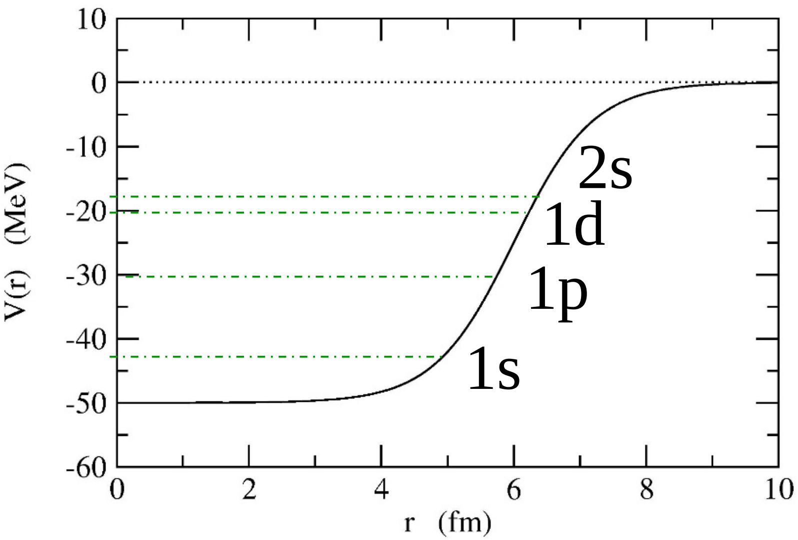
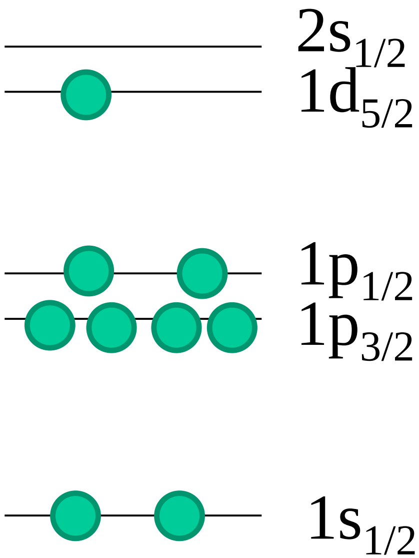
# 殻模型

