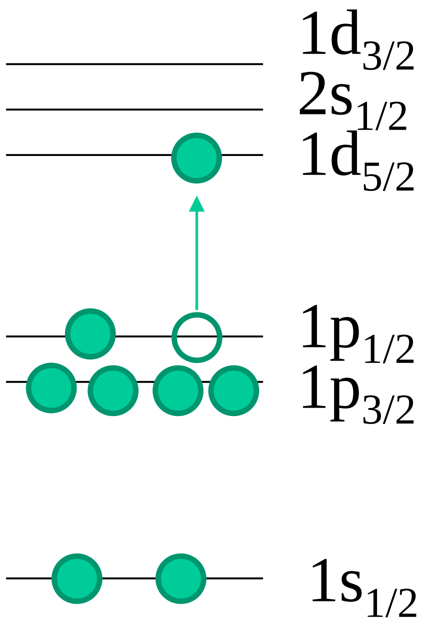
Excited states of atomic nuclei

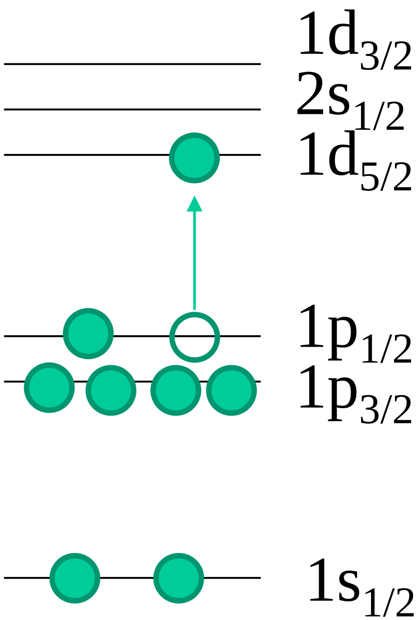
✓ single-particle excitations (only one nucleon involved)



an example of
s.p. excitations

Excited states of atomic nuclei

- ✓ single-particle excitations (only one nucleon involved)
- ✓ collective excitations (many nucleons contribute coherently)



an example of s.p. excitations

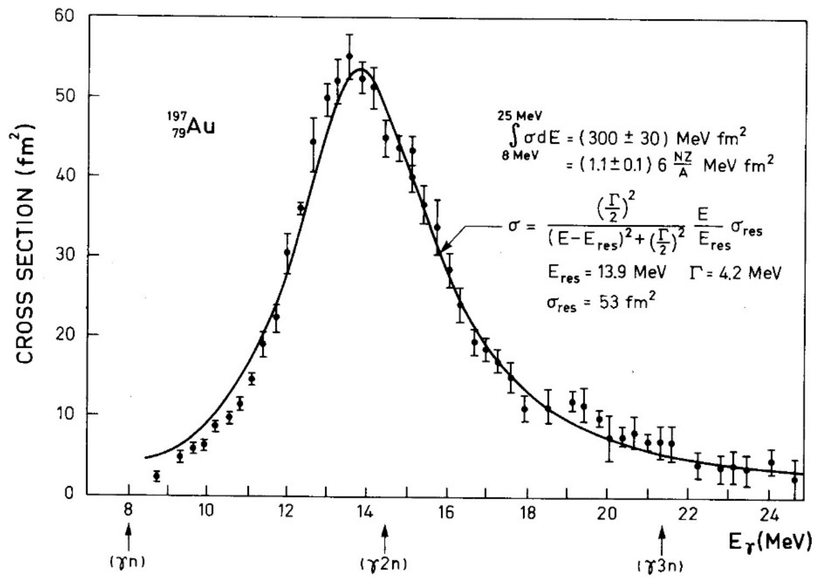
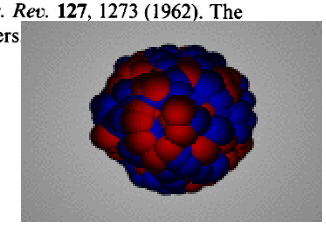
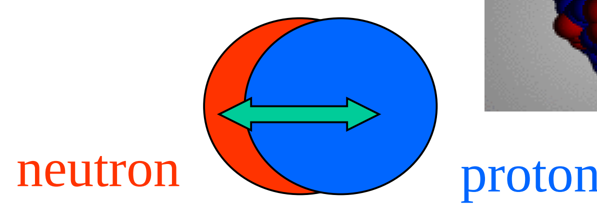


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.





an example of collective excitations: GDR

Excited states of atomic nuclei

- ✓ single-particle excitations (only one nucleon involved)
- ✓ collective excitations (many nucleons contribute coherently)

microscopic understanding of collective excitations?

how can one describe collective excitations microscopically?

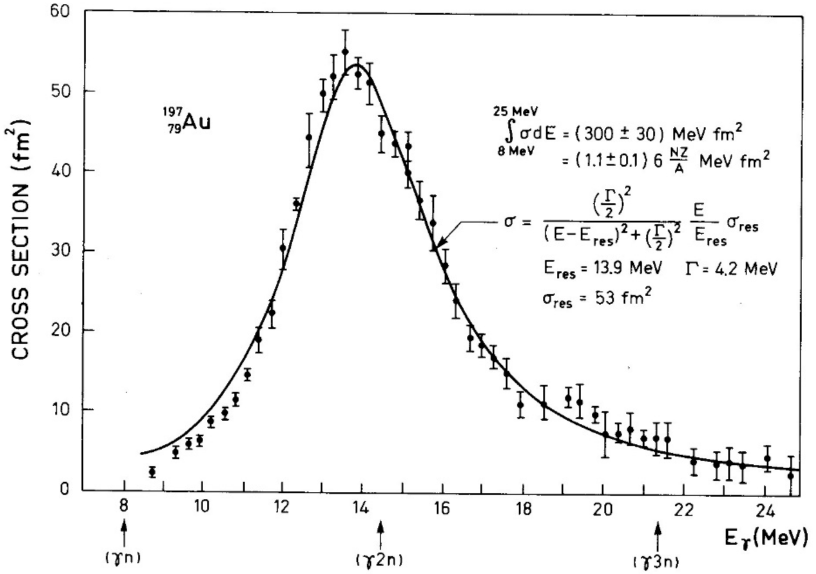
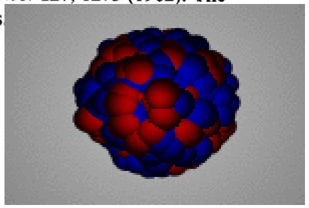
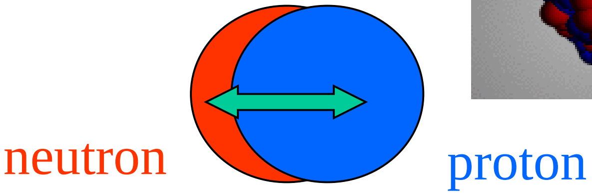


