Nuclear Reactions

Shape, interaction, and excitation structures of nuclei ⟵ scattering expt.

cf. Experiment by Rutherford (α scatt.)

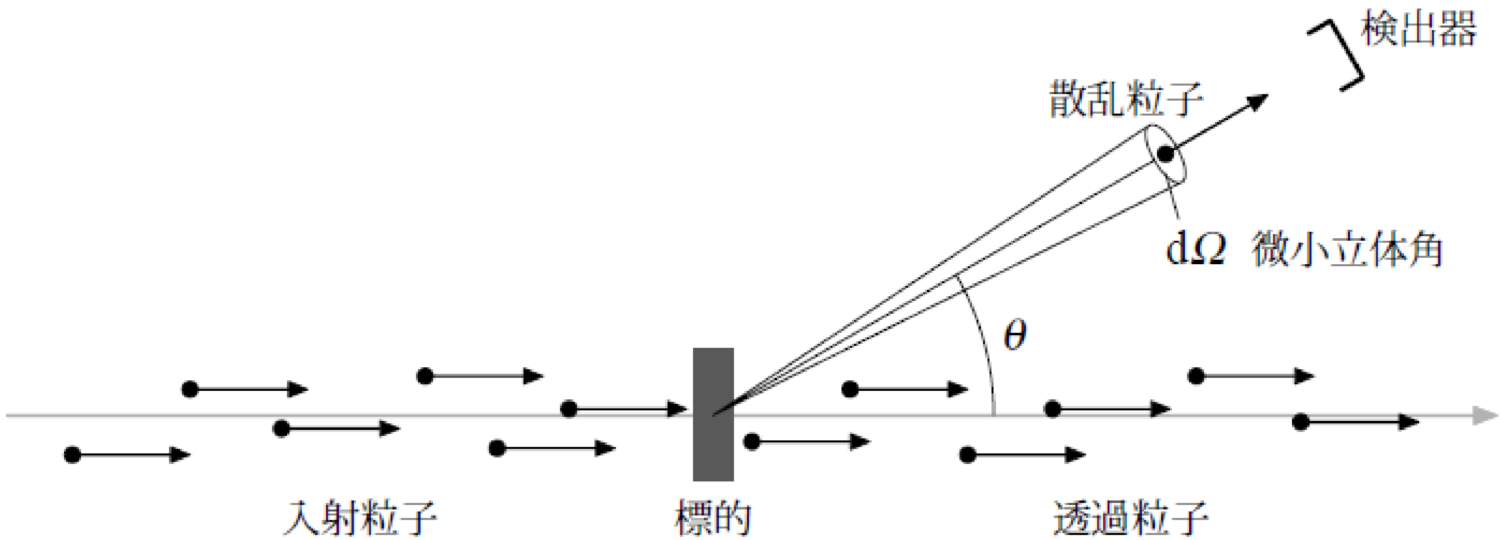
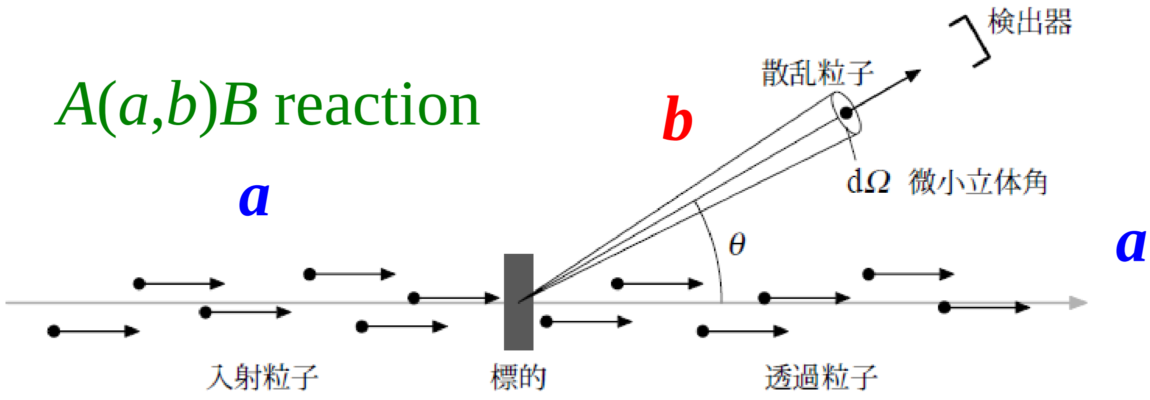


図 21.1: 散乱実験

http://www.th.phys.titech.ac.jp/~muto/lectures/QMII11/QMII11_chap21.pdf

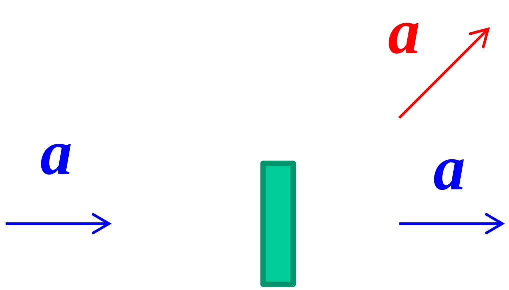
武藤一雄氏（東工大）

A(*a*,*b*)*B* reaction



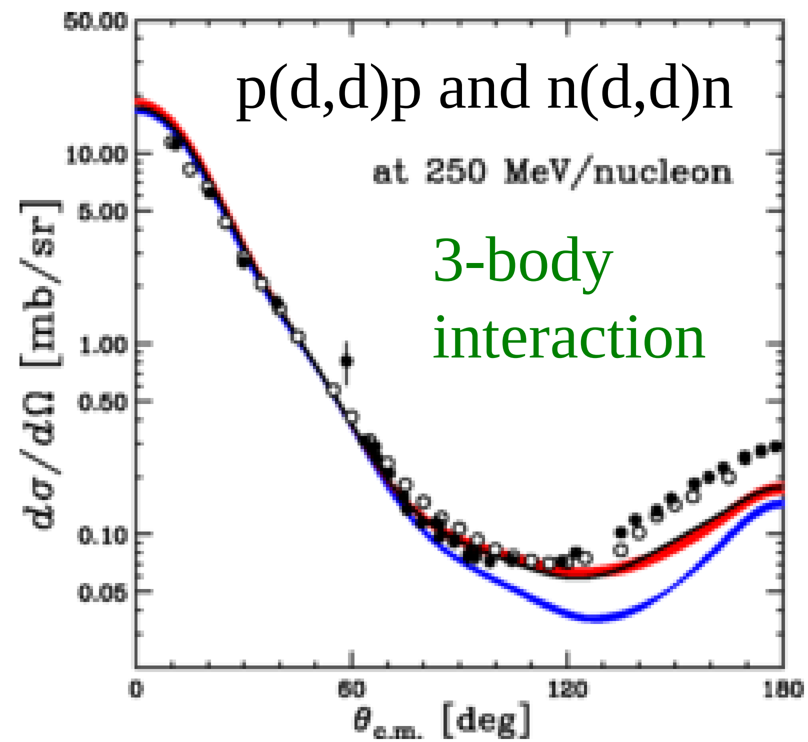
$A \rightarrow B$

✓elastic scattering



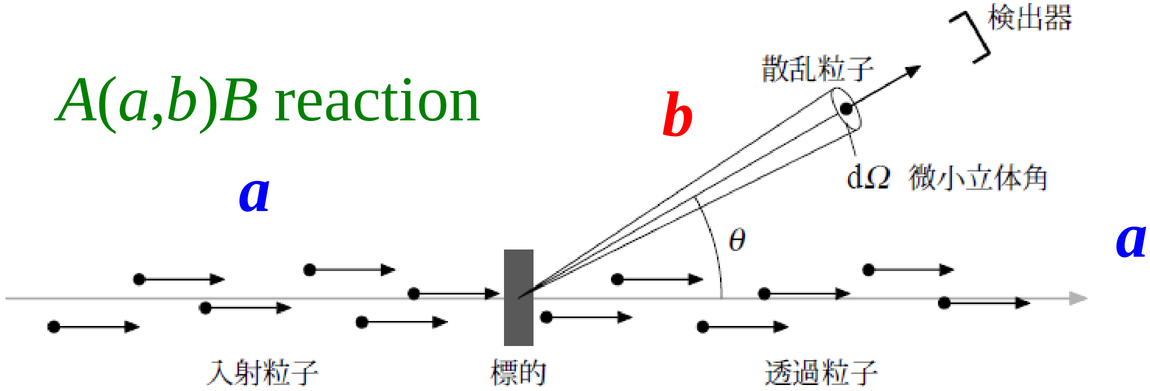
$A_{gs} \rightarrow A_{gs}$

fundamental interaction
between *a* and *A*



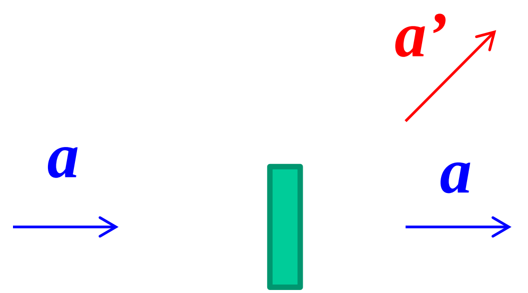
K. Sekiguchi et al., PRC89('14)064007

$A(a,b)B$ reaction

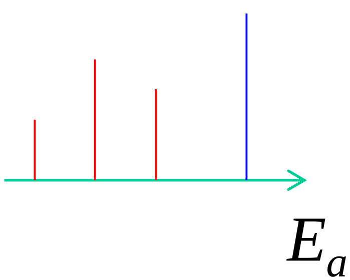


$A \rightarrow B$

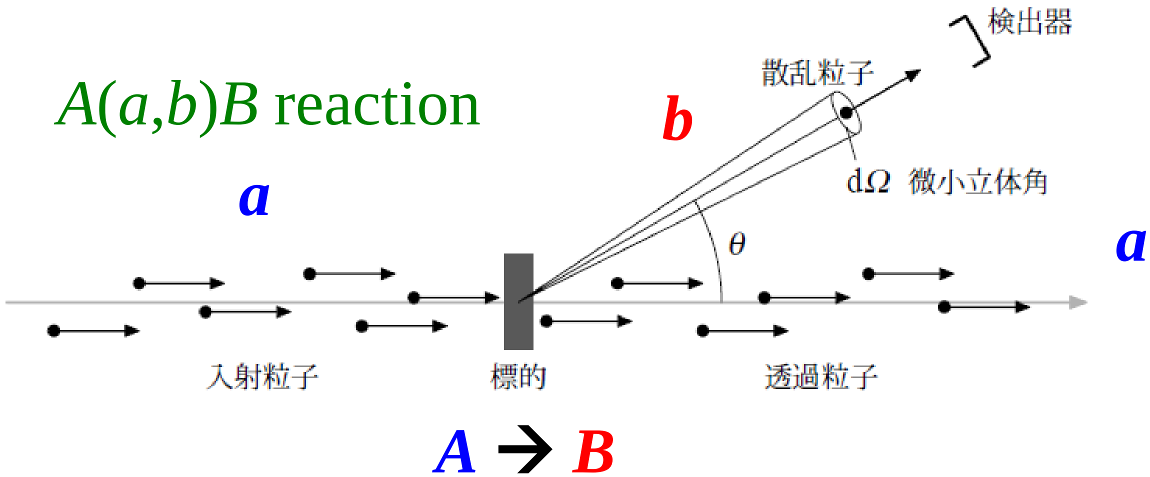
✓inelastic scattering



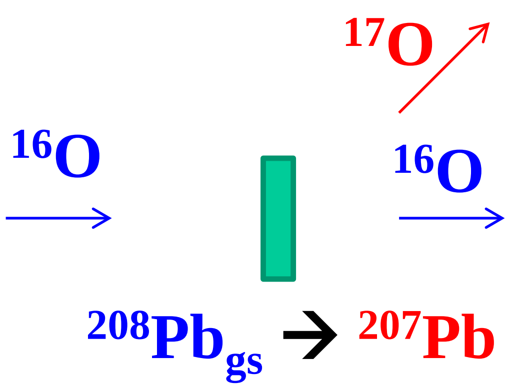
$A_{gs} \rightarrow A^*$



excitation spectrum
of a nucleus A

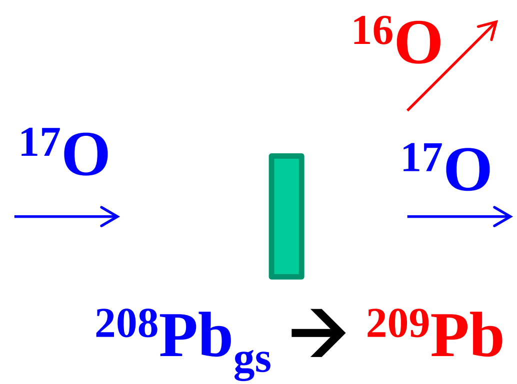


✓transfer reaction
(pick-up reaction)



