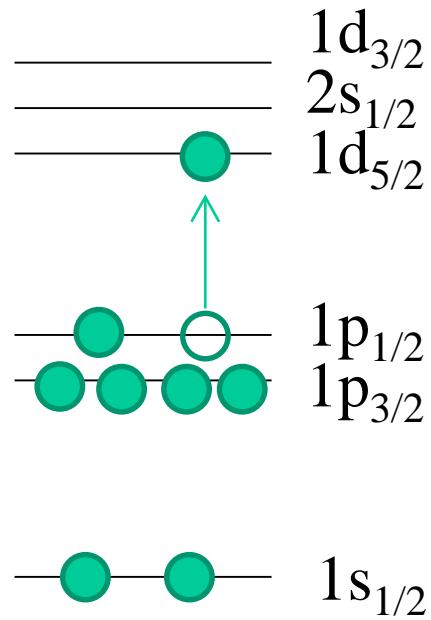


# 原子核の励起状態

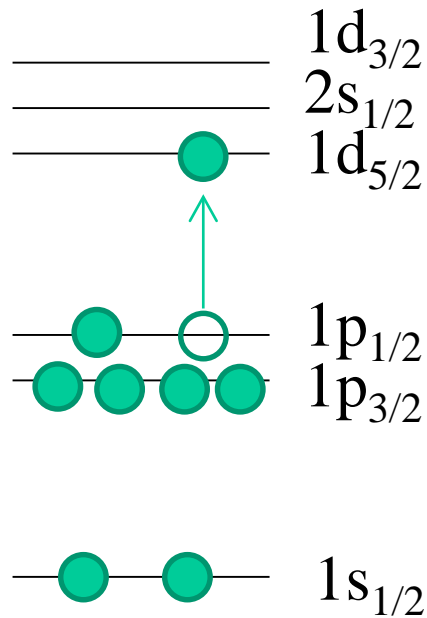
✓ 一粒子励起(一つの核子が励起に関与)



一粒子励起の例

# 原子核の励起状態

- ✓ 一粒子励起(一つの核子が励起に参与)
- ✓ 集団励起(多くの核子が集団として励起に参与)



一粒子励起の例

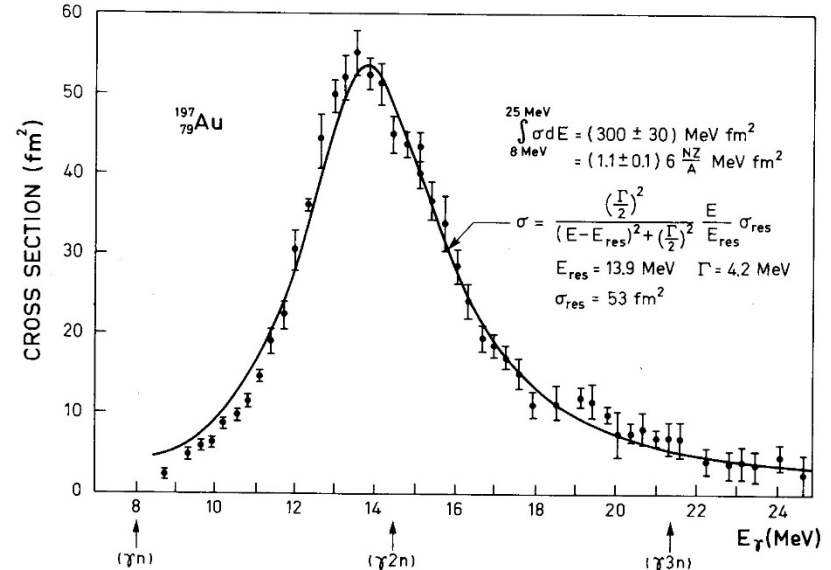
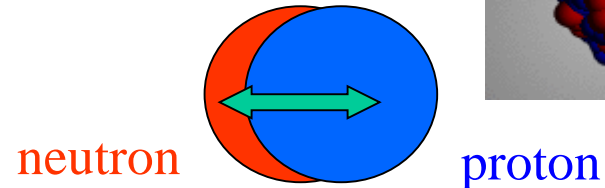


Figure 6-18 Total photoabsorption cross section for  $^{197}\text{Au}$ . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.



集団励起の例: 巨大双極子共鳴

# 原子核の励起状態

- ✓ 一粒子励起(一つの核子が励起に関与)
- ✓ 集団励起(多くの核子が集団として励起に関与)

集団励起を微視的に理解  
 してみる  
 (集団励起をミクロに見て  
 みるとどうなっているのか?)

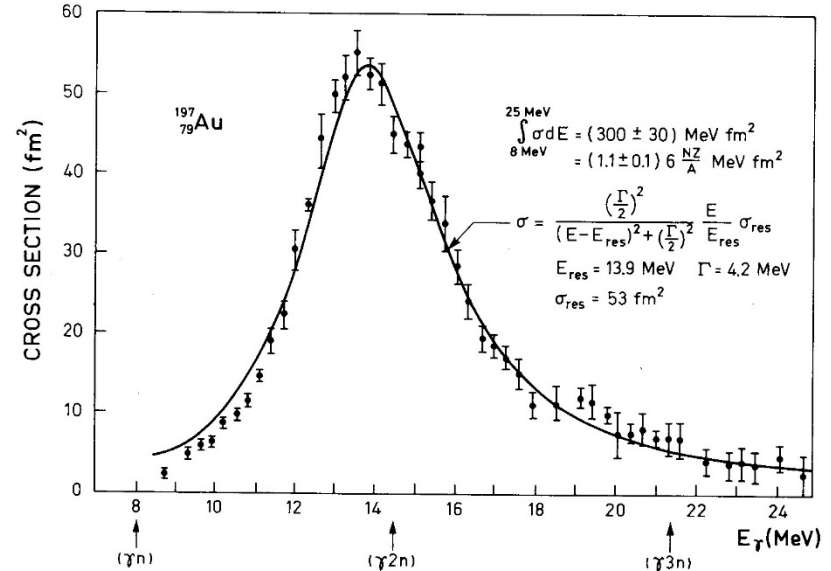
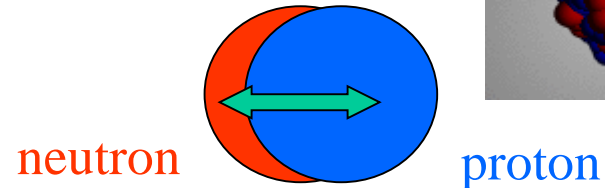


Figure 6-18 Total photoabsorption cross section for  $^{197}\text{Au}$ . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.



集団励起の例: 巨大双極子共鳴

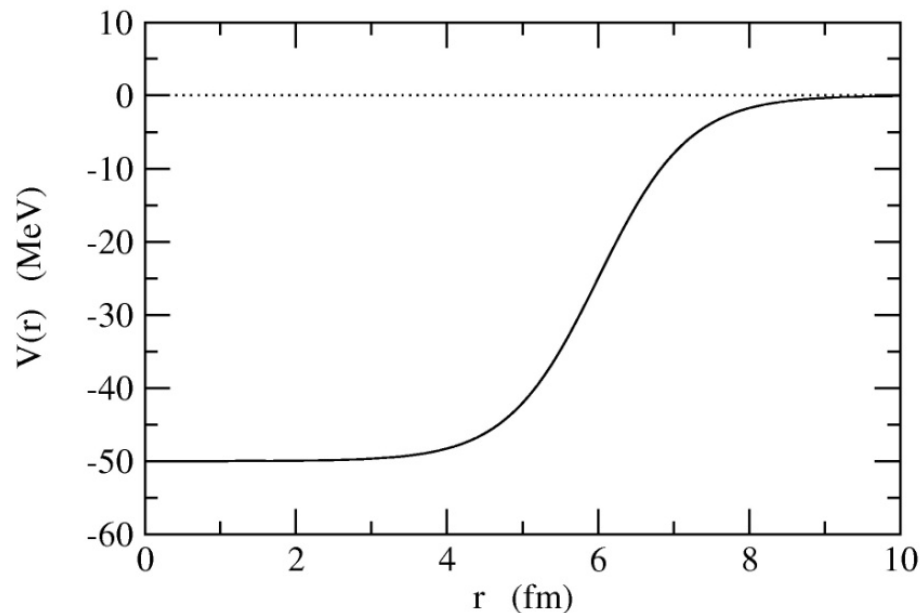
(余談)

