excited states
excited states of nuclei

the ground state
excited states of nuclei

\[ \begin{align*}
\text{ground state} & \quad 1s_{1/2} \\
& \quad 2s_{1/2} \\
& \quad 1d_{5/2} \\
& \quad 1p_{3/2} \\
& \quad 1p_{1/2} \\
\end{align*} \]

the ground state
excited states of nuclei

the ground state
In atomic nuclei
\[ \hbar \omega \sim 41 A^{-1/3} \text{ (MeV)} \]
\[ R \sim 1.2 A^{1/3} \text{ (fm)} \]
16.27 MeV for \( A=16 \)

cf. Actually, there is an \( 1^- \) state at 16.2 MeV in \( ^{16}\text{O} \)

.... but this is not the whole story: collective excitations
Collective Vibrations

How does a nucleus respond to an external perturbation?

i) Photo absorption cross section

The state is strongly excited when

\[ E_f - E_i = E_\gamma. \]
Giant Dipole Resonance (GDR)

$^{197\text{Au}}$ 

\[
\int \sigma \, dE = (300 \pm 30) \text{ MeV fm}^2 \\
8 \text{ MeV} = (1.1 \pm 0.1) \frac{N}{A} \text{ MeV fm}^2
\]

\[\sigma = \frac{\left( \frac{\Gamma}{2} \right)^2}{(E-E_{\text{res}})^2 + \left( \frac{\Gamma}{2} \right)^2} \frac{E}{E_{\text{res}}} \sigma_{\text{res}}
\]

$E_{\text{res}} = 13.9 \text{ MeV}$  
$\Gamma = 4.2 \text{ MeV}$  
$\sigma_{\text{res}} = 53 \text{ fm}^2$

**Figure 6-18** Total photoabsorption cross section for $^{197\text{Au}}$. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* 127, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.
Remarks

i) Photon interaction \( \xrightarrow{\text{dipole excitation}} \)

\[
H_{\text{int}} = \frac{1}{2mc} (p \cdot A + A \cdot p)
\]

\[
A(r, t) = \sum_k \sum_{\alpha=1,2} \sqrt{\frac{2\pi c^2 \hbar}{\omega V}} (a_{k\alpha} \epsilon_{\alpha} e^{ik \cdot r - i\omega_k t} + h.c.)
\]

\[
e^{ik \cdot r} \sim 1 \quad \text{(dipole approximation)}
\]

\[
\sigma_{\text{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c} (E_f - E_i) |\langle \phi_f | \tilde{z} | \phi_i \rangle|^2 \delta(E_\gamma - E_f + E_i)
\]

ii) Isospin

Isoscalar dipole motion

\[
\tilde{z} = \sum_p (z_p - Z_{cm})
\]

Isovector type

(proton) → (neutron)

(note)

iii) Collective motion

Motion of the whole nucleus rather than a single-particle motion
Fig. 1.2. The photo-neutron cross section \( \sigma(\gamma, n) \) as a function of the photon energy for the three nuclei \(^{208}\text{Pb}, ^{120}\text{Sn} \) and \(^{65}\text{Cu} \). Note that for these nuclei \( \sigma(\gamma, n) \approx \sigma_{\alpha\beta}(\gamma) \). From reference (BER75).
Bohr-Mottelson

“Nuclear Structure vol. II”

Isovector type

\[ E_{\text{res}} \sim 79A^{-1/3} \text{ (MeV)} \]

\[ E_{\text{GDR}} \propto A^{-1/3} \propto 1/R \]
proton
neutron

\[ E_{\text{GDR}} \propto \frac{1}{R} \]

deformed nucleus

Figure 6.18 Total photoabsorption cross section for \(^{197}\text{Au}\). The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, \textit{Phys. Rev.} 127, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.
spherical nucleus

(prolate deformation)

deformed nucleus

$E_{GDR} \propto 1/R$
M.N. Harakeh and A. van der Woude, “Giant Resonances”

(prolate deformation)

deformed nucleus

\[
\begin{align*}
\text{deformed nucleus} & \quad (\text{prolate deformation}) \\
& \quad E \quad x, y \quad z
\end{align*}
\]
Deformation effect

\[ \hbar \omega \sim A^{-1/3} \sim 1/R \]

