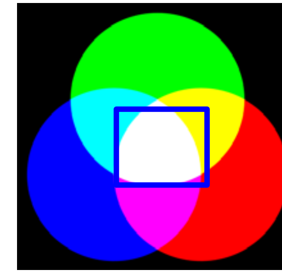
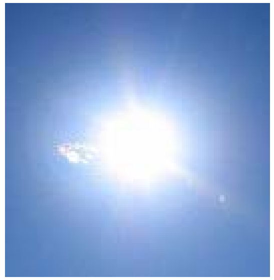
Nuclear Reactions

Shape, interaction, and excitation structures of nuclei ⟵ scattering expt.

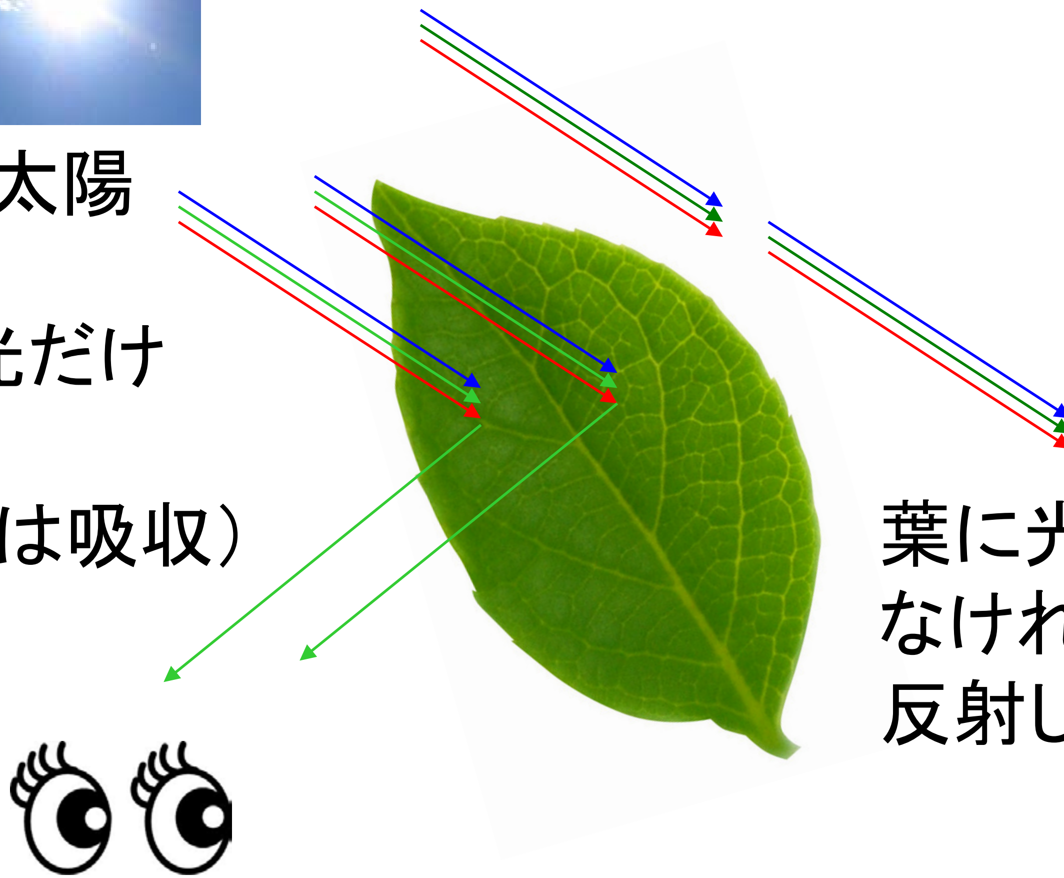
cf. Experiment by Rutherford (α scatt.)