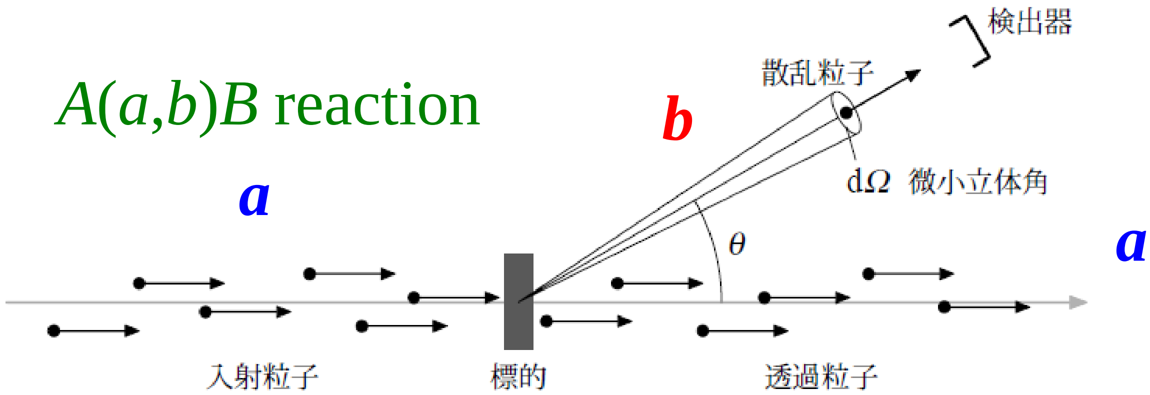
level schem of ^{207}Pb

✓transfer reaction
(stripping reaction)



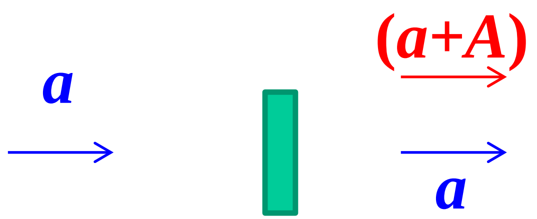
level schem of ^{209}Pb

$A(a,b)B$ reaction



$A \rightarrow B$

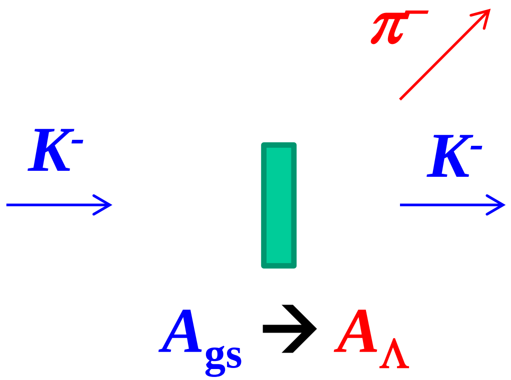
✓fusion reaction



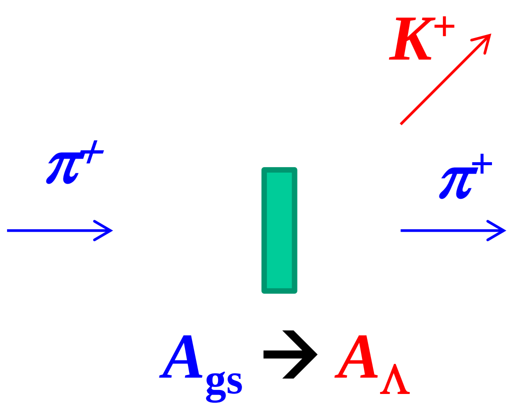
$A_{gs} \rightarrow X$

- interaction between a and A
- structure of a and A

✓(K^-,π^-) reaction

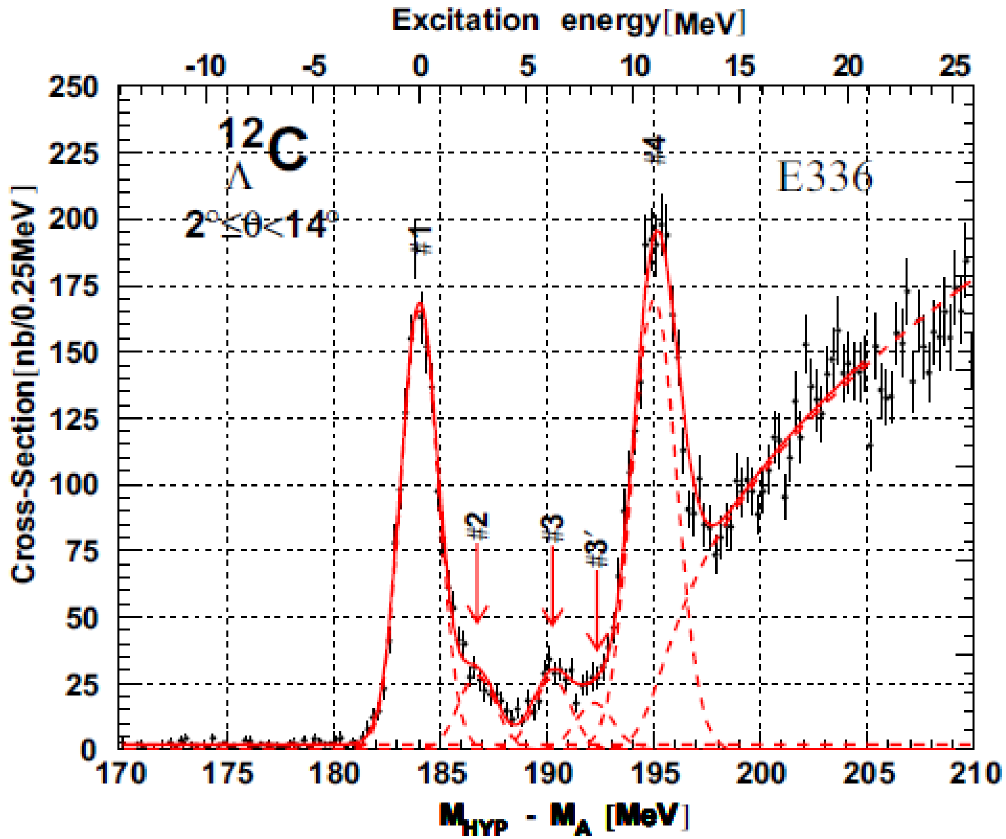


✓(π^+,K^+) reaction



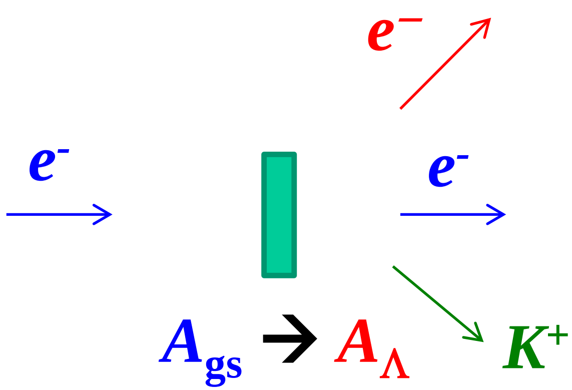
excitation spectrum
of a hypernucleus A_Λ

$^{12}C\ (\pi^+,K^+)\ ^{12}{}_{\Lambda}C$ reaction



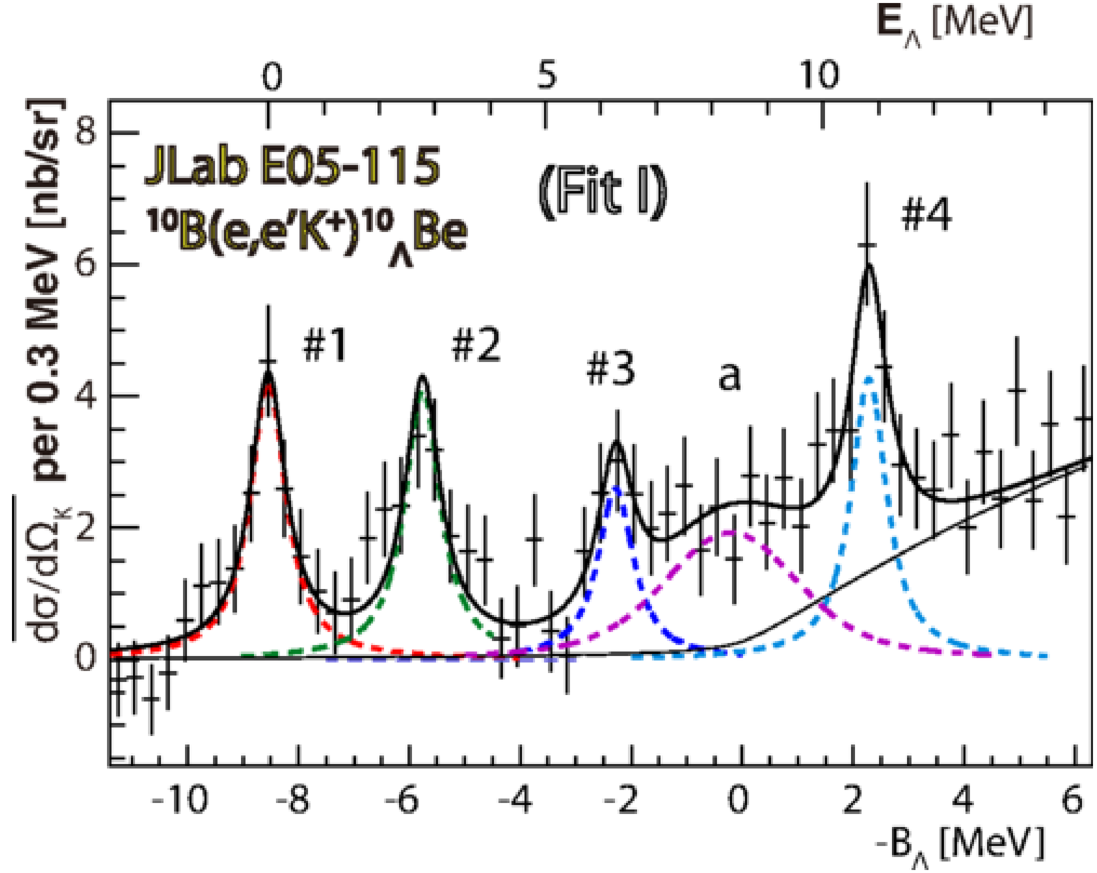
O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 (‘06)564

