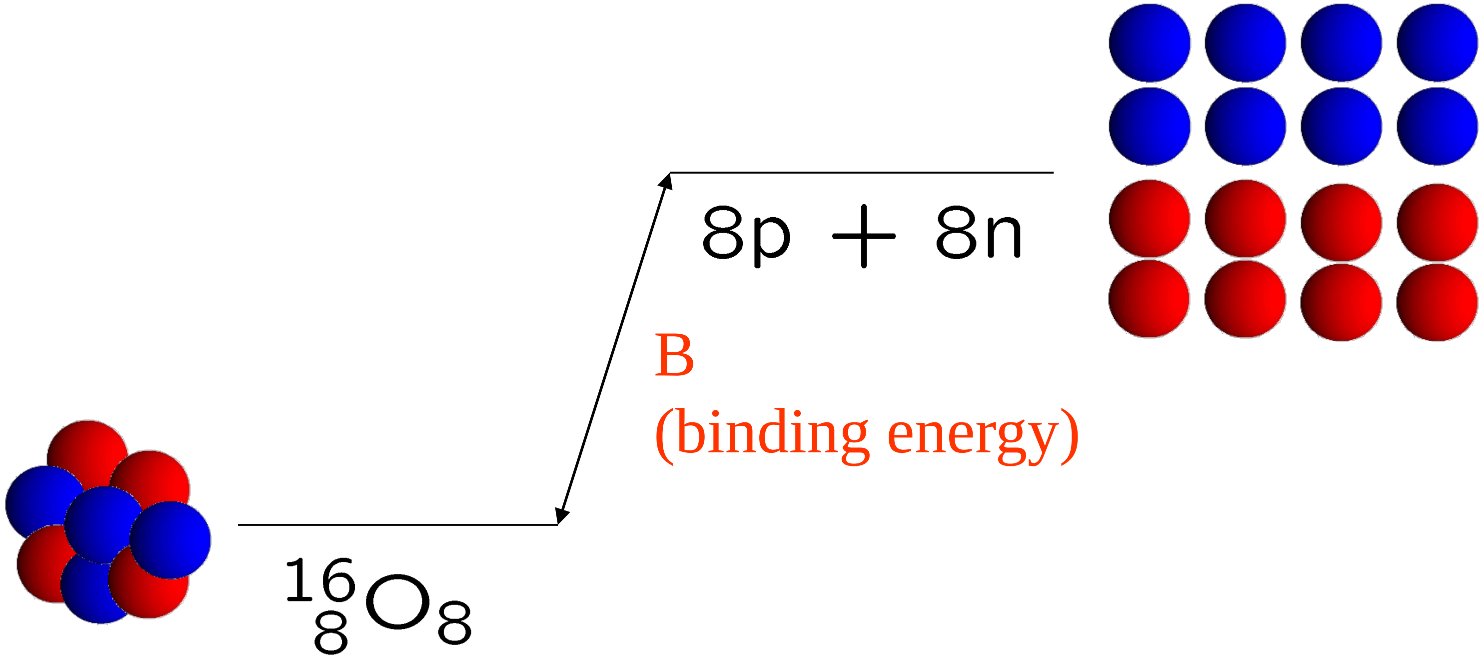
Nuclear Mass

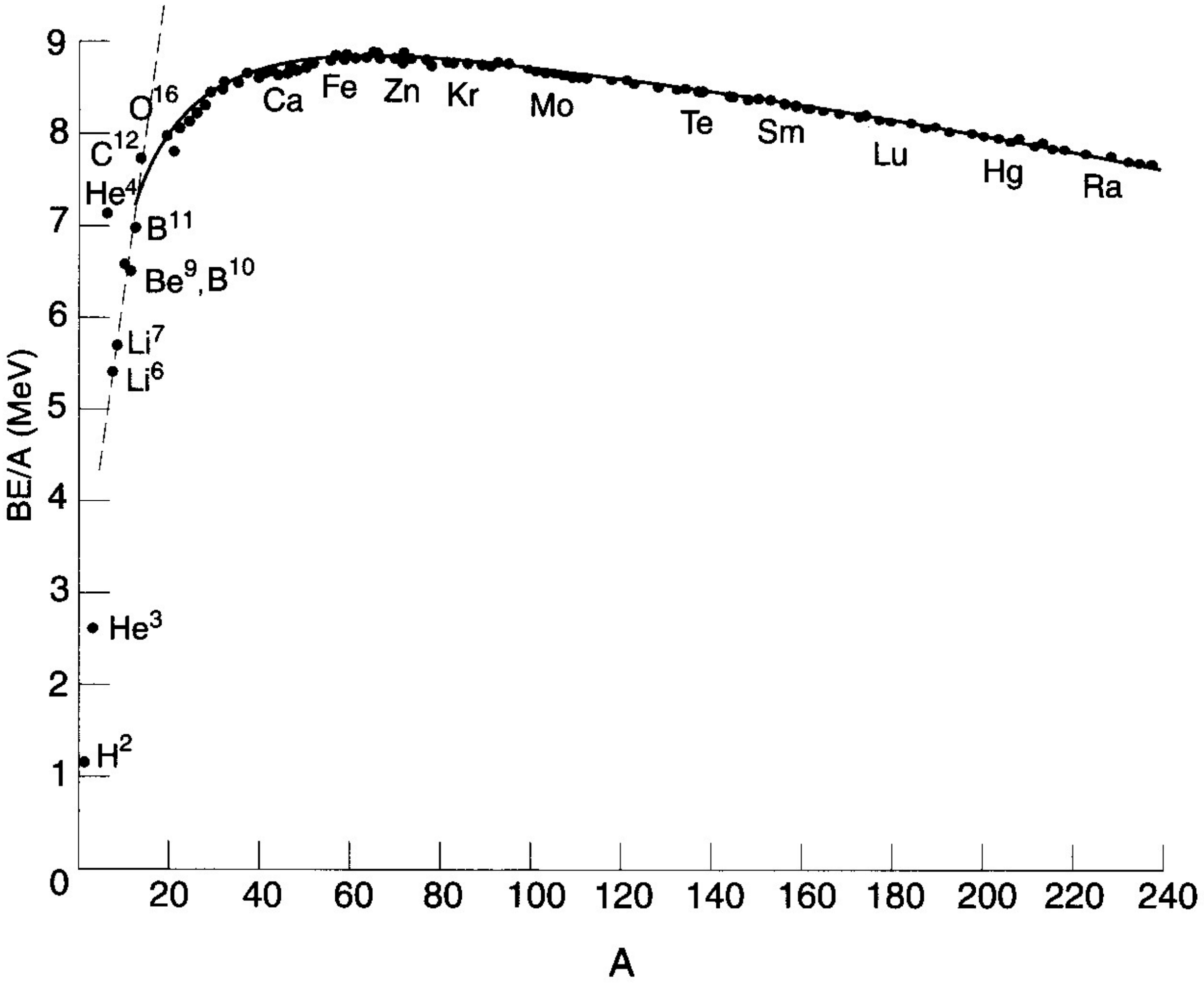


$8p + 8n$

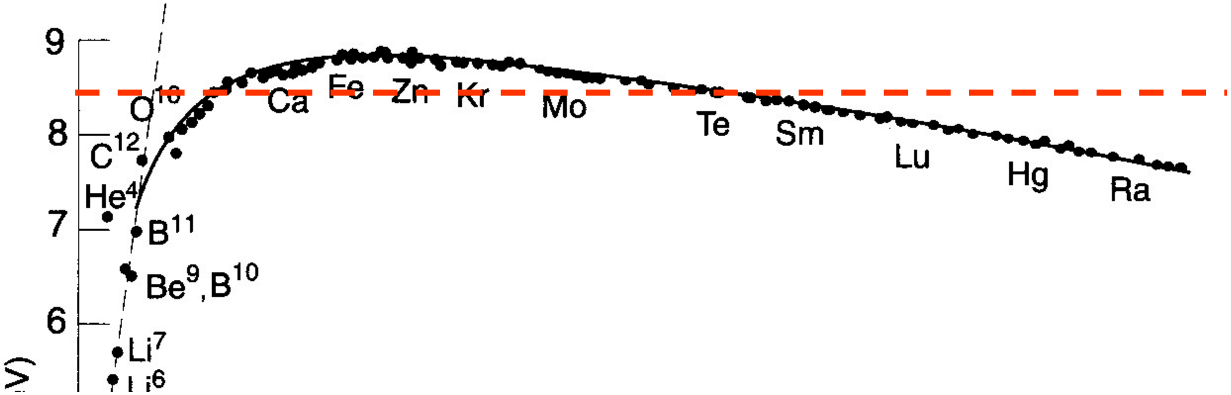
B
(binding energy)

$^{16}_{8}O_{8}$

$$m(N,Z)c^2 = Zm_pc^2 + Nm_nc^2 - B$$

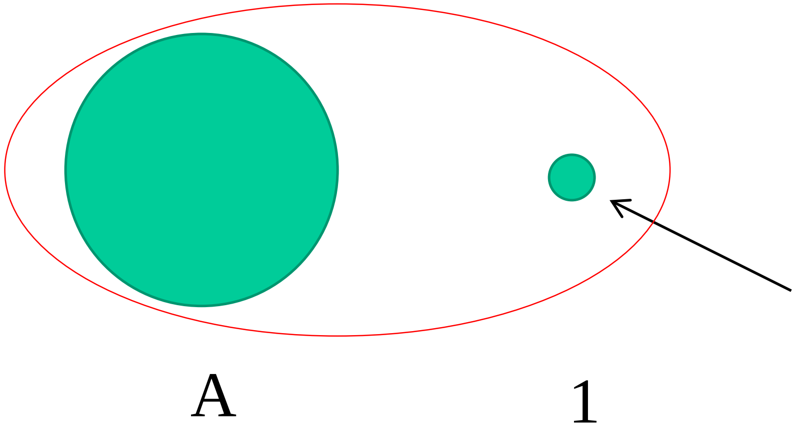


1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force



1. $B(N,Z)/A \sim 8.5$ MeV ($A > 12$)

Binding energy: increases only by a fixed amount (~ 8.5 MeV)
by adding one particle