ハミルトニアン固有状態や固有値をどうやって求めるか？

$$H|\phi_\nu\rangle = E_\nu|\phi_\nu\rangle$$

例えば、

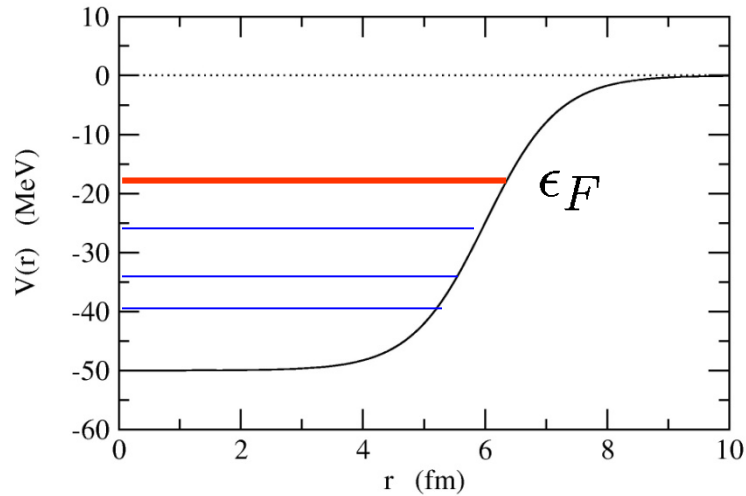
$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(r)$$





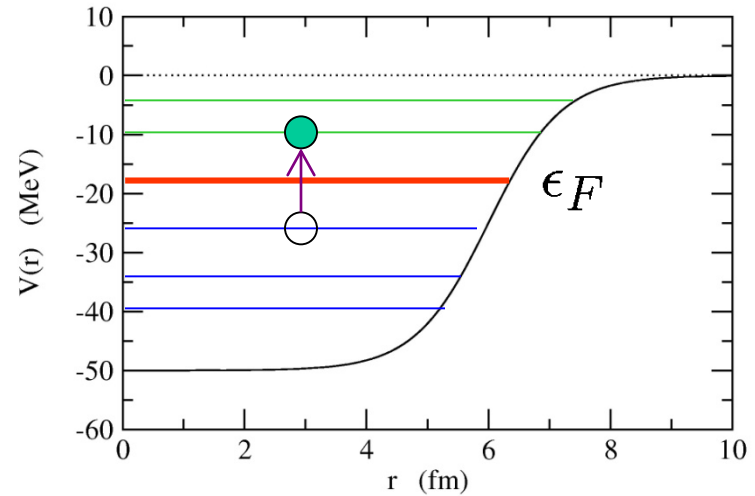
# Particle-Hole excitations

## Hartree-Fock state



$$|HF\rangle$$

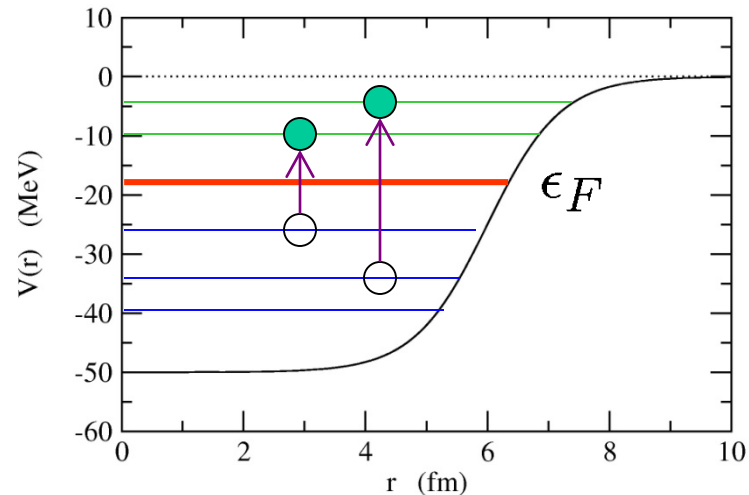
## 1 particle-1 hole (1p1h) state



$$a_p^\dagger a_h |HF\rangle$$

## 2 particle-2 hole (2p2h) state

$$a_p^\dagger a_{p'}^\dagger a_h a_{h'} |HF\rangle$$

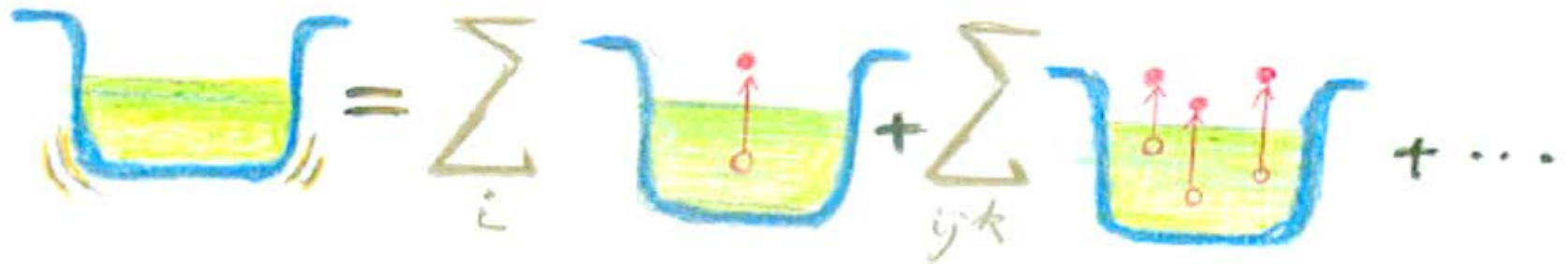
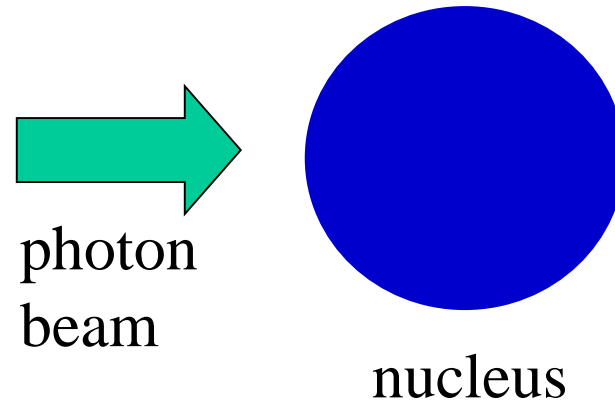


# Tamm-Dancoff Approximation

$$\begin{aligned}\text{Assume: } |\nu\rangle = Q_\nu^\dagger |HF\rangle &= \sum_{ph} X_{ph} a_p^\dagger a_h |HF\rangle \\ &\equiv \sum_{ph} X_{ph} |ph^{-1}\rangle\end{aligned}$$

(superposition of 1p1h states)

原子核を外場により揺らしてみると何が起こるのか？



スライド：松柳研一氏



# Tamm-Dancoff Approximation

$$\begin{aligned}\text{Assume: } |\nu\rangle = Q_\nu^\dagger |HF\rangle &= \sum_{ph} X_{ph} a_p^\dagger a_h |HF\rangle \\ &\equiv \sum_{ph} X_{ph} |ph^{-1}\rangle\end{aligned}$$

(superposition of 1p1h states)

$$H|\nu\rangle = E_\nu|\nu\rangle$$

$$H = \sum_i T_i + \sum_{i<j} v_{ij} \rightarrow H = \sum_i (T_i + V_i) + \underbrace{\sum_{i<j} v_{ij} - \sum_i V_i}_{\text{平均からのずれ (残留相互作用)}}$$