Figure 6.21 Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssière, *Nuclear Phys.* **A172**, 437 (1971). The solid curves represent Lorentzian fits with the parameters given in Table 6.6.
Giant Dipole Resonances

- Goldhaber-Teller type

\[ \hat{Q} = r Y_{1\mu}(\hat{r}) \tau_z \]

\[ \hbar \omega \sim A^{-1/6} \]

Inconsistent with expt. (except for light nuclei)

\[ E_{\text{res}} \sim 79A^{-1/3} \text{ (MeV)} \]
Giant Dipole Resonances

- **Goldhaber-Teller type**

\[
\hat{Q} = r Y_{1\mu}(\hat{r})\tau_z
\]

\[
\hbar\omega \sim A^{-1/6}
\]

- **Steinwedel-Jensen type**

\[
\hat{Q} = j_1(kr)Y_{1\mu}(\hat{r})\tau_z
\]

\[
\hbar\omega \sim A^{-1/3}
\]

\[kR = 2.08\]

\[
j_1(x) = (\sin x - x \cos x)/x^2
\]
FIG. 1. Schematic drawings that serve to illustrate the general features of the Goldhaber-Teller (Ref. 3) (GT) and Steinwedel-Jensen (Ref. 4) (SJ) dipole modes.

\[ \hbar \omega \sim 31A^{-1/3} + 21A^{-1/6} \text{ (MeV)} \]

J.D. Myers et al., PRC15(’77)2032
ii) Inelastic scattering

(e,e’), (p,p’), (α,α’), Heavy-ion → Higher multipolarities

\[ \Delta L = 0 \]
(IS)GMR

\[ \Delta L = 1 \]
IVGMR

\[ \Delta L = 2 \]
ISGQR

\[ \Delta T = 0 \quad \Delta S = 0 \]
\[ \Delta T = 1 \quad \Delta S = 0 \]
\[ \Delta T = 0 \quad \Delta S = 1 \]
\[ \Delta T = 1 \quad \Delta S = 1 \]

(note) \( \Delta L = 2 \) → \( \Delta N = 2 \)    Giant Resonance (GQR)

\( \Delta N = 0 \)    Low-lying state
(e,e'), (p,p'), (α,α'), Heavy-ion → Higher multipolarities

ΔL = 1

ΔL = 2

ΔT = 0
ΔS = 0

ΔT = 1
ΔS = 0

ΔT = 0
ΔS = 1

ΔT = 1
ΔS = 1

movies: H.-J. Wollersheim,
https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html
ii) Inelastic scattering

\((e,e')\), \((p,p')\), \((\alpha,\alpha')\), Heavy-ion \rightarrow \text{Higher multipolarities}

\[\Delta L = 0\]

(IS)GMR\hspace{1cm}IVGMR\hspace{1cm}IVSGMR

\[\Delta L = 1\]

\[\Delta L = 2\]

ISGQR\hspace{1cm}IVGQR

\[\Delta T = 0\] 
\[\Delta T = 1\]
\[\Delta T = 0\] 
\[\Delta T = 1\]

\[\Delta S = 0\] 
\[\Delta S = 0\]
\[\Delta S = 1\] 
\[\Delta S = 1\]

movies: H.-J. Wollersheim,
https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html
ii) Inelastic scattering

$(\text{e,e}'$, $(p,p')$, $(\alpha,\alpha')$, Heavy-ion $\rightarrow$ Higher multipolarities

\[ \Delta L = 0 \quad \begin{array}{c}
\text{(IS)GMR} \\
\text{IVGMR} \\
\text{IVSGMR}
\end{array} \]

$\Delta L = 1$

\[ \begin{array}{c}
\text{ISGQR} \\
\text{IVGQR}
\end{array} \]

movies: H.-J. Wollersheim, 
https://web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html
Giant Multipole Resonances in $^{90}$Zr Observed by Inelastic Electron Scattering

S. Fukuda and Y. Torizuka

*Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan*

(Received 24 August 1972)

Inelastic electron scattering from the giant dipole resonance region in $^{96}$Zr was measured. In addition to the usual dipole resonance we have found new resonances at 14.0 MeV and around 28 MeV. The spins and parities and transition strengths of these states are discussed.