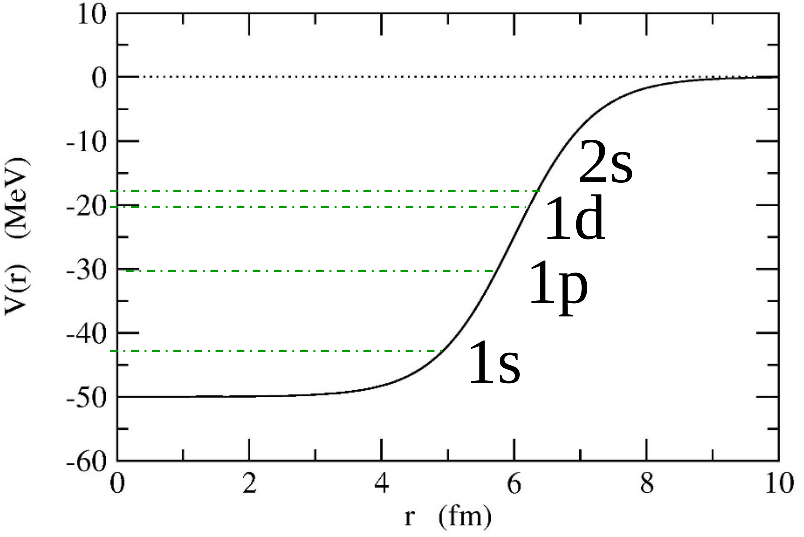
核子の感じるポテンシャル　→　核子の入る準位
→　その準位に核子をつめていく

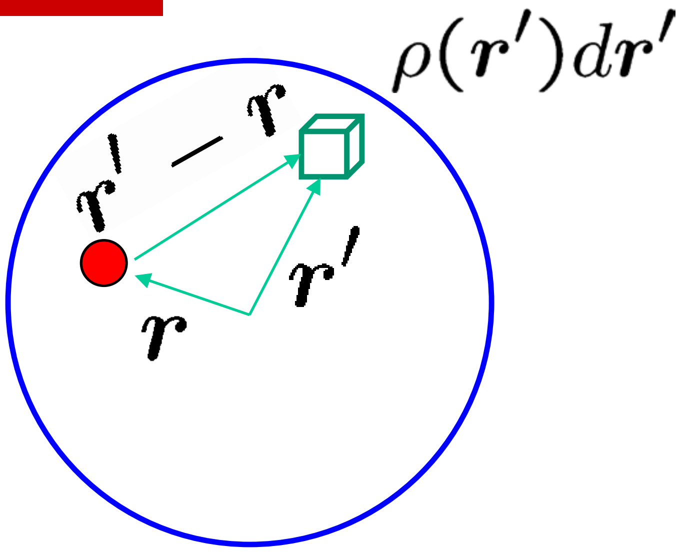


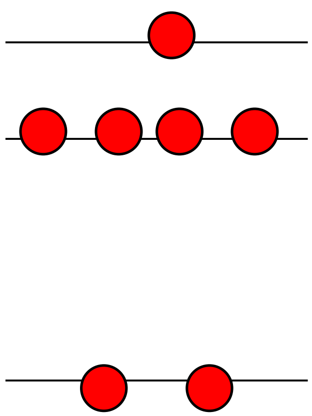




＊核子の感じるポテンシャルは球形でなくてもよい
（変形しててもよい）
cf. $^{11}$Be のレベルの説明

# Mean-field (Hartree-Fock) Theory



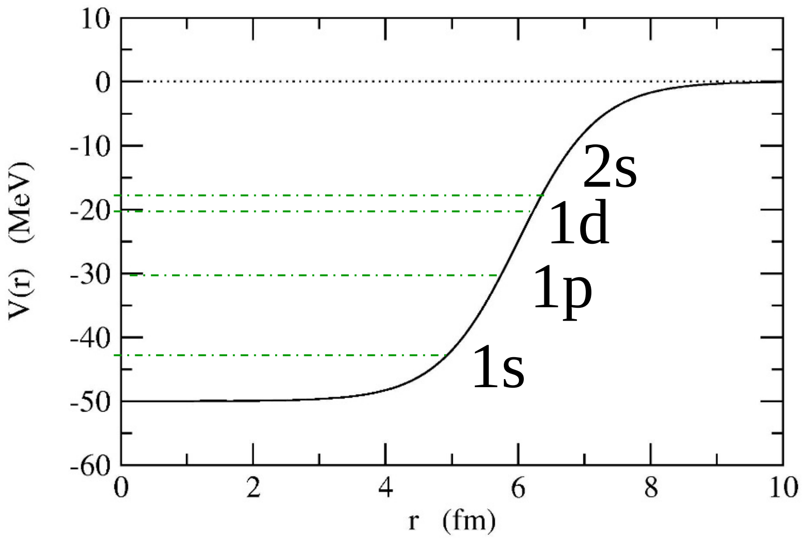



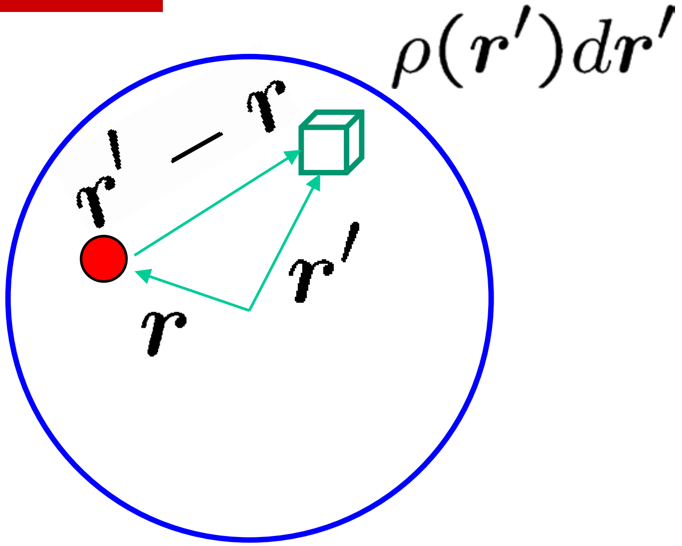

naively speaking,

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

shell model

# Mean-field (Hartree-Fock) Theory

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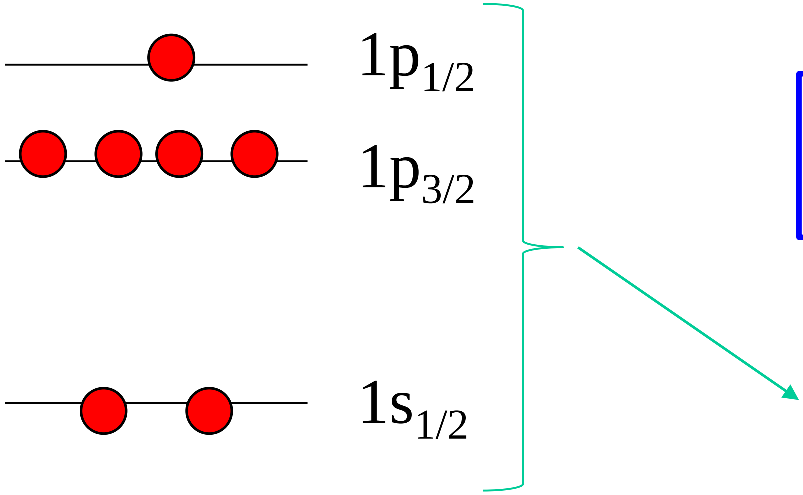
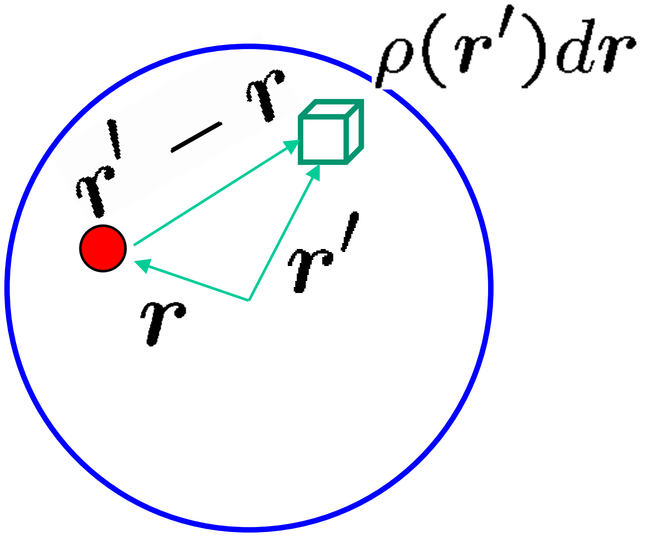



shell model

$$\rho(\boldsymbol{r}) = \sum_i |\psi_i(\boldsymbol{r})|^2$$

# Mean-field (Hartree-Fock) Theory

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$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}, \boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

$$\rho(\boldsymbol{r}) = \sum_i |\psi_i(\boldsymbol{r})|^2$$

$$\begin{aligned} 0 &= \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - \epsilon_i\right]\psi_i(\boldsymbol{r}) \\ &= \left[-\frac{\hbar^2}{2m}\nabla^2 + \int v(\boldsymbol{r}, \boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r}) \end{aligned}$$

the potential depends on the solutions

# Mean-field (Hartree-Fock) Theory

$$
\begin{aligned}
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&= \left[-\frac{\hbar^2}{2m}\nabla^2 + \int v(\boldsymbol{r},\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right)d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r})
\end{aligned}
$$

the potential depends on the solutions

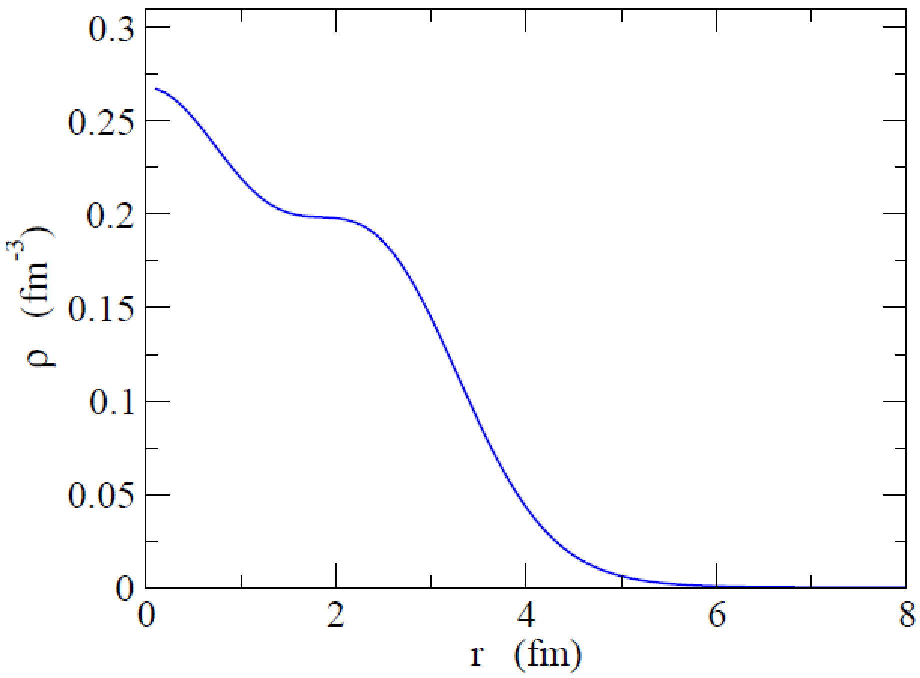
→ self-consistent solutions

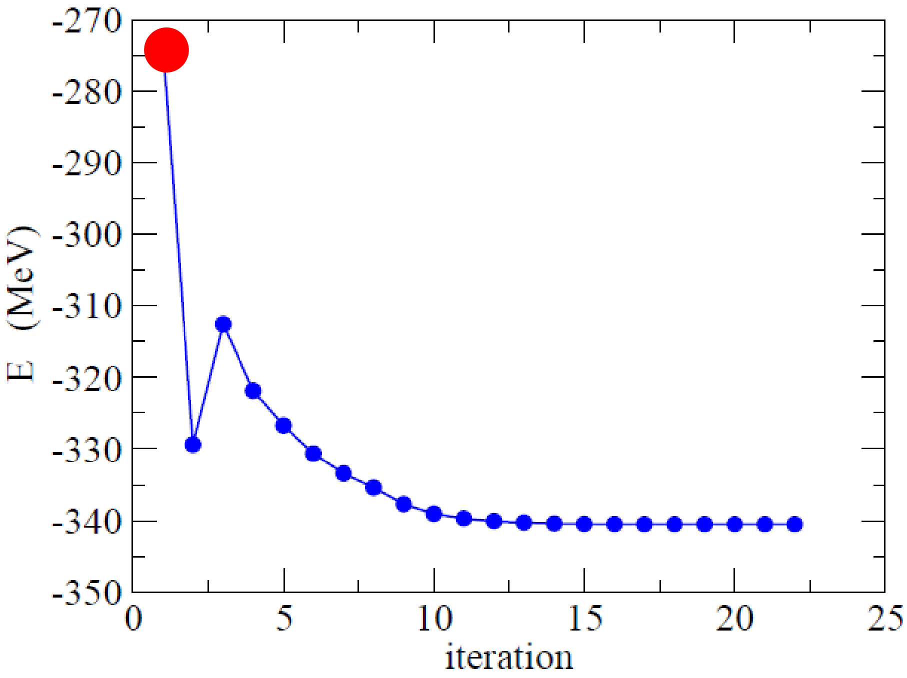
Iteration: $\{\psi_i\} \to \rho \to V \to \{\psi_i\} \to \cdots$

repeat until the first and the last wave functions are the same.