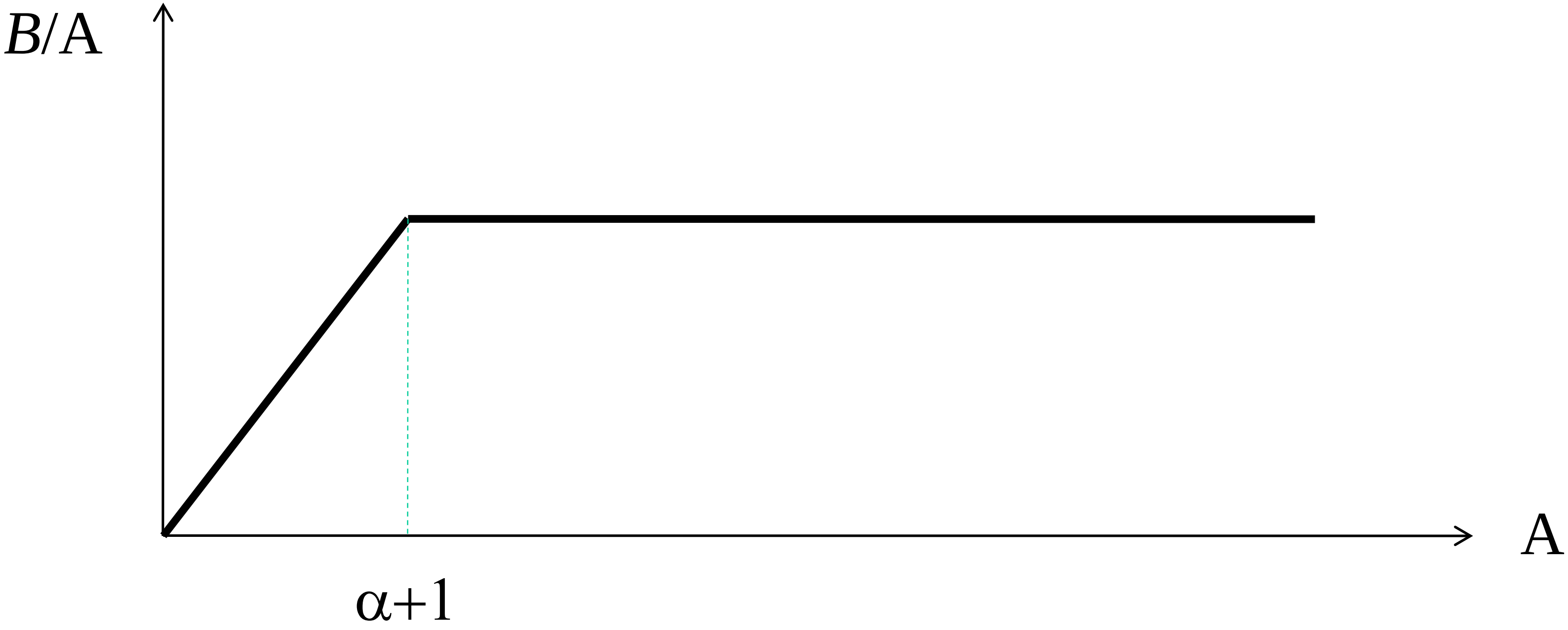
This nucleon interacts with only
a fixed number of nucleons.

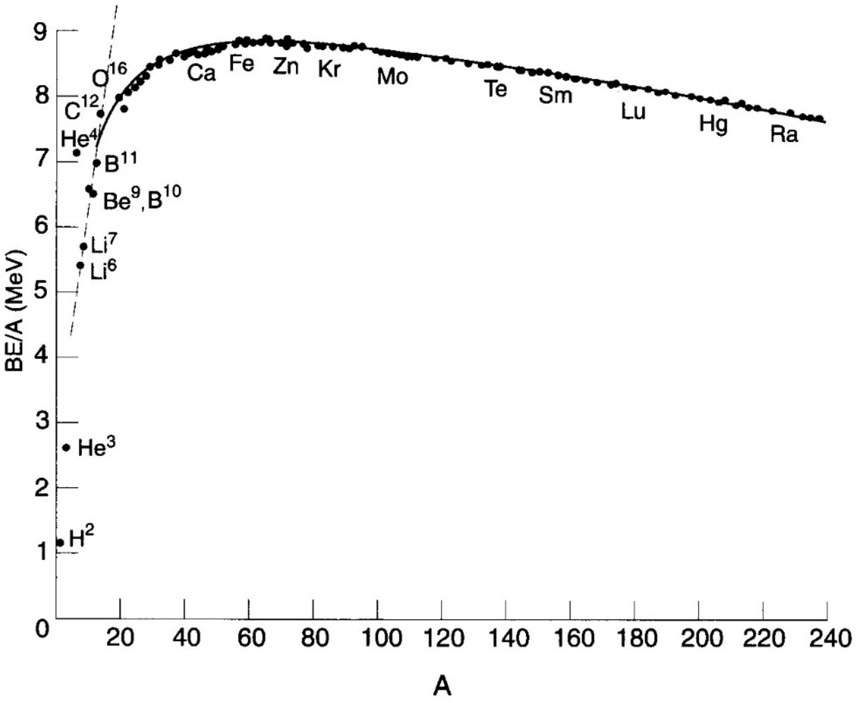
If one nucleon interacts only with surrounding α nucleons

$B \sim \alpha\, A/2 \longrightarrow B/A \sim \alpha/2$ (const.)

For A < α+1, one nucleon interacts with all the other nucleons

$$\longrightarrow \quad B/A \propto A$$





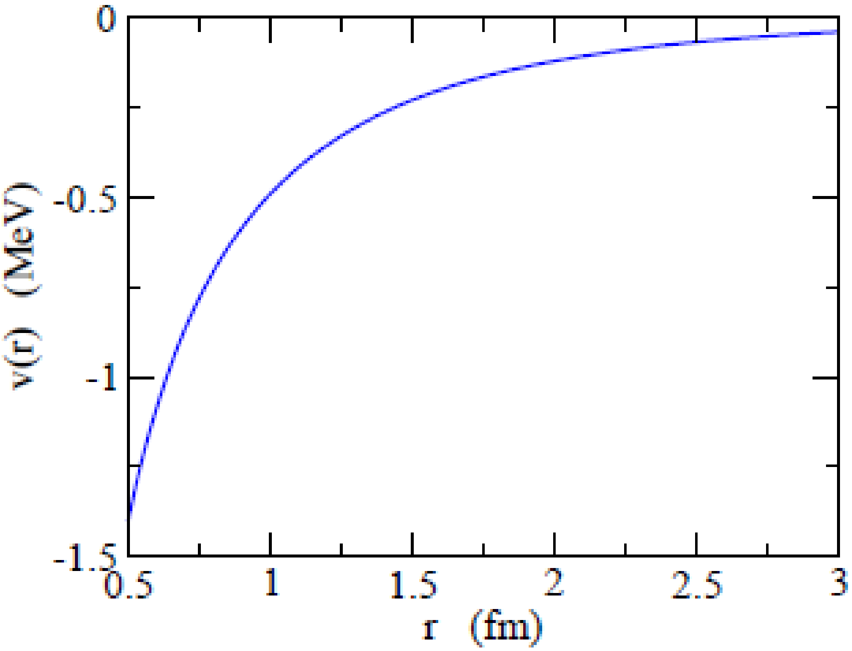
from this figure, one can read off the value of α to be around ~ 10

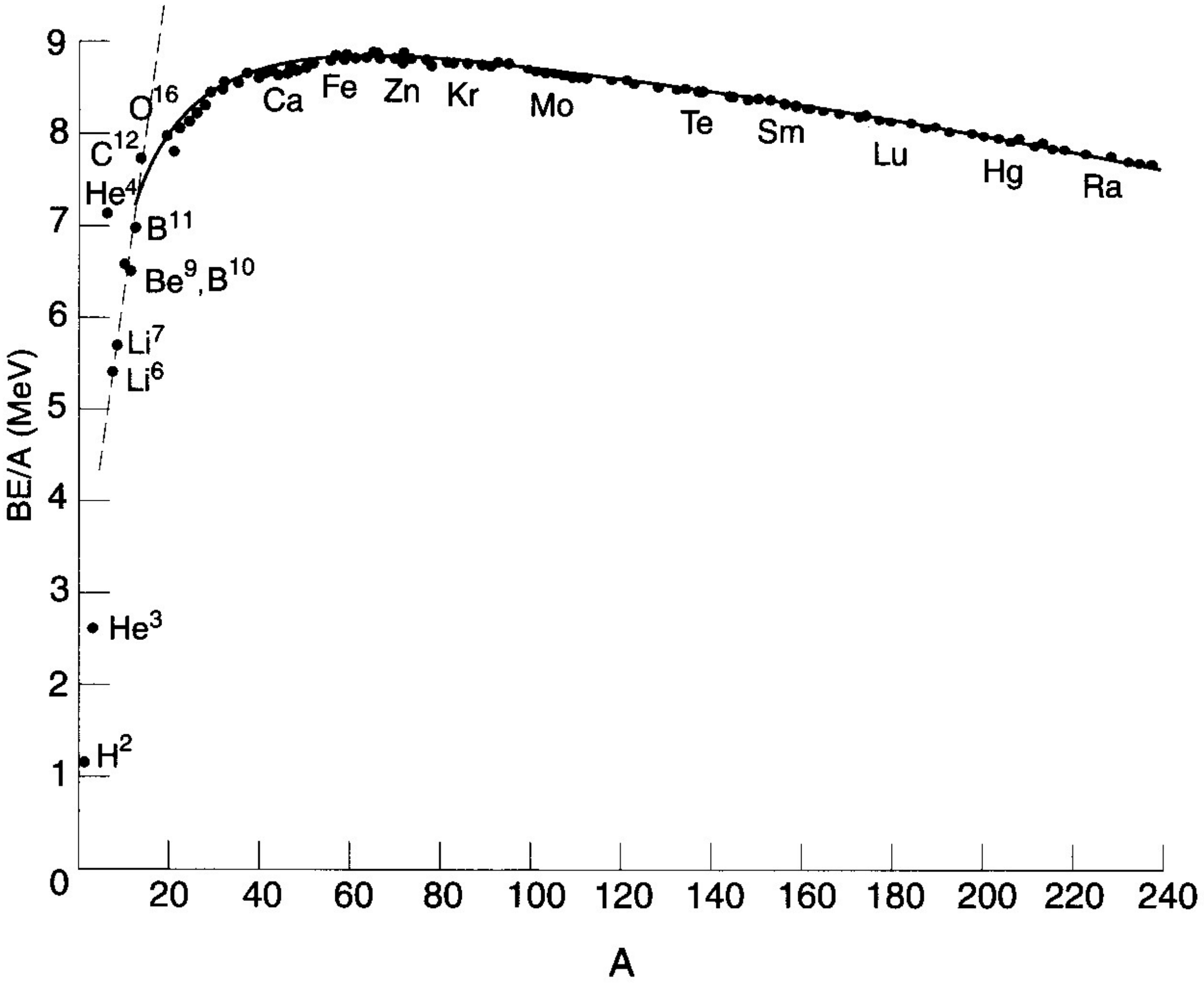
the range of nuclear interaction:
~ $1.1 \times 10^{1/3}$ = 2.37 fm