Electroexcitation of Giant Resonances in $^{208}$Pb

M. Nagao and Y. Torizuka

*Laboratory of Nuclear Science, Tohoku University, Tomizawa, Sendai, Japan*

(Received 27 February 1973)

The giant-resonance region in $^{208}$Pb was observed by inelastic electron scattering. We present evidence for the existence of a $2^+$ (or $0^+$) state at $\sim 22$ MeV and a $3^+$ state at $\sim 19$ MeV with giant-resonance character. The resonance states between 8.6 and 11.6 MeV are confirmed to be $2^+$ (or $0^+$) and the sum of their strengths exhausts about 50% of the $E2$ sum rule or 100% of $E0$. 

Now: Research Center for Electron Photon Science (ELPH)
EOS of infinite nuclear matter

\[ K_\infty = 9\rho^2 \frac{d^2 [E(\rho)/\rho]}{d\rho^2} \bigg|_{\rho_0} \]

\[ E[\rho] = E[\rho_0] + \frac{1}{18} K_\infty \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 \]
Isoscalar giant monopole resonances (breathing mode)

\[ E_{\text{ISGMR}} \sim \sqrt{\frac{\hbar^2 K}{m\langle r^2 \rangle}} \]

J.P. Blaizot, Phys. Rep. 64 (‘80) 171

\[ K \sim 231 \pm 5 \text{ MeV} \]

D.H. Youngblood, H.L. Clark, and Y.-W. Lui, PRL82 (‘99) 691
\[ |\psi\rangle = F|0\rangle \]
\[ = \sum_n |n\rangle \langle n|F|0\rangle \]

Sum Rule

\( F \) (external field)

\[ |0\rangle \]

\[ |\langle n|F|0\rangle|^2 \]

\[ E_1 \quad E_2 \quad E_3 \quad E_4 \quad \cdots \]

\[ E \]
**Sum Rule**

**Strength function:**

\[ S(E) = \sum_n |\langle n | F | 0 \rangle|^2 \delta(E_n - E_0 - E) \]

\( F \) (external field)

\[ |\langle n | F | 0 \rangle|^2 \]

\( E_1 \quad E_2 \quad E_3 \quad E_4 \quad \rightarrow \quad E \)

\( |1\rangle \quad + \quad |2\rangle \quad + \quad |3\rangle \quad + \ldots \)
Sum Rule

Strength function:

\[
S(E) = \sum_n |\langle n|F|0 \rangle|^2 \times \delta(E_n - E_0 - E)
\]

- **non-energy weighted sum rule**

\[
S_0 \equiv \int S(E) dE = \sum_n |\langle n|F|0 \rangle|^2
\]

- **energy weighted sum rule**

\[
S_1 \equiv \int ES(E) dE = \sum_n (E_n - E_0) |\langle n|F|0 \rangle|^2
\]
\( S_0 \equiv \int S(E) \, dE = \sum_n |\langle n | F | 0 \rangle|^2 = \langle 0 | F^2 | 0 \rangle \)

cf. geometry of Borromean nuclei

\[
B(E1) = \sum_i B(E1; gs \to i) = \frac{3}{\pi} \left( \frac{Ze}{A} \right)^2 \langle R^2 \rangle
\]

\[
\langle \theta_{nn} \rangle = 65.2^{+11.4}_{-13.0} \quad (^{11}\text{Li})
\]

\[
= 74.5^{+11.2}_{-13.1} \quad (^{6}\text{He})
\]

K.H. and H. Sagawa,
PRC76(’07)047302
energy weighted sum rule

\[ S_1 \equiv \int E S(E) dE \]
\[ = \sum_n (E_n - E_0) |\langle n | F | 0 \rangle|^2 \]
\[ = \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle \]

\[ S(E) = \sum_n |\langle n | F | 0 \rangle|^2 \times \delta(E_n - E_0 - E) \]

\[ \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle = \frac{1}{2} \langle F(HF - FH) - (HF - FH)H | 0 \rangle \]
\[ = \langle FHF - E_0 F^2 | 0 \rangle \]
\[ = \sum_n E_n |\langle 0 | F | n \rangle|^2 - E_0 \langle 0 | F^2 | 0 \rangle \]
\[ = \sum_n (E_n - E_0) |\langle n | F | 0 \rangle|^2 \]
Energy weighted sum rule:

\[ S_1 = \sum_n (E_n - E_0)|\langle n|F|0\rangle|^2 \]
\[ = \frac{1}{2}\langle 0|[F, [H, F]]|0\rangle \]