そもそも、ものが見えるとはどういうことか？





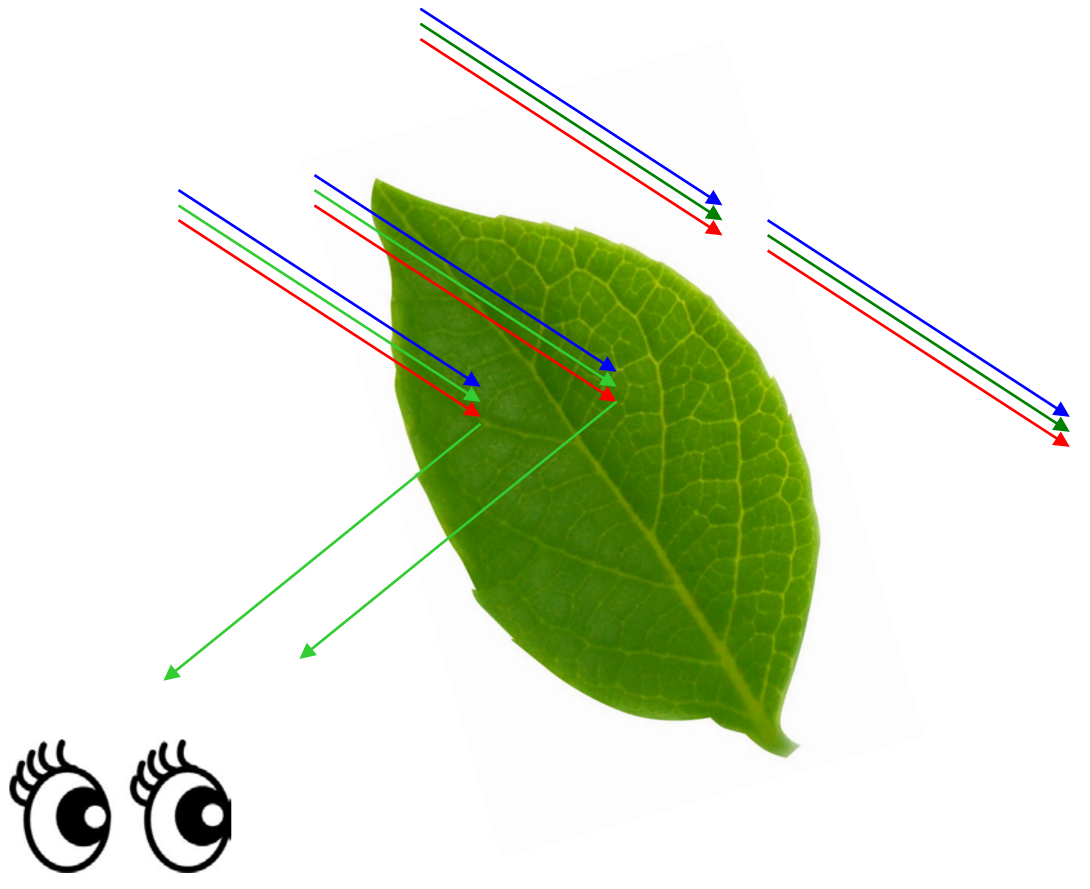
太陽



緑色の光だけ
が反射
（他の色は吸収）

葉に光が当たら
なければ緑は
反射しない
⇩
葉の形

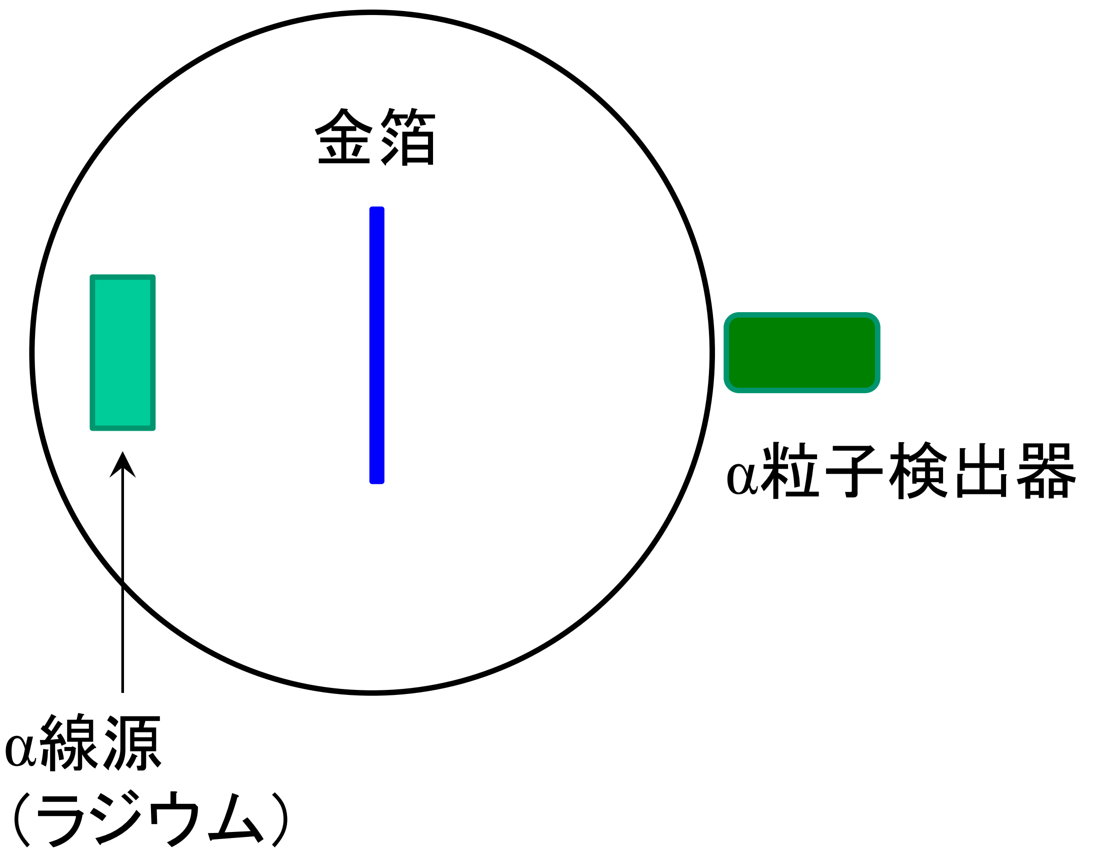
そもそも、ものが見えるとはどういうことか？



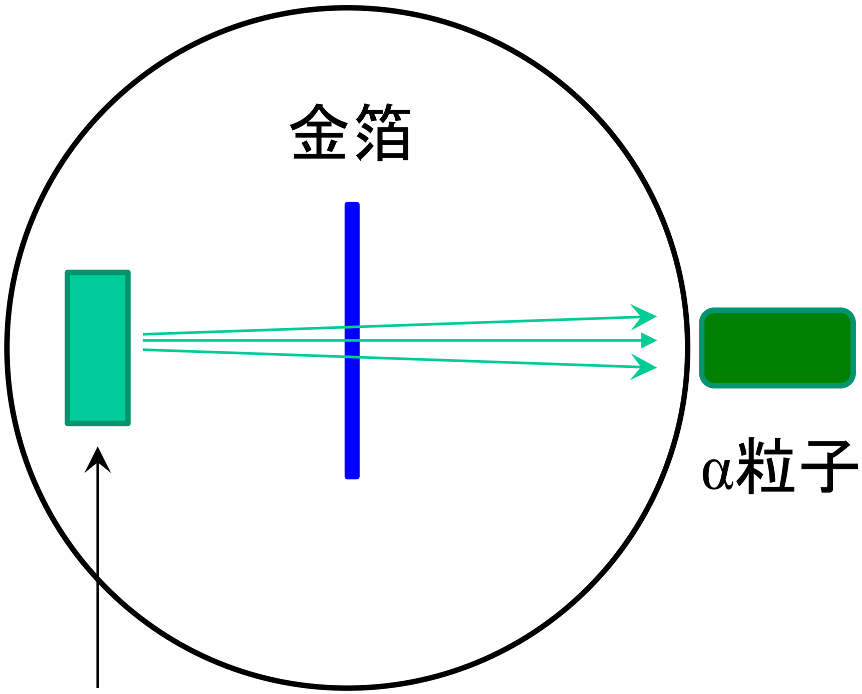
原子核のようなミクロなものの大きさを測るのも基本的には同じ

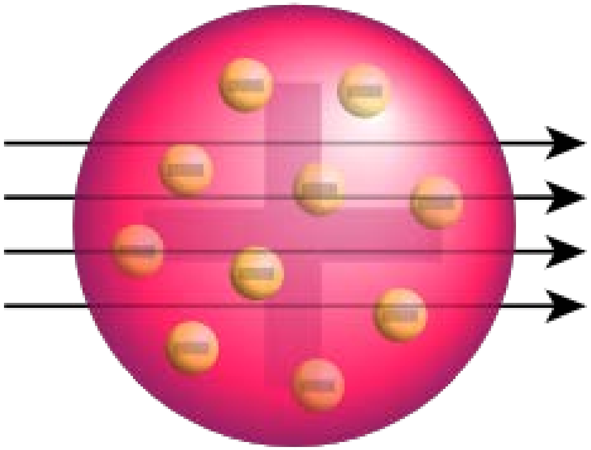
何かをぶつけて、どのように散乱されるか観測する

ラザフォード散乱（ラザフォード、ガイガー、マースデン：1909年）



ラザフォード散乱（ラザフォード、ガイガー、マースデン：1909年）





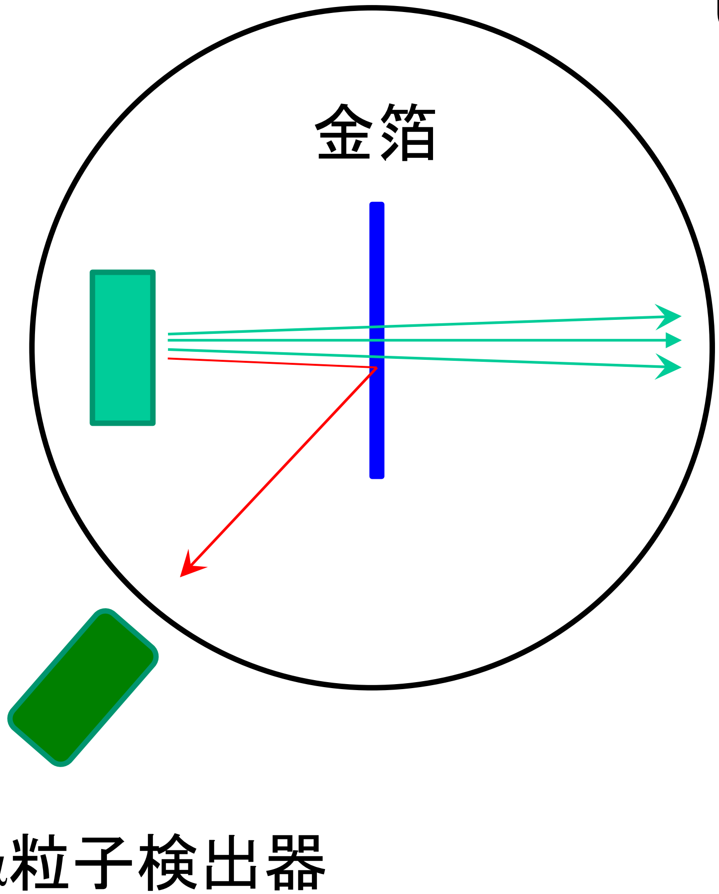
J.J.トンプソンのブドウパン模型を検証したい

⇩

散乱の角度は高々 0.01 度

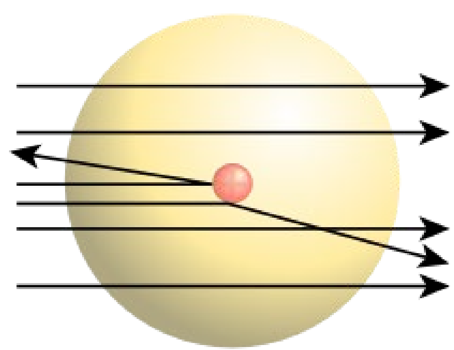
観測：たいていのα粒子はほとんど曲げられずに検出器に入る　→ ブドウパン模型は正しそうだ（?）

ラザフォード散乱 （ラザフォード、ガイガー、マースデン：1909年）



試しに検出器を後方角度に置いて見た
（ブドウパン模型が正しければ、何も観測しないはず）

8千個に1個の割合で後方に跳ね返ってくるα粒子を観測
（驚愕の事実）



→ 原子核の大きさは約 $2\text{x}10^{-14}$ m 以下

STAFF AND RESEARCH STUDENTS OF MANCHESTER UNIVERSITY PHYSICS DEPARTMENT, 1910

W. Eccles S. Kinoshita R. Rossi W. Kay G. N. Antonoff E. Marsden W. C. Lantsberry
F. W. Whaley H. C. Greenwood W. Wilson W. Borodowsky Miss M. White E. J. Evans H. Geiger T. Tuomikoski
S. Russ H. Stansfield H. Bateman Prof. Schuster Prof. Rutherford R. Beattie J. N. Pring W. Makower
R. E. Slade W. A. Harwood

STA S OF MANCHESTER UNIVERSITY PHYSICS
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Nuclear Reactions

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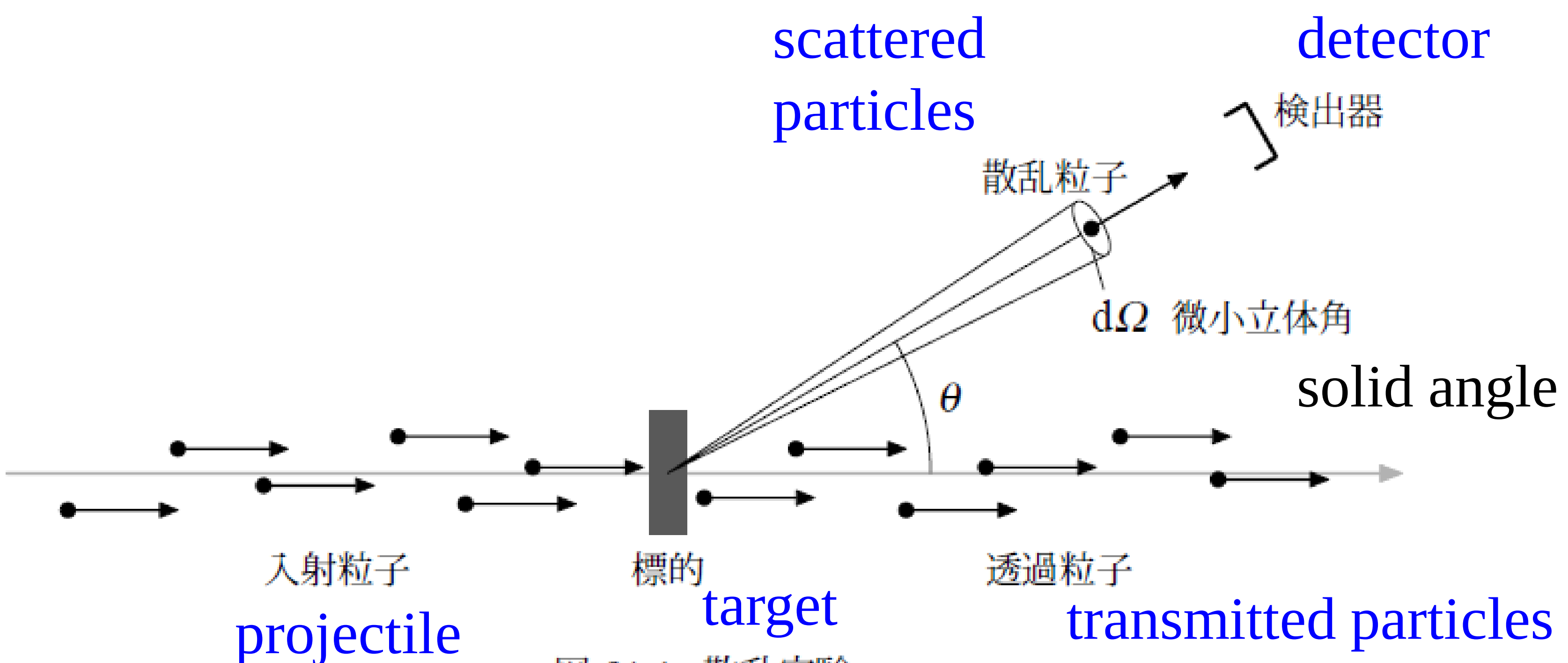
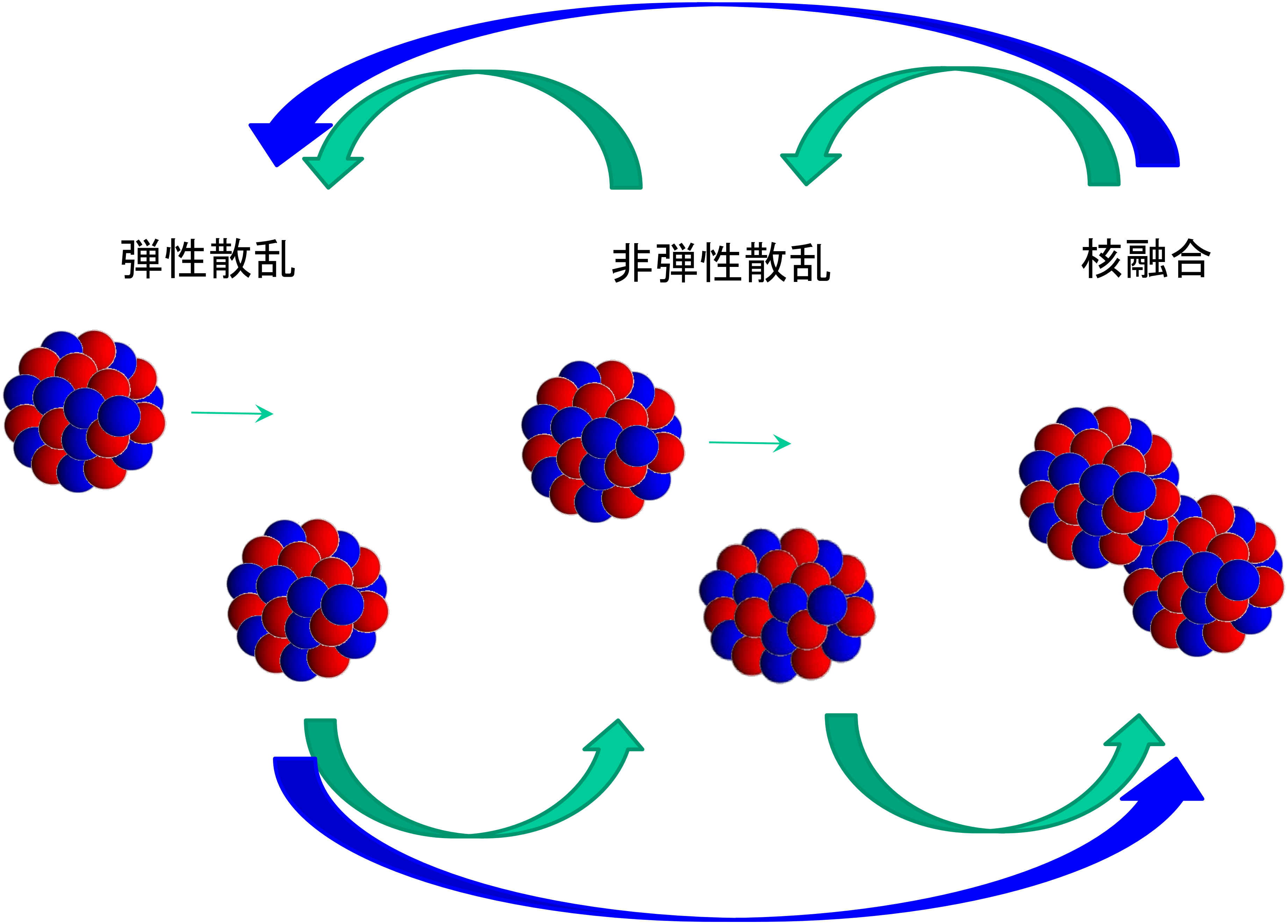


