Advanced Nuclear Physics

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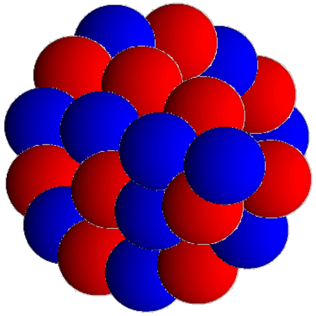
原子核理論特論

東北大学
原子核理論研究室
萩野浩一

Contents

Nuclei: aggregate of nucleons (protons and neutrons)

→ ***Nuclear Many-Body Problems***



(Low-energy) Nuclear Physics:

to understand rich nature of atomic nuclei starting from nucleon-nucleon interactions

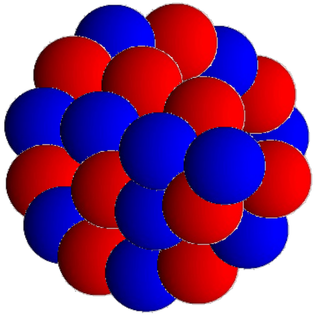
- size, mass, density, shape
- excitations
- decays
- nuclear reactions

two kinds of particle: protons and neutrons

Contents

Nuclei: aggregate of nucleons (protons and neutrons)

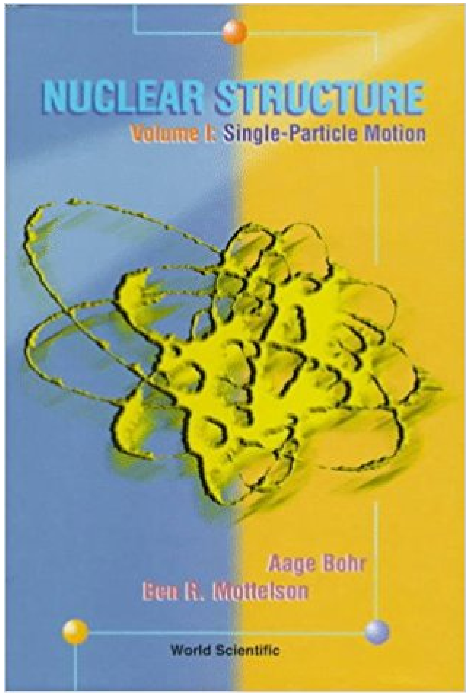
→ ***Nuclear Many-Body Problems***



microscopic descriptions of atomic nuclei

- Liquid drop model
- Single-particle motion and shell structure
- **Hartree-Fock approximation**
- Bruckner theory
- Pairing correlations and superfluid Nuclei
- Angular momentum and number projections
- 1-neutron and 2-neutron halo nuclei
- **Random phase approximation (RPA)**
- **Nuclear reactions**

References


NUCLEAR STRUCTURE

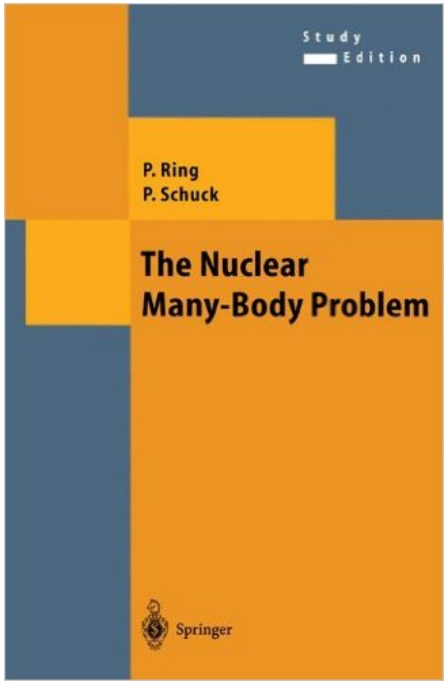
Volume I: Single-Particle Motion

Aage Bohr

Ben R. Mottelson

World Scientific

Bohr-Mottelson


Study Edition

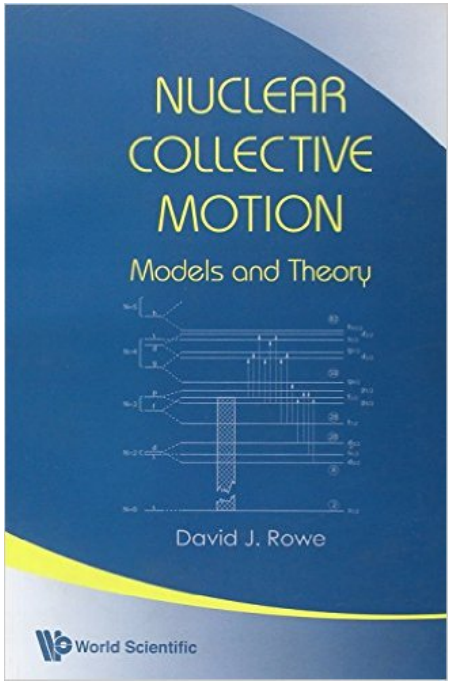
P. Ring

P. Schuck

The Nuclear Many-Body Problem

Springer

Ring-Schuck

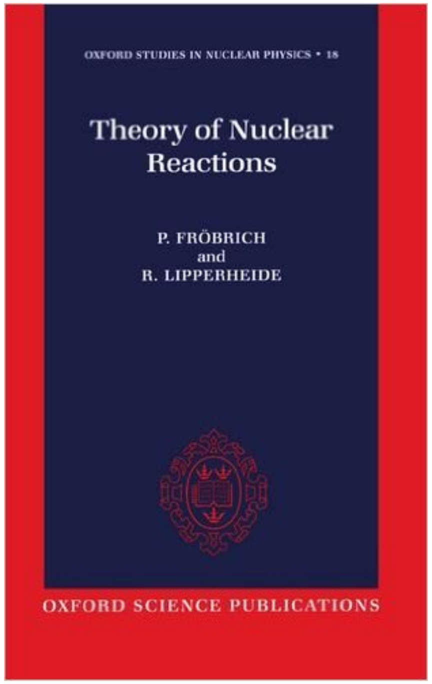

NUCLEAR COLLECTIVE MOTION

Models and Theory

David J. Rowe

World Scientific

Rowe


OXFORD STUDIES IN NUCLEAR PHYSICS • 18

Theory of Nuclear Reactions

P. FRÖBRICH
and
R. LIPPERHEIDE

OXFORD SCIENCE PUBLICATIONS

Frobrich
-Lipperheide

Lecture notes:

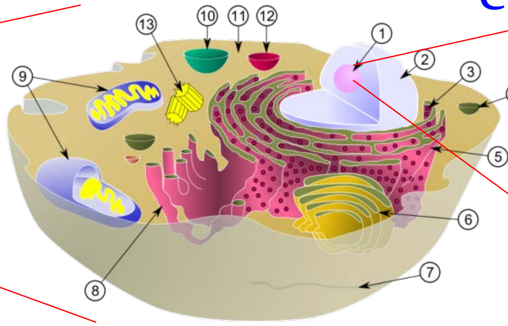
http://www.nucl.phys.tohoku.ac.jp/~hagino/lecture.html

(Tohoku University → Physics → Nuclear Theory
→ Kouichi Hagino → Lectures)

Introduction: atoms and atomic nuclei

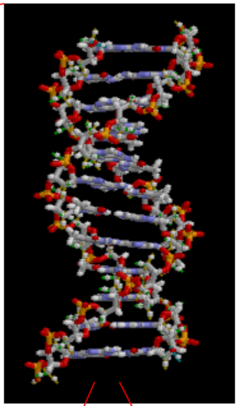
~ 50 cm

cells



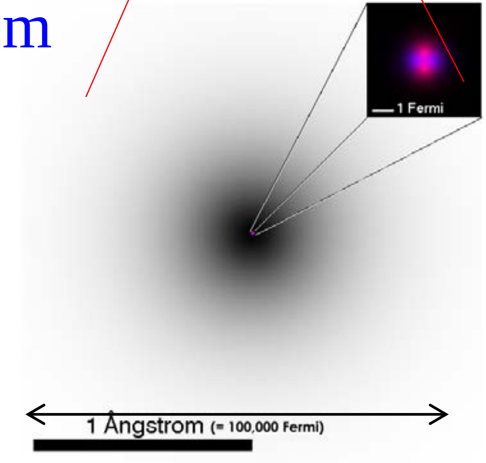
~ μm = 10^{-6} m

DNA



~ 10^{-8} m

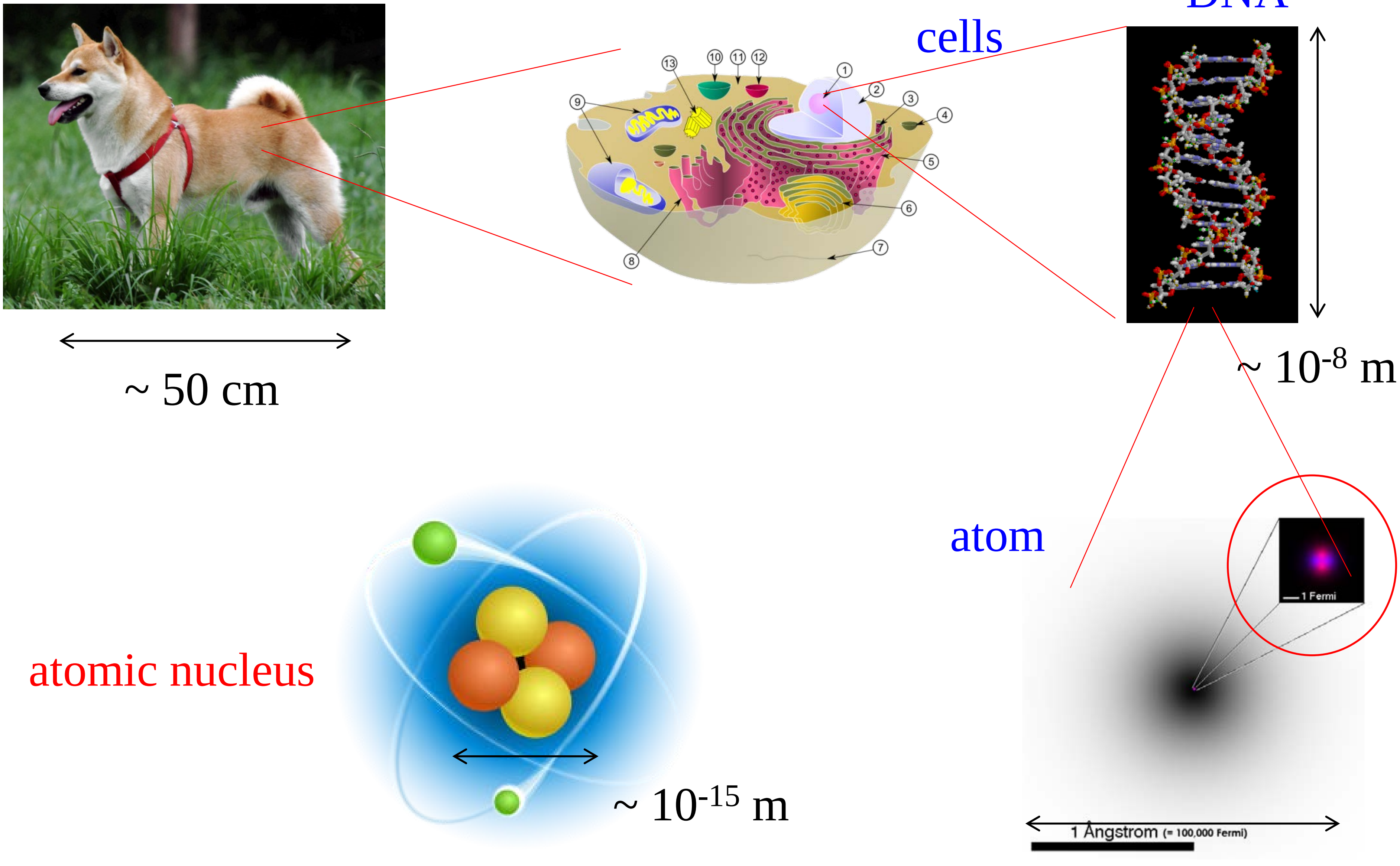
atom



~ 10^{-10} m

Everything is made of atoms.

