Advanced Nuclear Physics

Nuclear Theory Group,
Tohoku University
Kouichi Hagino

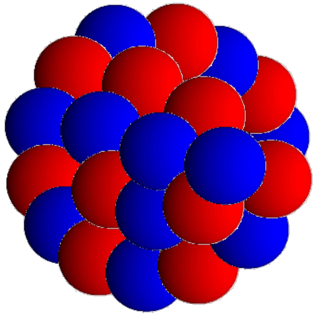
原子核理論特論

東北大学
原子核理論研究室
萩野浩一

Contents

Nuclei: many-body systems of nucleons (protons and neutrons)

→ ***Nuclear Many-Body Problems***



(Low-energy) Nuclear Physics

to understand rich nature of atomic nuclei starting from nucleon-nucleon interactions

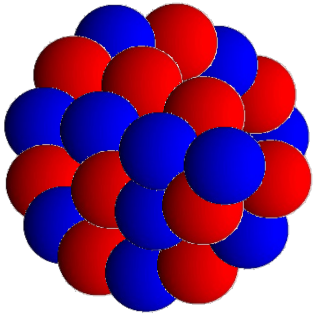
- size, mass, density, shape
- excitations
- decays
- nuclear reactions

two kinds of particle: protons and neutrons

Contents

Nuclei: many-body systems of nucleons (protons and neutrons)

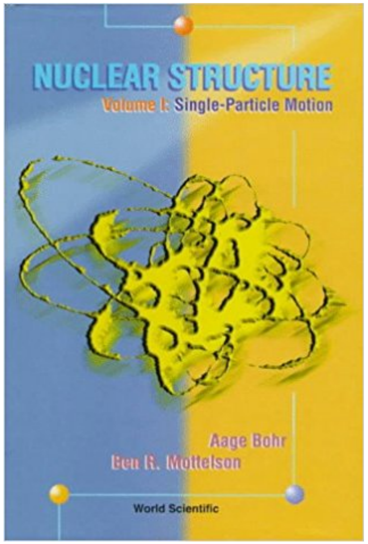
→ ***Nuclear Many-Body Problems***



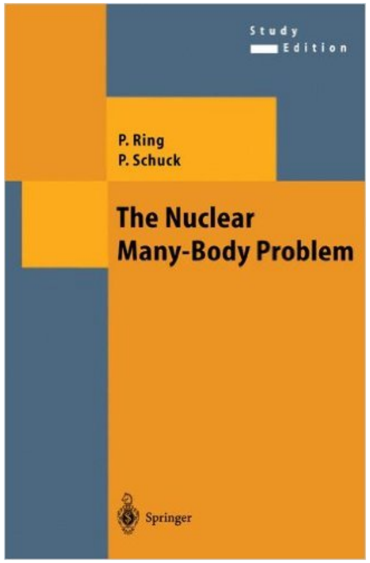
this lecture: microscopic descriptions of atomic nuclei

- Liquid drop model
- Single-particle motion and shell structure
- **Hartree-Fock approximation**
- Bruckner theory
- Pairing correlations and superfluid Nuclei
- physics of neutron-rich nuclei
- **Random phase approximation (RPA)**
- **Nuclear reactions and superheavy elements (physics of Nihonium)**

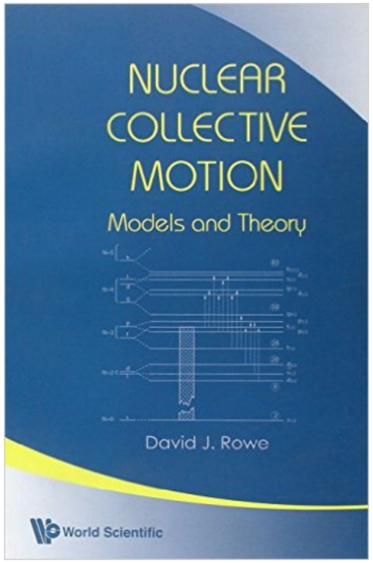
References



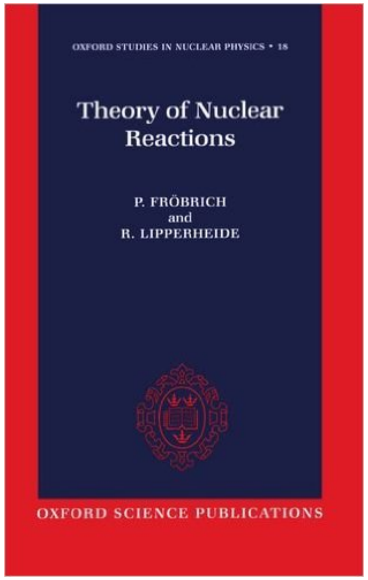
Bohr-Mottelson



Ring-Schuck



Rowe



Frobrich
-Lipperheide

Lecture notes:

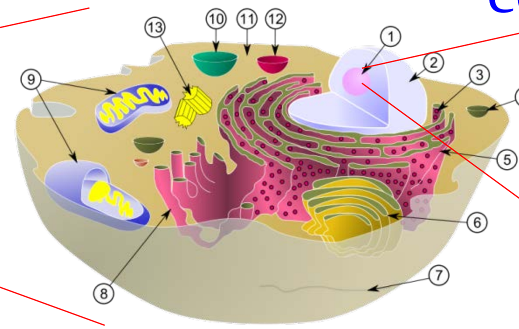
http://www.nucl.phys.tohoku.ac.jp/~hagino/lecture.html

(Tohoku University → Physics → Nuclear Theory
→ Kouichi Hagino → Lectures)

Introduction: atoms and atomic nuclei

~ 50 cm

cells

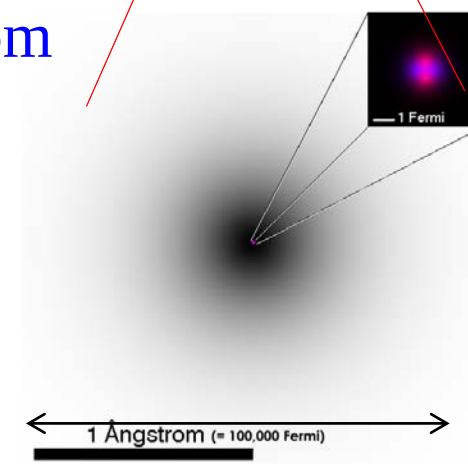


~ μm = 10^{-6} m

DNA

~ 10^{-8} m

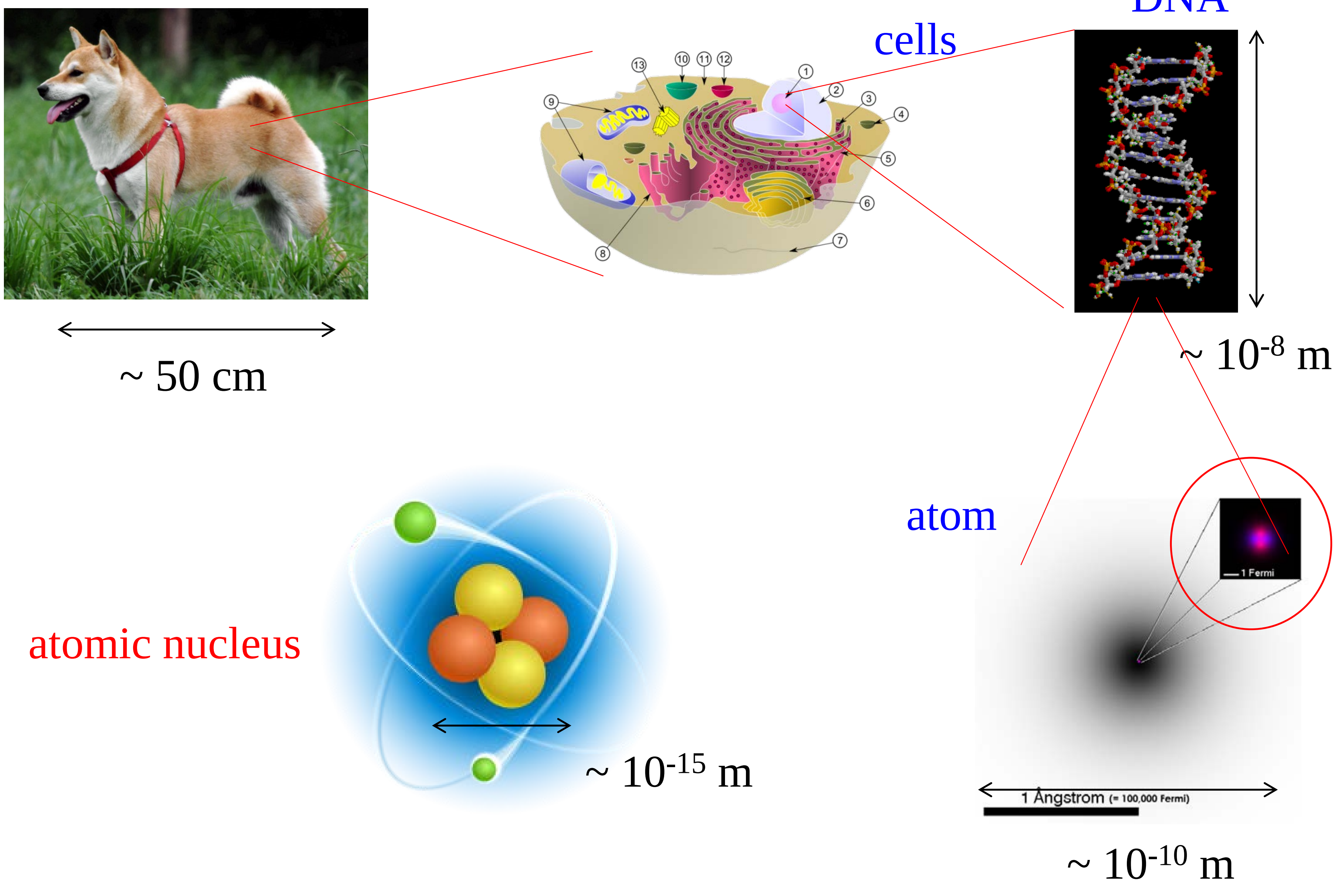
atom



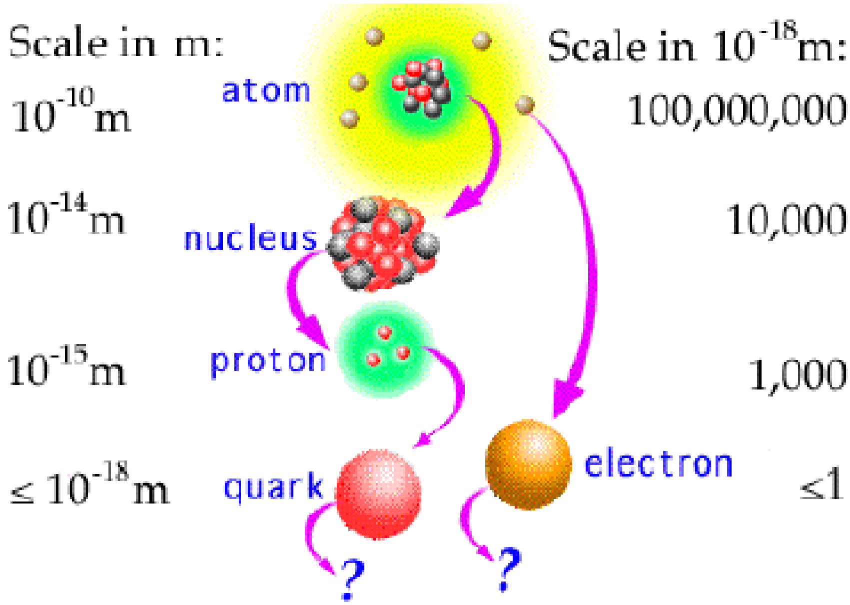
~ 10^{-10} m

Everything is made of atoms.

Introduction: atoms and atomic nuclei



Nuclear Physics



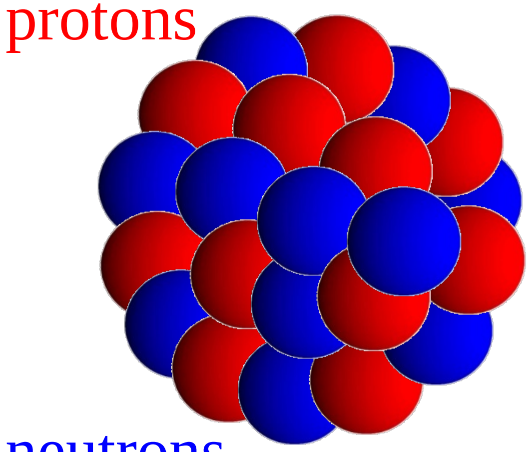
1 fm = 10^{-15} m

Nucleus as a *quantum many body system*

Basic ingredients:

	charge	mass (MeV)	spin, parity
Proton	+e	938.256	$½^+$
Neutron	0	939.550	$½^+$

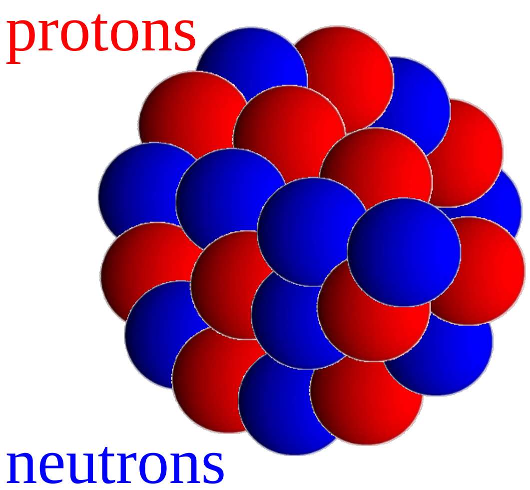
(note) $n \rightarrow p + e^- + \bar{\nu}_e$ (10.4 min)



- Nucleons are not stopping inside a nucleus.
  (they move relatively freely)

- Yet, they are not completely independent.
  A nucleus keeps its shape while nucleons influence among themselves so that a nucleon does not escape.

a self-bound system

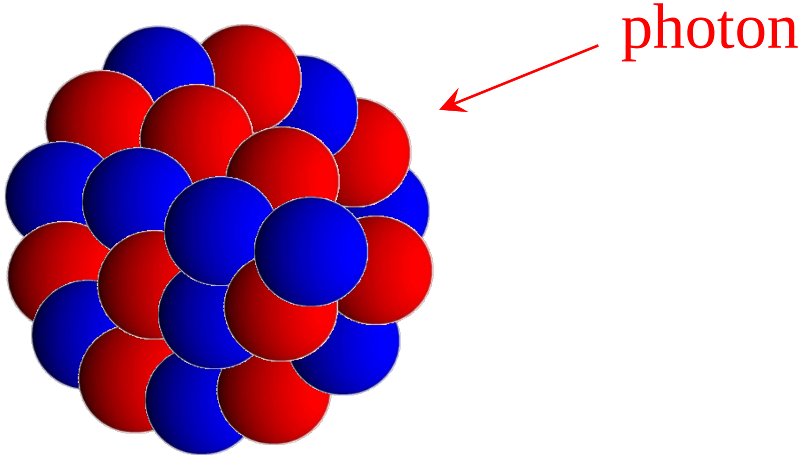


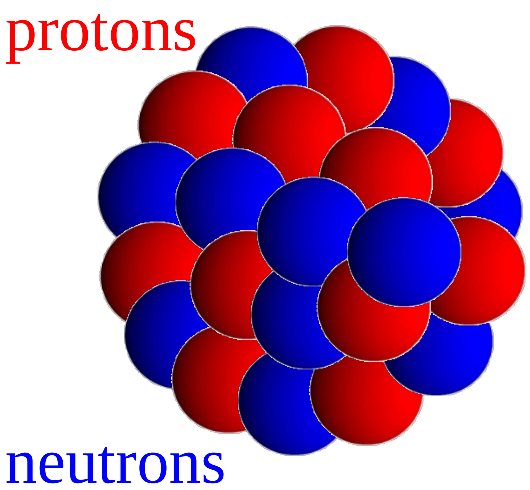
- Nucleons are not stopping inside a nucleus. (they move relatively freely)

- Yet, they are not completely independent. A nucleus keeps its shape while nucleons influence among themselves so that a nucleon does not escape.

a self-bound system

What happens if a photon is absorbed into a nucleus?
- one nucleon simply starts moving faster?



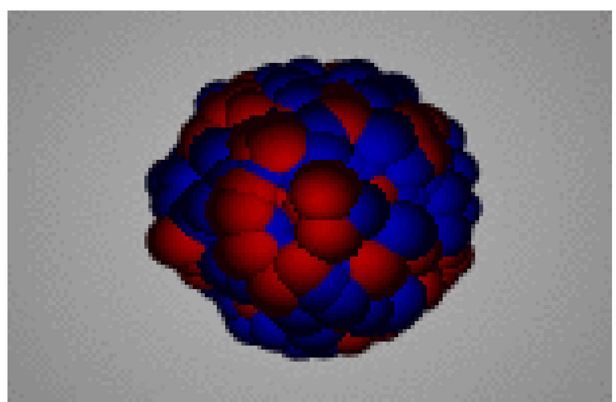


- Nucleons are not stopping inside a nucleus.
  (they move relatively freely)

- Yet, they are not completely independent.
  A nucleus keeps its shape while nucleons influence among themselves so that a nucleon does not escape.

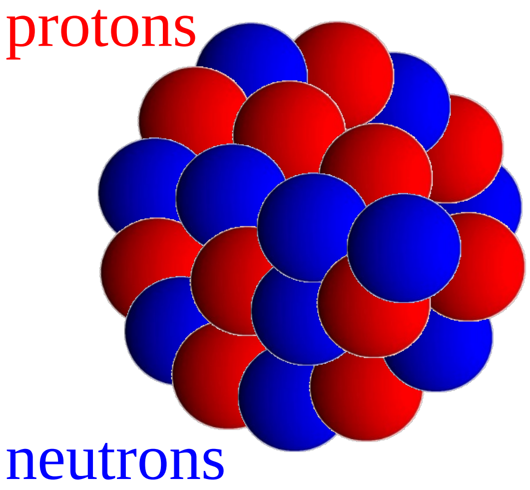
a self-bound system

What happens if a photon is absorbed into a nucleus?
- one nucleon simply starts moving faster?



