Advanced Nuclear Physics

Nuclear Theory Group,
Tohoku University
Kouichi Hagino

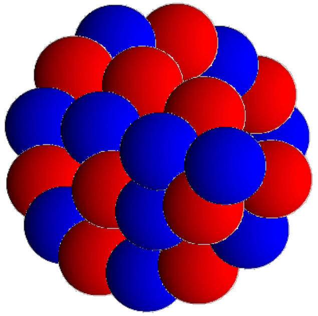
原子核理論特論

東北大学
原子核理論研究室
萩野浩一

Contents

Nuclei: many-body systems of nucleons (protons and neutrons)

→ ***Nuclear Many-Body Problems with strong interaction***



(Low-energy) Nuclear Physics

to understand rich nature of atomic nuclei starting from nucleon-nucleon interactions

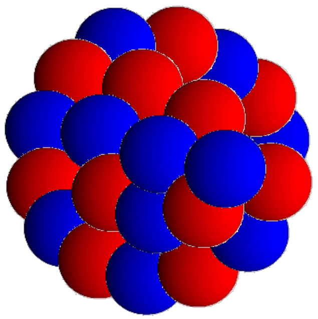
- size, mass, density, shape
- excitations
- decays
- nuclear reactions

two kinds of particle: protons and neutrons

Contents

Nuclei: many-body systems of nucleons (protons and neutrons)

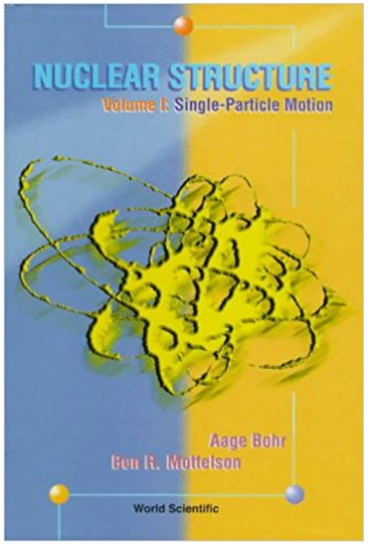
→ ***Nuclear Many-Body Problems***



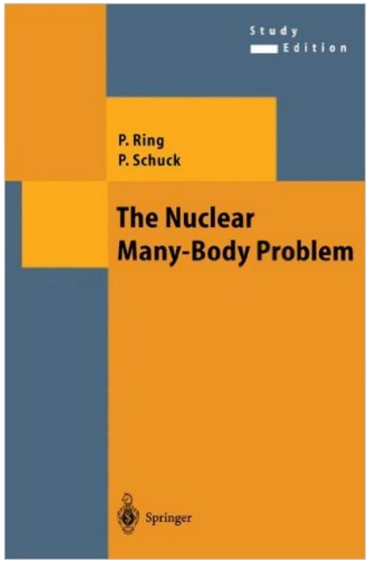
this lecture: microscopic descriptions of atomic nuclei

- Liquid drop model
- Single-particle motion and shell structure
- **Hartree-Fock approximation**
- Bruckner theory
- Pairing correlations and superfluid Nuclei
- nucleosynthesis and physics of neutron-rich nuclei
- **Random phase approximation (RPA)**
- **Nuclear reactions and superheavy elements (physics of Nihonium)**

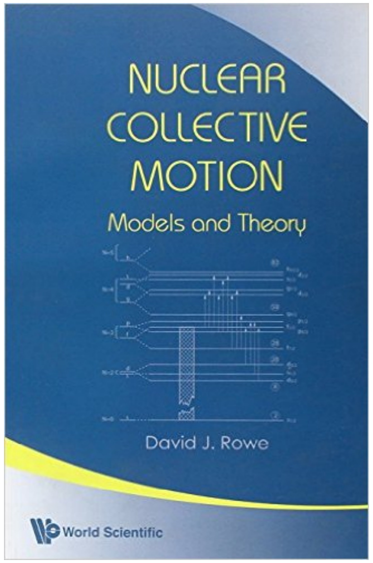
References



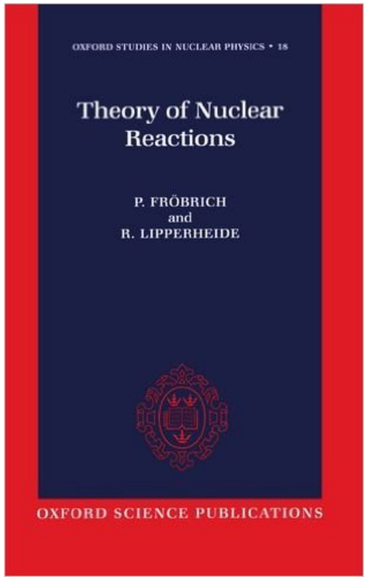
Bohr-Mottelson



Ring-Schuck



Rowe



Frobrich
-Lipperheide

Lecture notes:

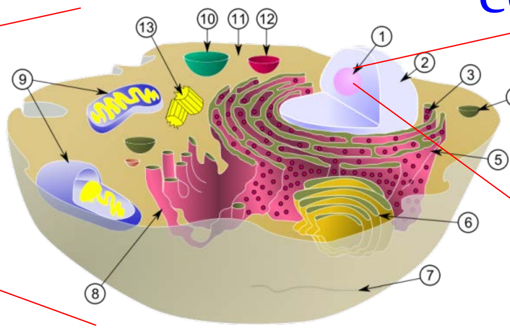
http://www.nucl.phys.tohoku.ac.jp/~hagino/lecture.html

(Tohoku University → Physics → Nuclear Theory
→ Kouichi Hagino → Lectures)

Introduction: atoms and atomic nuclei

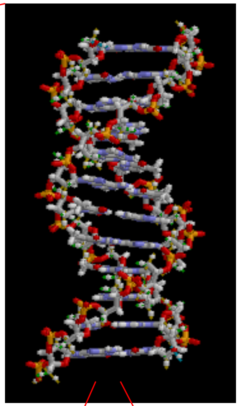
~ 50 cm

cells



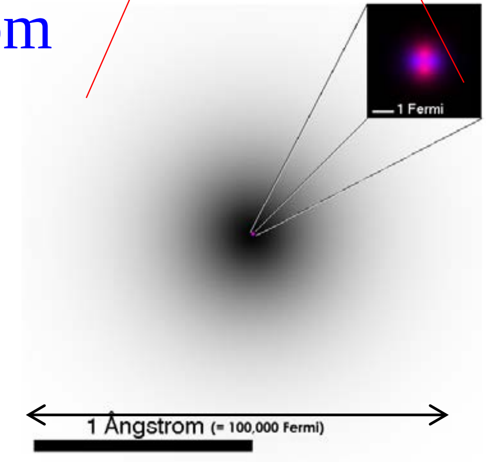
~ μm = 10^{-6} m

DNA



~ 10^{-8} m

atom



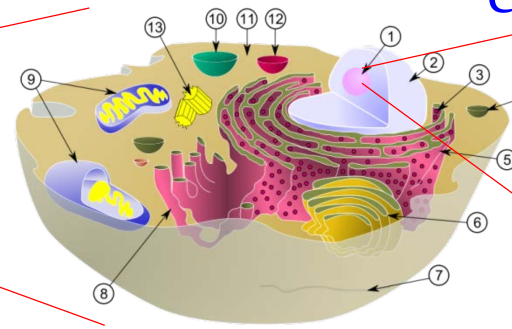
~ 10^{-10} m

everything is made of atoms.

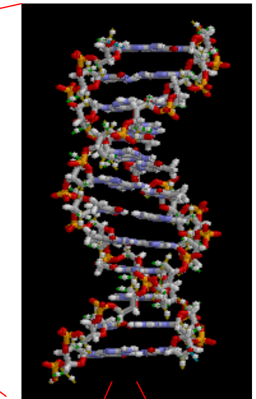
Introduction: atoms and atomic nuclei

~ 50 cm

cells

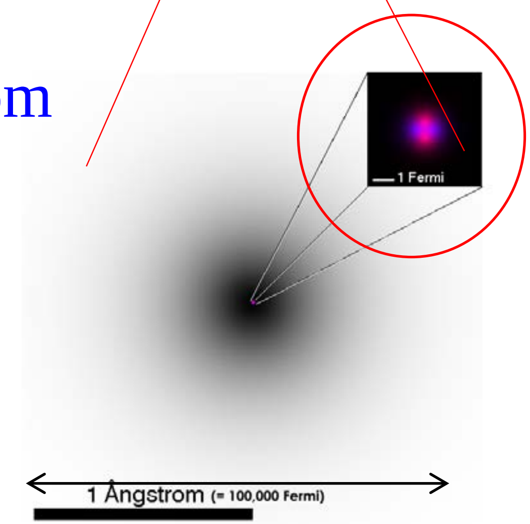


DNA



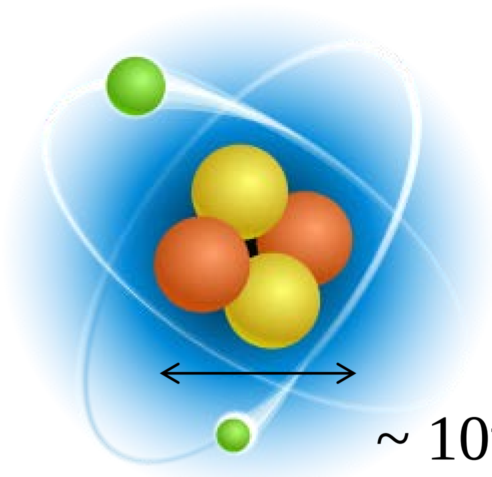
~ 10^{-8} m

atom



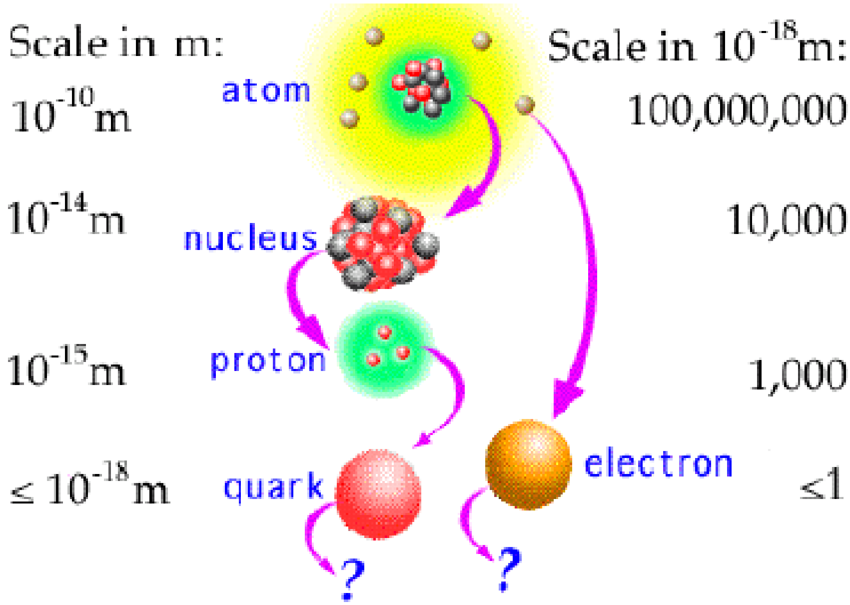
~ 10^{-10} m

atomic nucleus



~ 10^{-15} m

Nuclear Physics



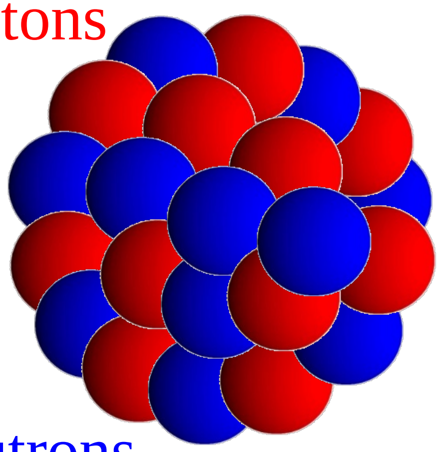
$1 \text{ fm} = 10^{-15} \text{ m}$

Nucleus as a *quantum many body system*

Basic ingredients:

	charge	mass (MeV)	spin, parity
Proton	+e	938.256	$\frac{1}{2}^+$
Neutron	0	939.550	$\frac{1}{2}^+$

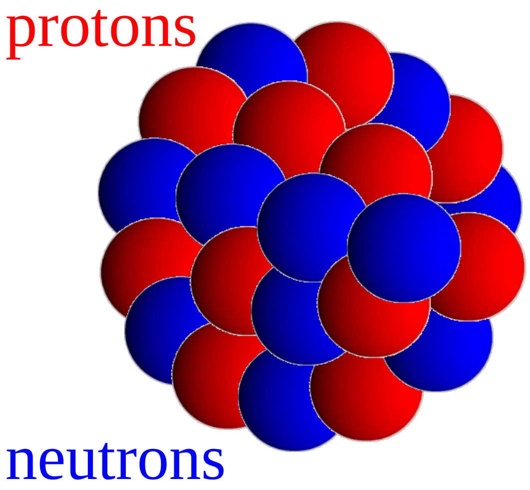
(note) $n \rightarrow p + e^- + \bar{\nu}_e$ (10.4 min)



- Nucleons are not stopping inside a nucleus.
  (they move relatively freely)

- Yet, they are not completely independent.
  A nucleus keeps its shape while nucleons influence among themselves so that a nucleon does not escape.

  a self-bound system



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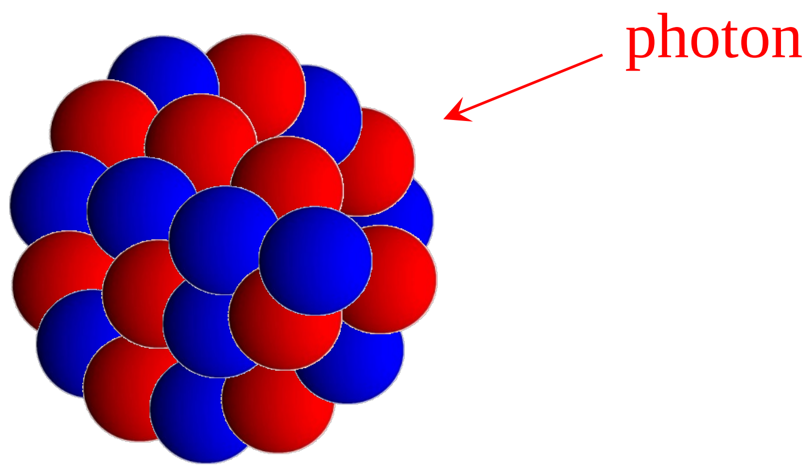
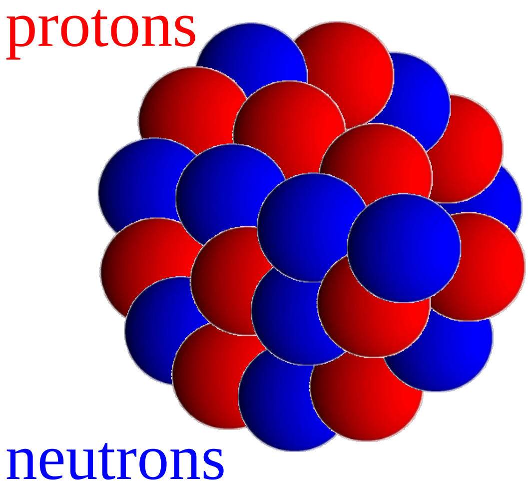
a self-bound system

What happens if a photon is absorbed into a nucleus?
- one nucleon simply starts moving faster?