Introduction: atoms and atomic nuclei



~ 50 cm

cells

DNA

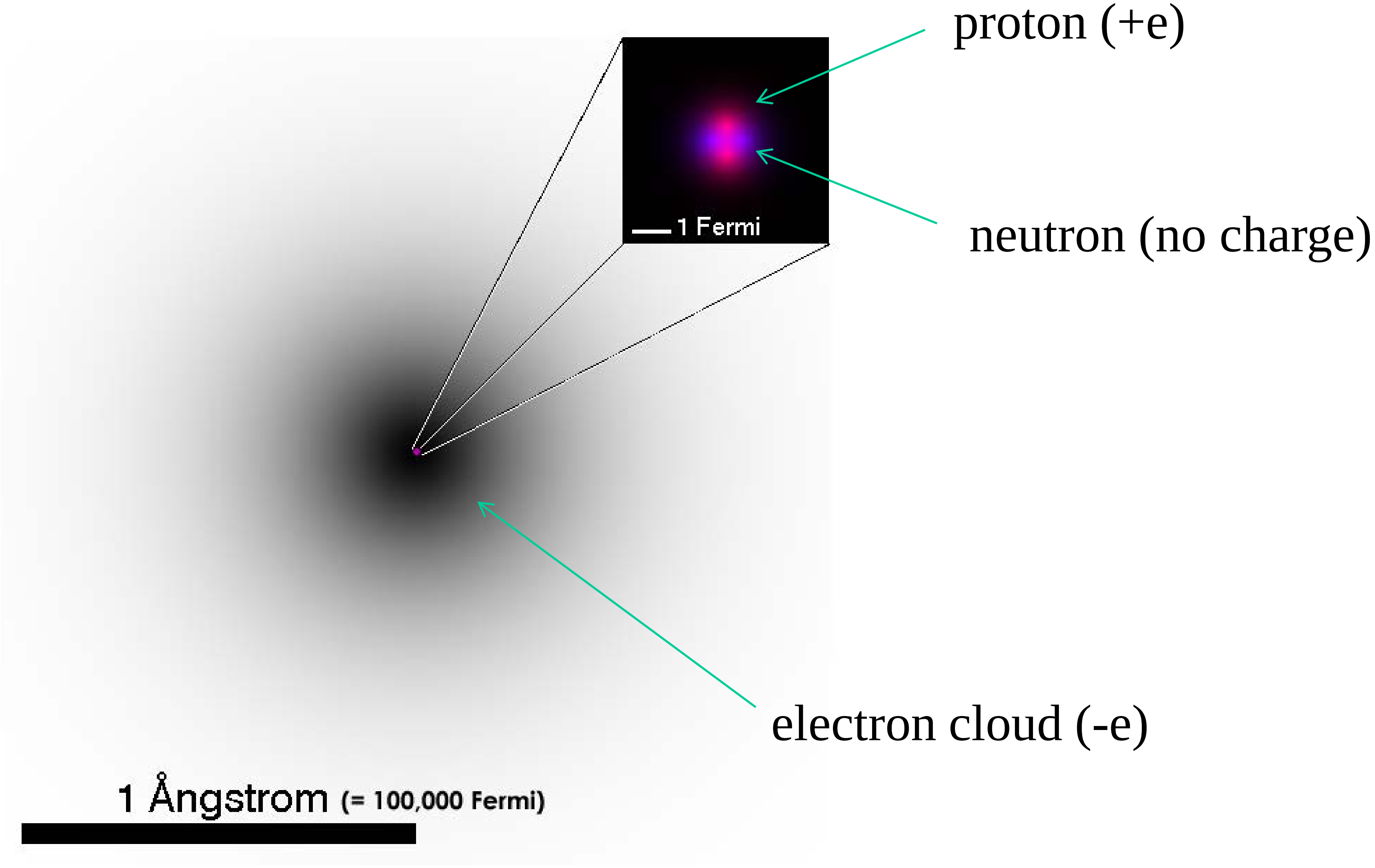
~ 10^{-8} m

atom

~ 10^{-10} m

atomic nucleus

~ 10^{-15} m



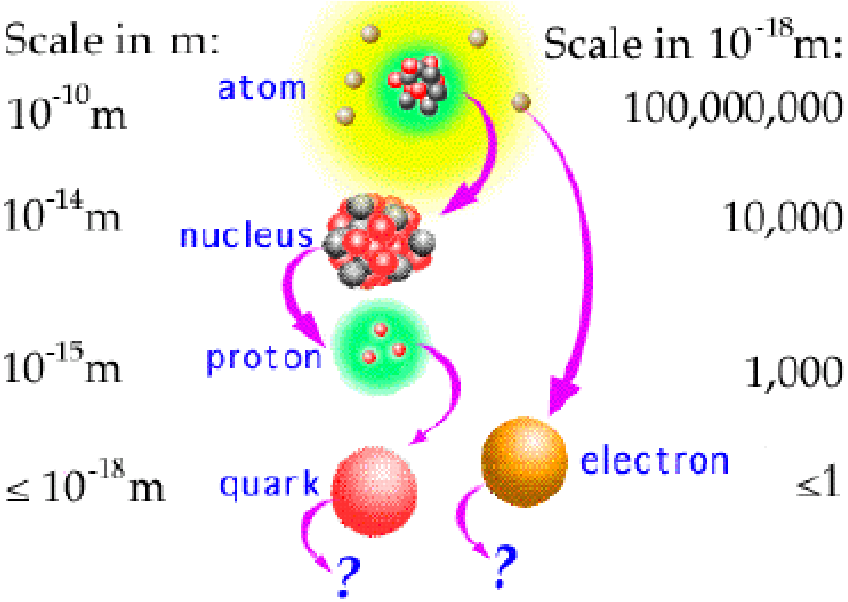
proton (+e)

neutron (no charge)

electron cloud (-e)

- Neutral atoms: # of protons = # of electrons
- Chemical properties of atoms ⟶ # of electrons
- $M_p \sim M_n \sim 2000\, M_e \longrightarrow$ the mass of atom ~ the mass of nucleus

Nuclear
Physics



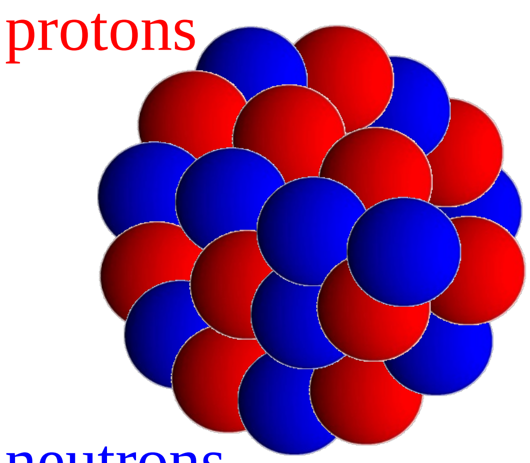
1 fm = 10^{-15} m

Nucleus as a *quantum many body system*

Basic ingredients:

	charge	mass (MeV)	spin
Proton	+e	938.256	½+
Neutron	0	939.550	½+

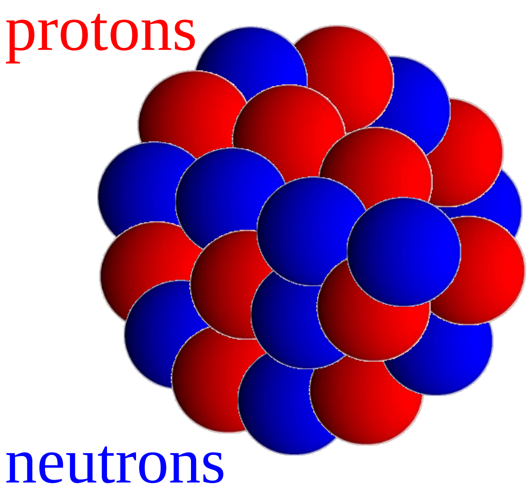
(note) $n \longrightarrow p + e^- + \bar{\nu}$ (10.4 min)



- Nucleons are not stopping inside a nucleus.
  (they move relatively freely)

- Yet, they are not completely independent.
  A nucleus keeps its shape while nucleons influence among themselves so that a nucleon does not escape.

a self-bound system



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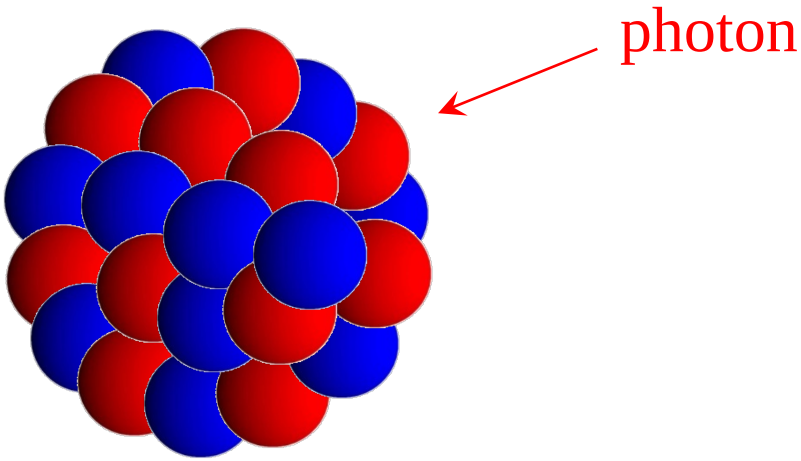
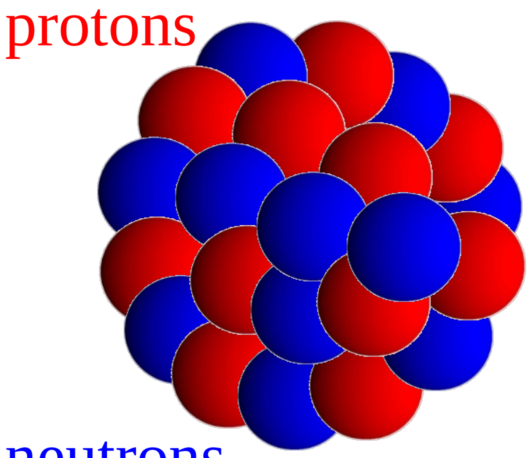
a self-bound system

What happens if a photon is absorbed into a nucleus?
- one nucleon simply starts moving faster?





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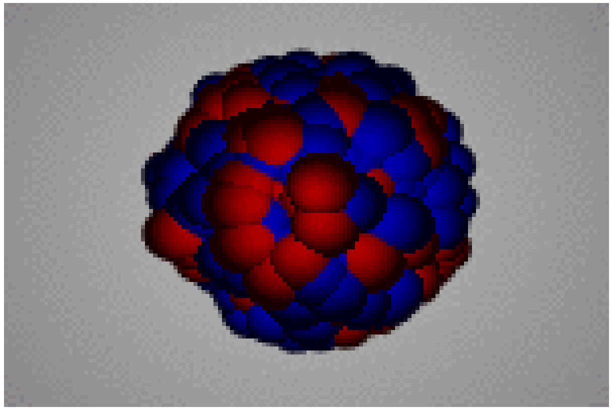
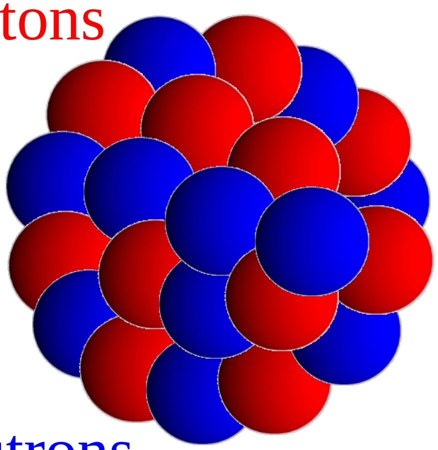


Very coherent motion can happen

Collective motions



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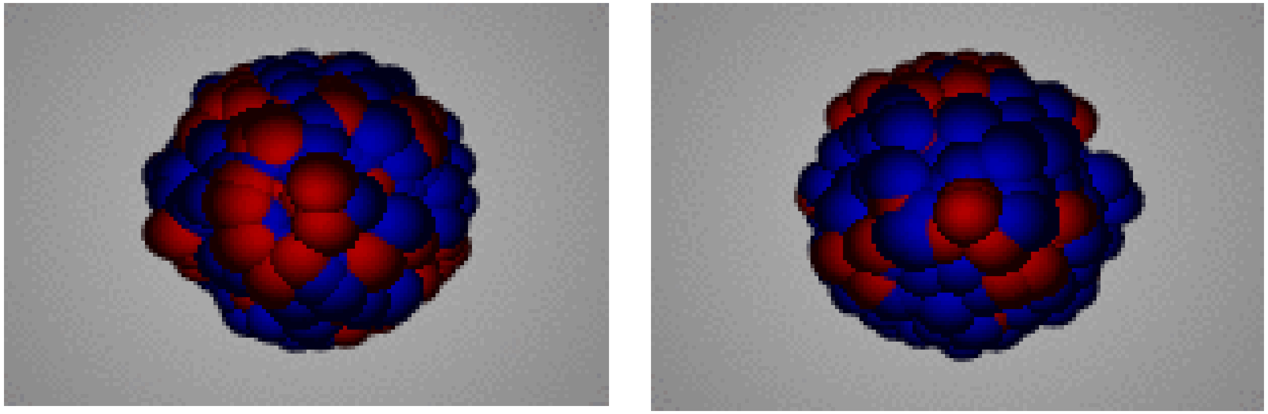

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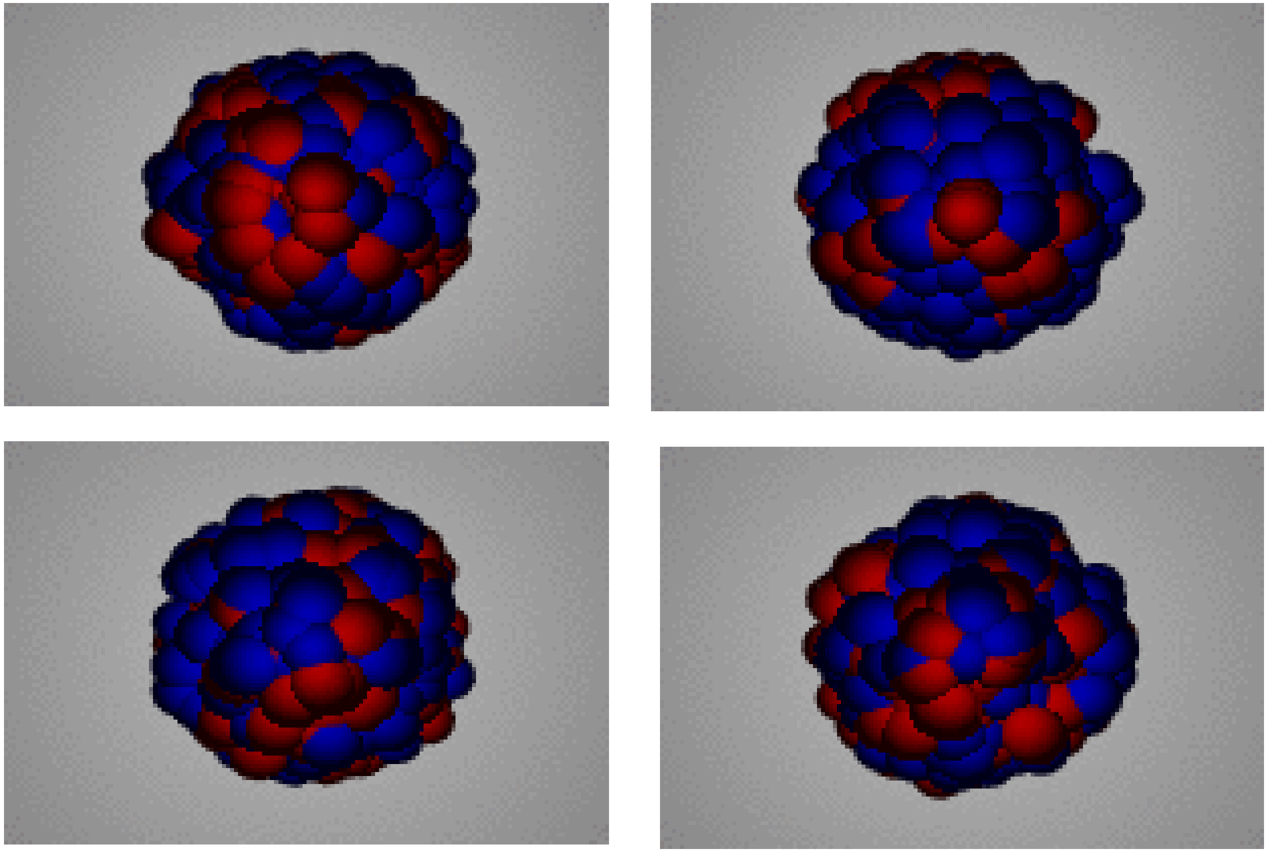
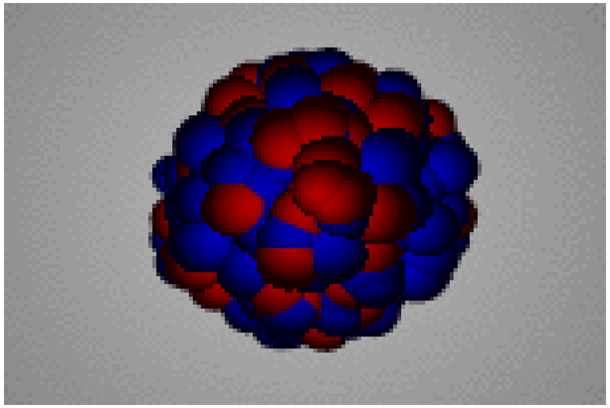
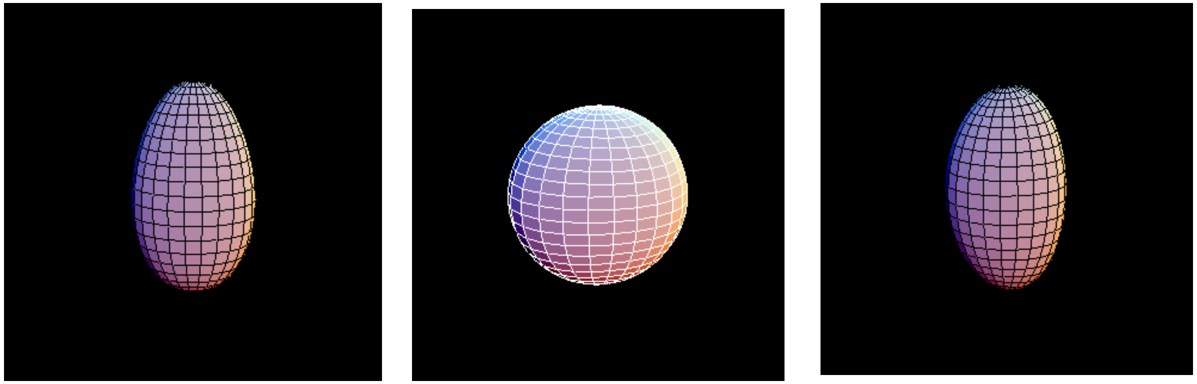