✓(e,e’K$^+$) reaction



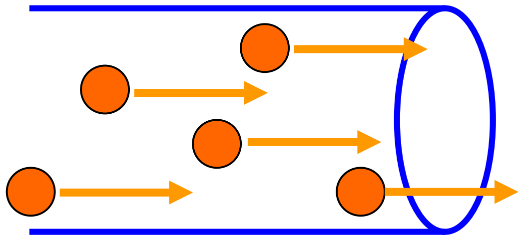
S.N. Nakamura et al.,
PRL110(‘13)012502

$^{10}B(e,e'K^+)\ ^{10}_{\Lambda}Be$



T. Gogami et al.,
PRC93 (‘16) 034314

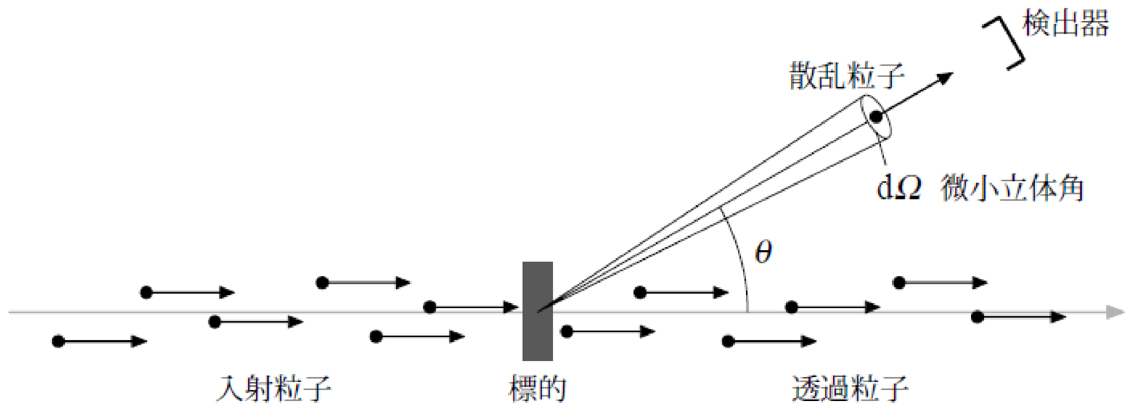
Cross sections



incident beam

flux = the number of particles
crossing unit area
per unit time

$$j = \rho_{\mathrm{P}} \cdot v$$

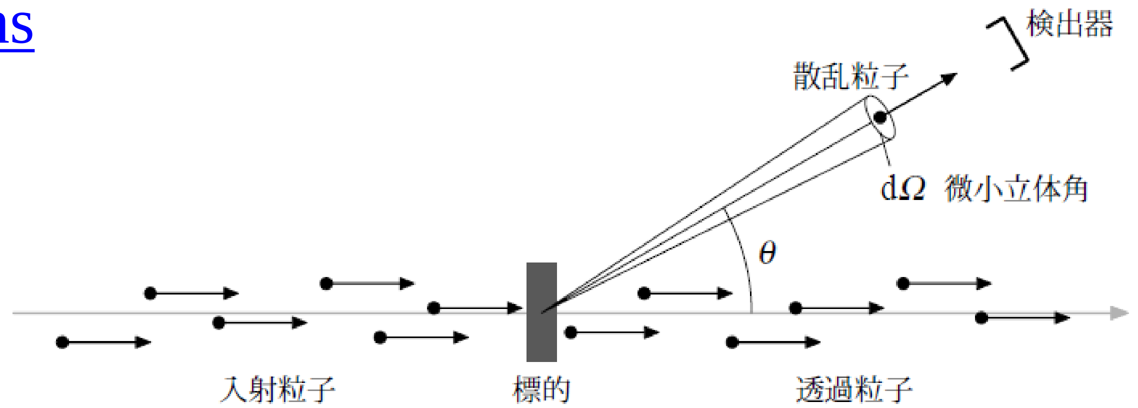


event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

$$\longrightarrow \quad R = N_{\mathrm{T}} \cdot \sigma \cdot j$$

cross section

Cross sections



event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

cross section

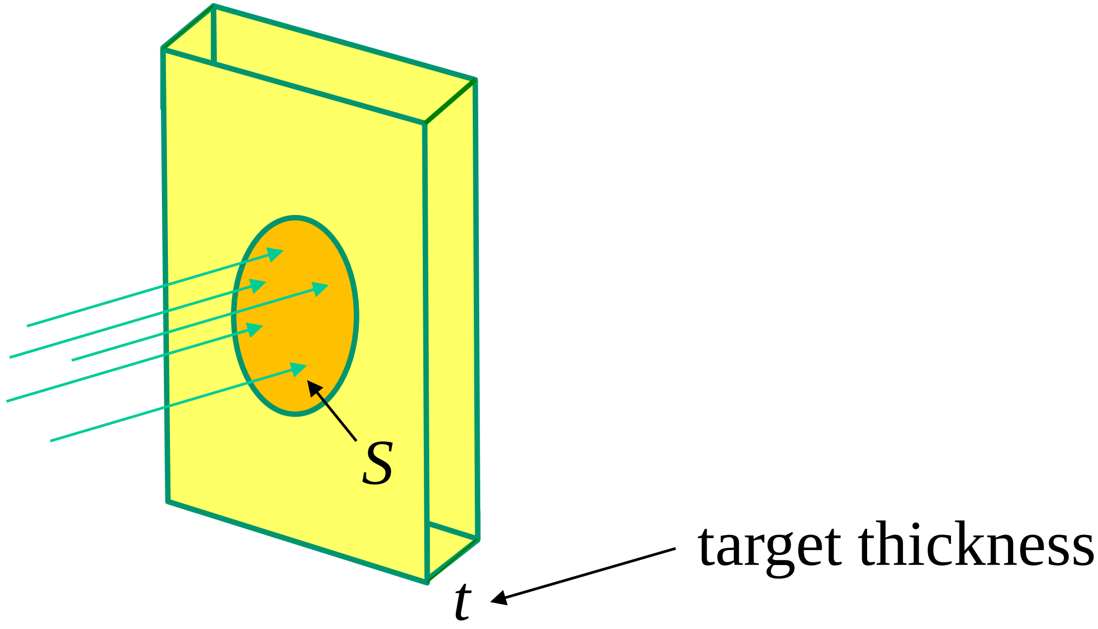
$$\longrightarrow \quad R = N_T \cdot \sigma \cdot j$$

differential cross sections (angular distribution)

$$dR(\theta, \phi) = N_T \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega \qquad \sigma = \int d\Omega \frac{d\sigma}{d\Omega}$$

units: 1 barn = 10^{-24} cm^2 = 100 fm^2 (1 mb = 10^{-3} b = 0.1 fm^2)

Cross sections (experiments)



$$dR(\theta, \phi) = N_{\mathsf{T}} \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega$$

beam intensity: $I = j \cdot S$

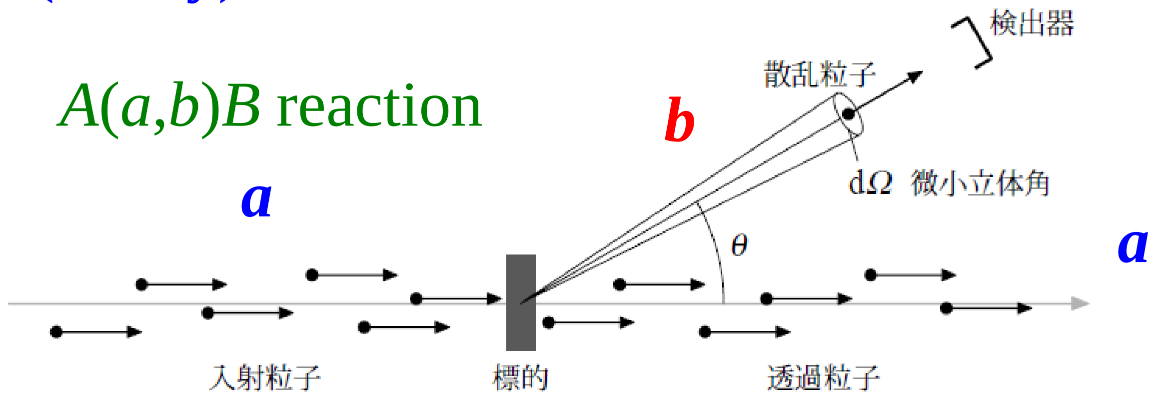
the number of target nucleus: $N_{\mathsf{T}} = S \cdot t \cdot \rho_{\mathsf{T}}$

$$dR(\theta, \phi) = I \cdot \frac{d\sigma}{d\Omega} \cdot t \cdot \rho_{\mathsf{T}} \cdot d\Omega \cdot \epsilon$$

detection efficiency

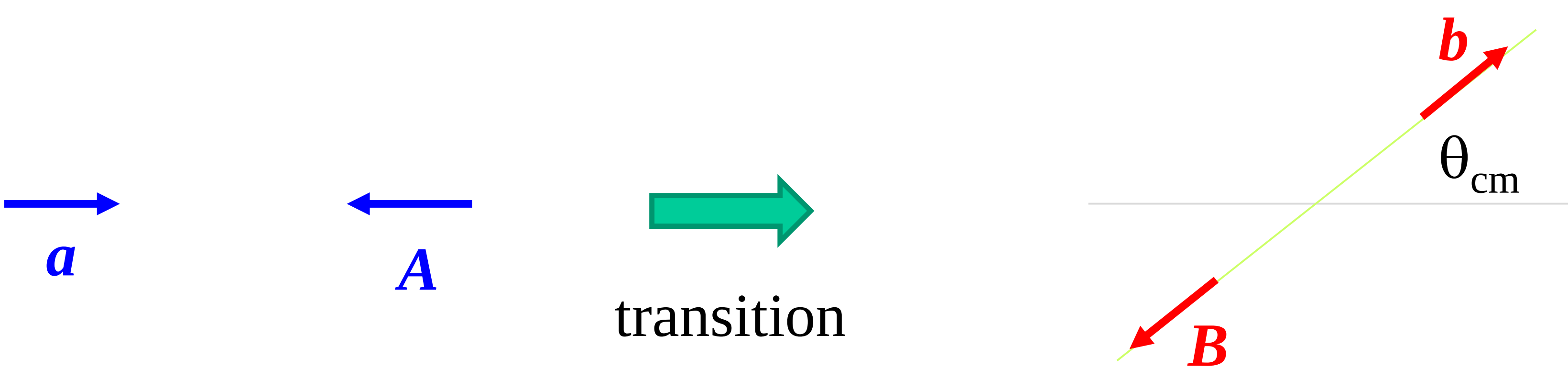
Cross sections (theory)