"self-consistent mean-field theory"

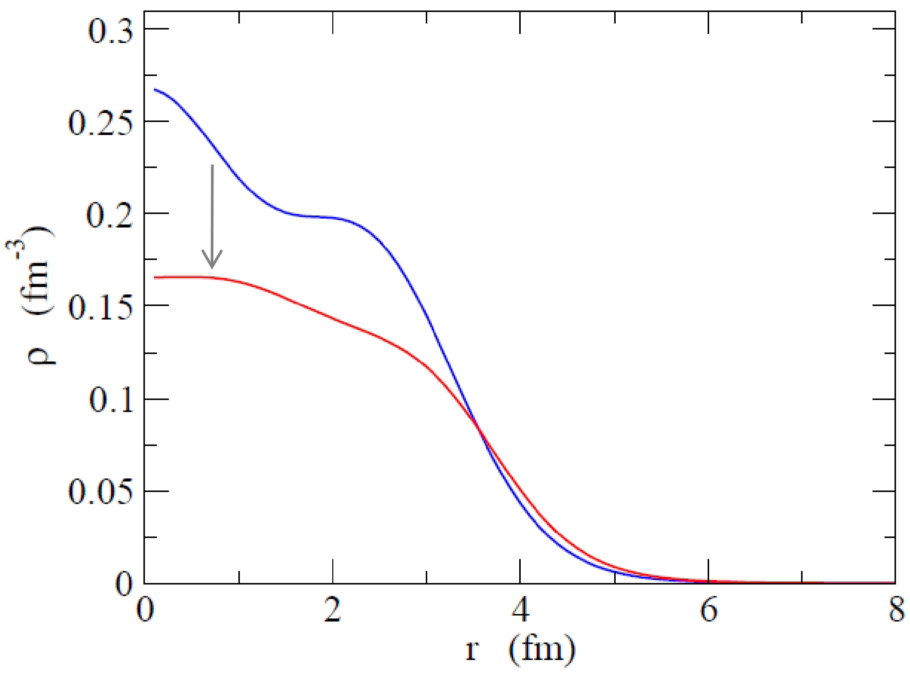
# Skyrme-Hartree-Fock calculations for $^{40}Ca$

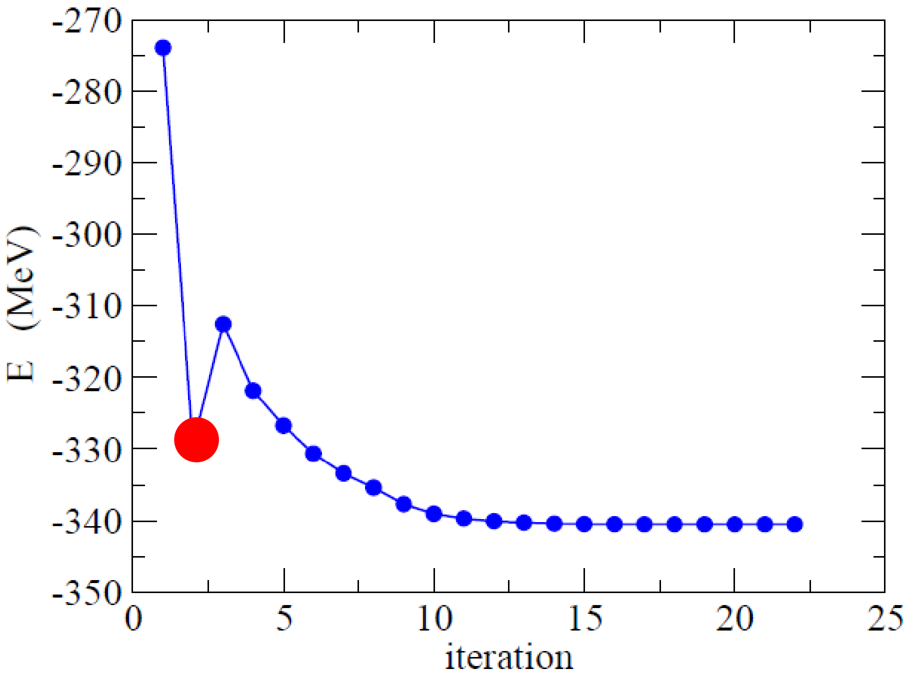



optimize the density by taking into account the nucleon-nucleon interaction

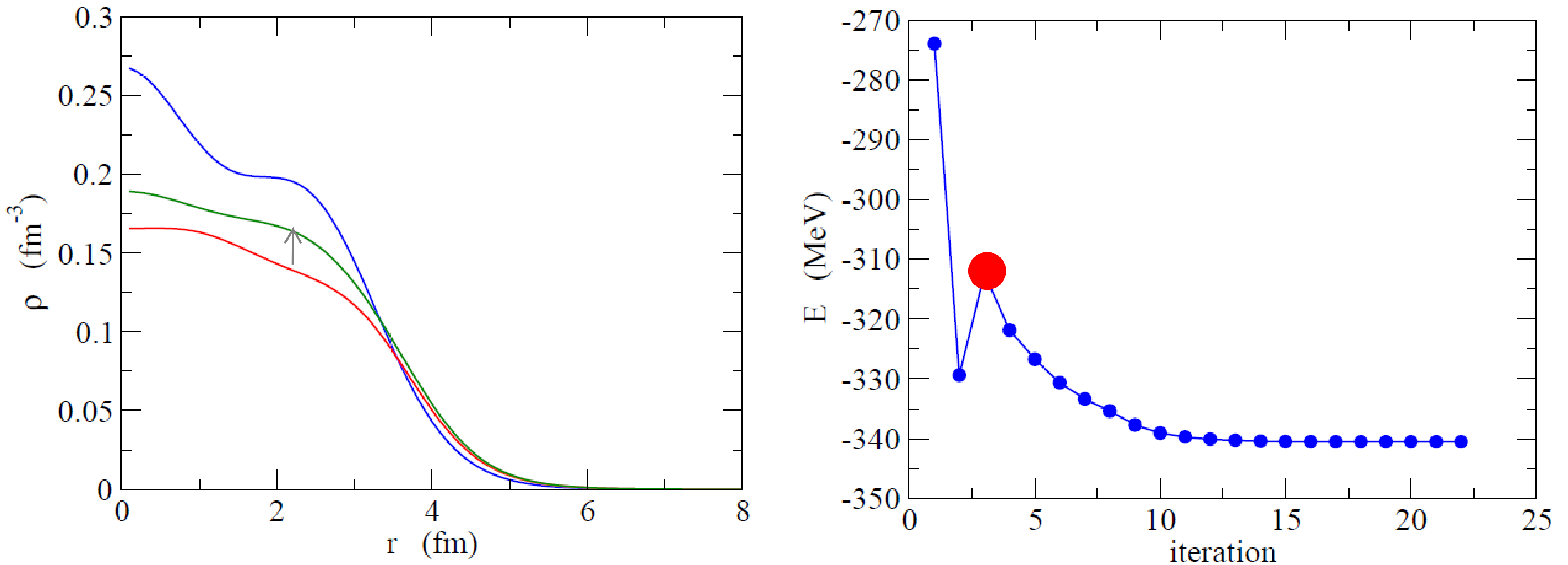
# Skyrme-Hartree-Fock calculations for $^{40}$Ca