Very coherent motion can happen

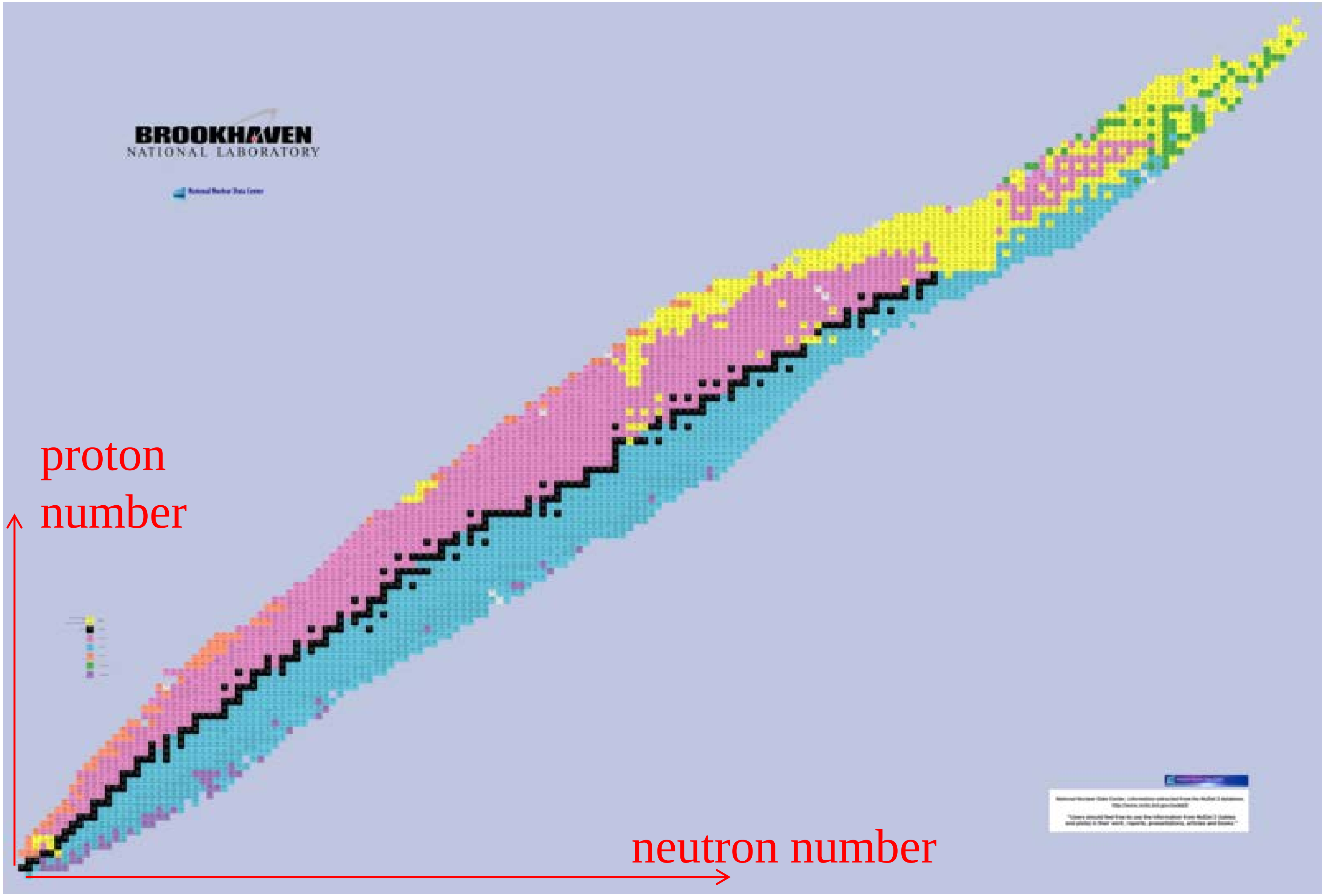
Collective motions

a variety of motions
→ very rich!

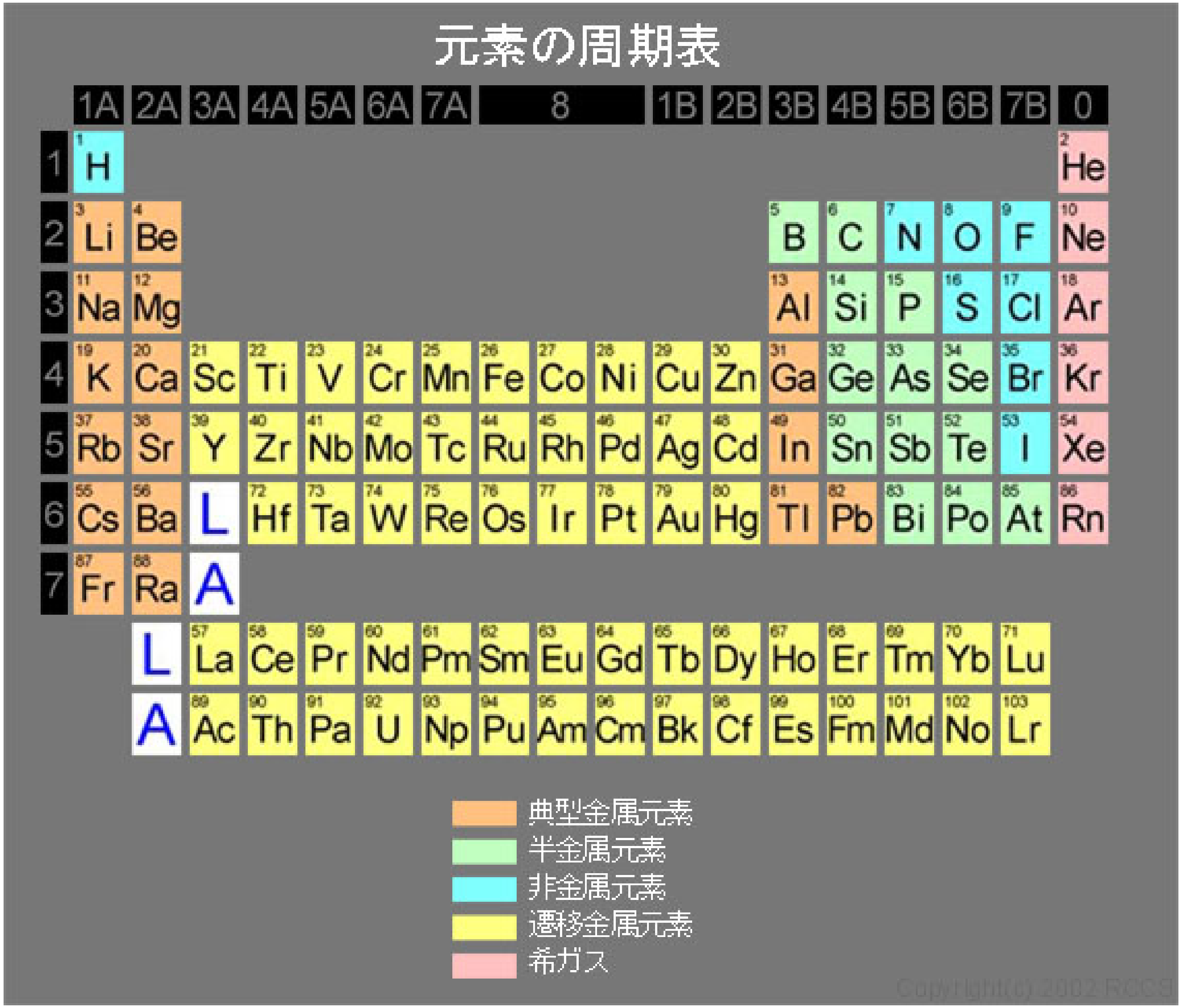




Nuclear Chart: 2D map of atomic nuclei



Periodic table: protons only, no neutrons

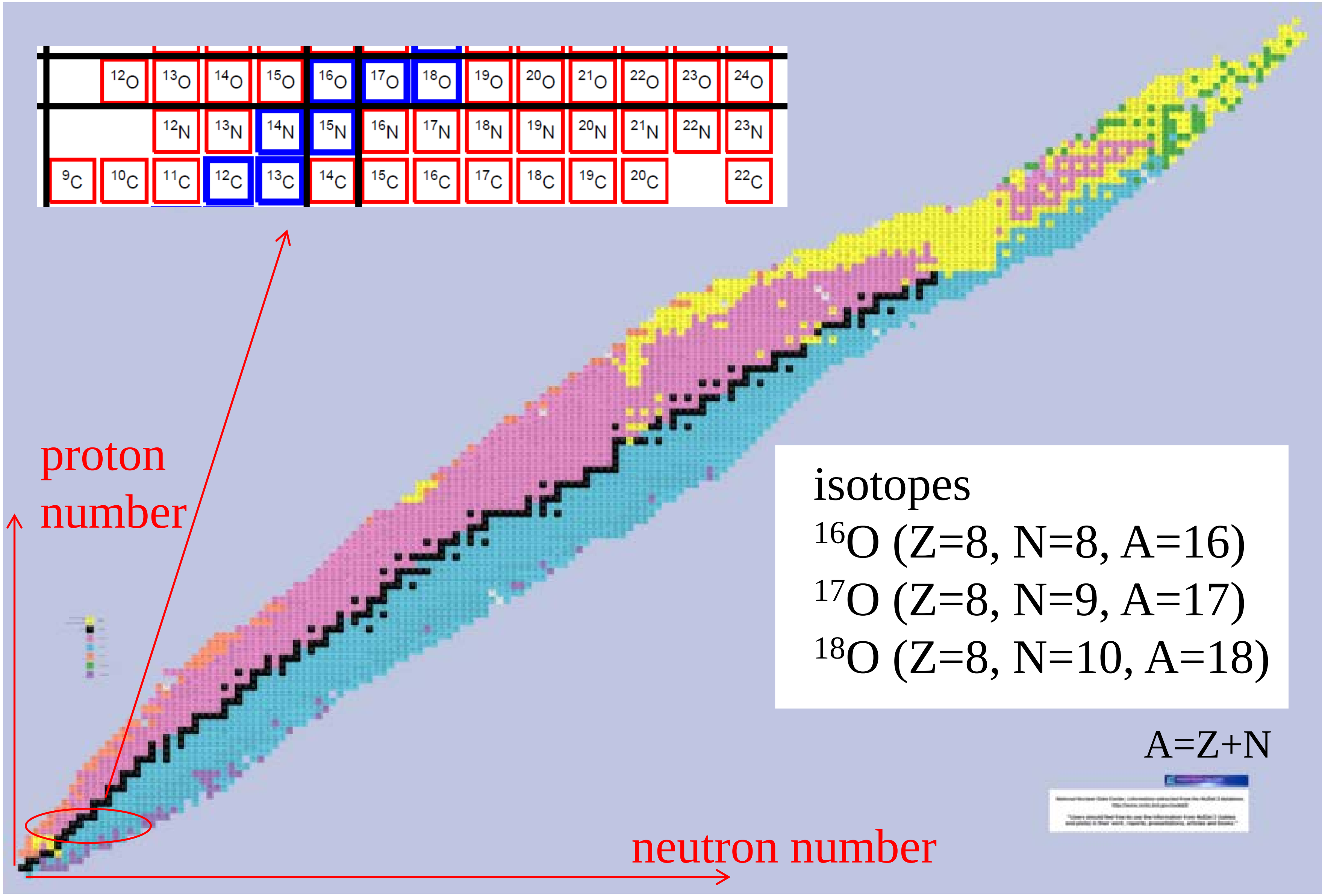


元素の周期表

	1A	2A	3A	4A	5A	6A	7A	8			1B	2B	3B	4B	5B	6B	7B	0
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	L	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	A															
		L	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
		A	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

🟧 典型金属元素
🟩 半金属元素
🟦 非金属元素
🟨 遷移金属元素
🟥 希ガス

Nuclear Chart: 2D map of atomic nuclei



isotopes

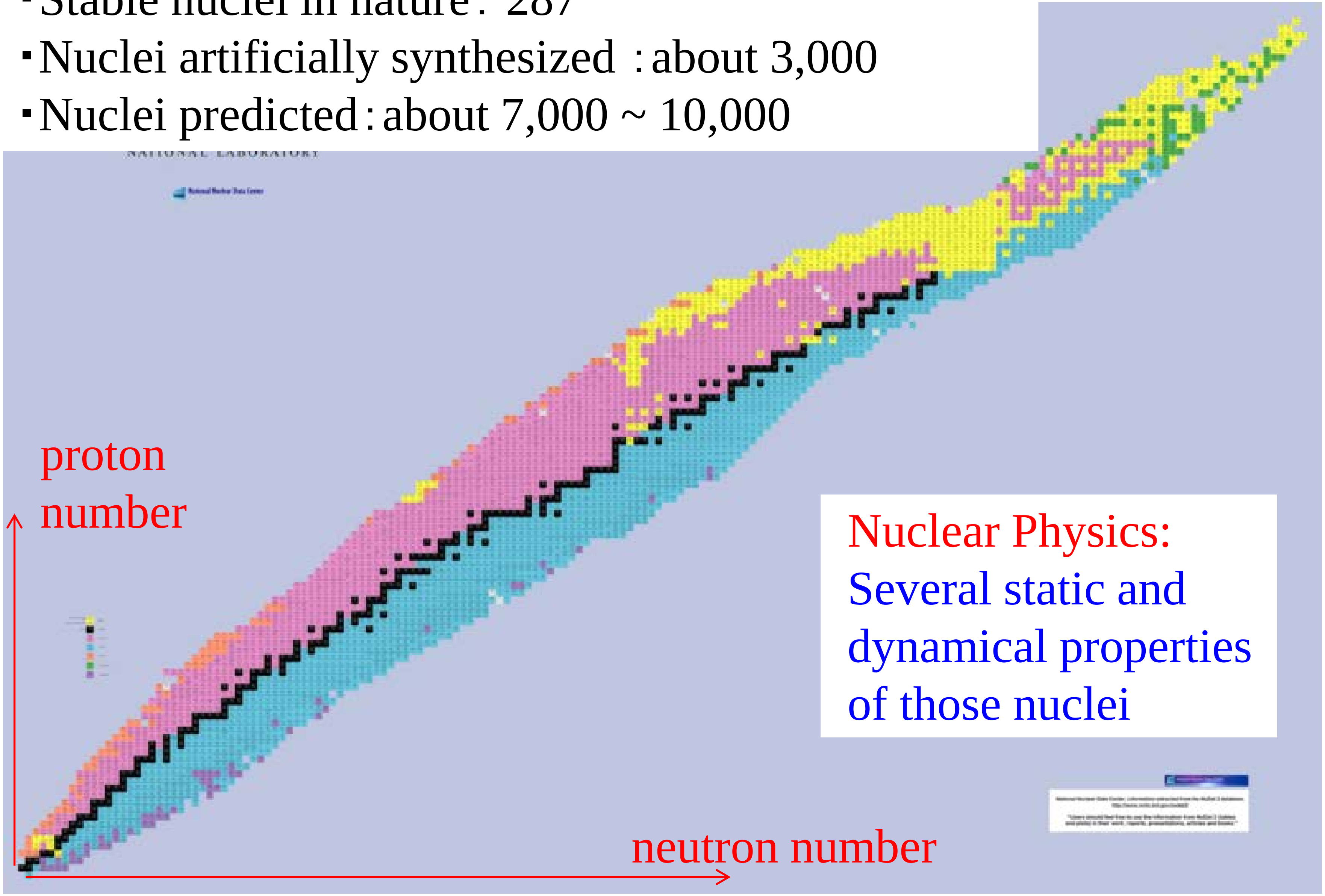
^{16}O (Z=8, N=8, A=16)