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

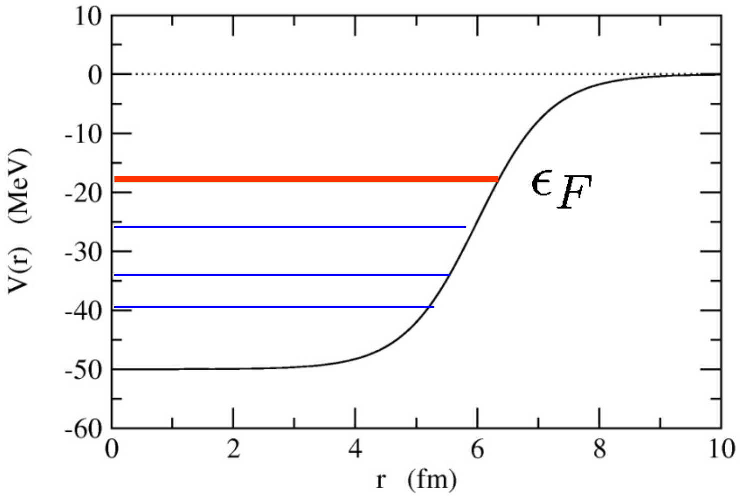




an example of collective excitations: GDR

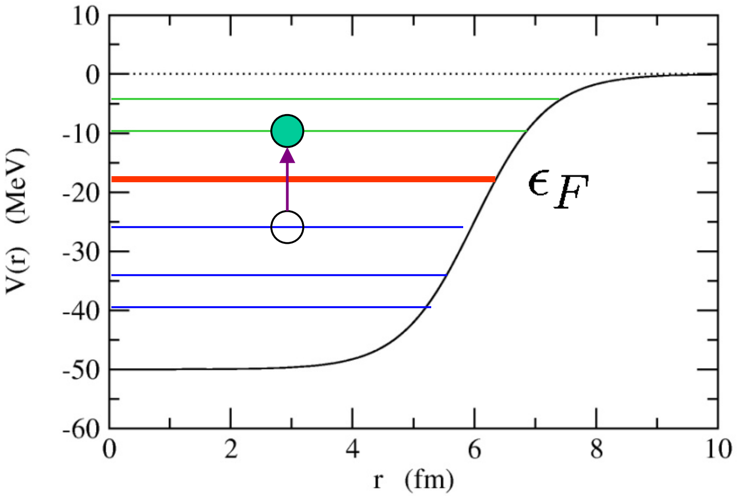
Particle-Hole excitations

Hartree-Fock state



$$|HF\rangle$$

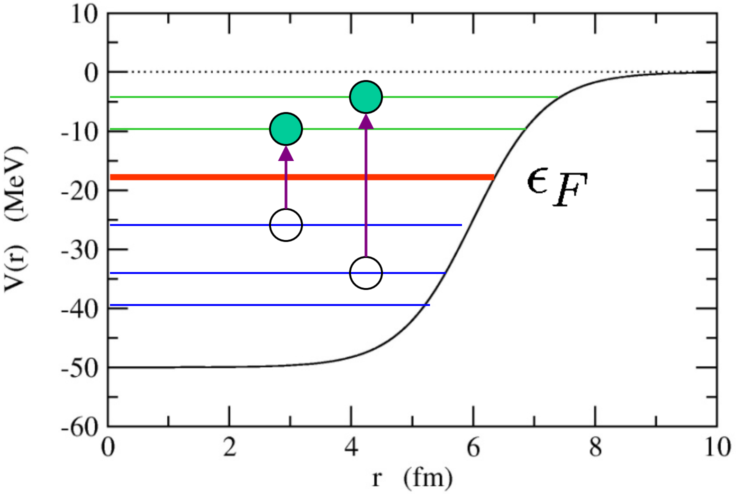
1 particle-1 hole (1p1h) state



$$a_p^\dagger a_h |HF\rangle$$

2 particle-2 hole (2p2h) state

$$a_p^\dagger a_{p'}^\dagger a_h a_{h'} |HF\rangle$$

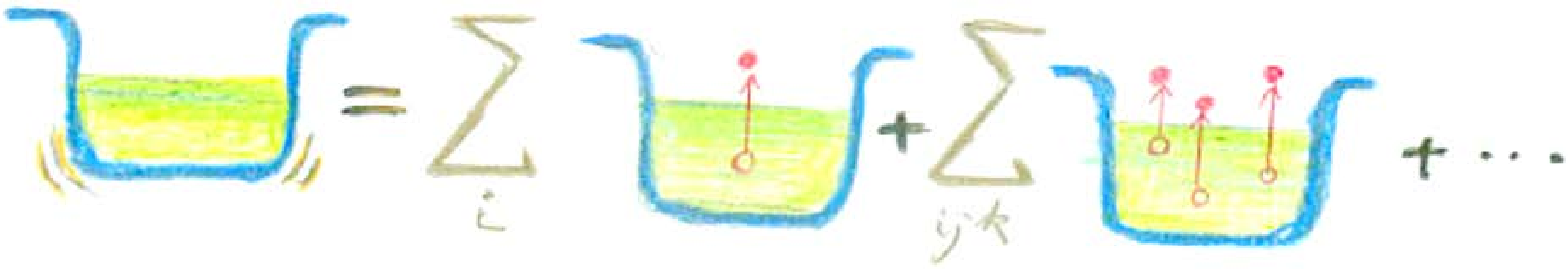


Tamm-Dancoff Approximation

Assume: $|\nu\rangle = Q^\dagger_\nu|HF\rangle = \sum_{ph} X_{ph}\, a^\dagger_p a_h|HF\rangle$

$$= \sum_{ph} X_{ph}|ph^{-1}\rangle$$

(superposition of 1p1h states)



Slide: K. Matsuyanagi

Tamm-Dancoff Approximation

Assume: $|\nu\rangle = Q_\nu^\dagger|HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|HF\rangle$

$$= \sum_{ph} X_{ph}|ph^{-1}\rangle$$

(superposition of 1p1h states)