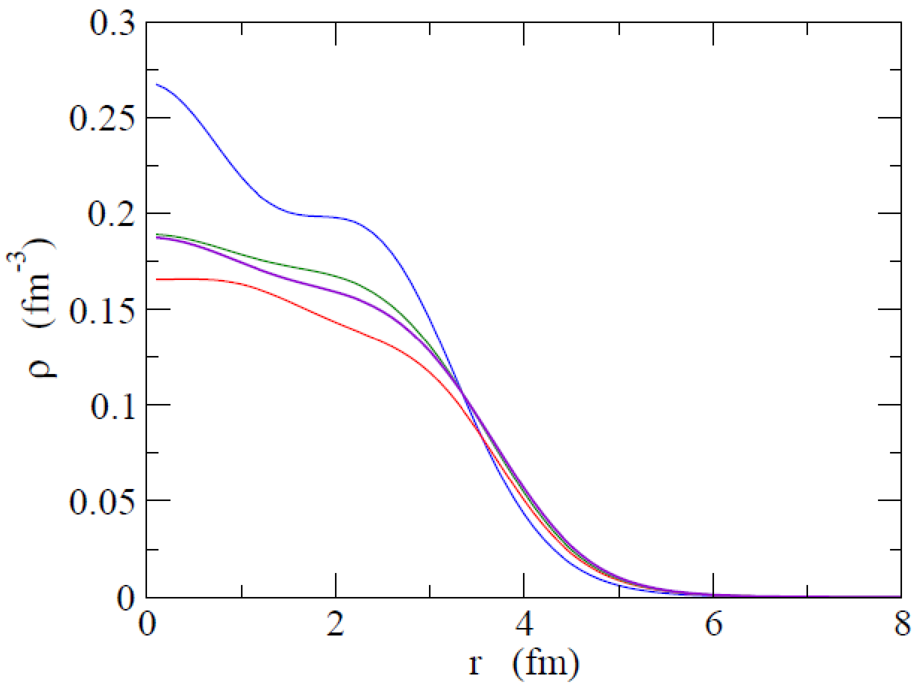

optimize the density by taking into account the nucleon-nucleon interaction

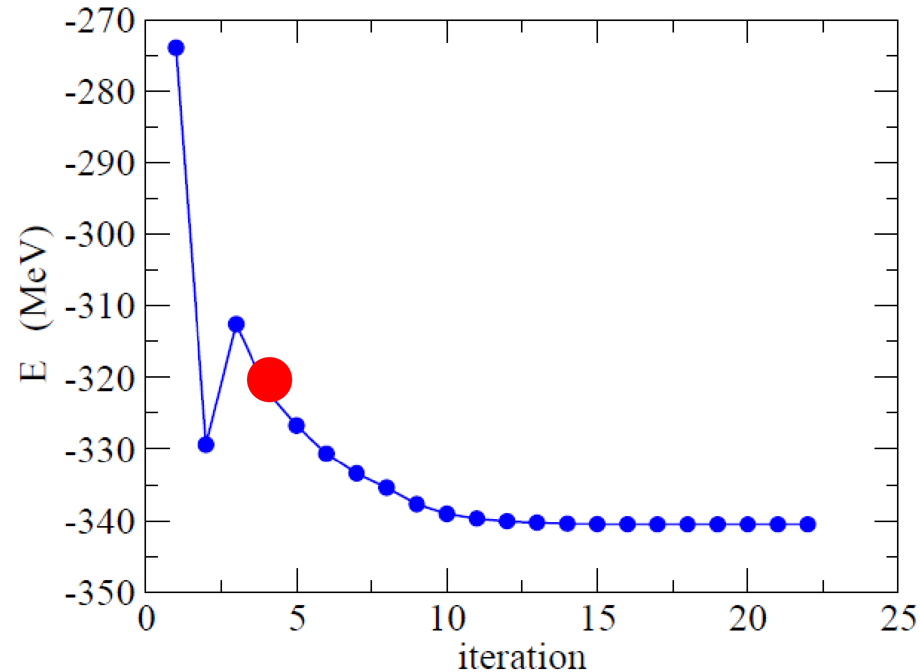
# Skyrme-Hartree-Fock calculations for $^{40}$Ca




optimize the density by taking into account the nucleon-nucleon interaction

# Skyrme-Hartree-Fock calculations for $^{40}$Ca




optimize the density by taking into account the nucleon-nucleon interaction

→ optimized density (and shape) can be determined automatically

# Variational Principle （変分原理）

optimization ⟷ variational principle

$$\frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \geq E_{\mathrm{g.s.}}$$

$H$ : many-body Hamiltonian

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots) = \psi_1(\boldsymbol{r}_1) \cdot \psi_2(\boldsymbol{r}_2) \cdot \psi_3(\boldsymbol{r}_3) \cdots$$

← many-body wave function for independent particles

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + \int v(\boldsymbol{r}, \boldsymbol{r}') \rho(\boldsymbol{r}') d\boldsymbol{r}' - \epsilon_i \right] \psi_i(\boldsymbol{r}) = 0$$

＊全エネルギーが最少になるようにちょっとずつ
一粒子ポテンシャルを変えていく

＊変形した方がエネルギーが下がるのであれば変形させる

many-body Hamiltonian: $$H = -\sum_{i=1}^{A}\frac{\hbar^2}{2m}\nabla_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i, \boldsymbol{r}_j)$$

$$\langle\Psi|H|\Psi\rangle = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\int \psi_i^*(\boldsymbol{r})\nabla^2\psi_i(\boldsymbol{r})\,d\boldsymbol{r} + \frac{1}{2}\sum_{i,j}^{A}\int \psi_i^*(\boldsymbol{r})\psi_j^*(\boldsymbol{r}')v(\boldsymbol{r},\boldsymbol{r}')\psi_i(\boldsymbol{r})\psi_j(\boldsymbol{r}')\,d\boldsymbol{r}d\boldsymbol{r}'$$