$A(a,b)B$ reaction



$A \rightarrow B$

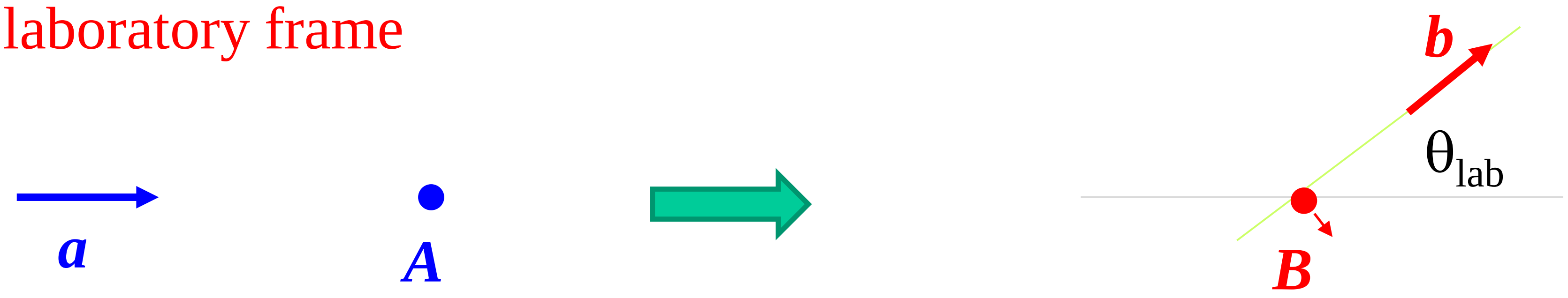
center of mass frame



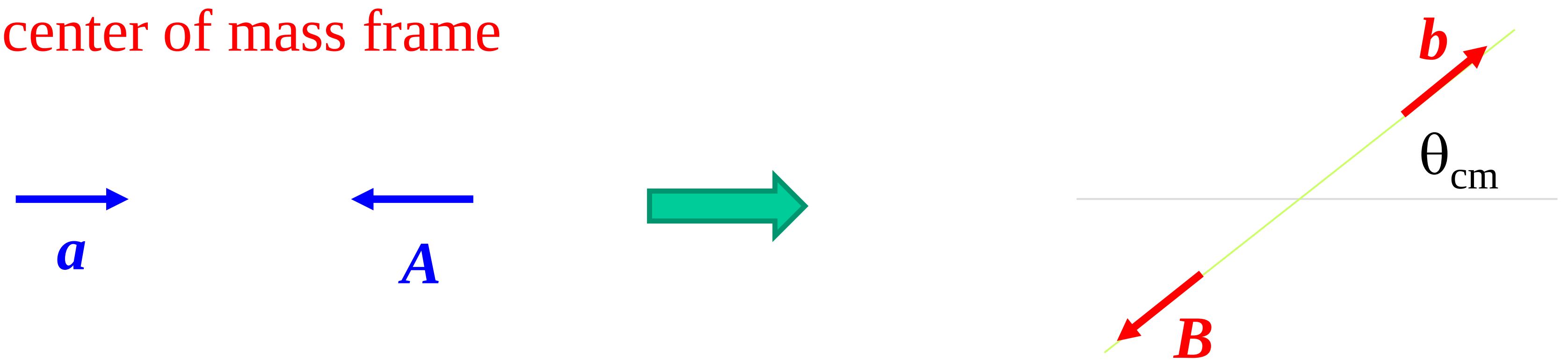
$$\frac{d\sigma}{d\Omega} = \frac{R}{j_{\text{in}}}$$

Cross sections

✓ laboratory frame



✓ center of mass frame



□ transformation ⟵ energy and momentum conservations

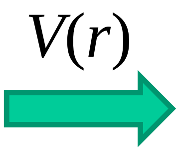
$$\tan\theta_{\mathrm{lab}} = \frac{\sin\theta_{\mathrm{cm}}}{\gamma+\cos\theta_{\mathrm{cm}}}, \qquad d\Omega_{\mathrm{lab}} = \frac{|1+\gamma\cos\theta_{\mathrm{cm}}|}{(1+\gamma^2+2\gamma\cos\theta_{\mathrm{cm}})^{3/2}}\, d\Omega_{\mathrm{cm}}$$

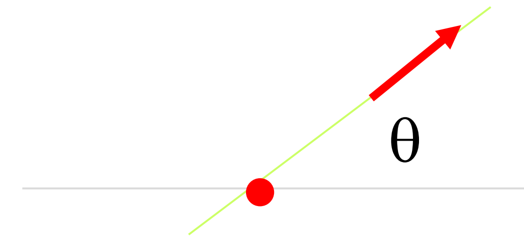
$$E_{\mathrm{cm}} = \frac{M_A}{M_a+M_A} E_{\mathrm{lab}}, \qquad \gamma = \sqrt{\frac{M_a M_b}{M_A M_B}\frac{E_{\mathrm{cm}}}{E_{\mathrm{cm}}+Q}}$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$





$$\left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

transition rate for elastic scattering:

$$\begin{aligned} W_{fi} &= \frac{2\pi}{\hbar}\int \frac{d\boldsymbol{p}_f}{(2\pi\hbar)^3}\,|\langle\psi_f|V|\psi_i\rangle|^2\delta(E_f - E_i) \\ &= \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega\,\left|\tilde{V}(\boldsymbol{q})\right|^2 \end{aligned}$$

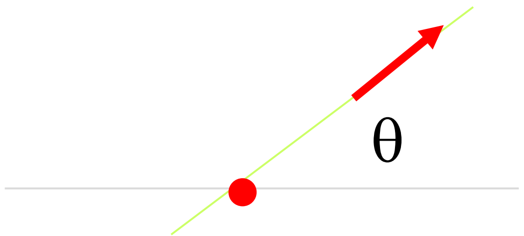
$$\begin{aligned} \tilde{V}(\boldsymbol{q}) &= \int d\boldsymbol{r}\,\psi_f^*(\boldsymbol{r})V(r)\psi_i(\boldsymbol{r}) \\ &= \int d\boldsymbol{r}e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar}V(r) \equiv \int d\boldsymbol{r}e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(r) \end{aligned}$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

V(r)



momentum transfer

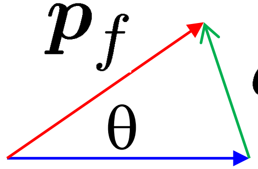
$$W_{fi} = \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega \left|\tilde{V}(\boldsymbol{q})\right|^2$$

$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r} e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar} V(r) \equiv \int d\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} V(r)$$

incident flux: $j_{\text{inc}} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\text{inc}}} = \int d\Omega \boxed{\frac{\mu^2}{4\pi^2\hbar^4}\left|\tilde{V}(\boldsymbol{q})\right|^2}$$

$$= \frac{d\sigma}{d\Omega}$$



$$q\hbar = 2p_i \sin\frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E \sin^2\theta/2)^2} |F(\boldsymbol{q})|^2$$

$$= \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega}\right) |F(\boldsymbol{q})|^2$$

Form factor

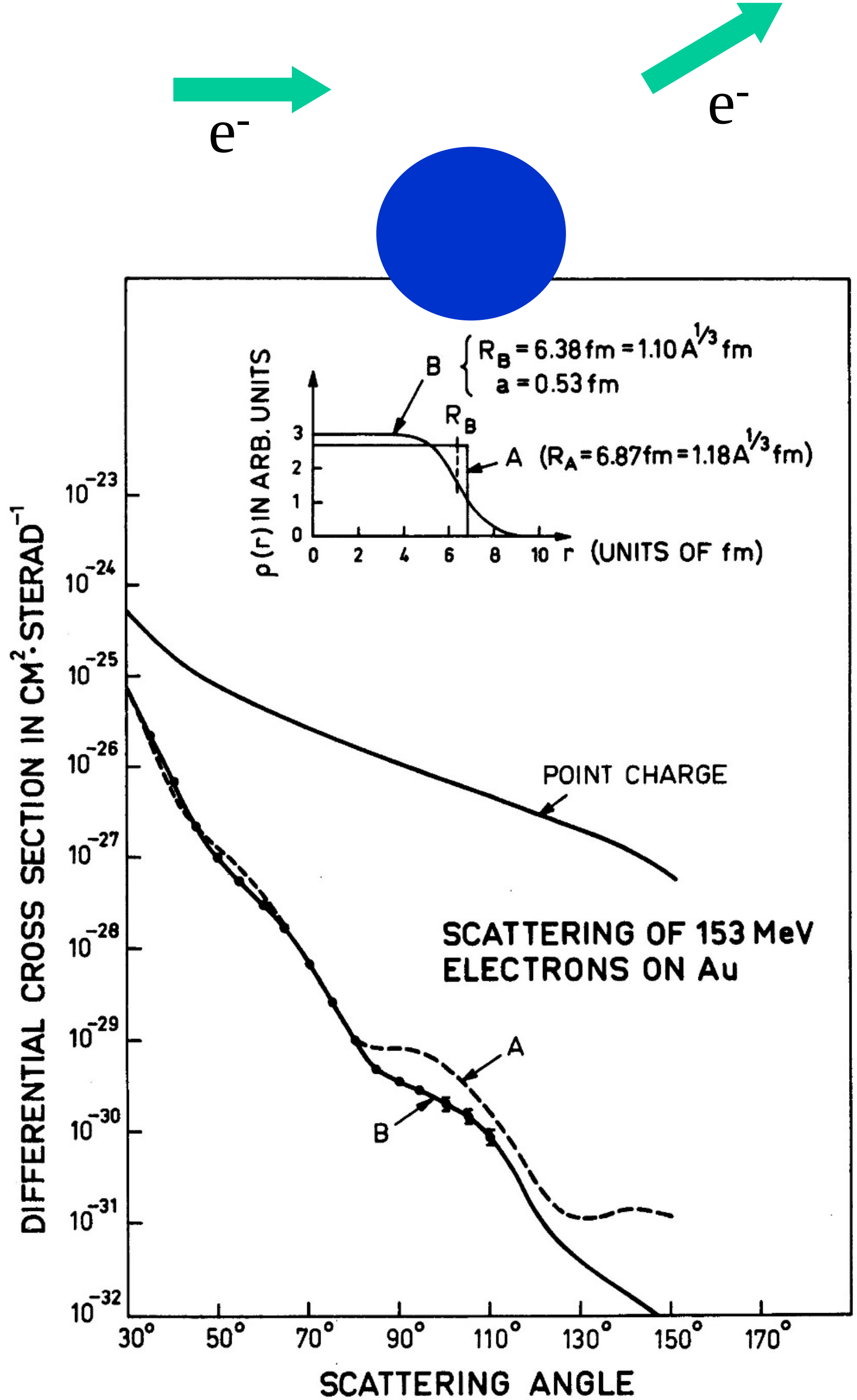
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})\, d\boldsymbol{r}$$

* relativistic correction:

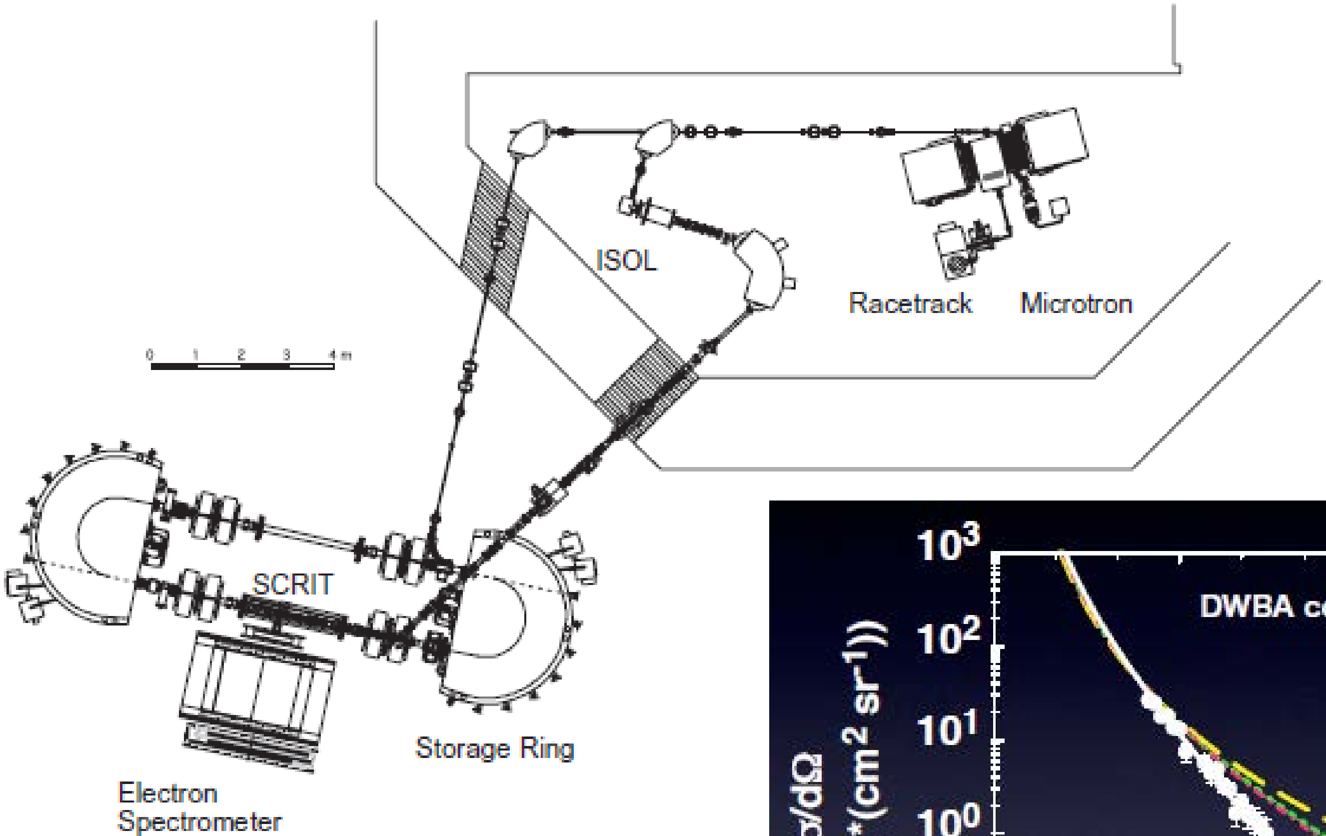
$$\frac{d\sigma_{\text{Ruth}}}{d\Omega} \rightarrow \frac{d\sigma_{\text{Mott}}}{d\Omega}$$