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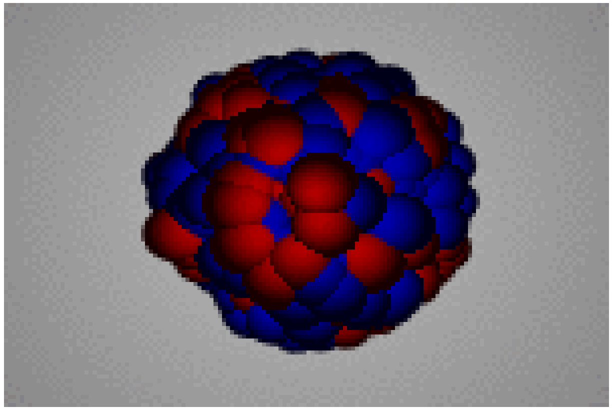
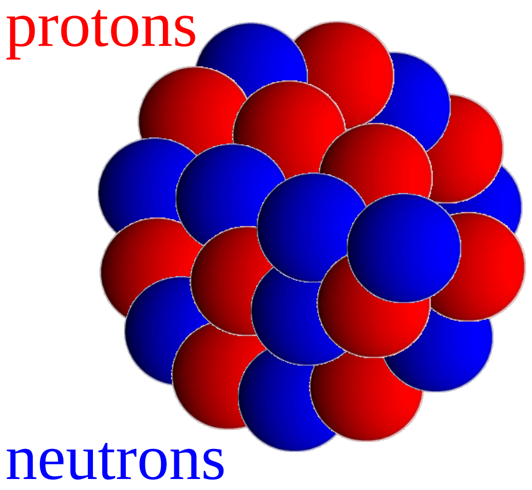


Very coherent motion can happen

Collective motions



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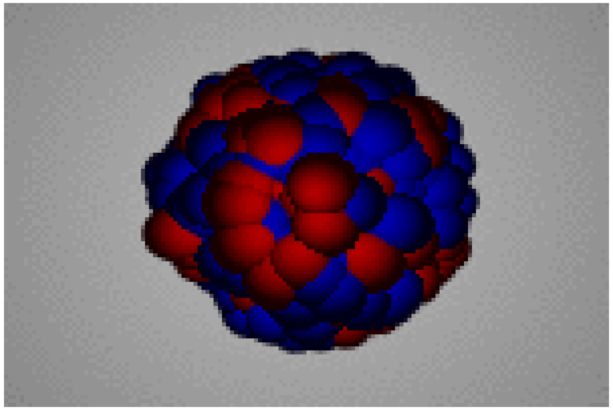

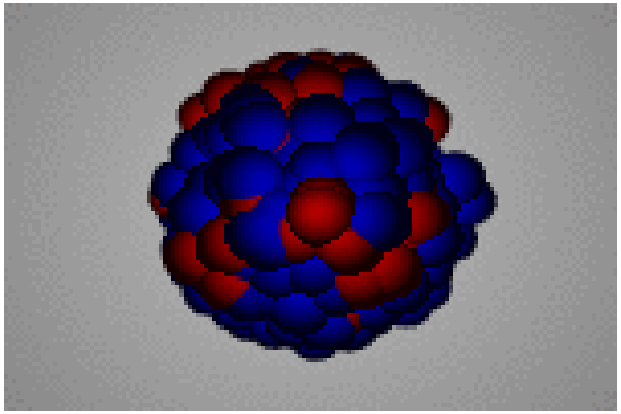

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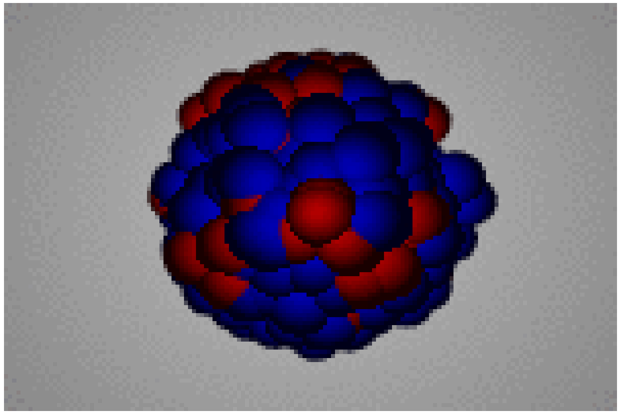

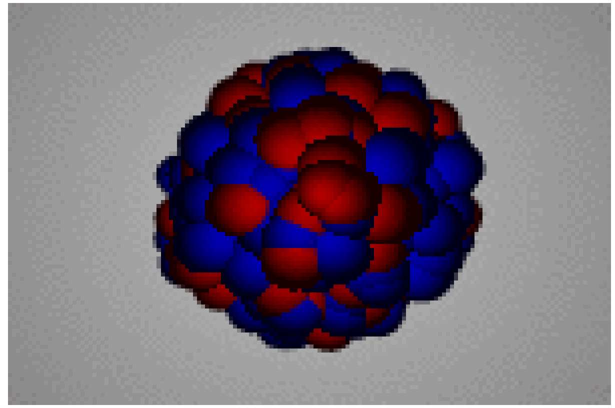

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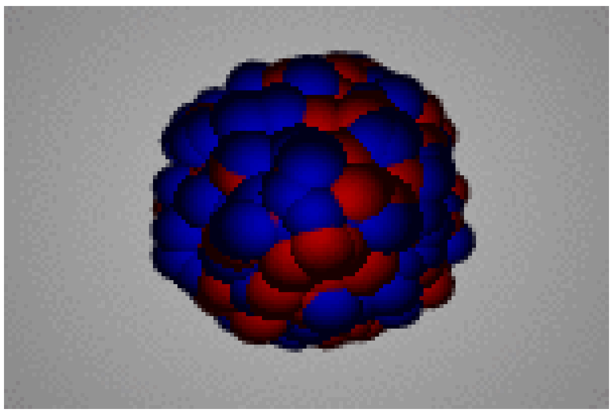
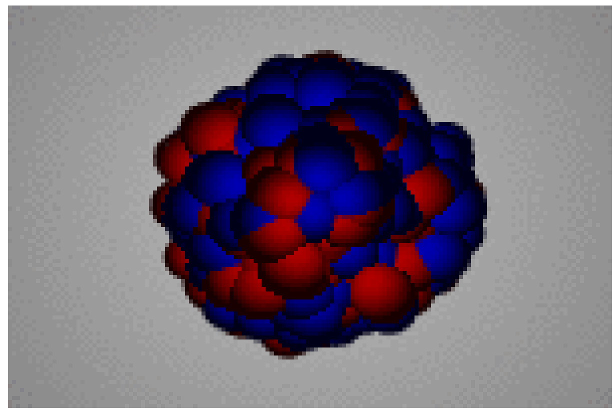
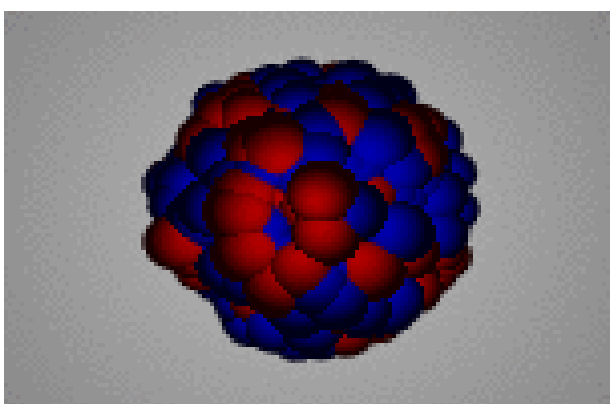
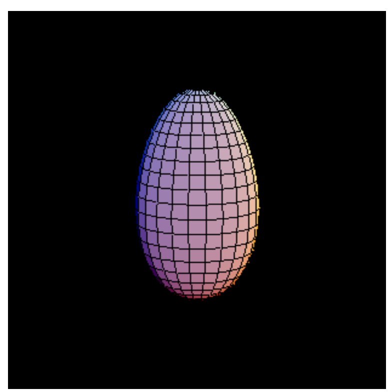
Collective motions

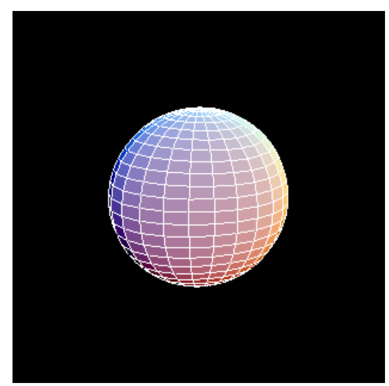


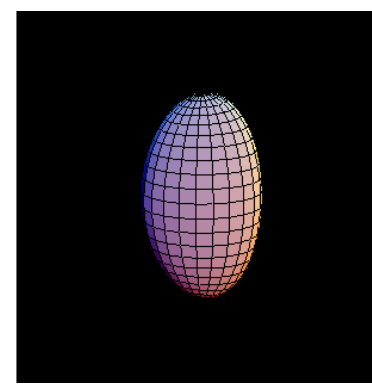


a variety of motions
→ very rich!

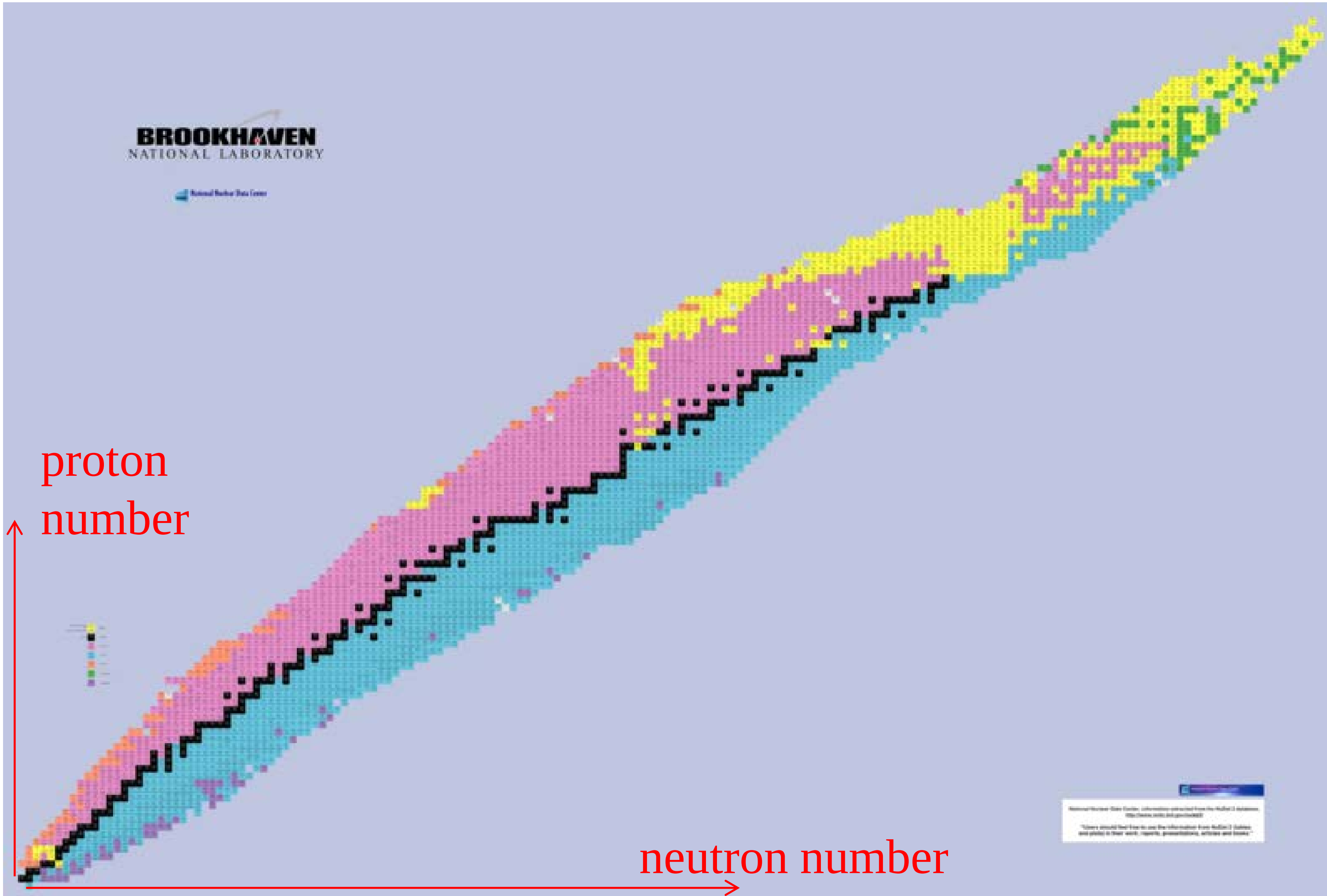






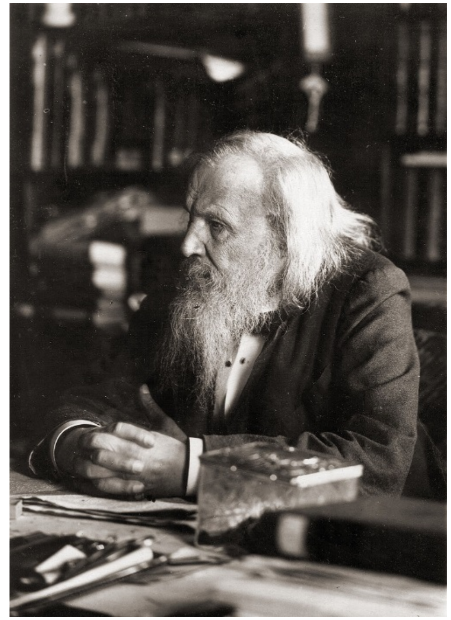


Nuclear Chart: 2D map of atomic nuclei



Periodic Table of elements (1869)

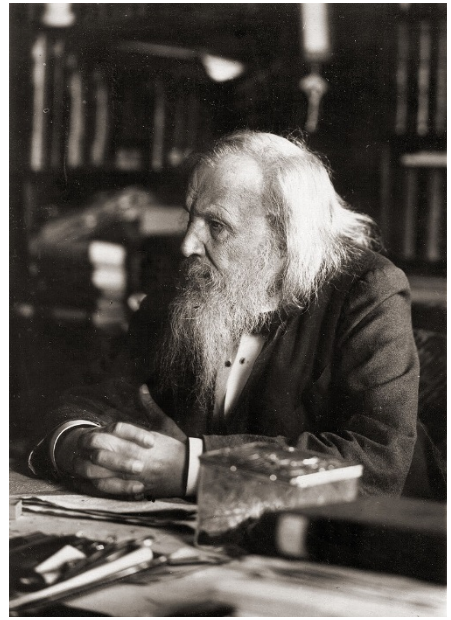
International Year
of the Periodic Table
of Chemical Elements



Mendeleev
(1834-1907)

Group → / Period ↓	1	2												13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc		22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y		40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

Periodic Table of elements (1869)



Mendeleev
(1834-1907)

Group / Period						5	6	7	13	14	15	16	17	18
1														2 He
2									5 B	6 C	7 N	8 O	9 F	10 Ne
3	Na	Mg							13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
														6 r
														4 e
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W							6 n
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg							8 g
				*	58 Ce	59 Pr	60 Nd							
				**	90 Th	91 Pa	92 U							

Taka and Fuji (a grand
-daughter of Mendeleev)