Yukawa interaction:

$$v(r) = -g\frac{e^{-\kappa r}}{r}$$

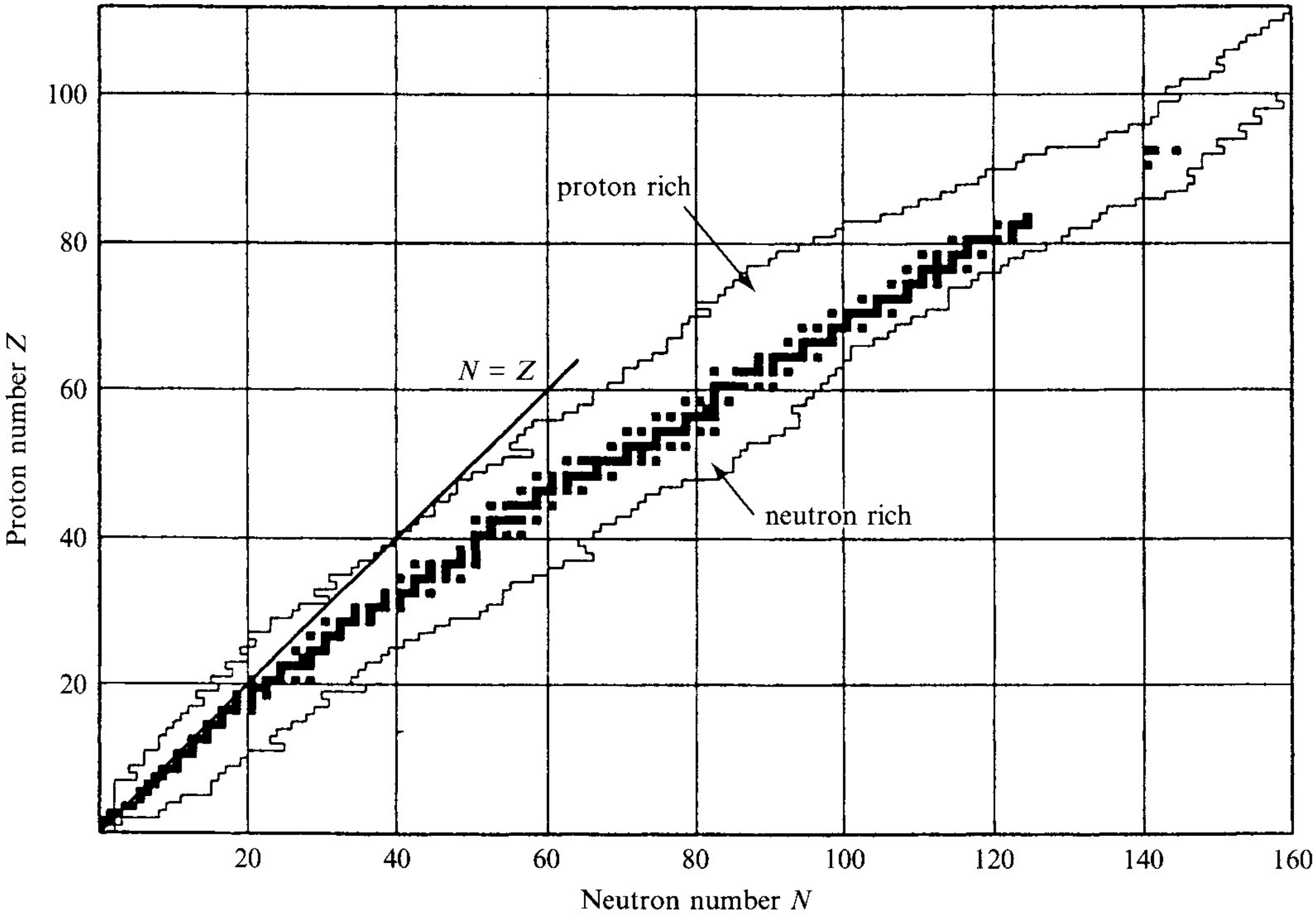
$$\frac{1}{\kappa} = \frac{\hbar}{m_\pi c} = 1.41 \text{ fm}$$



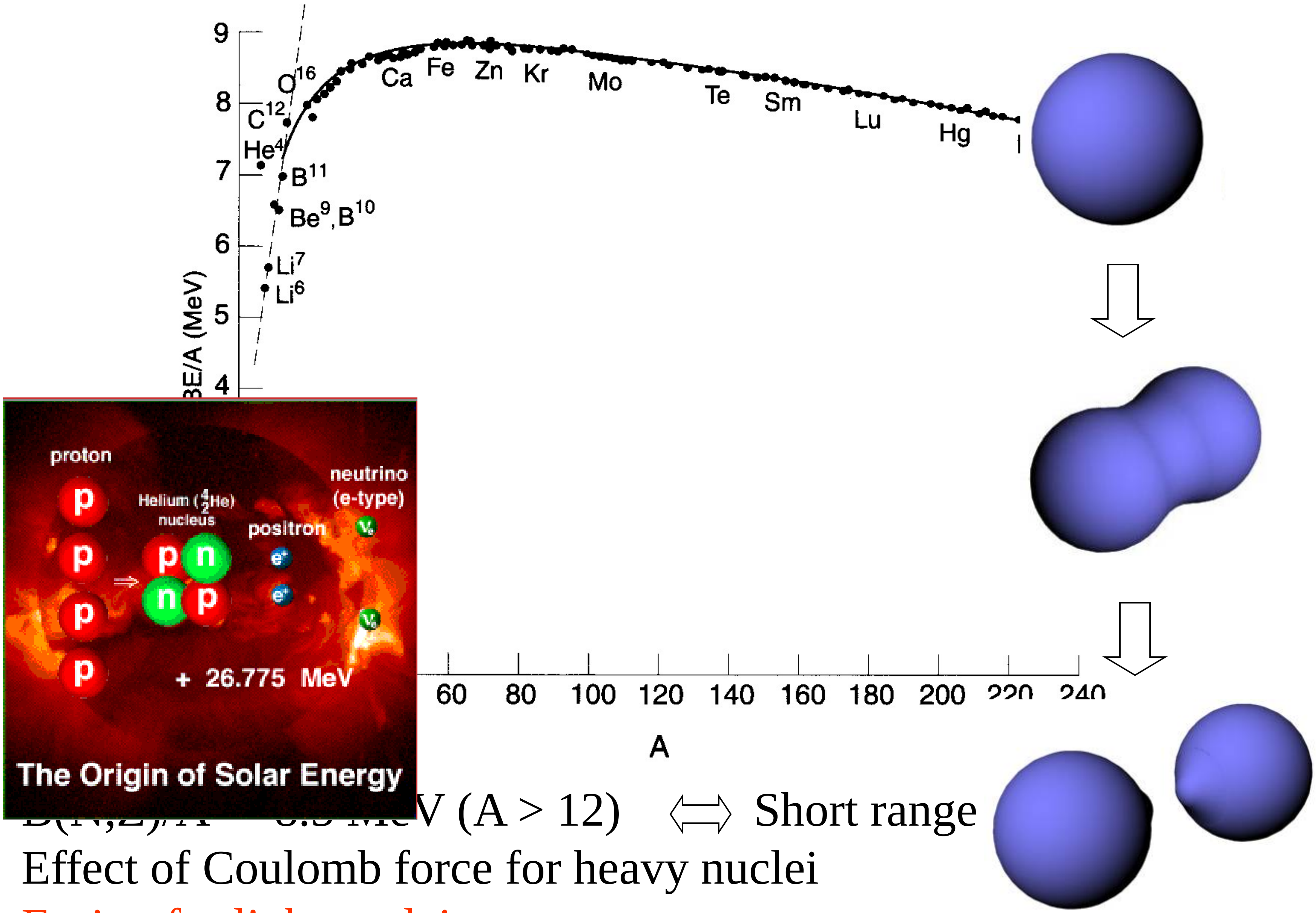


1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force
2. Effect of Coulomb force for heavy nuclei

Nuclear Chart



Stable nuclei: $N \geq Z$



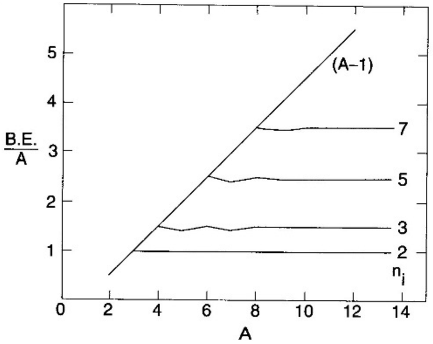
1. $B(N,Z)/A \sim 8.5$ MeV ($A > 12$) ⟺ Short range
2. Effect of Coulomb force for heavy nuclei
3. Fusion for light nuclei
4. Fission for heavy nuclei

Semi-empirical mass formula

(Bethe-Weizacker formula: Liquid-drop model)

$$\boxed{B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}}$$

- Volume energy: $a_v A$

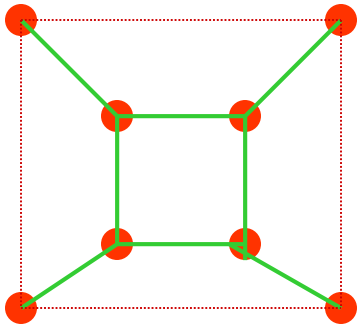


$$R_0 \sim 1.1 \times A^{1/3} \rightarrow V \propto A$$

$$S \propto A^{2/3}$$

- Surface energy: $-a_s A^{2/3}$

A nucleon near the surface
interacts with fewer nucleons.



$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$

- Coulomb energy: $-a_C\, Z^2/A^{1/3}$

$$E_C = \frac{3}{5}\frac{Z^2 e^2}{R_C}$$ for a uniformly charged sphere

- Symmetry energy: $-a_{\mathrm{sym}}\,(N-Z)^2/A$

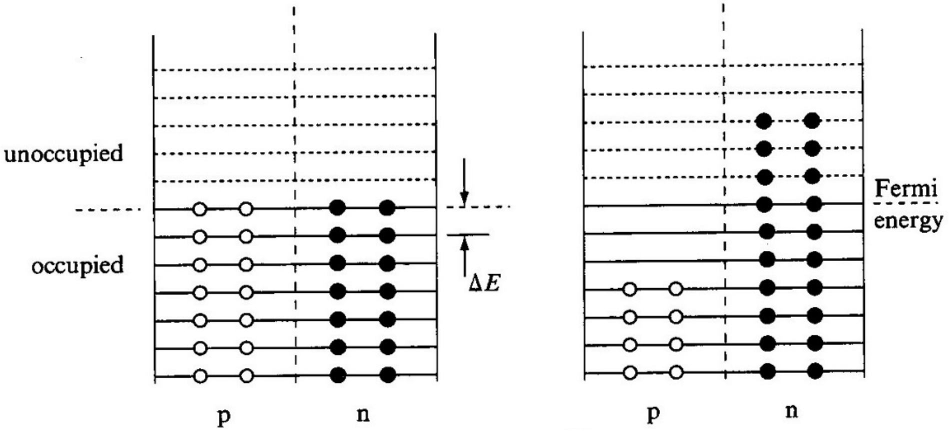
Potential energy $v_{nn} = v_{pp} = v, \quad v_{np} \sim 2v$

a nucleon interacting with nuclear matter:

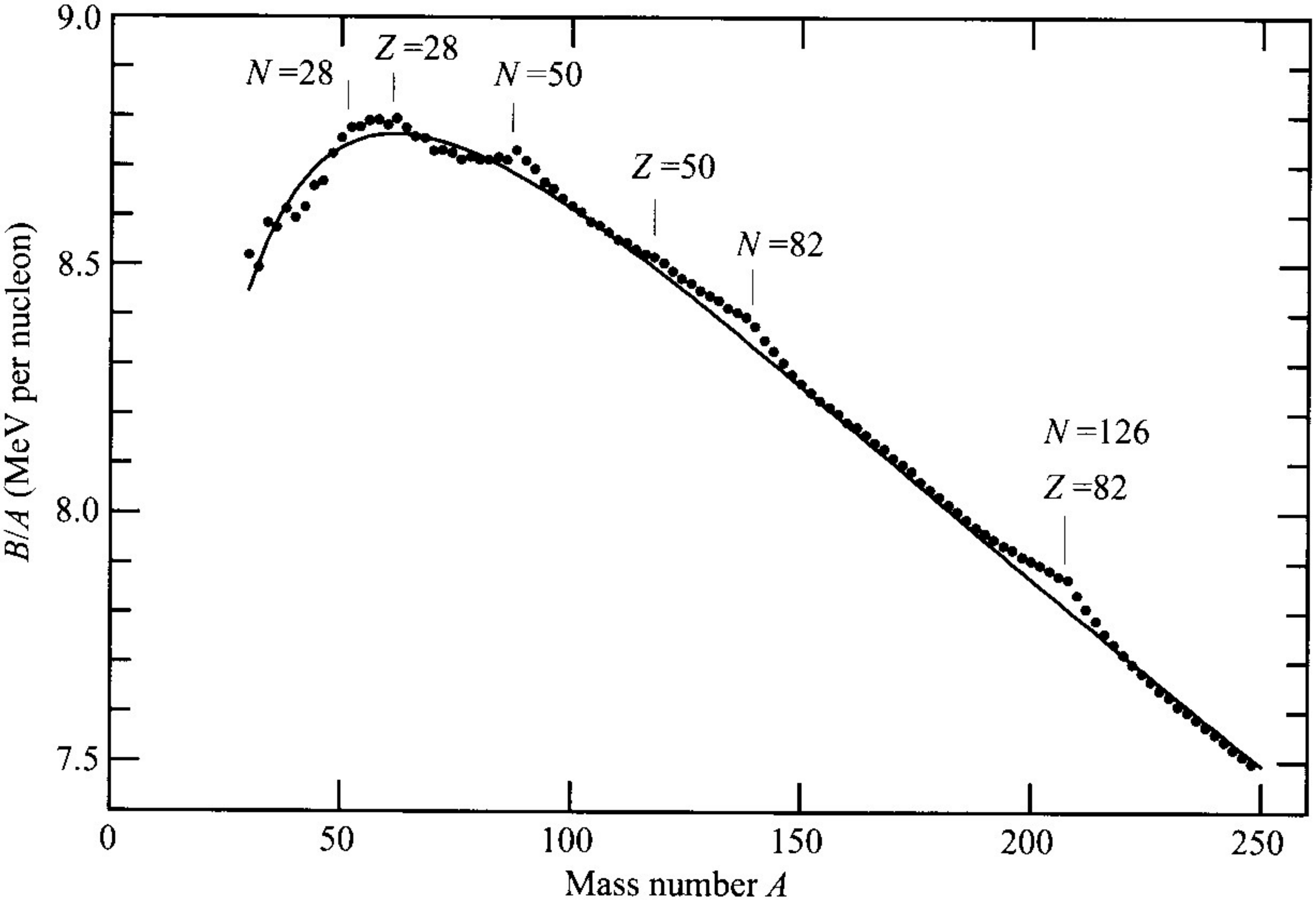
$$N(v_{nn}N/A + v_{pn}Z/A) + Z(v_{pn}N/A + v_{pp}Z/A) = \frac{v}{2}\left(3A - (N-Z)^2/A\right)$$

Kinetic energy

Pauli exclusion principle



How well does the Bethe-Weizacker formula reproduce the data?

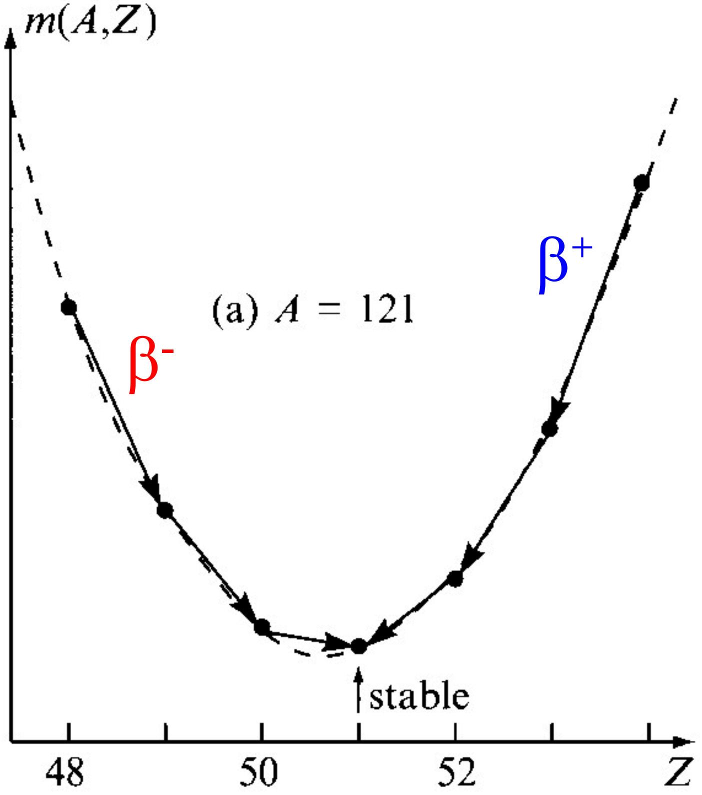


not so bad

β-stability line

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\rm sym}\frac{(N-Z)^2}{A}$$

$$\boxed{m(A,Z) = f(A) + a_C \frac{Z^2}{A^{1/3}} + a_{\rm sym}\frac{(A-2Z)^2}{A}}$$



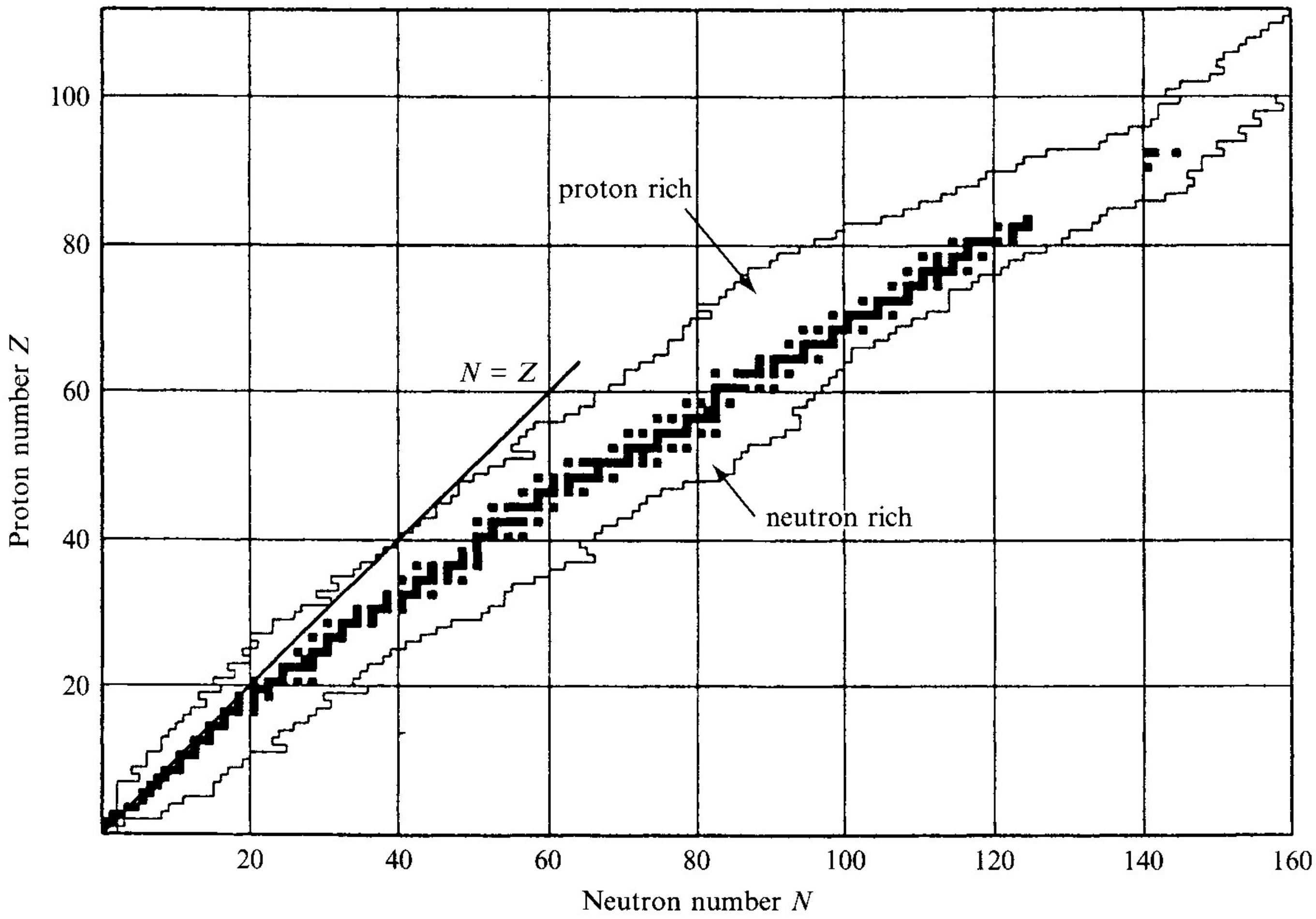
Stable nuclei (beta-stability line)

$$\left.\frac{\partial m}{\partial Z}\right|_{A=const.} = 0$$

$$Z = \frac{4a_{\rm sym}}{2a_C/A^{1/3} + 8a_{\rm sym}/A}$$

⇨ $Z < A/2$

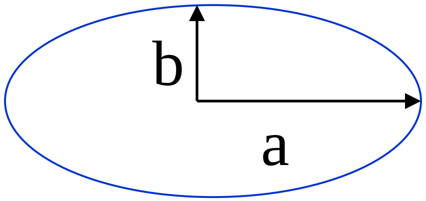
Nuclear Chart



Stable nuclei: $N \geq Z$

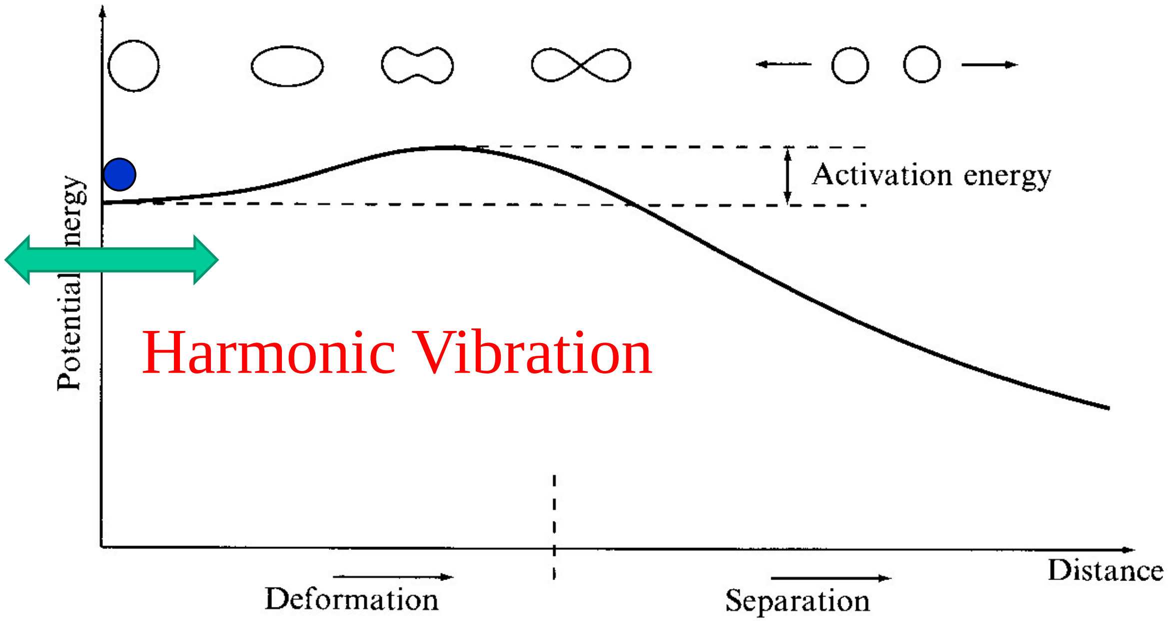
Collective Vibrations

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$



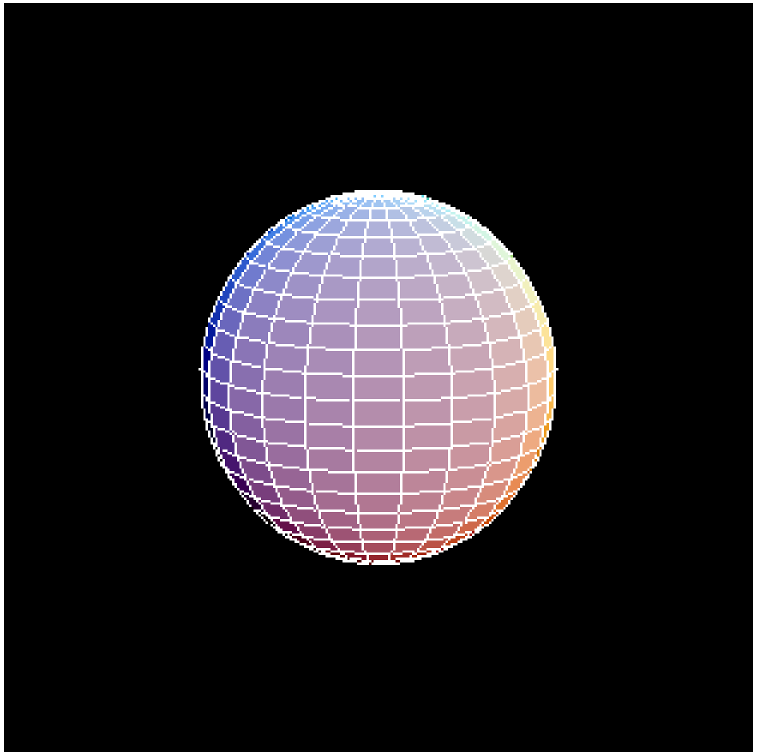
$$\begin{aligned} a &= R \cdot (1+\epsilon) \\ b &= R \cdot (1+\epsilon)^{-1/2} \end{aligned}$$