図 21.1: 散乱実験

http://www.th.phys.titech.ac.jp/~muto/lectures/QMII11/QMII11_chap21.pdf

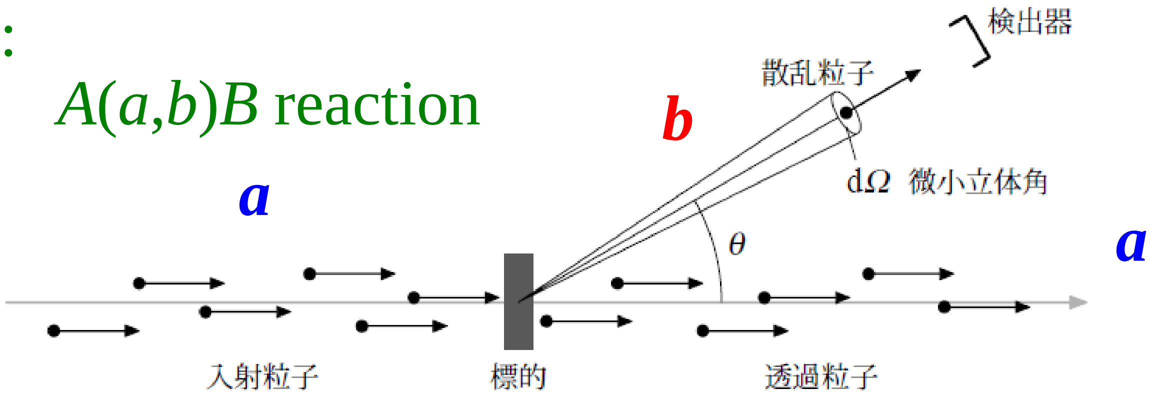
K. Muto (TIT)

量子多体系のダイナミックス（原子核反応）



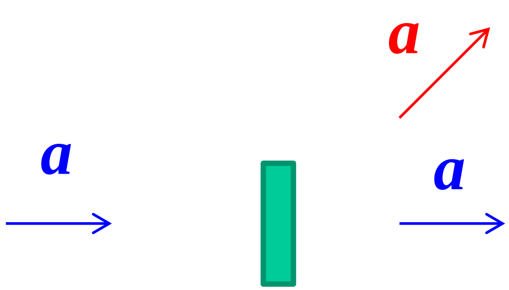
notation:

A(*a*,*b*)*B* reaction



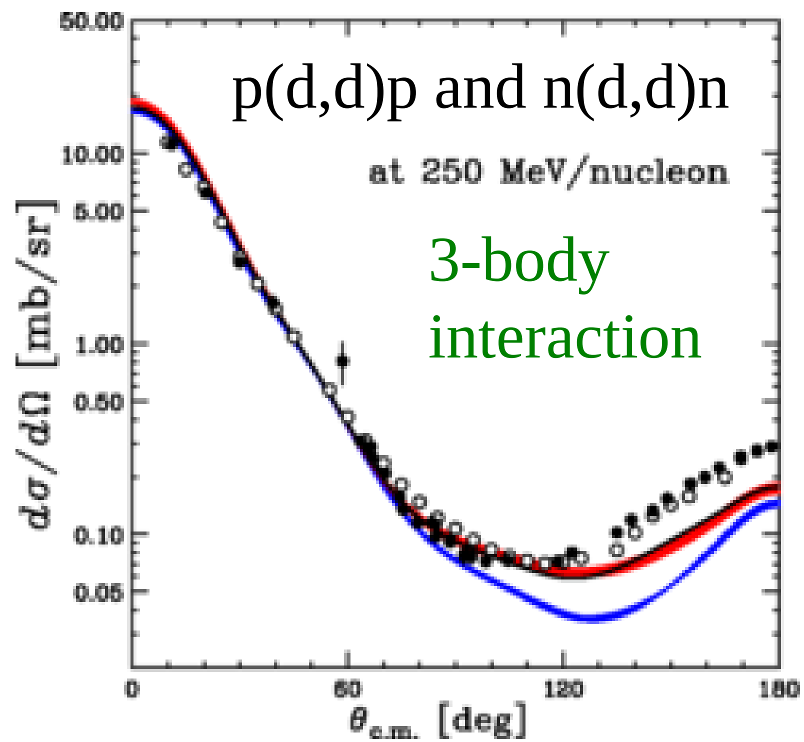
$A \rightarrow B$

✓elastic scattering



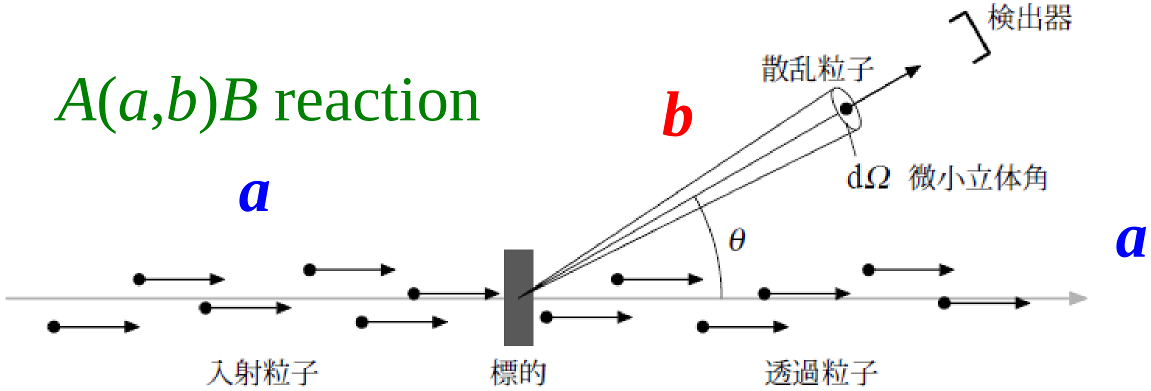
$A_{gs} \rightarrow A_{gs}$

fundamental interaction between *a* and *A*



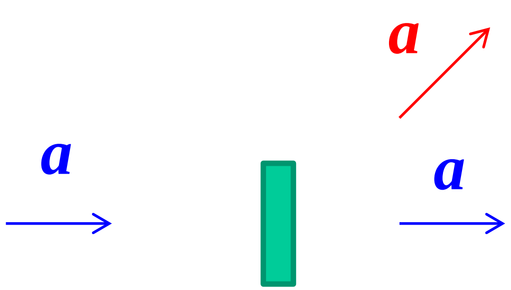
K. Sekiguchi et al., PRC89('14)064007

$A(a,b)B$ reaction



$A \rightarrow B$

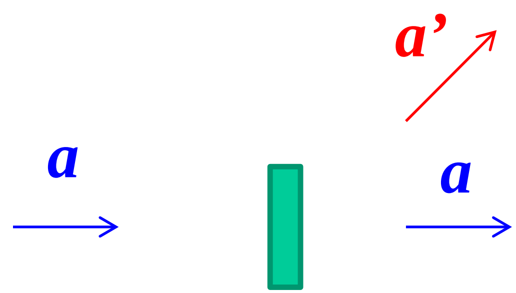
✓elastic scattering



$A_{gs} \rightarrow A_{gs}$

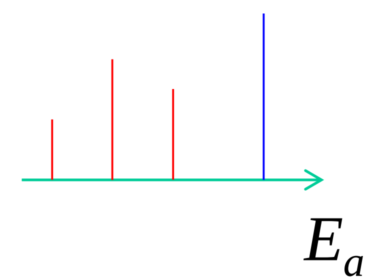
fundamental interaction between a and A

✓inelastic scattering

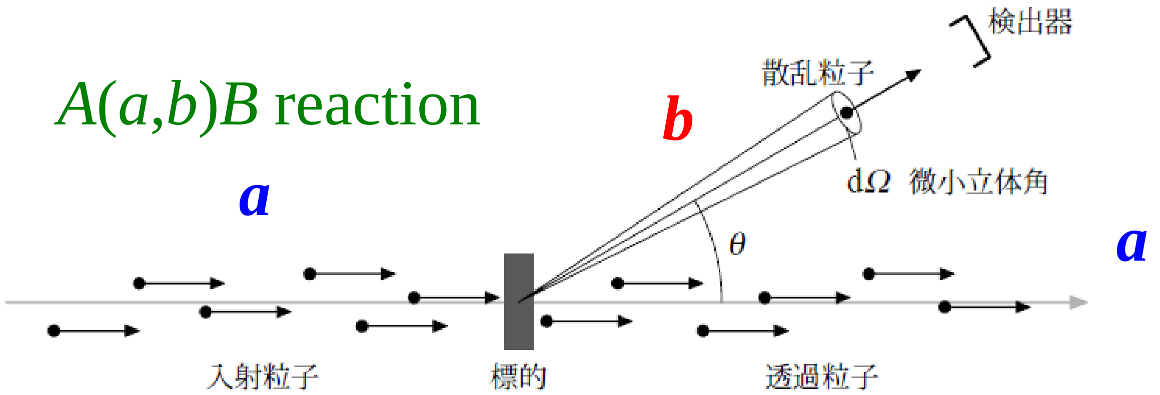


$A_{gs} \rightarrow A^*$

excitation spectrum of a nucleus A



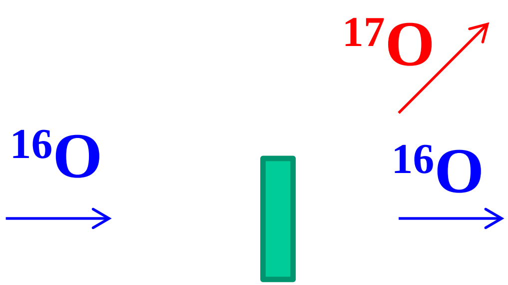
$A(a,b)B$ reaction



$A \rightarrow B$

transfer reactions

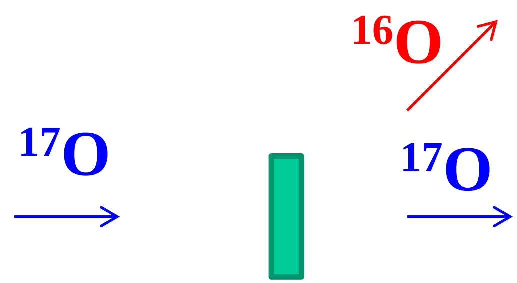
✓transfer reaction (pick-up reaction)



$^{208}Pb_{gs} \rightarrow {}^{207}Pb$

level schem of ^{207}Pb

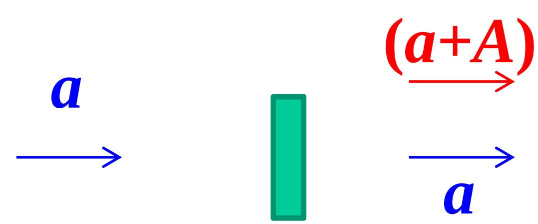
✓transfer reaction (stripping reaction)



$^{208}Pb_{gs} \rightarrow {}^{209}Pb$

level schem of ^{209}Pb

✓fusion reaction

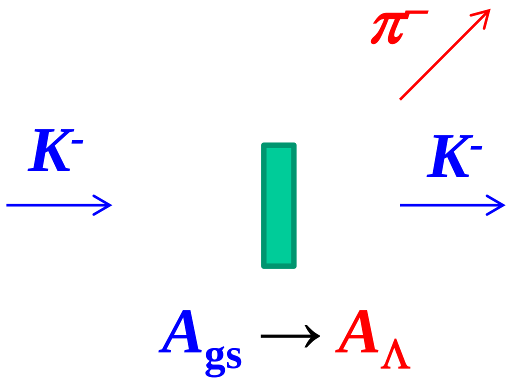


$A_{gs} \rightarrow X$

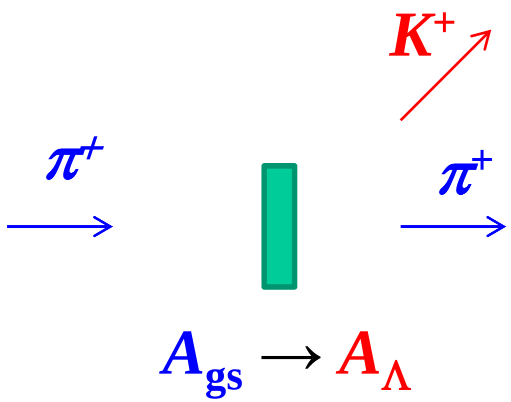
- interaction between *a* and *A*
- structure of *a* and *A*

hypernucleus production reactions

✓(K^-,π^-) reaction

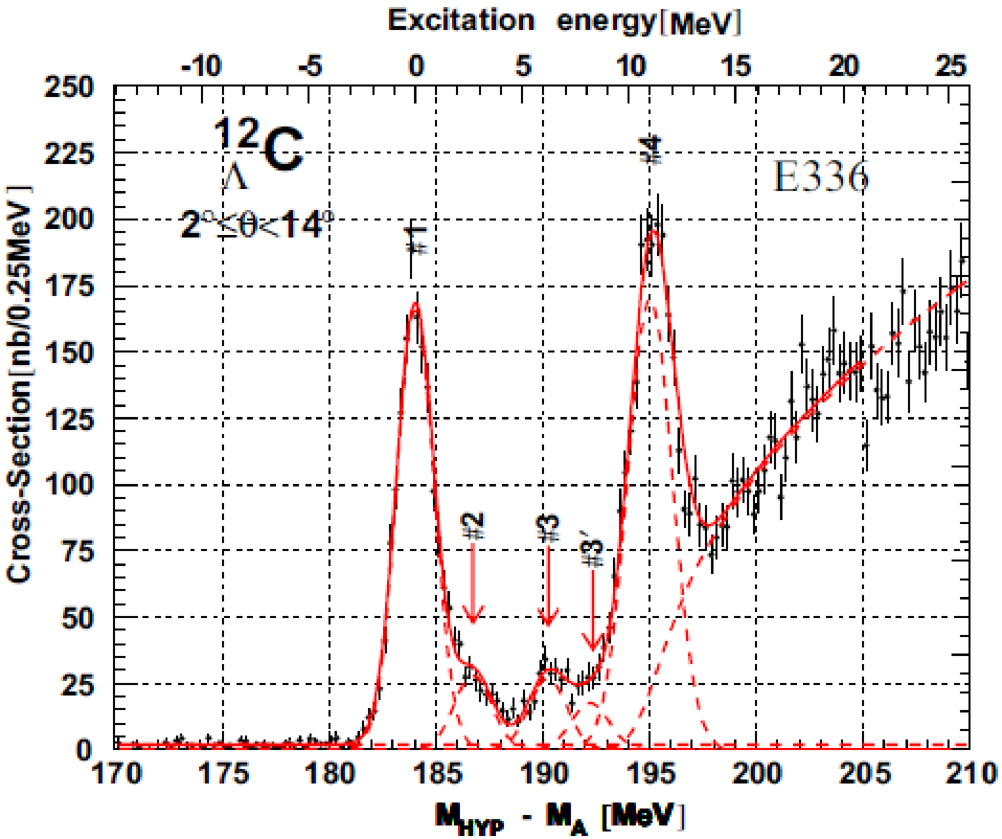


✓(π^+,K^+) reaction



excitation spectrum
of a hypernucleus A_Λ

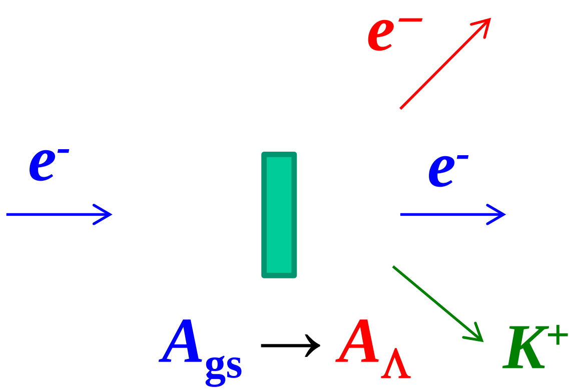
$^{12}C\ (\pi^+,K^+)\ ^{12}{}_{\Lambda}C$ reaction



O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 ('06)564

"reaction spectroscopy"