Periodic Table of elements (1869)

protons only, no information on neutrons

International Year
of the Periodic Table
of Chemical Elements

Period	1	2	3		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
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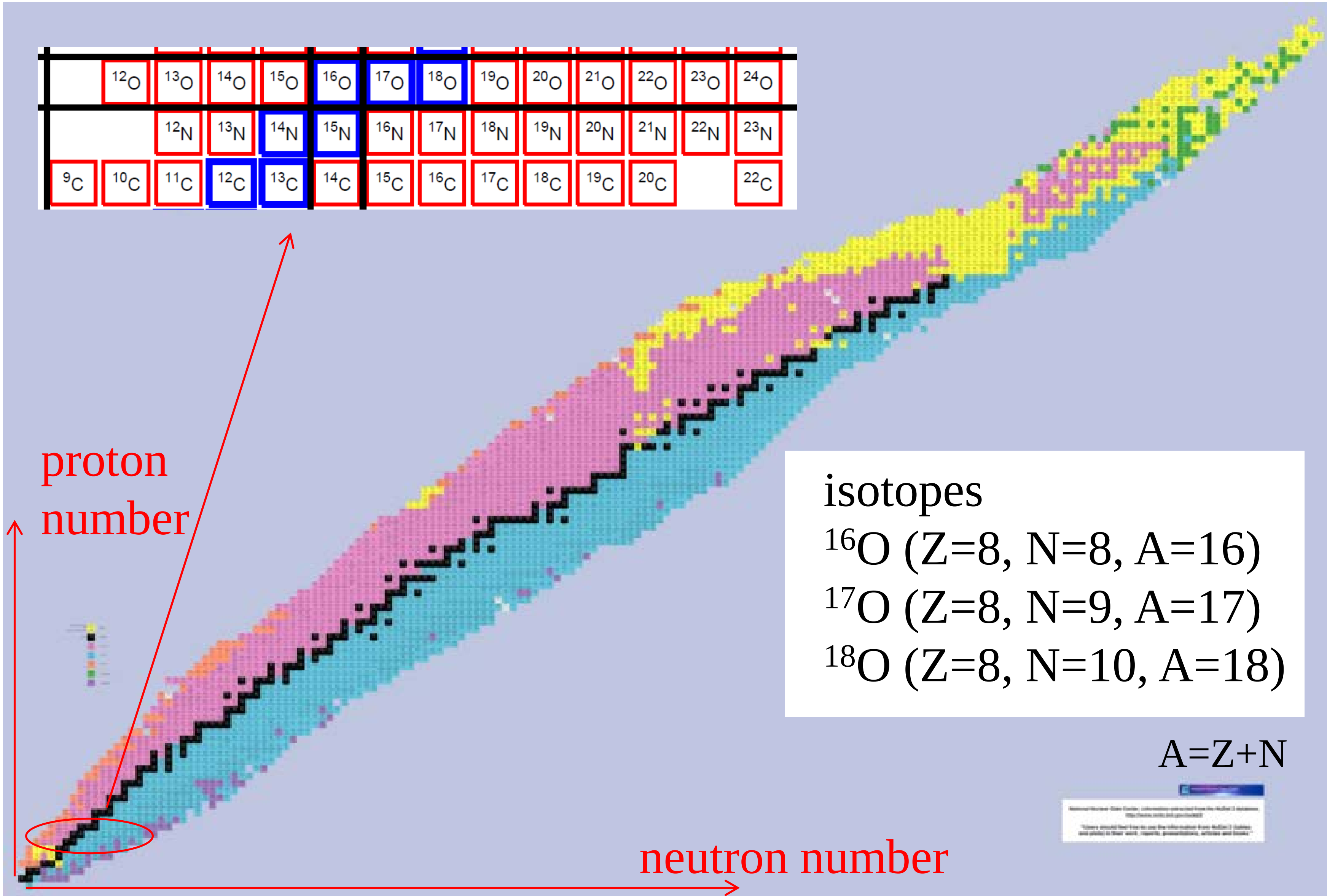
Nuclear Chart: an extended version of periodic table



isotopes

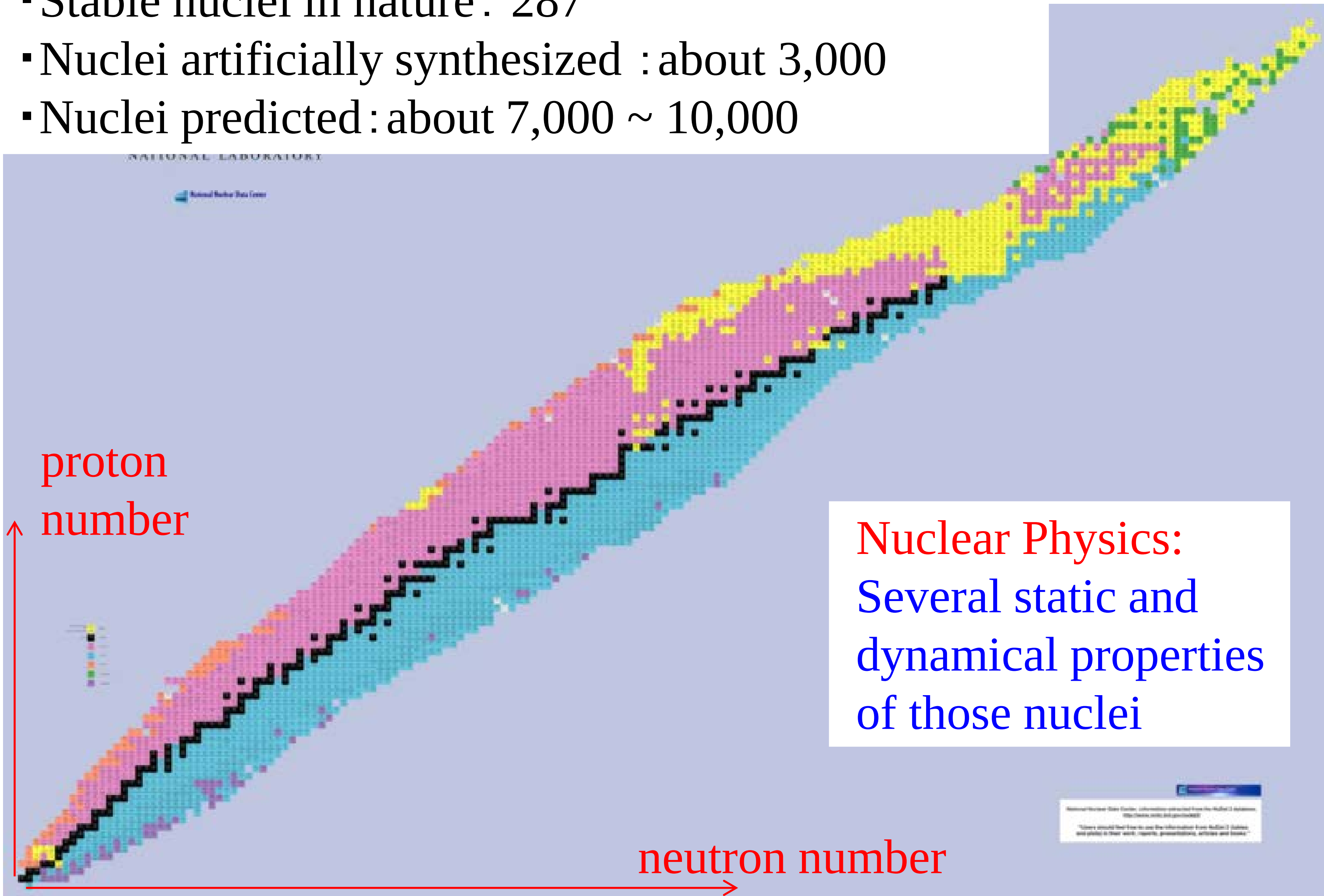
^{16}O (Z=8, N=8, A=16)

^{17}O (Z=8, N=9, A=17)

^{18}O (Z=8, N=10, A=18)

A=Z+N

- Stable nuclei in nature： 287
- Nuclei artificially synthesized ：about 3,000
- Nuclei predicted：about 7,000 ~ 10,000



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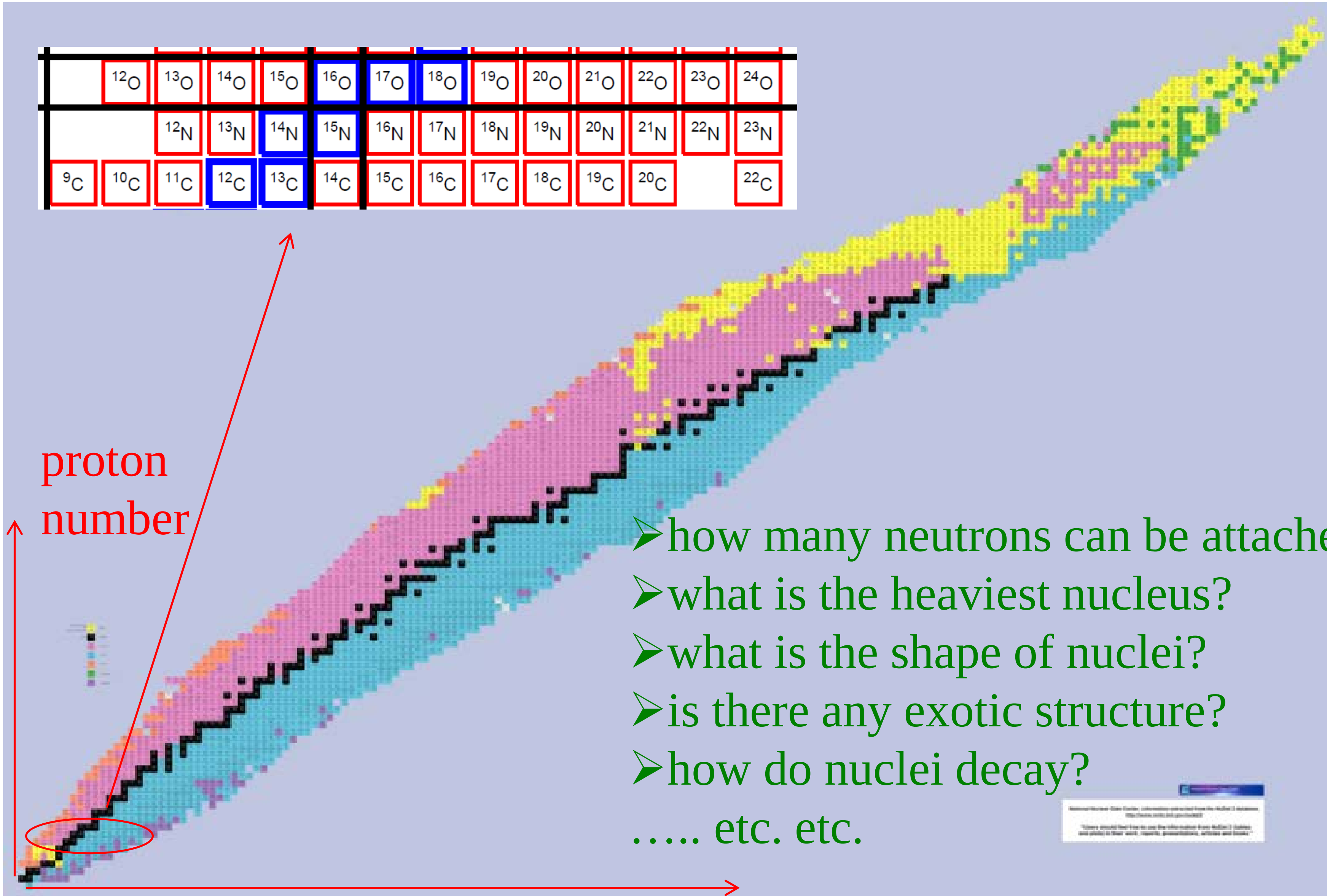
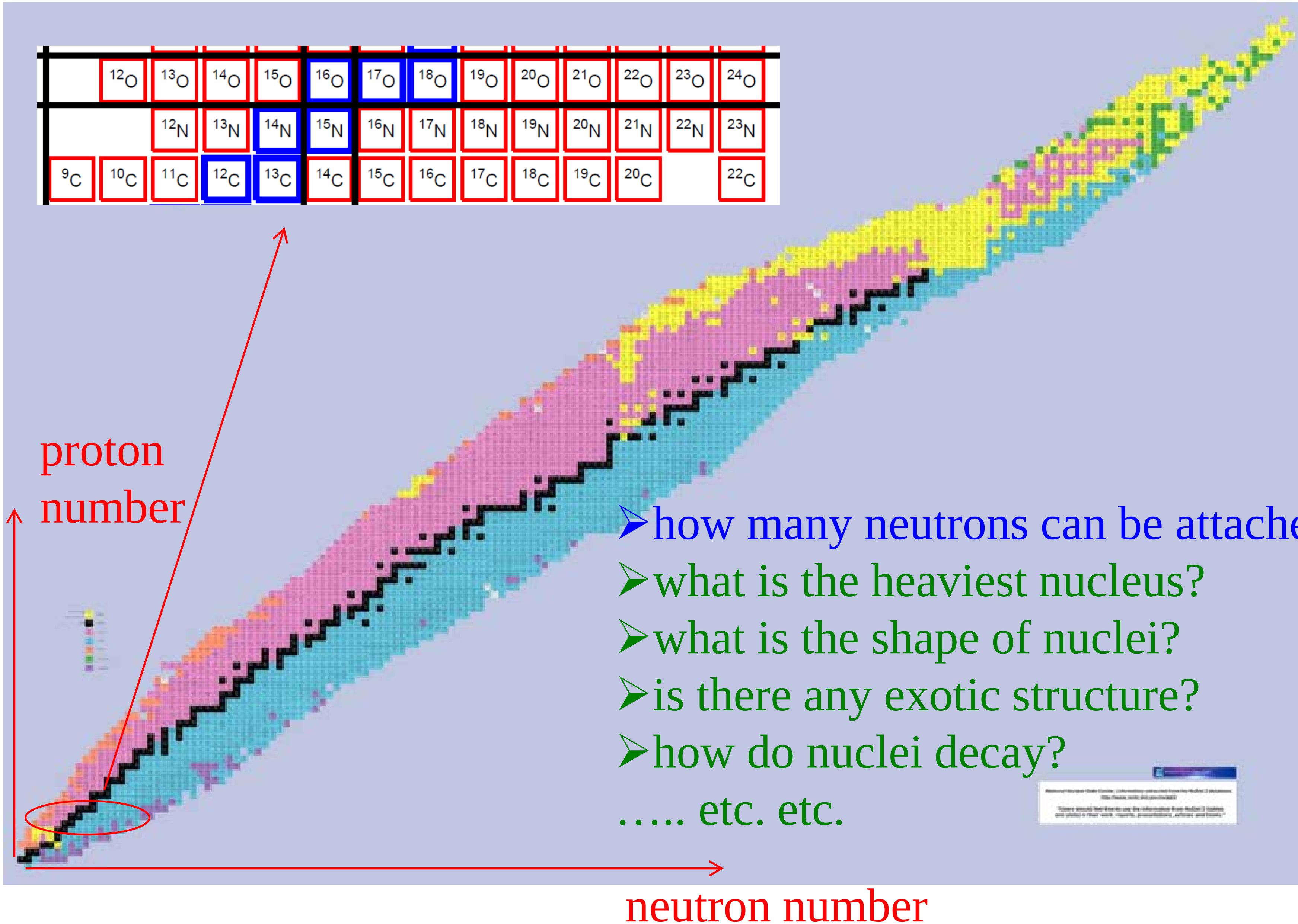


- how many neutrons can be attached?
- what is the heaviest nucleus?
- what is the shape of nuclei?
- is there any exotic structure?
- how do nuclei decay?

….. etc. etc.