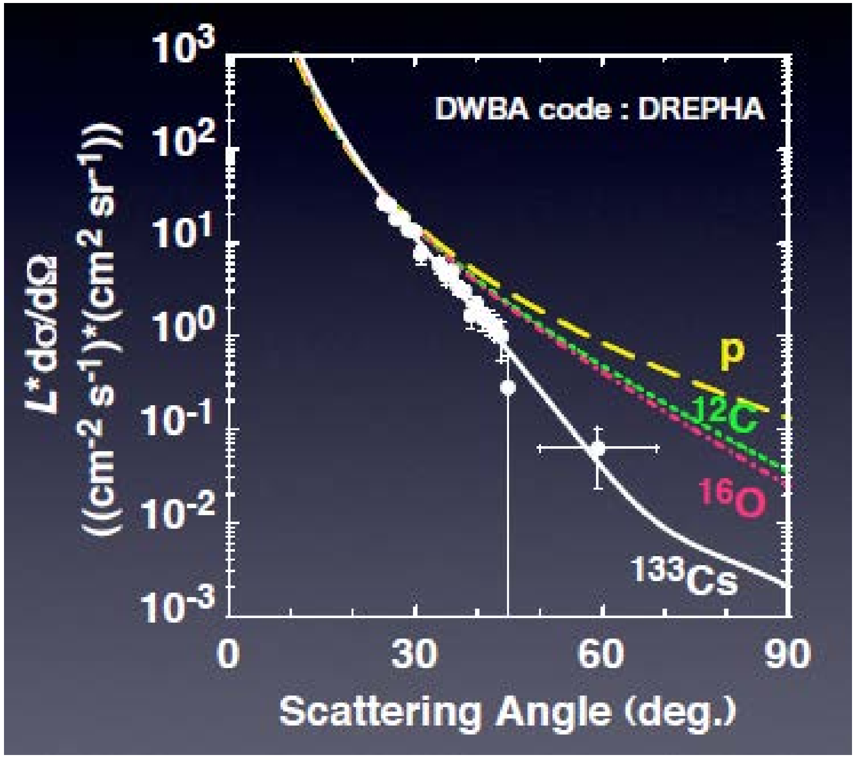
$$= \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \left(1 - \frac{v^2}{c^2} \sin^2\frac{\theta}{2}\right)$$

$$\sim \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \cos^2\frac{\theta}{2} \qquad (v \rightarrow c)$$



cf. electron scattering off unstable nuclei (SCRIT)





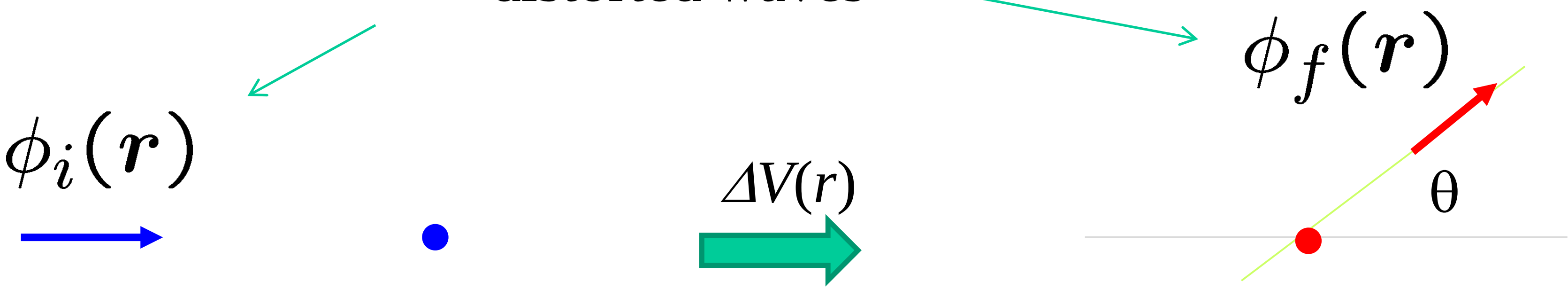
T. Suda et al.,
PTEP 2012, 03C008 (2012)
PRL102, 102501 (2009)

Distorted Wave Born approximation (DWBA)

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \underbrace{V(r)}_{\text{perturbation}} - E\right)\psi(\boldsymbol{r}) = 0$$

$$\Rightarrow \left(-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \underbrace{V(r) - V_0(r)}_{\text{perturbation}} - E\right)\psi(\boldsymbol{r}) = 0$$

"distorted waves"

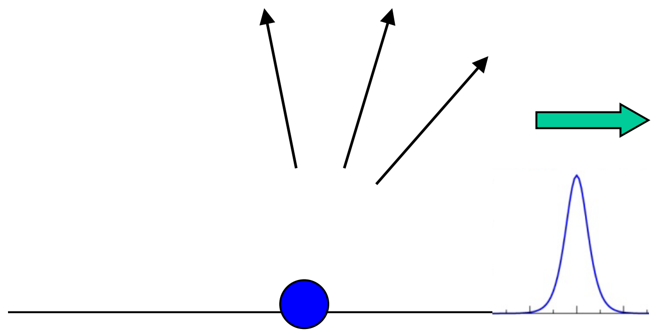


✓inelastic scattering
✓transfer reactions

Optical model

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux (absorption)

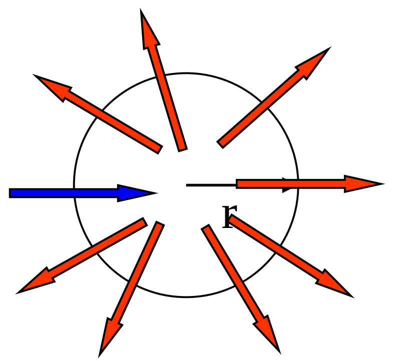
Optical potential

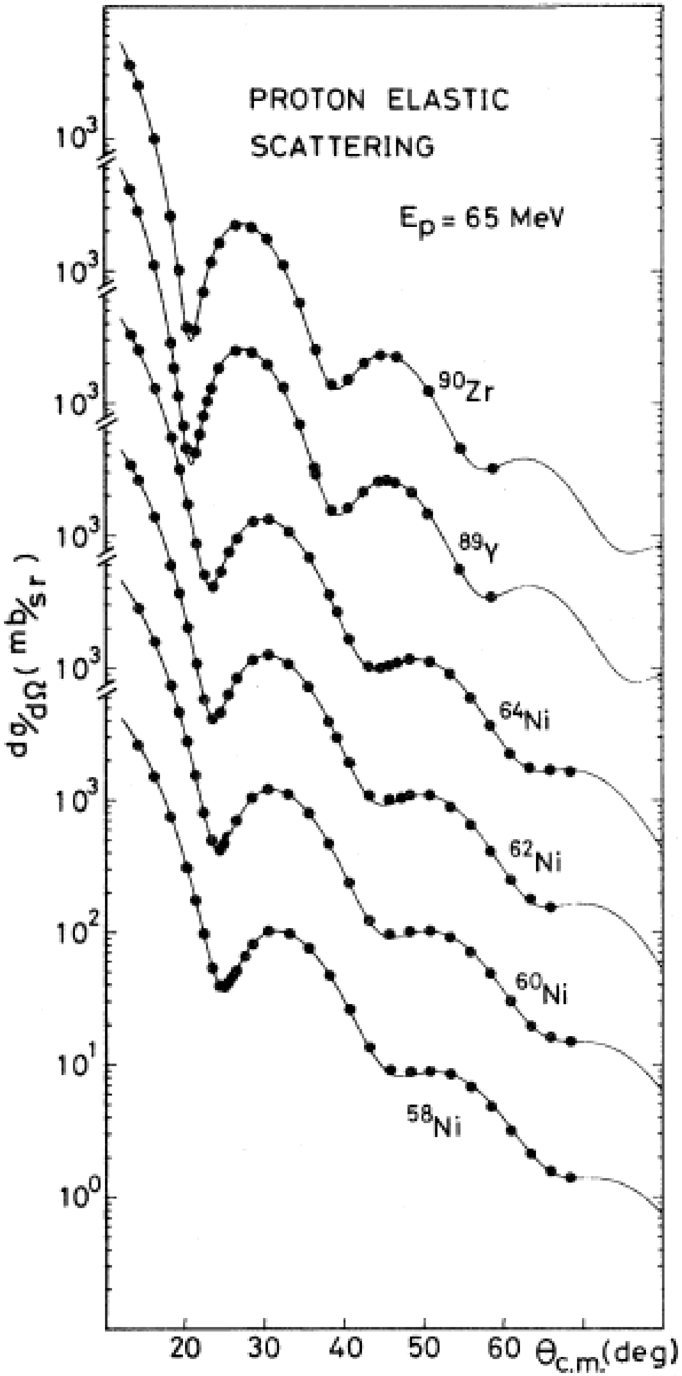
$$V_{\text{opt}}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)$$

$$\Longrightarrow \quad \boldsymbol{\nabla} \cdot \boldsymbol{j} = \cdots = -\frac{2}{\hbar} W |\psi|^2$$

(note) Gauss's theorem

$$\int_S \boldsymbol{j} \cdot \boldsymbol{n} \, dS = \int_V \boldsymbol{\nabla} \cdot \boldsymbol{j} \, dV$$





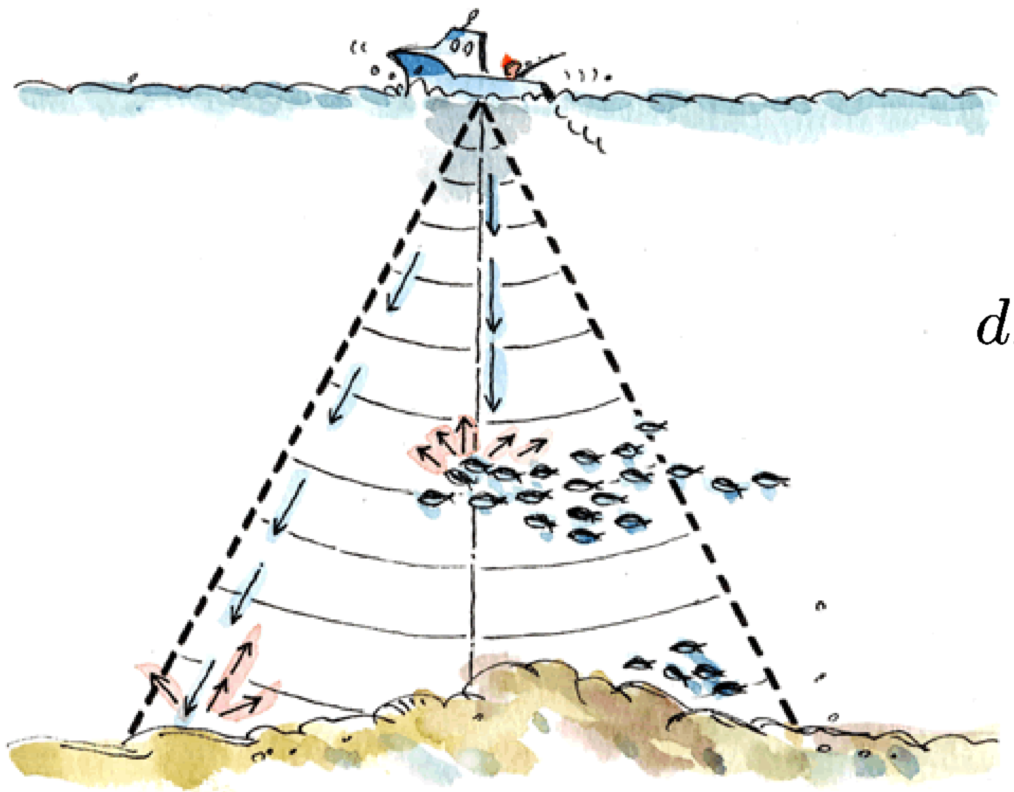
$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \frac{Z_P Z_T e^2}{r} + V_{\rm opt}(r) - E\right)\psi(\boldsymbol{r}) = 0$$