$$\begin{aligned} E_{\mathrm{surf}} &= E_{\mathrm{surf}}^{(0)} \left(1 + 2\epsilon^2/5 + \cdots\right) \\ E_C &= E_C^{(0)} \left(1 - \epsilon^2/5 + \cdots\right) \end{aligned}$$

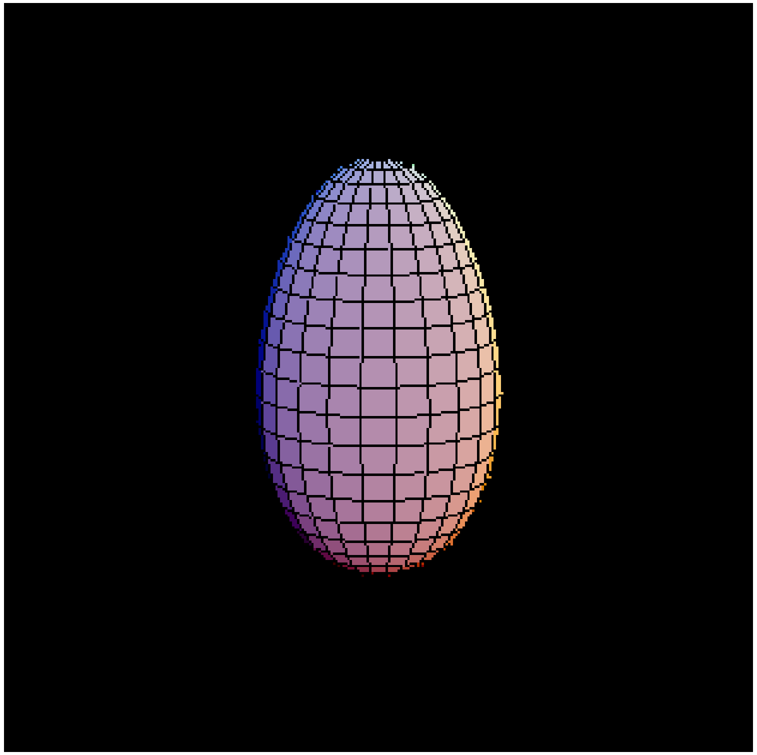


Harmonic Vibration

$$R(\theta,\phi) = R_0\left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}\right) \qquad V = \frac{1}{2}\sum_{\lambda,\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$



λ=2: Quadrupole vibration



λ=3: Octupole vibration

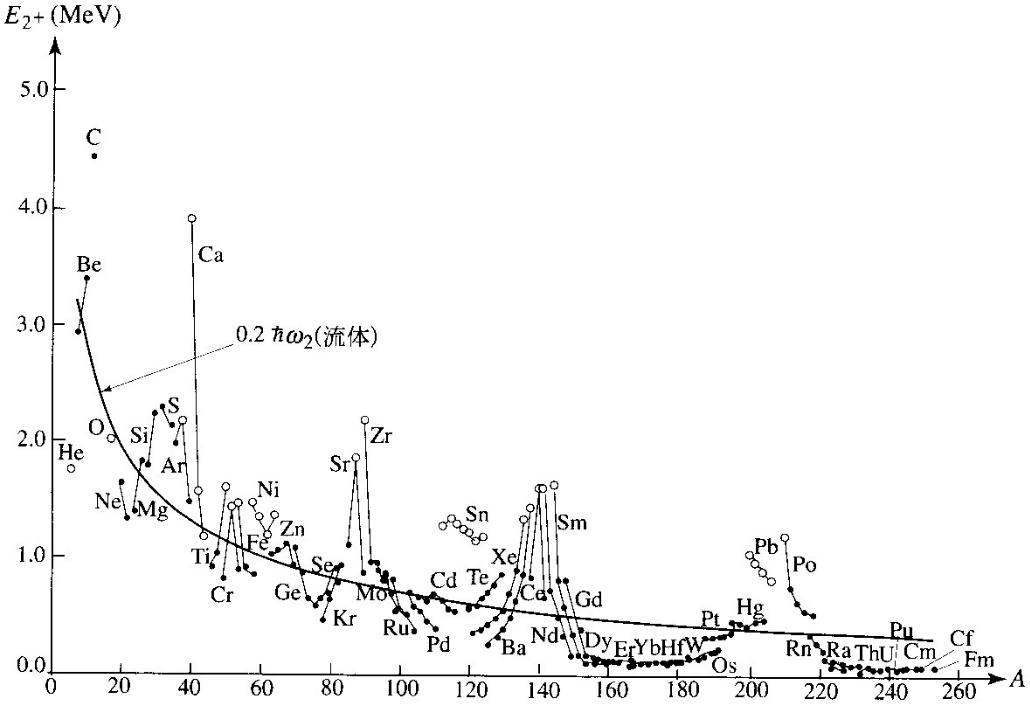
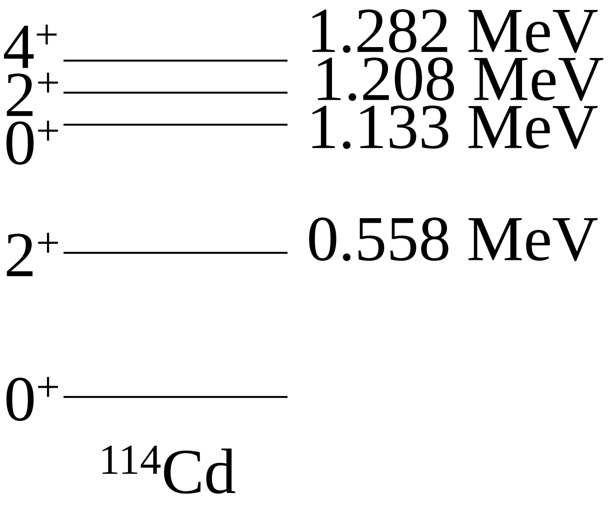


図 3.2 偶々核の第 1 励起 2^+ 状態の励起エネルギー

Double phonon states

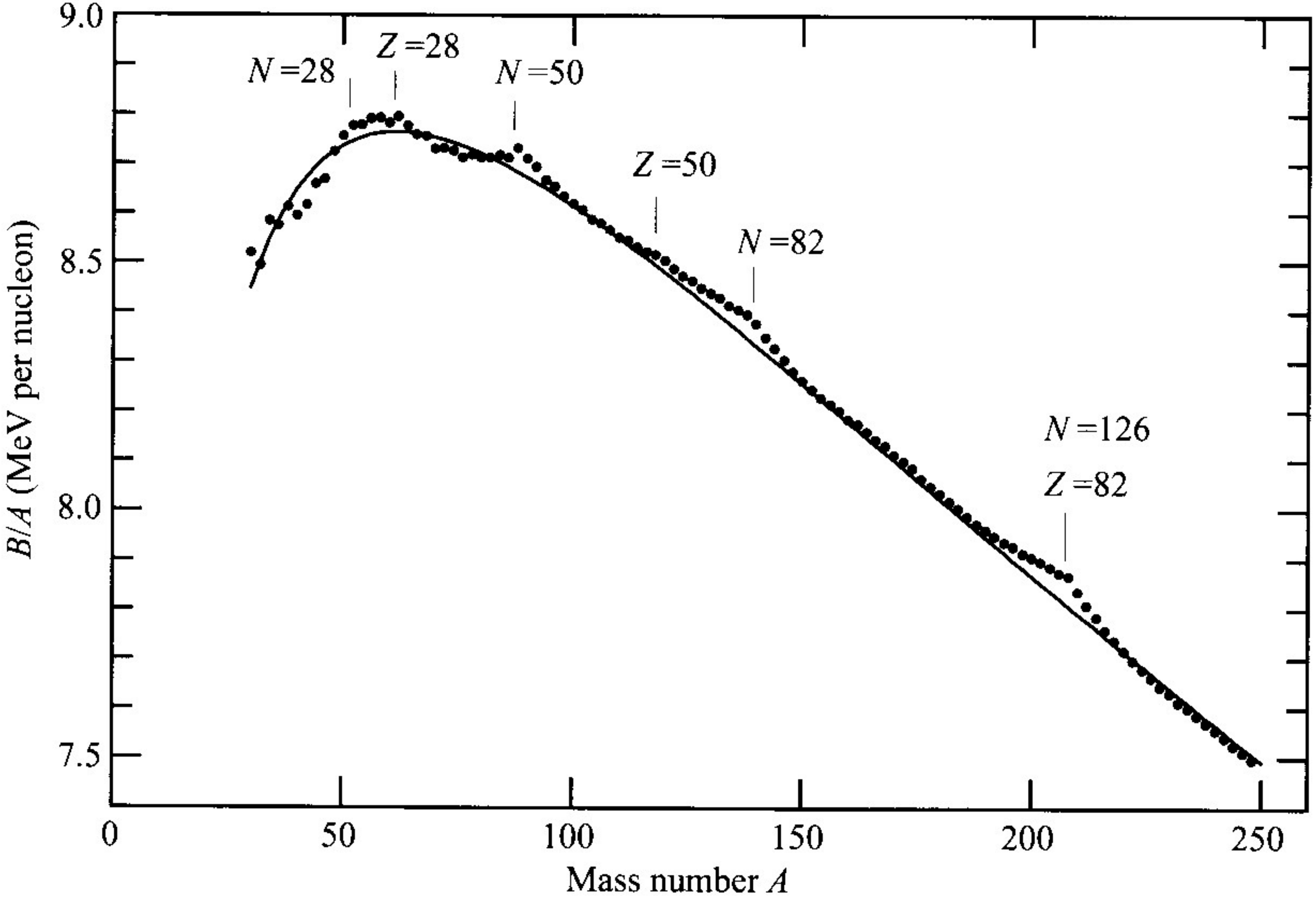


Microscopic description

⇒ Random phase approximation (RPA)

[later in this lecture]

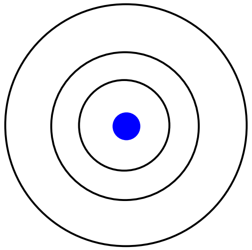
How well does the Bethe-Weizacker formula reproduce the data?