neutron number

Nuclear Chart: an extended version of periodic table



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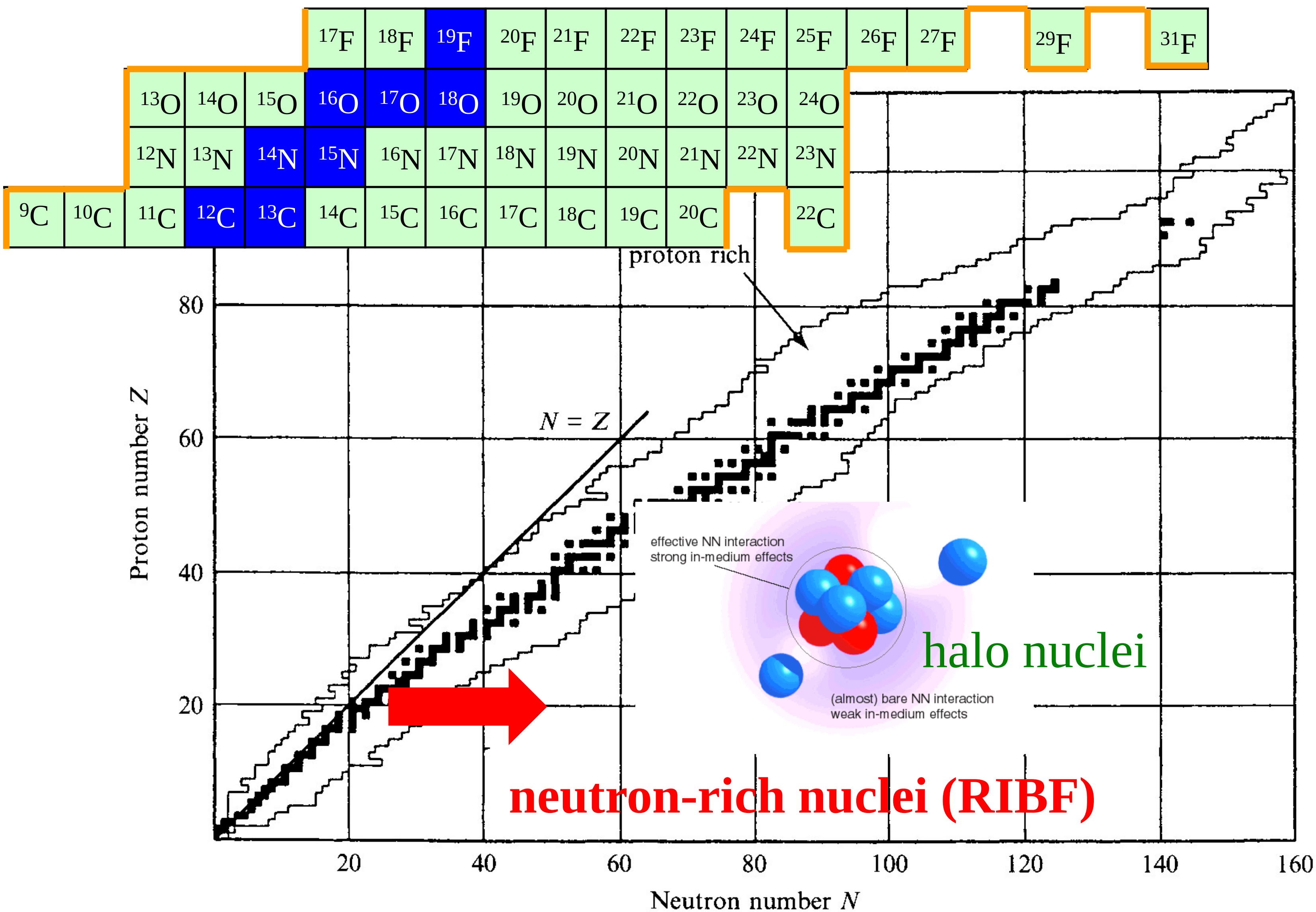
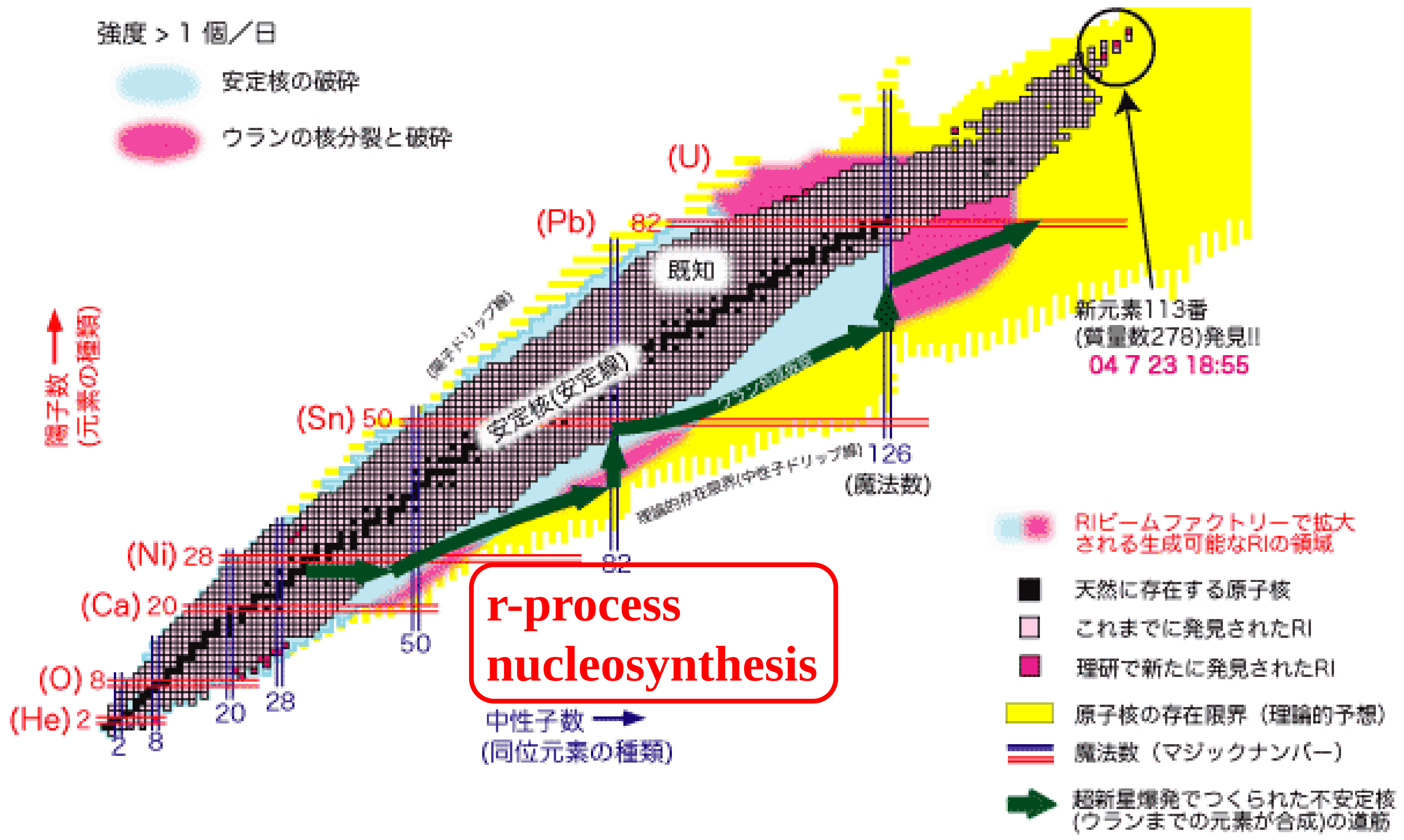
Extension of nuclear chart: frontier of nuclear physics



Neutron-rich nuclei (RIBF at RIKEN)



October 17, 2017

星の合体 重力波で観測

発生源からの光も確認

米欧の研究グループ発表

地球から1・3億光年離れた二つの「中性子星」が合体した様子を、重力波と光で観測することに成功したと、米欧の研究グループが16日、発表した。宇宙のかなたからやってくる重力波を手がかりに、望遠鏡などで光をとらえたのは世界で初めて。天文観測の新たな手法として期待される。

元素の起源 解明に期待

中性子星合体による重力波を観測

過去4回の重力波の観測
ブラックホール同士の合体
質量：太陽の8～36倍
13億～30億光年

今回の重力波の観測
(8月17日)
中性子星同士の合体
質量：太陽の1.2～1.6倍
1億3千万光年

中性子星

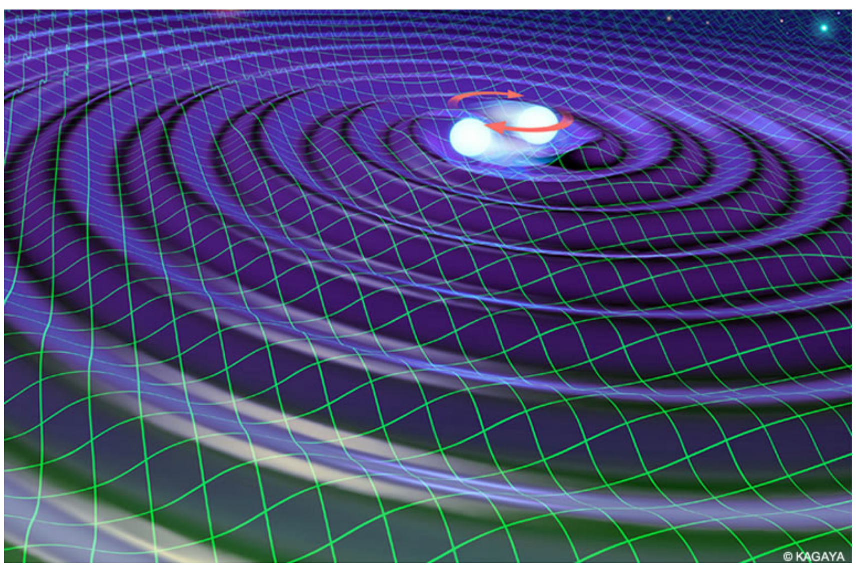
2017年(平成29年)
10月17日
火曜日

朝日新聞

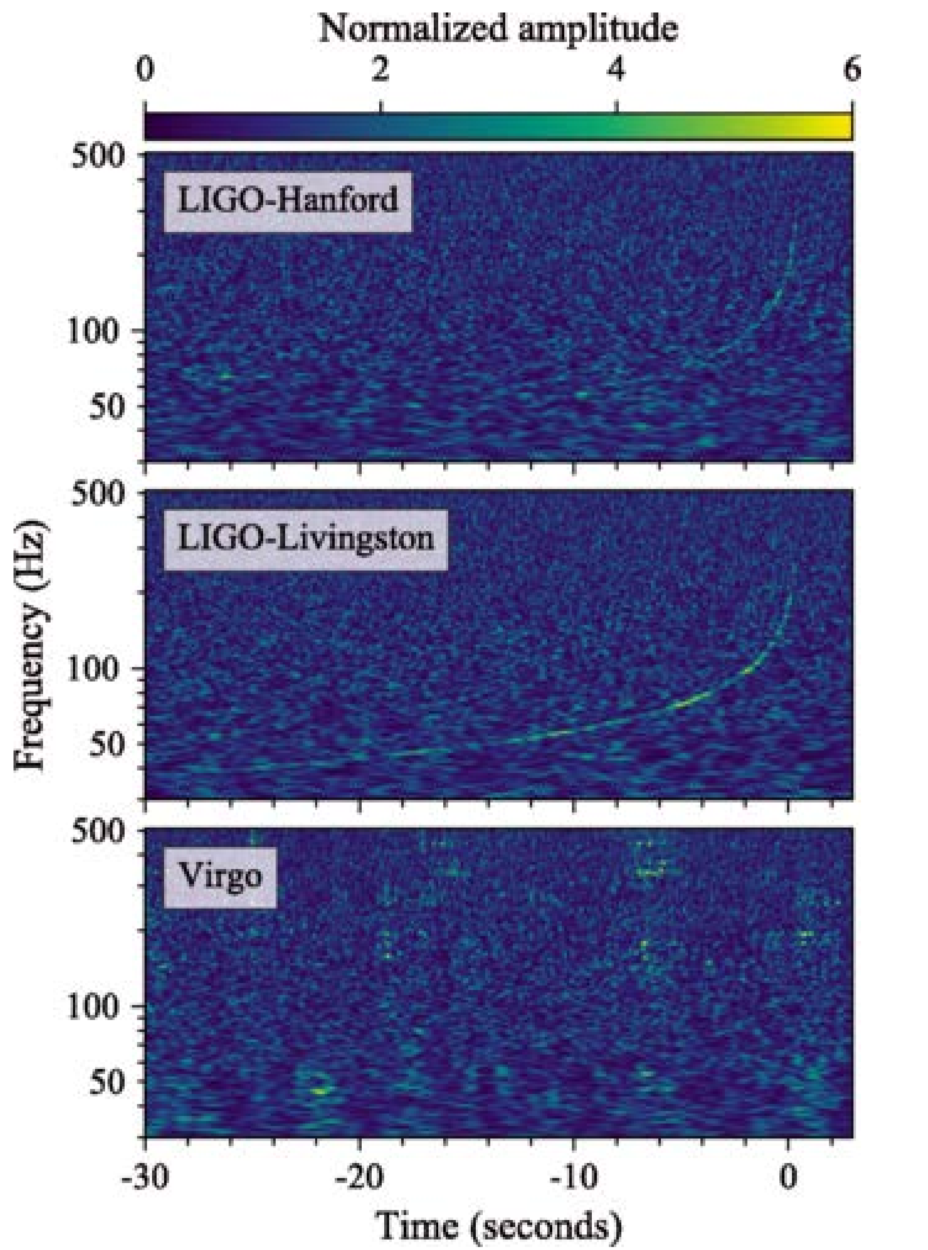
朝日新聞東京本社

信頼の技術を、医薬品に。
NIPRO

2017 衆院選 投票まで 5日



NAOJ



nucleosynthesis at this time?

B.P. Abbott et al., PRL119 ('17) 161101

October 17, 2017

Ele.-mag. wave from the source of GW


星の合体 重力波で観測

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改憲への動きシミュレーション 2

安倍政治 その先⑥ 核政策 3

原発再稼働 争点化の攻防 4

金融緩和 語られない「出口」 7

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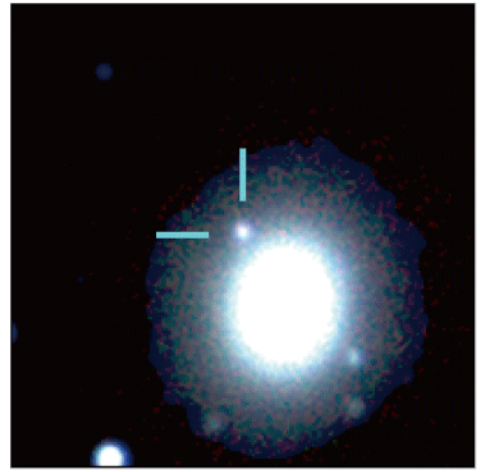
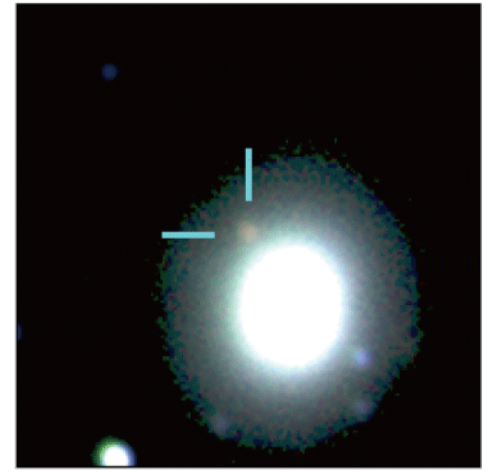
耕論 社会保障は国難か 15