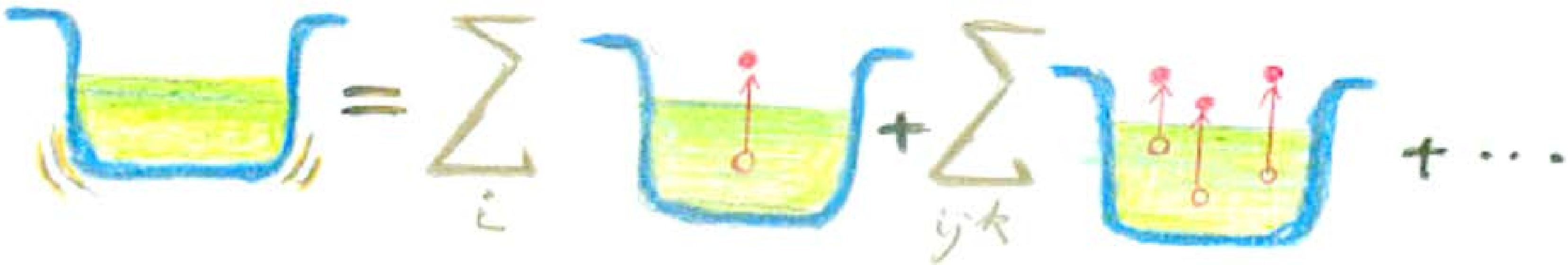
$$H|\nu\rangle = E_\nu|\nu\rangle$$

$$\sum_{p'h'} H_{ph,p'h'}X_{p'h'} = E_\nu X_{ph}$$

residual interaction

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

Tamm-Dancoff equation



Slide: K. Matsuyanagi

$$V(\boldsymbol{r}) \sim \int d\boldsymbol{r}'\, v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')$$

vibration: $\rho = \rho_0(\boldsymbol{r}) \rightarrow \rho_0(\boldsymbol{r}) + \delta\rho(\boldsymbol{r},t)$

$$v_{\mathrm{res}}(\boldsymbol{r}) \sim \int d\boldsymbol{r}'\, v(\boldsymbol{r},\boldsymbol{r}')\delta\rho(\boldsymbol{r}')$$

residual interaction

TDA on a schematic model

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \quad \langle ph'|\bar{v}|hp'\rangle = g$$

for three ph configurations:

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

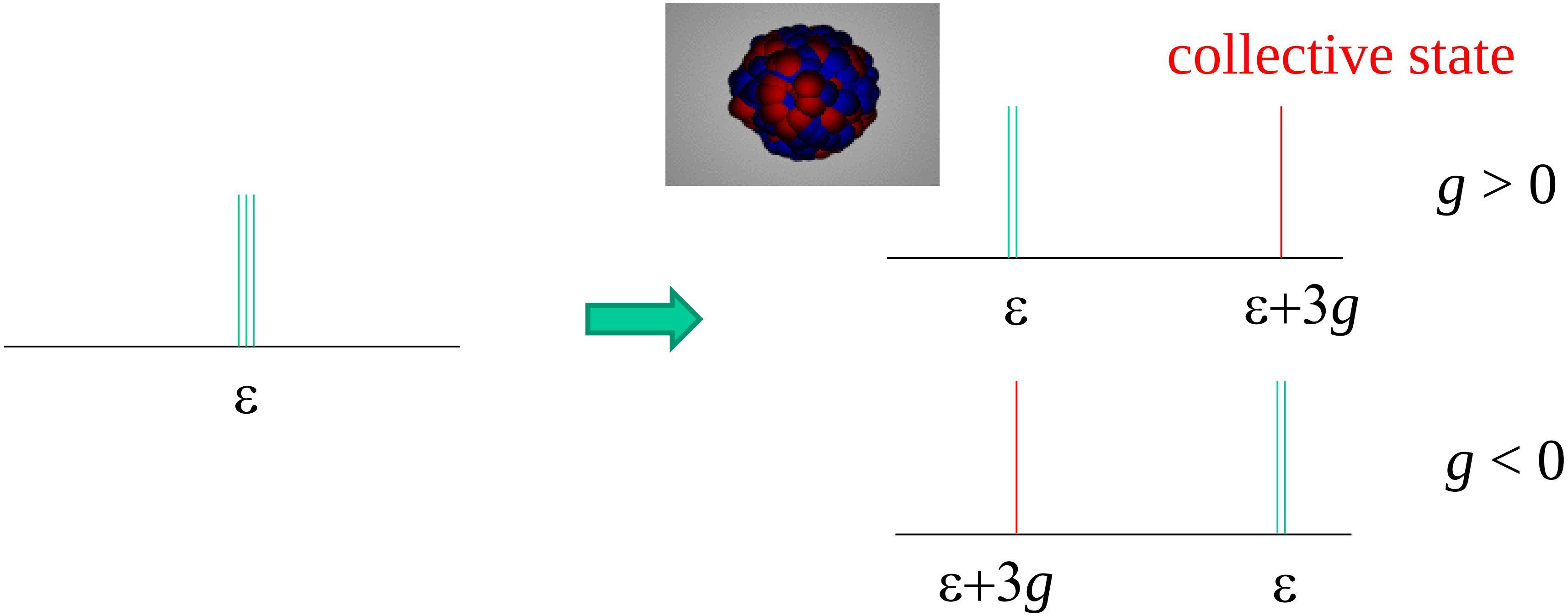
→ Diagonalization:

$$\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$$

TDA on a schematic model

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$\rightarrow$ Diagonalization: $\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$



TDA on a schematic model

Separable interaction: $\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D^*_{p'h'}$

(separable interaction)

$$H\psi = E\psi; \qquad \psi = \sum_i C_i \phi_i$$

$$\longrightarrow \quad \sum_j H_{ij} C_j = E C_i; \qquad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose $H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j$ (separable form)

$$(\epsilon_i - E) C_i + \lambda f_i^* \underbrace{\sum_j f_j C_j}_{\equiv T} = 0$$

$$C_i = -\lambda \frac{T f_i^*}{\epsilon_i - E}$$

$$T = -\lambda \sum_j \frac{|f_j|^2}{\epsilon_j - E} T \quad \longrightarrow \quad \boxed{\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}}$$

TDA on a schematic model

Separable interaction: $\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$

Tamm-Dancoff equation: $\sum_{p'h'} A_{ph,p'h'} X_{p'h'} = E X_{ph}$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$(\epsilon_{ph} - E)X_{ph} + \lambda D_{ph} \cdot T = 0 \qquad T \equiv \sum_{ph} D^*_{ph} X_{ph}$$

$$X_{ph} = -\lambda \frac{D_{ph} T}{\epsilon_{ph} - E}$$

$$T = \sum_{ph} D^*_{ph} X_{ph} = -\lambda \sum_{ph} \frac{|D_{ph}|^2}{\epsilon_{ph} - E} T$$

or

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

(TDA dispersion relation)