Very coherent motion can happen

Collective motions

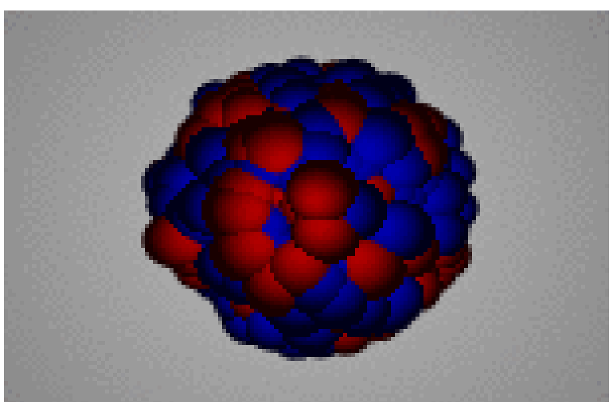


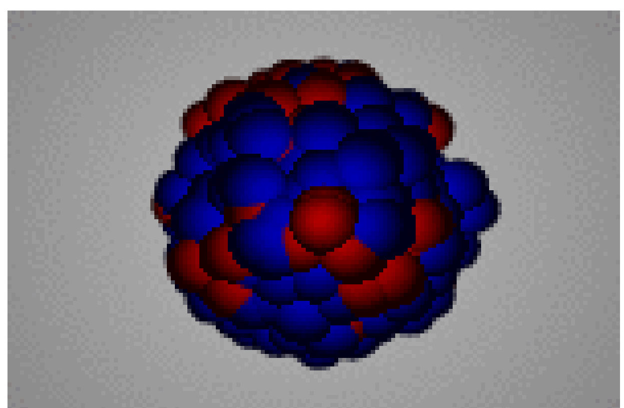
- Nucleons are not stopping inside a nucleus.
  (they move relatively freely)

- Yet, they are not completely independent.
  A nucleus keeps its shape while nucleons influence among themselves so that a nucleon does not escape.

a self-bound system

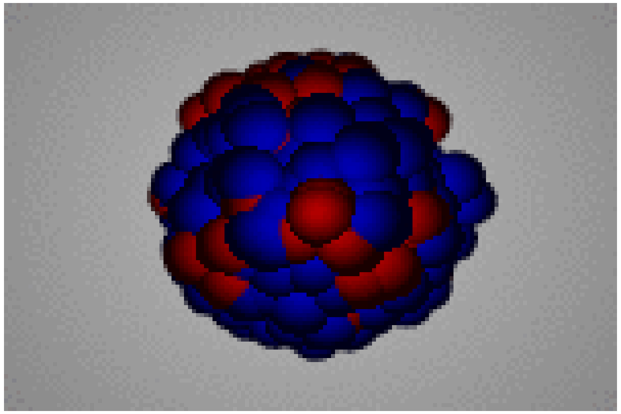
What happens if a photon is absorbed into a nucleus?
- one nucleon simply starts moving faster?

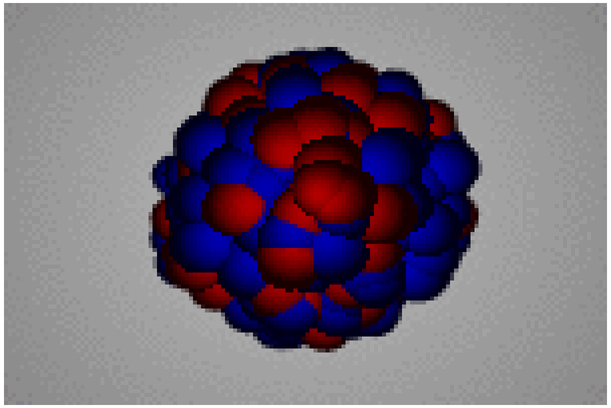




Very coherent motion can happen

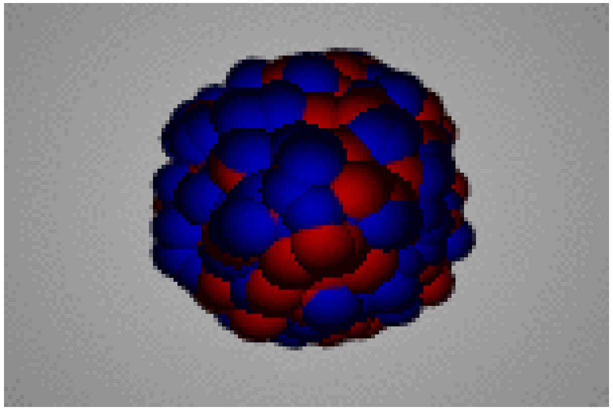
Collective motions

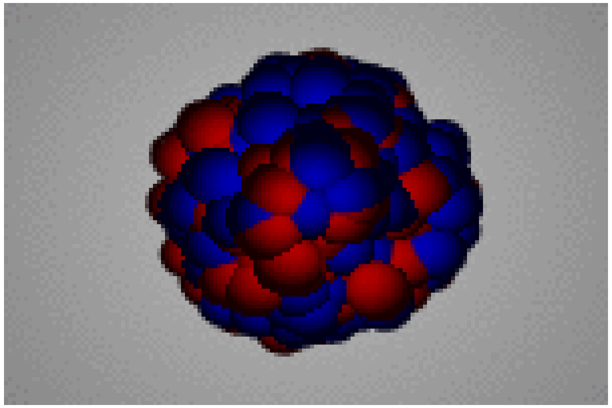




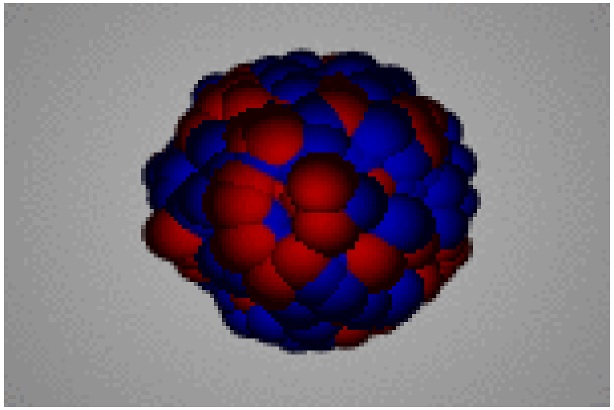
Very coherent motion can happen

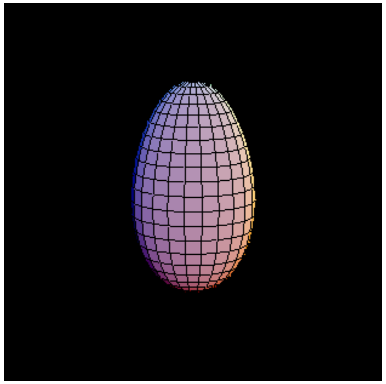
Collective motions

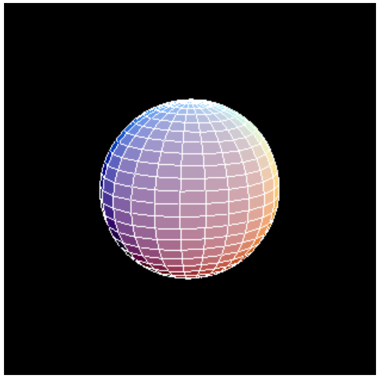


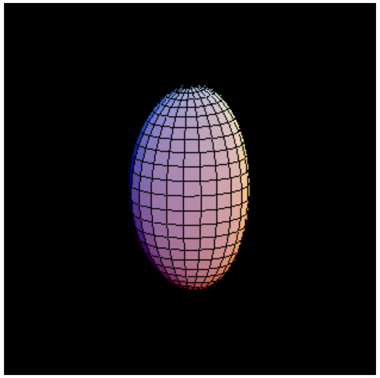


a variety of motions
→ very rich!

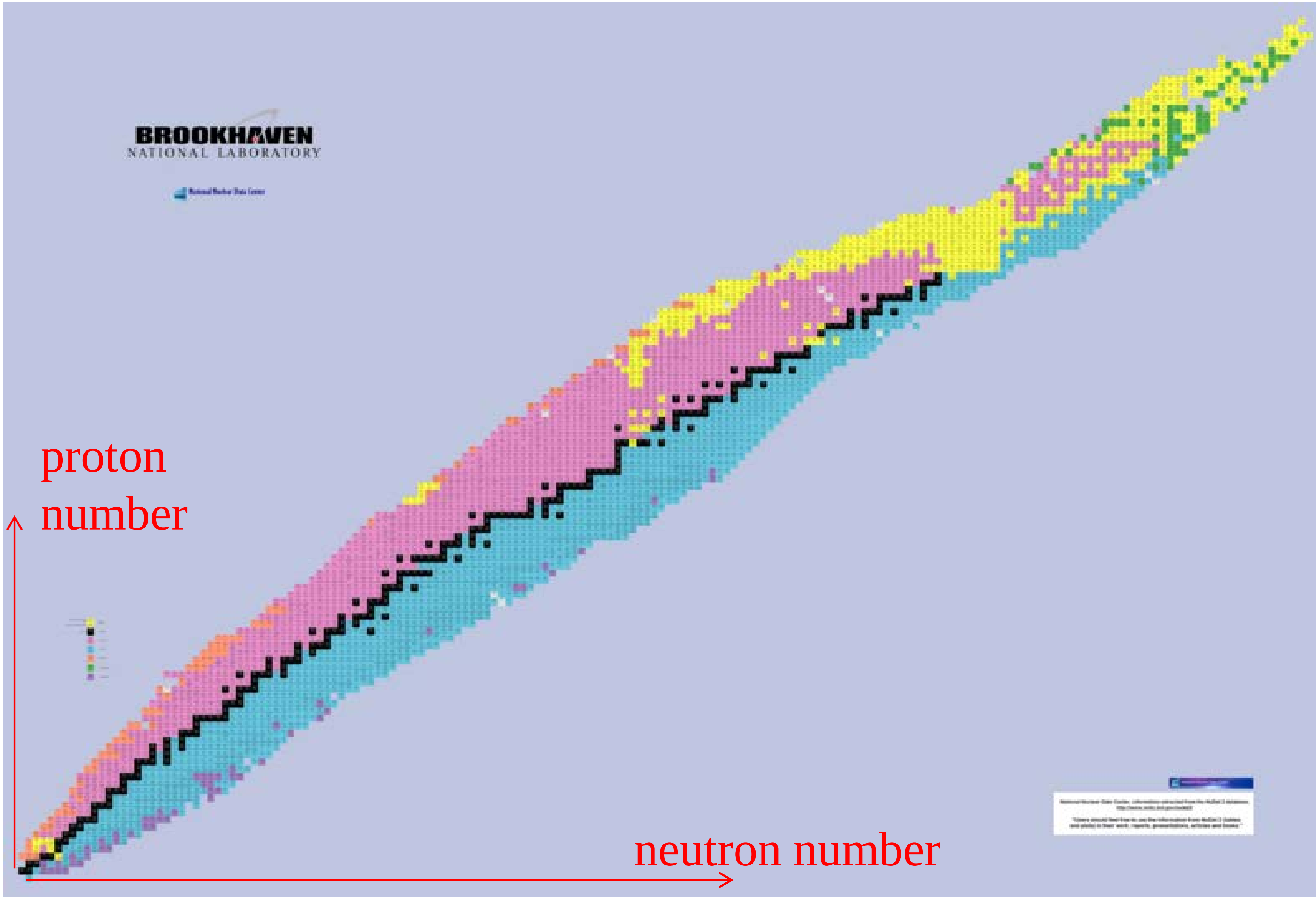




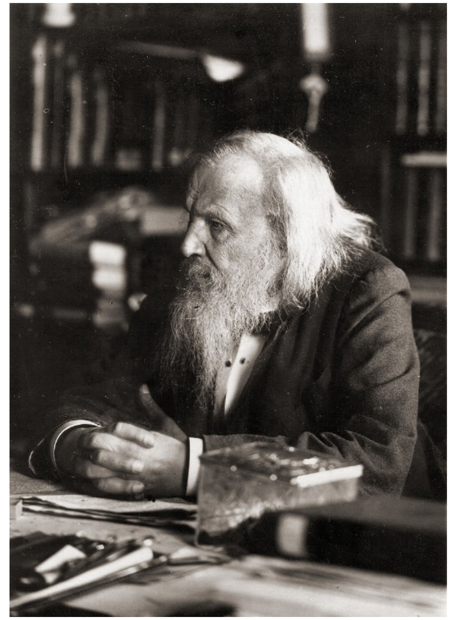




Nuclear Chart: 2D map of atomic nuclei



Periodic Table of elements (1869)

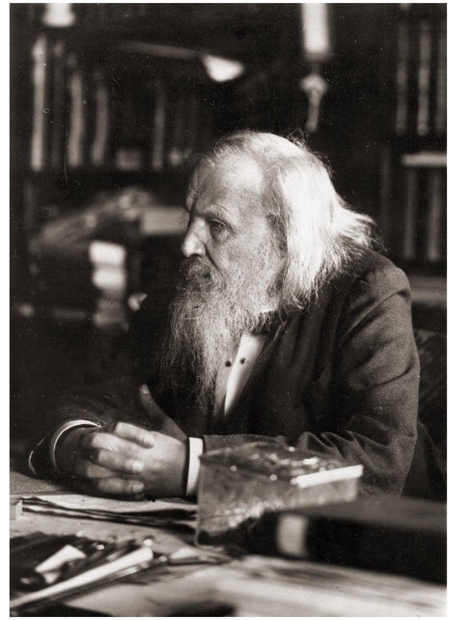


Mendeleev
(1834-1907)

International Year
of the Periodic Table
of Chemical Elements

Group → / Period ↓	1	2												13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc		22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y		40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

Periodic Table of elements (1869)



Mendeleev
(1834-1907)

タカとフジ（メンデレーエフの孫）

Group
1
2
3
5 6 7
13 14 15 16 17 18
2 He
5 B 6 C 7 N 8 O 9 F 10 Ne
13 Al 14 Si 15 P 16 S 17 Cl 18 Ar
6
55 Cs 56 Ba 57 La * 72 Hf 73 Ta 74 W
7
87 Fr 88 Ra 89 Ac ** 104 Rf 105 Db 106 Sg
* 58 Ce 59 Pr 60 Nd
** 90 Th 91 Pa 92 U

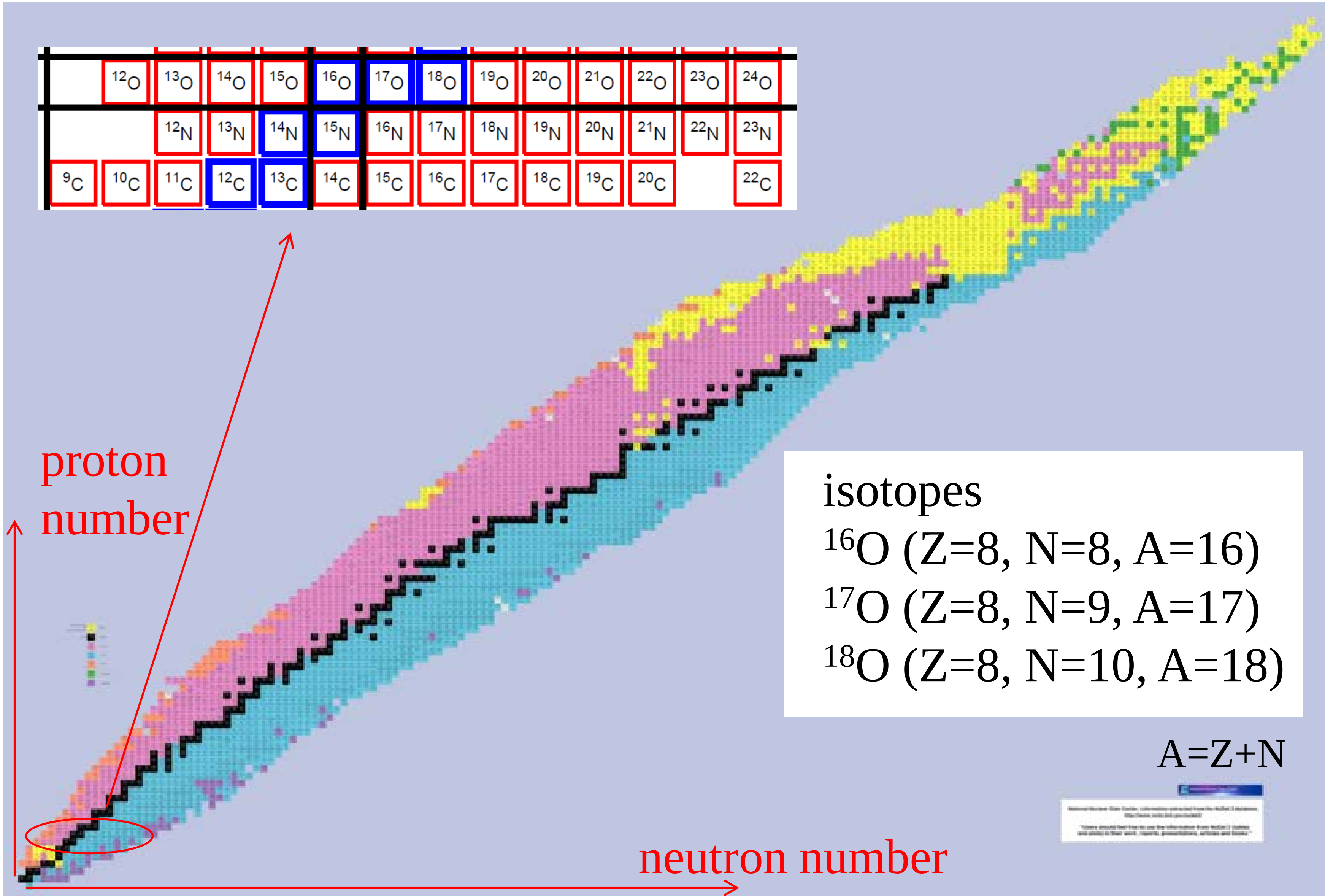
Periodic Table of elements (1869)