^{17}O (Z=8, N=9, A=17)

^{18}O (Z=8, N=10, A=18)

A=Z+N

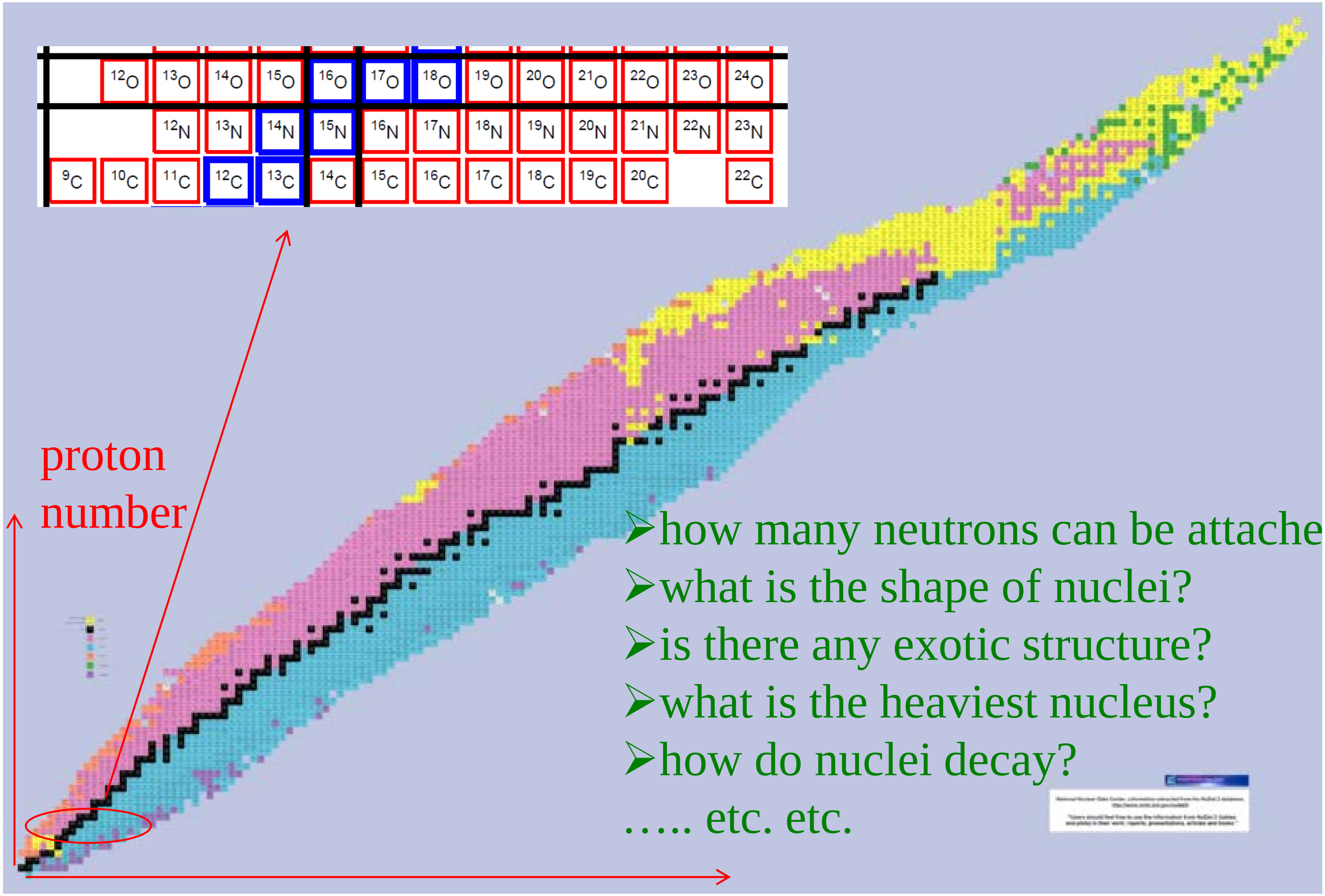
- Stable nuclei in nature： 287
- Nuclei artificially synthesized ：about 3,000
- Nuclei predicted：about 7,000 ~ 10,000



Nuclear Physics:

Several static and dynamical properties of those nuclei

Nuclear Chart: 2D map of atomic nuclei



	^{12}O	^{13}O	^{14}O	^{15}O	^{16}O	^{17}O	^{18}O	^{19}O	^{20}O	^{21}O	^{22}O	^{23}O	^{24}O
		^{12}N	^{13}N	^{14}N	^{15}N	^{16}N	^{17}N	^{18}N	^{19}N	^{20}N	^{21}N	^{22}N	^{23}N
^{9}C	^{10}C	^{11}C	^{12}C	^{13}C	^{14}C	^{15}C	^{16}C	^{17}C	^{18}C	^{19}C	^{20}C		^{22}C

proton number

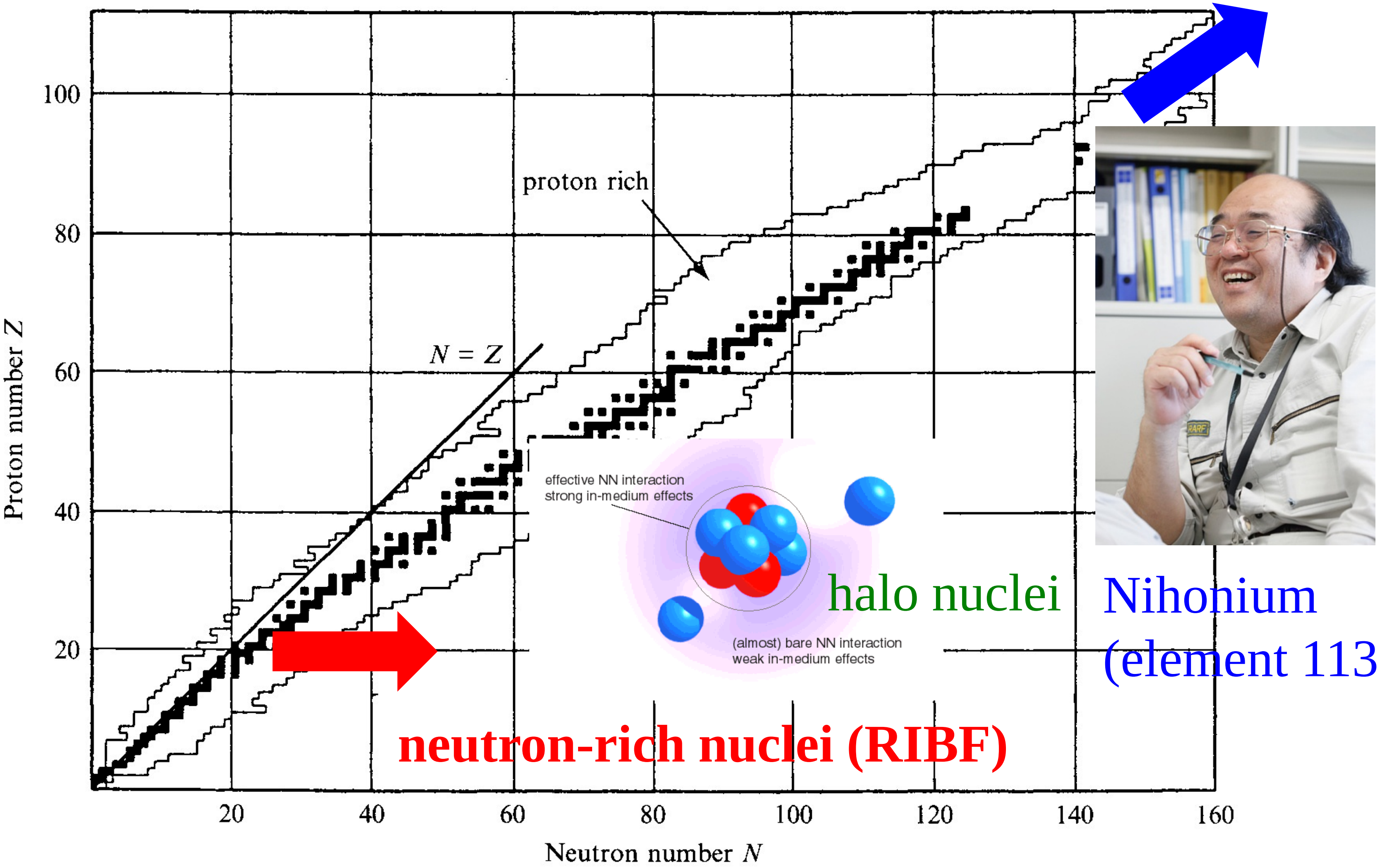
neutron number

- how many neutrons can be attached?
- what is the shape of nuclei?
- is there any exotic structure?
- what is the heaviest nucleus?
- how do nuclei decay?

….. etc. etc.

Extension of nuclear chart: frontier of nuclear physics

superheavy elements

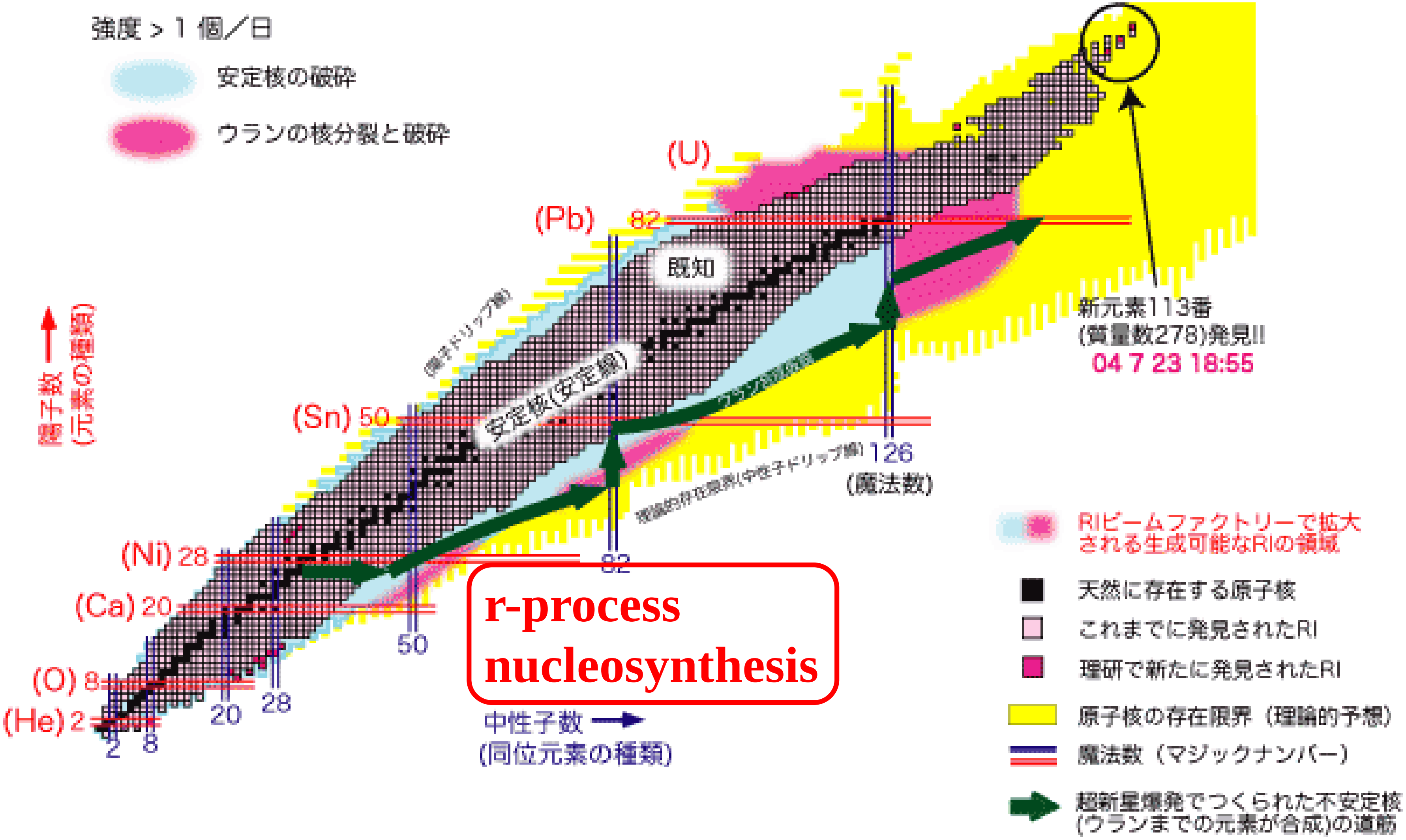


halo nuclei

Nihonium (element 113)