Graphical solutions

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

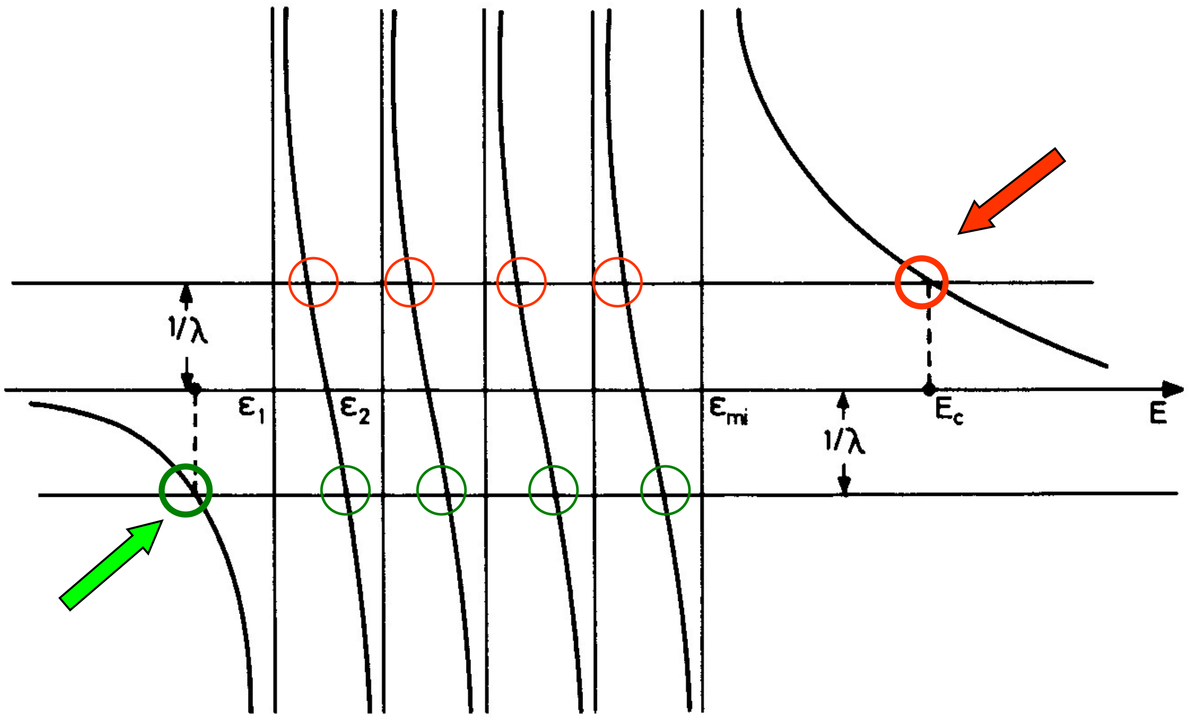


Figure 8.4. Graphical solution of Eq. (8.18).

(note) in the degenerate limit: $\epsilon_{ph} \sim \epsilon$

$$E = \epsilon_{ph} + \lambda \sum_{ph} |D_{ph}|^2, \qquad |\nu\rangle = \sum_{ph} D_{ph} a_p^\dagger a_h |HF\rangle$$

coherent superpositon of 1p1h states

Iso-scalar type modes: $E < \epsilon_{ph} \rightarrow \lambda < 0$ (attractive)

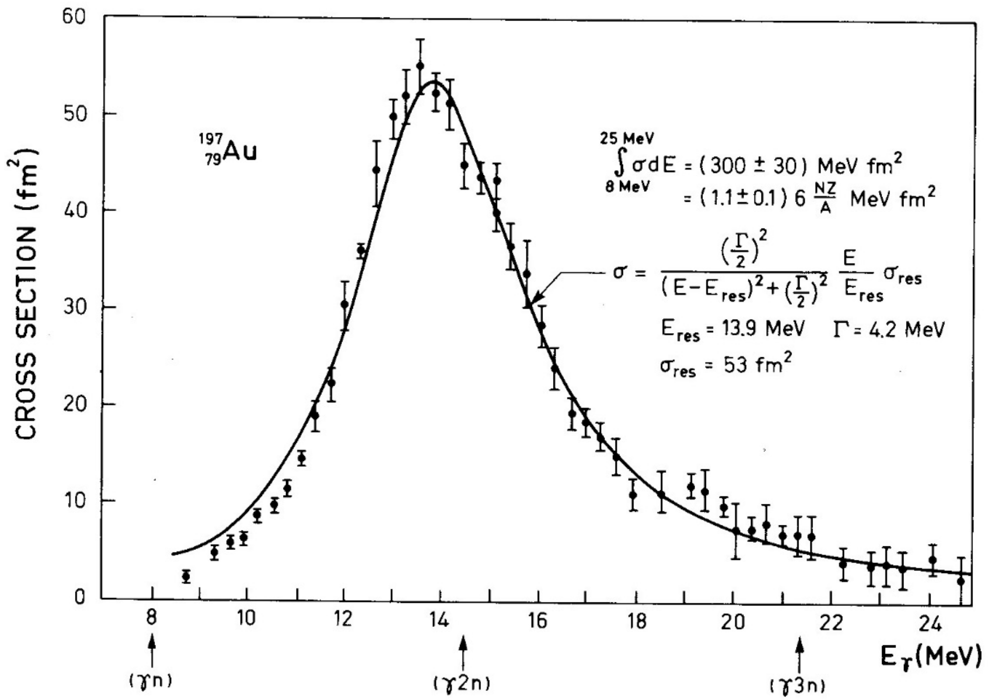
Iso-vector type modes: $E > \epsilon_{ph} \rightarrow \lambda > 0$ (repulsive)

Experimental systematics:

IV GDR: $E \sim 79 A^{-1/3}$ (MeV) ⟺ $\epsilon_{ph} \sim 41 A^{-1/3}$

IS GQR: $E \sim 65 A^{-1/3}$ (MeV) ⟺ $\epsilon_{ph} \sim 82 A^{-1/3}$

(note) single particle potential: $\hbar\omega \sim 41 A^{-1/3}$ (MeV)