protons only, no information on neutrons

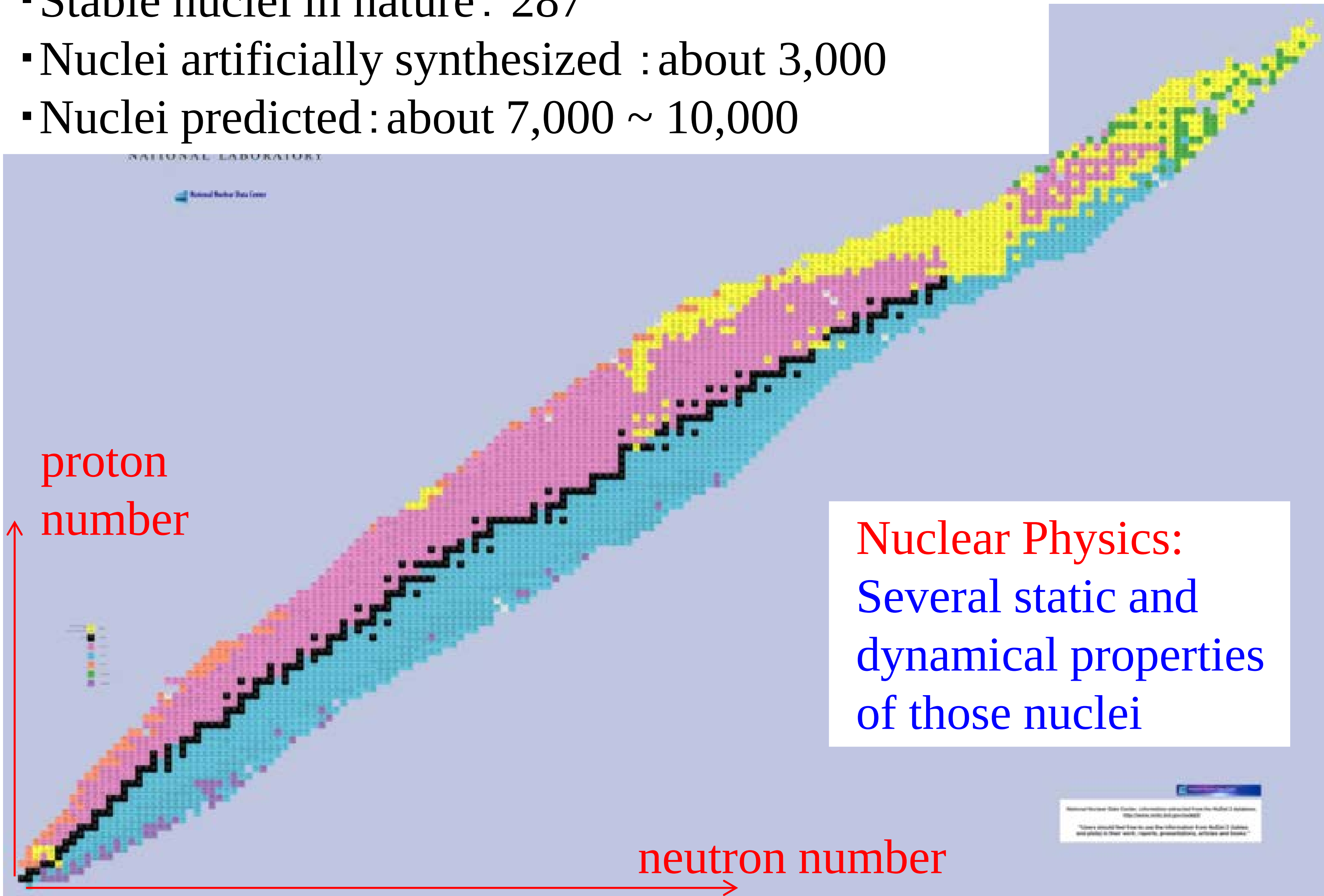
International Year
of the Periodic Table
of Chemical Elements

Period ↓ / Group →	1	2	3		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc		22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y		40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

Nuclear Chart: an extended version of periodic table

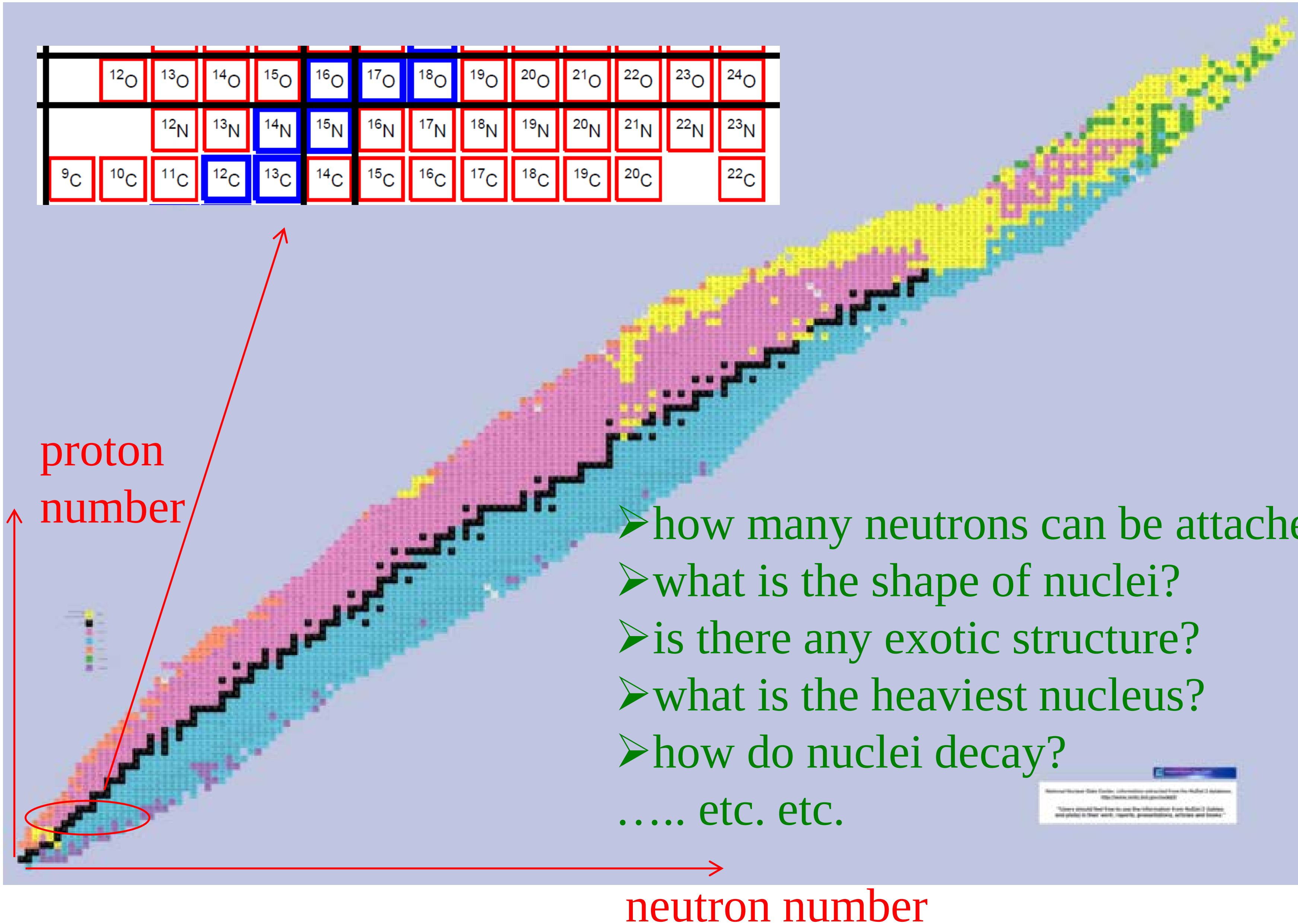


- Stable nuclei in nature： 287
- Nuclei artificially synthesized ：about 3,000
- Nuclei predicted：about 7,000 ~ 10,000



Nuclear Physics:
Several static and
dynamical properties
of those nuclei

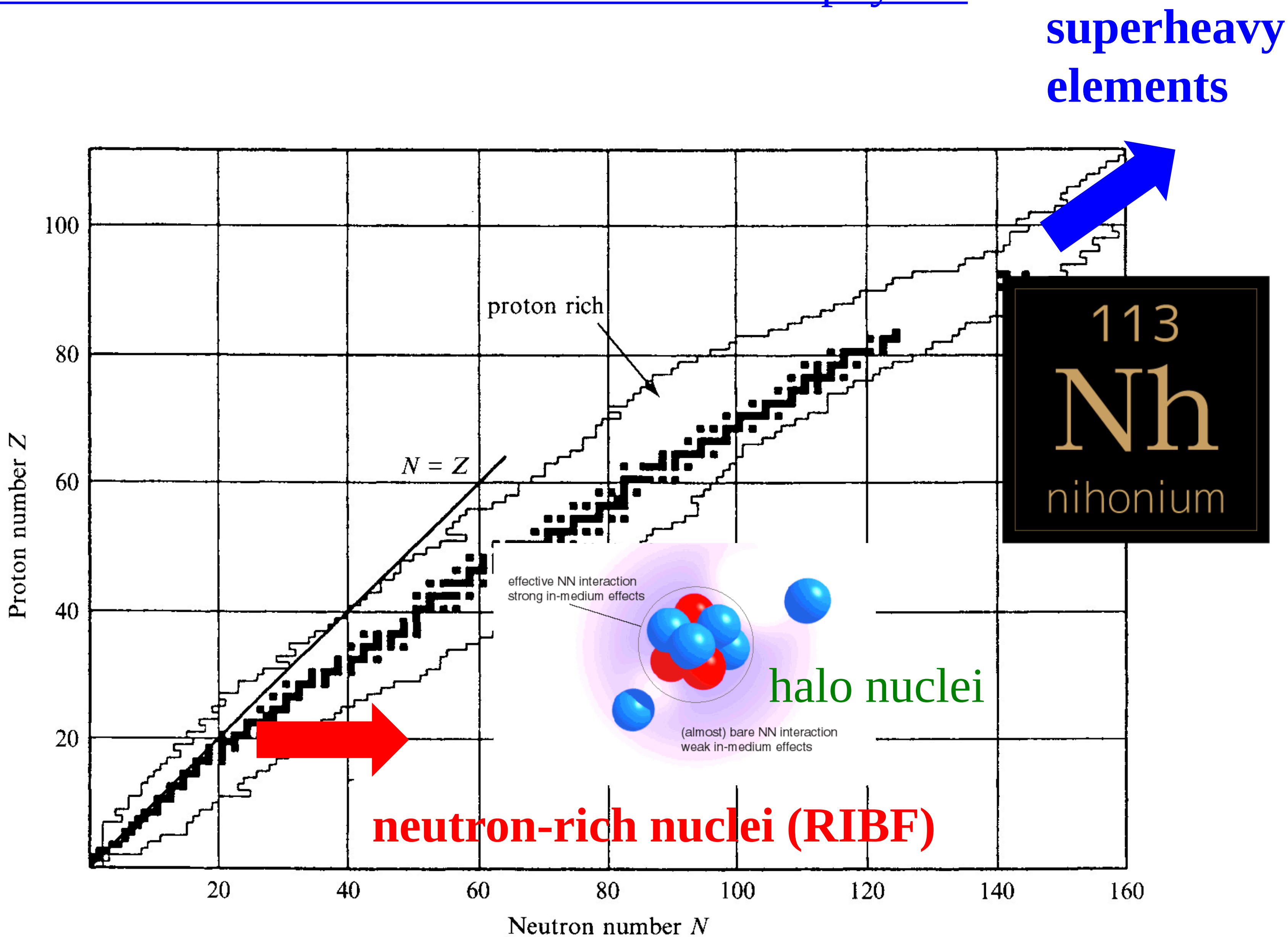
Nuclear Chart: an extended version of periodic table



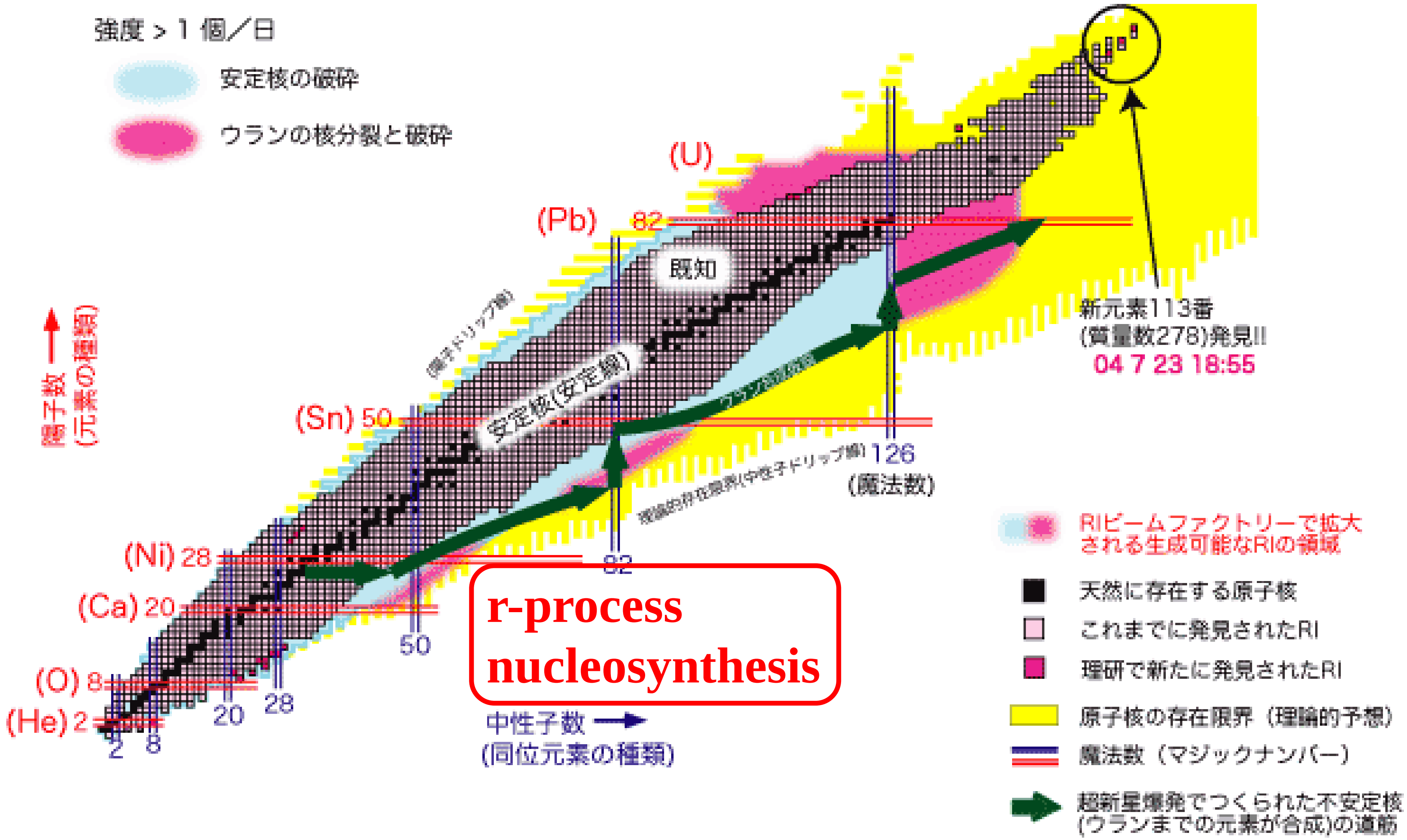
- how many neutrons can be attached?
- what is the shape of nuclei?
- is there any exotic structure?
- what is the heaviest nucleus?
- how do nuclei decay?

….. etc. etc.

Extension of nuclear chart: frontier of nuclear physics



Neutron-rich nuclei (RIBF at RIKEN)