Woods-Saxon + volume &surface imaginary parts

H. Sakaguchi et al., PRC26 (1982) 944

おまけ：海洋音響学におけるDWBA

魚群探知機



散乱体（魚など）による
（超）音波の（後方）散乱

$$dR(\theta, \phi) = N_{\mathrm{T}} \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega$$

$$\downarrow$$

$$N_{\mathrm{T}} = \frac{\frac{dR}{d\Omega}}{j \cdot \frac{d\sigma}{d\Omega}}$$

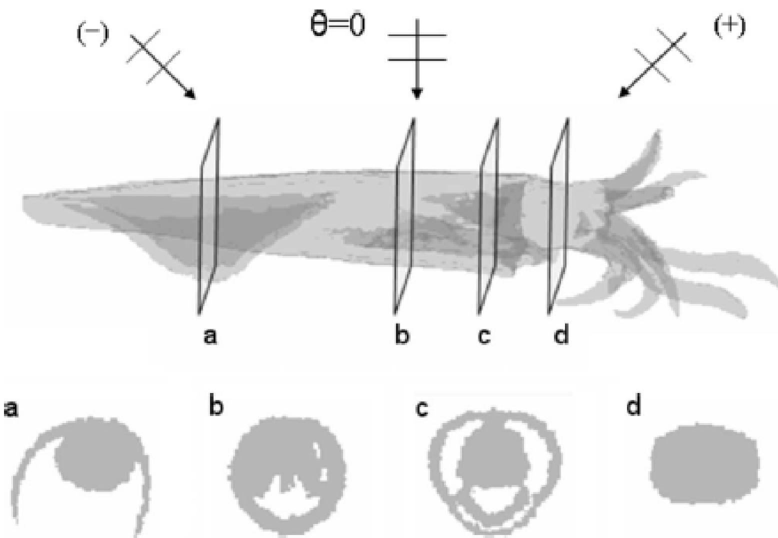
微分散乱断面積を知って
いれば魚の数 N_{T} がわかる

https://www.furuno.co.jp/technology/about/fishfinder1.html

Use of the distorted wave Born approximation to predict scattering by inhomogeneous objects: Application to squid

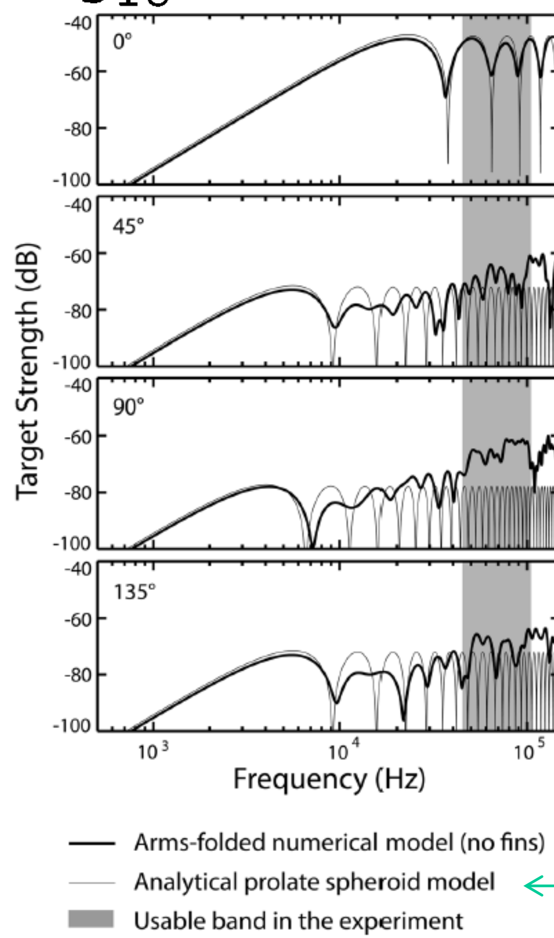
Benjamin A. Jones,[a)] Andone C. Lavery, and Timothy K. Stanton
Department of Applied Ocean Physics and Engineering, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts 02543-1053