Variation with respect to $\psi_i^*$

$\{\psi_i^*\} \to \{\psi_i^* + \delta\psi_i^*\}$ としてもエネルギーが変わらない

（ただし波動関数の規格化が変わらないための拘束をつける）

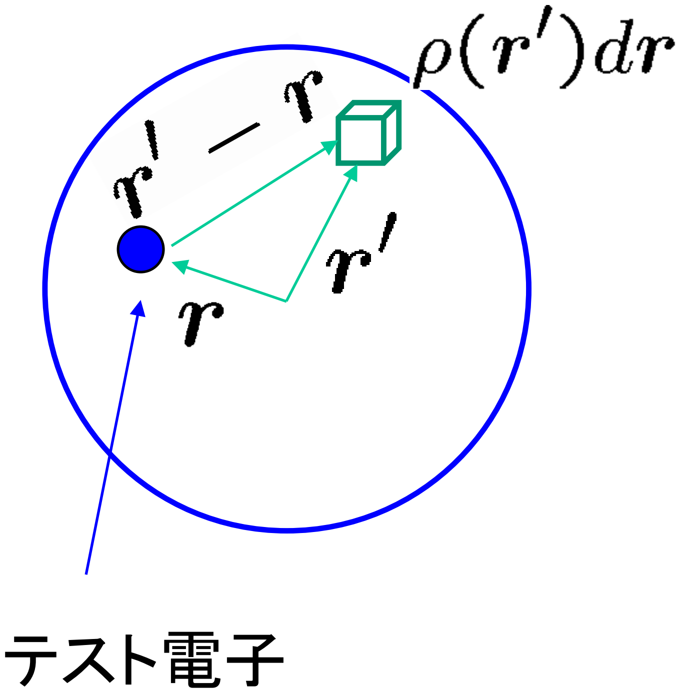
$$\delta E = \int d\boldsymbol{r}\,\delta\psi_i^*(\boldsymbol{r})\left[-\frac{\hbar^2}{2m}\nabla^2\psi_i(\boldsymbol{r}) + \sum_j\int \psi_j^*(\boldsymbol{r}')v(\boldsymbol{r},\boldsymbol{r}')\psi_j(\boldsymbol{r}')\psi_i(\boldsymbol{r})\,d\boldsymbol{r}' - \epsilon\psi_i(\boldsymbol{r})\right] = 0$$

Hartree equation:

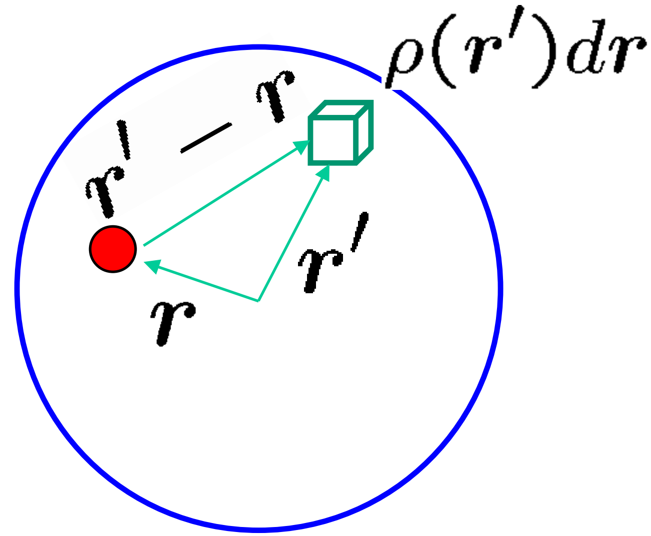
$$-\frac{\hbar^2}{2m}\nabla^2\psi_i(\boldsymbol{r}) + \sum_j\int \psi_j^*(\boldsymbol{r}')v(\boldsymbol{r},\boldsymbol{r}')\psi_j(\boldsymbol{r}')\psi_i(\boldsymbol{r})\,d\boldsymbol{r}' = \epsilon_i\psi_i(\boldsymbol{r})$$

# Mean-field (Hartree-Fock) Theory

電磁気の場合




原子核の場合




同種粒子間の相互作用
→反対称化が必要

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}'$$

anti-symmetrization

nucleon: fermion

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \cdots) = -\Psi(\boldsymbol{r}_2, \boldsymbol{r}_1, \boldsymbol{r}_3 \cdots)$$

$$\psi_1(\boldsymbol{r}_1)\psi_2(\boldsymbol{r}_2) \rightarrow \frac{1}{\sqrt{2}}[\psi_1(\boldsymbol{r}_1)\psi_2(\boldsymbol{r}_2) - \psi_2(\boldsymbol{r}_1)\psi_1(\boldsymbol{r}_2)]$$

Slater determinat

$$0 = \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \int v(\boldsymbol{r}, \boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r})$$

$$\rightarrow \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \int v(\boldsymbol{r}, \boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right) d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r})$$

$$-\int v(\boldsymbol{r}, \boldsymbol{r}')\left(\sum_j \psi_j^*(\boldsymbol{r}')\psi_i(\boldsymbol{r}')\right) d\boldsymbol{r}'\psi_j(\boldsymbol{r})$$

exchange term

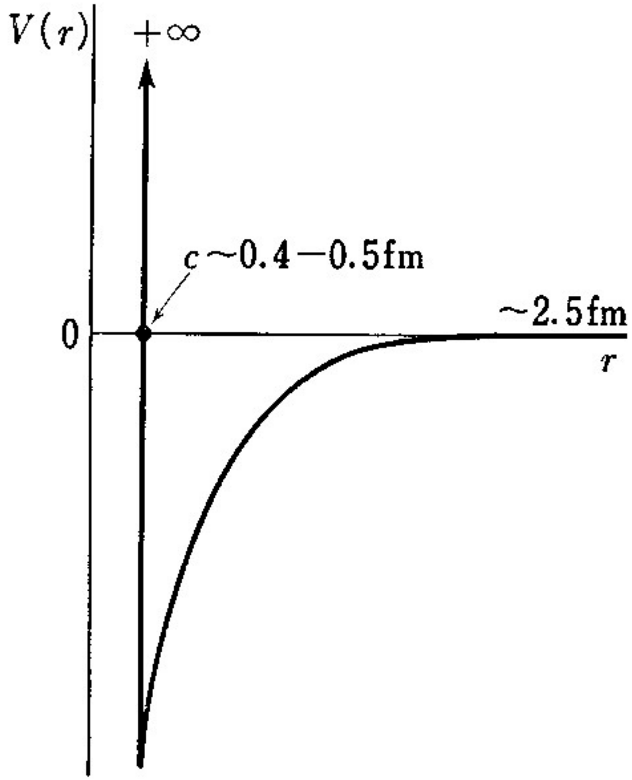
Hartree-Fock theory

# anti-symmetrization

$$
\begin{aligned}
0 &= \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \int v(\boldsymbol{r},\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right)d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r}) \\
&\to \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \int v(\boldsymbol{r},\boldsymbol{r}')\left(\sum_j |\psi_j(\boldsymbol{r}')|^2\right)d\boldsymbol{r}' - \epsilon_i\right]\psi_i(\boldsymbol{r}) \\
&\quad - \int v(\boldsymbol{r},\boldsymbol{r}')\left(\sum_j \psi_j^*(\boldsymbol{r}')\psi_i(\boldsymbol{r}')\right)d\boldsymbol{r}'\psi_j(\boldsymbol{r}) \\
&= \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V(\boldsymbol{r}) - \epsilon_i\right]\psi_i(\boldsymbol{r}) + \int d\boldsymbol{r}' V_{\mathrm{NL}}(\boldsymbol{r},\boldsymbol{r}')\psi_i(\boldsymbol{r}')
\end{aligned}
$$