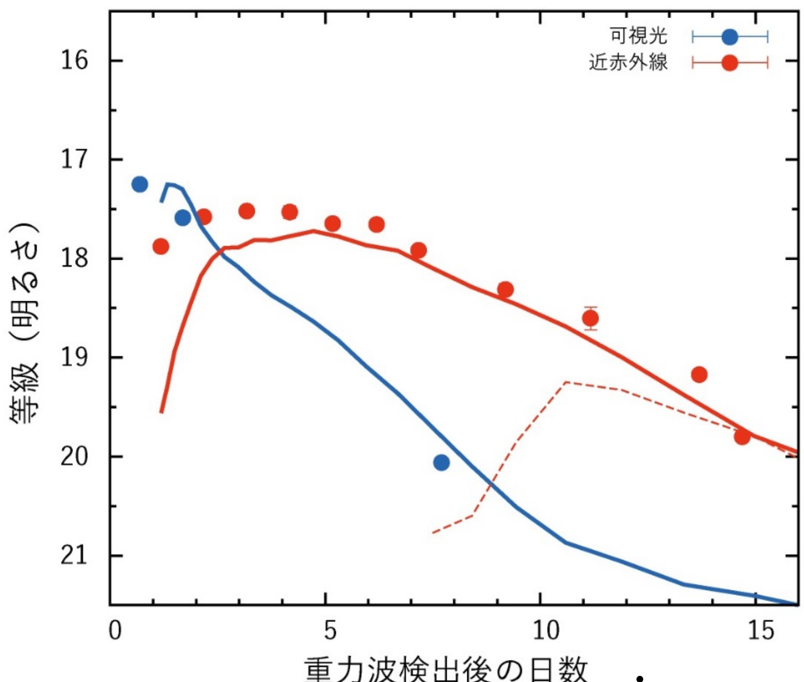
2017.08.18-19



2017.08.24-25



opacity



time

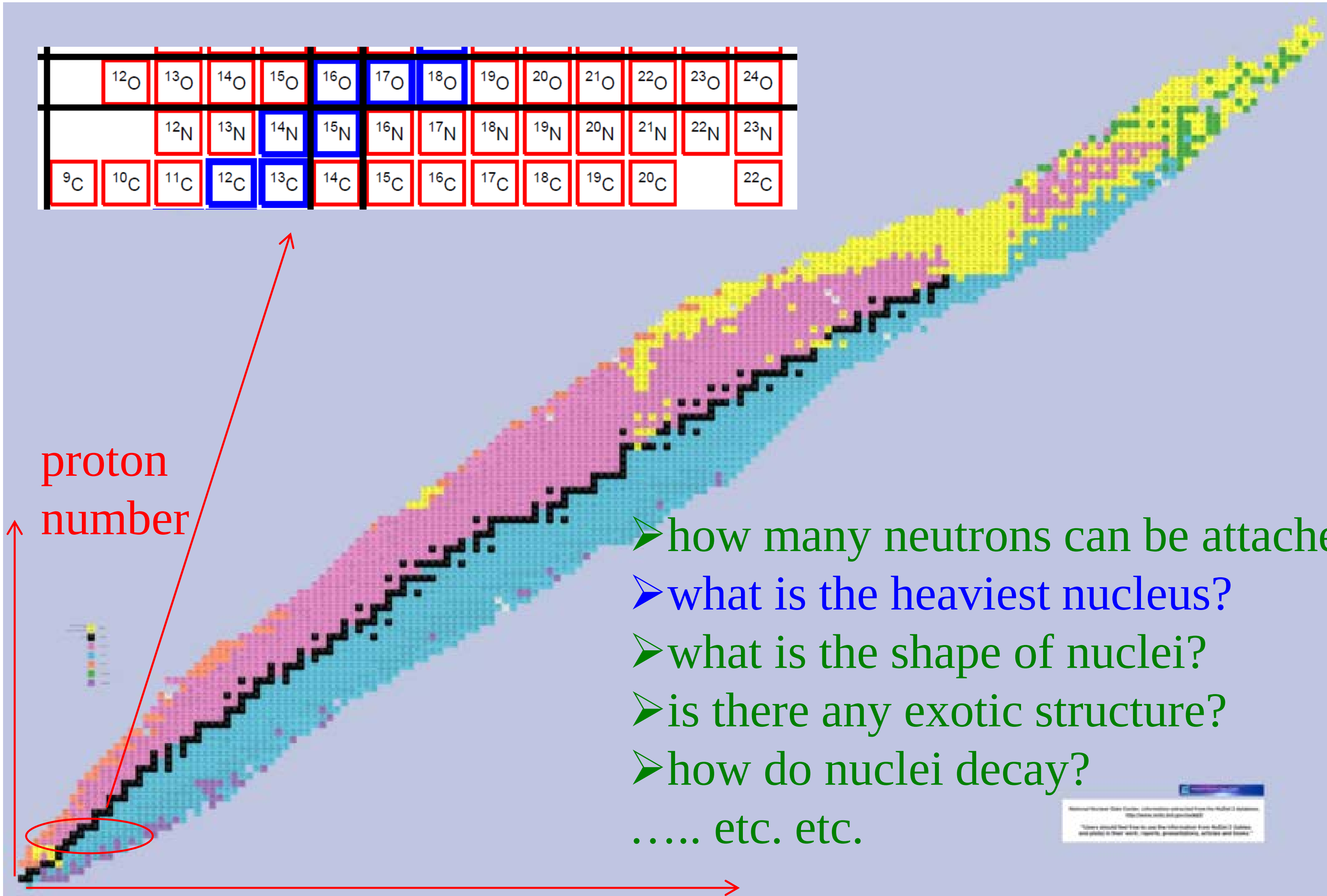
solid lines：with r-process nucleosynthesis

dashed line：no r-process nucleosynthesis

M. Tanaka et al.,
Astron. Soc. Jpn. 69 (‘17) 102

http://www.cfca.nao.ac.jp/pr/20171016

Nuclear Chart: an extended version of periodic table



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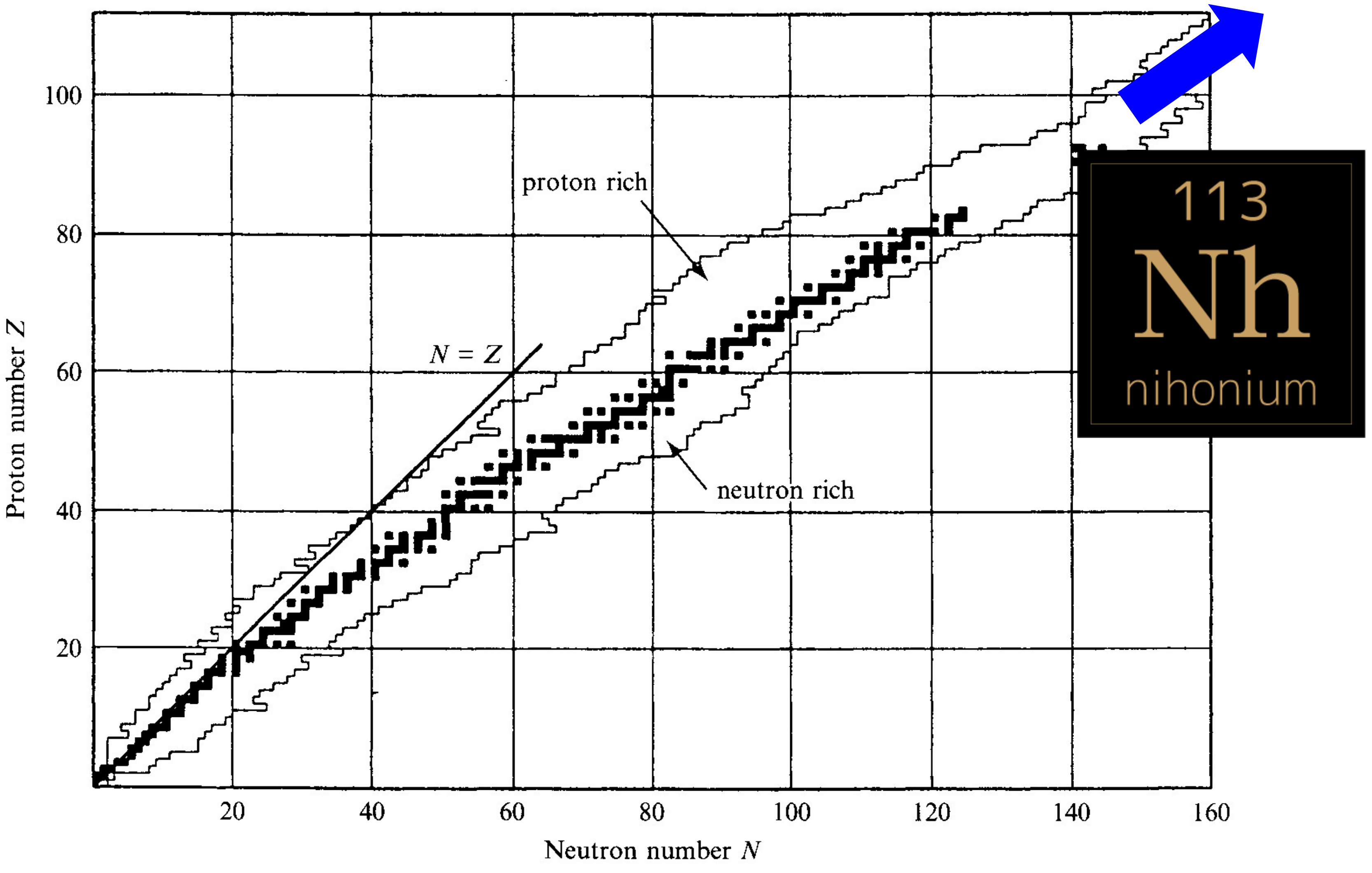
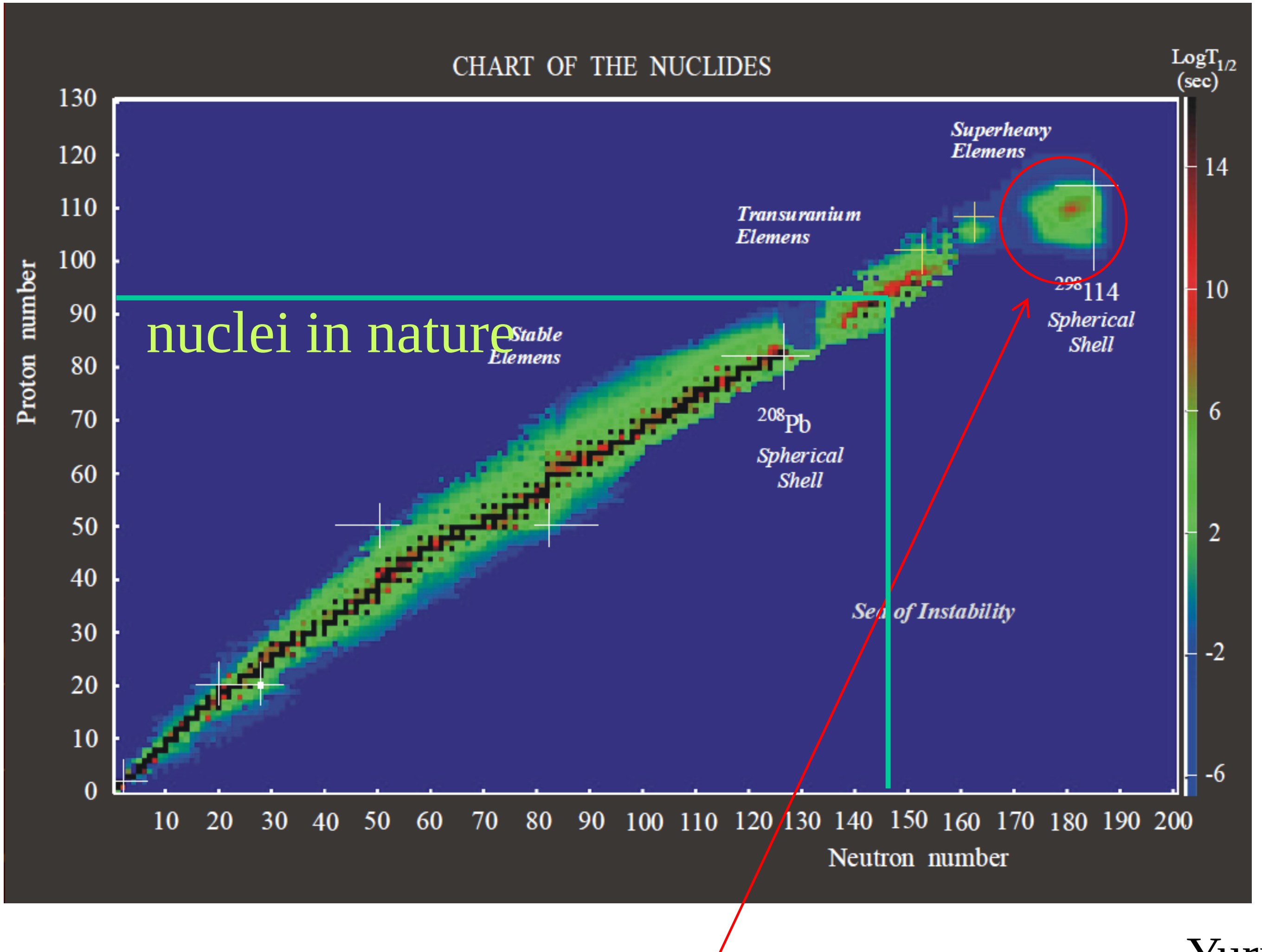
neutron number

Extension of nuclear chart: frontier of nuclear physics

superheavy elements



Prediction of island of stability: an important motivation of SHE study

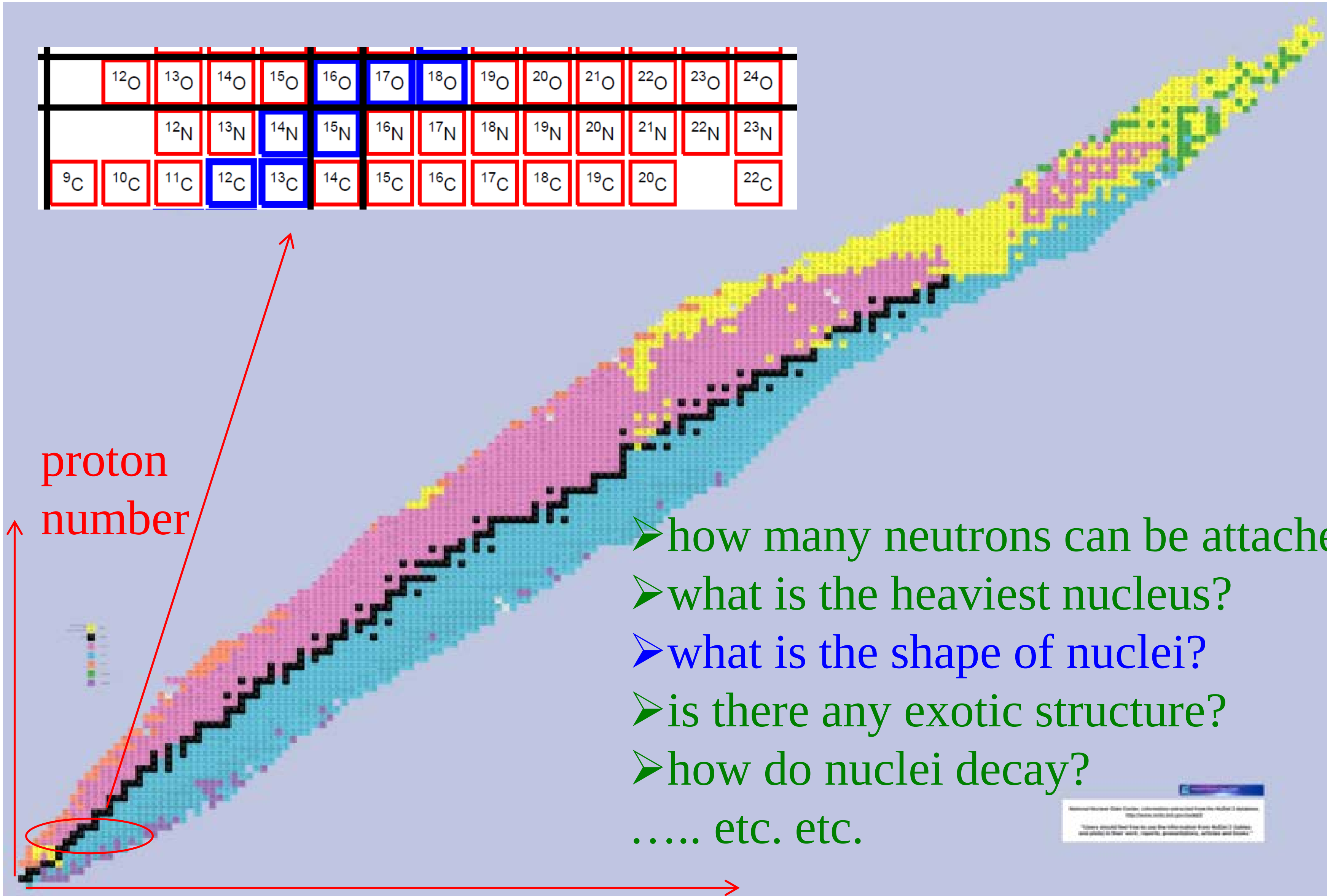


Yuri Oganessian

island of stability around Z=114, N=184

W.D. Myers and W.J. Swiatecki (1966), A. Sobiczewski et al. (1966)

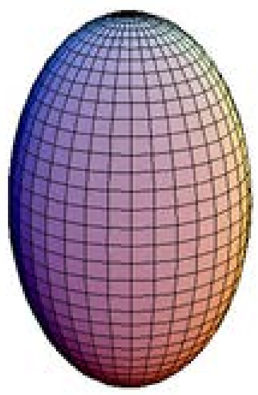
Nuclear Chart: an extended version of periodic table

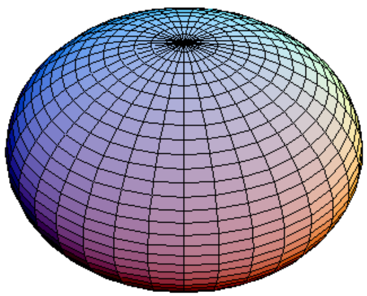


- how many neutrons can be attached?
- what is the heaviest nucleus?
- what is the shape of nuclei?
- is there any exotic structure?
- how do nuclei decay?

