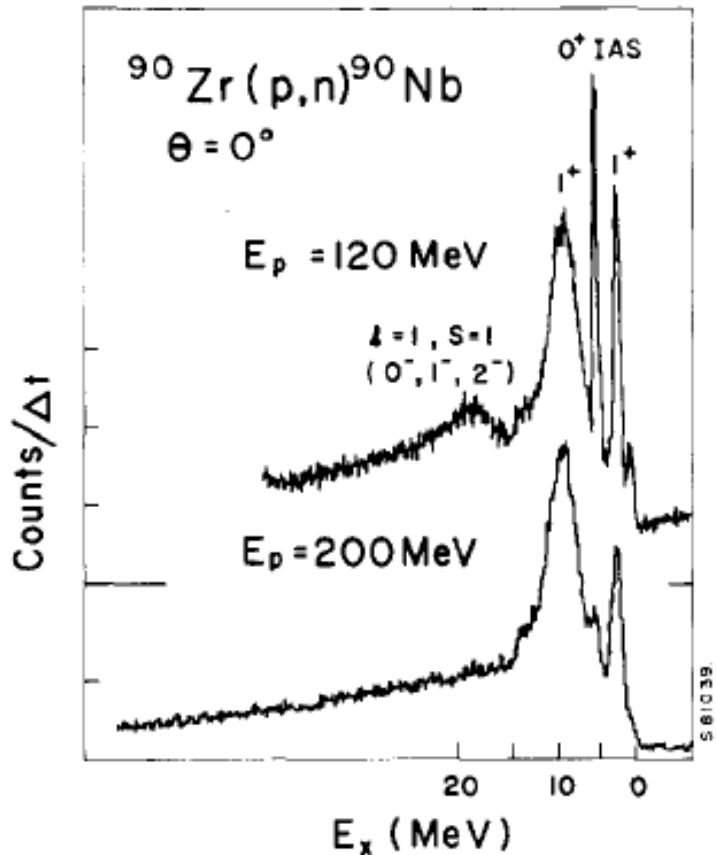
charge exchange reactions: Gamow-Teller transitions



Ikeda sum rule

$$S_0(\sigma\tau_-) - S_0(\sigma\tau_+) = 3(N - Z)$$

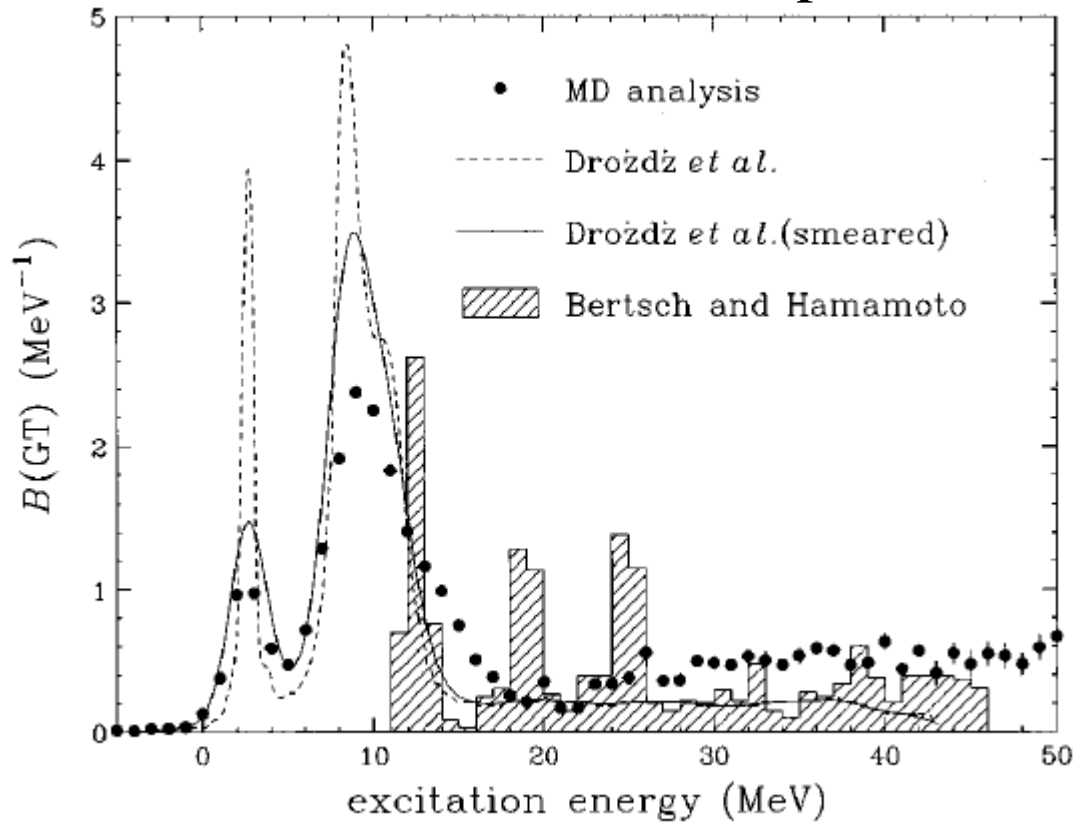
situation before 1997



the “quenching problem”  
 of GT strength

quark ( $\Delta$  resonance)?

# $^{90}\text{Zr} (p,n) ^{90}\text{Nb}$



T. Wakasa et al.,  
PRC55 ('97) 2909

$$S_- - S_+ = 27.0 \pm 1.6 = (90 \pm 5)\% \text{ of Ikeda sum rule}$$

→ quark contribution: small

(proof of Ikeda sum rule)

$$(Y_{\pm})_{\mu} \equiv \sum_i \tau_{\pm}(i) \sigma_{\mu}(i)$$

$$[(Y_{\pm})_{\mu}]^{\dagger} = (-)^{\mu} (Y_{\mp})_{-\mu}$$

$$\begin{aligned} S_- - S_+ &= \langle 0 | Y_-^{\dagger} Y_- | 0 \rangle - \langle 0 | Y_+^{\dagger} Y_+ | 0 \rangle \\ &= \sum_{\mu} (-)^{\mu} \langle 0 | [(Y_+)_{\mu}, (Y_-)_{-\mu}] | 0 \rangle \\ &= \langle 0 | \sum_i [\tau_+(i), \tau_-(i)] \left( \sum_{\mu} (-)^{\mu} \sigma_{\mu} \sigma_{-\mu} \right) | 0 \rangle \\ &= 3 \langle 0 | \sum_i [\tau_+(i), \tau_-(i)] | 0 \rangle \\ &= 3 \langle 0 | \sum_i \tau_z(i) | 0 \rangle = 3(N - Z) \end{aligned}$$