non-local potential

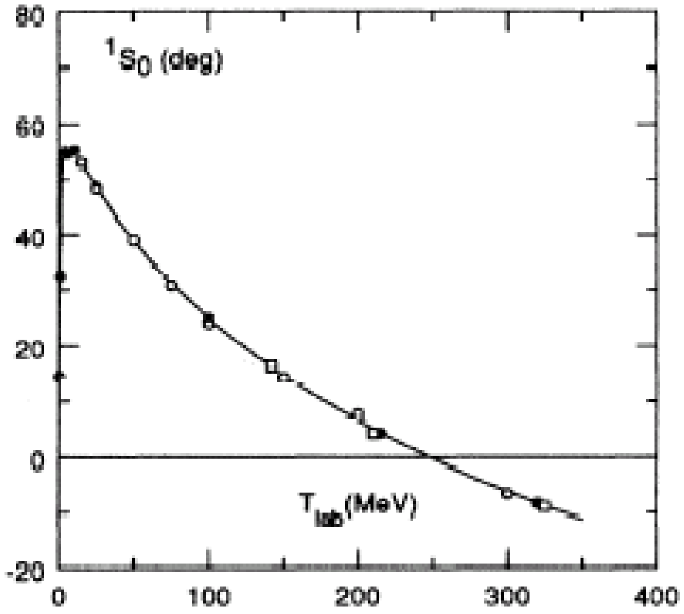
# Bare nucleon-nucleon interaction

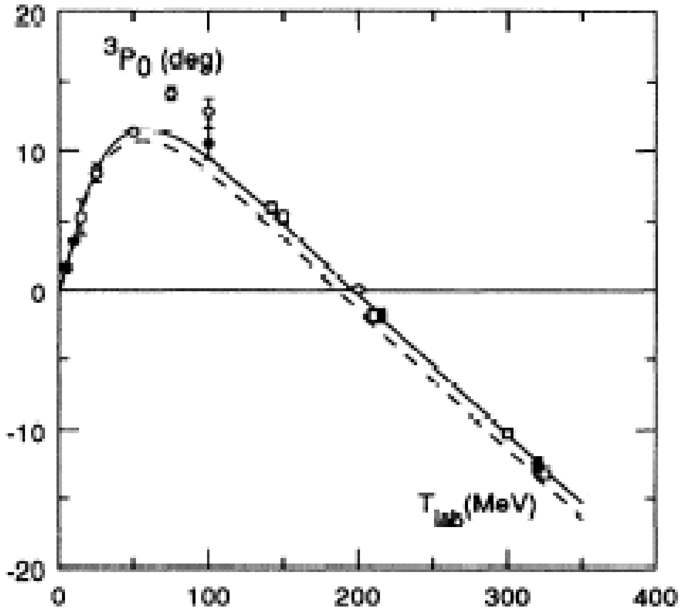



Existence of short range
repulsive core

# Bare nucleon-nucleon interaction

Phase shift for p-p scattering




(V.G.J. Stoks et al., PRC48('93)792)

# Phase shift:

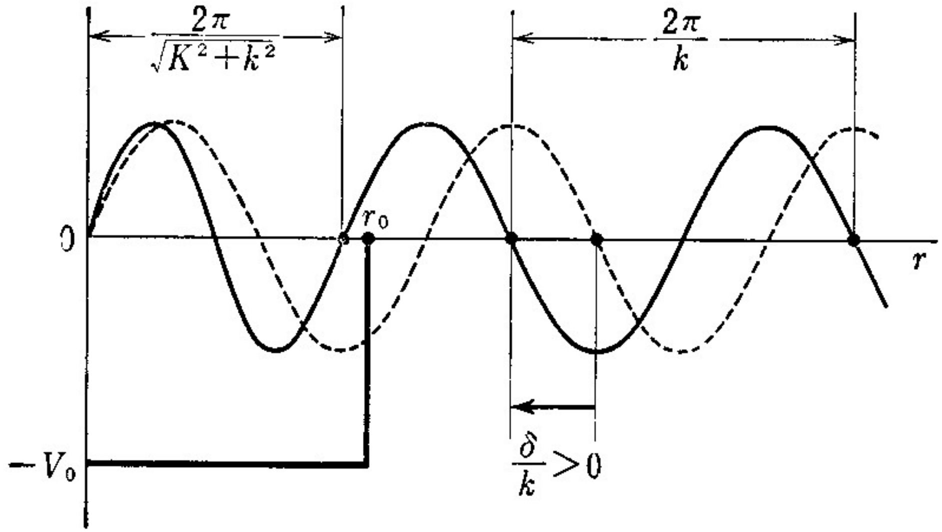
Radial wave function

$$\Psi_l(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}})$$

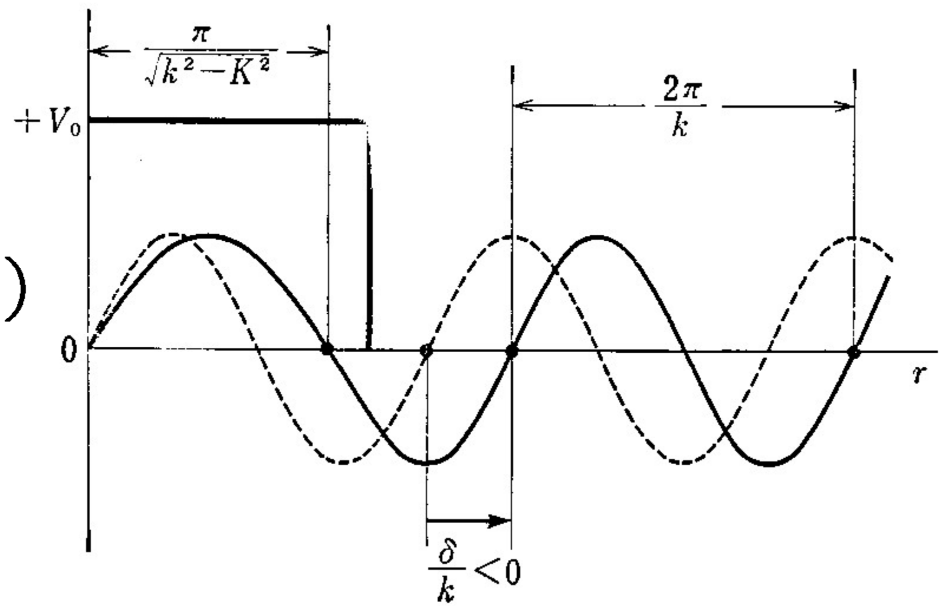
$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + V(r) + \frac{l(l+1)\hbar^2}{2mr^2} - E\right] u_l(r) = 0$$

Asymptotic form:

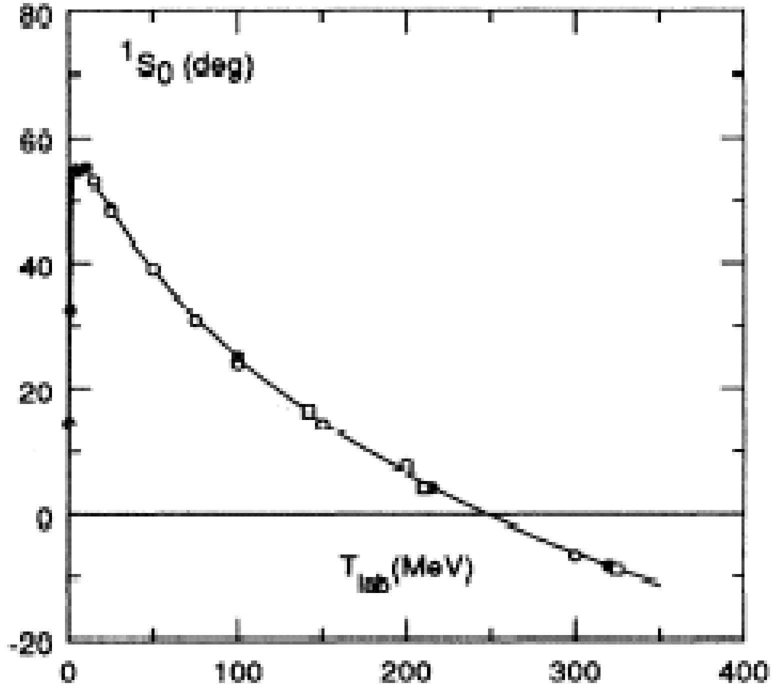
$$u_l(r) \rightarrow \sin(kr - l\pi/2 + \delta_l) \quad (r \rightarrow \infty)$$