2017年10月17日朝日新聞


2017年（平成29年）10月17日 火曜日
朝日新聞

星の合体 重力波で観測

発生源からの光も確認

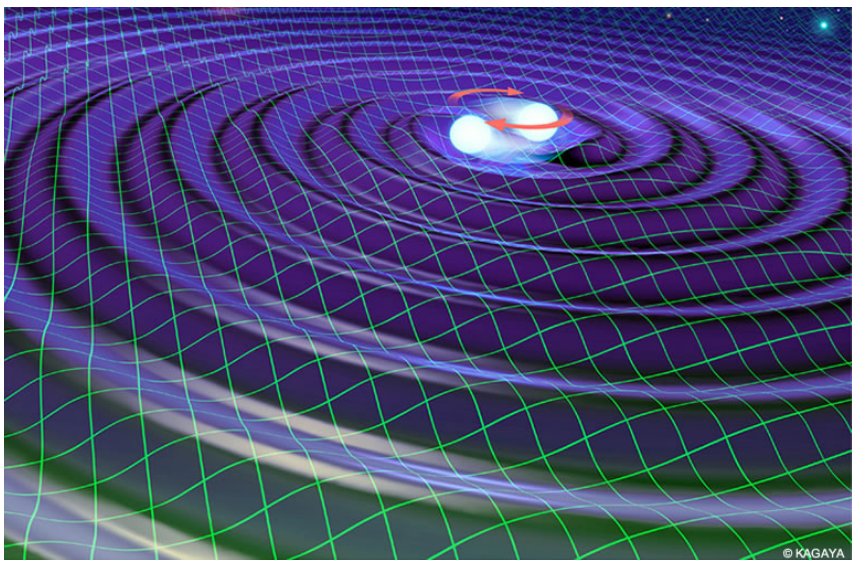
米欧の研究グループ発表

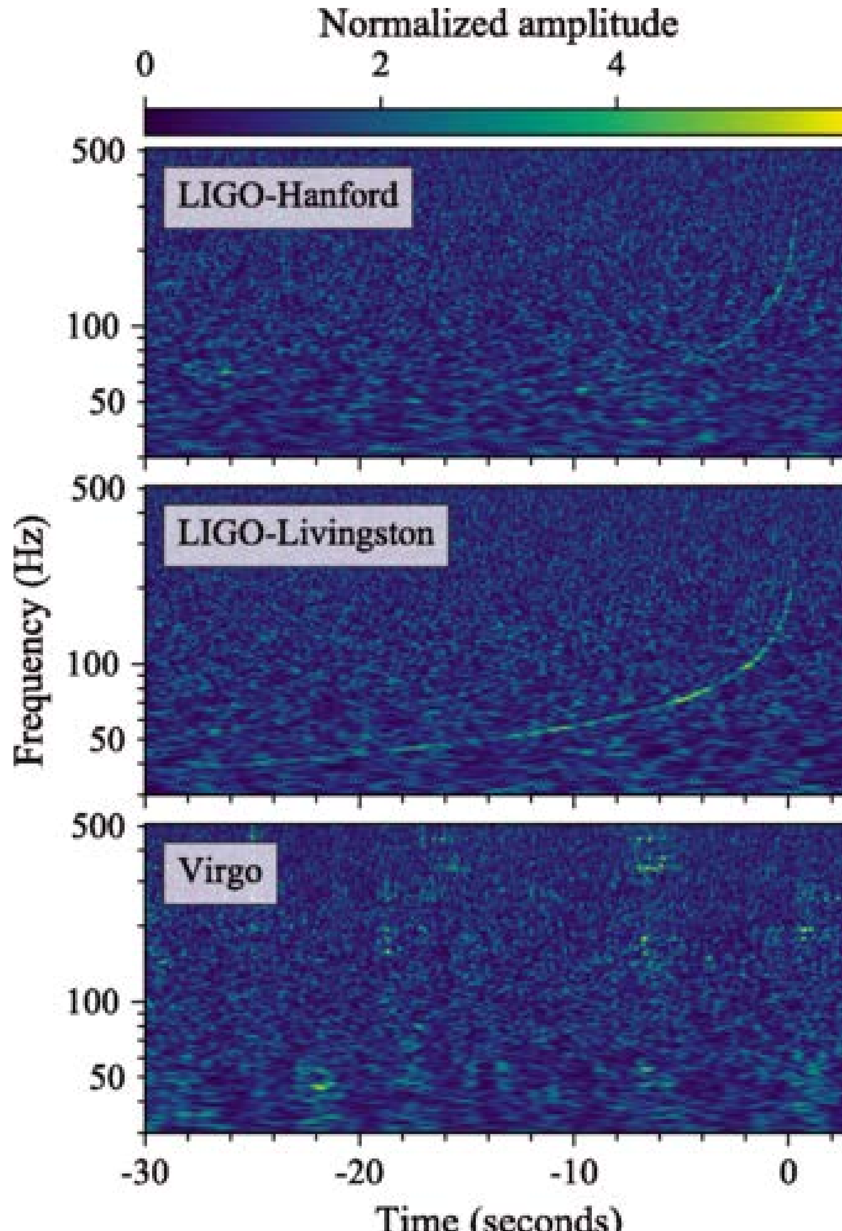
元素の起源 解明に期待

中性子星

中性子星合体による重力波を観測

信頼の技術を、医薬品に。 NIPRO





このときに元素合成が? NAOJ

B.P. Abbott et al., PRL119 ('17) 161101

2017年10月17日朝日新聞

朝日新聞

2017年(平成29年) 10月17日 火曜日

星の合体 重力波で観測

発生源からの光も確認

米欧の研究グループ発表

元素の起源 解明に期待

中性子星

中性子星合体による重力波を観測

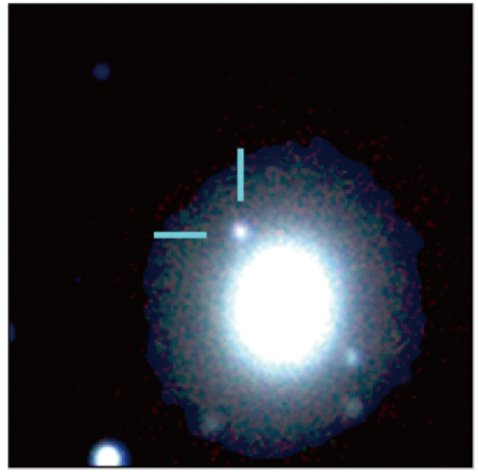
2017衆院選 投票まで 5日

改憲への動きシミュレーション 2
安倍政治 その先⑥ 核政策 3
原発再稼働 争点化の攻防 4
金融緩和 語られない「出口」 7
社説 教育無償化／知る権利 14
耕論 社会保障は国難か 15

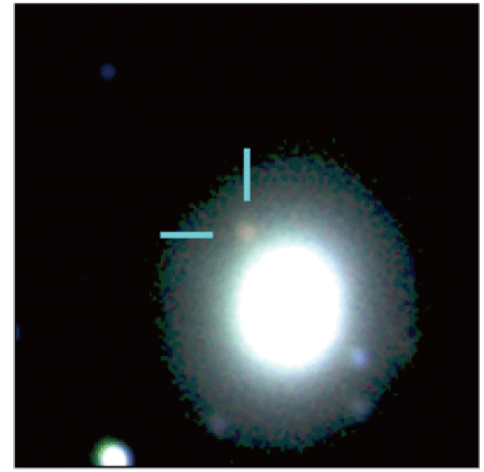
信頼の技術を、医薬品に。 NIPRO

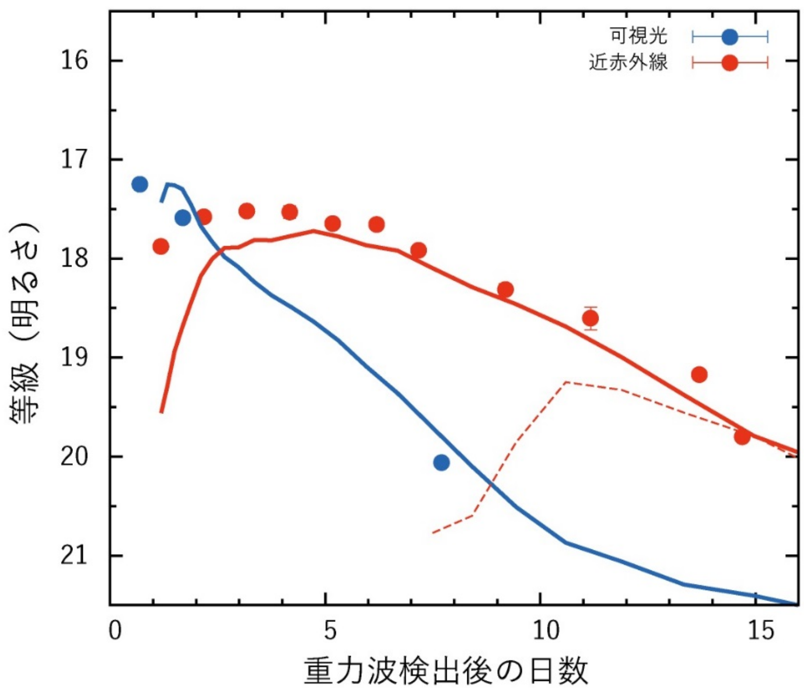
重力波源から来た電磁波（の時間変化）

2017.08.18-19



2017.08.24-25





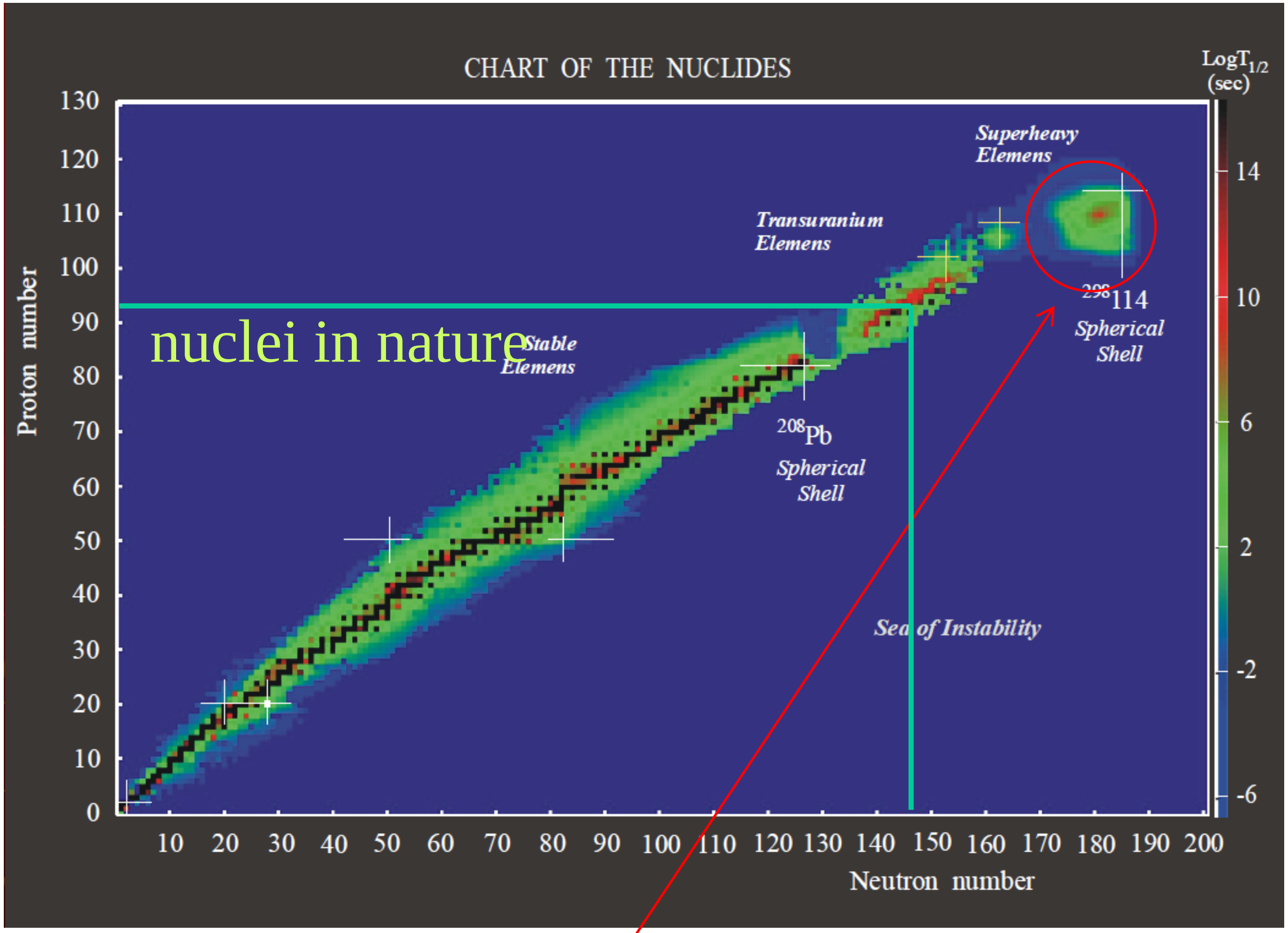
実線：r-プロセスが起こった場合

破線：起こらなかった場合

M. Tanaka et al.,
Astron. Soc. Jpn. 69 (‘17) 102

http://www.cfca.nao.ac.jp/pr/20171016

Prediction of island of stability: an important motivation of SHE study



island of stability around Z=114, N=184

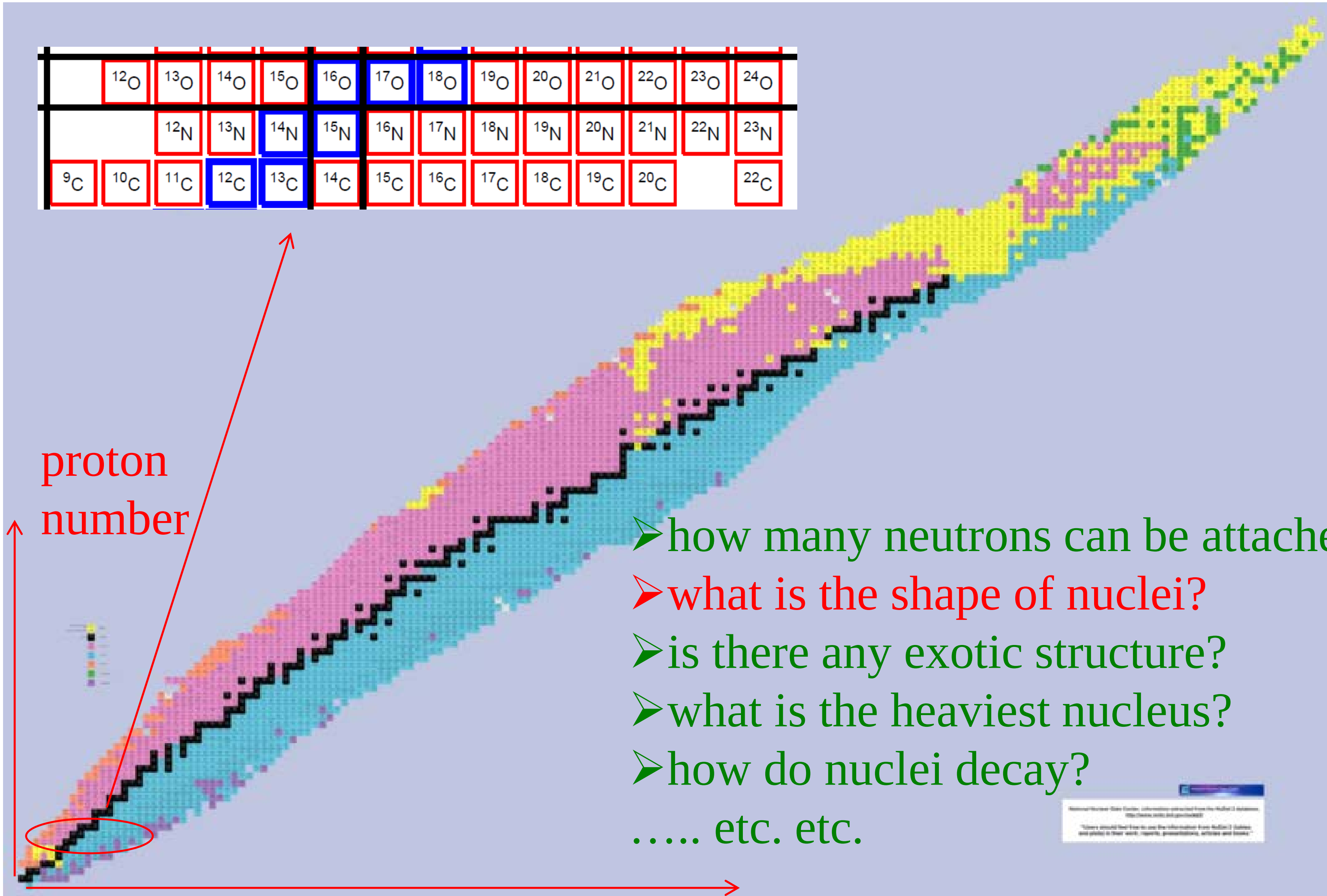
Yuri Oganessian

W.D. Myers and W.J. Swiatecki (1966), A. Sobiczewski et al. (1966)

⟶ modern calculations: Z=114,120, or 126, N=184

e.g., H. Koura et al. (2005)

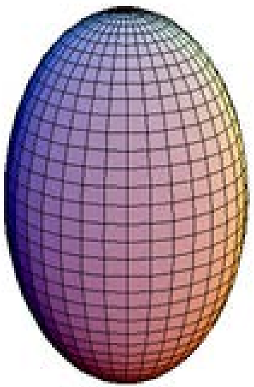
Nuclear Chart: an extended version of periodic table

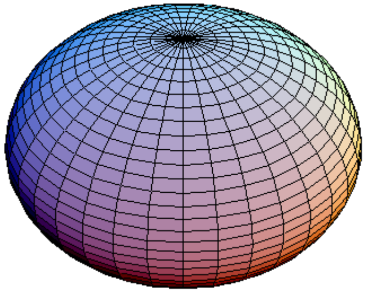


- how many neutrons can be attached?
- what is the shape of nuclei?
- is there any exotic structure?
- what is the heaviest nucleus?
- how do nuclei decay?

….. etc. etc.

a nucleus is not always spherical



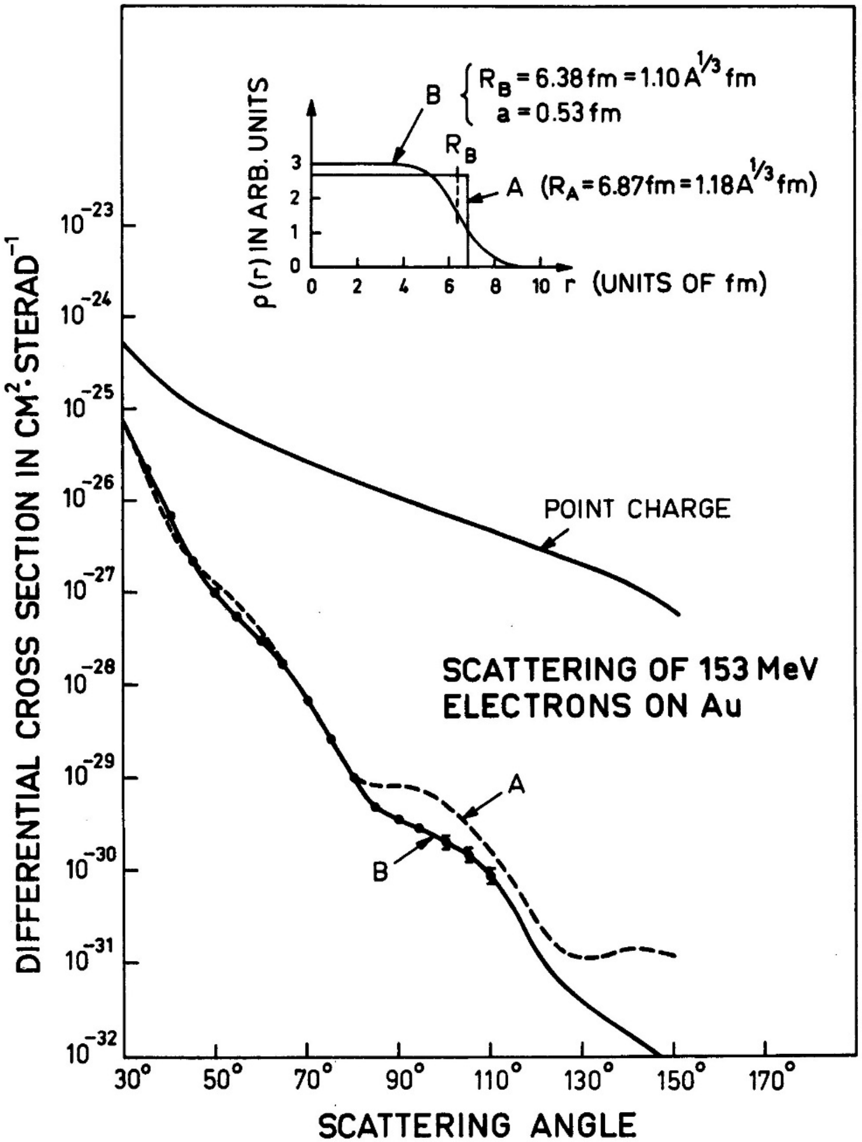


Quantum shape dynamics

「形の量子論」

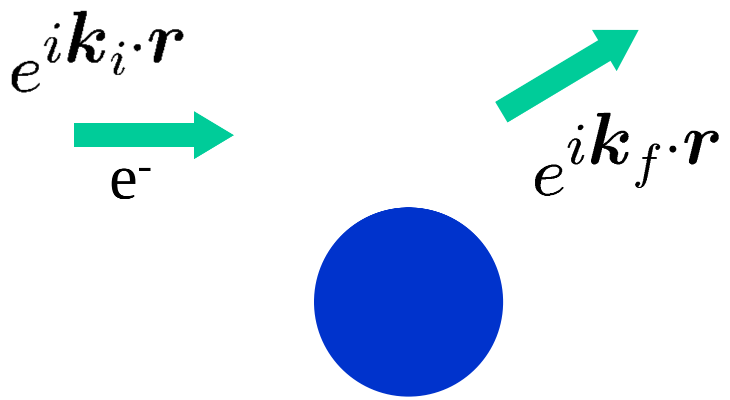
Density Distribution

Density Distribution



High energy electron scattering

Born approximation:



$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2}\,|F(\boldsymbol{q})|^2$$

Form factor

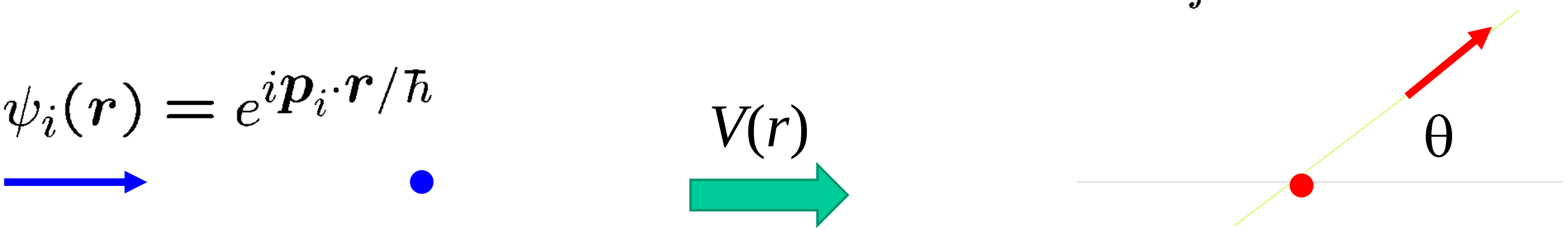
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho(\boldsymbol{r})\,d\boldsymbol{r}$$

(Fourier transform of the density)

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$



$$W_{fi} = \frac{2\pi}{\hbar}|\langle\psi_f|V|\psi_i\rangle|^2\rho(E_f)$$

incident flux: $j_{\text{inc}} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\text{inc}}} = \int d\Omega \frac{d\sigma}{d\Omega}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E \sin^2 \theta/2)^2} |F(\boldsymbol{q})|^2$$

$$= \left(\frac{d\sigma_{\mathrm{Ruth}}}{d\Omega}\right) |F(\boldsymbol{q})|^2$$

Form factor

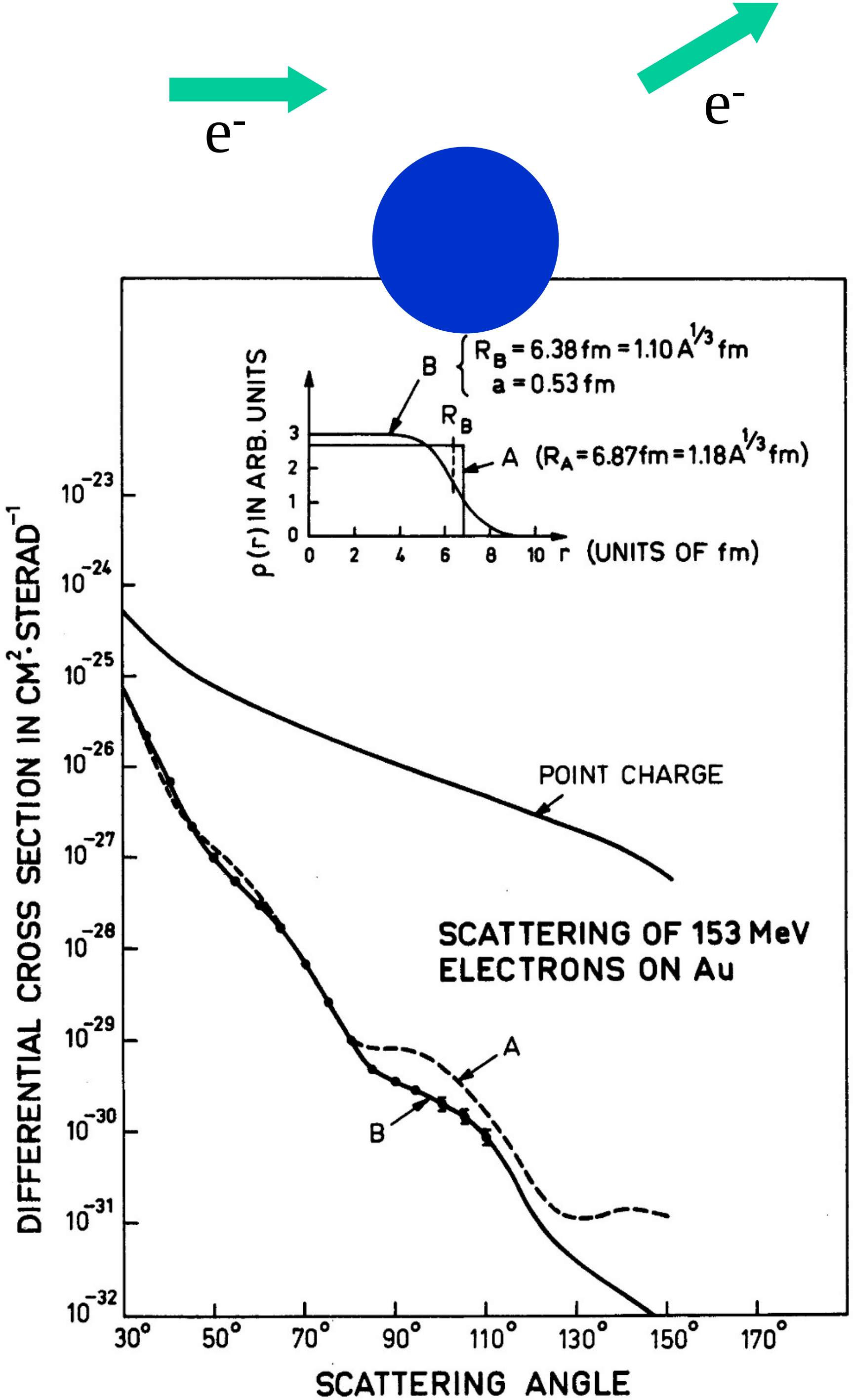
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})\, d\boldsymbol{r}$$

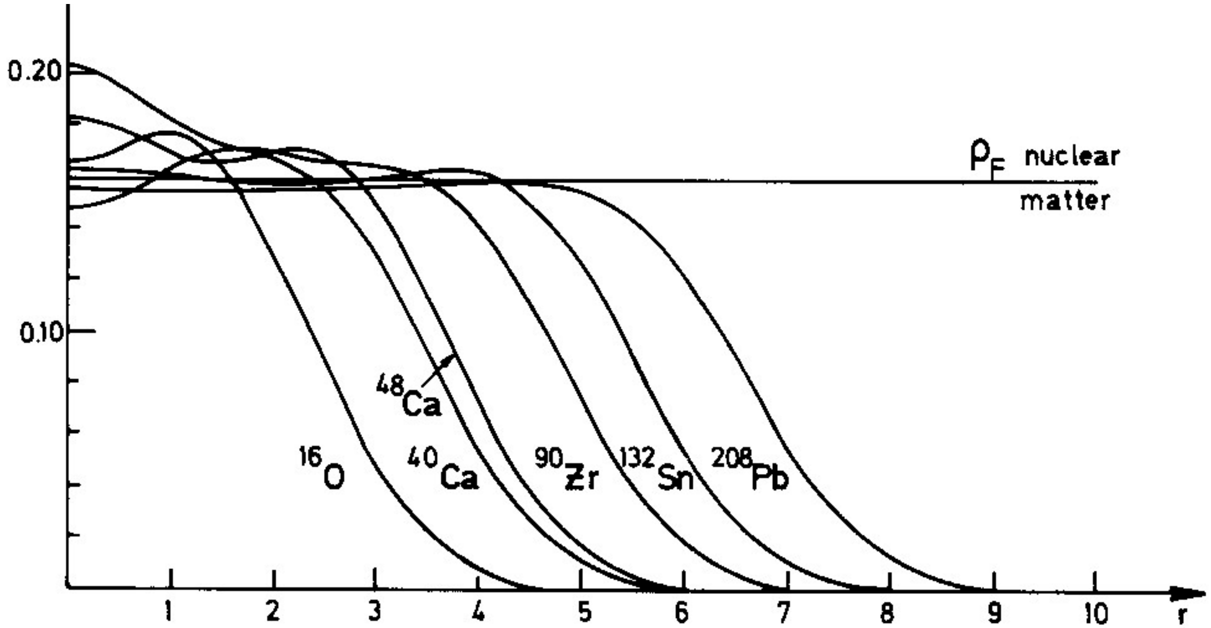
* relativistic correction:

$$\frac{d\sigma_{\mathrm{Ruth}}}{d\Omega} \to \frac{d\sigma_{\mathrm{Mott}}}{d\Omega}$$

$$= \frac{d\sigma_{\mathrm{Ruth}}}{d\Omega} \cdot \left(1 - \frac{v^2}{c^2} \sin^2 \frac{\theta}{2}\right)$$

$$\sim \frac{d\sigma_{\mathrm{Ruth}}}{d\Omega} \cdot \cos^2 \frac{\theta}{2} \qquad (v \to c)$$





Fermi distribution

$$\rho(r) = \frac{\rho_0}{1 + \exp[(r - R_0)/a]}$$

$$\rho_0 \sim 0.17 \ (\mathrm{fm}^{-3})$$

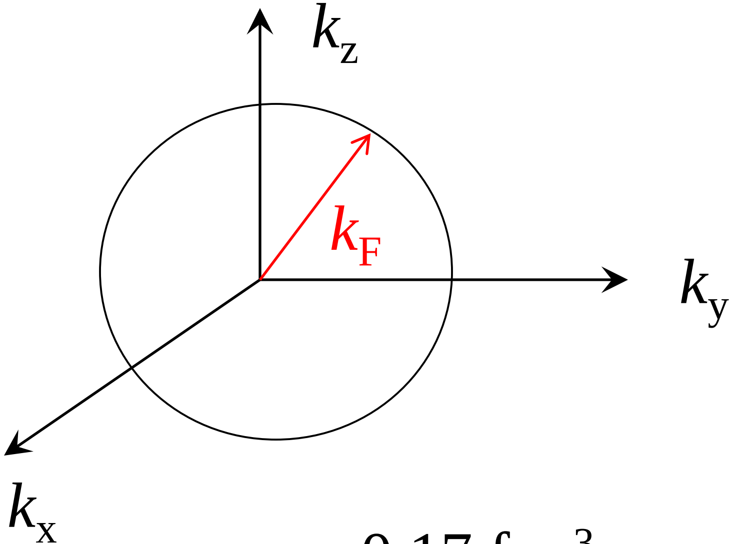
← Saturation property

$$R_0 \sim 1.1 \times A^{1/3} \ (\mathrm{fm})$$

$$a \sim 0.57 \ (\mathrm{fm})$$

Momentum Distribution

Fermi gas approximation



$$\rho = 2 \times 2 \times 4\pi \int_0^{k_F} \frac{k^2 dk}{(2\pi)^3}$$

$$= \frac{2}{3\pi^2} k_F^3$$

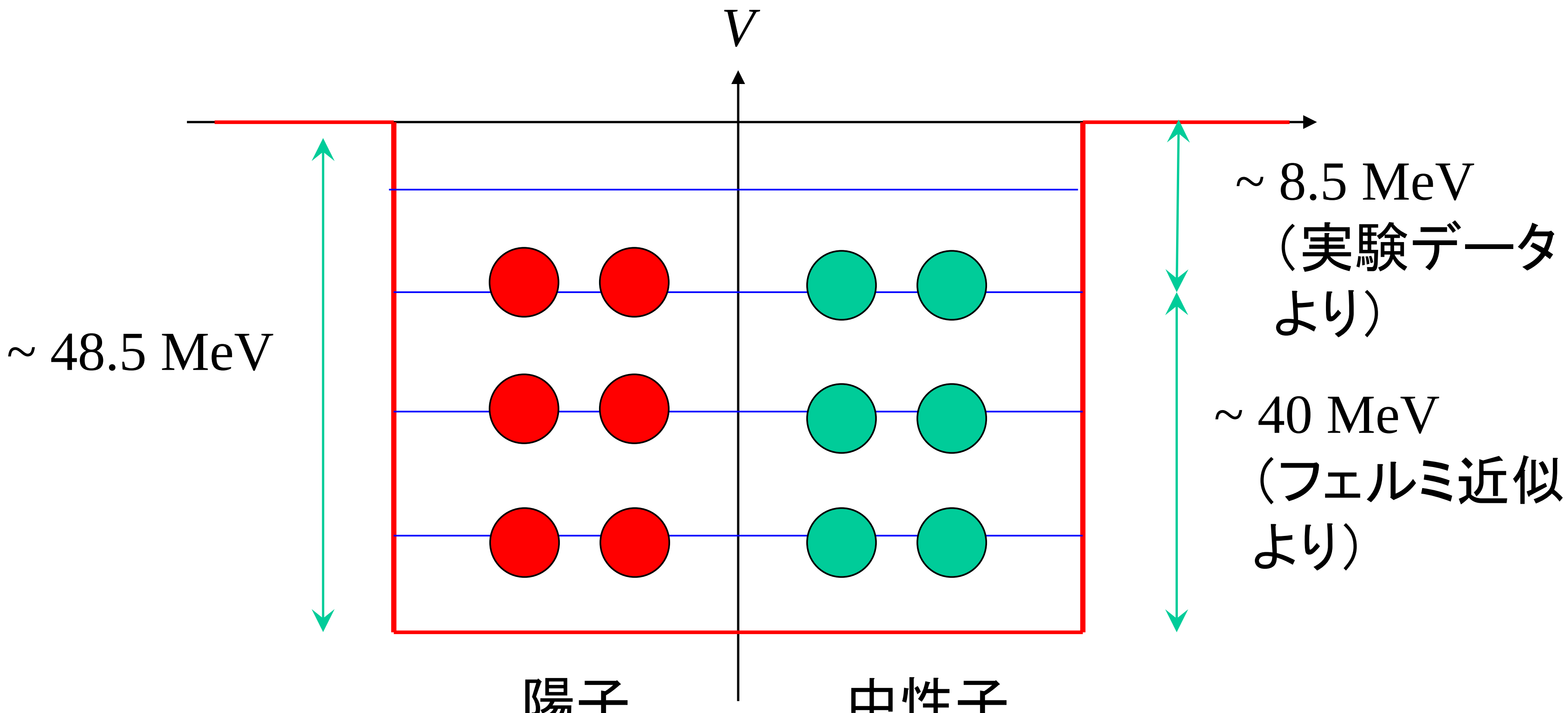
(note: spin-isospin degeneracy)

ρ = 0.17 fm^{-3} ⟶ $k_F \sim$ 1.36 fm^{-1}

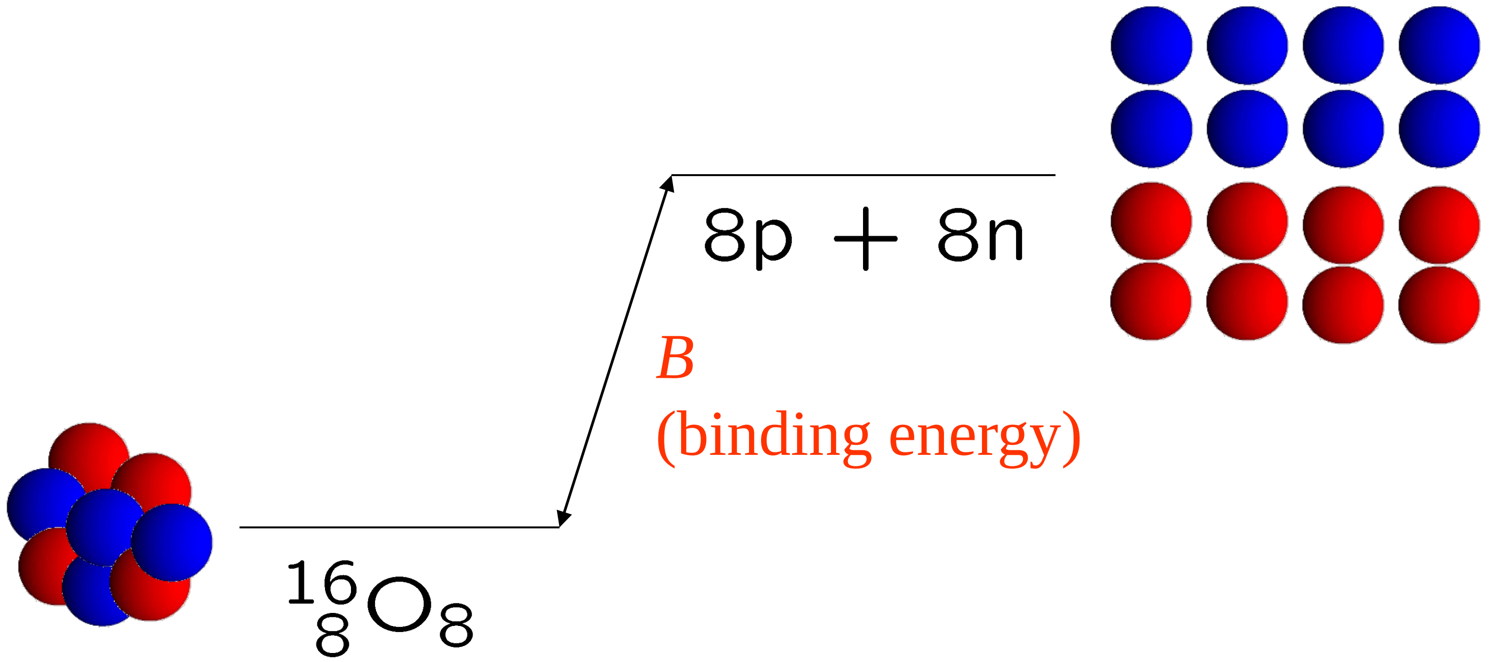
$$\Longleftrightarrow \quad \frac{v_F}{c} = \frac{k_F \cdot \hbar c}{mc^2} = 0.285$$

Fermi energy: $\epsilon_F = \dfrac{k_F^2 \hbar^2}{2m} \sim 37$ (MeV)