平均からのずれ  
(残留相互作用)

# Tamm-Dancoff Approximation

$$\begin{aligned}\text{Assume: } |\nu\rangle = Q_\nu^\dagger |HF\rangle &= \sum_{ph} X_{ph} a_p^\dagger a_h |HF\rangle \\ &\equiv \sum_{ph} X_{ph} |ph^{-1}\rangle\end{aligned}$$

(superposition of 1p1h states)

$$H|\nu\rangle = E_\nu|\nu\rangle$$

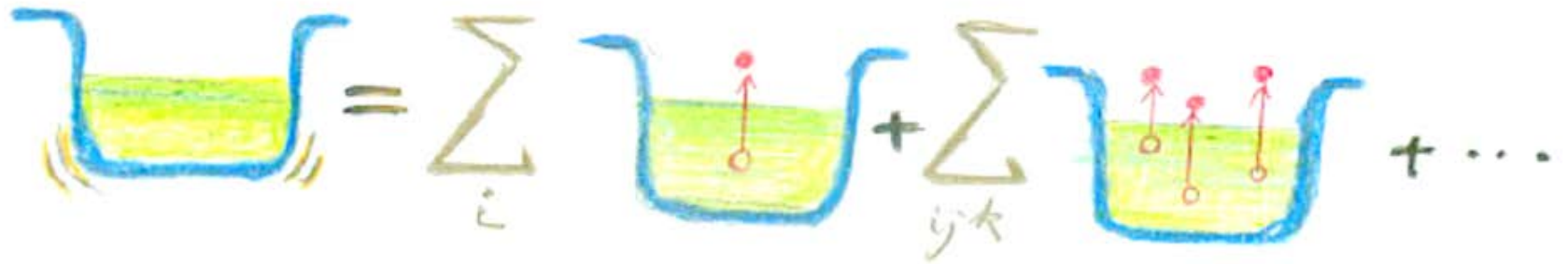


$$\sum_{p'h'} H_{ph,p'h'} X_{p'h'} = E_\nu X_{ph}$$

residual  
interaction

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$

Tamm-Dancoff equation



スライド: 松柳研一氏

$$V(\mathbf{r}) \sim \int d\mathbf{r}' v(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}')$$

vibration:  $\rho = \rho_0(\mathbf{r}) \rightarrow \rho_0(\mathbf{r}) + \delta\rho(\mathbf{r}, t)$

---

↓  
residual  
interaction

# TDA on a schematic model

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$


Separable interaction:  $\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^*$


(separable interaction)


$$H\psi = E\psi; \quad \psi = \sum_i C_i \phi_i$$


$$\longrightarrow \sum_j H_{ij} C_j = E C_i; \quad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose  $H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j$  (separable form)


$$(\epsilon_i - E) C_i + \lambda f_i^* \underbrace{\sum_j f_j C_j}_{\equiv T} = 0$$


$$C_i = -\lambda \frac{T f_i^*}{\epsilon_i - E}$$


$$T = -\lambda \sum_j \frac{|f_j|^2}{\epsilon_j - E} T$$


$$\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}$$

(separable interaction)

$$H\psi = E\psi; \quad \psi = \sum_i C_i \phi_i$$

$$\longrightarrow \sum_j H_{ij} C_j = E C_i; \quad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose  $H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j \longrightarrow$

$$\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}$$

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} + \lambda D_{ph} D_{p'h'}^*$$

$$\longrightarrow \frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

(TDA dispersion relation)

# Graphical solutions

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

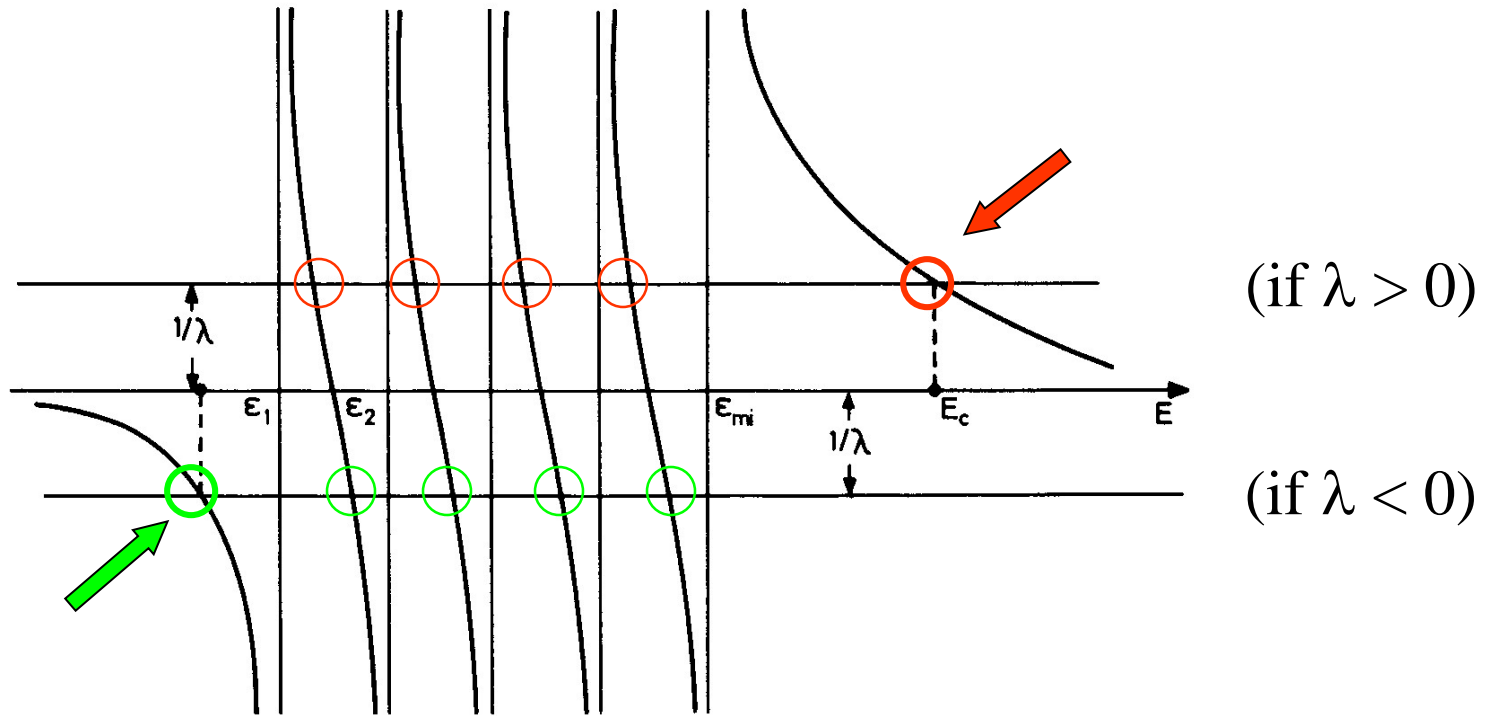


Figure 8.4. Graphical solution of Eq. (8.18).