For \( F = F(r) \) (local operator)

\[ [H, F] = \left[ -\frac{\hbar^2}{2m} \nabla^2, F \right] \]
\[ = -\frac{\hbar^2}{2m}(\nabla^2 F + 2\nabla F \cdot \nabla) \]

\[ [F, [H, F]] = \frac{\hbar^2}{m}(\nabla F)^2 \]

\[ S_1 = \frac{\hbar^2}{2m} \int dr \rho(r) \cdot (\nabla F)^2 \]
\[
S_1 = \sum_n (E_n - E_0)|\langle n|F|0 \rangle|^2 = \frac{\hbar^2}{2m} \int dr \, \rho(r) \cdot (\nabla F)^2
\]

For \( F=z \)

\[
S_1 = \sum_n (E_n - E_0)|\langle n|z|0 \rangle|^2 = \frac{\hbar^2 N_{sys}}{2m}
\]

[TRK (Thomas-Reiche-Kuhn) Sum Rule]

Model independent

For \( F = r^\lambda Y_{\lambda\mu}(\hat{r}) \)

\[
S_1 = \frac{\lambda(2\lambda + 1)\hbar^2}{8\pi m} A \langle r^{2\lambda-2} \rangle
\]
Photo absorption cross section:

\[ \sigma_{\text{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c} (E_f - E_i) |\langle \phi_f | \tilde{z} | \phi_i \rangle|^2 \delta(E_\gamma - E_f + E_i) \]

\[ \tilde{z} = \sum_p (z_p - Z_{cm}) = \sum_p \left\{ z_p - \frac{1}{A} \left[ \sum_{p'} z_{p'} + \sum_n z_n \right] \right\} \]

\[ = \frac{NZ}{A} \left( \frac{1}{Z} \sum_p z_p - \frac{1}{N} \sum_n z_n \right) \]

\[ \int \sigma_{\text{abs}}(E_\gamma) dE_\gamma = \frac{4\pi^2 e^2}{\hbar c} \cdot \frac{\hbar^2}{2m} \cdot \frac{NZ}{A} \]

\[ = \frac{2\pi^2 e^2 \hbar}{mc} \cdot \frac{NZ}{A} \]
Giant Dipole Resonance (GDR)

\[ \int \sigma dE = (300 \pm 30) \text{ MeV fm}^2 \]
\[ = (1.1 \pm 0.1) \frac{N^2}{A} \text{ MeV fm}^2 \]

\[ \sigma = \frac{\left( \frac{\Gamma}{2} \right)^2}{(E - E_{\text{res}})^2 + \left( \frac{\Gamma}{2} \right)^2} \frac{E}{E_{\text{res}}} \sigma_{\text{res}} \]

\[ E_{\text{res}} = 13.9 \text{ MeV} \quad \Gamma = 4.2 \text{ MeV} \]
\[ \sigma_{\text{res}} = 53 \text{ fm}^2 \]

Figure 6-18  Total photoabsorption cross section for $^{197}$Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, Phys. Rev. 127, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.
Figure 6-20 Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with $A > 50$, the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic $\gamma$ rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in Proc. Intern. Nuclear Physics Conference, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of ($\gamma p$) processes must be included and the data are from: $^{12}$C and $^{27}$Al (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, Phys. Rev. 143, 790, 1966); $^{16}$O (Dolbilkin et al., loc.cit., Fig. 6-26). For the heavy nuclei ($A > 50$), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssiére et al., 1970).
Nice features of sum rule

\[ S_0 = \langle 0 | F^2 | 0 \rangle \]
\[ S_1 = \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle \]

Sum rule:
(Some) information on excited states
← only with the ground state
(no need to know the excited states)

- a missing strength in the region outside the measurement?
- extraction of radii etc. by measuring a strength distribution
- a check of numerical calculations