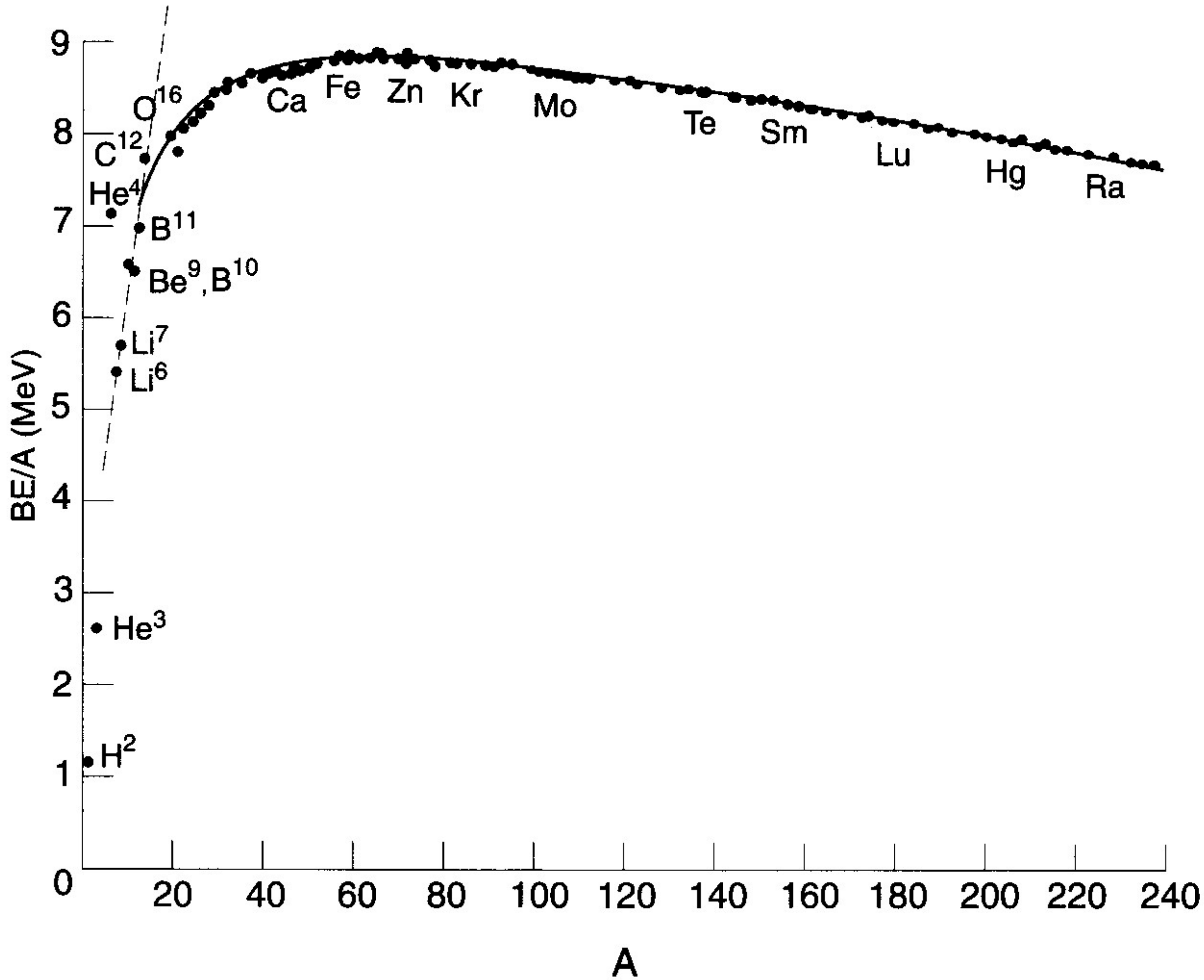
原子核の中で核子が感じるポテンシャル



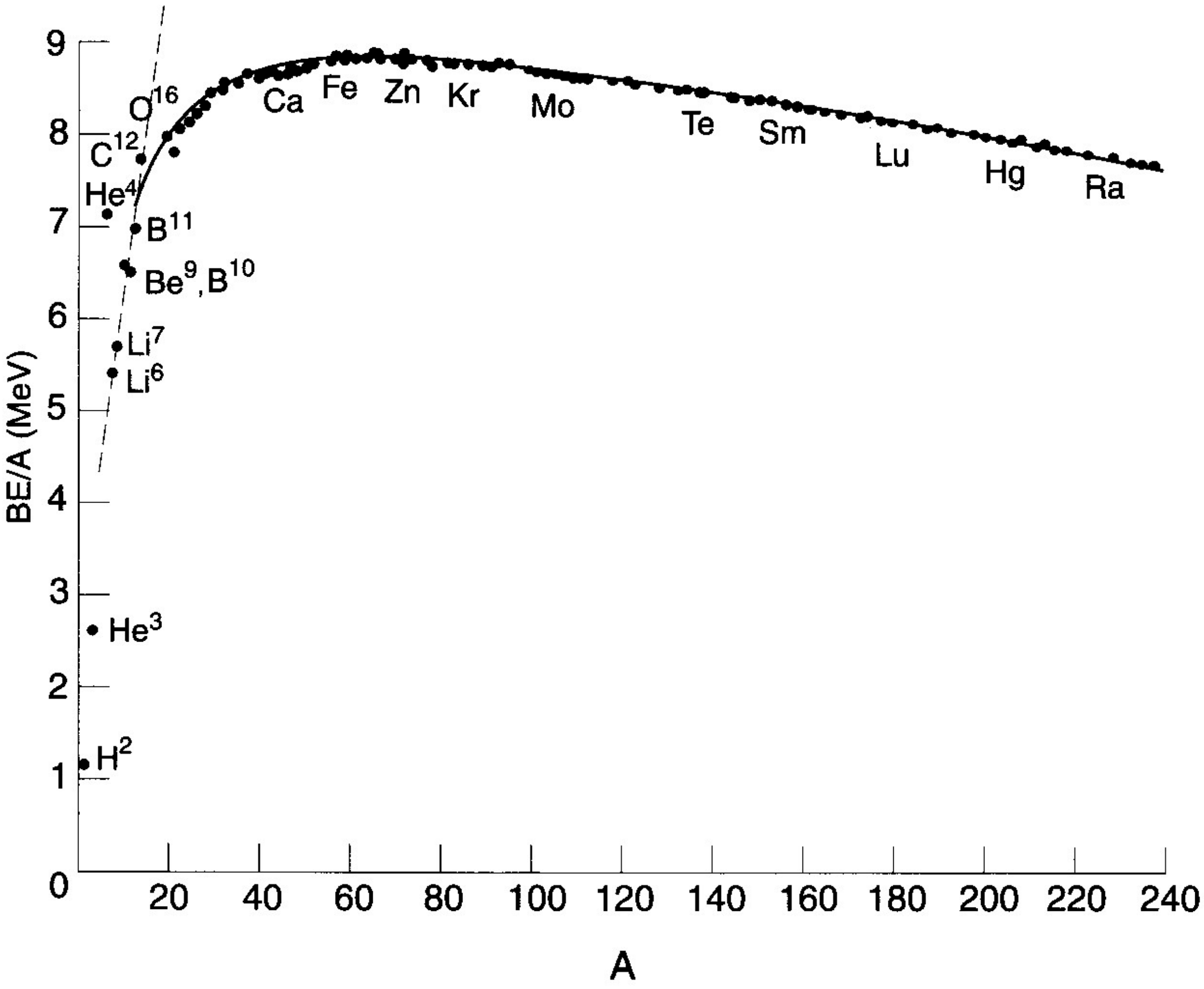
Nuclear Mass



$$m(N,Z)c^2 = Zm_pc^2 + Nm_nc^2 - B$$



BE/A (MeV)
A
H^2
He^3
He^4
Li^6
Li^7
Be^9,B^{10}
B^{11}
C^{12}
O^{16}
Ca
Fe
Zn
Kr
Mo
Te
Sm
Lu
Hg
Ra

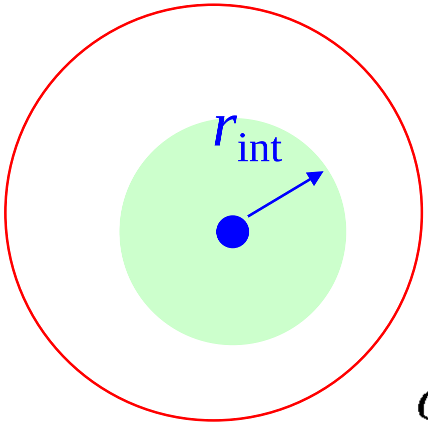


B(N,Z)/A ~ 8.5 MeV (A > 12) $\Longleftrightarrow$ Short range nuclear force

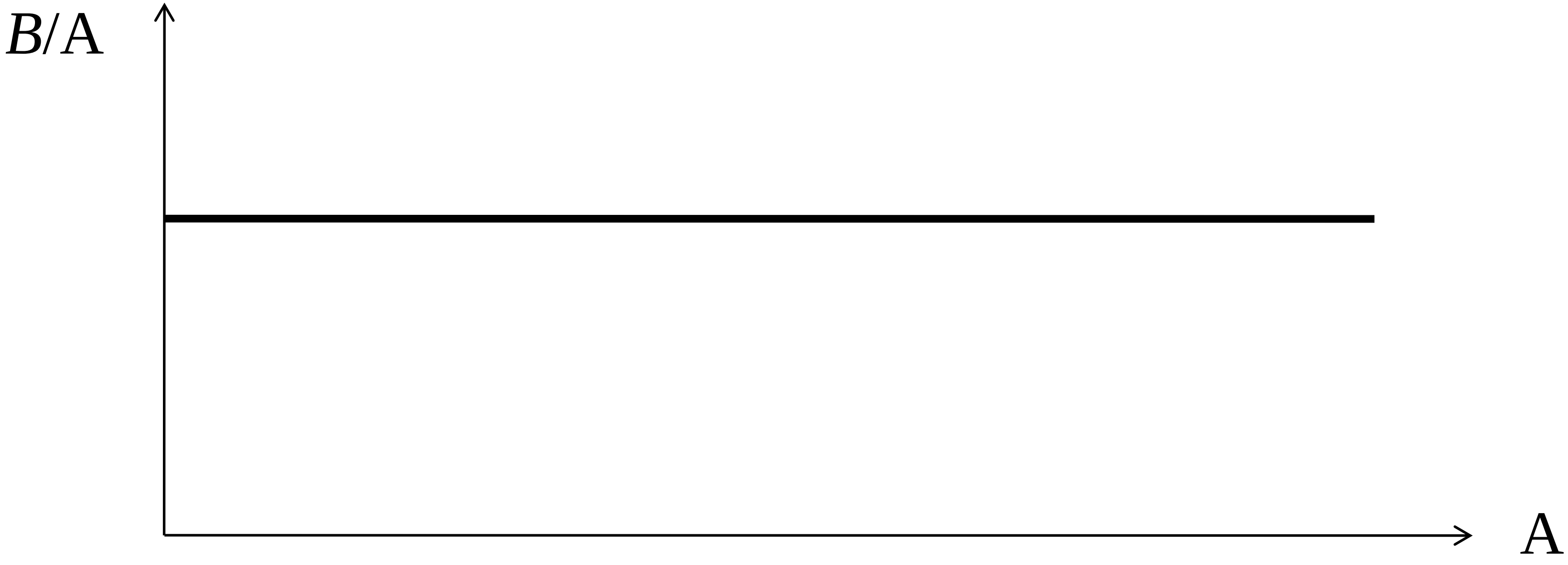
a long range interaction: $B \propto A(A-1) \sim A^2 \quad \rightarrow B/A \sim A$

if each nucleon can interact only α-nucleons close by:

$$B \sim \alpha\, A/2 \longrightarrow B/A \sim \alpha/2 \text{ (const.)}$$

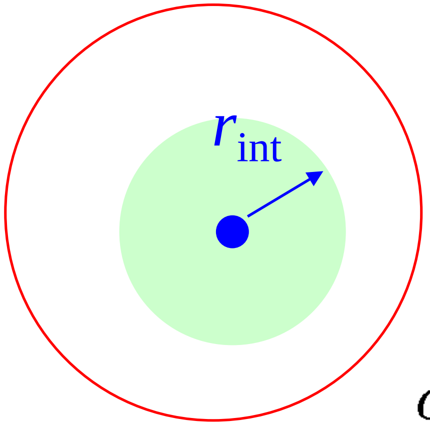


$$\alpha = \frac{4\pi}{3} r_{\rm int}^3 \cdot \rho$$

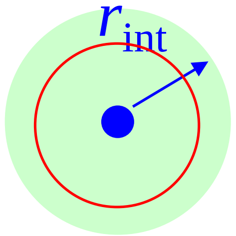


if each nucleon can interact only α-nucleons close by:

$B \sim \alpha\, A/2 \longrightarrow B/A \sim \alpha/2$ (const.)

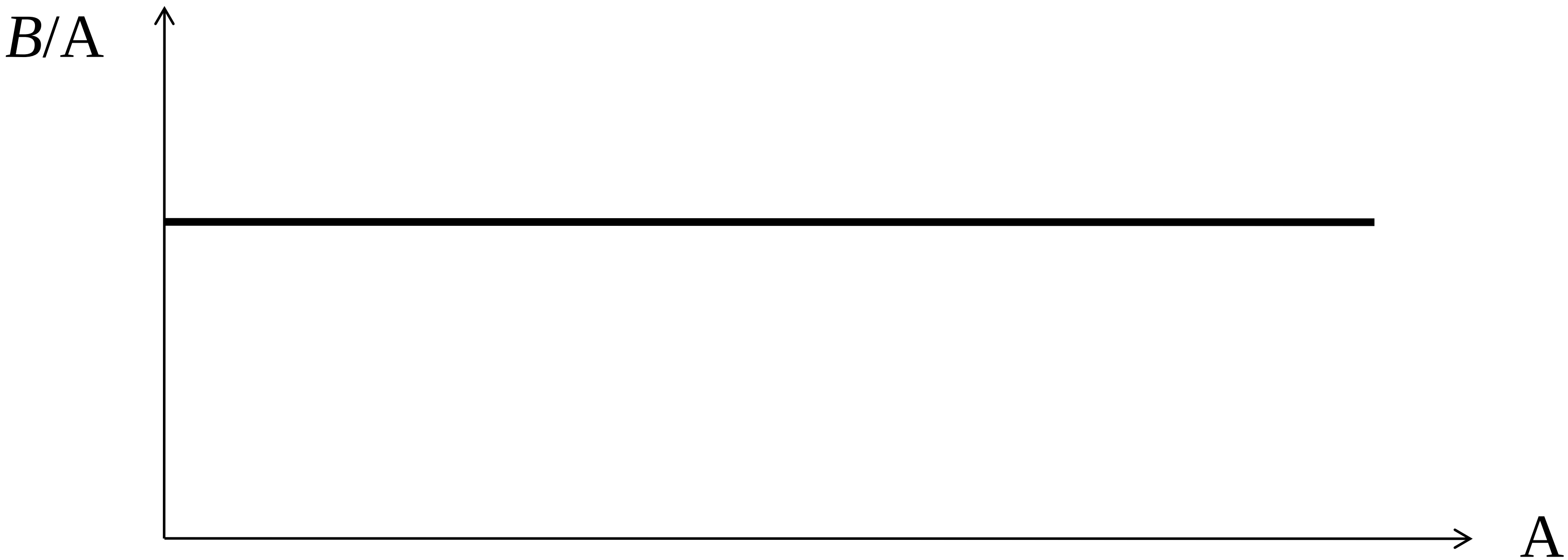


a small nucleus



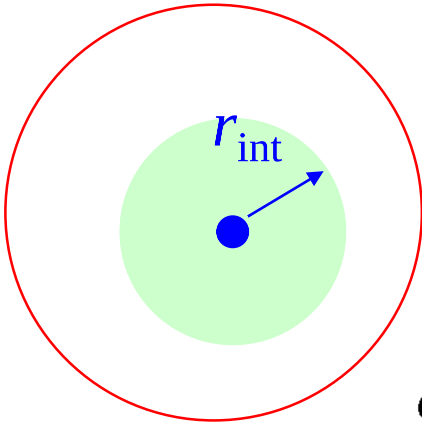
$$\alpha = \frac{4\pi}{3} r_{\text{int}}^3 \cdot \rho$$

$$\rightarrow B/A \propto A - 1$$

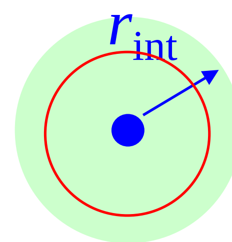


if each nucleon can interact only α-nucleons close by:

$B \sim \alpha A/2 \longrightarrow B/A \sim \alpha/2$ (const.)