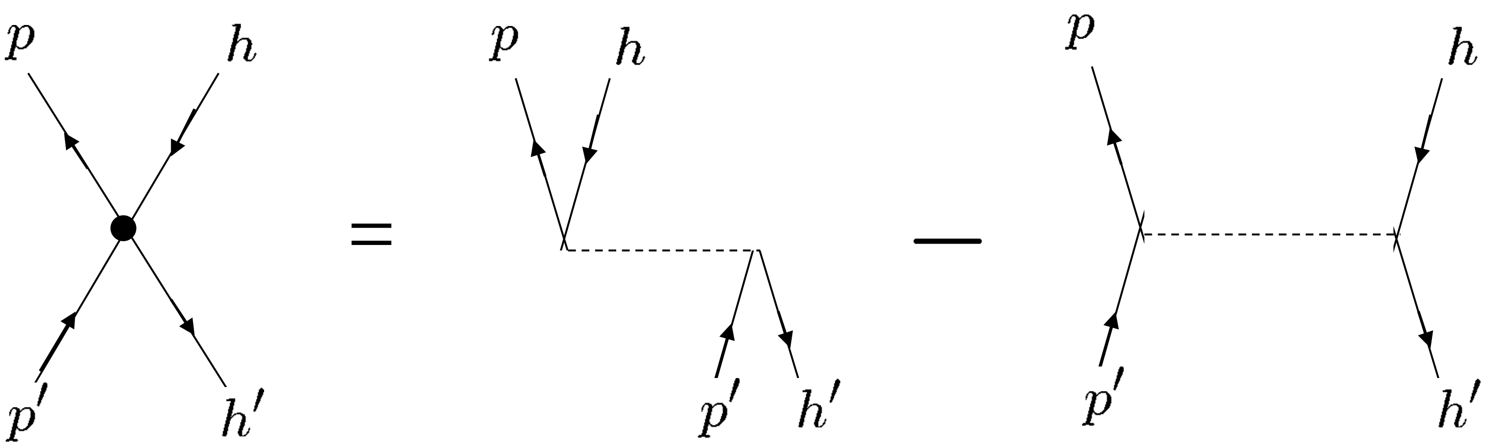
^{197}Au

$$E_{\rm GDR} = 14 \ \ (\text{MeV})$$

$$\epsilon_{ph} \sim 41 \cdot 197^{-1/3}$$

$$\sim 7 \ \ (\text{MeV})$$

$$\langle ph^{-1}|\bar{v}|p'h'^{-1}\rangle = \langle ph'|\bar{v}|hp'\rangle = \langle ph'|v|hp'\rangle - \langle ph'|v|p'h\rangle$$



Direct term — Exchange term

$$\langle PP^{-1}|\bar{v}|PP^{-1}\rangle \sim \langle NN^{-1}|\bar{v}|NN^{-1}\rangle = D - E$$

$$\langle PP^{-1}|\bar{v}|NN^{-1}\rangle = D$$ (no charge exchange)

$$\langle IS|\bar{v}|IS\rangle = 2D - E \sim D$$

$$\langle IV|\bar{v}|IV\rangle = -E \sim -D$$

$$|IS\rangle \propto |NN^{-1}\rangle + |PP^{-1}\rangle$$

$$|IV\rangle \propto |NN^{-1}\rangle - |PP^{-1}\rangle$$

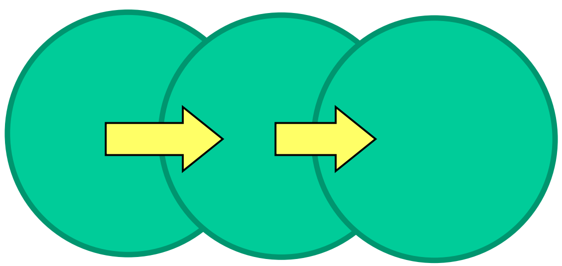
Spurious motion and RPA

Mean-Field Approximation ⟷ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇒ Zero energy mode (Nambu-Goldstone mode)



does not require an extra energy → zero energy mode

A drawback of TDA:

Zero modes appear at finite excitation energies.

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|0\rangle \qquad \text{(TDA)}$$

A better approximation:

the random phase approximation (RPA)

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p\right)|0\rangle$$

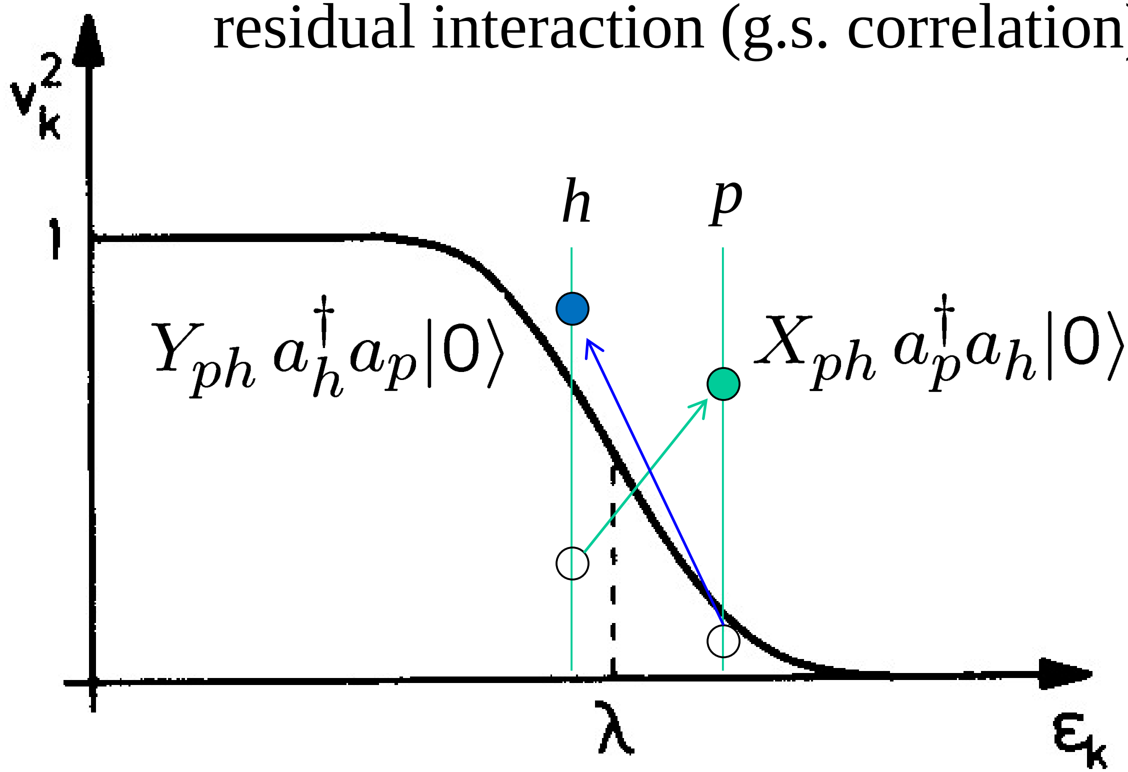
(superposition of 1p1h states)

A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p\right)|0\rangle$$

(superposition of 1p1h states)

smearing of Fermi surface due to the residual interaction (g.s. correlation)



A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\,a_p^\dagger a_h - Y_{ph}\,a_h^\dagger a_p\right)|0\rangle$$

(superposition of 1p1h states)

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

⇓

$$\langle HF|[\delta Q,[H,Q_\nu^\dagger]]|HF\rangle = E_\nu\langle HF|[\delta Q, Q_\nu^\dagger]|HF\rangle$$

$$\langle HF|[\delta Q,[H,Q_\nu^\dagger]]|HF\rangle = E_\nu \langle HF|[\delta Q, Q_\nu^\dagger]|HF\rangle$$

$$Q_\nu^\dagger = \sum_{ph} X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p \qquad \delta Q = a_h^\dagger a_p, \quad a_p^\dagger a_h$$