J. Accoust. Soc. Am. 125 (‘09) 73



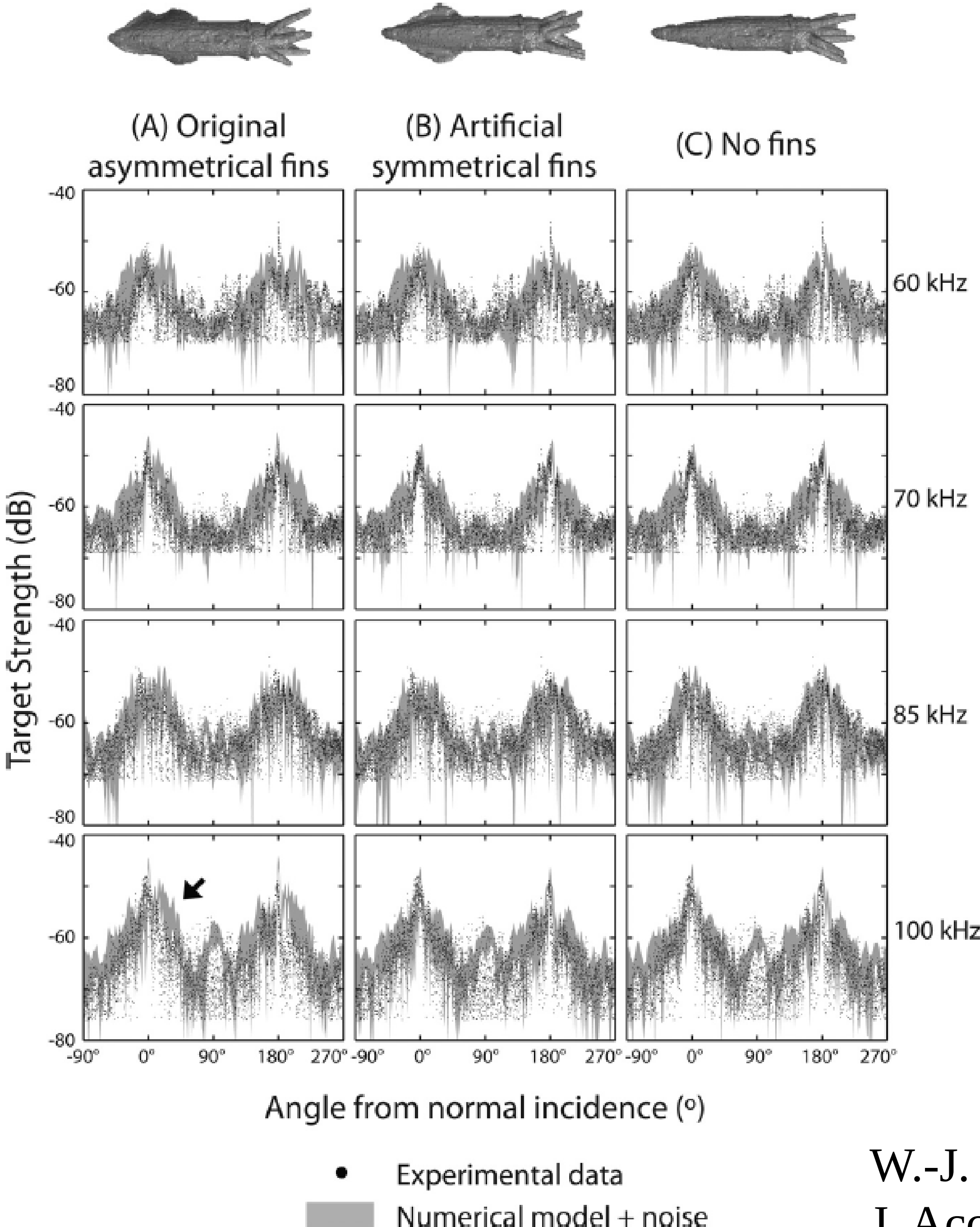
イカのモデル化

$10\log_{10}\sigma$

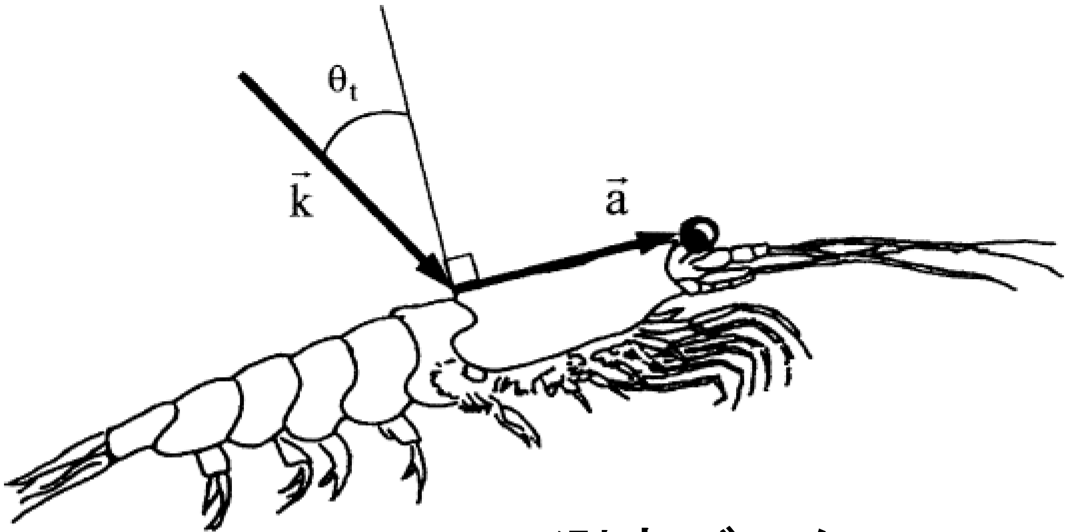


← ！

DWBA:イカの内部では局所的な波数を用いる

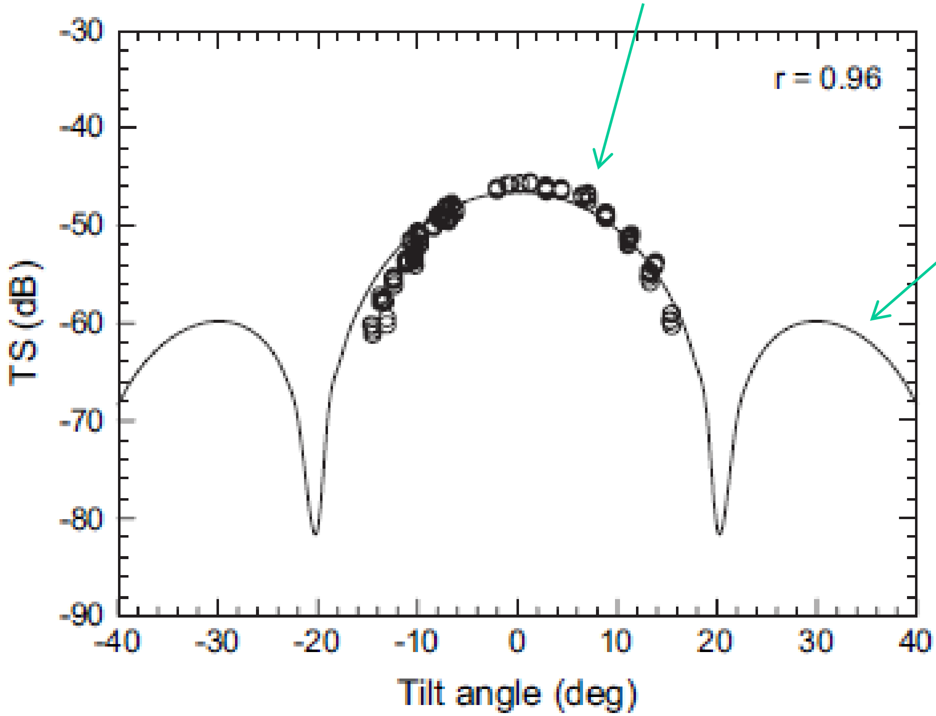


W.-J. Lee, A.C. Lavery, T. Stanton,
J. Accoust. Soc. Am. 131 (‘12) 4461



オキアミ

測定データ

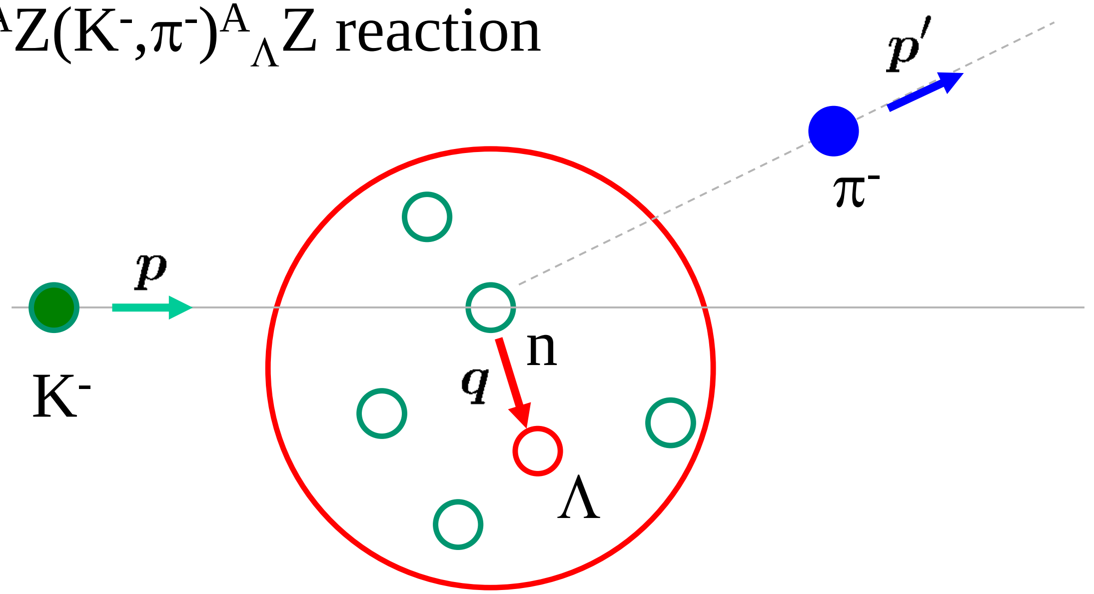


DWBA

K. Akamatsu and M. Furusawa,
ICES J. of Marine Science 63 (‘06) 36

Impulse approximation

example: $^{A}Z(K^{-},\pi^{-})^{A}_{\Lambda}Z$ reaction



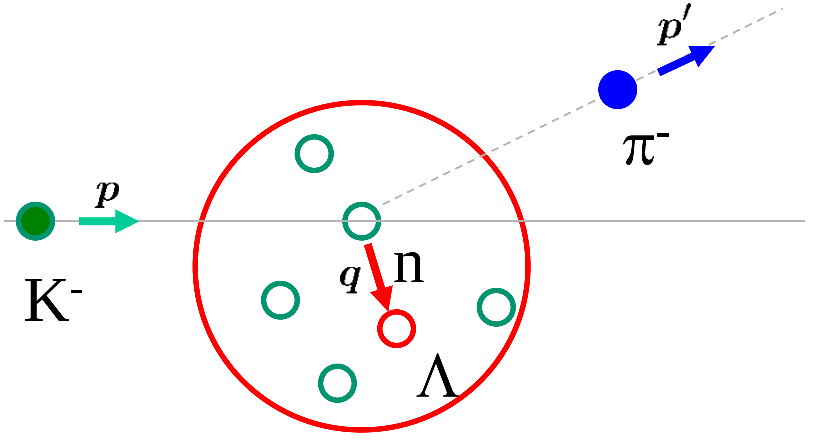
✓ high energy
✓ single scattering approximation
✓ (other nucleons: spectator)

$$T_{fi} \sim \left\langle \psi_{\pi^-} \middle| \left\langle \Psi_{^{A}_{\Lambda}Z} \middle| \sum_j v_{eff}(j) \middle| \Psi_{^{A}Z} \right\rangle \middle| \psi_{K^-} \right\rangle$$

effective K-n interaction
(including multiple scattering)

Impulse approximation

example: $^{A}Z(K^{-},\pi^{-})^{A}_{\Lambda}Z$ reaction



- ✓ high energy
- ✓ single scattering approximation

$$T_{fi} \sim \left\langle \psi_{\pi^-} \left| \left\langle \Psi_{^A_\Lambda Z} \left| \sum_j v_{eff}(j) \right| \Psi_{^A Z} \right\rangle \right| \psi_{K^-} \right\rangle$$

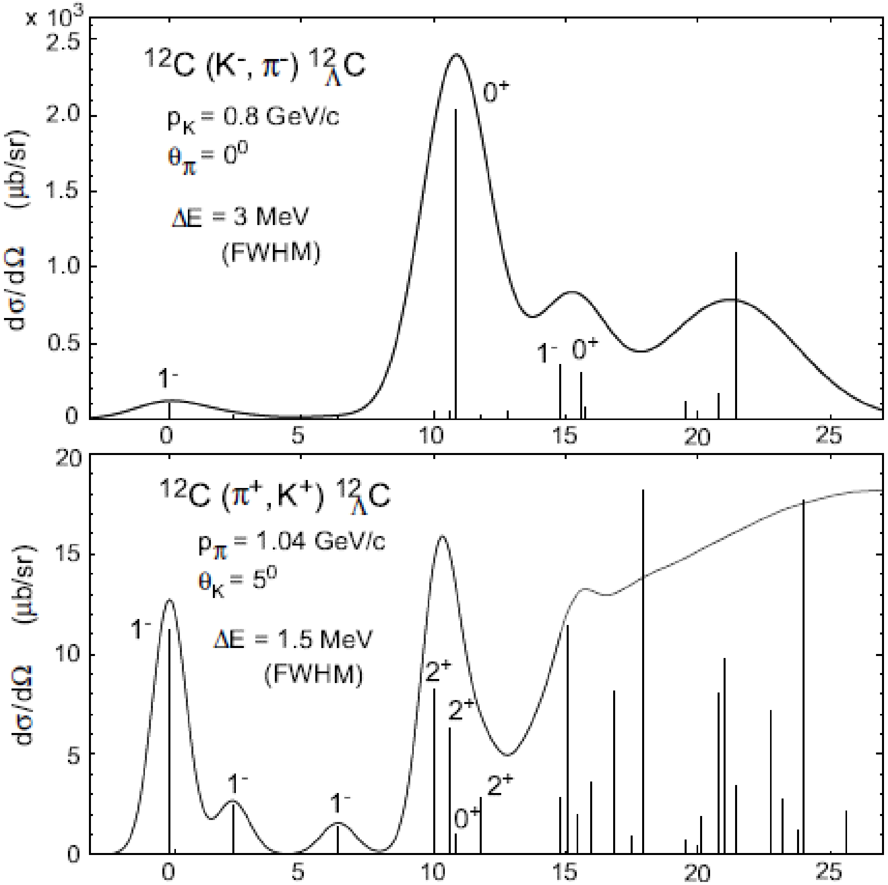
$$\frac{d\sigma}{d\Omega} \sim \alpha_{\rm kin} \left(\frac{d\sigma}{d\Omega}\right)_{K^- n \to \pi^- \Lambda} N_{\rm eff}(\theta; i \to f)$$

kinematical factor

elementary process

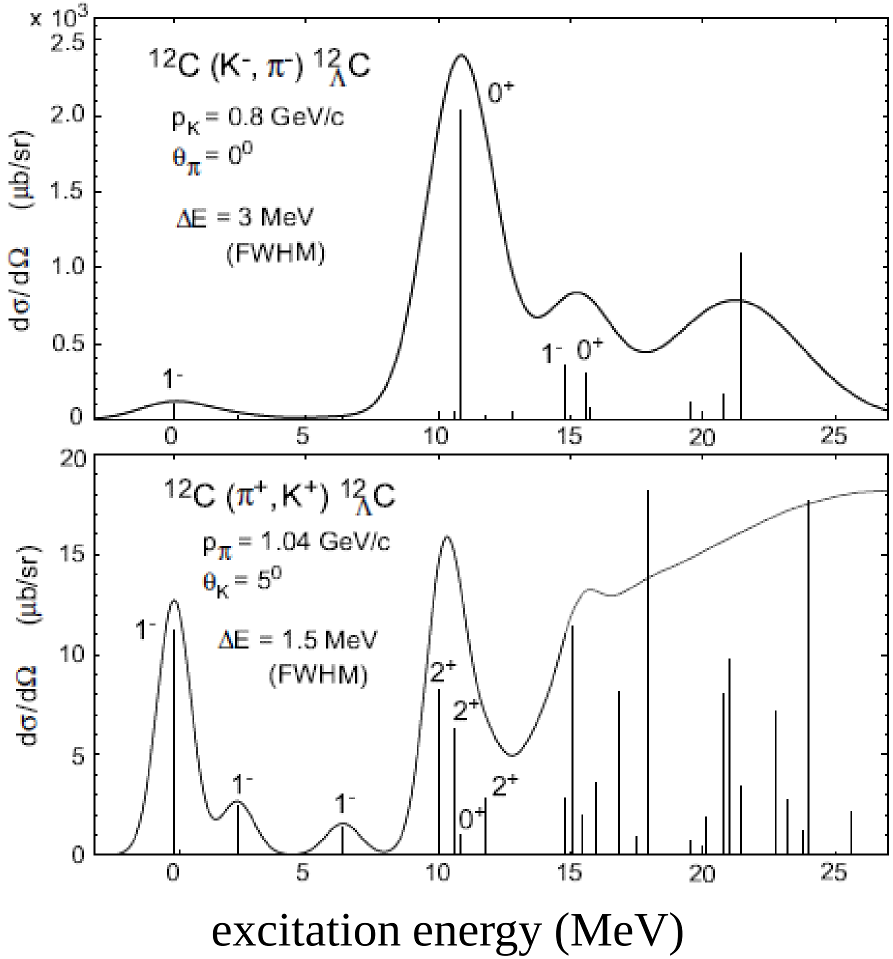
$$N_{\rm eff}(\theta; i \to f) \sim \left| \int d\boldsymbol{r}\, \psi^*_{\pi^-}(\boldsymbol{r})\, \varphi^{(\Lambda)*}_{j_\Lambda l_\Lambda m_\Lambda}(\boldsymbol{r}) \varphi^{(n)}_{j_n l_n m_n}(\boldsymbol{r})\, \psi_{K^-}(\boldsymbol{r}) \right|^2$$