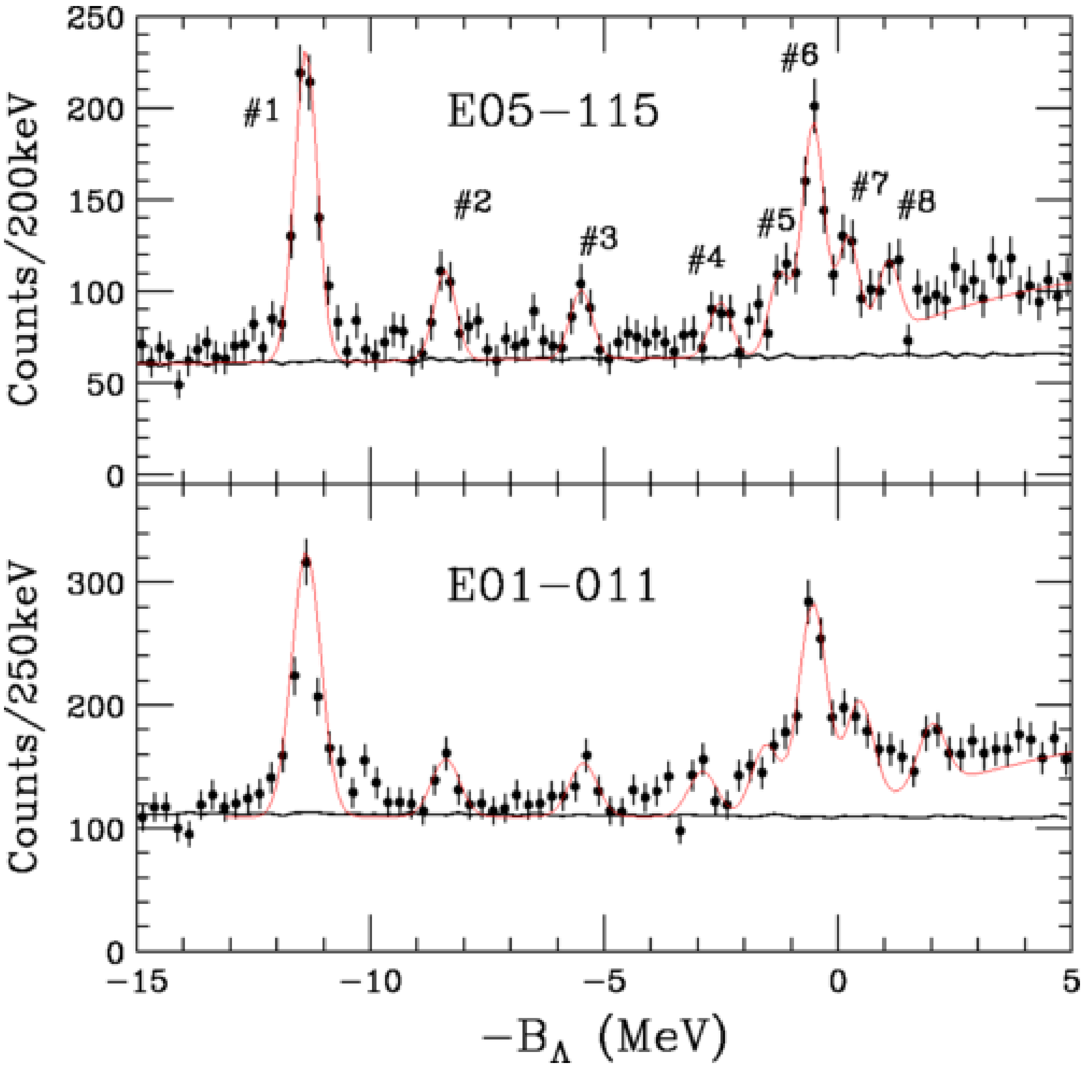
✓(e,e'K^+) reaction



S.N. Nakamura et al.,
PRL110('13)012502

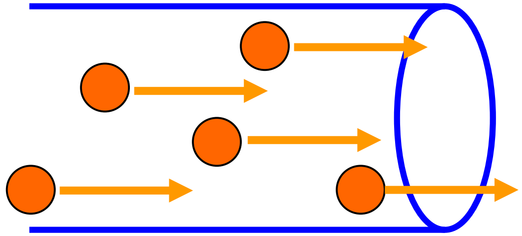
T. Gogami,
Ph.D. Thesis (Tohoku U.)
2014

$^{12}C(e,e'K^+)\,^{12}_{\Lambda}B$



L. Tang et al., PRC90('14)034320

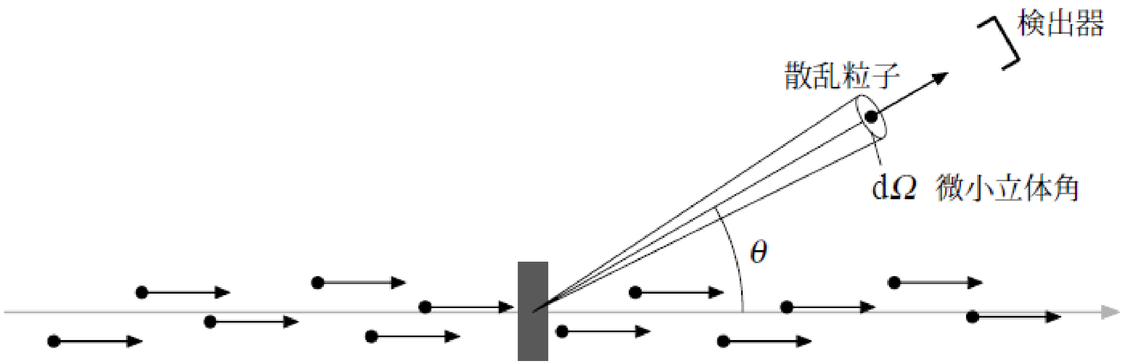
Cross sections



incident beam

flux = the number of particles
crossing unit area
per unit time

$$j = \rho_P \cdot v$$

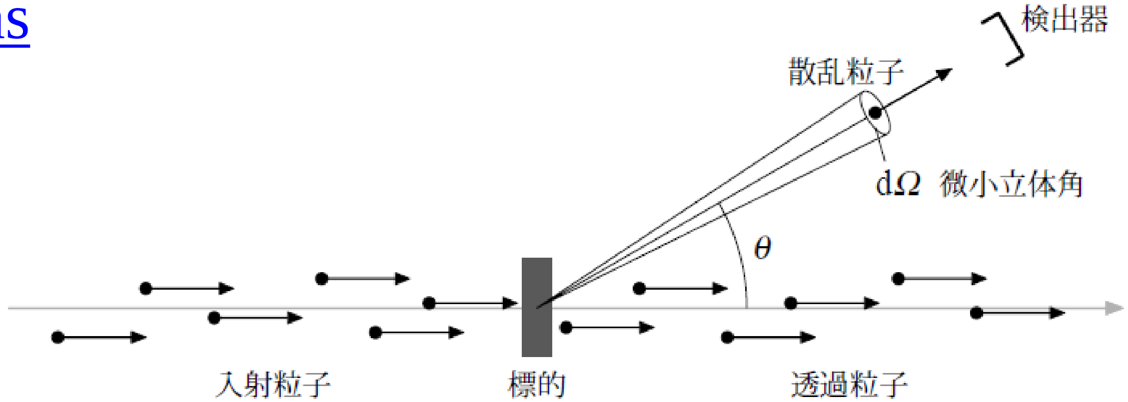


event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

$$R = N_T \cdot \sigma \cdot j$$

cross section

Cross sections



event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

$$\longrightarrow \quad R = N_T \cdot \sigma \cdot j$$

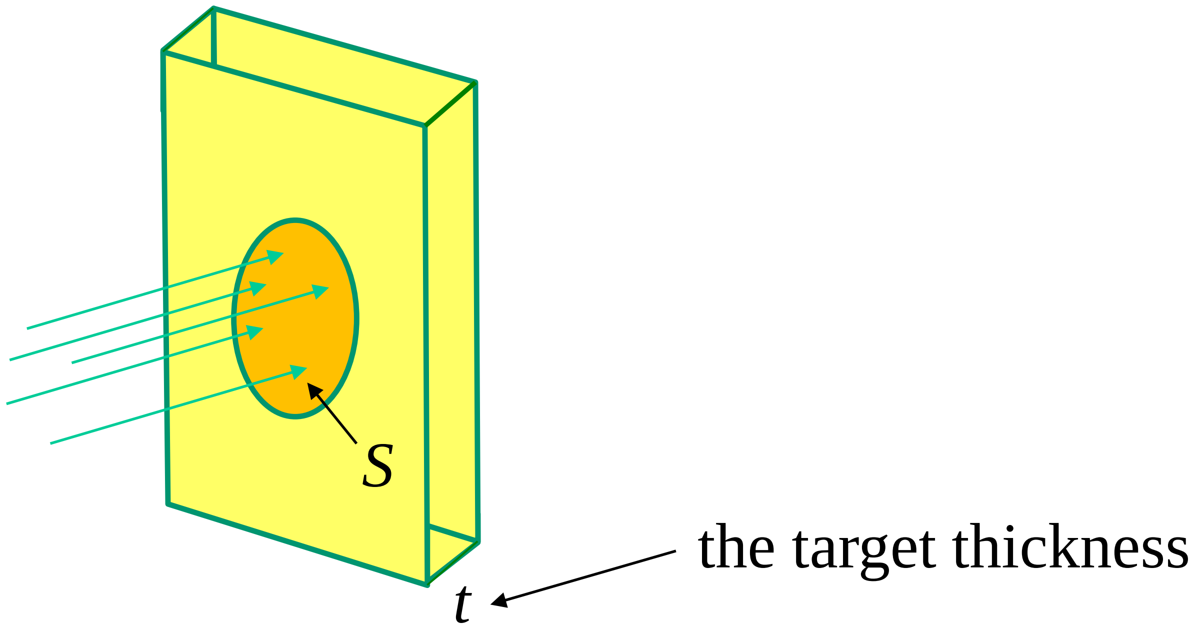
cross section

differential cross sections (angular distribution)

$$dR(\theta, \phi) = N_T \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega, \qquad \sigma = \int d\Omega \frac{d\sigma}{d\Omega}$$

units: 1 barn = 10^{-24} cm^2 = 100 fm^2 (1 mb = 10^{-3} b = 0.1 fm^2)

Cross sections (experiments)



$$dR(\theta,\phi) = N_{\mathsf{T}} \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega$$

the number of target nucleus: $N_{\mathsf{T}} = S \cdot t \cdot \rho_{\mathsf{T}}$

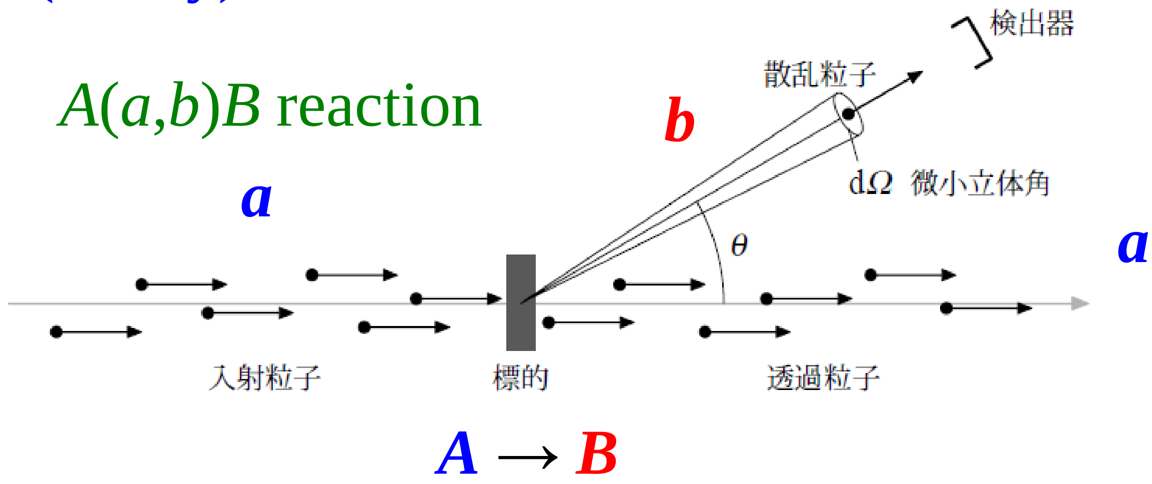
beam intensity: $I = j \cdot S$

$$dR(\theta,\phi) = I \cdot \frac{d\sigma}{d\Omega} \cdot t\rho_{\mathsf{T}} \cdot d\Omega \cdot \epsilon$$