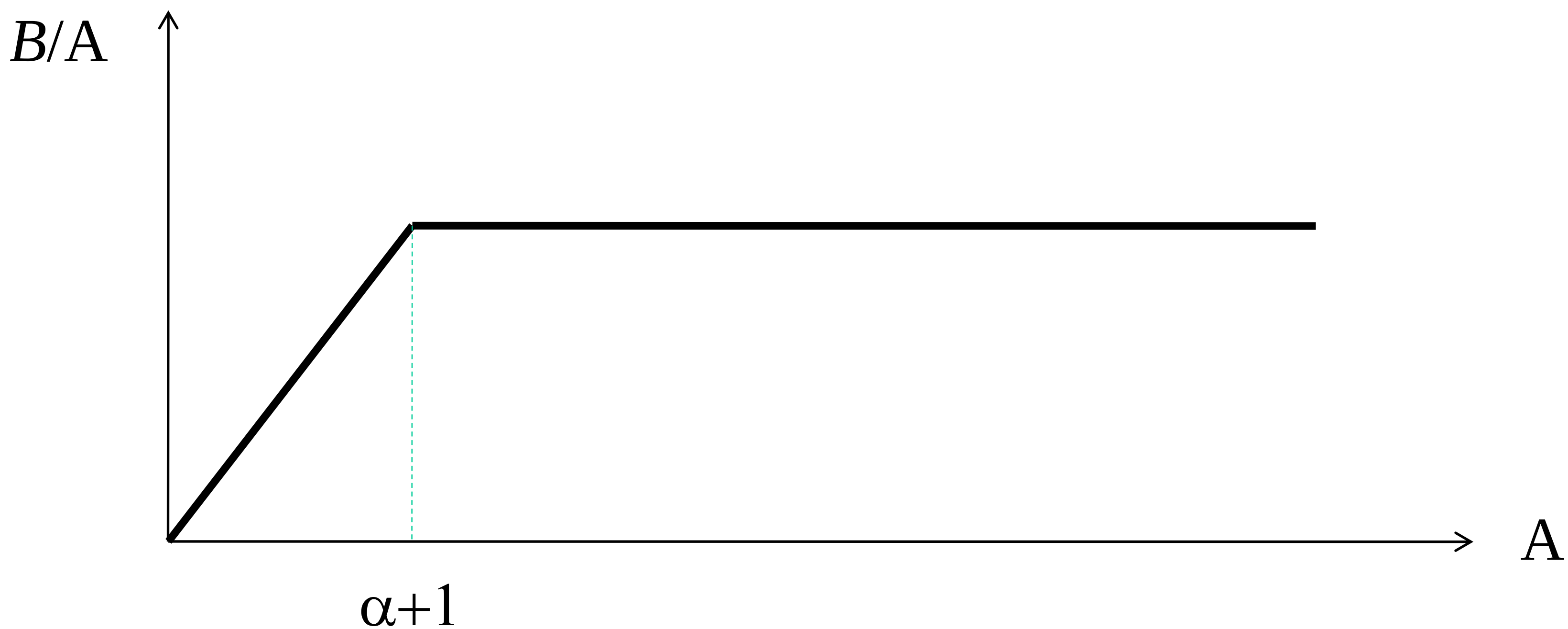


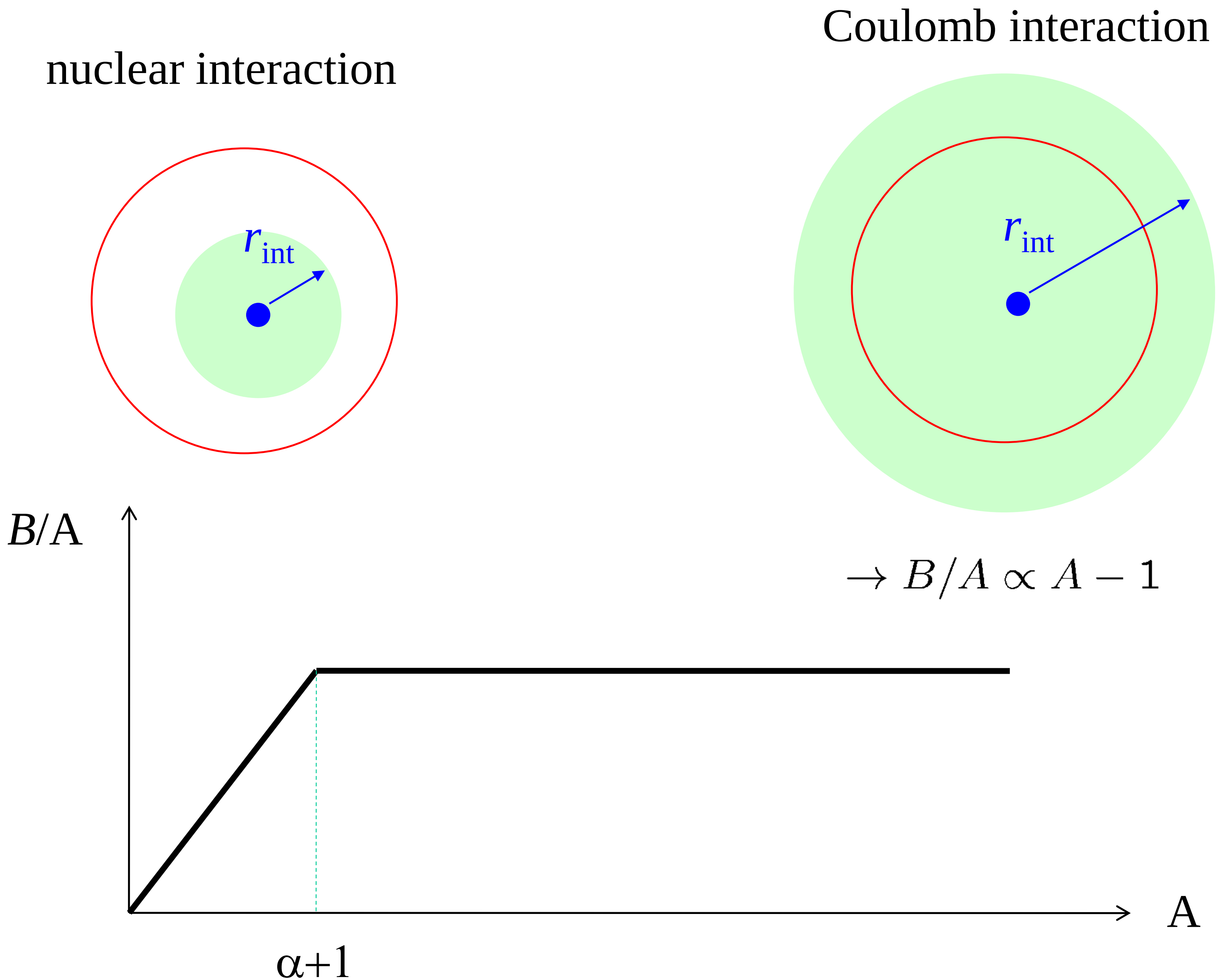
a small nucleus

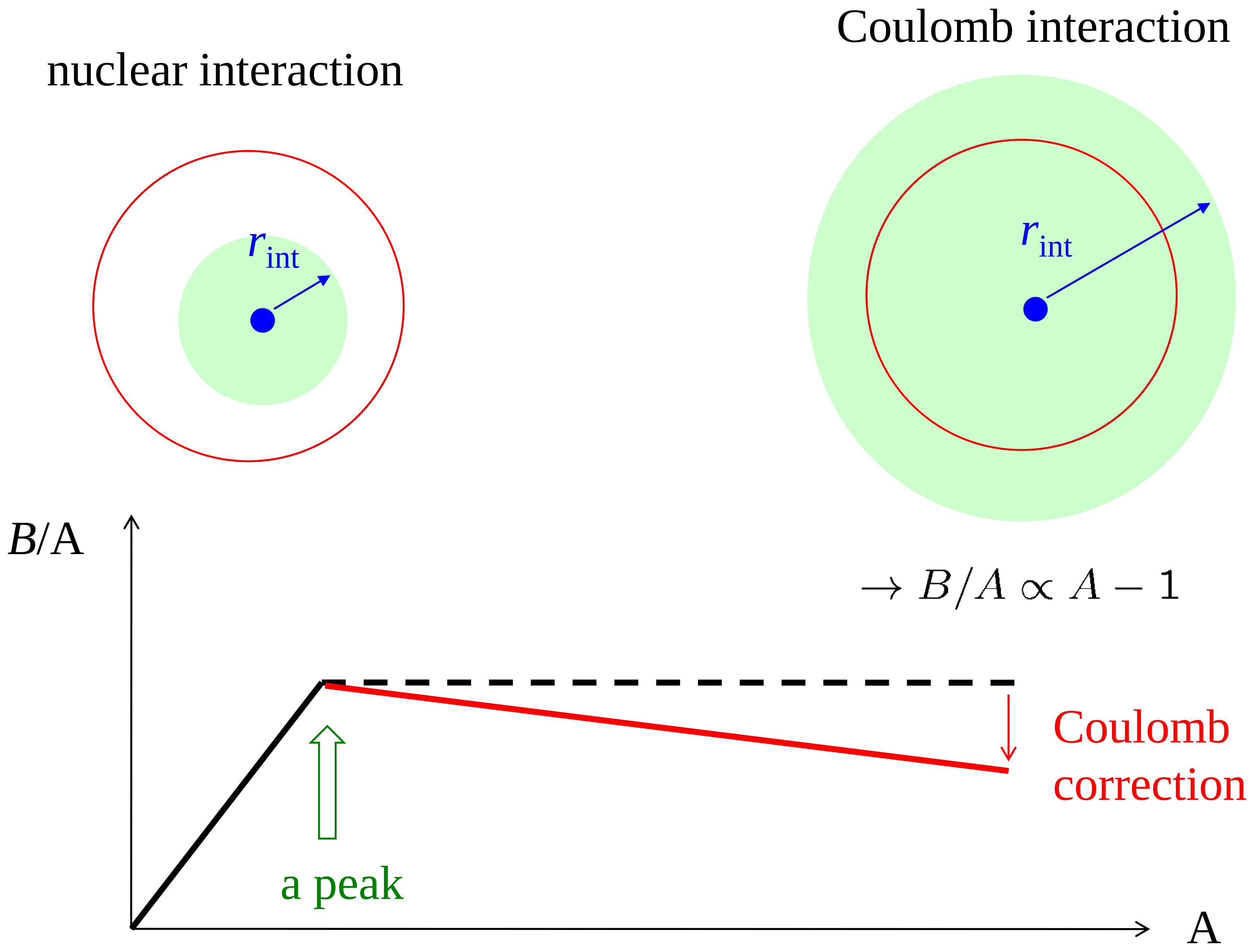


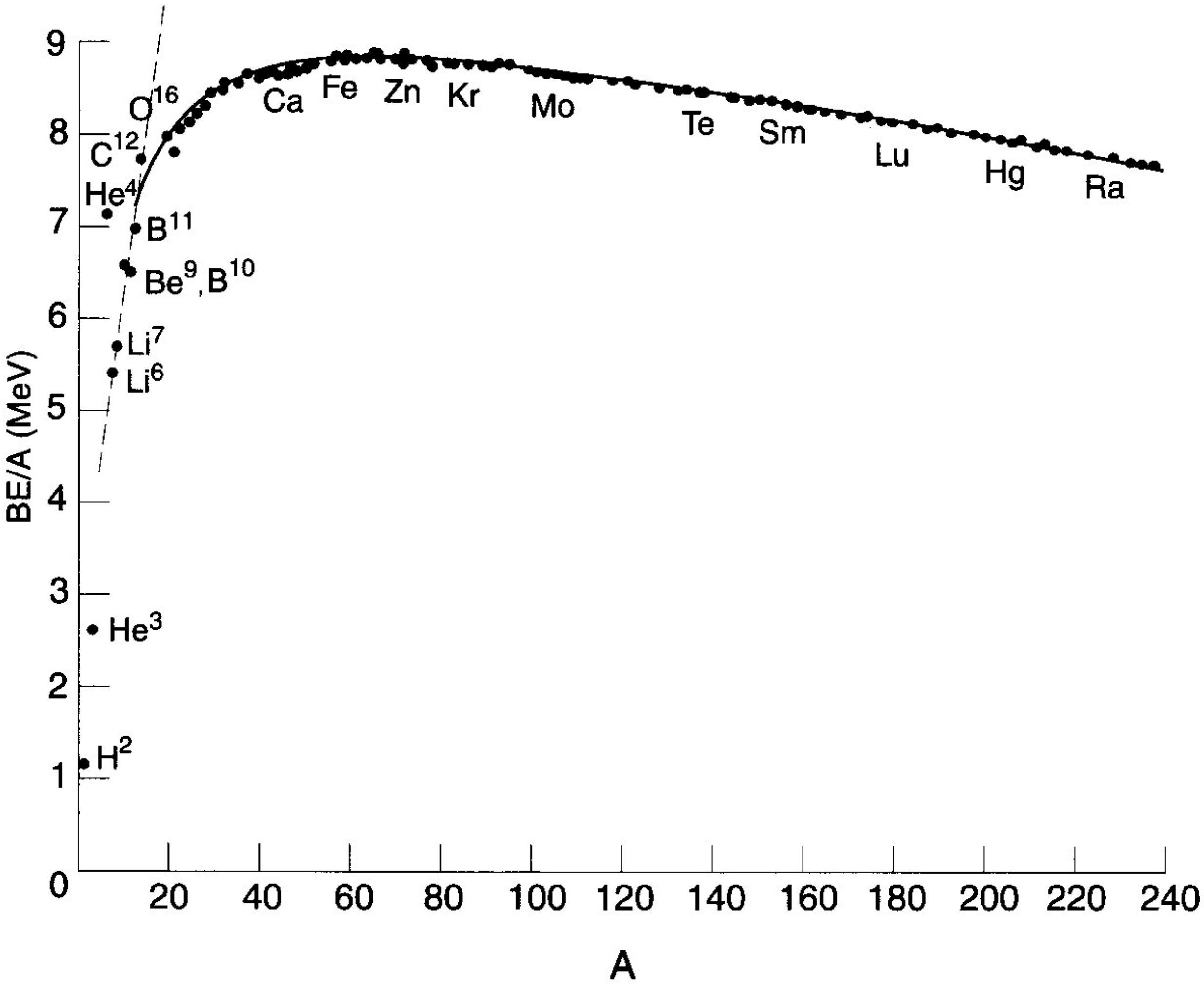
$$\alpha = \frac{4\pi}{3} r_{\text{int}}^3 \cdot \rho$$

$$\rightarrow B/A \propto A - 1$$



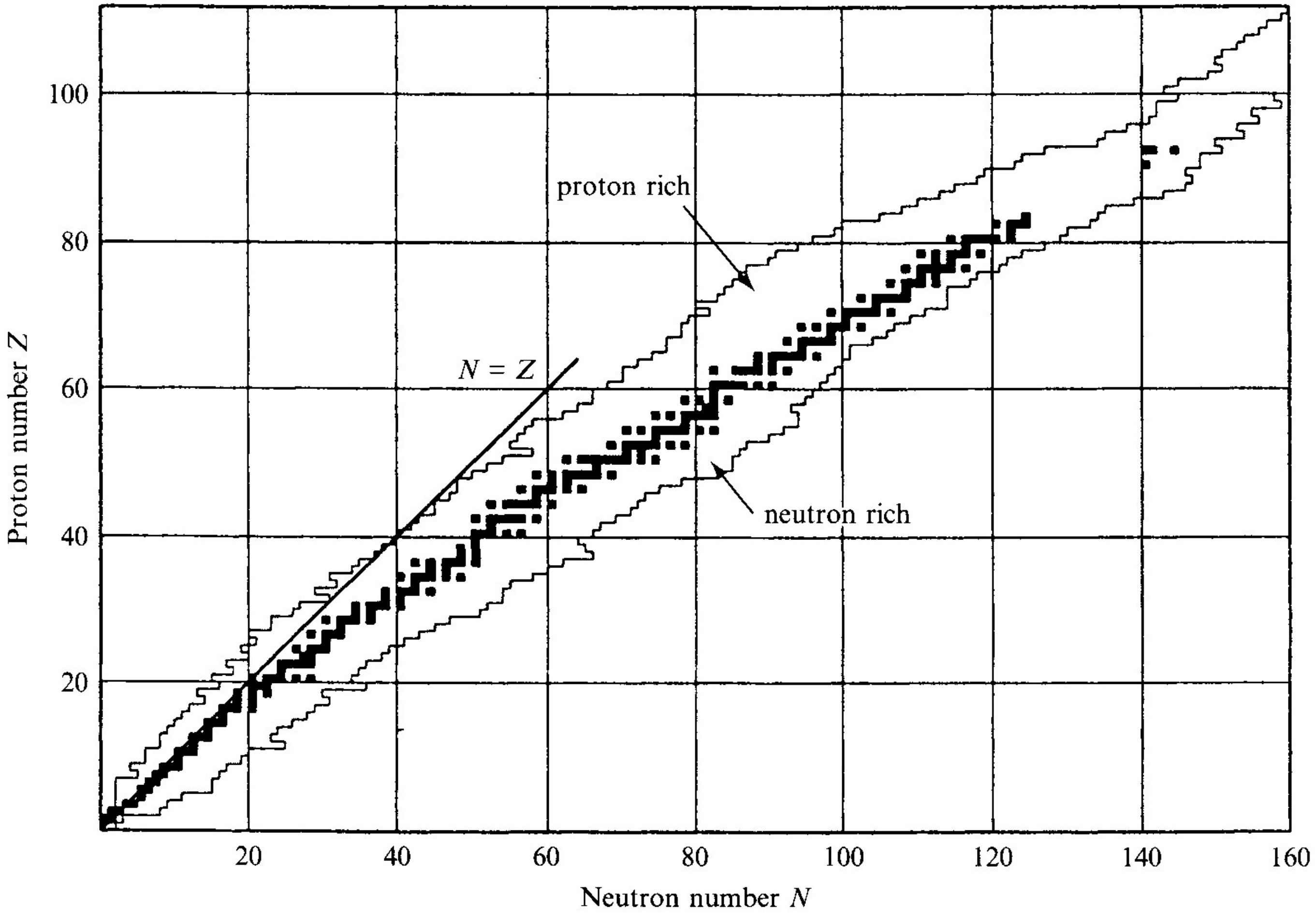

nuclear interaction
Coulomb interaction
r int
r int
→ B/A ∝ A − 1
B/A
A
α+1


nuclear interaction
Coulomb interaction
$r_{\rm int}$
$r_{\rm int}$
B/A
$\rightarrow B/A \propto A-1$
Coulomb
correction
a peak
A