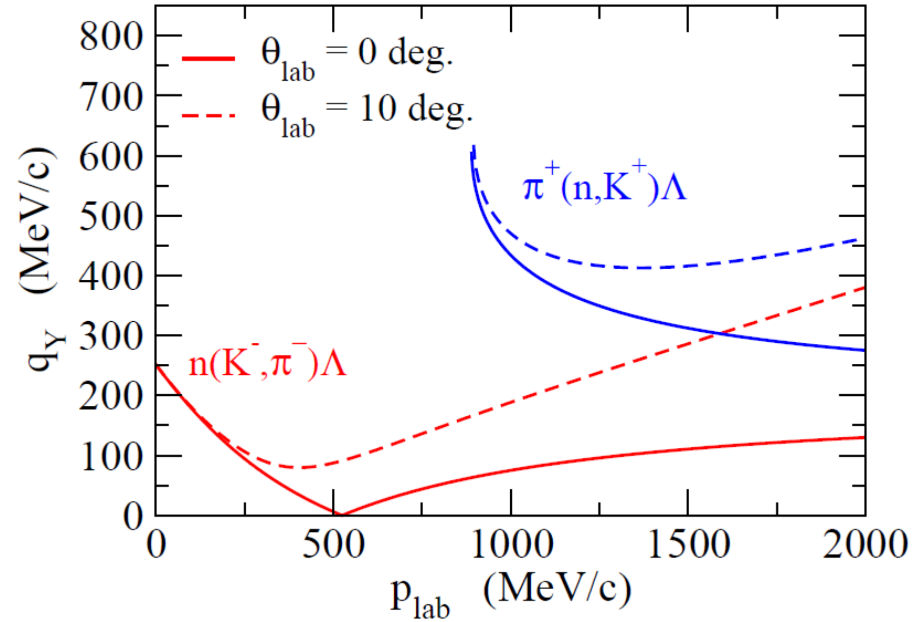
- Plane wave impulse approximation (PWIA)
- Distorted wave impulse approximation (DWIA)



O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 (‘06)564

T. Motoba et al., PRC38(‘88)1322





$m_n + m_K$ = 1432 MeV

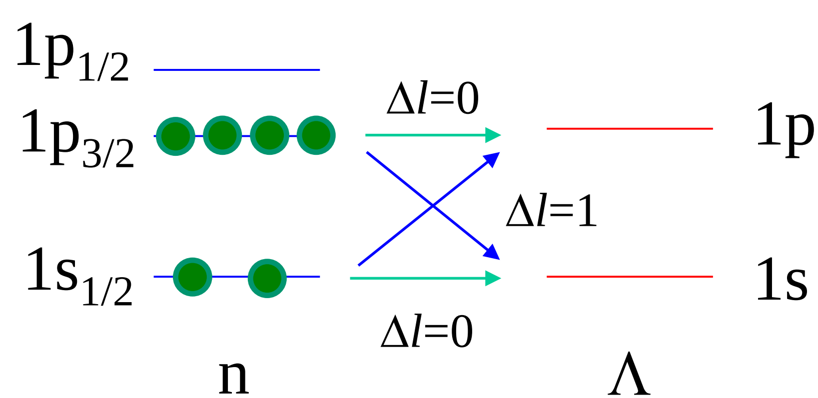
$m_\pi + m_\Lambda$ = 1255.3 MeV

Q > 0

$m_\pi + m_n$ = 1079.2 MeV

$m_K + m_\Lambda$ = 1609.4 MeV

Q < 0



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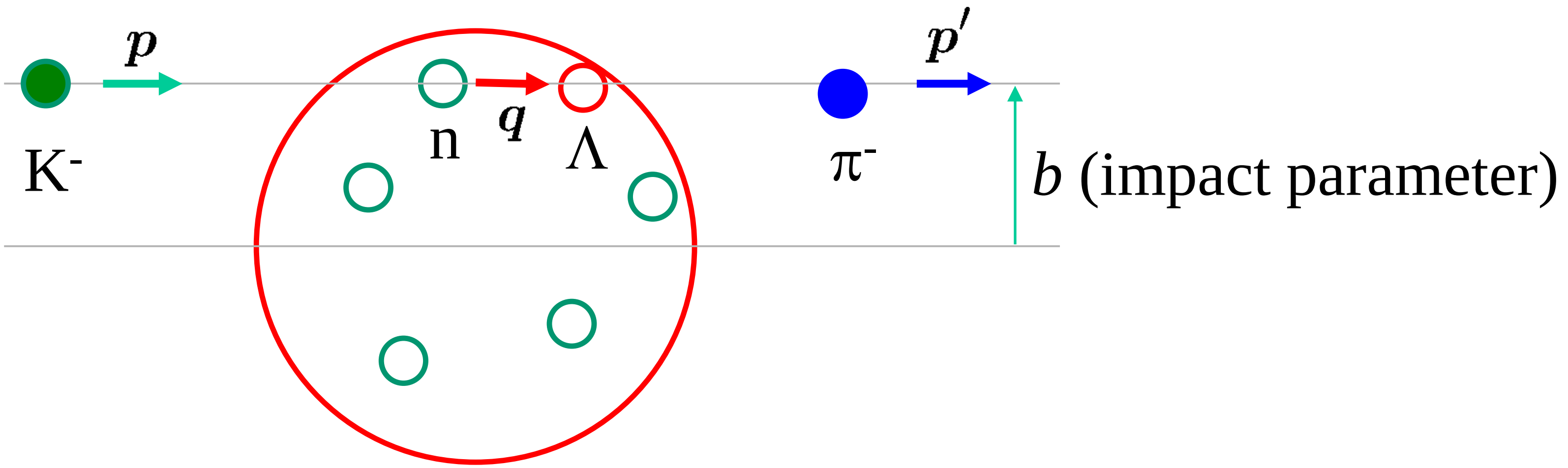
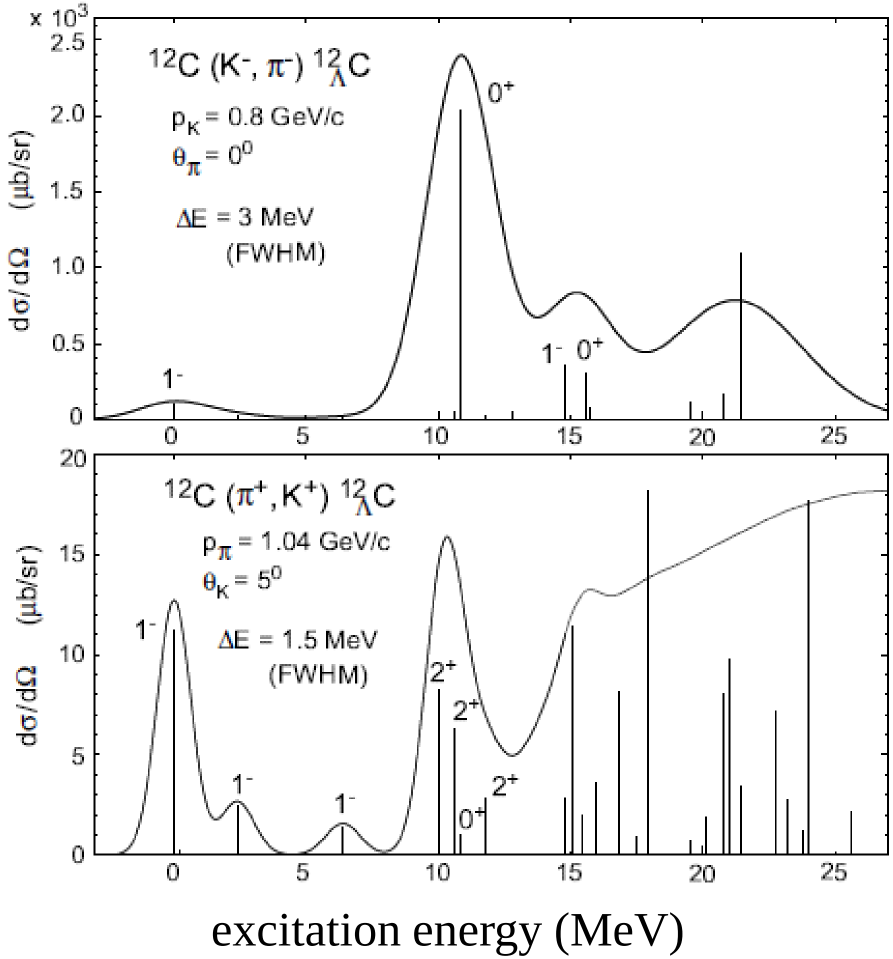
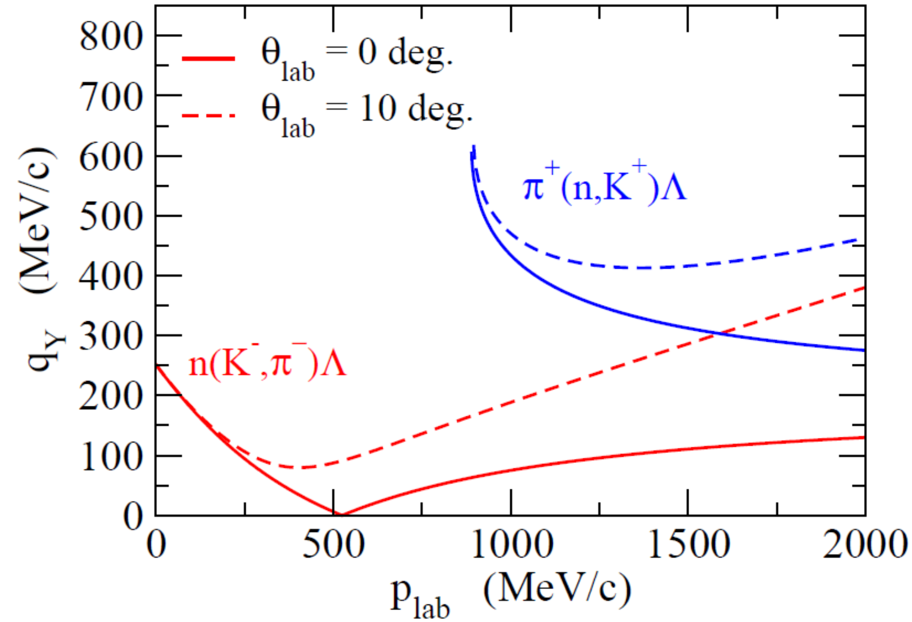
relation between q and Δl

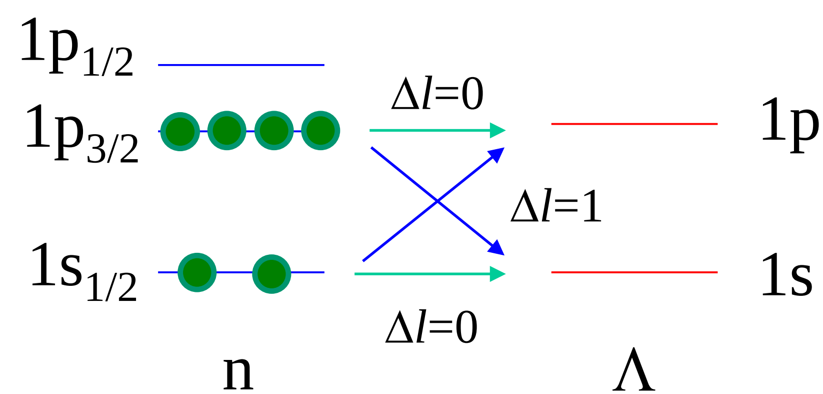


$l \sim kb$ (classically)

$\Longrightarrow$ $\Delta l \sim b(p'-p) = bq$







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$$\Delta l \sim b(p' - p) = bq$$