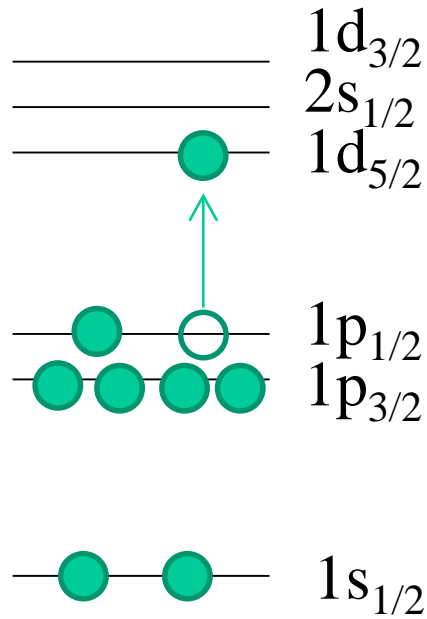
(note) in the degenerate limit:  $\epsilon_{ph} \sim \epsilon$

$$E = \epsilon + \lambda \sum_{ph} |D_{ph}|^2, \quad |\nu\rangle = \sum_{ph} D_{ph} a_p^\dagger a_h |HF\rangle$$

*coherent superposition of 1p1h states*

# 原子核の励起状態

- ✓ 一粒子励起(一つの核子が励起に参与)
- ✓ 集団励起(多くの核子が集団として励起に参与)



一粒子励起の例

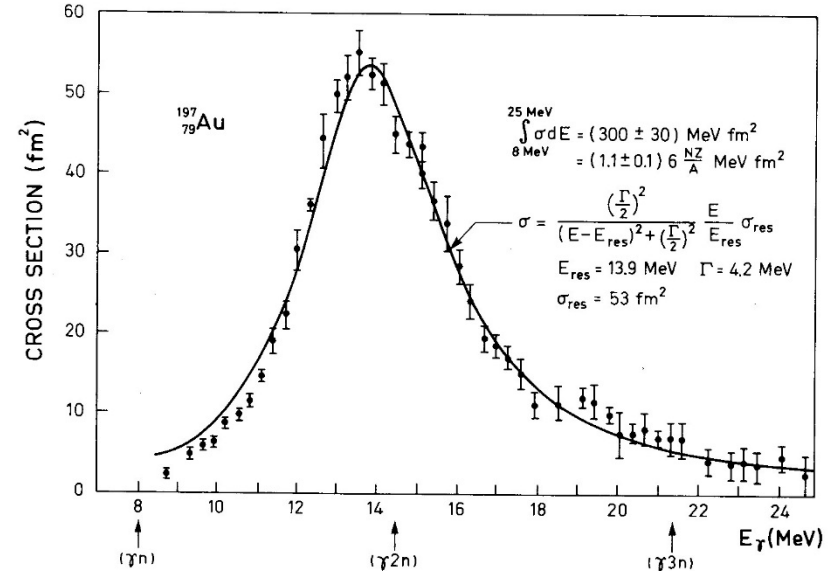
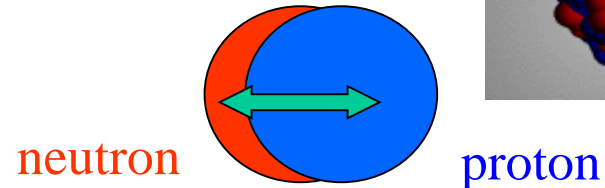
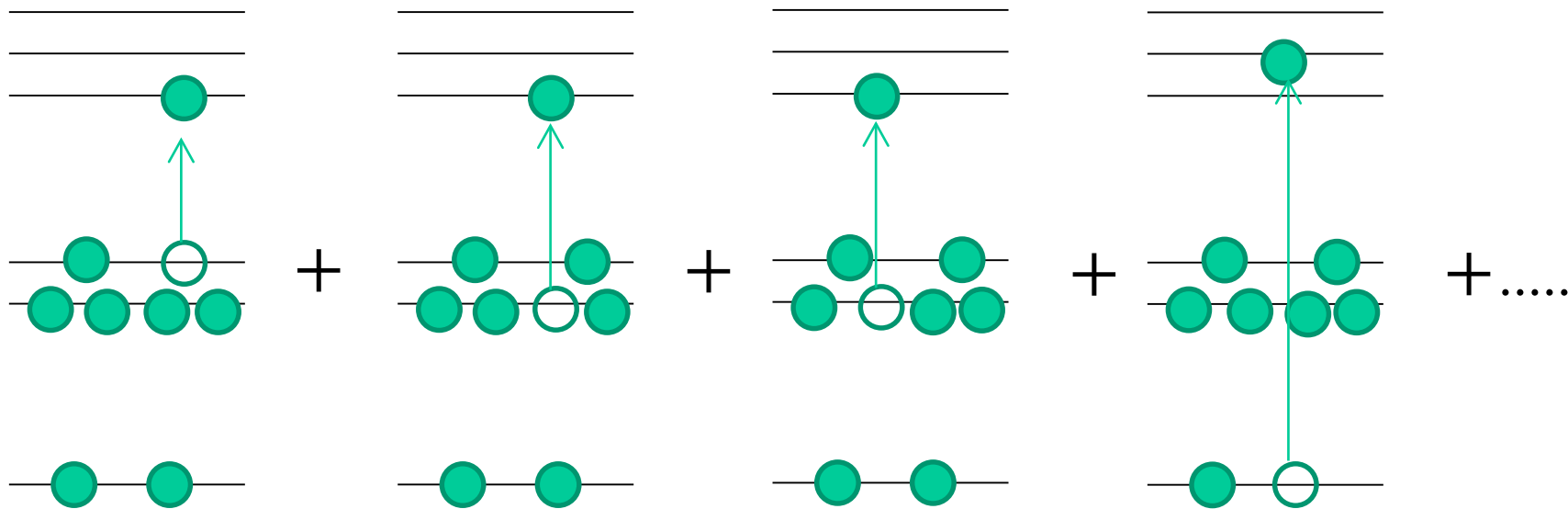


Figure 6-18 Total photoabsorption cross section for  $^{197}\text{Au}$ . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.



集団励起の例: 巨大双極子共鳴





複数の粒子・空孔状態を**コヒーレント**に重ね合わせることによって  
 多数の核子が励起に関与していることを表現する

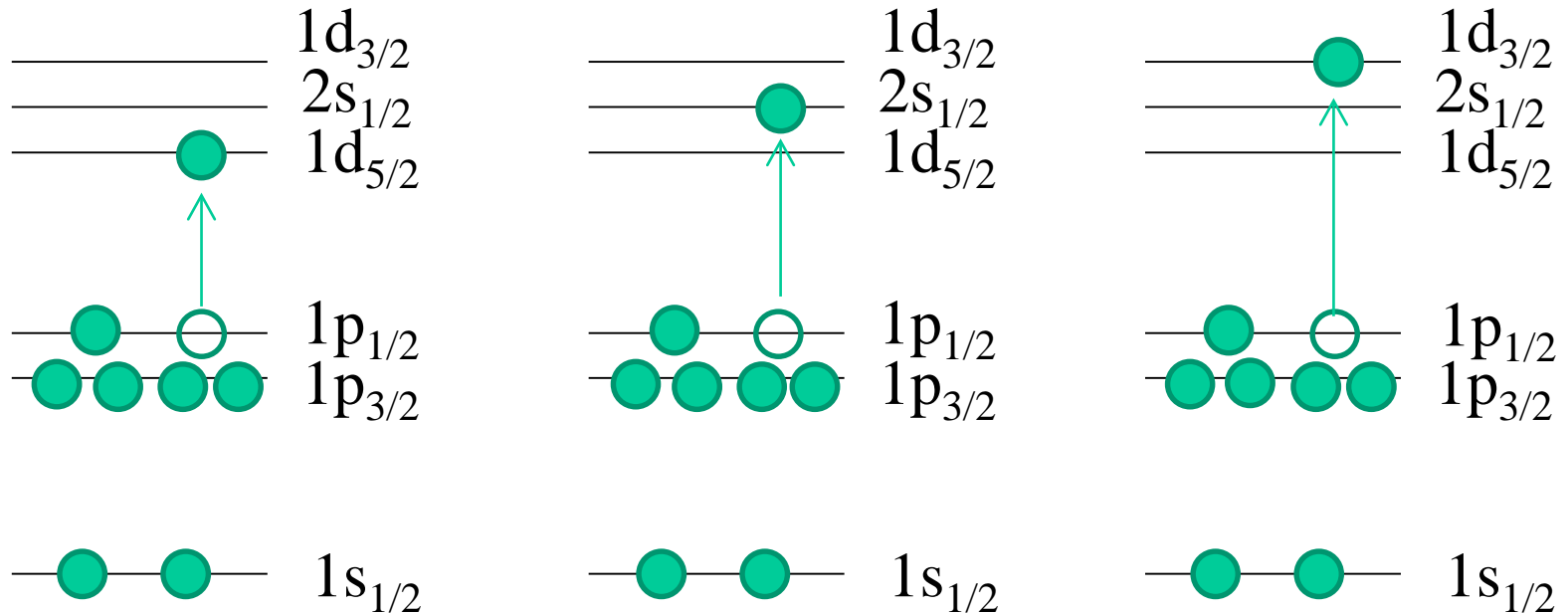
# TDA on a schematic model

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \quad \langle ph' | \bar{v} | hp' \rangle = g$$

for three ph configurations:

(例えば)



## TDA on a schematic model

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \quad \langle ph' | \bar{v} | hp' \rangle = g$$

for three ph configurations:

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization:

$$\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$$

# TDA on a schematic model

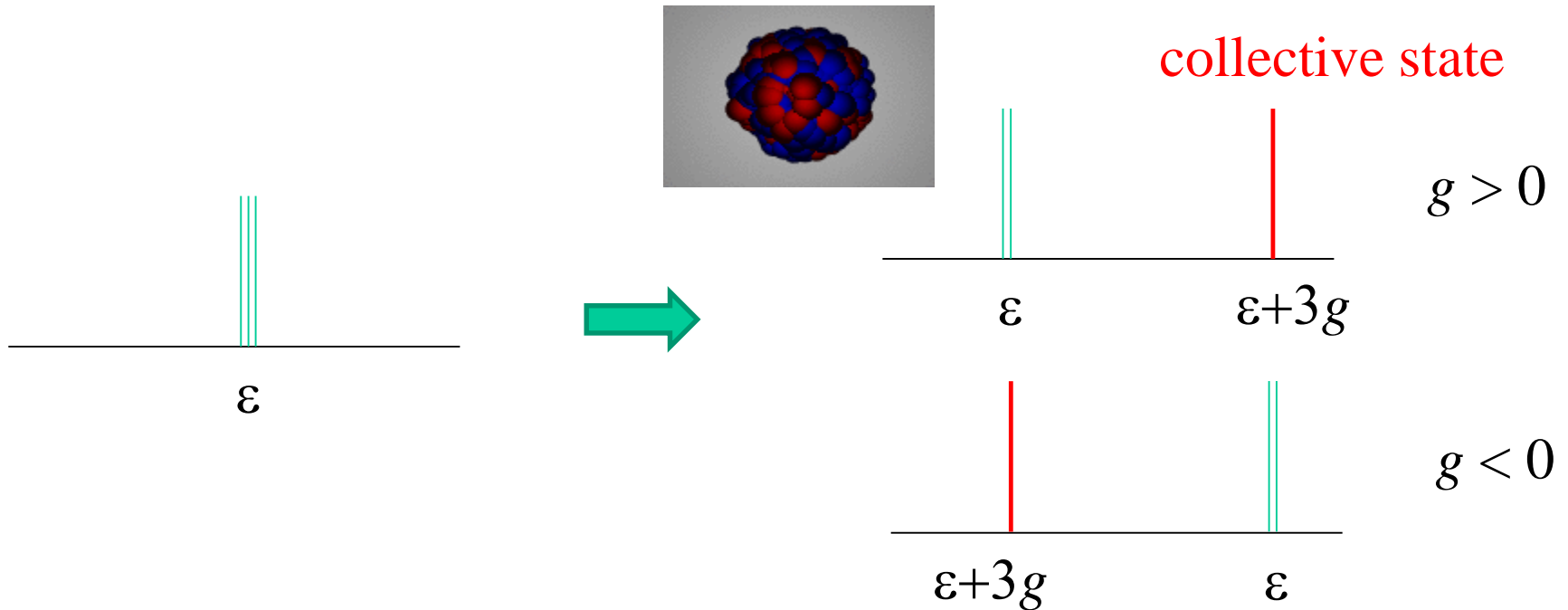
$$\begin{pmatrix} \epsilon + g & g & g \\ g & \epsilon + g & g \\ g & g & \epsilon + g \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = (\epsilon + 3g) \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

全ての状態が同位相で寄与  
=コヒーレントな重ね合わせ

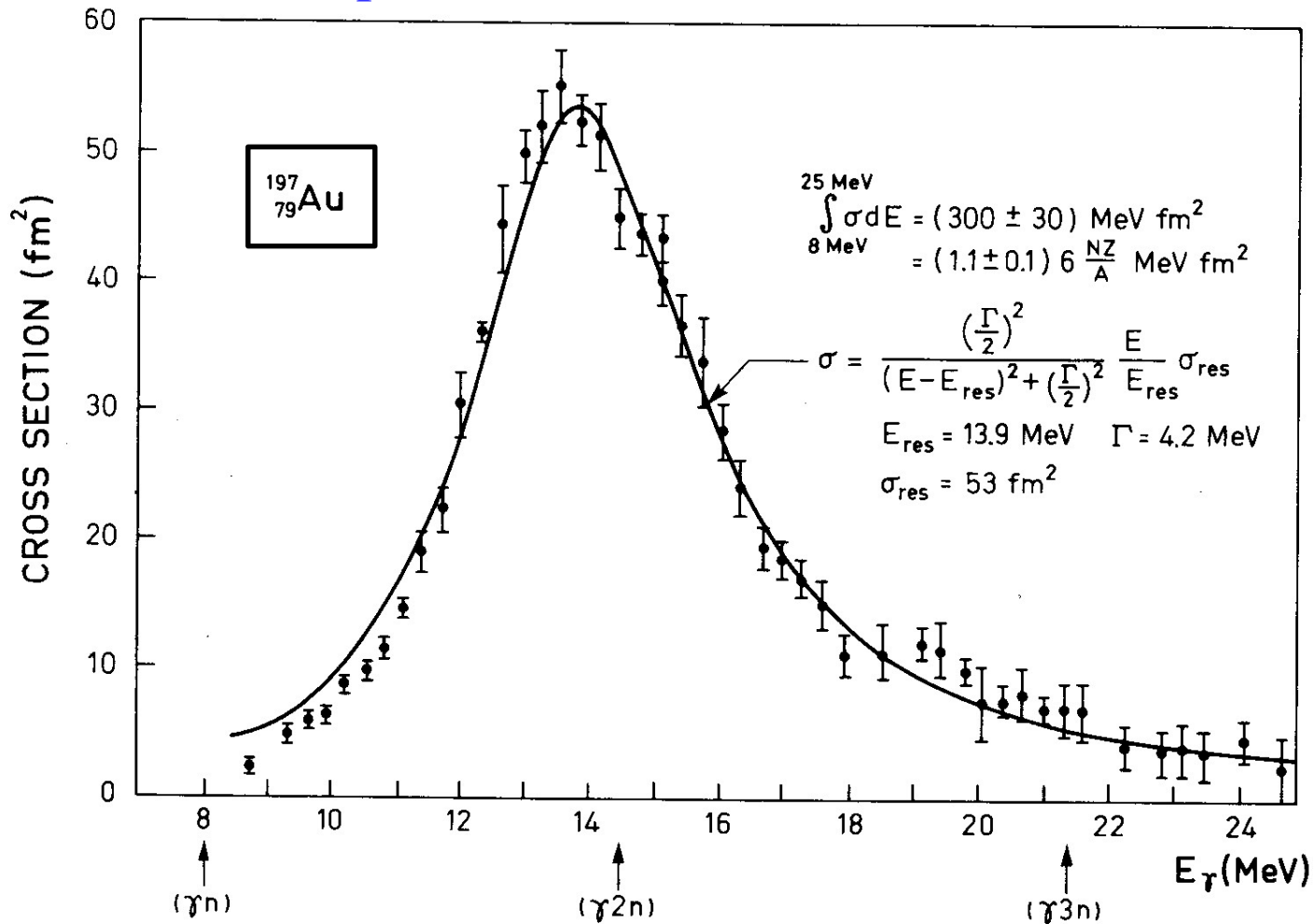
# TDA on a schematic model

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization:  $\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$