- ✓ N,Z = 2, 8, 20, 28, 50, 82, 126: large binding energy
  “magic numbers” → next week
- ✓ Pairing

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)



shell structure

元素の周期表

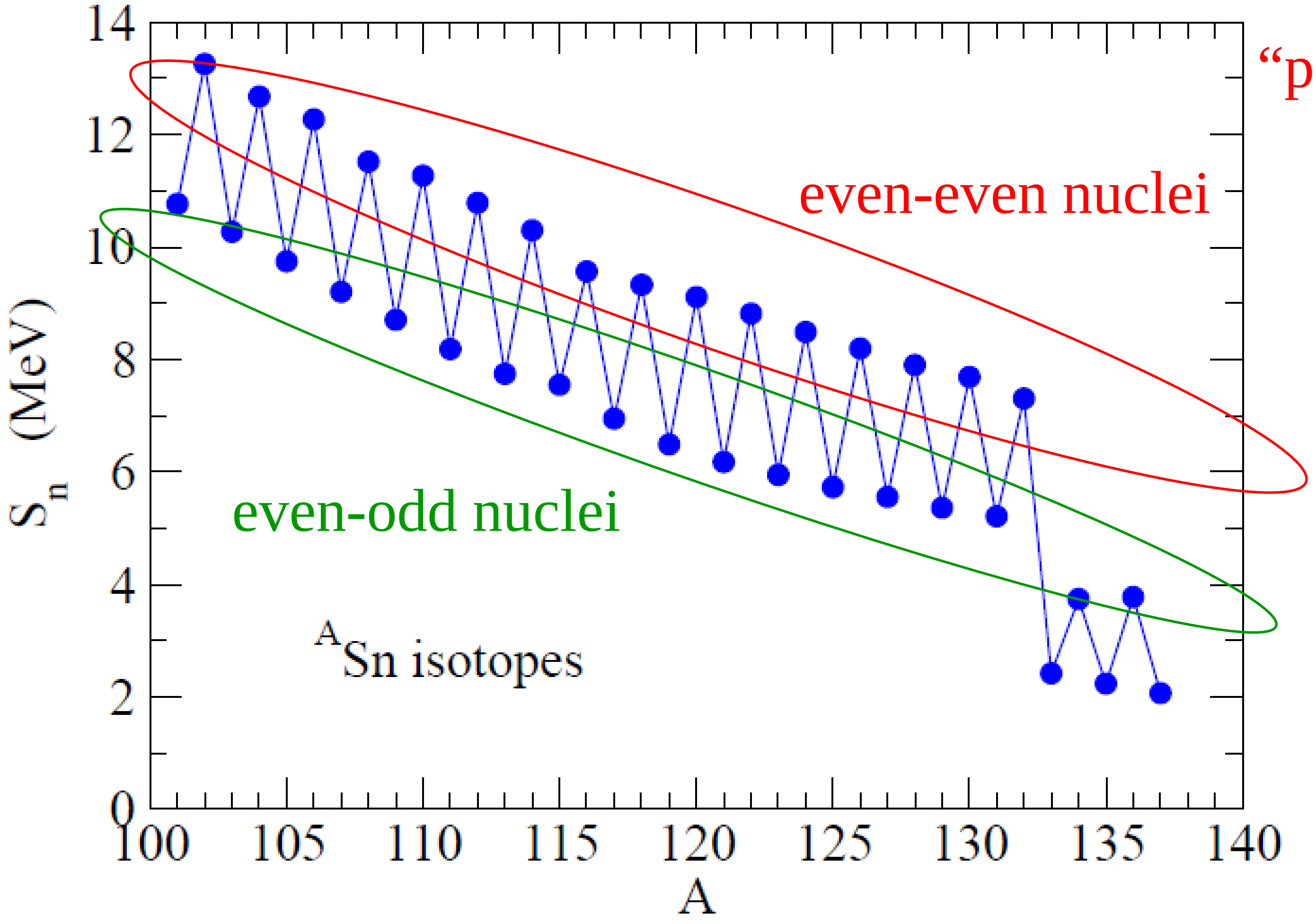
	1A	2A	3A	4A	5A	6A	7A	8			1B	2B	3B	4B	5B	6B	7B	0
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 C	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 B	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	L	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 A	86 Rn
7	87 Fr	88 Ra	A															
		L	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
		A	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

- 🟧 典型金属元素
- 🟩 半金属元素
- 🟦 非金属元素
- 🟨 遷移金属元素
- 🟥 希ガス

Pairing energy

A larger energy required to remove one neutron from nuclei with even A than from those with odd A