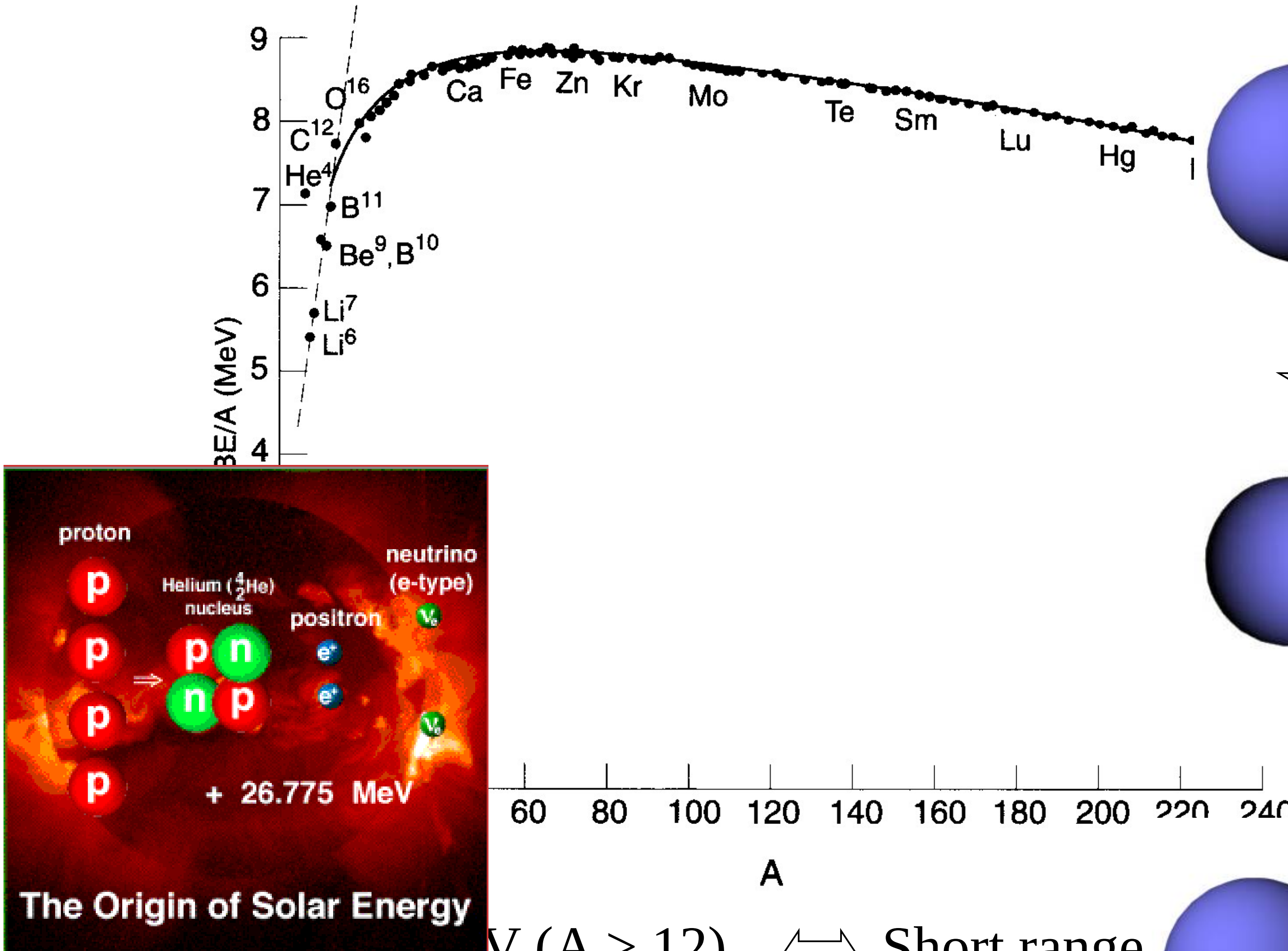


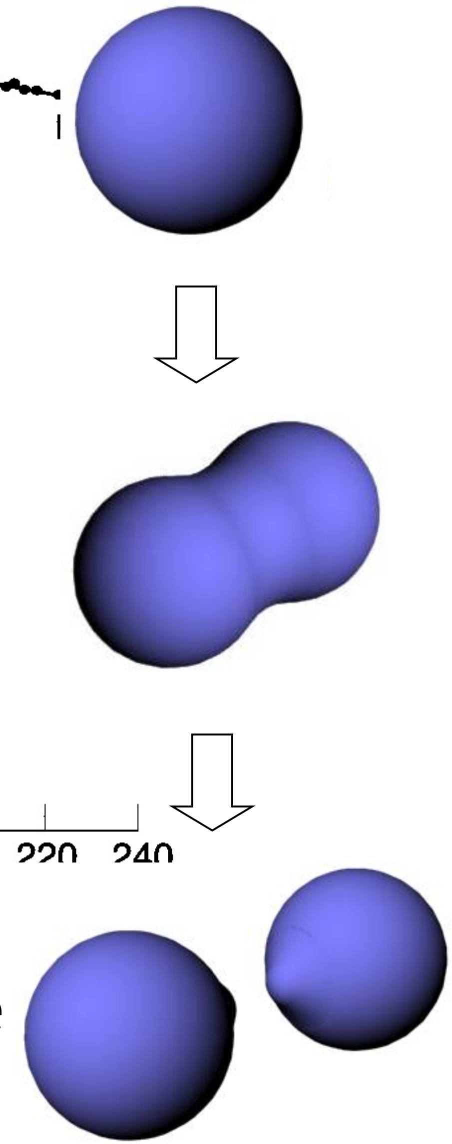
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force
2. Effect of Coulomb force for heavy nuclei

Nuclear Chart



Stable nuclei: $N \geq Z$





1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range
2. Effect of Coulomb force for heavy nuclei
3. Fusion for light nuclei
4. Fission for heavy nuclei

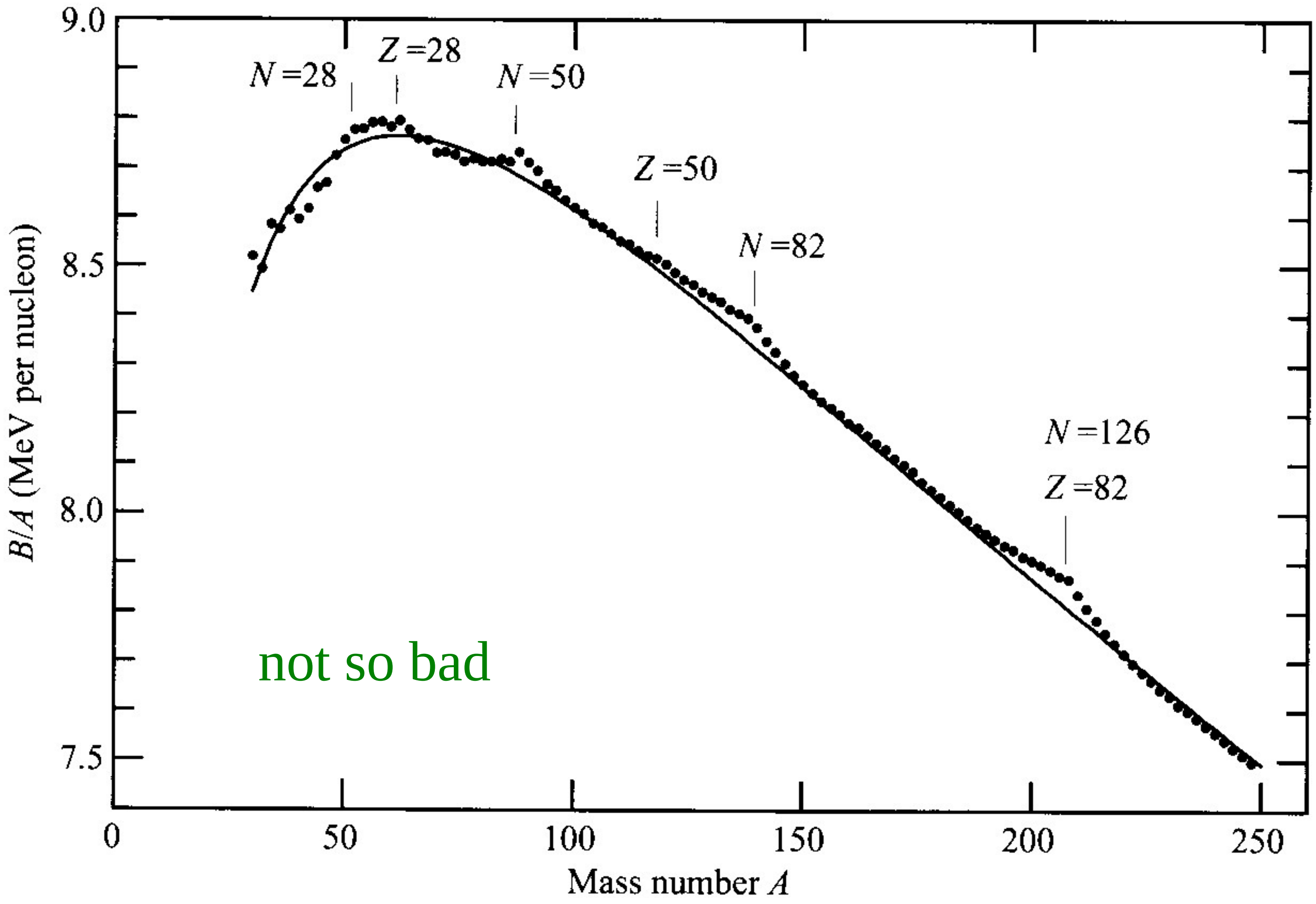
Semi-empirical mass formula

(Bethe-Weizacker formula: Liquid-drop model)

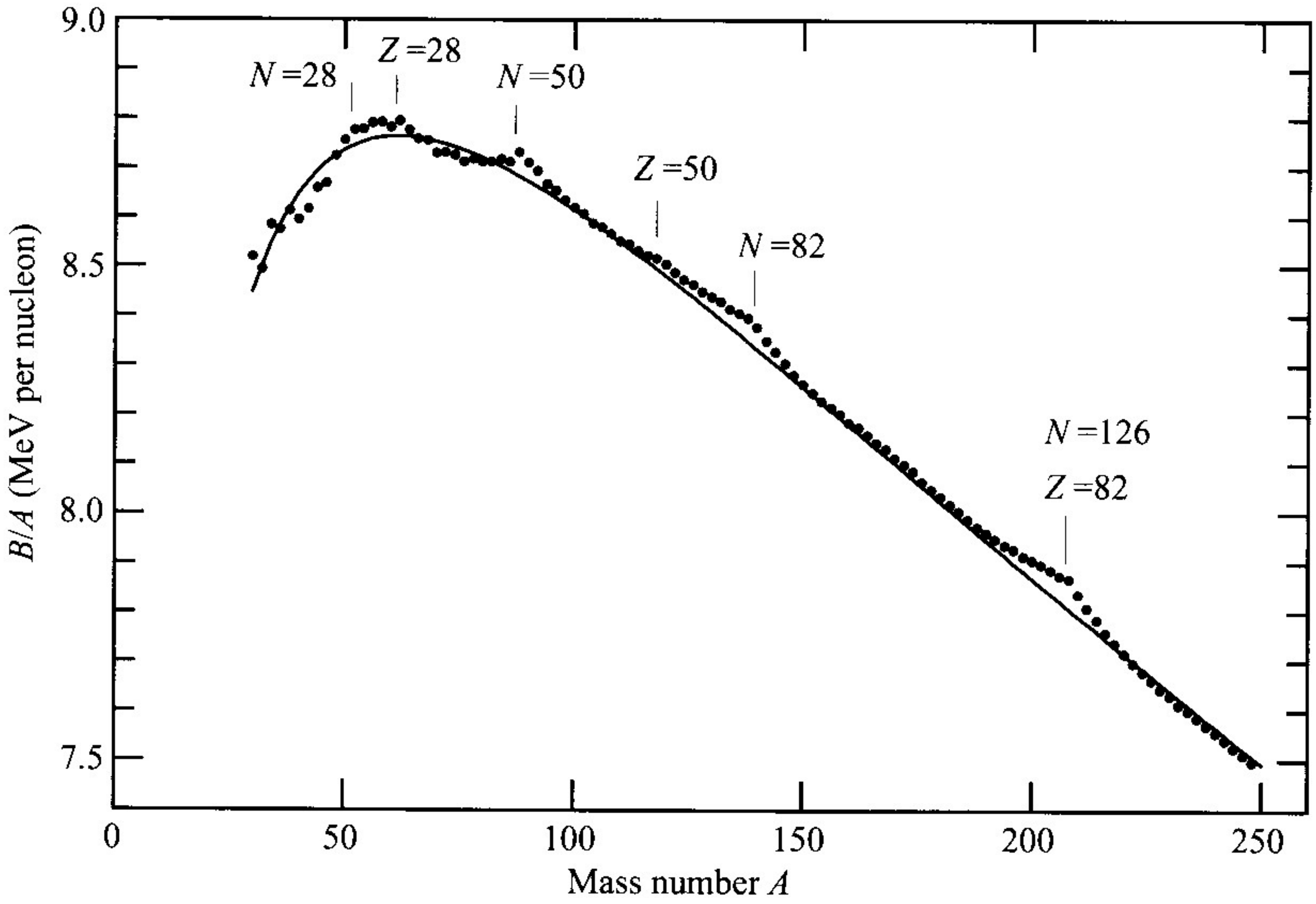
$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$

- Volume energy: $a_v A$
- Surface energy: $-a_s A^{2/3}$
- Coulomb energy: $-a_C Z^2 / A^{1/3}$
- Symmetry energy: $-a_{\mathrm{sym}} (N-Z)^2 / A$

How well does the Bethe-Weizacker formula reproduce the data?



How well does the Bethe-Weizacker formula reproduce the data?

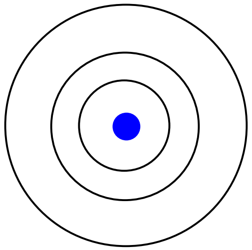


cf. N,Z = 2, 8, 20, 28, 50, 82, 126: large binding energy

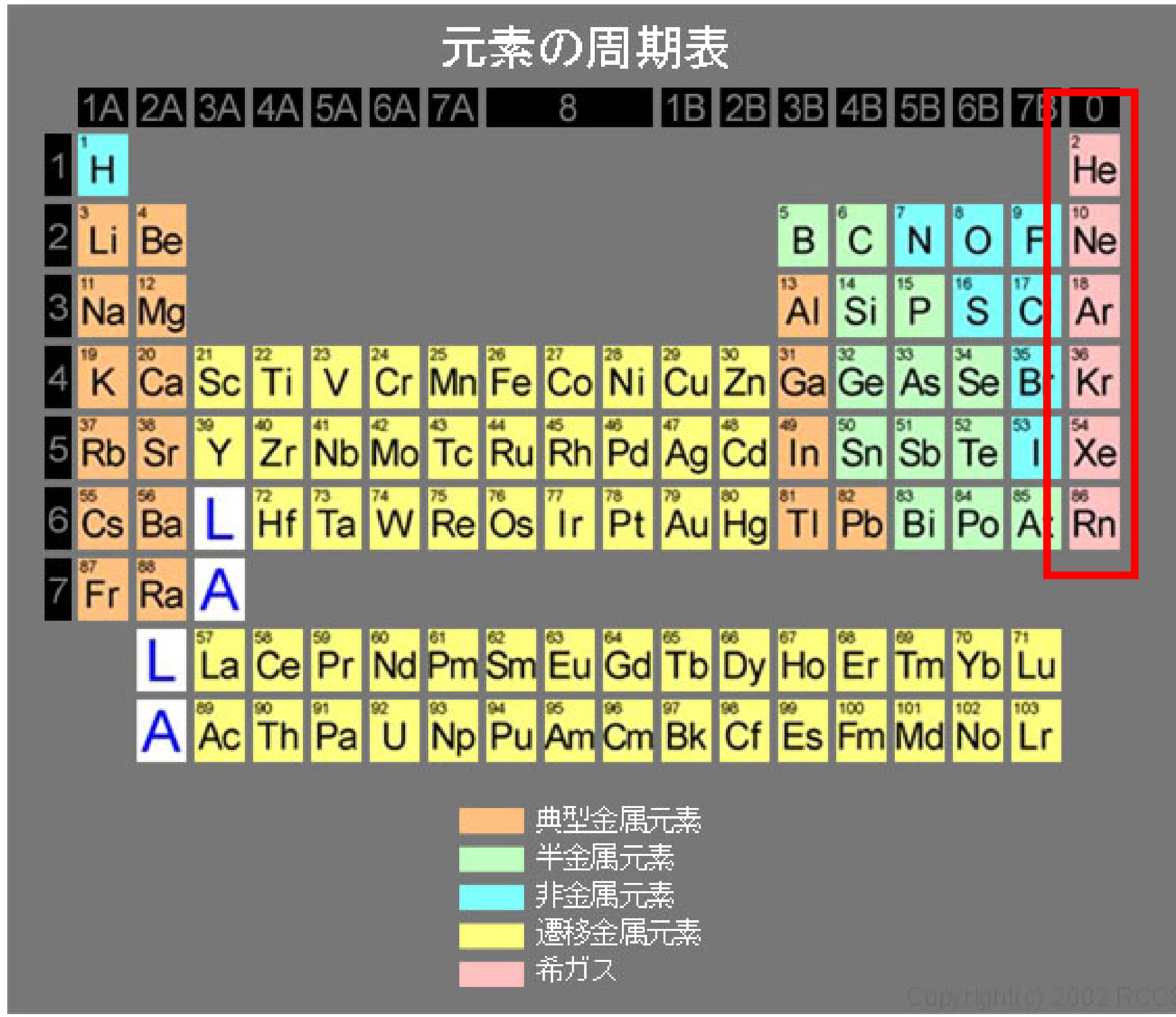
“magic numbers”

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)

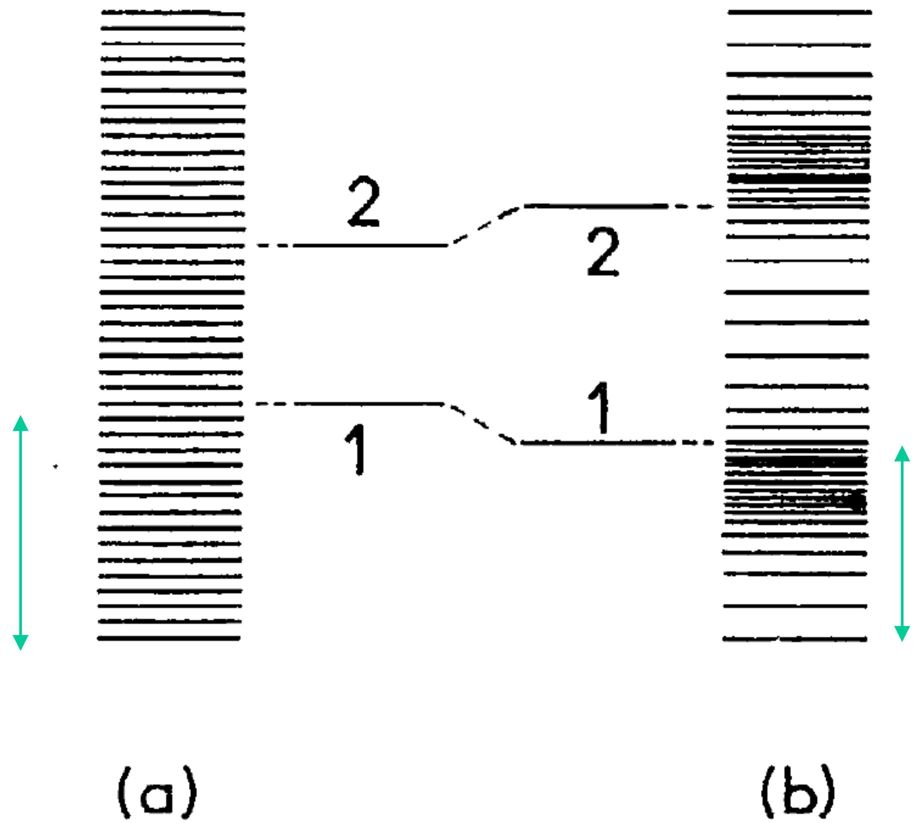


shell structure



Why do closed-shell-nuclei become stable?

level density



uniform

non-uniform

smaller total energy (more stable)