# Giant Dipole Resonance (GDR) 巨大双極子共鳴



**Figure 6-18** Total photoabsorption cross section for  $^{197}\text{Au}$ . The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

$$\text{cf. } 41 \times 197^{-1/3} = 7.05 \text{ MeV} \rightarrow 14 \text{ MeV}$$

Iso-scalar type modes:  $E < \epsilon_{ph} \rightarrow \lambda < 0$  (attractive)

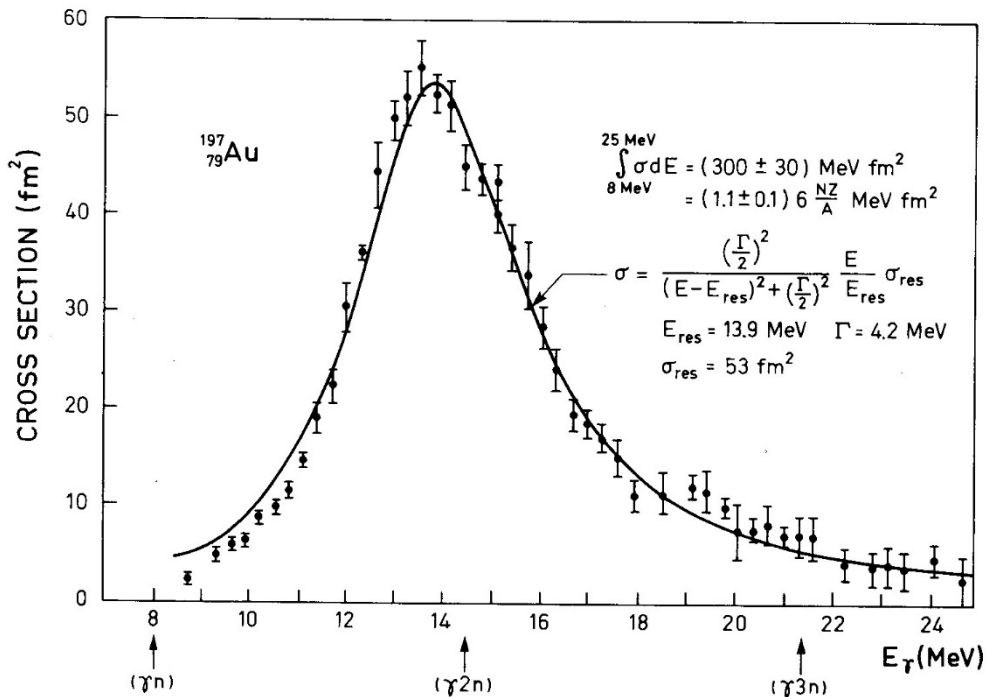
Iso-vector type modes:  $E > \epsilon_{ph} \rightarrow \lambda > 0$  (repulsive)

### Experimental systematics:

**IV GDR:**  $E \sim 79 A^{-1/3}$  (MeV)  $\longleftrightarrow \epsilon_{ph} \sim 41 A^{-1/3}$

**IS GQR:**  $E \sim 65 A^{-1/3}$  (MeV)  $\longleftrightarrow \epsilon_{ph} \sim 82 A^{-1/3}$

(note) single particle potential:  $\hbar\omega \sim 41 A^{-1/3}$  (MeV)



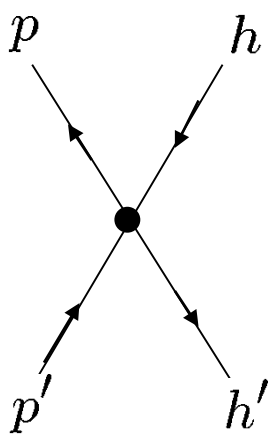
<sup>197</sup>Au

$$E_{\text{GDR}} = 14 \text{ (MeV)}$$

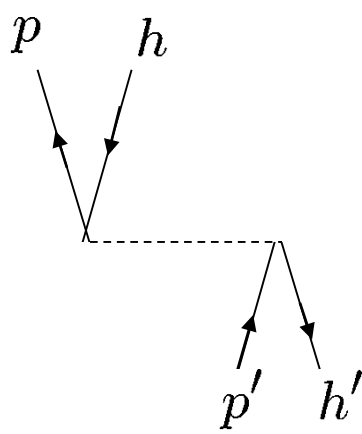
$$\epsilon_{ph} \sim 41 \cdot 197^{-1/3}$$

$$\sim 7 \text{ (MeV)}$$

$$\langle ph^{-1} | \bar{v} | p'h'^{-1} \rangle = \langle ph' | \bar{v} | hp' \rangle = \langle ph' | v | hp' \rangle - \langle ph' | v | p'h \rangle$$

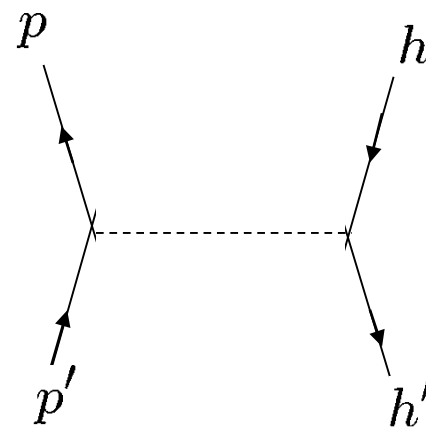


=



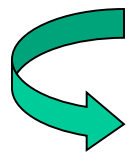
Direct term

-



Exchange term

$$\left\{ \begin{array}{l} \langle PP^{-1} | \bar{v} | PP^{-1} \rangle \sim \langle NN^{-1} | \bar{v} | NN^{-1} \rangle = D - E \\ \langle PP^{-1} | \bar{v} | NN^{-1} \rangle = D \quad (\text{no charge exchange}) \end{array} \right.$$