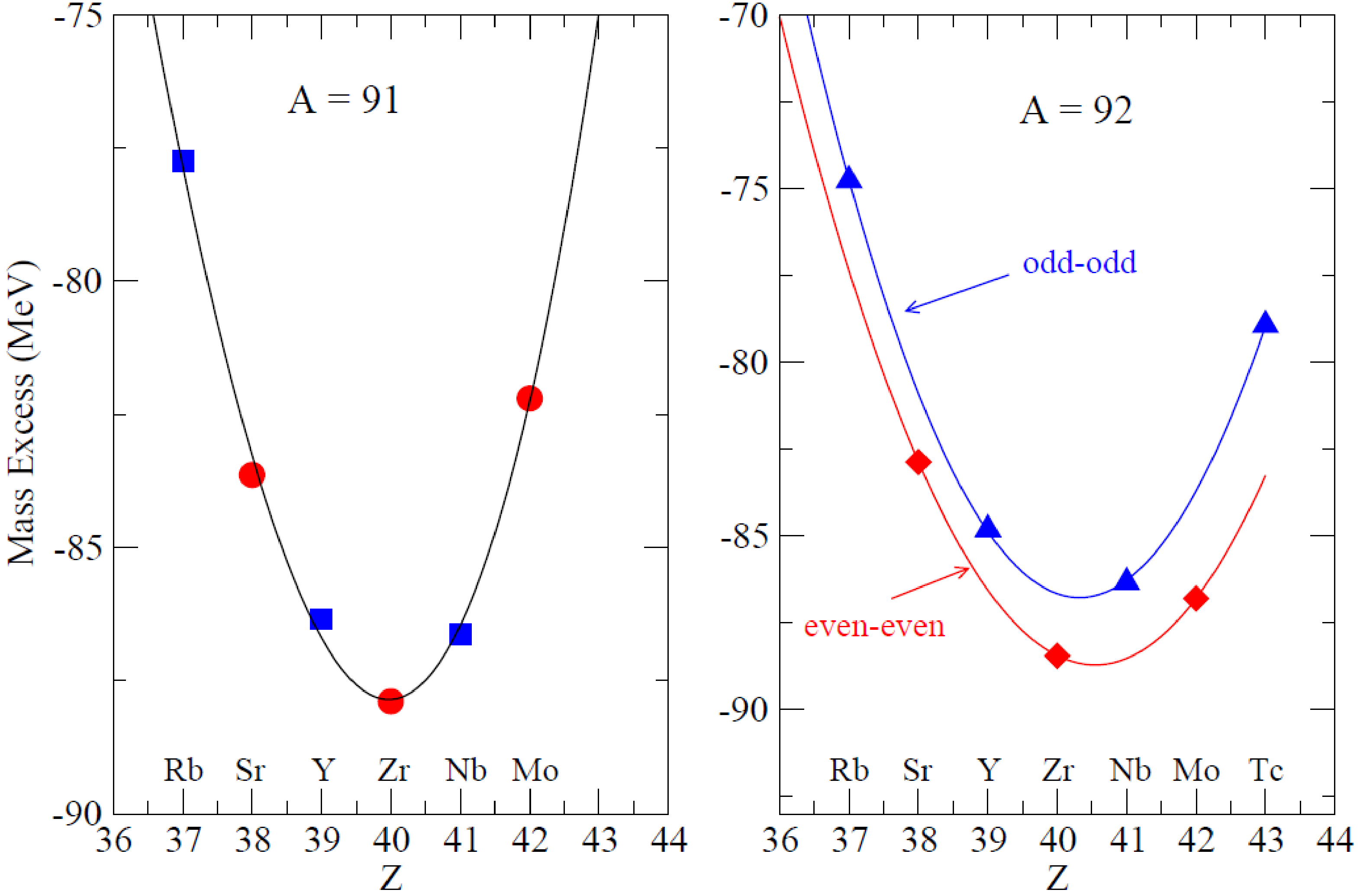
even-odd staggering

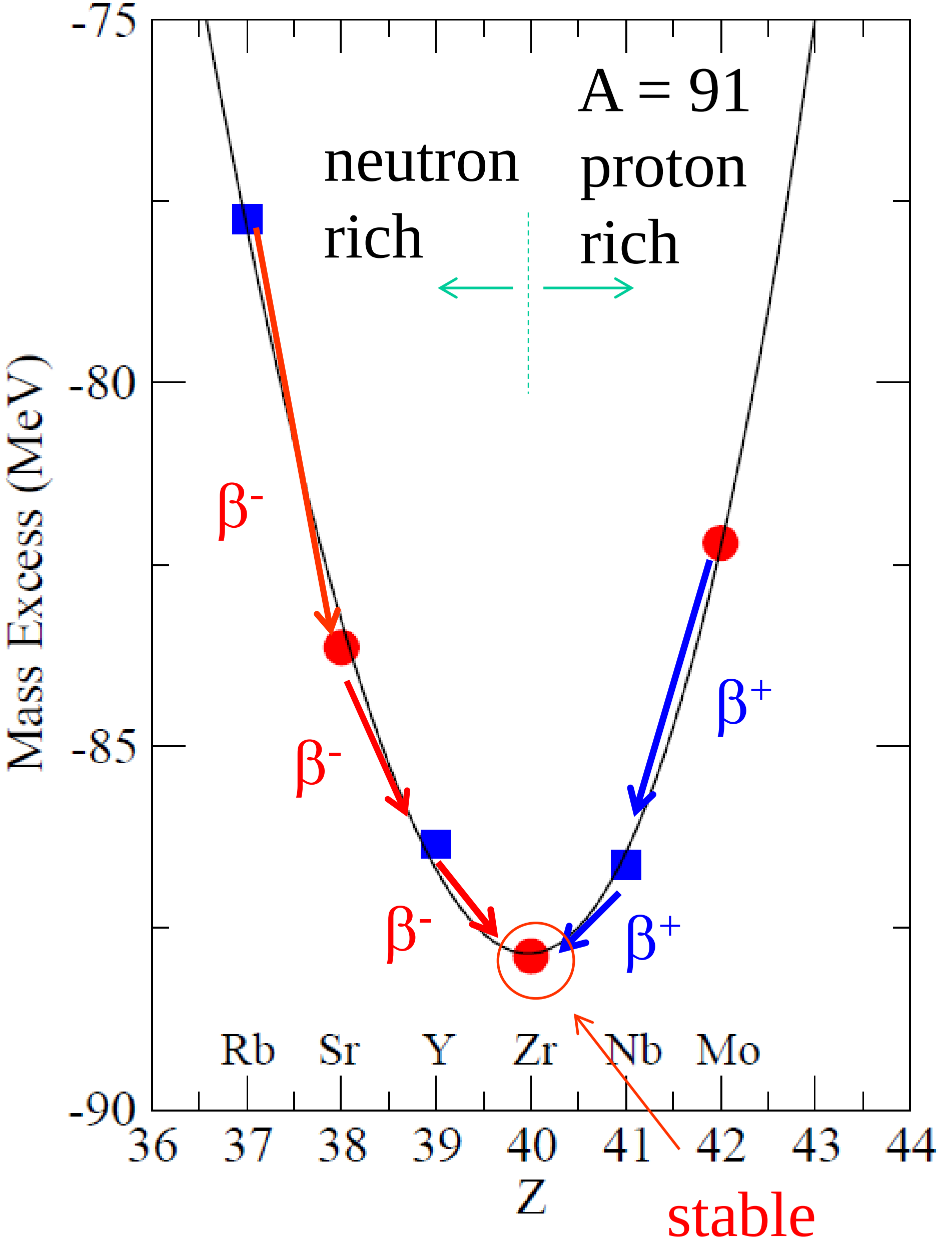


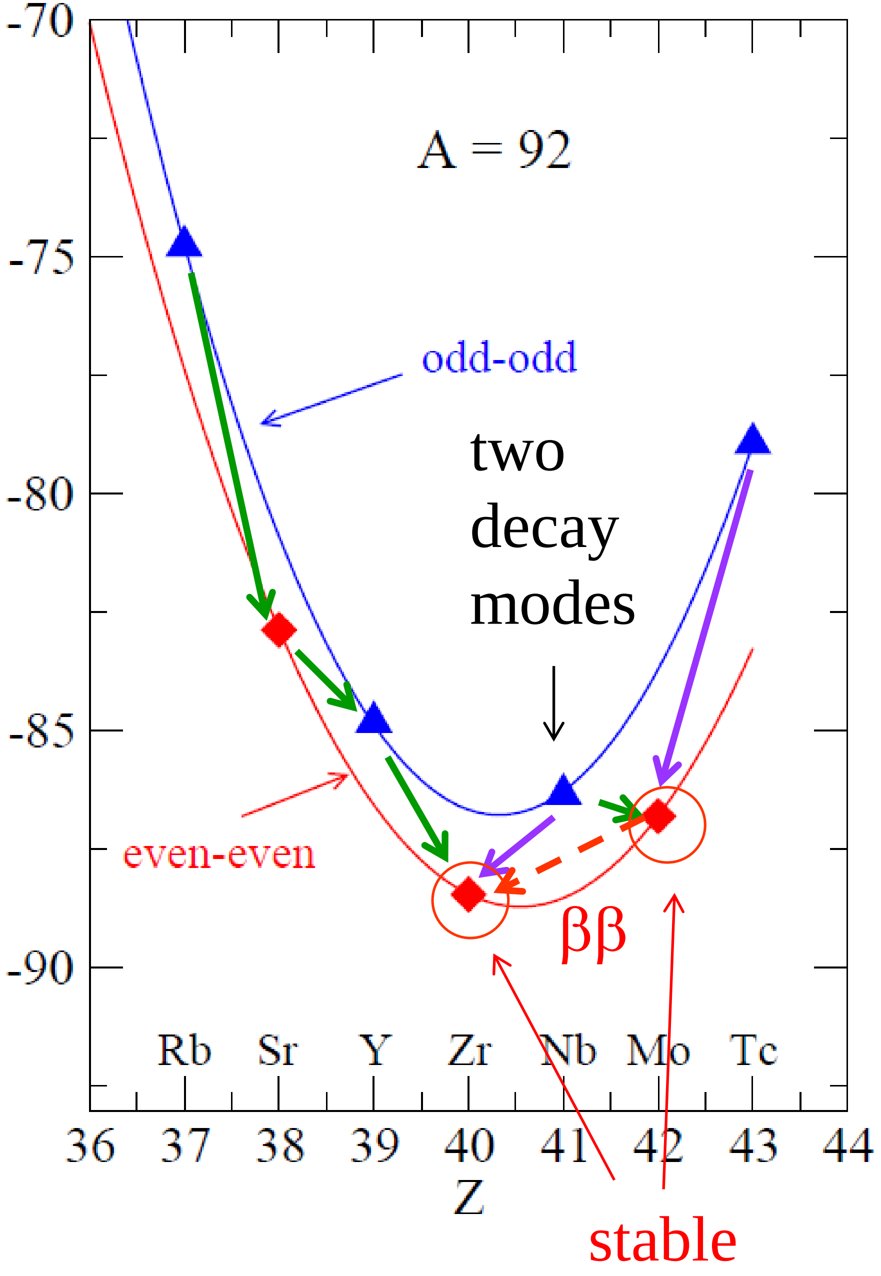
“pair correlation”

1n separation energy: $S_n(A,Z) = B(A,Z) - B(A-1,Z)$

$$
\begin{aligned}
B_{\text{pair}} &= \Delta && \text{(for even – even)} \\
&= 0 && \text{(for even – odd)} \\
&= -\Delta && \text{(for odd – odd)}
\end{aligned}
$$

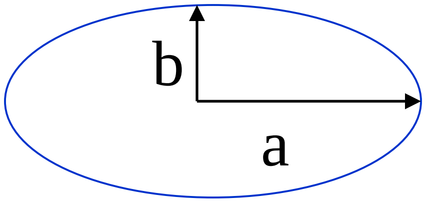



A = 91
neutron rich
proton rich
β-
β-
β-
β+
β+
Rb
Sr
Y
Zr
Nb
Mo
stable
Mass Excess (MeV)
-75
-80
-85
-90
36
37
38
39
40
41
42
43
44
Z


A = 92
odd-odd
even-even
two decay modes
ββ
Rb
Sr
Y
Zr
Nb
Mo
Tc
stable
-70
-75
-80
-85
-90
36
37
38
39
40
41
42
43
44
Z

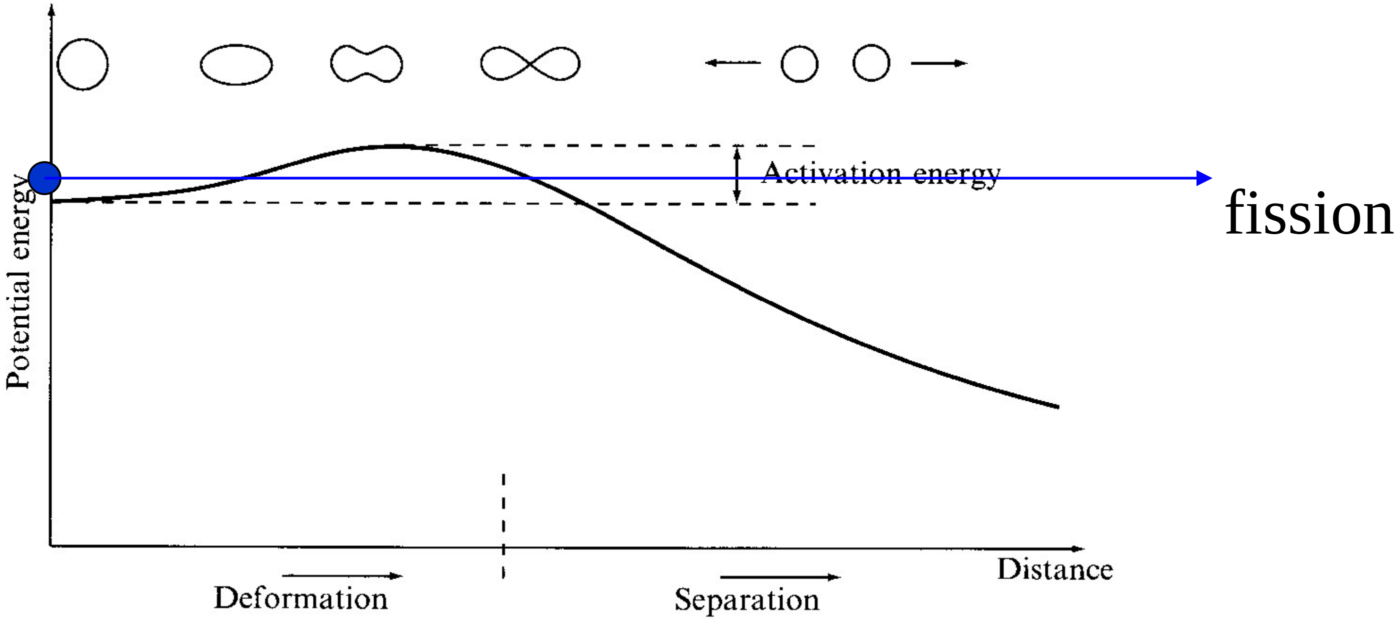
Nuclear Fission

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$



$$\begin{aligned} a &= R \cdot (1+\epsilon) \\ b &= R \cdot (1+\epsilon)^{-1/2} \end{aligned}$$

$$\begin{aligned} E_{\mathrm{surf}} &= E_{\mathrm{surf}}^{(0)} (1 + 2\epsilon^2/5 + \cdots) \\ E_C &= E_C^{(0)} (1 - \epsilon^2/5 + \cdots) \end{aligned}$$



Why is ^{235}U, rather than ^{238}U, used in a nuclear power plant ?

Natural Uranium

^{238}U	99.2742%
^{235}U	0.7204%
^{234}U	0.0054%

Only ^{235}U can be used for nuclear power plants

$$^{235}\mathrm{U} + \mathrm{n} \rightarrow {}^{236}\mathrm{U}^* \rightarrow \text{ fission}$$

$$^{238}\mathrm{U} + \mathrm{n} \rightarrow {}^{239}\mathrm{U}^* \rightarrow \text{ negligible fission probability}$$

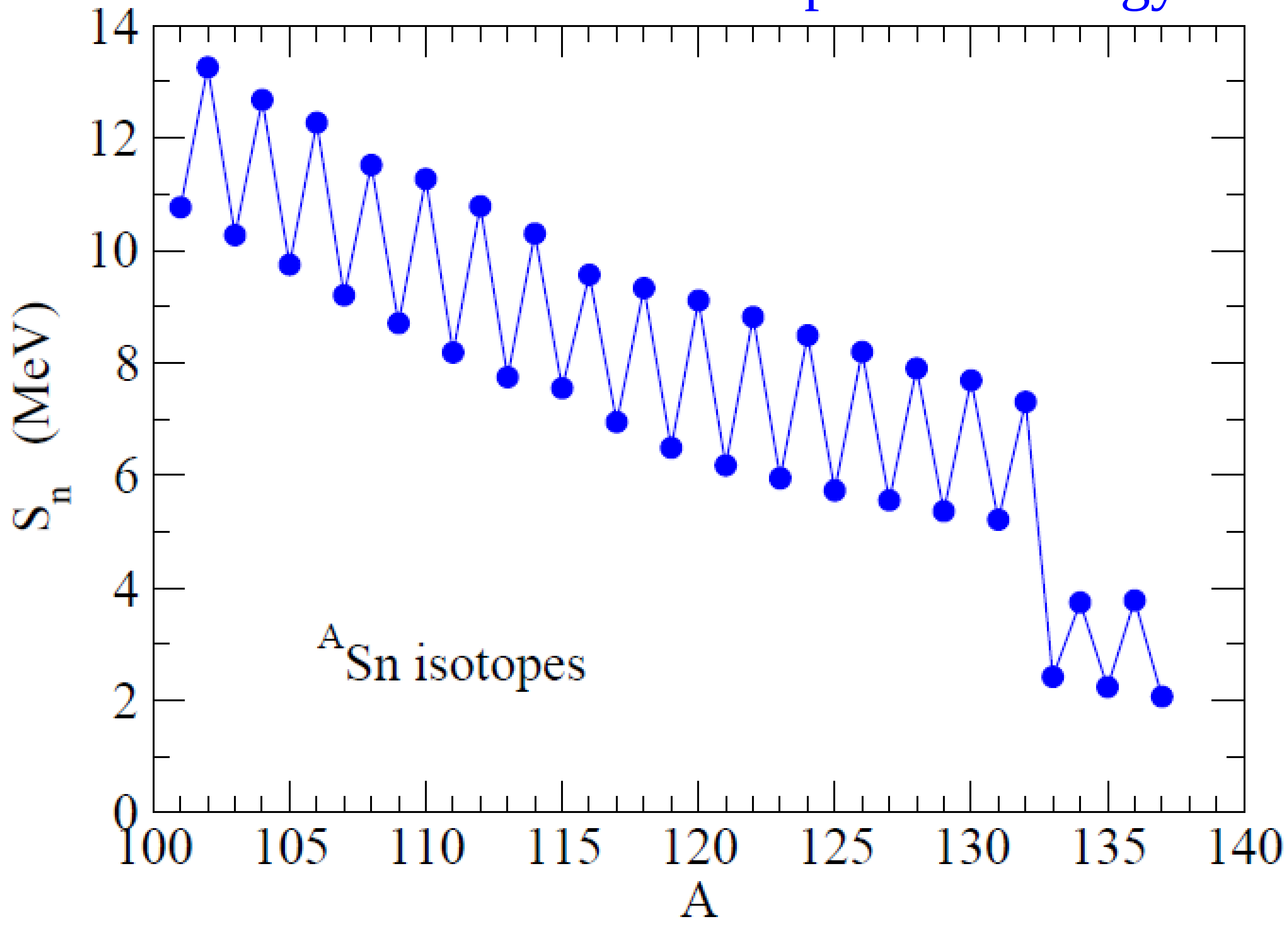
Why?