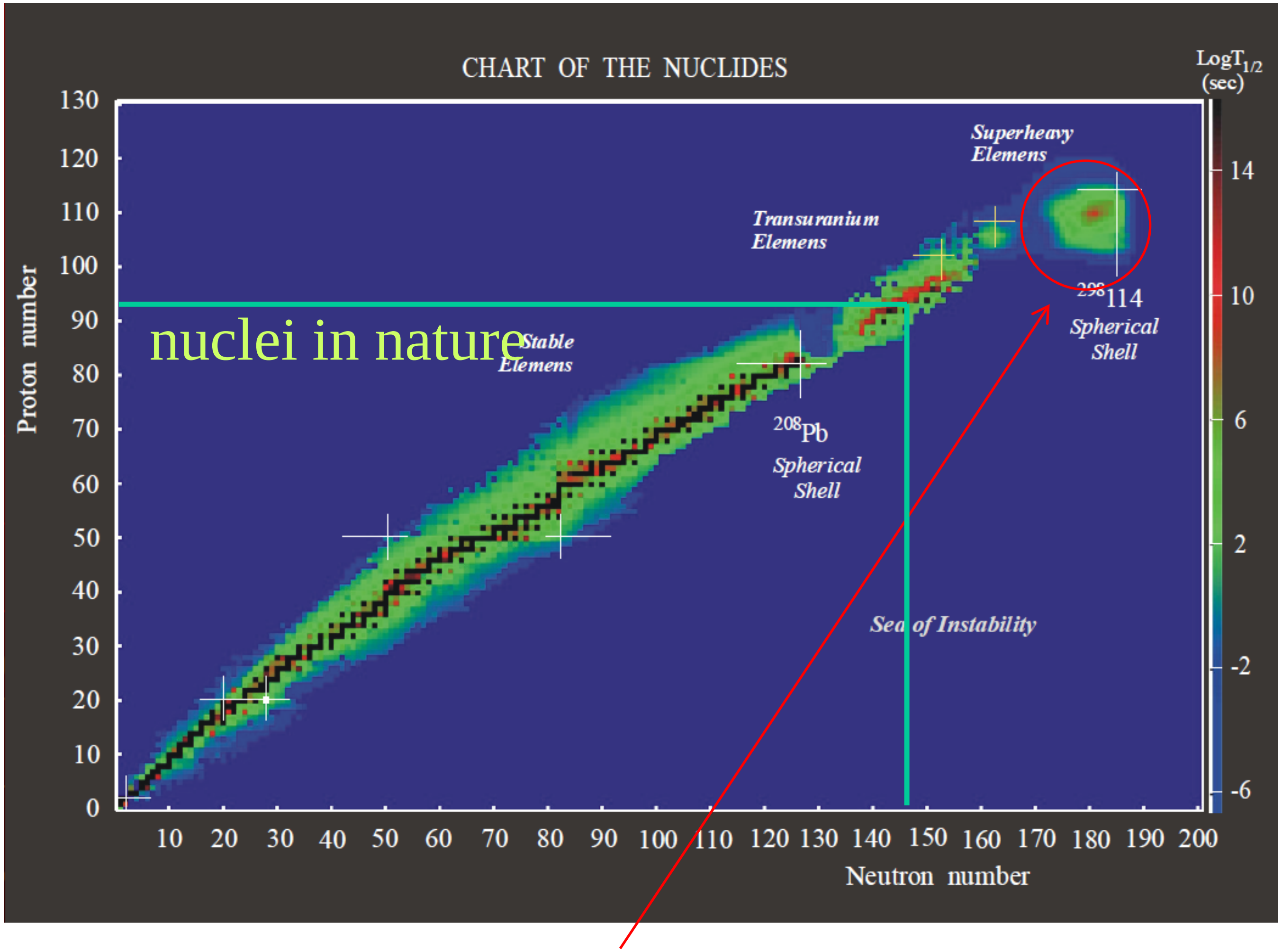
neutron-rich nuclei (RIBF)

Neutron-rich nuclei (RIBF at RIKEN)



r-process nucleosynthesis

Prediction of island of stability: an important motivation of SHE study



island of stability around Z=114, N=184

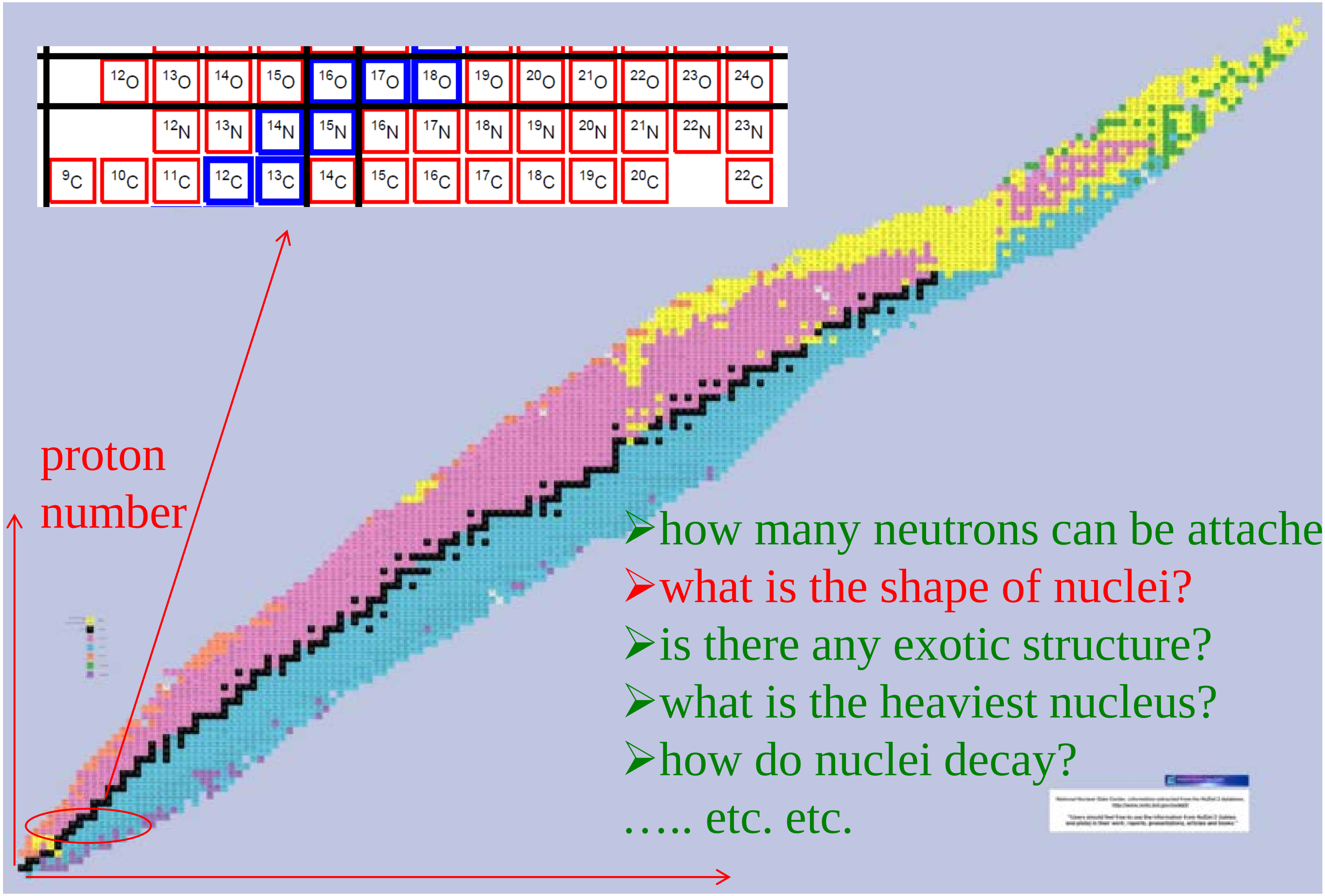
Yuri Oganessian

W.D. Myers and W.J. Swiatecki (1966), A. Sobiczewski et al. (1966)

→ modern calculations: Z=114,120, or 126, N=184

e.g., H. Koura et al. (2005)

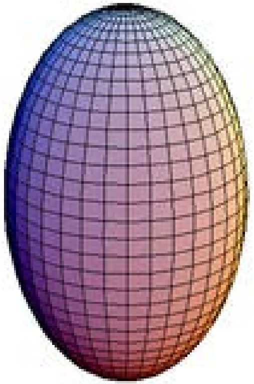
Nuclear Chart: 2D map of atomic nuclei

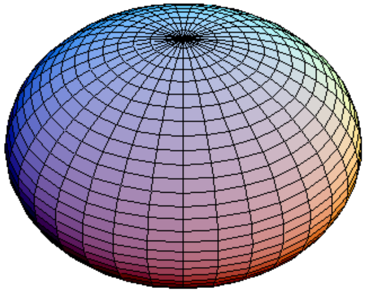


- how many neutrons can be attached?
- what is the shape of nuclei?
- is there any exotic structure?
- what is the heaviest nucleus?
- how do nuclei decay?

….. etc. etc.

a nucleus is not always spherical

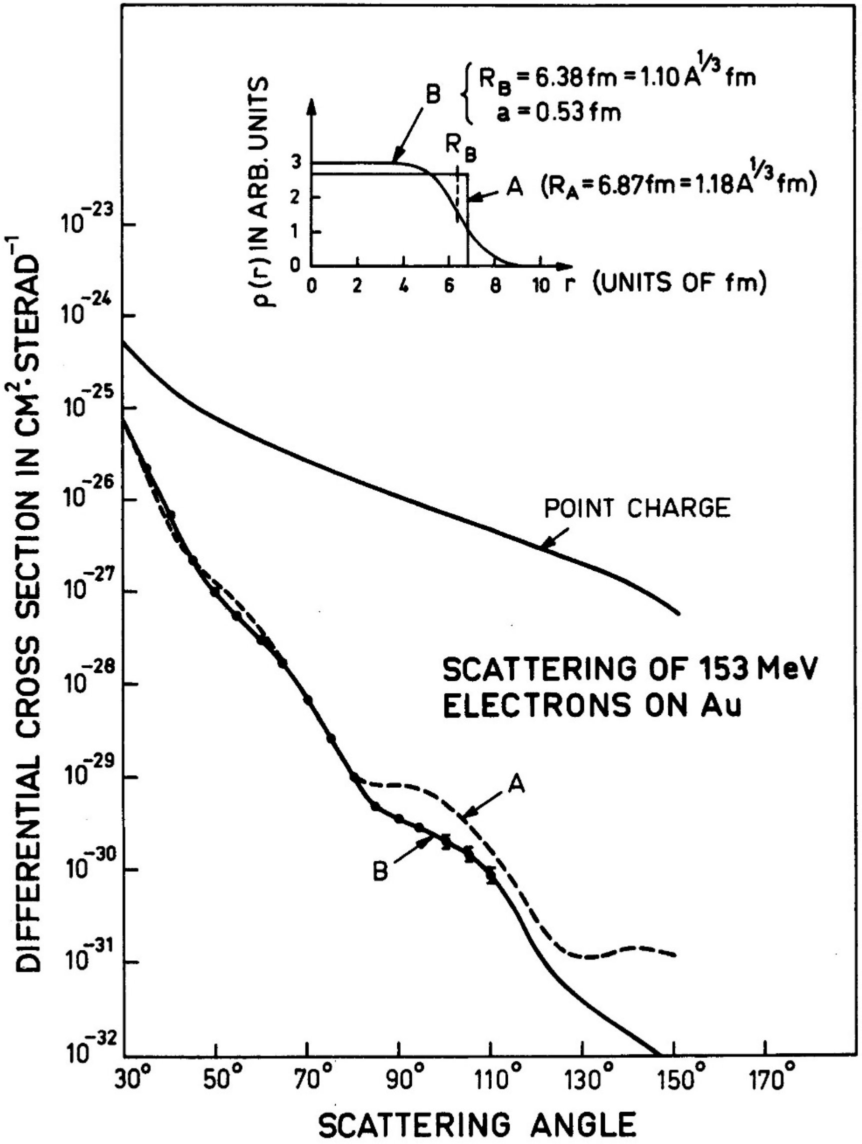




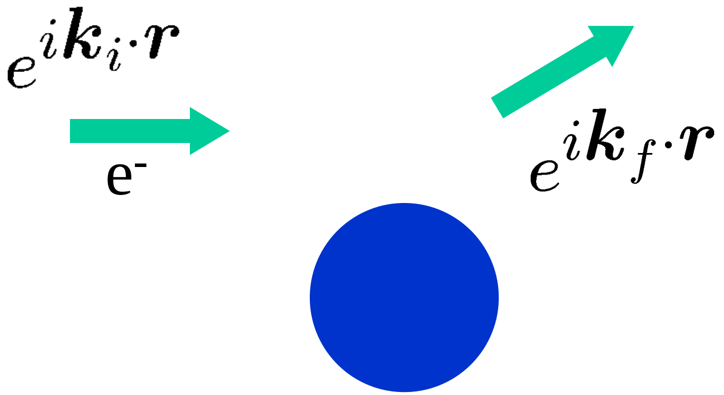
Quantum shape dynamics

Density Distribution

High energy electron scattering



Born approximation:



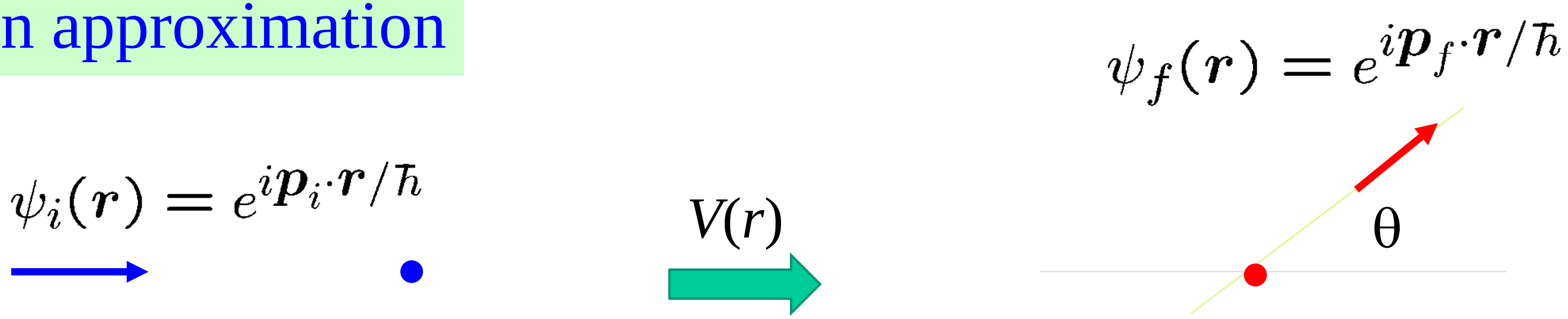
$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2}\,|F(\boldsymbol{q})|^2$$

Form factor

$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho(\boldsymbol{r})\,d\boldsymbol{r}$$

(Fourier transform of the density)

Born approximation



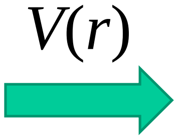
$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

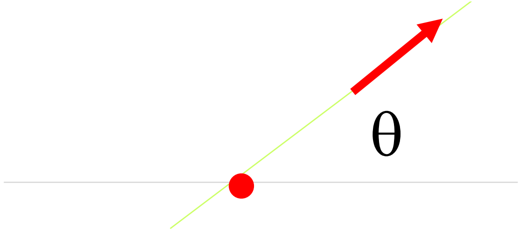
$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$





$$\boxed{W_{fi} = \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega \left|\tilde{V}(\boldsymbol{q})\right|^2}$$

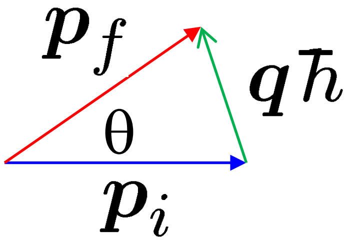
momentum transfer

$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r}\, e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar} V(r) \equiv \int d\boldsymbol{r}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} V(r)$$

incident flux: $j_{\rm inc} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\rm inc}} = \int d\Omega \boxed{\frac{\mu^2}{4\pi^2\hbar^4}\left|\tilde{V}(\boldsymbol{q})\right|^2}$$

$$= \frac{d\sigma}{d\Omega}$$



$$q\hbar = 2p_i \sin\frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E \sin^2 \theta/2)^2} |F(\boldsymbol{q})|^2$$

$$= \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega}\right) |F(\boldsymbol{q})|^2$$