….. etc. etc.

a nucleus is not always spherical

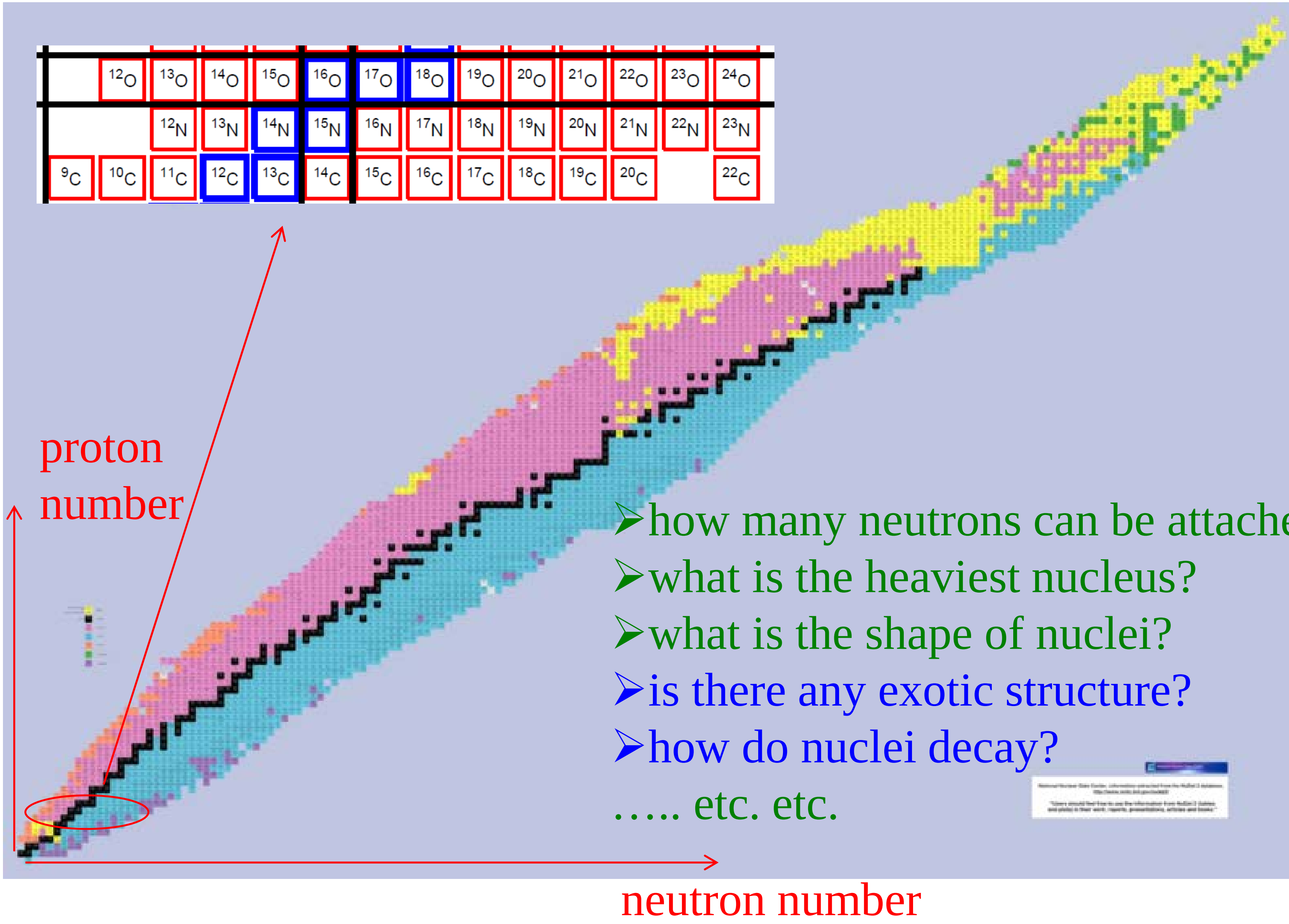




Quantum shape dynamics

「形の量子論」

Nuclear Chart: an extended version of periodic table



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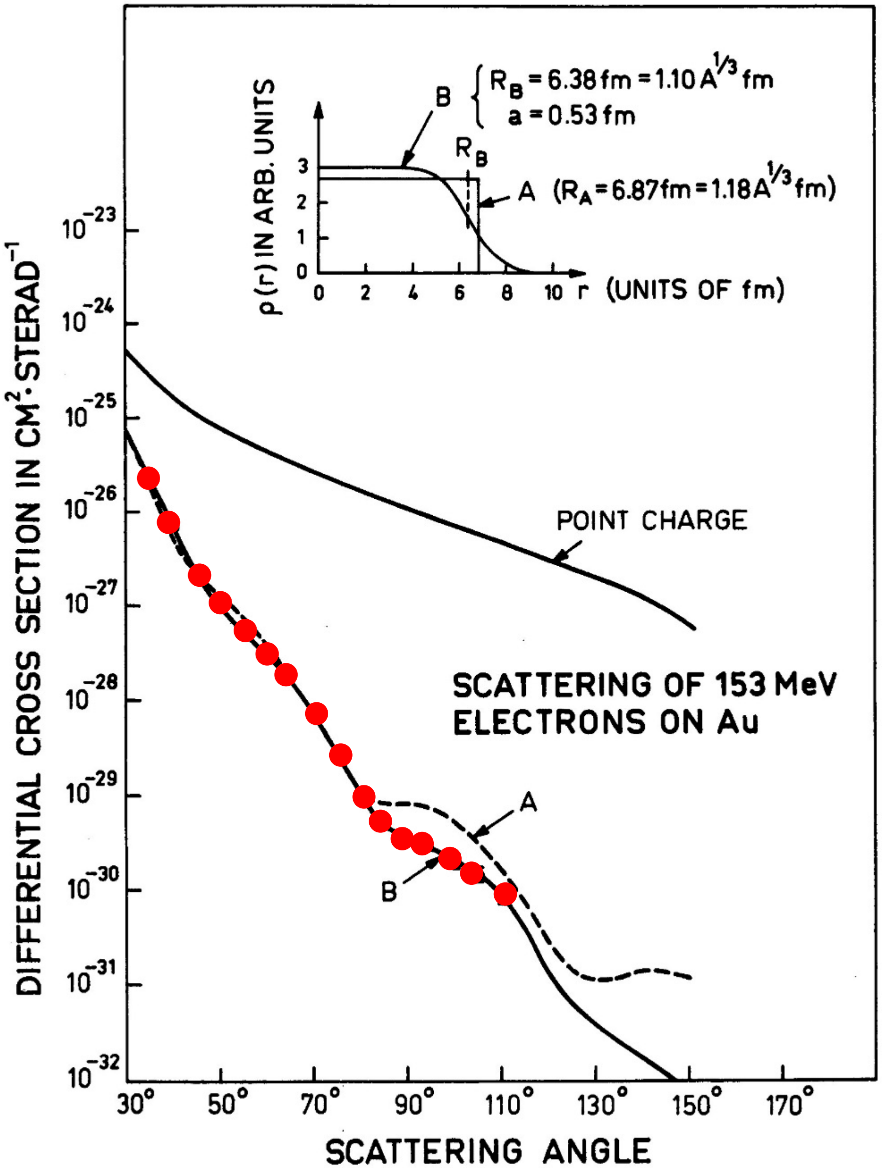
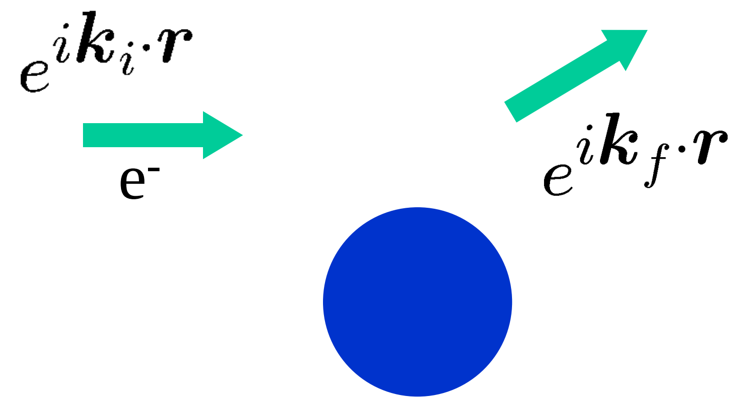
Density Distribution

Density Distribution

High energy electron scattering



Born approximation:



$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2}\,|F(\boldsymbol{q})|^2$$

Form factor

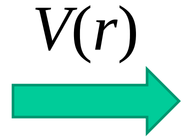
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho(\boldsymbol{r})\,d\boldsymbol{r}$$

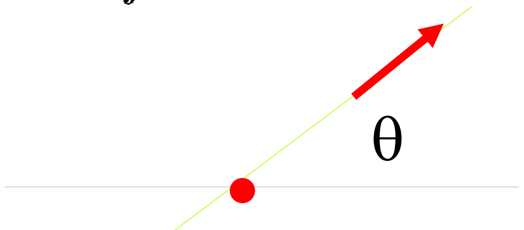
(Fourier transform of the density)

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$





$$W_{fi} = \frac{2\pi}{\hbar}|\langle\psi_f|V|\psi_i\rangle|^2\,\rho(E_f)$$

incident flux: $j_{\text{inc}} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\text{inc}}} = \int d\Omega \boxed{\frac{d\sigma}{d\Omega}}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

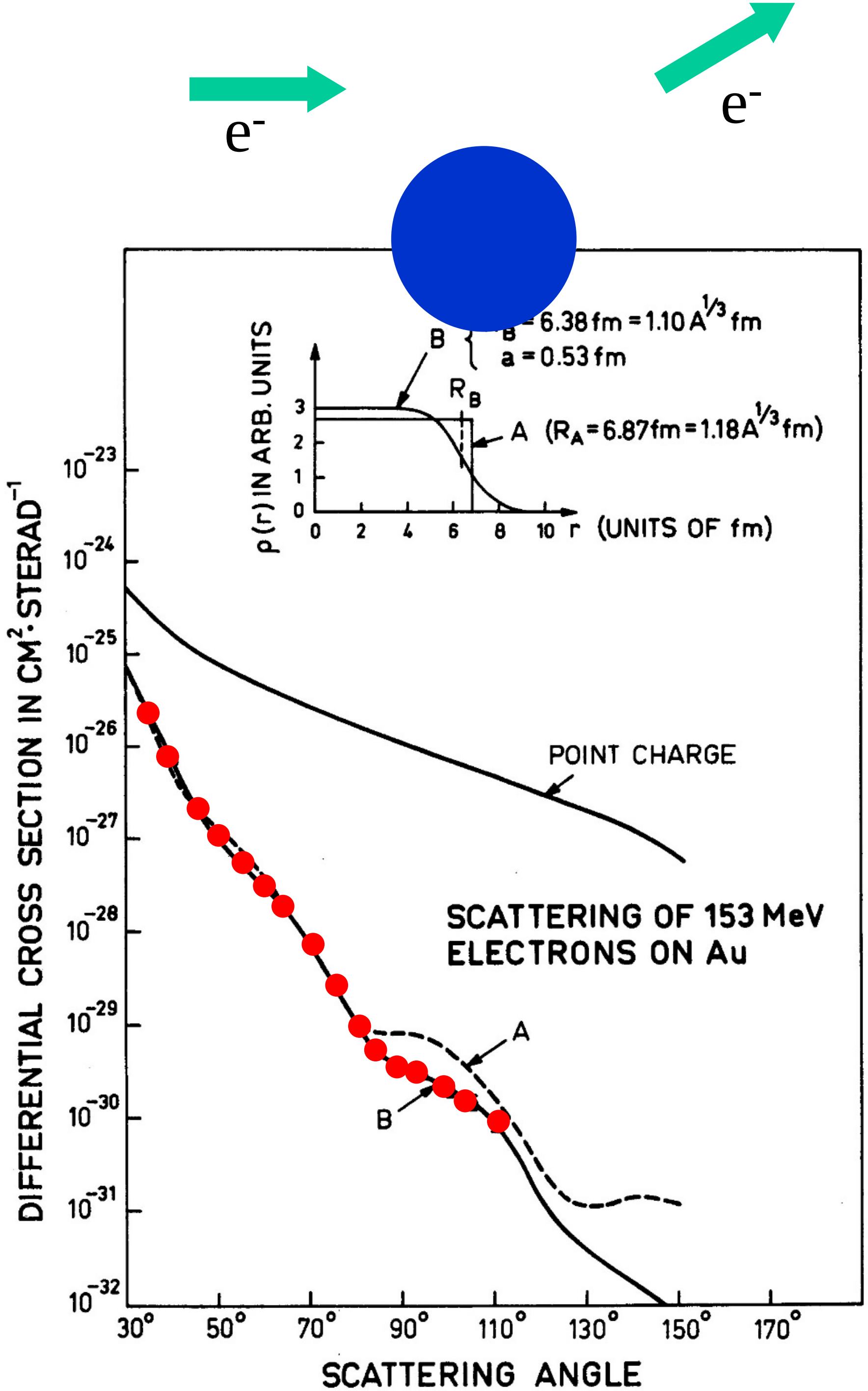
$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2}|F(\boldsymbol{q})|^2 = \left(\frac{d\sigma_{\mathrm{Ruth}}}{d\Omega}\right)|F(\boldsymbol{q})|^2$$

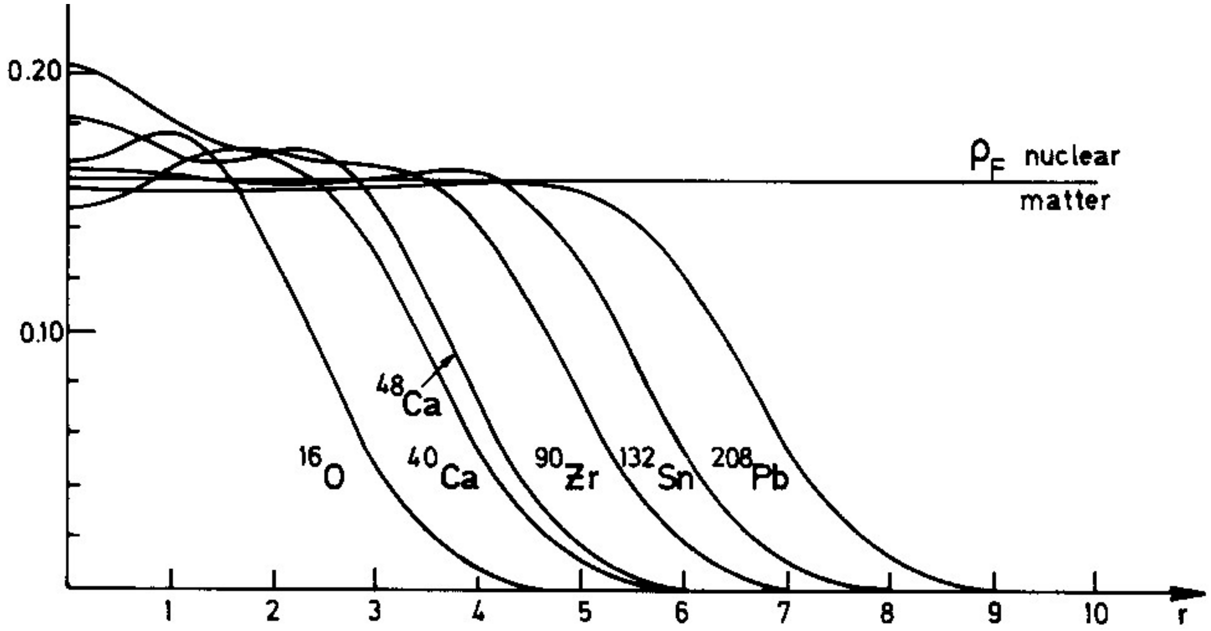
Form factor

$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\rho(\boldsymbol{r})\,d\boldsymbol{r}$$

* relativistic correction:

$$\frac{d\sigma_{\mathrm{Ruth}}}{d\Omega} \to \frac{d\sigma_{\mathrm{Mott}}}{d\Omega} = \frac{d\sigma_{\mathrm{Ruth}}}{d\Omega}\cdot\left(1-\frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right) \sim \frac{d\sigma_{\mathrm{Ruth}}}{d\Omega}\cdot\cos^2\frac{\theta}{2} \qquad (v\to c)$$





Fermi distribution

$$\rho(r) = \frac{\rho_0}{1 + \exp[(r - R_0)/a]}$$

$$\rho_0 \sim 0.17 \ (\mathrm{fm}^{-3})$$

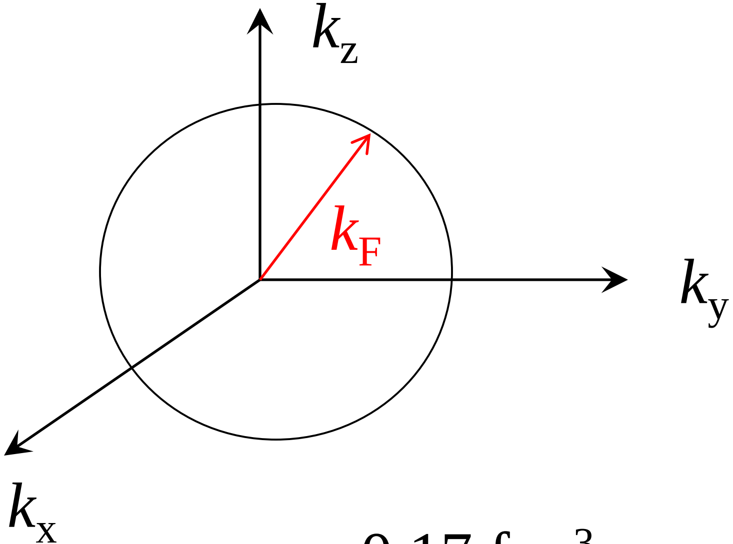
Saturation property

$$R_0 \sim 1.1 \times A^{1/3} \ (\mathrm{fm})$$

$$a \sim 0.57 \ (\mathrm{fm})$$

Momentum Distribution

Fermi gas approximation



$$\rho = 2 \times 2 \times 4\pi \int_0^{k_F} \frac{k^2 dk}{(2\pi)^3}$$
$$= \frac{2}{3\pi^2} k_F^3$$