$$\langle IS | \bar{v} | IS \rangle = 2D - E \sim D$$

$$\langle IV | \bar{v} | IV \rangle = -E \sim -D$$

$$|IS\rangle \propto |NN^{-1}\rangle + |PP^{-1}\rangle$$

$$|IV\rangle \propto |NN^{-1}\rangle - |PP^{-1}\rangle$$

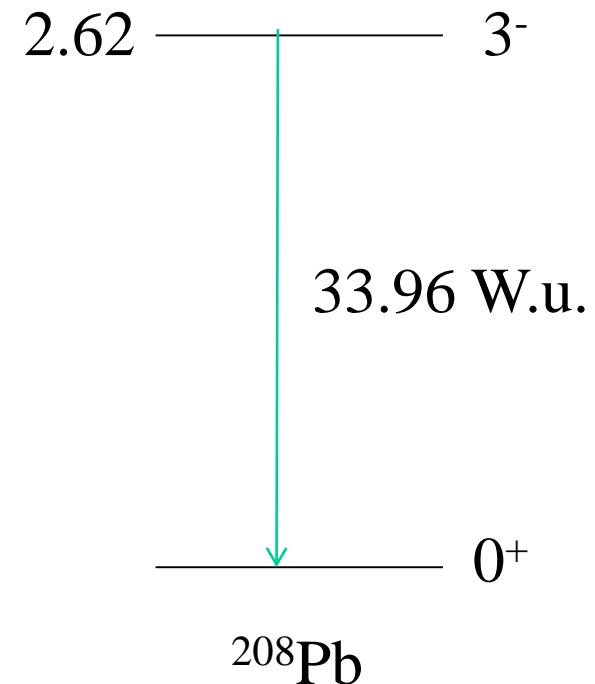
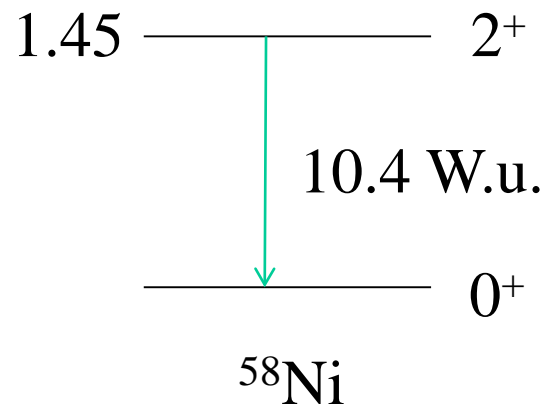
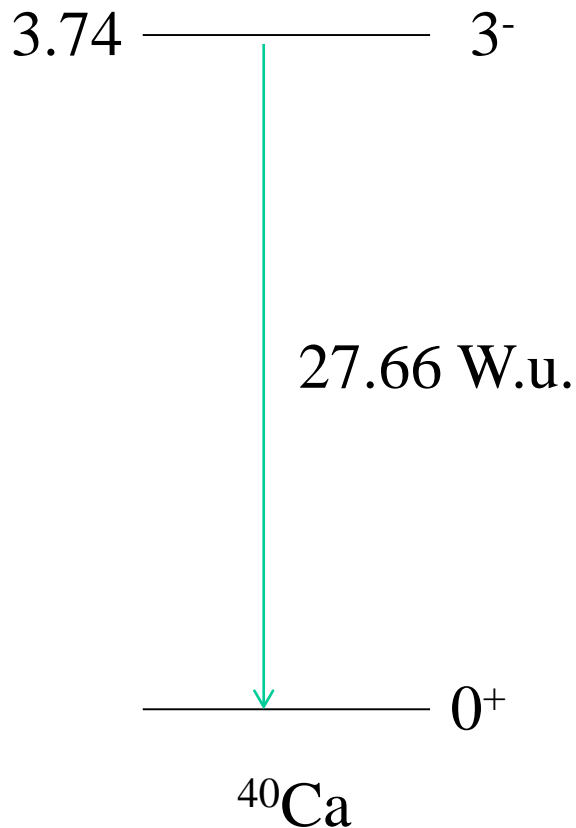


## どれだけの核子が励起に参与しているのか?

Single-particle transition (Weisskopf unit)

$$B(E\lambda : I_i \rightarrow I_{gs}) = \frac{(1.2A^{1/3})^{2\lambda}}{4\pi} \left( \frac{3}{\lambda + 3} \right)^2 (e^2 \text{fm}^{2\lambda})$$

exp data:



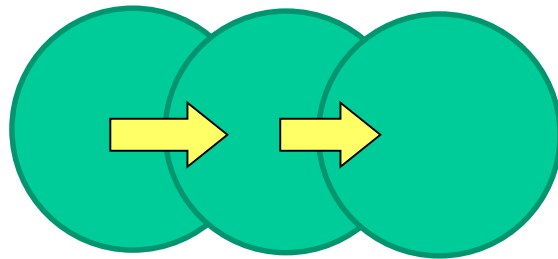
# Spurious motion and RPA

Mean-Field Approximation  $\longleftrightarrow$  Broken symmetries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

$\longrightarrow$  Zero energy mode (Nambu-Goldstone mode)



does not require an extra energy  $\rightarrow$  zero energy mode

A drawback of TDA:

Zero modes appear at finite excitation energies.

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} X_{ph} a_p^\dagger a_h |0\rangle \quad (\text{TDA})$$

 A better approximation:

**the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} \left( X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \right) |0\rangle$$

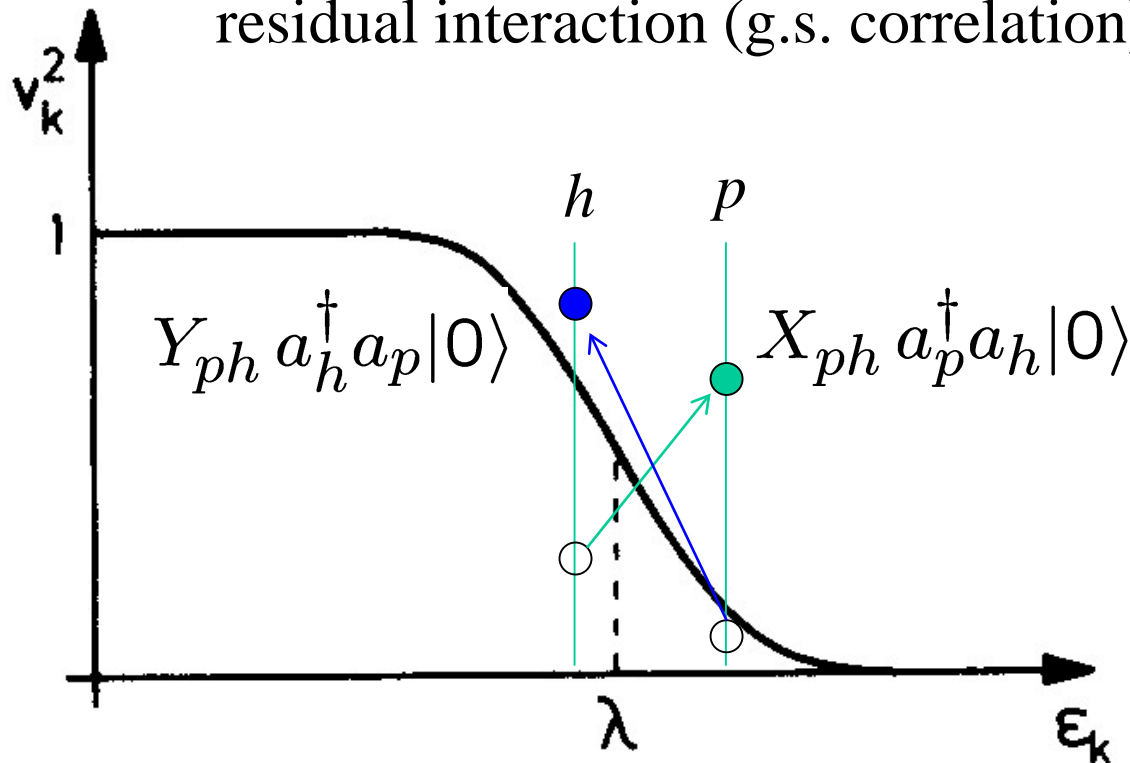
(superposition of 1p1h states)

A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} \left( X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \right) |0\rangle$$

(superposition of 1p1h states)

smearing of Fermi surface due to the residual interaction (g.s. correlation)



A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger |0\rangle = \sum_{ph} \left( X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \right) |0\rangle$$

(superposition of 1p1h states)

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$



$$\langle HF | [\delta Q, [H, Q_\nu^\dagger]] | HF \rangle = E_\nu \langle HF | [\delta Q, Q_\nu^\dagger] | HF \rangle$$

→ coupled equations for  $X$  and  $Y$

$$\langle HF | [\delta Q, [H, Q_\nu^\dagger]] | HF \rangle = E_\nu \langle HF | [\delta Q, Q_\nu^\dagger] | HF \rangle$$

$$Q_\nu^\dagger = \sum_{ph} X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p \quad \delta Q = a_h^\dagger a_p, \quad a_p^\dagger a_h$$

RPA equation:

$$\sum_{p'h'} A_{ph,p'h'} X_{p'h'} + B_{ph,p'h'} Y_{p'h'} = E_\nu X_{ph}$$

$$\sum_{p'h'} B_{ph,p'h'}^* X_{p'h'} + A_{ph,p'h'}^* Y_{p'h'} = -E_\nu Y_{ph}$$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} + \langle ph' | \bar{v} | hp' \rangle$$

$$B_{ph,p'h'} = \langle pp' | \bar{v} | hh' \rangle$$

or

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E_\nu \begin{pmatrix} X \\ Y \end{pmatrix}$$

# Spurious motion in RPA

Mean-Field Approximation  $\longleftrightarrow$  Broken symmetries

- Center of mass localization (single center)
- Rotational motion

## Restoration of broken symmetries

$\longrightarrow$  Zero mode (Nambu-Goldstone mode)

$$[H, \hat{O}] = 0$$

RPA

$$[H, Q_{\nu}^{\dagger}] \sim E_{\nu} Q_{\nu}^{\dagger}$$



$\hat{O}$  is a solution of RPA with  $E=0$

$$Q^{\dagger} = \hat{O} = \sum_{ph} (O_{ph} a_p^{\dagger} a_h + O_{hp} a_h^{\dagger} a_p)$$

(note)  $Q_{\text{TDA}}^{\dagger} = \sum_{ph} O_{ph} a_p^{\dagger} a_h \longrightarrow [H, Q_{\text{TDA}}^{\dagger}] \neq 0$

# Spurious motion in RPA

Mean-Field Approximation  $\longleftrightarrow$  Broken symmetries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

$\longrightarrow$  Zero mode (Nambu-Goldstone mode)

RPA

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$



if  $[H, \hat{O}] = 0$

Then  $\hat{O}$  is a solution of RPA with  $E=0$



The physical solutions are exactly separated out from the spurious modes.