Form factor

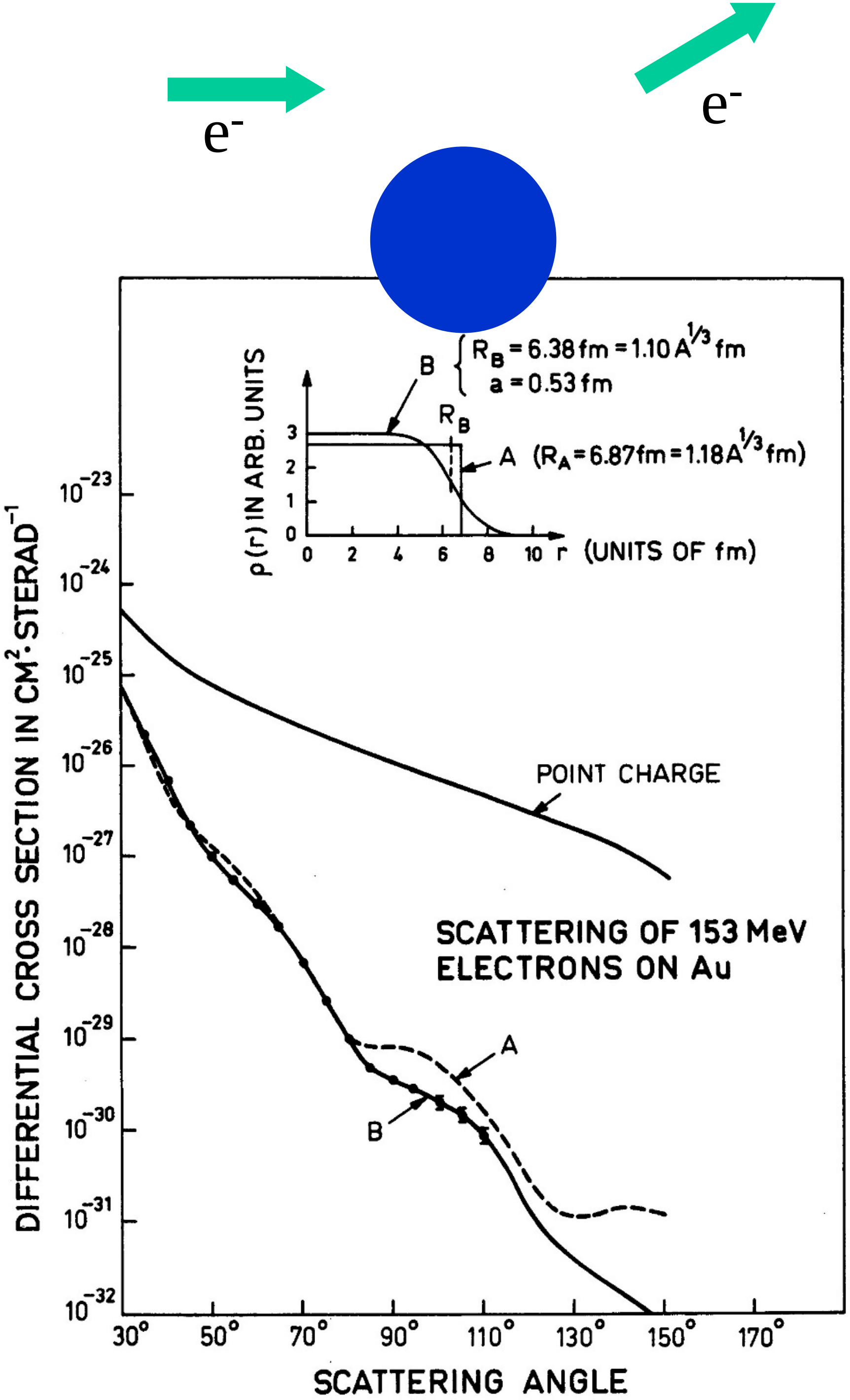
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})\, d\boldsymbol{r}$$

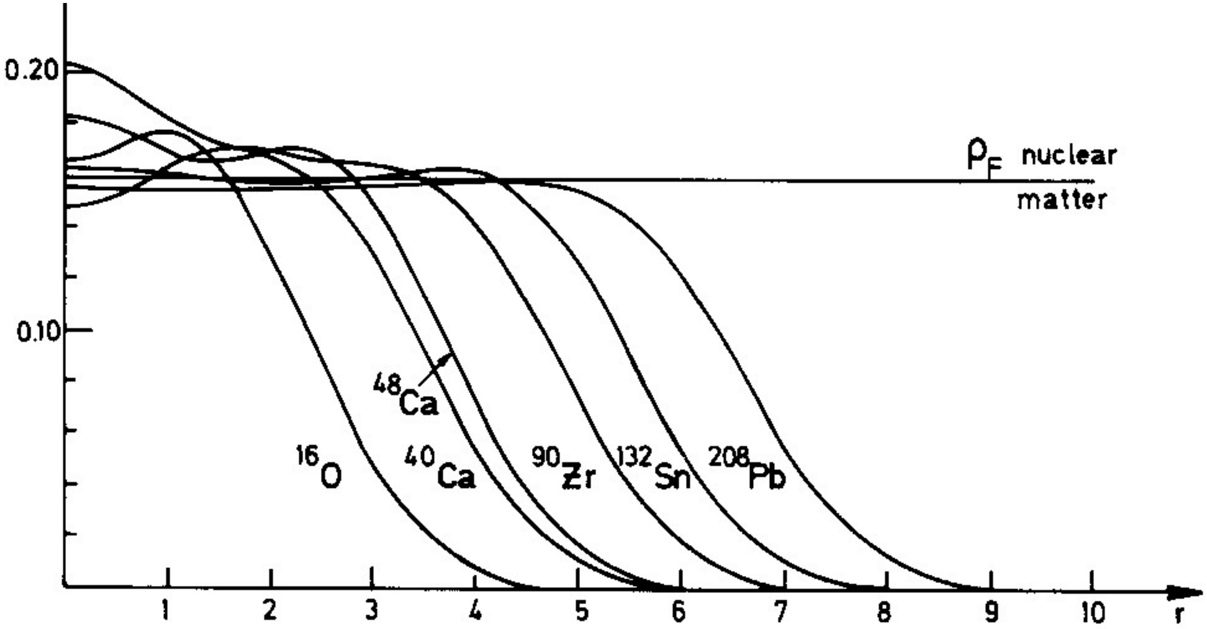
* relativistic correction:

$$\frac{d\sigma_{\text{Ruth}}}{d\Omega} \rightarrow \frac{d\sigma_{\text{Mott}}}{d\Omega}$$

$$= \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \left(1 - \frac{v^2}{c^2} \sin^2 \frac{\theta}{2}\right)$$

$$\sim \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \cos^2 \frac{\theta}{2} \qquad (v \rightarrow c)$$





Fermi distribution

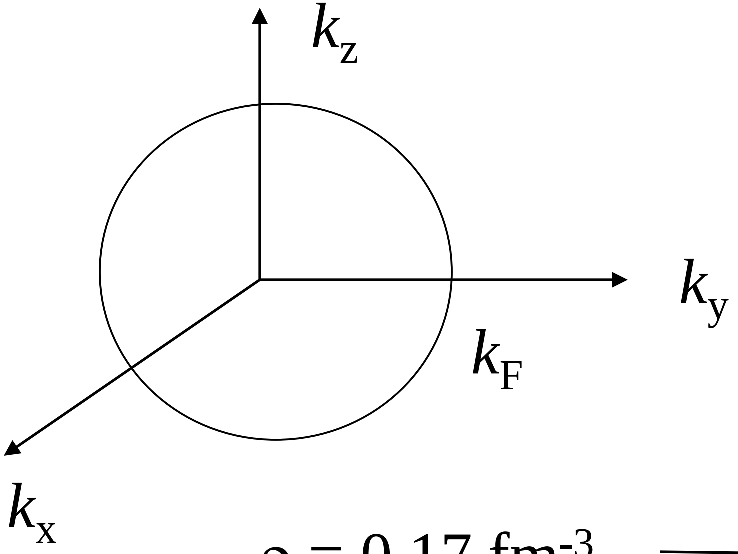
$$\boxed{\rho(r) = \frac{\rho_0}{1 + \exp[(r - R_0)/a]}}$$

$$\begin{aligned} \rho_0 &\sim 0.17 \ (\mathrm{fm}^{-3}) \\ R_0 &\sim 1.1 \times A^{1/3} \ (\mathrm{fm}) \\ a &\sim 0.57 \ (\mathrm{fm}) \end{aligned}$$

← Saturation property

Momentum Distribution

Fermi gas approximation



$$\rho = 2 \times 2 \times 4\pi \int_0^{k_F} \frac{k^2 dk}{(2\pi)^3}$$
$$= \frac{2}{3\pi^2} k_F^3$$

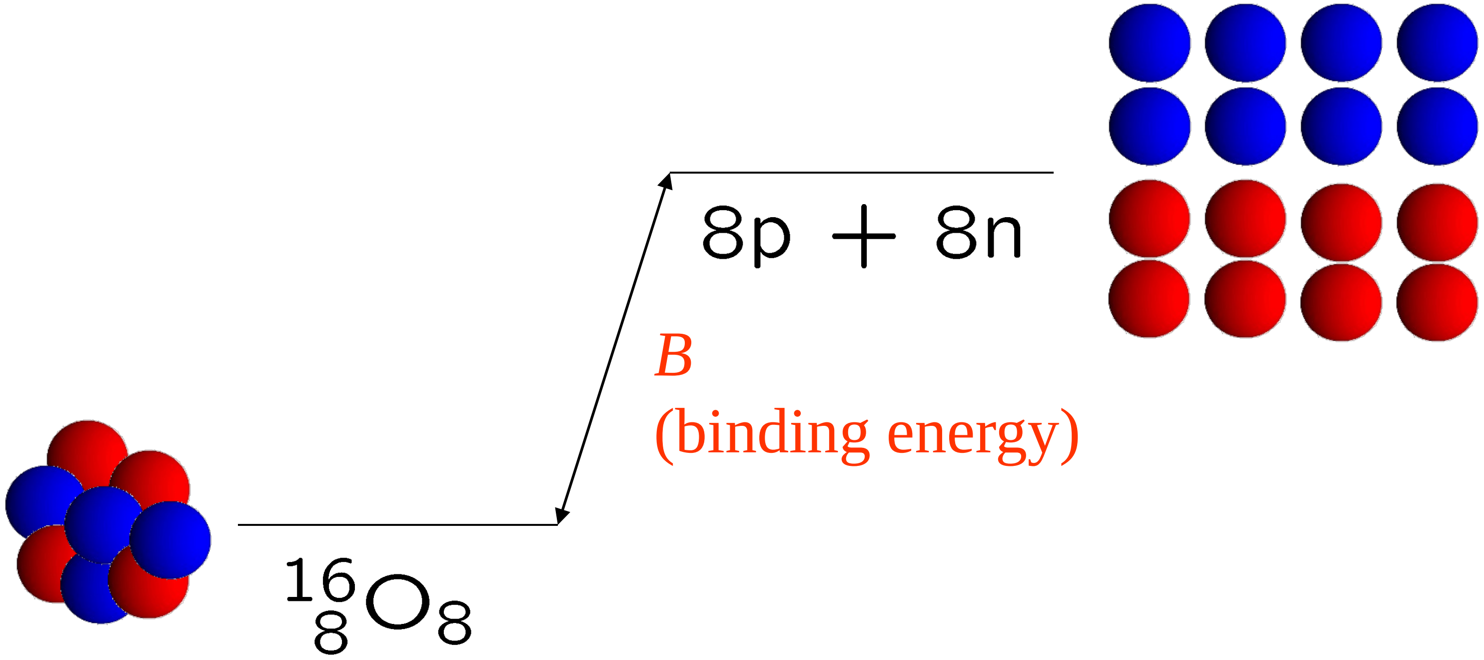
(note: spin-isospin degeneracy)

$\rho = 0.17\ \mathrm{fm}^{-3}$ → $k_F \sim 1.36\ \mathrm{fm}^{-1}$

$$\Longleftrightarrow \quad \frac{v_F}{c} = \frac{k_F \cdot \hbar c}{mc^2} = 0.285$$

Fermi energy: $\epsilon_F = \dfrac{k_F^2 \hbar^2}{2m} \sim 37$ (MeV)

Nuclear Mass



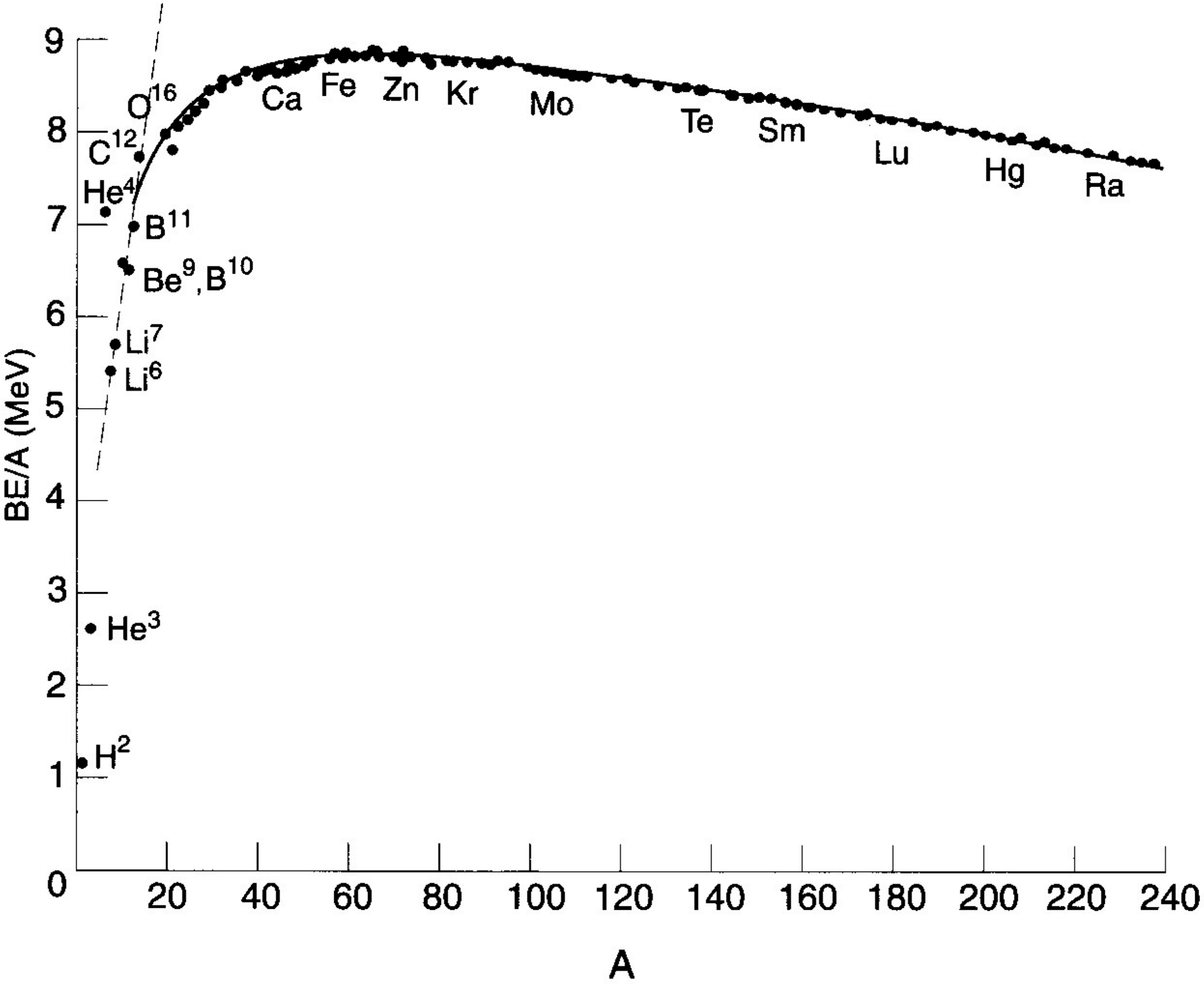
8p + 8n

B

(binding energy)