RPA equation:

$$\begin{aligned} \sum_{p'h'} A_{ph,p'h'} X_{p'h'} + B_{ph,p'h'} Y_{p'h'} &= E_\nu X_{ph} \\ \sum_{p'h'} B^*_{ph,p'h'} X_{p'h'} + A^*_{ph,p'h'} Y_{p'h'} &= -E_\nu Y_{ph} \end{aligned}$$

$$\begin{aligned} A_{ph,p'h'} &= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle \\ B_{ph,p'h'} &= \langle pp'|\bar{v}|hh'\rangle \end{aligned}$$

or

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E_\nu \begin{pmatrix} X \\ Y \end{pmatrix}$$

Spurious motion in RPA

Mean-Field Approximation ⟷ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇒ Zero mode (Nambu-Goldstone mode)

$$[H, \hat{O}] = 0$$

RPA

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

$\hat{O}$ is a solution of RPA with *E=0*

$$Q^\dagger = \hat{O} = \sum_{ph} (O_{ph} a_p^\dagger a_h + O_{hp} a_h^\dagger a_p)$$

(note) $Q^\dagger_{\mathrm{TDA}} = \sum_{ph} O_{ph} a_p^\dagger a_h$ ⇒ $[H, Q^\dagger_{\mathrm{TDA}}] \neq 0$

Spurious motion in RPA

Mean-Field Approximation ⟷ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇒ Zero mode (Nambu-Goldstone mode)

RPA $$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

if $[H, \hat{O}] = 0$

Then $\hat{O}$ is a solution of RPA with *E=0*

$$\hat{O} = \sum_{ph} (O_{ph} a_p^\dagger a_h + O_{hp} a_h^\dagger a_p)$$

The physical solutions are exactly separated out from the spurious modes.

RPA on a schematic model

Separable interaction: $\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)

Cf. TDA dispersion relation: $\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$

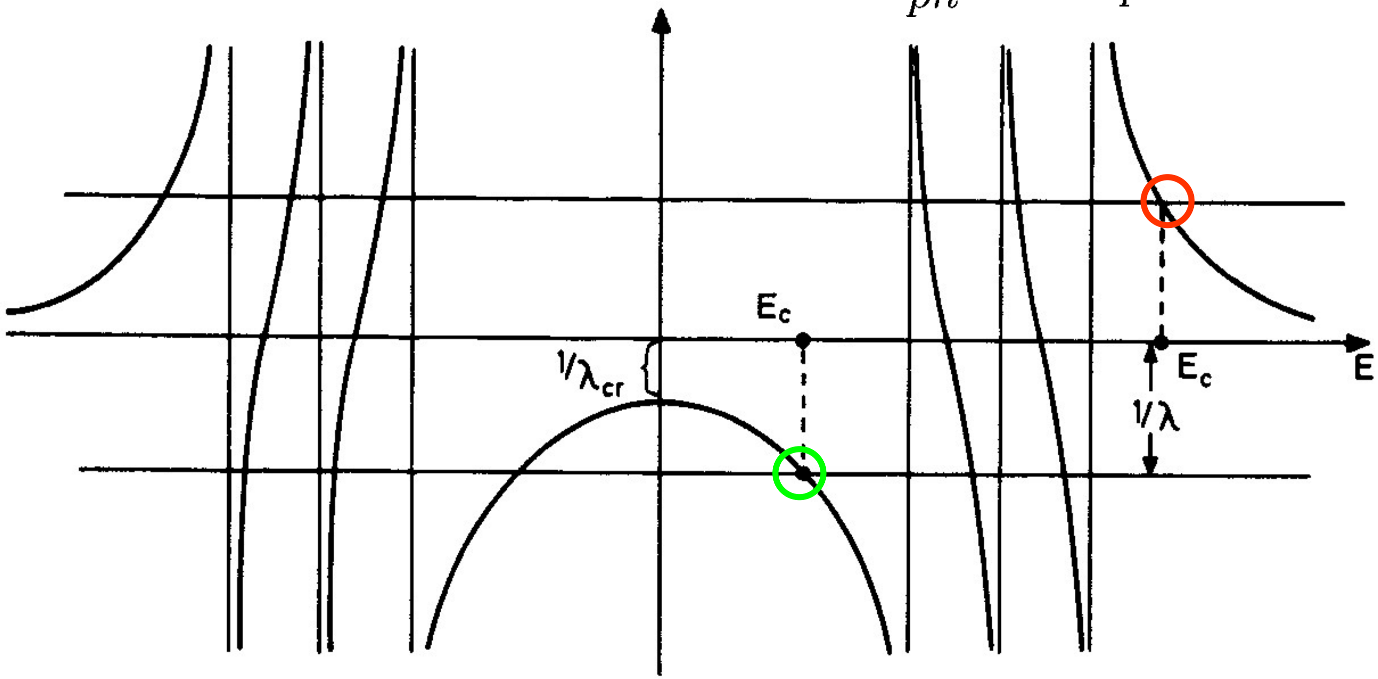


Figure 8.11. Graphical solution of the dispersion relation (8.135).