A big difference: 1n separation energy of ^{236}U and ^{239}U

$S_n(^{236}U) = 6.3$ MeV

$S_n(^{239}U) = 4.8$ MeV

Odd-even difference in separation energy



1n separation energy: $S_n(A,Z) = B(A,Z) – B(A-1,Z)$

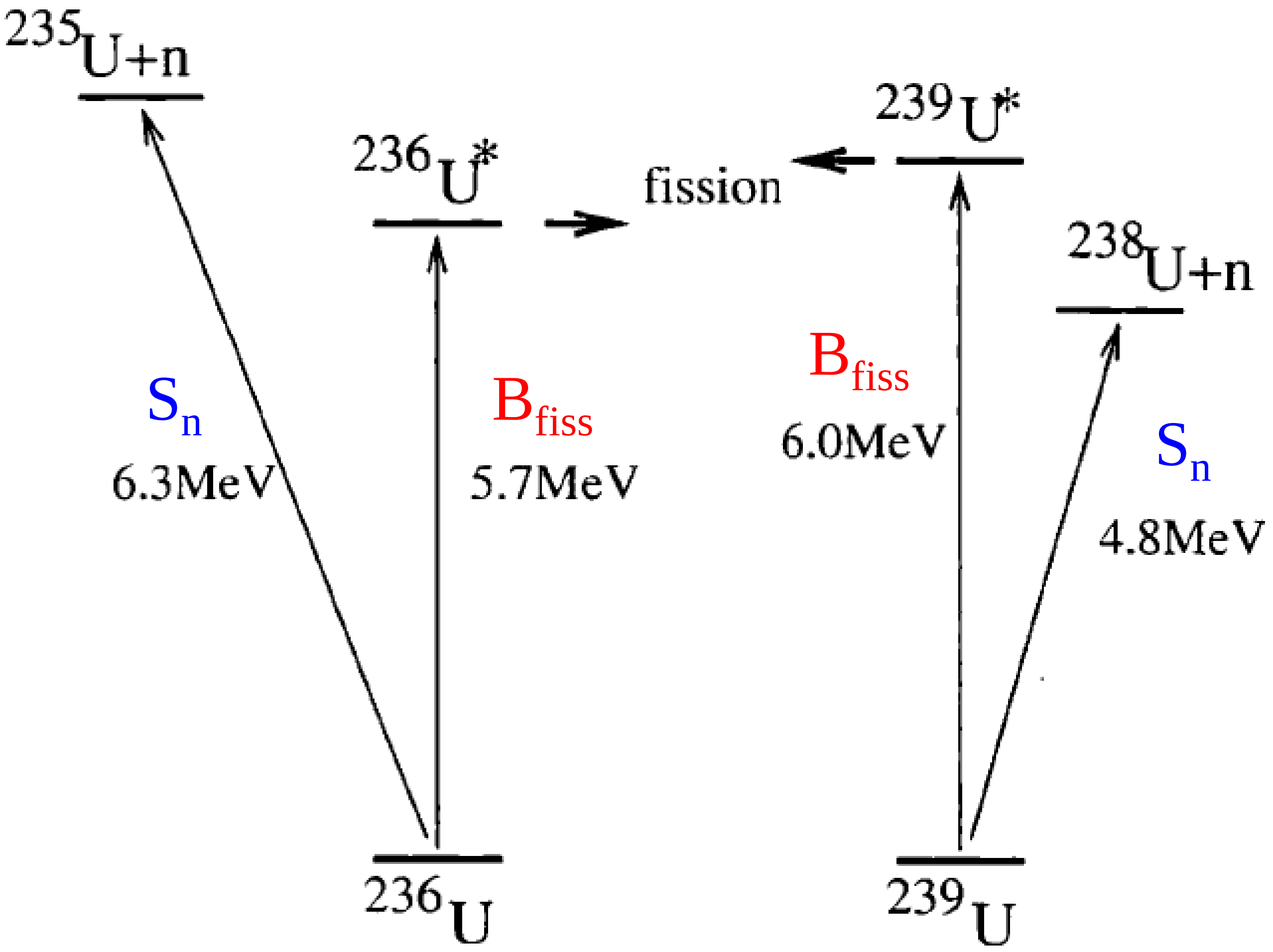
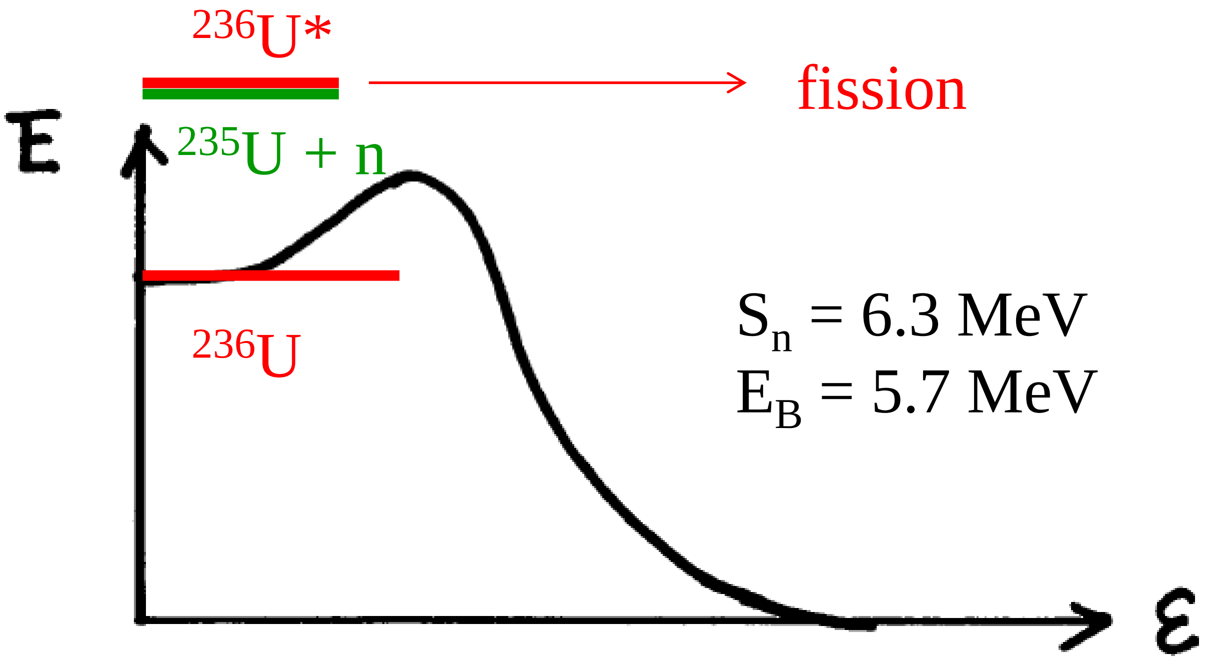


Fig. 6.6. Levels of the systems $A = 236$ and $A = 239$ involved in the fission of ^{236}U and ^{239}U. The addition of a motionless (or thermal) neutron to ^{235}U can lead to the fission of ^{236}U. On the other hand, fission of ^{239}U requires the addition of a neutron of kinetic energy $T_n = 6.0 - 4.8 = 1.2\,\text{MeV}$.

Relation between fission barrier height and 1n separation energy



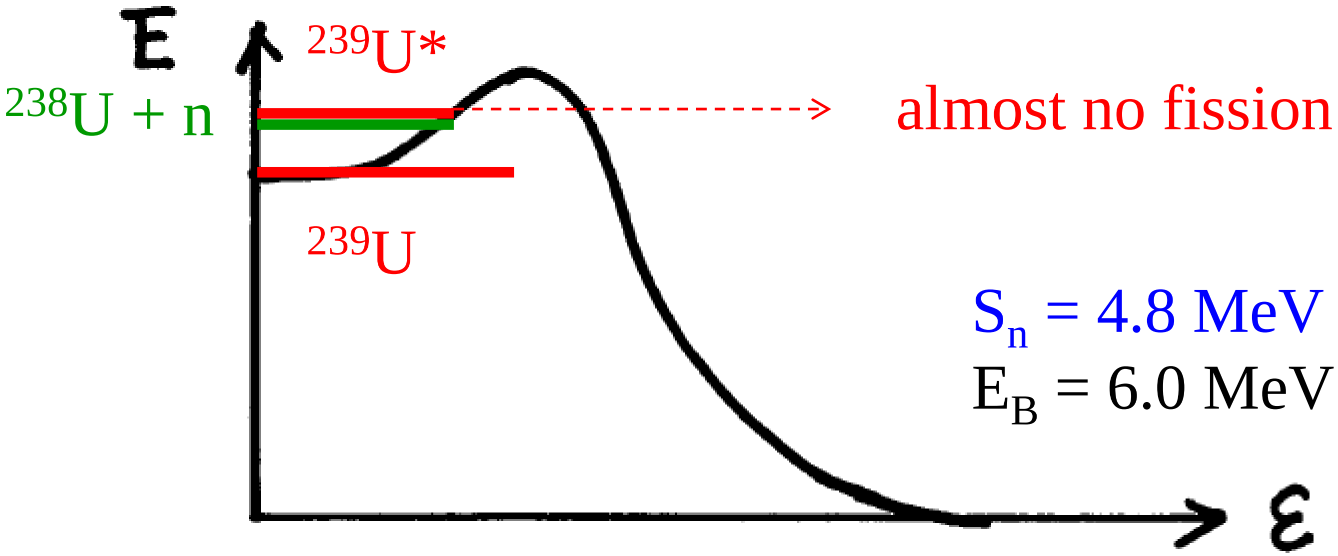
^{236}U*

^{235}U + n

fission

^{236}U

S_n = 6.3 MeV
E_B = 5.7 MeV



^{239}U*

^{238}U + n

almost no fission

^{239}U

S_n = 4.8 MeV
E_B = 6.0 MeV