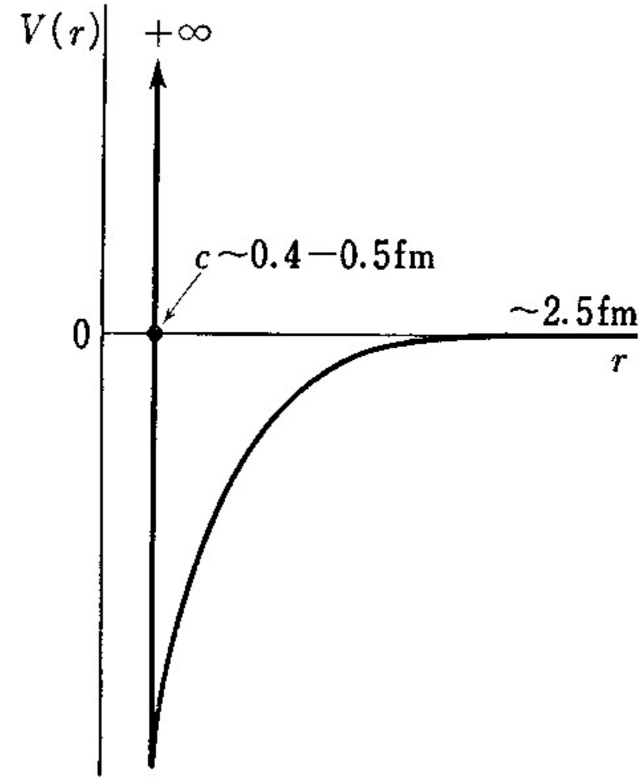


（a） 引 力



（b） 斥 力




Phase shift: positive → negative at high energies

Existence of short range repulsive core

# Bruckner's G-matrix

Nucleon-nucleon interaction *in medium*

Nucleon-nucleon interaction with a hard core

⇨ HF method: does not work

← Matrix elements: diverge

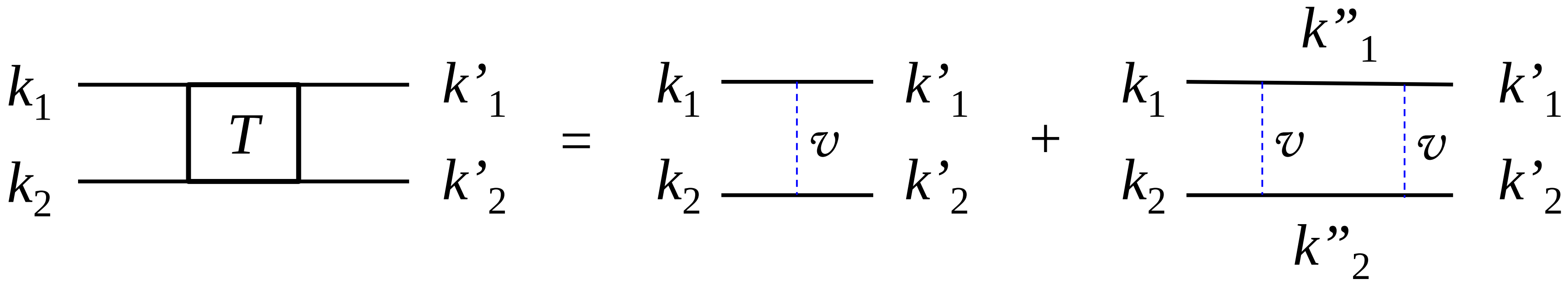
…..but the HF picture seems to work in nuclear systems

Solution: a nucleon-nucleon interaction *in medium* (effective interaction) rather than a bare interaction

⇧

Bruckner's G-matrix

## ➢ two-body (multiple) scattering *in vacuum*




+.......

Lippmann-Schwinger equation

$$T = v + v\frac{1}{E - H_0}T$$

(note) Lippmann-Schwinger equation

$$\left[-\frac{\hbar}{2m}\nabla^2 + V - E\right]\Psi = 0 \quad \text{or} \quad \left[-\frac{\hbar}{2m}\nabla^2 - E\right]\Psi = -V\Psi$$

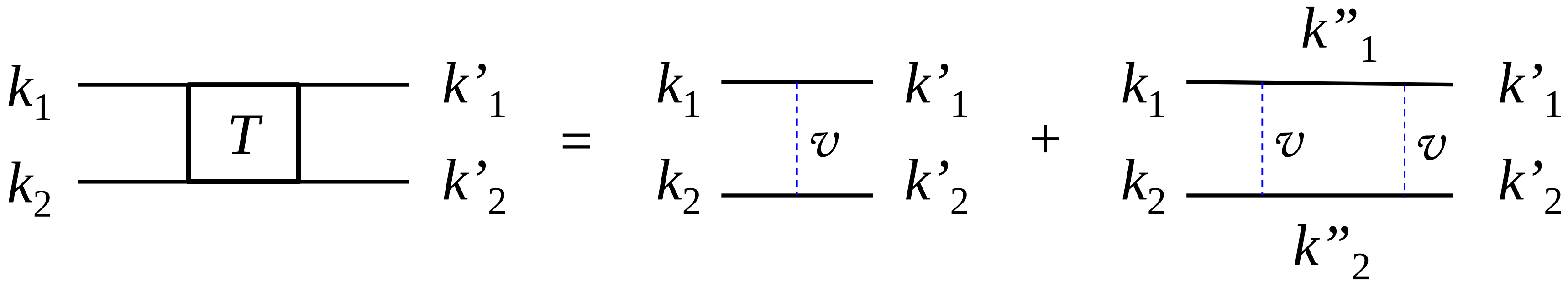
$$\Psi = \Phi - \frac{1}{H_0 - E - i\eta} V\Psi$$

define $T\Phi = V\Psi$ (T-matrix)

$$T\Phi = V\Phi - V\frac{1}{H_0 - E - i\eta} T\Phi$$

$$T = V - V\frac{1}{H_0 - E - i\eta} T$$

- two-body (multiple) scattering *in vacuum*




+.......

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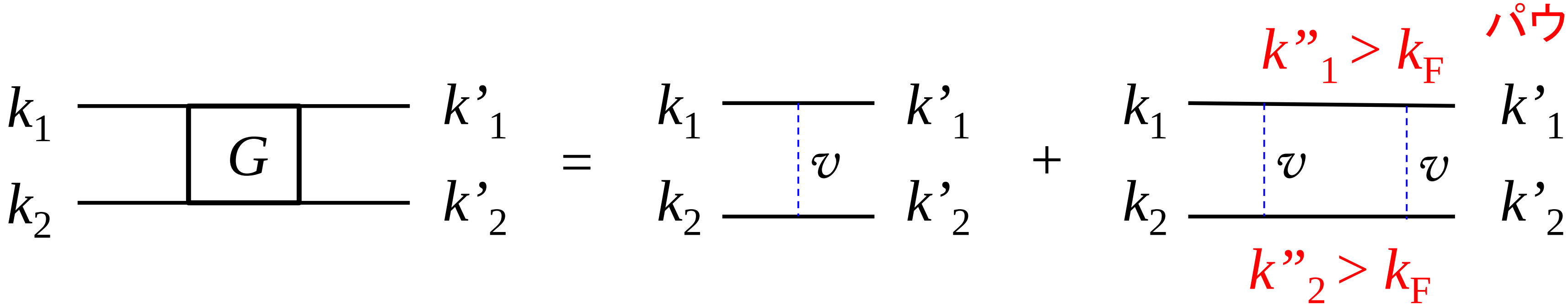
- two-body (multiple) scattering *in medium*

パウリ原理




+.......