Comparison between Skyrme-(Q)RPA calculation and exp. data

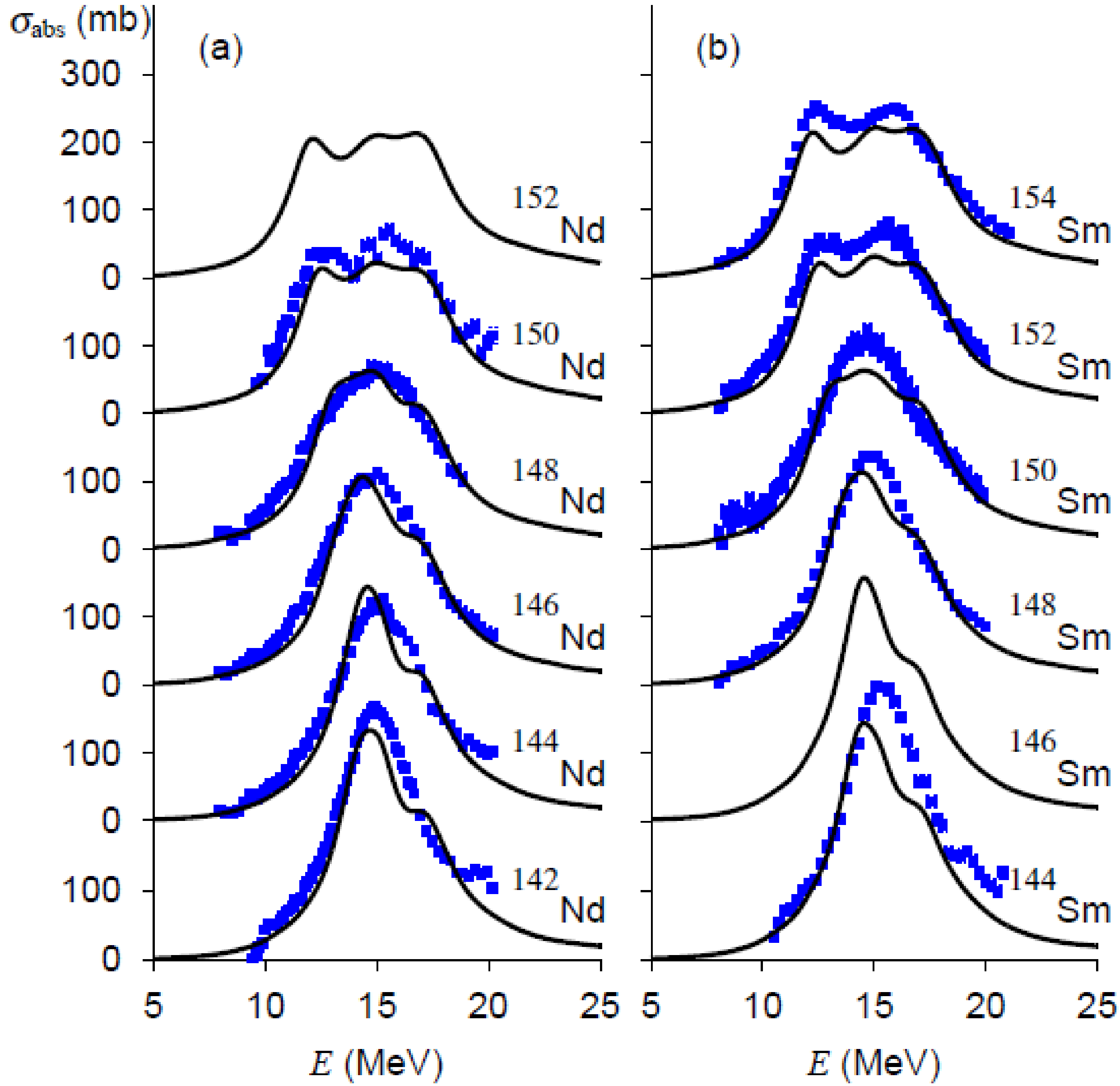
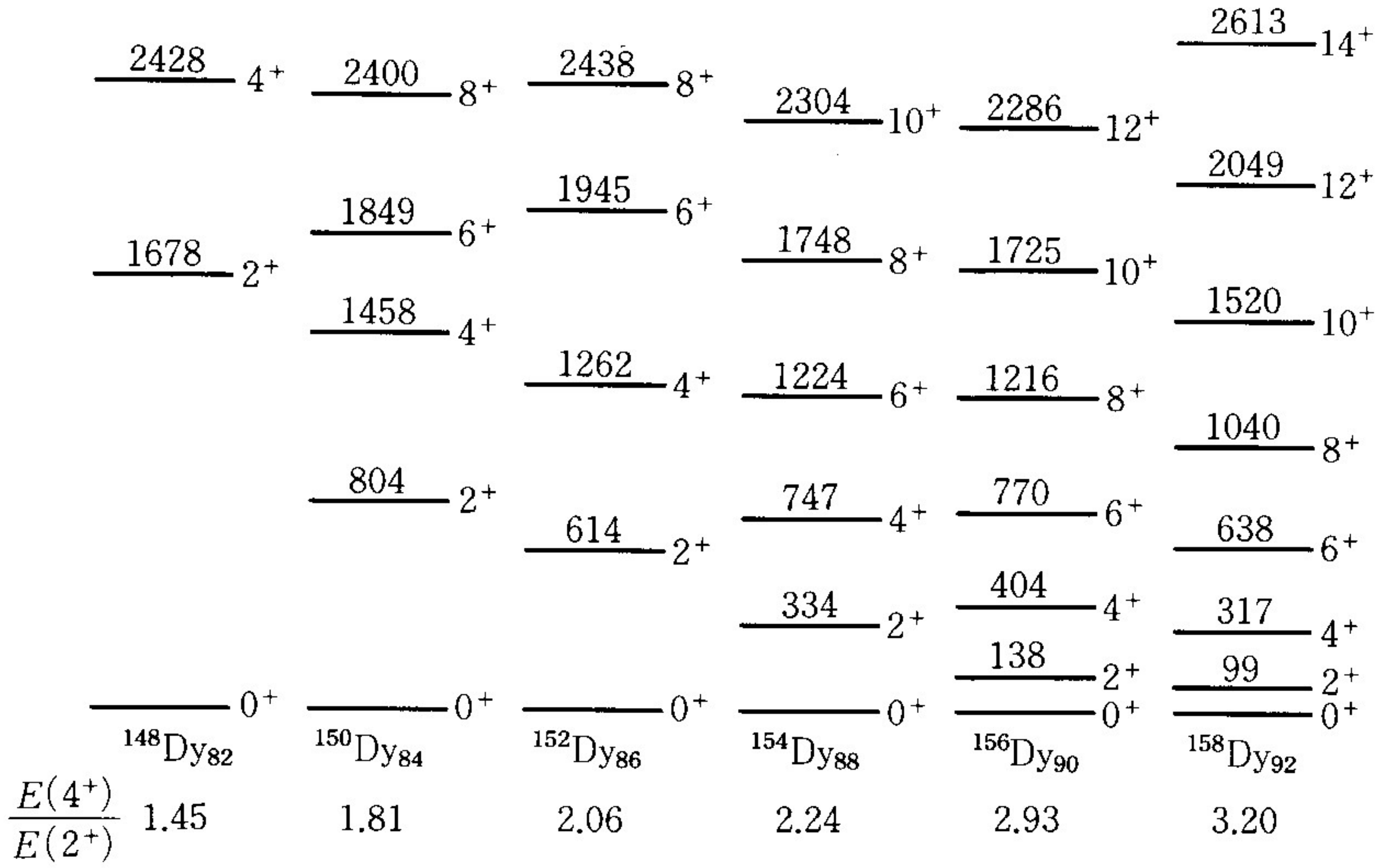


photo-absorption cross section (GDR)