# Comparison between Skyrme-(Q)RPA calculation and exp. data

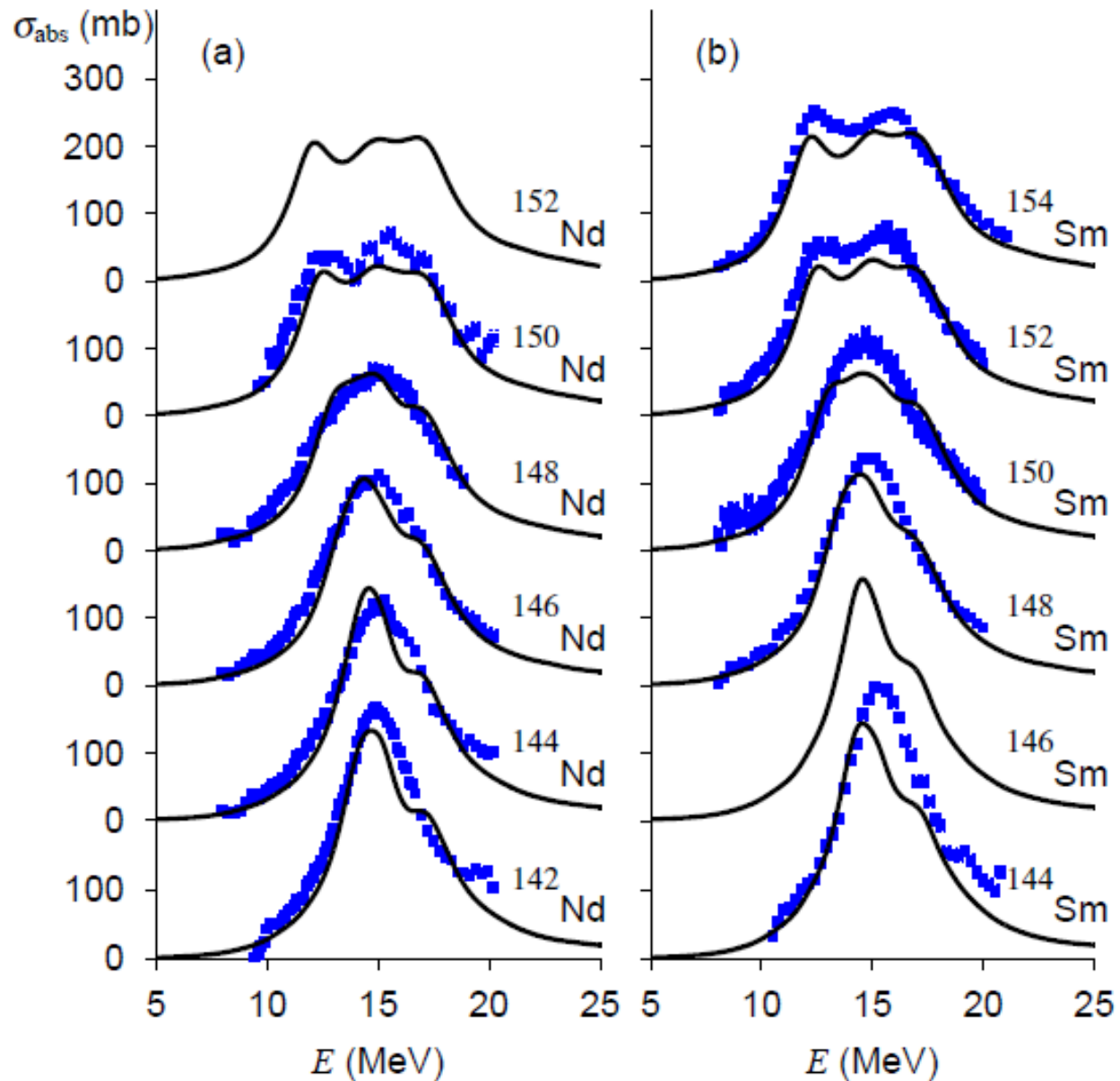


photo-absorption  
cross section  
(GDR)

K. Yoshida  
and T. Nakatsukasa,  
PRC83('11)021304

# RPA on a schematic model

Separable interaction:  $\langle ph' | \bar{v} | hp' \rangle = \lambda D_{ph} D_{p'h'}^*$

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)

Cf. TDA dispersion relation:  $\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$

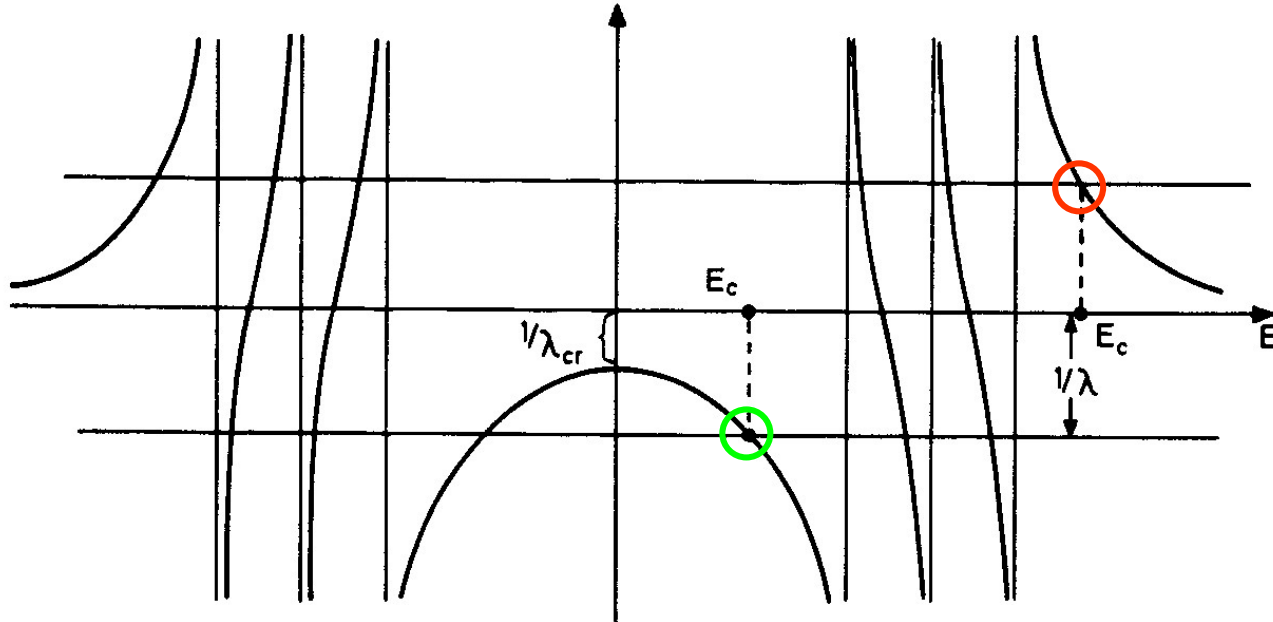


Figure 8.11. Graphical solution of the dispersion relation (8.135).