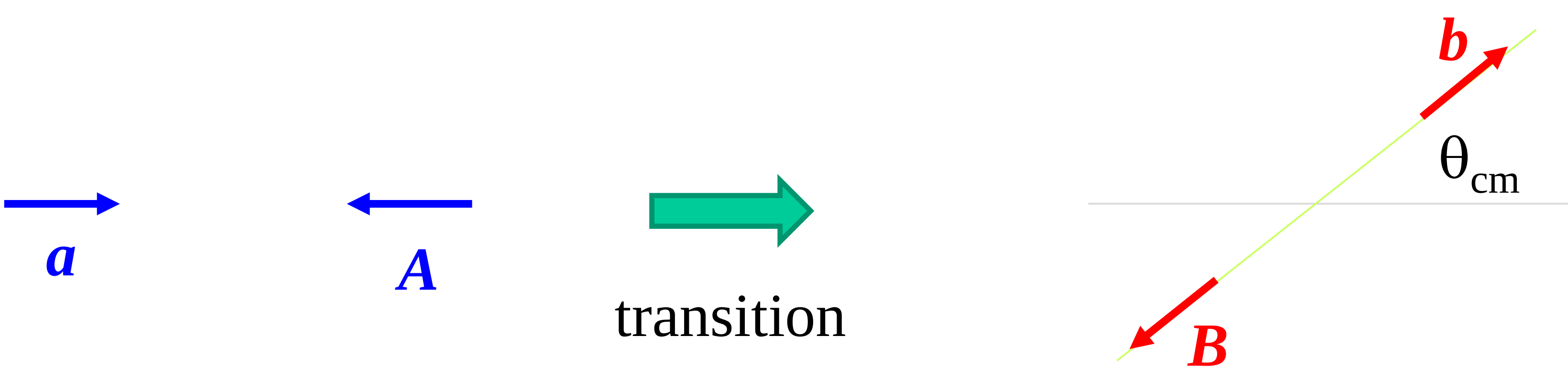
detection efficiency

Cross sections (theory)

$A(a,b)B$ reaction



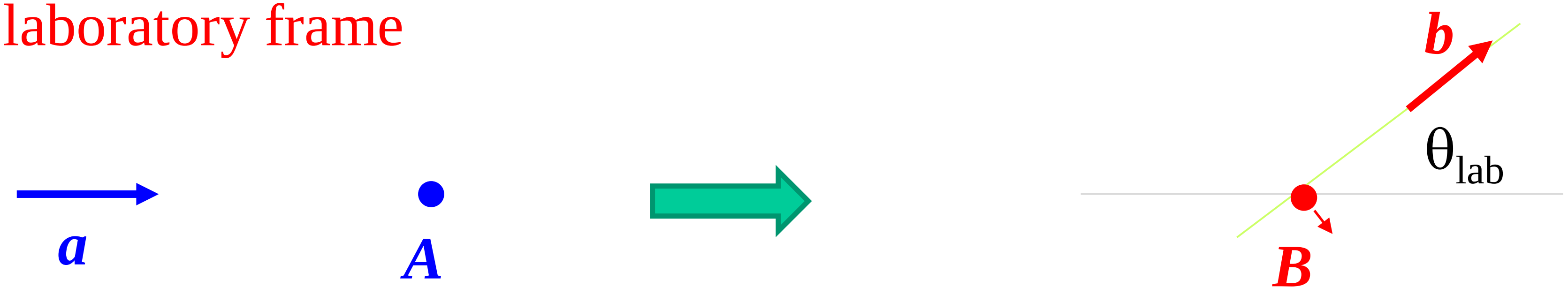
center of mass frame



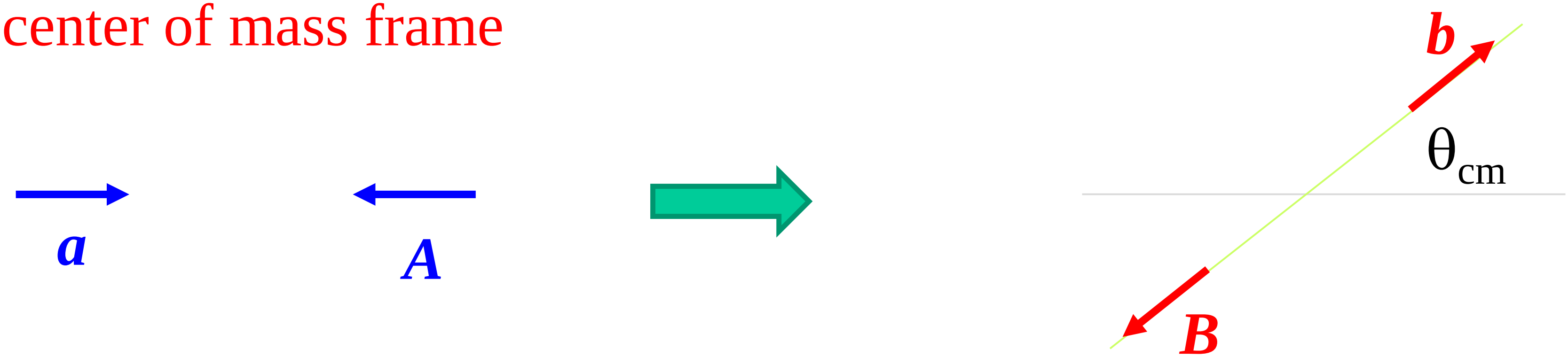
$$\frac{d\sigma}{d\Omega} = \frac{R}{j_{\rm in}}$$

Cross sections

✓ laboratory frame



✓ center of mass frame



□ transformation ⟵ energy and momentum conservations

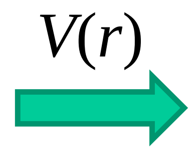
$$\tan\theta_{\rm lab} = \frac{\sin\theta_{\rm cm}}{\gamma + \cos\theta_{\rm cm}}, \qquad d\Omega_{\rm lab} = \frac{|1+\gamma\cos\theta_{\rm cm}|}{(1+\gamma^2+2\gamma\cos\theta_{\rm cm})^{3/2}}\,d\Omega_{\rm cm}$$

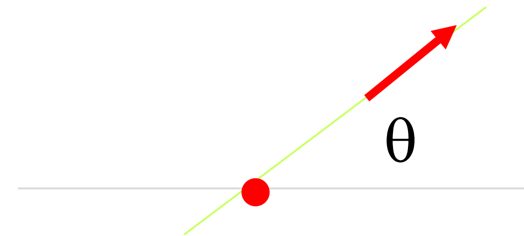
$$E_{\rm cm} = \frac{M_A}{M_a+M_A}E_{\rm lab}, \qquad \gamma = \sqrt{\frac{M_aM_b}{M_AM_B}\frac{E_{\rm cm}}{E_{\rm cm}+Q}}$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$





$$\left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

transition rate for elastic scattering:

$$W_{fi} = \frac{2\pi}{\hbar}\int \frac{d\boldsymbol{p}_f}{(2\pi\hbar)^3}\left|\langle\psi_f|V|\psi_i\rangle\right|^2\delta(E_f - E_i)$$

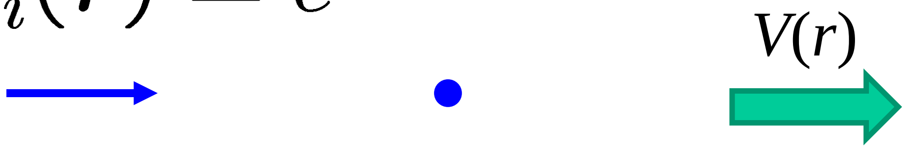
$$= \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega\left|\widetilde{V}(\boldsymbol{q})\right|^2$$

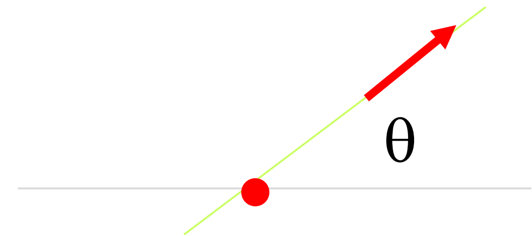
$$\widetilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r}e^{i(\boldsymbol{p}_i - \boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar}V(r) \equiv \int d\boldsymbol{r}e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(r)$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$





$$W_{fi} = \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega \left|\tilde{V}(\boldsymbol{q})\right|^2$$

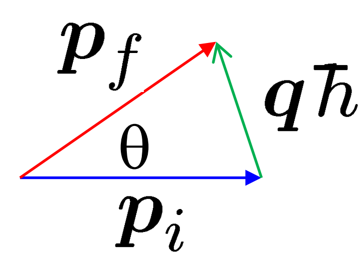
momentum transfer

$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r} e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar} V(r) \equiv \int d\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} V(r)$$

incident flux: $j_{\text{inc}} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\text{inc}}} = \int d\Omega \boxed{\frac{\mu^2}{4\pi^2\hbar^4}\left|\tilde{V}(\boldsymbol{q})\right|^2}$$

$$= \frac{d\sigma}{d\Omega}$$



$$q\hbar = 2p_i \sin\frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho_{\rm ch}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

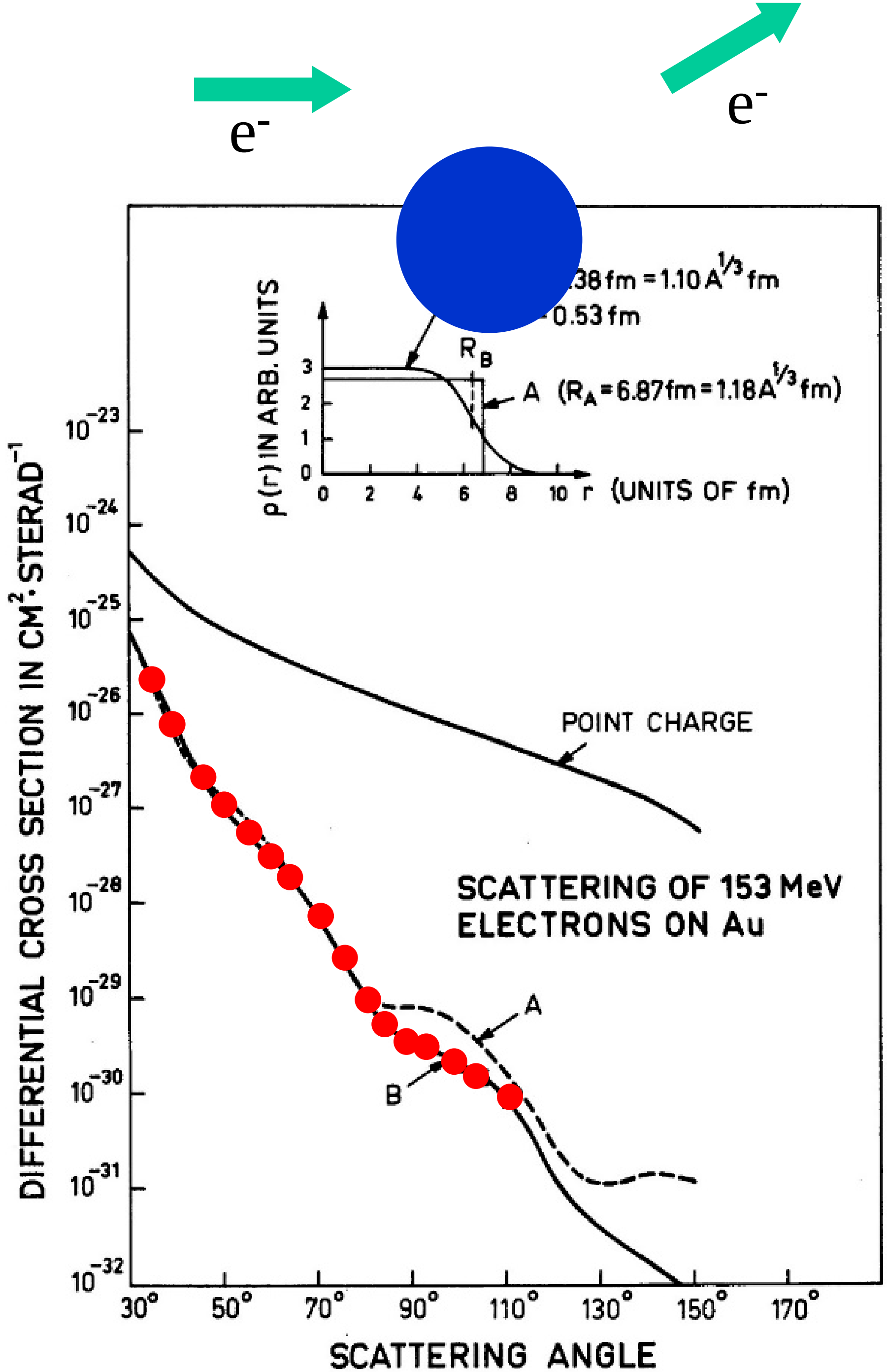
$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2} |F(\boldsymbol{q})|^2 = \left(\frac{d\sigma_{\rm Ruth}}{d\Omega}\right) |F(\boldsymbol{q})|^2$$