$^{16}_{8}\mathrm{O}_{8}$

$$m(N,Z)c^2 = Zm_pc^2 + Nm_nc^2 - B$$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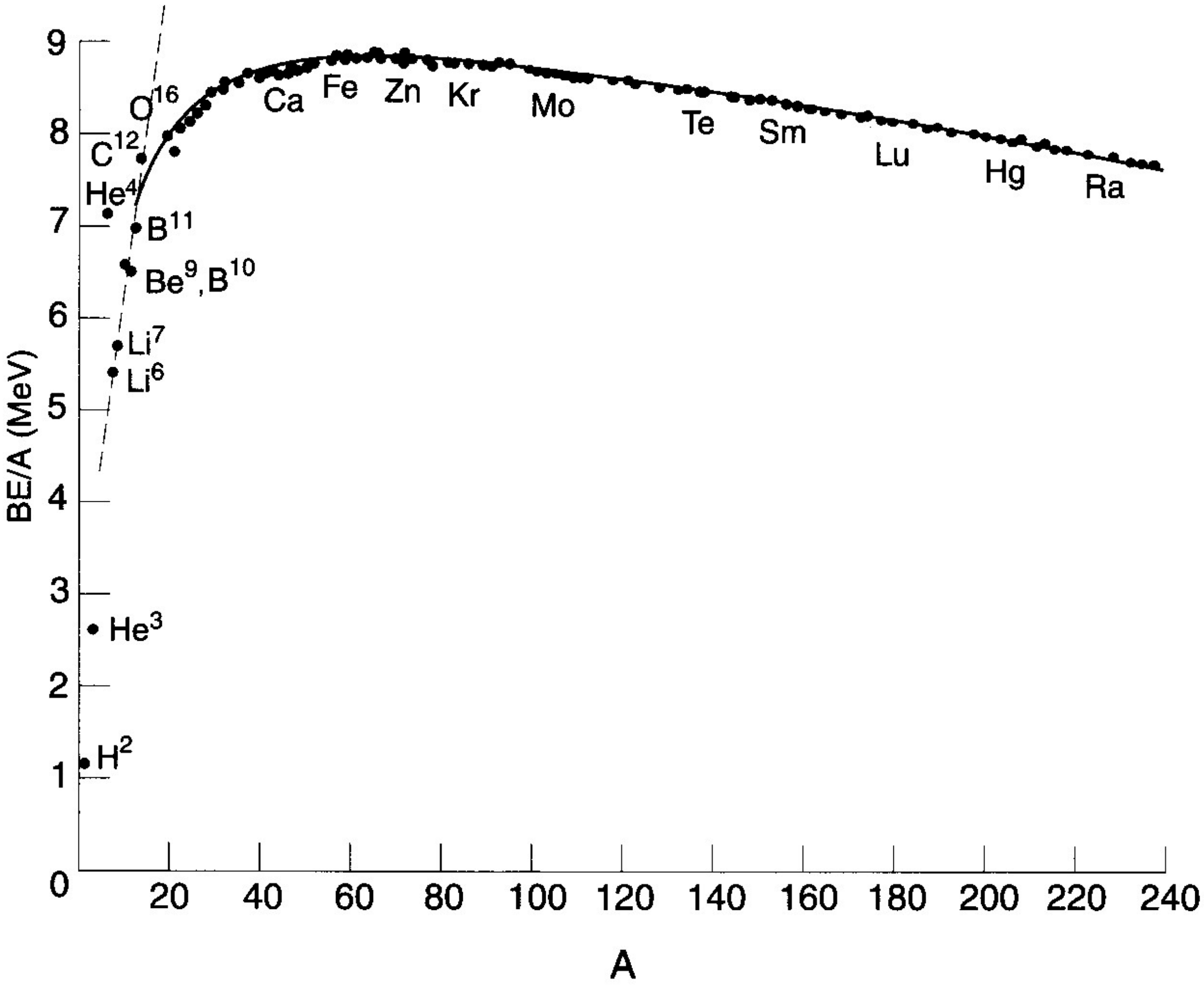
6
5
4
3
2
1
0
BE/A (MeV)
20
40
60
80
100
120
140
160
180
200
220
240
A
O16
C12
He4
B11
Be9, B10
Li7
Li6
He3
H2
Ca
Fe
Zn
Kr
Mo
Te
Sm
Lu
Hg
Ra



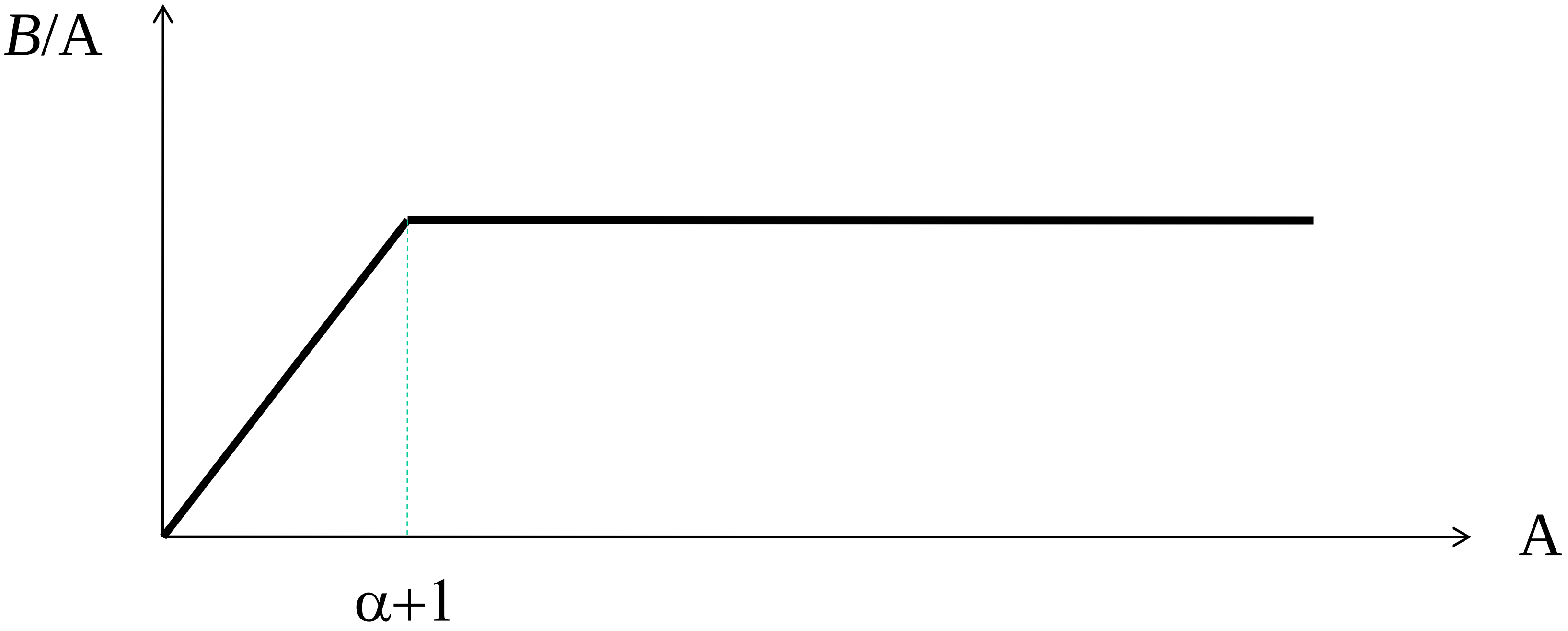
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force

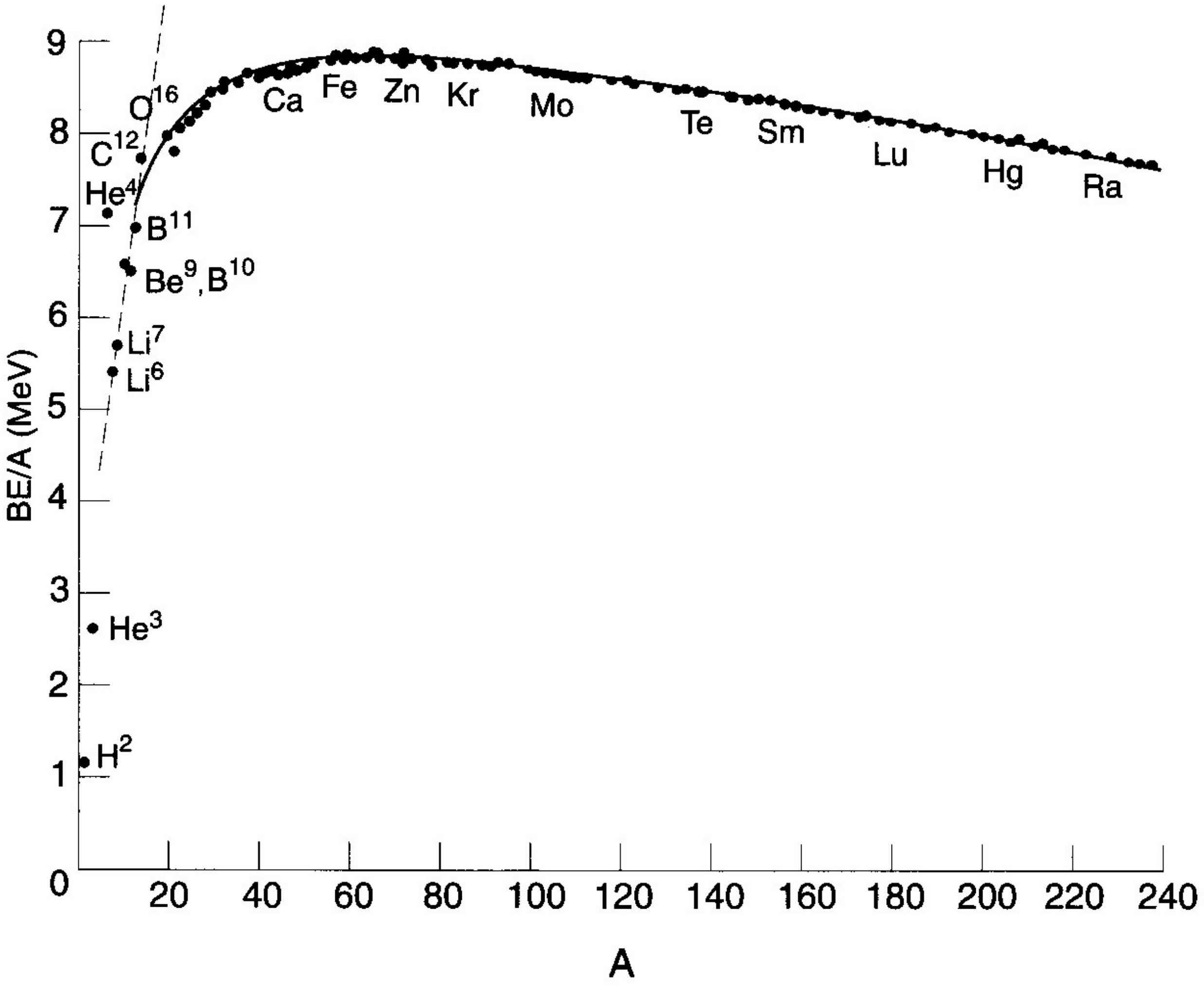
If one nucleon interacts only with surrounding α nucleons

$B \sim \alpha\, A/2 \longrightarrow B/A \sim \alpha/2$ (const.)