K. Yoshida and T. Nakatsukasa, PRC83('11)021304

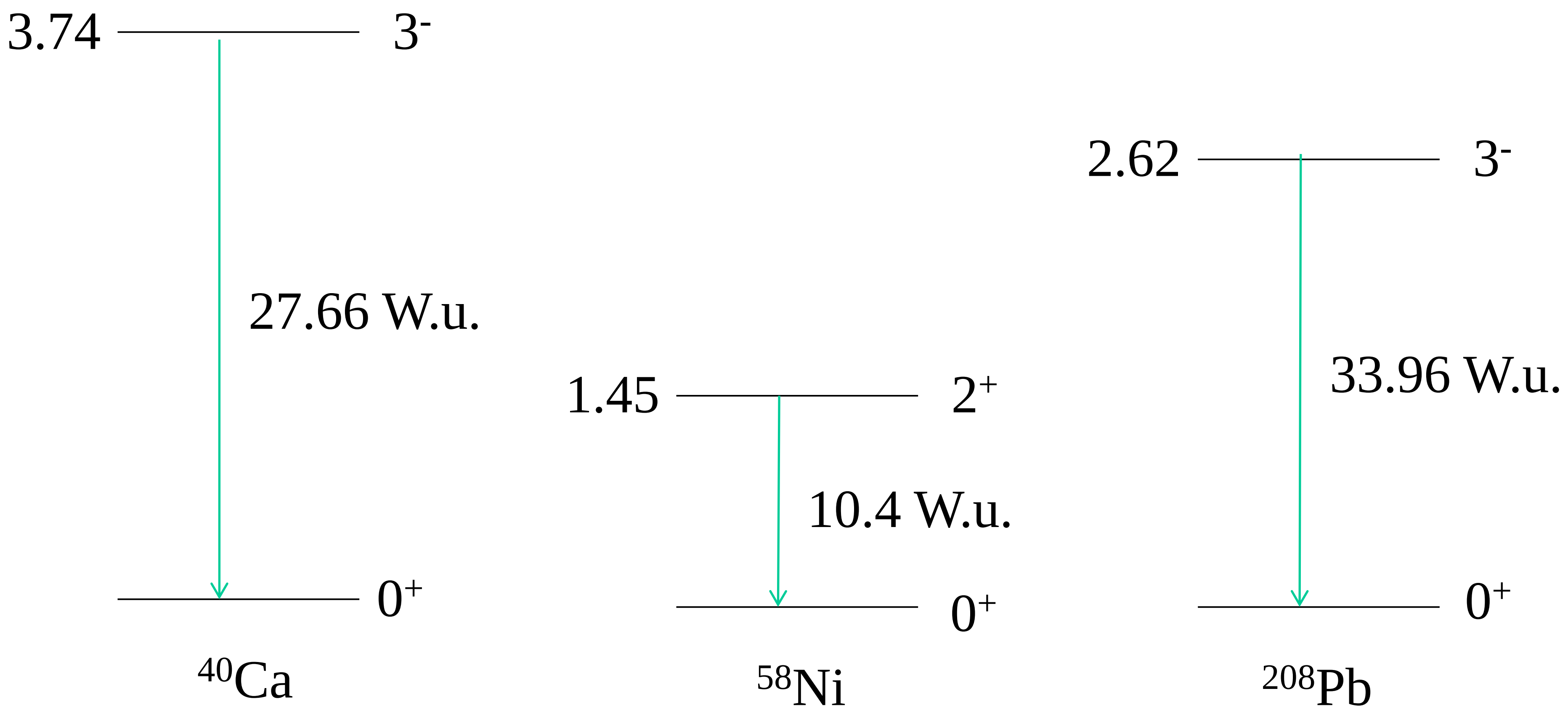
low-lying collective states

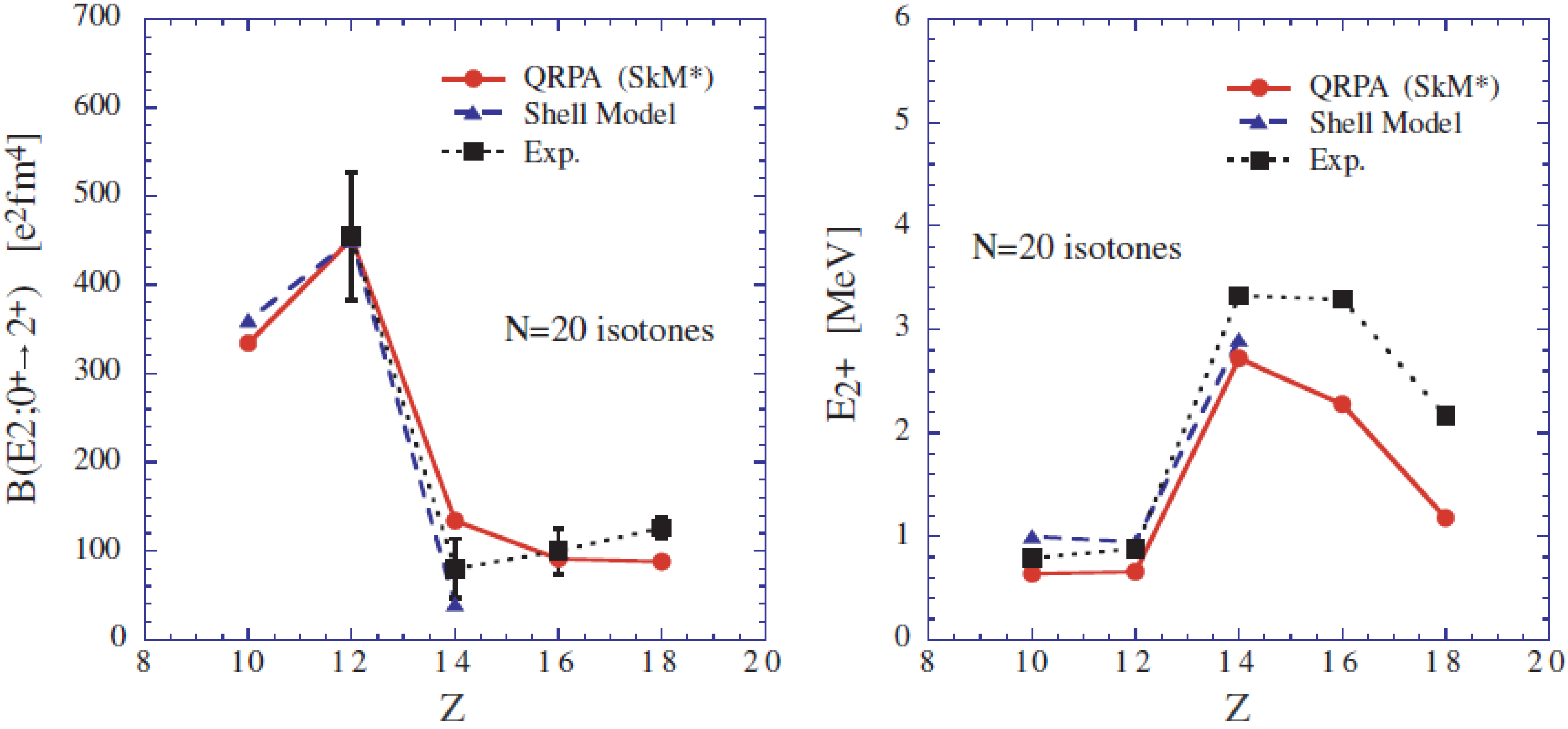
Low-lying excited states in even-even nuclei are collective excitations, and strongly reflect the pairing correlation and shell strucuture



Single-particle transition (Weisskopf unit)

$$B(E\lambda : I_i \to I_{gs}) = \frac{(1.2A^{1/3})^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 \qquad (e^2\mathrm{fm}^{2\lambda})$$





M. Yamagami and Nguyen Van Giai, PRC69 (‘04) 034301