Form factor

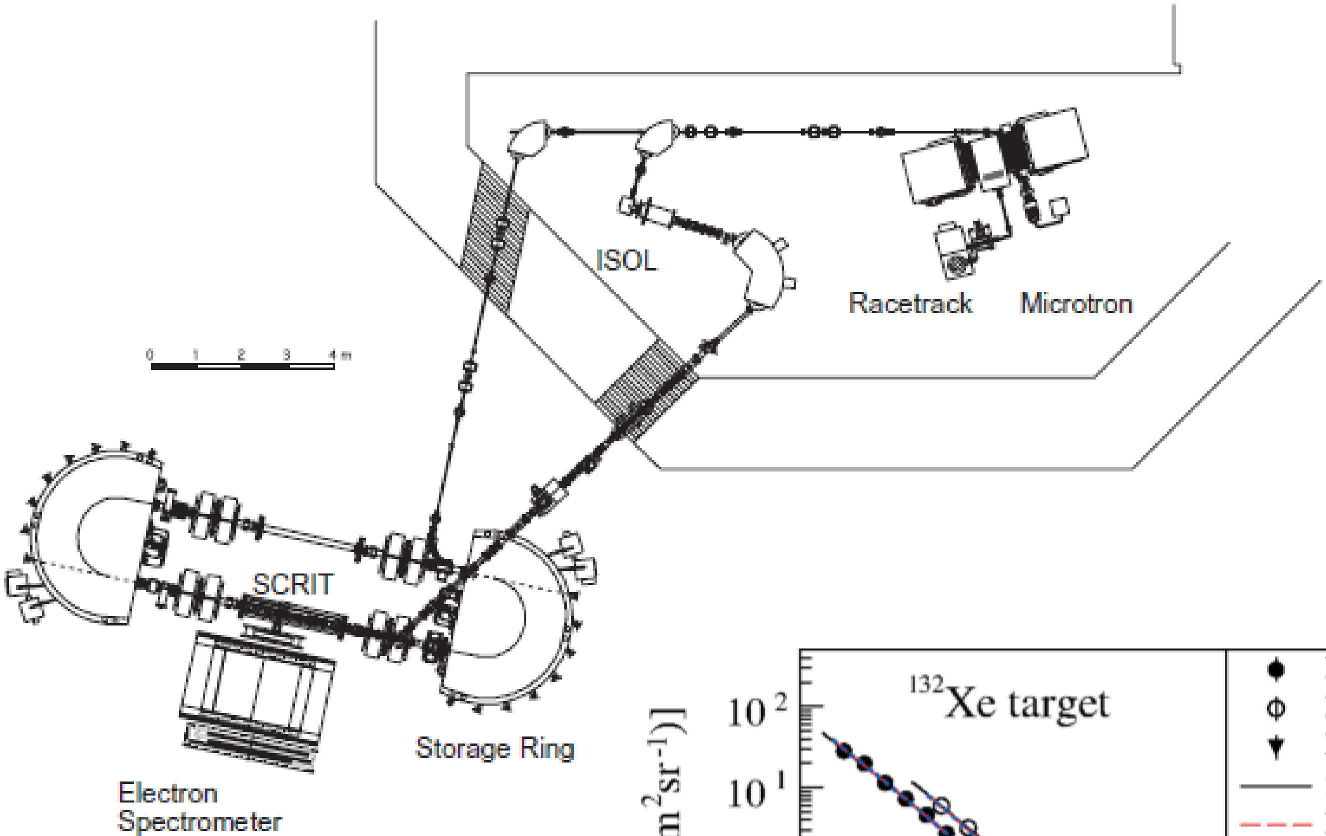
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \rho_{\rm ch}(\boldsymbol{r})\, d\boldsymbol{r}$$

* relativistic correction:

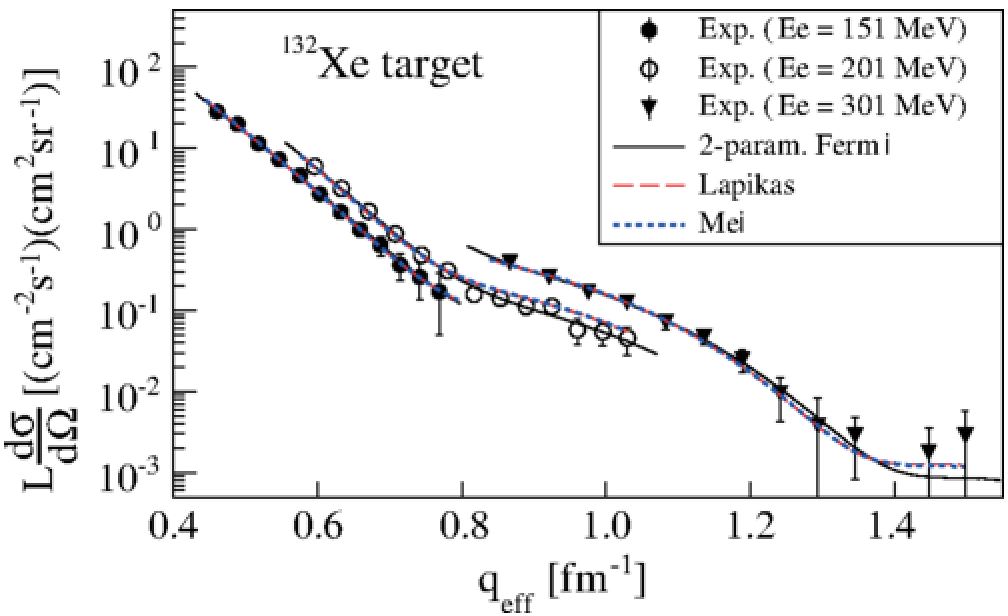
$$\frac{d\sigma_{\rm Ruth}}{d\Omega} \to \frac{d\sigma_{\rm Mott}}{d\Omega} = \frac{d\sigma_{\rm Ruth}}{d\Omega} \cdot \left(1 - \frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right) \sim \frac{d\sigma_{\rm Ruth}}{d\Omega} \cdot \cos^2\frac{\theta}{2} \qquad (v \to c)$$



cf. electron scattering off unstable nuclei (SCRIT)

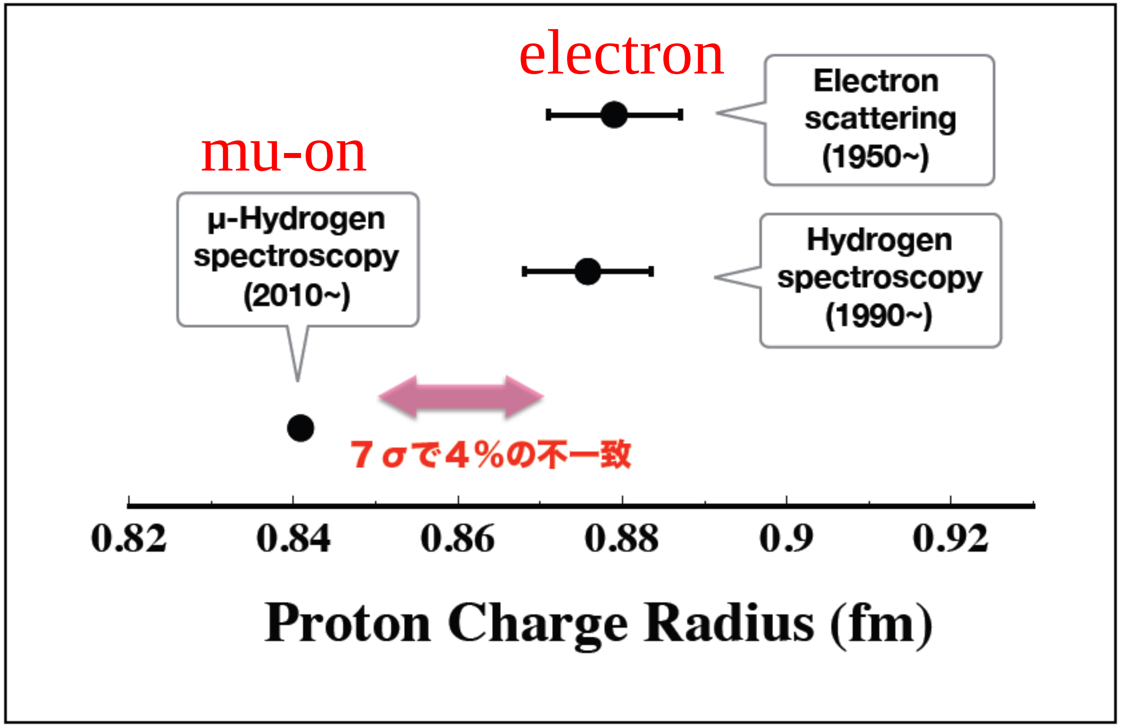


K. Tsukada et al.,
PRL118, 262501 (2017)



proton radius puzzle

$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\, \rho_{\mathrm{ch}}(\boldsymbol{r})\, d\boldsymbol{r}$$
$$\sim \int \left(1 - i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{(qr)^2}{2}\cos^2\theta + \cdots\right) \rho_{\mathrm{ch}}(\boldsymbol{r})\, d\boldsymbol{r}$$
$$\sim Z\left(1 - \frac{q^2}{6}\langle r^2\rangle + \cdots\right)$$