For A < α+1, one nucleon interacts with all the other nucleons

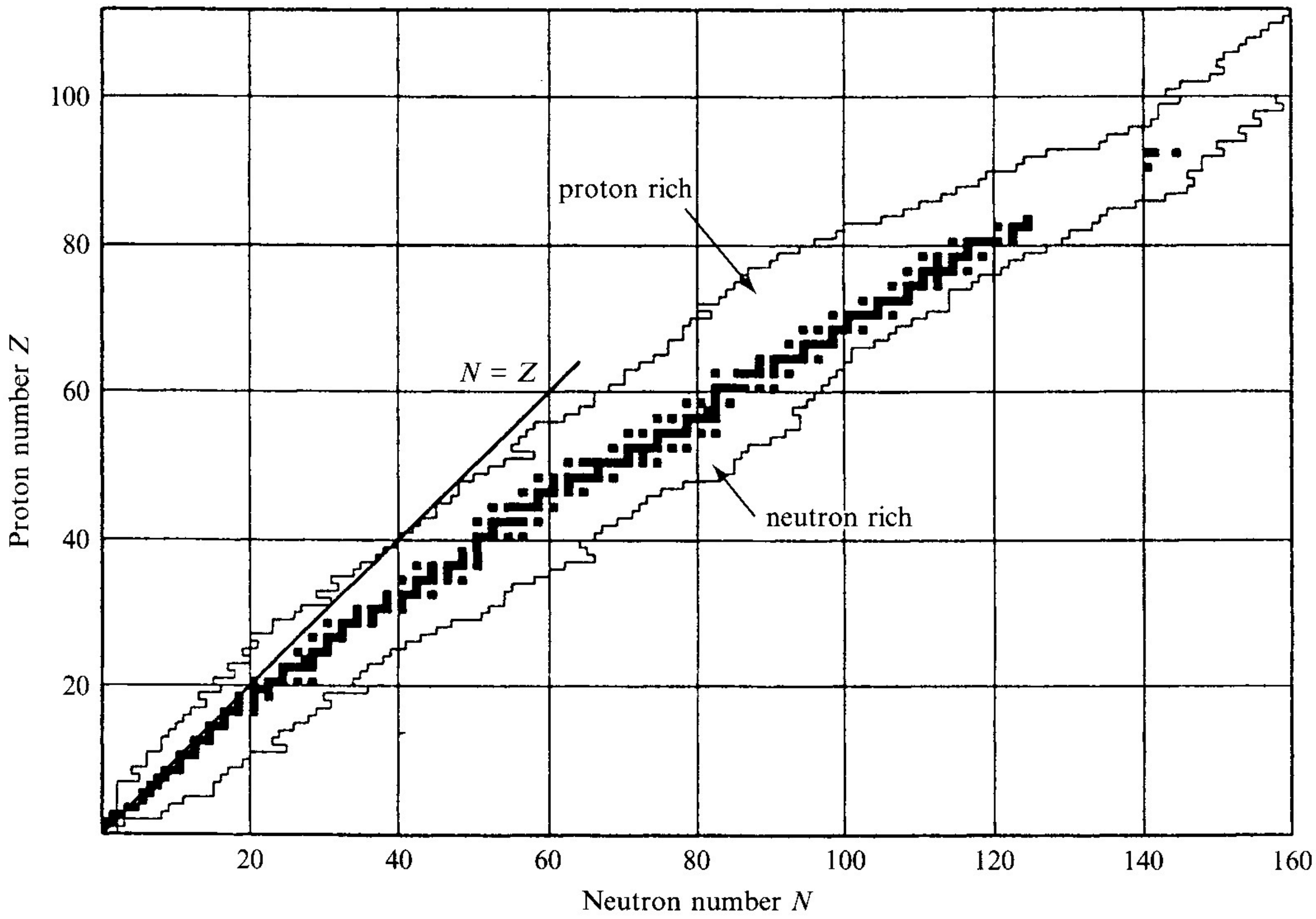
$$\longrightarrow\ B/A \propto A$$



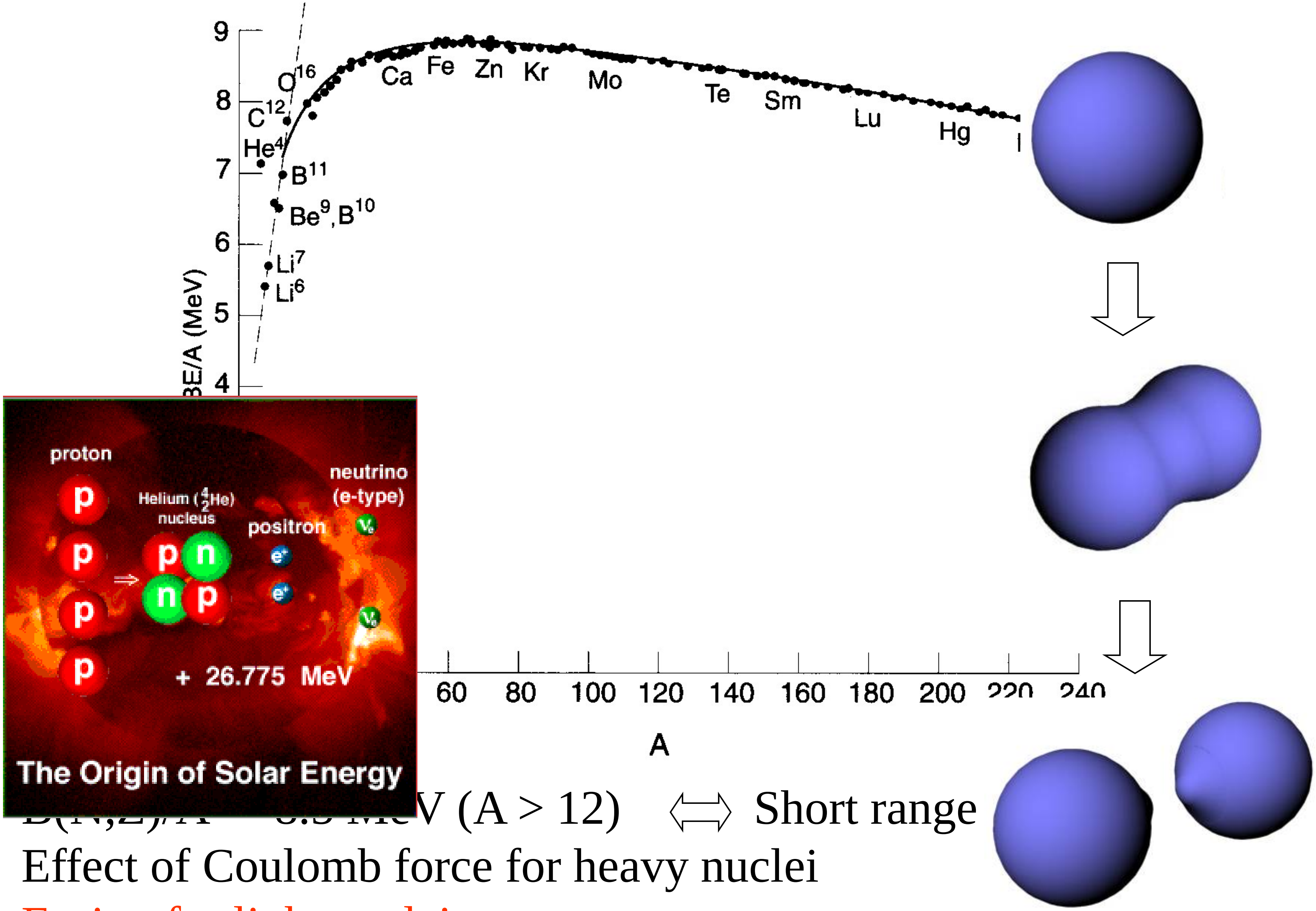


1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force
2. Effect of Coulomb force for heavy nuclei

Nuclear Chart



Stable nuclei: $N \geq Z$



1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range
2. Effect of Coulomb force for heavy nuclei
3. Fusion for light nuclei
4. Fission for heavy nuclei

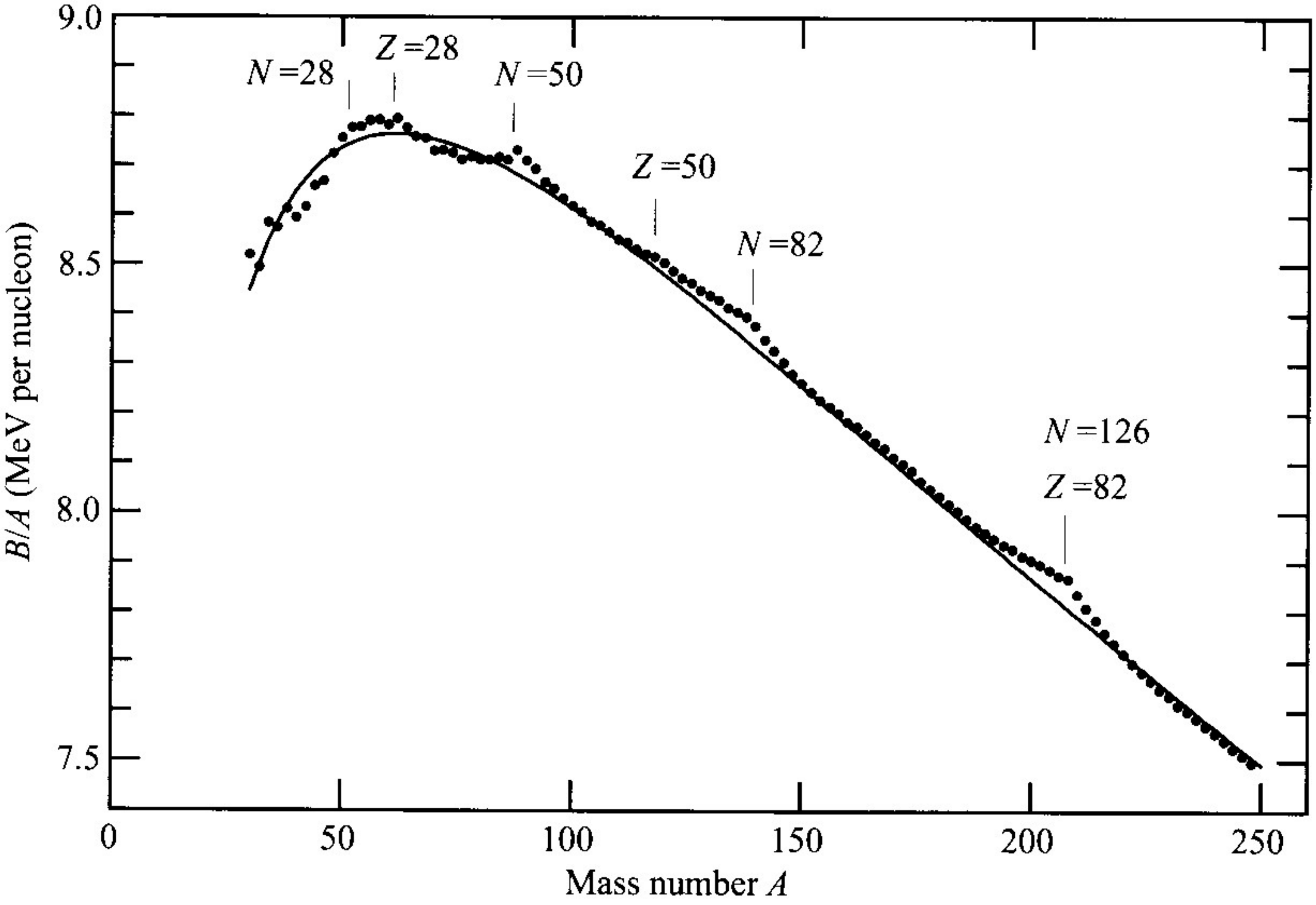
Semi-empirical mass formula

(Bethe-Weizacker formula: Liquid-drop model)

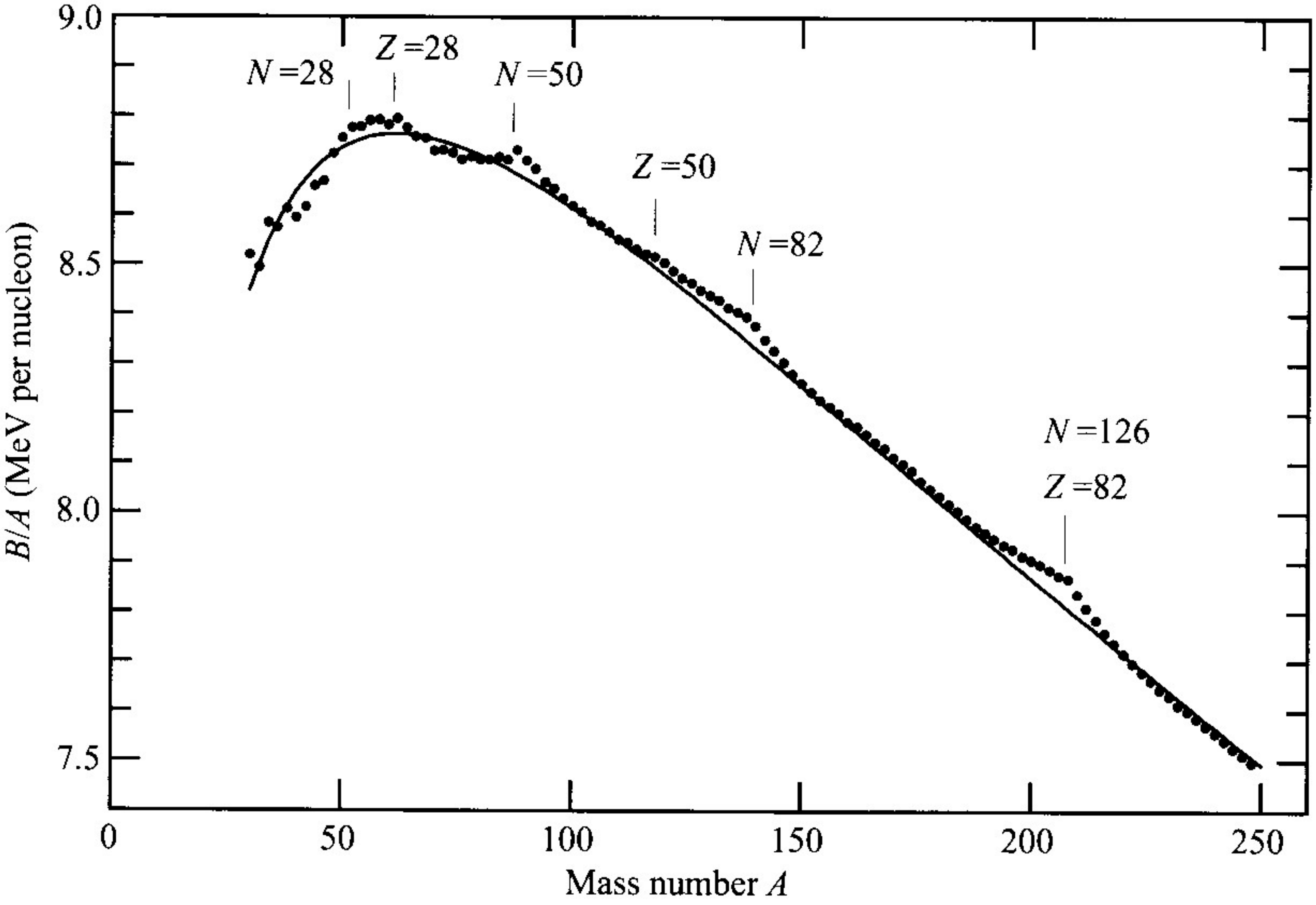
$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$

- Volume energy: $a_v A$
- Surface energy: $-a_s A^{2/3}$
- Coulomb energy: $-a_C Z^2/A^{1/3}$
- Symmetry energy: $-a_{\mathrm{sym}} (N-Z)^2/A$

How well does the Bethe-Weizacker formula reproduce the data?



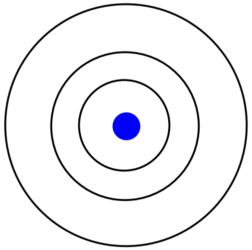
How well does the Bethe-Weizacker formula reproduce the data?



cf. N,Z = 2, 8, 20, 28, 50, 82, 126: large binding energy
“magic numbers”

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)



shell structure

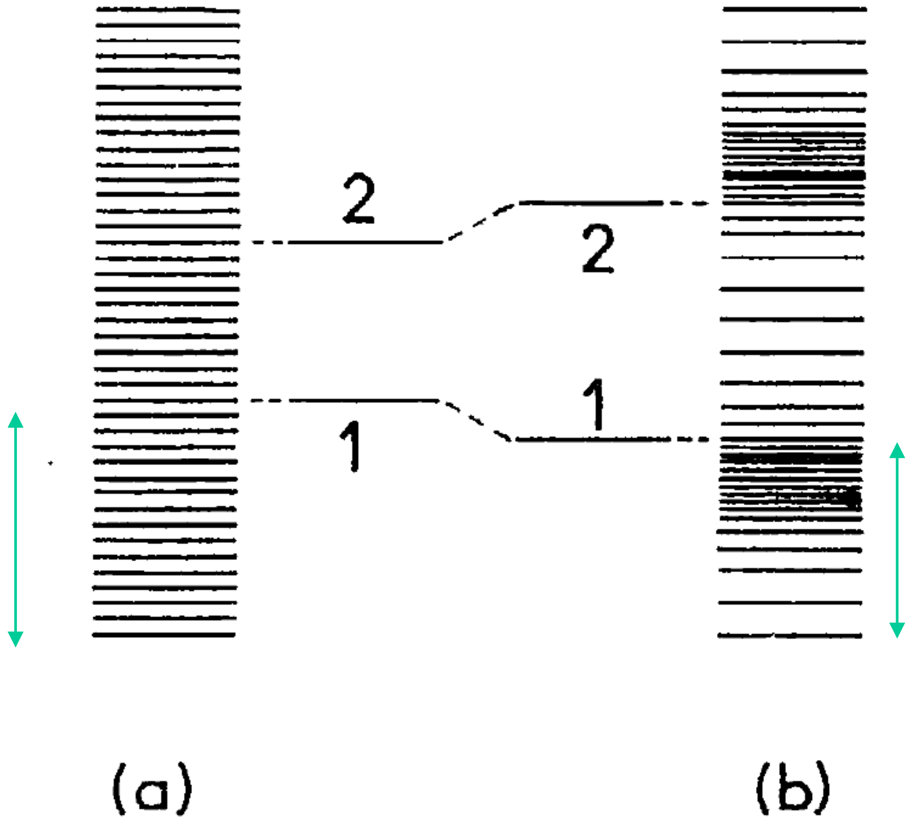
元素の周期表

	1A	2A	3A	4A	5A	6A	7A	8			1B	2B	3B	4B	5B	6B	7B	0
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	L	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	A															
		L	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
		A	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

🟧 典型金属元素
🟩 半金属元素
🟦 非金属元素
🟨 遷移金属元素
🟥 希ガス

Why do closed-shell-nuclei become stable?

level density



smaller total
energy
(more stable)

uniform

non-uniform