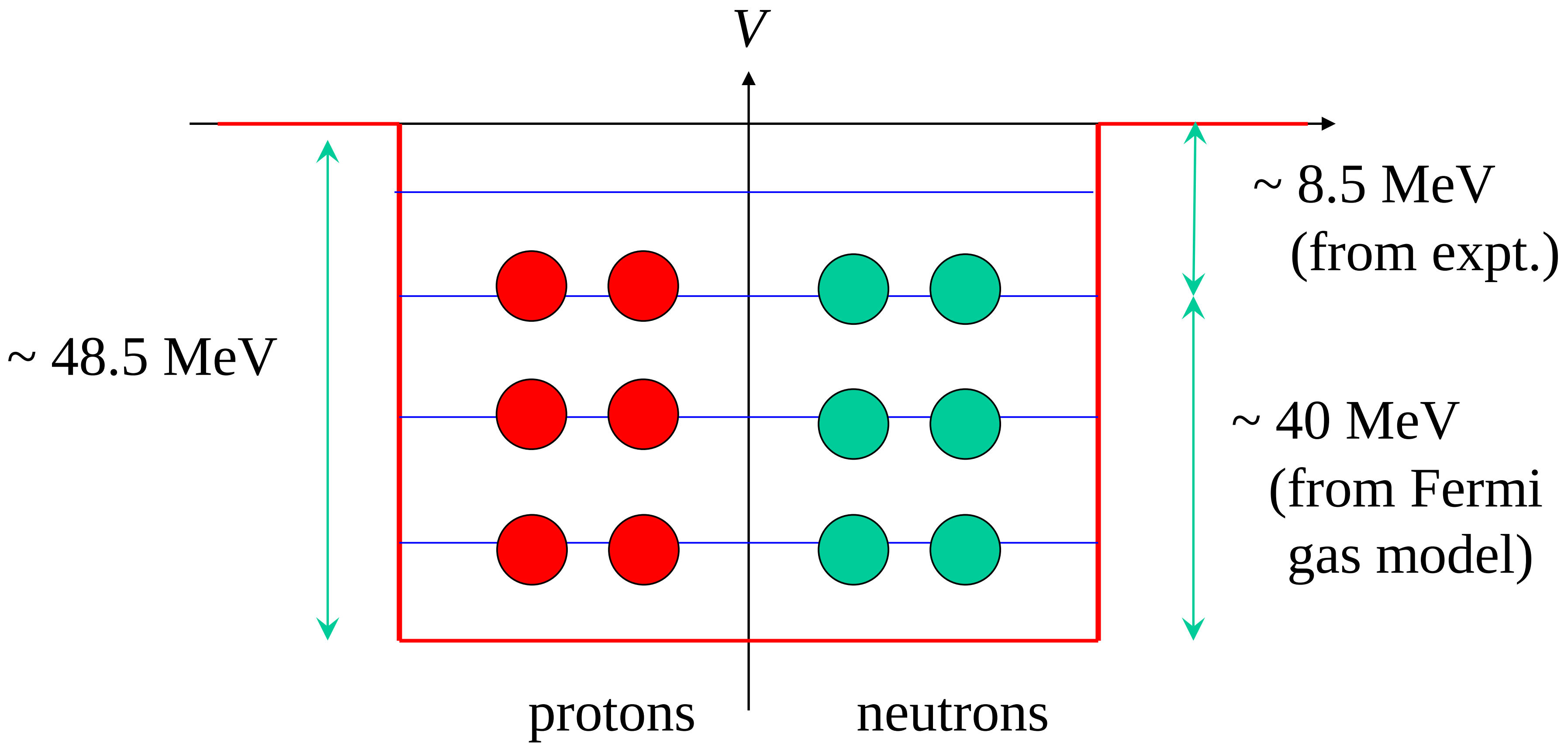
(note: spin-isospin degeneracy)

ρ = 0.17 fm^{-3} $\longrightarrow$ k_F ~ 1.36 fm^{-1}

$$\Longleftrightarrow \quad \frac{v_F}{c} = \frac{k_F \cdot \hbar c}{mc^2} = 0.285$$

Fermi energy: $\epsilon_F = \frac{k_F^2 \hbar^2}{2m} \sim 37$ (MeV)

A potential for nucleons inside a nucleus



Discussion: Electron scattering

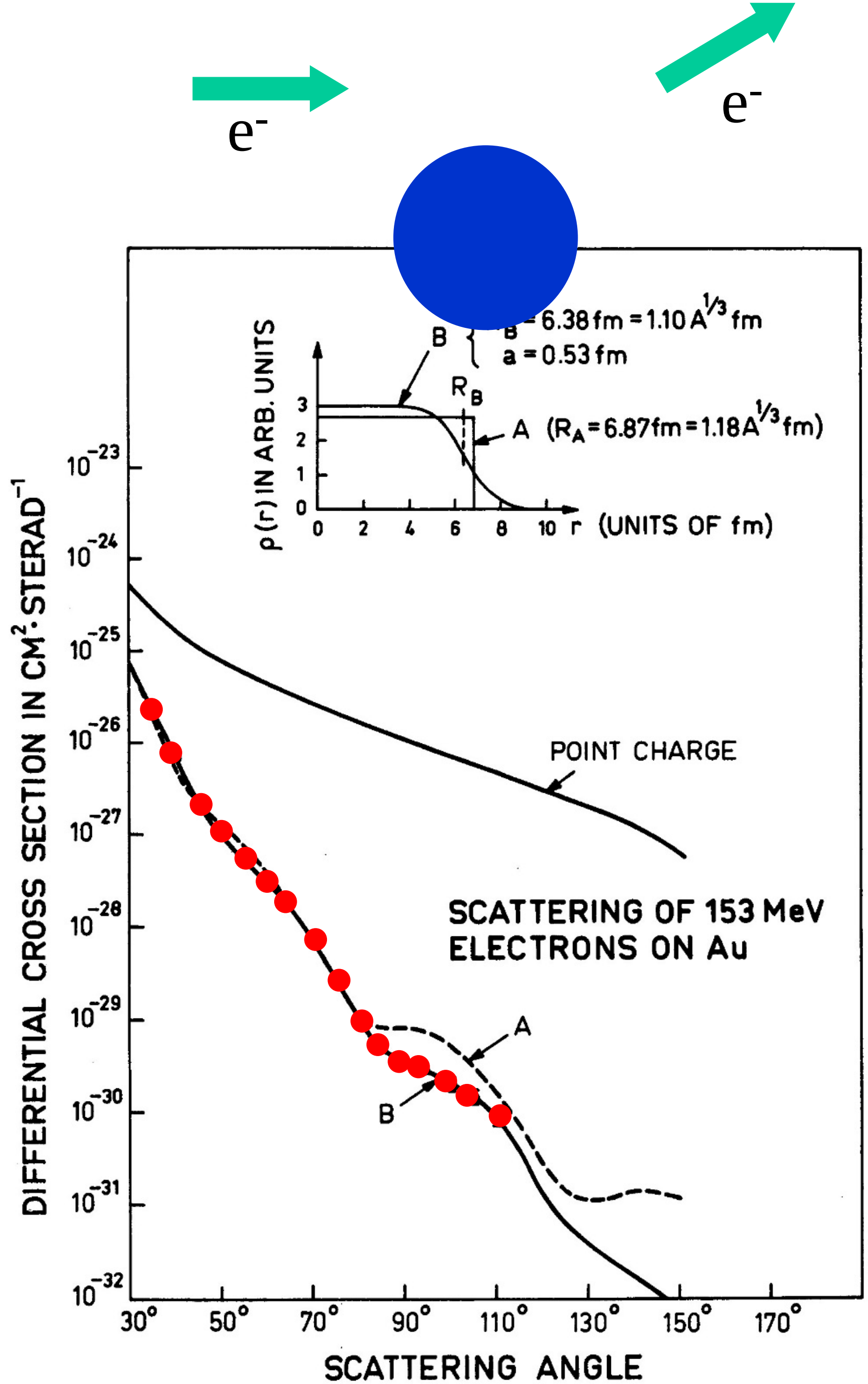
$$V(r) = -e^2 \int d\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E \sin^2\theta/2)^2} |F(\mathbf{q})|^2 = \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega}\right) |F(\mathbf{q})|^2$$

Form factor

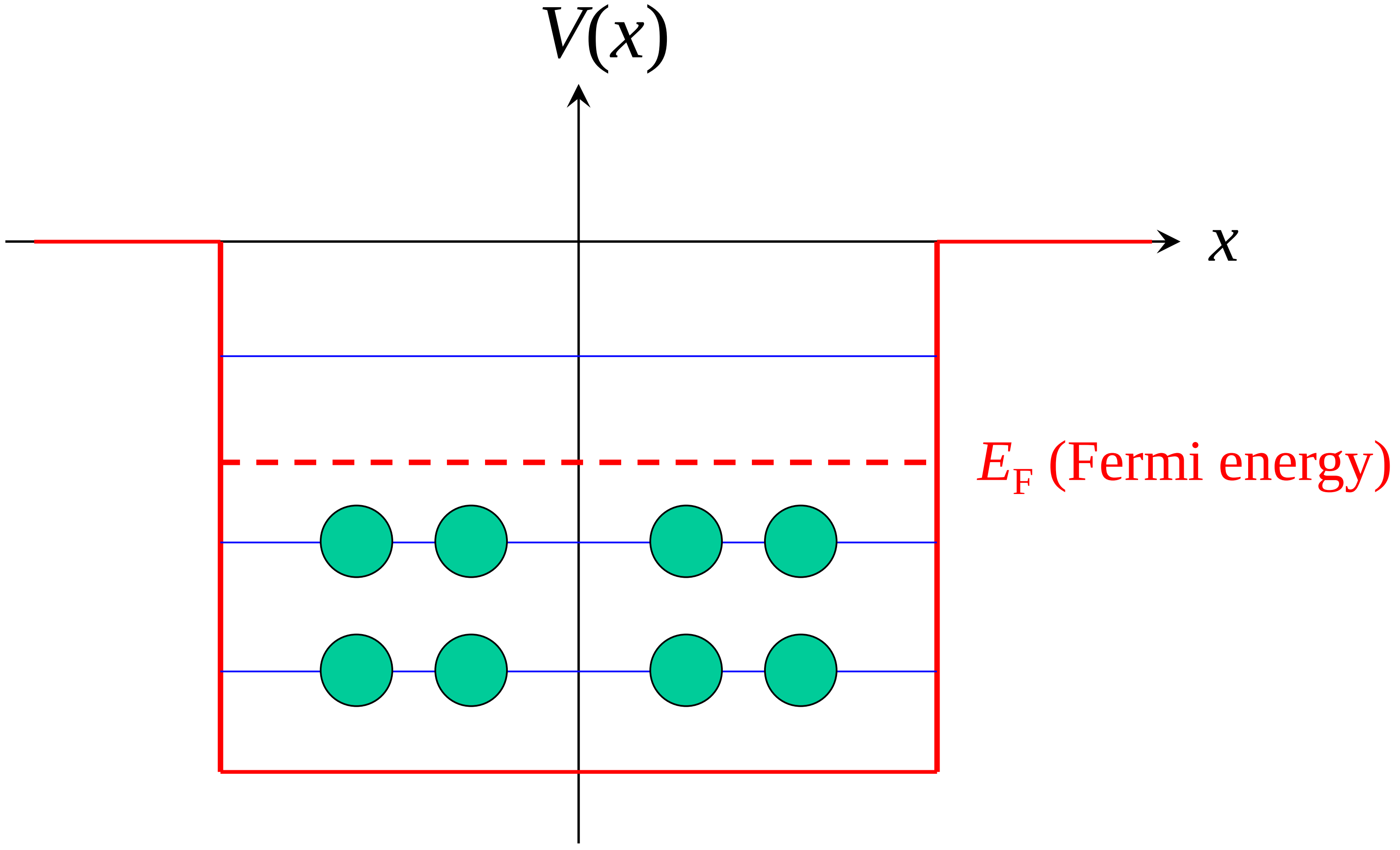
$$F(\mathbf{q}) = \int e^{-i\mathbf{q}\cdot\mathbf{r}} \rho(\mathbf{r})\, d\mathbf{r}$$

Why do cross sections decrease for an extended density distribution?



Appendix

Fermi gas model

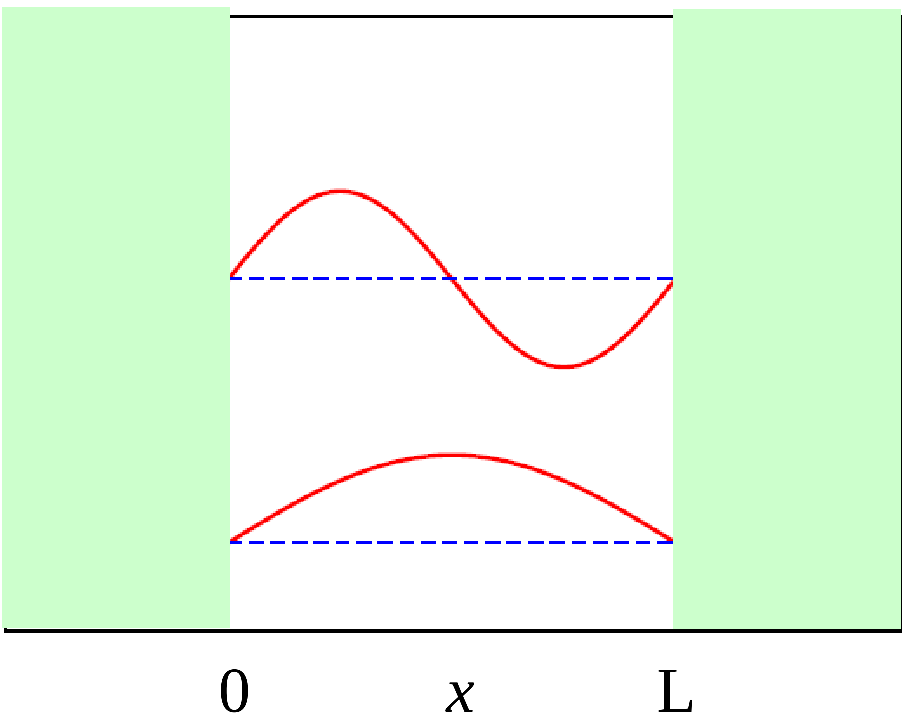


What is the relation between E_F and the particle number?