*中間状態で $k_F$ 以上
に飛ばなければならないので、
散乱が抑制 → 独立粒子描像

Bethe-Goldstone equation

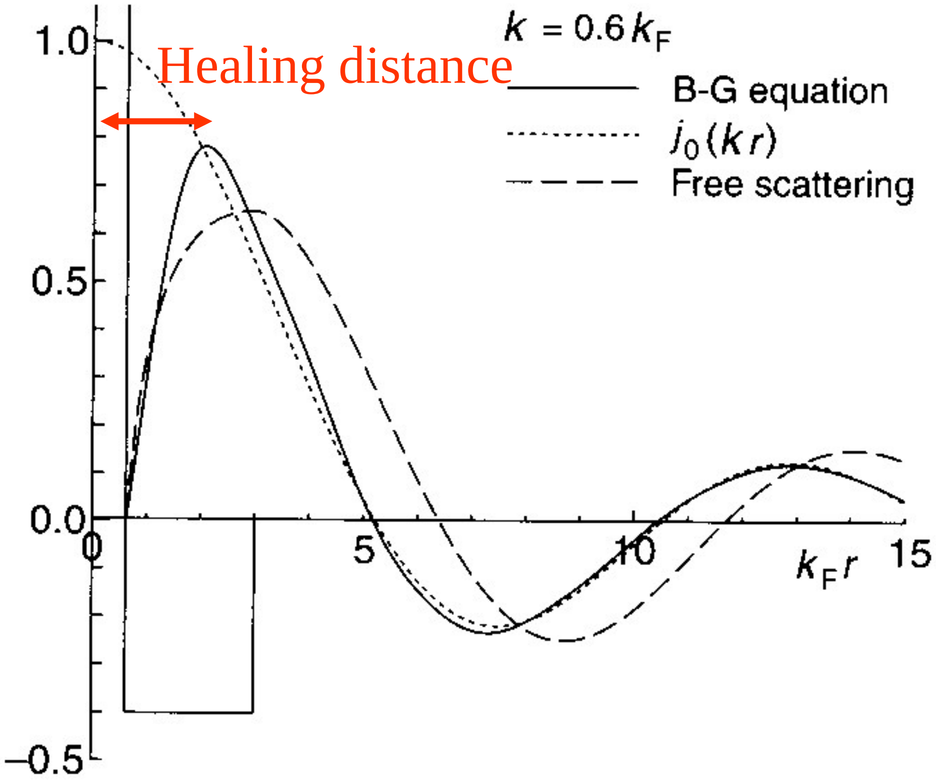
$$G = v + v\frac{Q_F}{E - H_0}G$$

◆Hard core

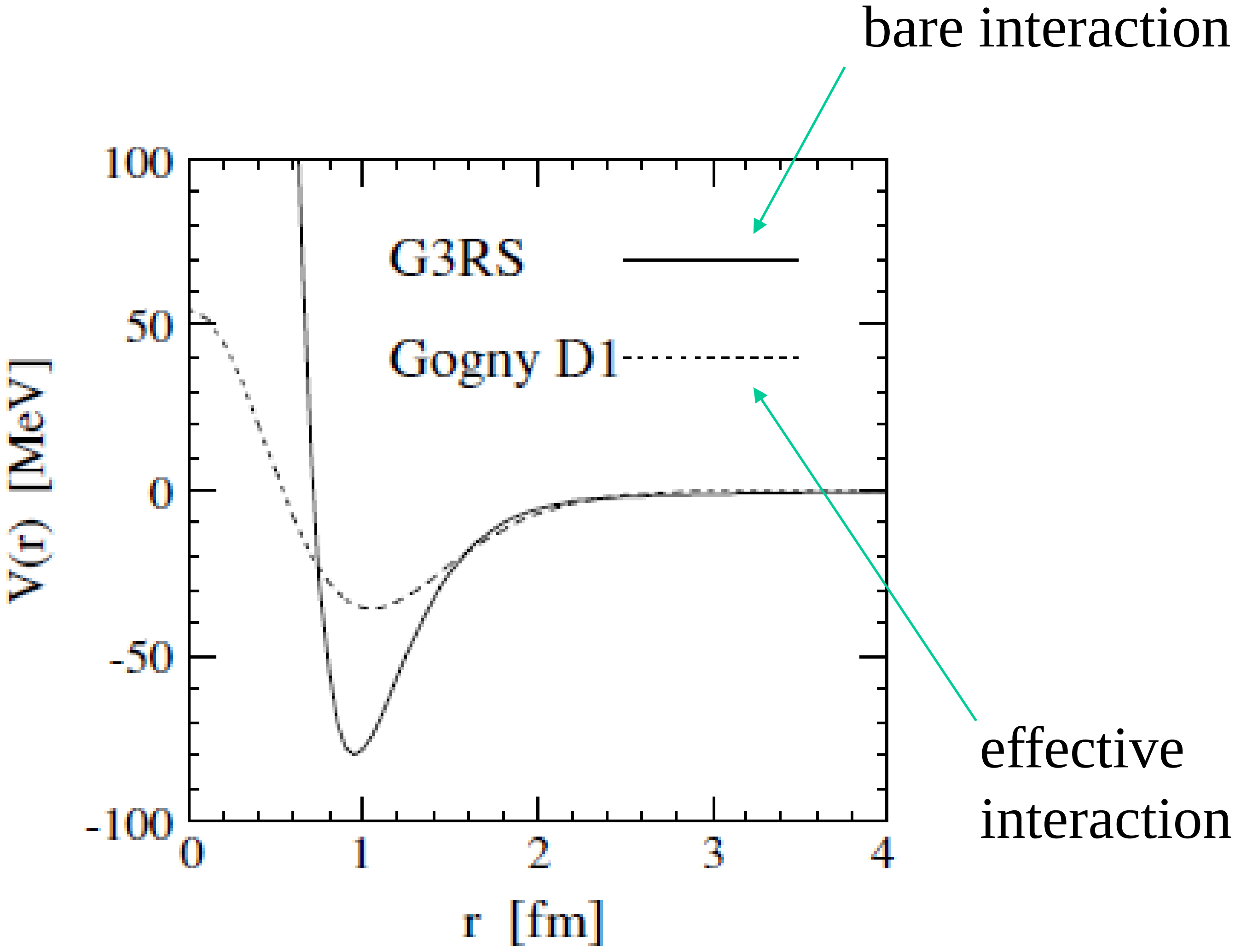
$$G = v + v\frac{Q_F}{E - H_0}G \quad \Longleftrightarrow \quad G = \frac{v}{1 - v\,\frac{Q_F}{E-H_0}}$$

Even if $v$ tends to infinity, $G$ may stay finite.

◆Independent particle motion




→ use $G$ instead of $v$ in mean-field calculations




M. Matsuo, Phys. Rev. C73(‘06)044309