→ Fermi gas model

Fermi gas model

non-interaction many Fermion system (with no external potential)



put infinite walls at $x = 0$ and $x = L$:

$$\longrightarrow \quad \psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$$

$$E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2$$

$$(n = 1, 2, \cdots)$$

three-dimensional case:

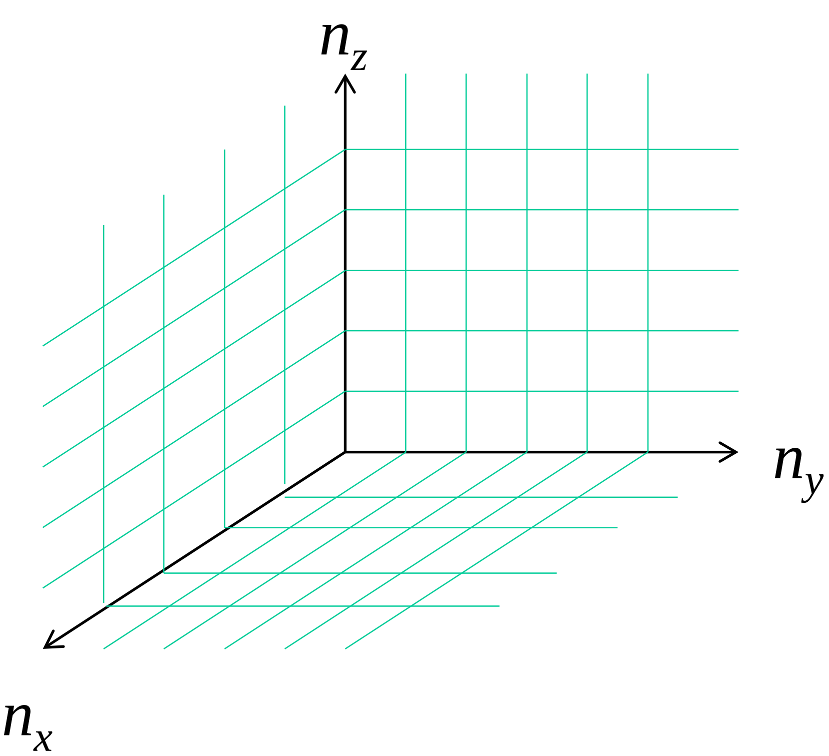
$$\psi_{n_x n_y n_z}(x) = \left(\frac{2}{L}\right)^{3/2} \sin\left(\frac{n_x\pi}{L}x\right) \sin\left(\frac{n_y\pi}{L}y\right) \sin\left(\frac{n_z\pi}{L}z\right)$$

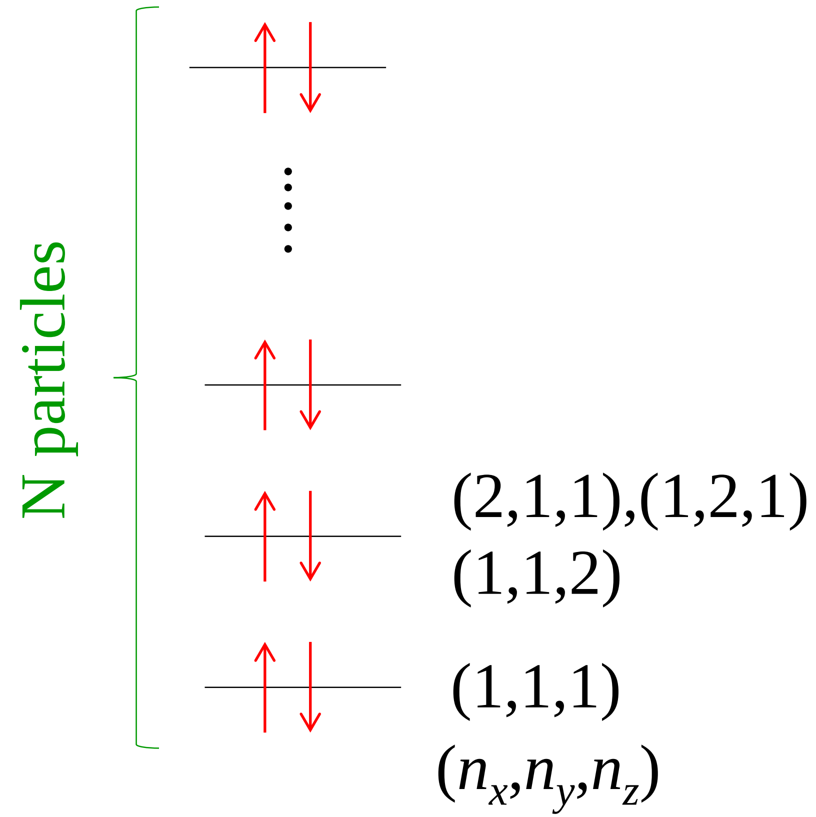
$$E_{n_x n_y n_z} = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 (n_x^2 + n_y^2 + n_z^2)$$

Fermi gas model

$$\psi_{n_x n_y n_z}(x) = \left(\frac{2}{L}\right)^{3/2} \sin\left(\frac{n_x \pi}{L} x\right) \sin\left(\frac{n_y \pi}{L} y\right) \sin\left(\frac{n_z \pi}{L} z\right)$$

$$E_{n_x n_y n_z} = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 (n_x^2 + n_y^2 + n_z^2)$$

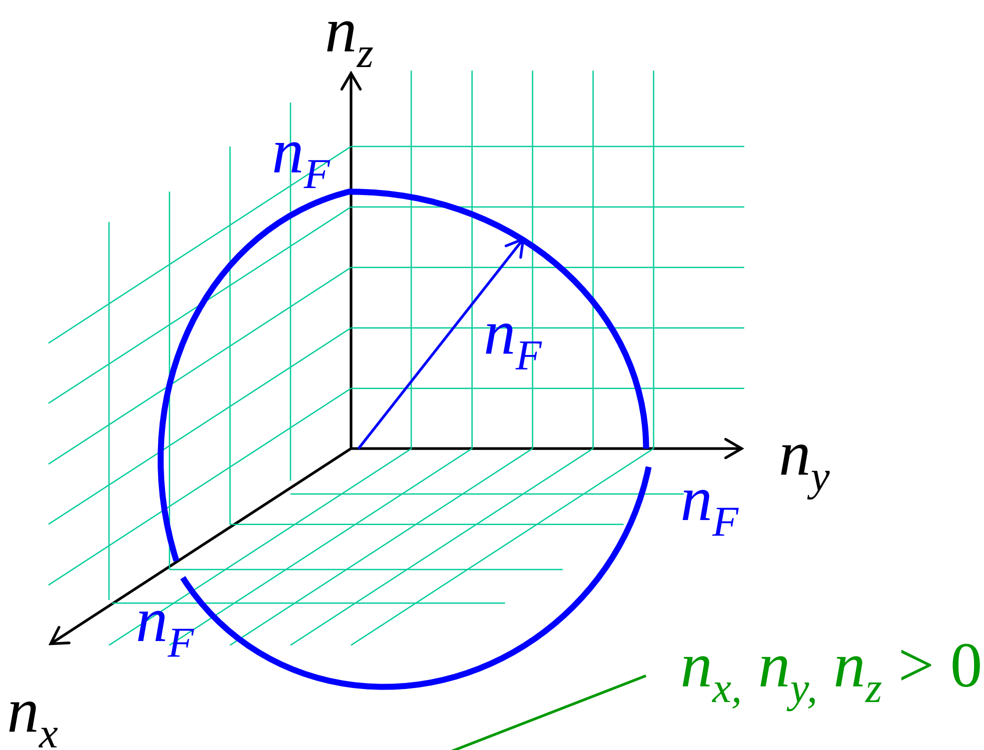


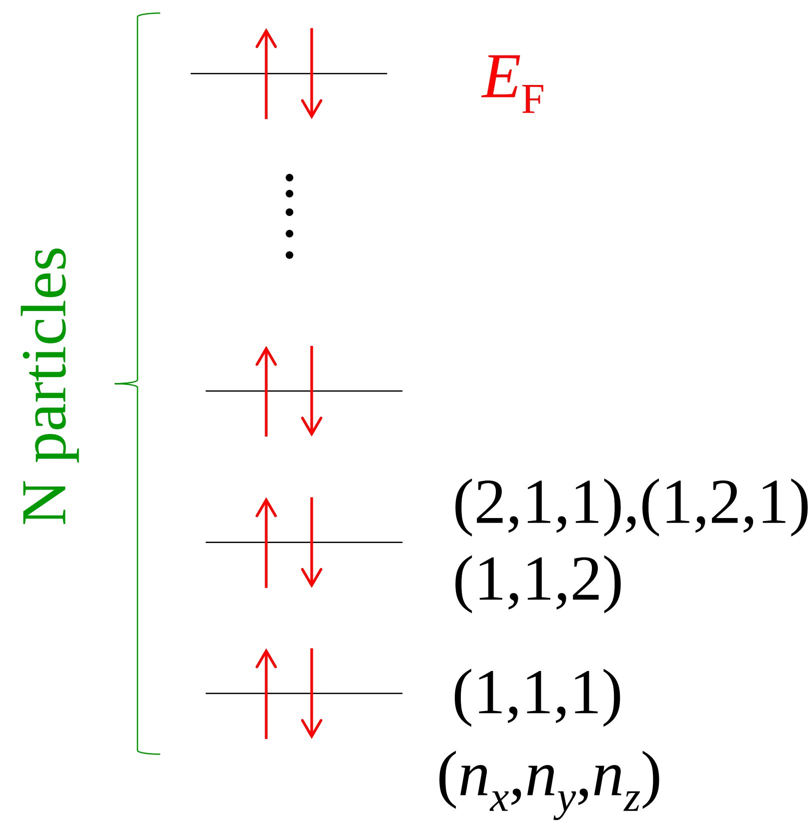


Fermi gas model

the highest energy: $$E_F = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 (n_x^2 + n_y^2 + n_z^2) \equiv \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 n_F^2$$

$$\longrightarrow \quad n_F^2 = \frac{2mE_F}{\hbar^2\pi^2} L^2$$





$$N = 2 \times \frac{1}{8} \times \frac{4}{3}\pi n_F^3$$

Fermi gas model

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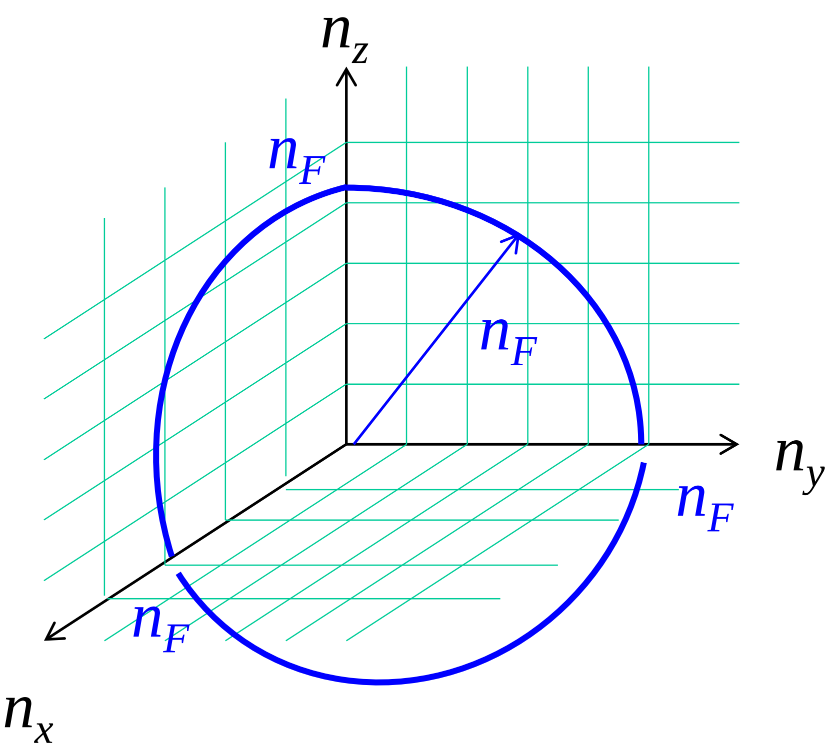


$$N = 2 \times \frac{1}{8} \times \frac{4}{3}\pi n_F^3 = \frac{\pi}{3}\left(\frac{2mE_F}{\hbar^2\pi^2} L^2\right)^{3/2} = \frac{\pi}{3}\left(\frac{2mE_F}{\hbar^2\pi^2}\right)^{3/2} L^3$$

Fermi gas model

$$N = 2 \times \frac{1}{8} \times \frac{4}{3}\pi n_F^3 = \frac{\pi}{3}\left(\frac{2mE_F}{\hbar^2\pi^2}L^2\right)^{3/2} = \frac{\pi}{3}\left(\frac{2mE_F}{\hbar^2\pi^2}\right)^{3/2} L^3$$

$$\longrightarrow \quad \rho = \frac{N}{V} = \frac{N}{L^3} = \frac{\pi}{3}\left(\frac{2mE_F}{\hbar^2\pi^2}\right)^{3/2}$$

or

$$E_F = \frac{\pi^2\hbar^2}{2m}\left(\frac{3}{\pi}\rho\right)^{2/3}$$

Fermi gas model

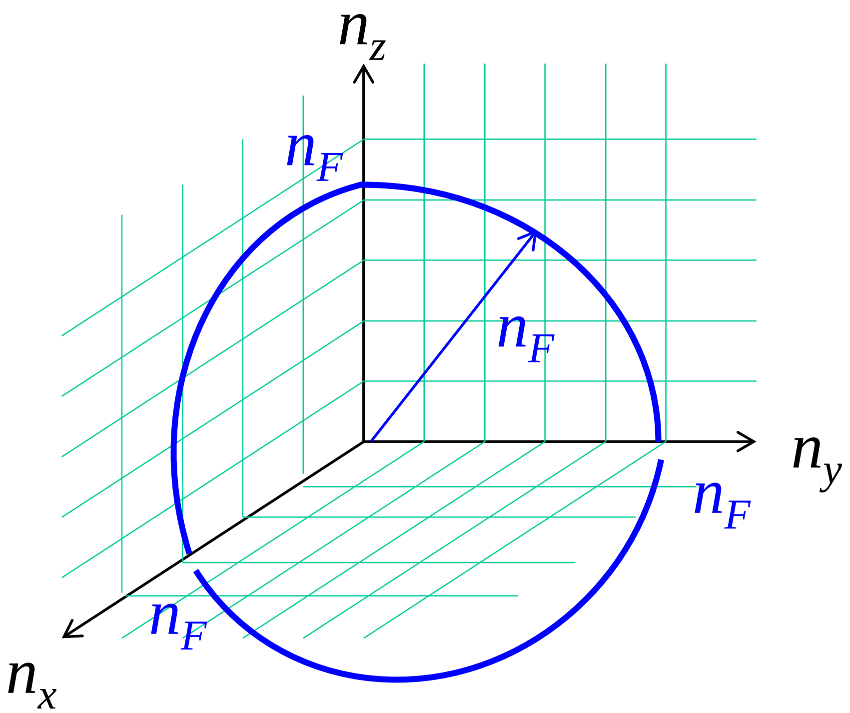
total energy

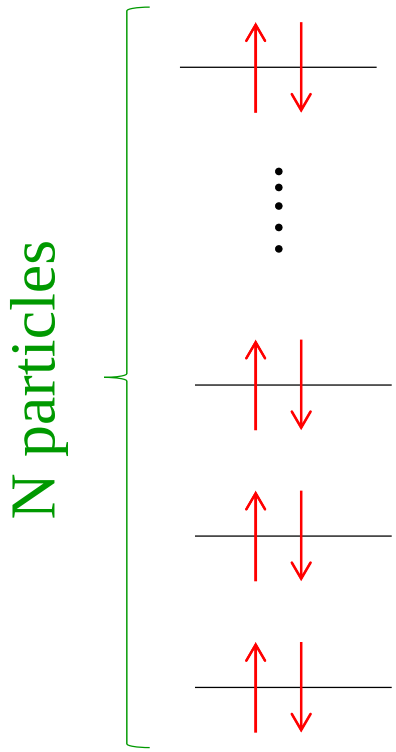
$$
\begin{aligned}
E_{\text{tot}} &= 2 \times \frac{1}{8} \int_{|\boldsymbol{n}| \le n_F} E_{n_x n_y n_z} \, d^3 n \\
&= 2 \times \frac{1}{8} \int_{|\boldsymbol{n}| \le n_F} \frac{\pi^2 \hbar^2}{2mL^2} (n_x^2 + n_y^2 + n_z^2) \, d^3 n \\
&= \cdots = \frac{\hbar^2 \pi^3}{10 m L^2} n_F^5
\end{aligned}
$$

$$
N = 2 \times \frac{1}{8} \times \frac{4}{3} \pi n_F^3
$$

$$
\longrightarrow \quad \boxed{E_{\text{tot}} = \frac{3}{5} N \cdot \frac{\hbar^2}{2m} \left(3\pi^2 \frac{N}{V} \right)^{2/3} = \frac{3}{5} N E_F}
$$

Fermi gas model





$$N = 2 \times \frac{1}{8} \times \frac{4}{3}\pi n_F^3$$

$$\begin{aligned} E_{\mathrm{tot}} &= 2 \times \frac{1}{8}\int_{|\boldsymbol{n}|\leq n_F} E_{n_x n_y n_z}\, d^3 n \\ &= \cdots = \frac{3}{5}N \cdot \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{2/3} \end{aligned}$$