Distorted Wave Born approximation (DWBA)

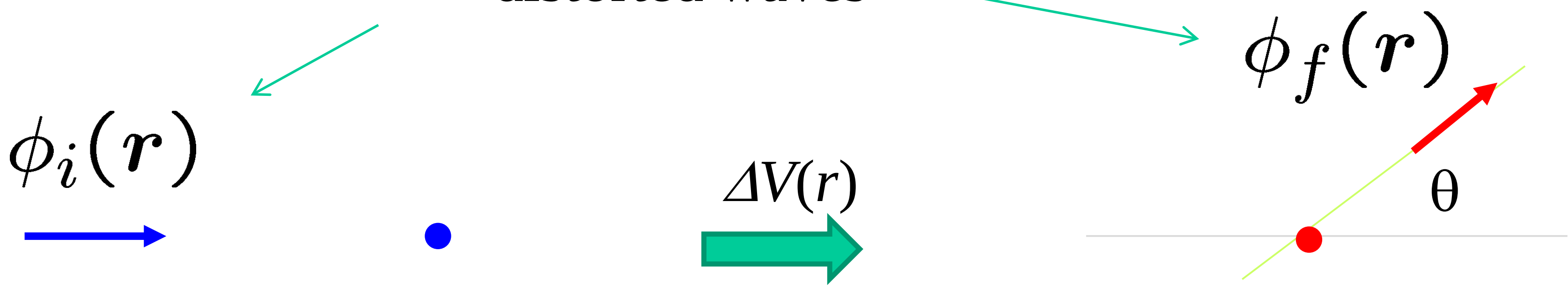
$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

$$\Longrightarrow \left(-\frac{\hbar^2}{2\mu}\nabla^2 + \underline{V_0(r)} + \underline{V(r) - V_0(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

“distorted waves”

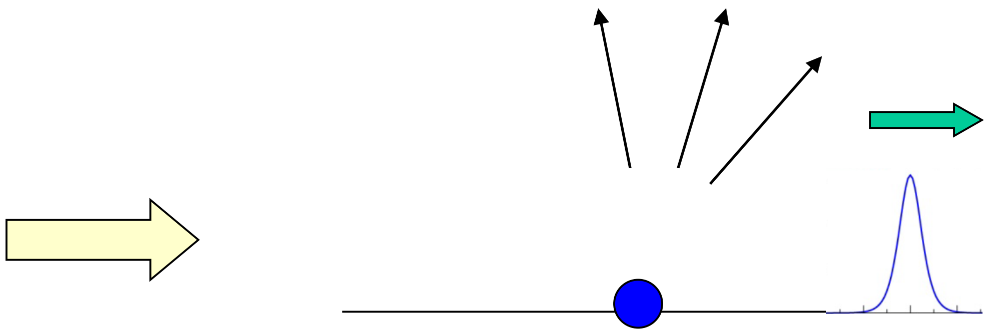


✓inelastic scattering
✓transfer reactions

How to choose $V_0(r)$? : Optical model

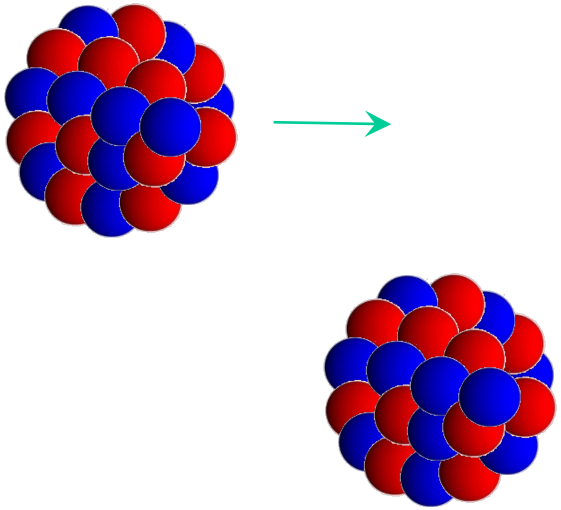
Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)

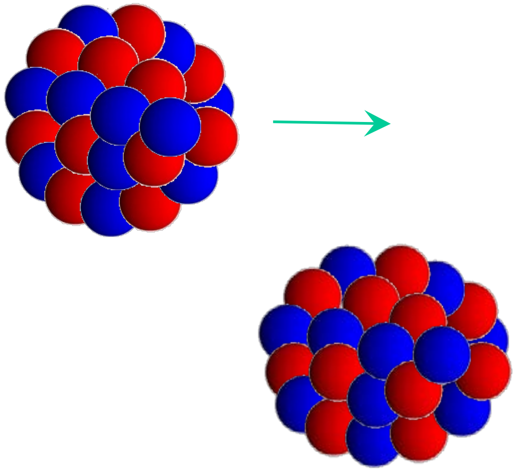


Loss of incident flux (absorption)

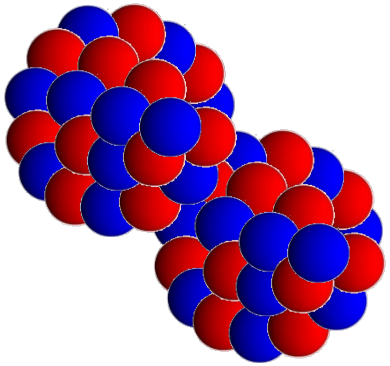
弾性散乱



非弾性散乱



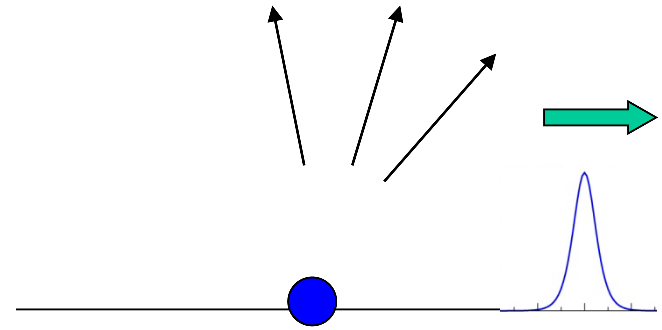
核融合



How to choose $V_0(r)$? : Optical model

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux (absorption)

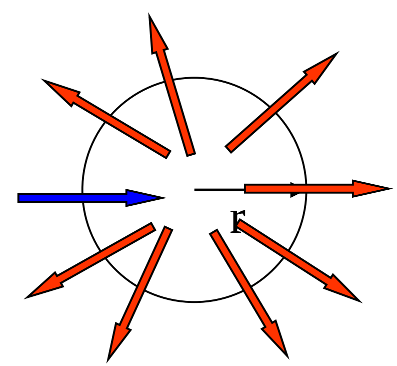
Optical potential

$$V_{\text{opt}}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)$$

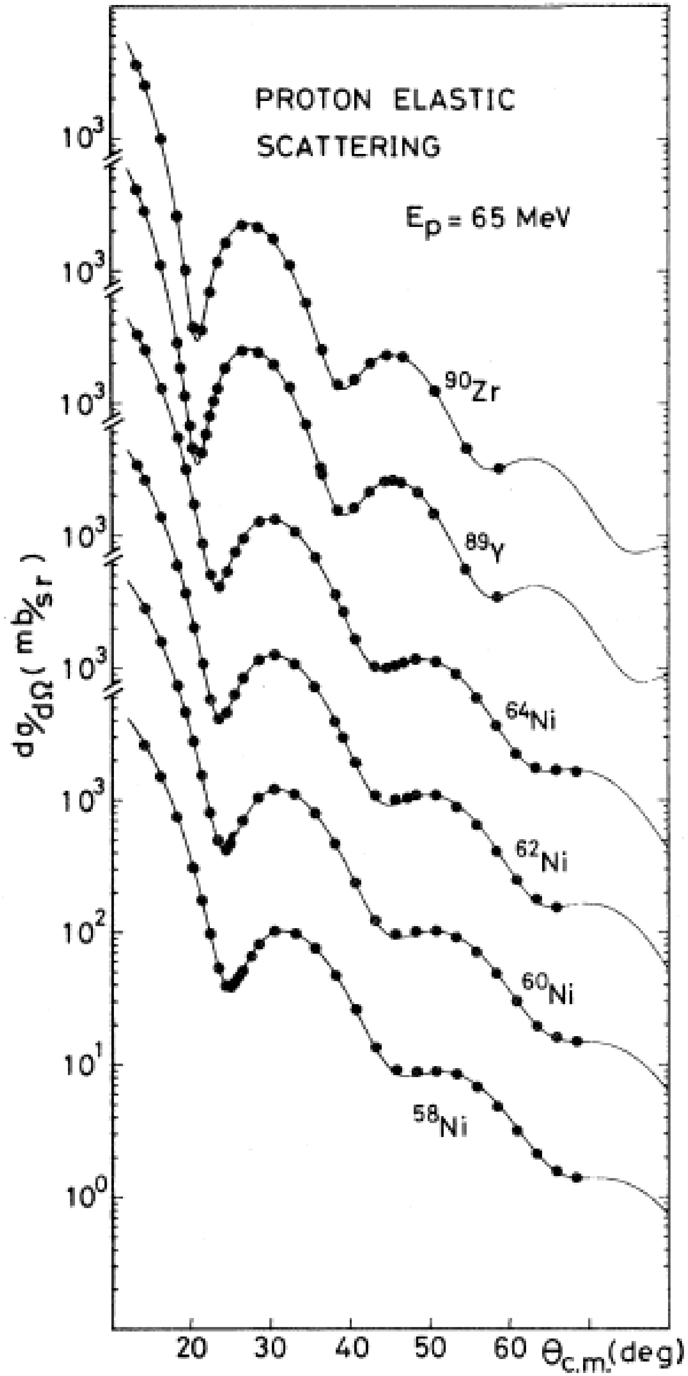
$$\Longrightarrow \quad \nabla \cdot \boldsymbol{j} = \cdots = -\frac{2}{\hbar} W |\psi|^2$$

(note) Gauss's theorem

$$\int_S \boldsymbol{j} \cdot \boldsymbol{n}\, dS = \int_V \nabla \cdot \boldsymbol{j}\, dV$$



$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \frac{Z_P Z_T e^2}{r} + V_{\rm opt}(r) - E\right)\psi(\boldsymbol{r}) = 0$$

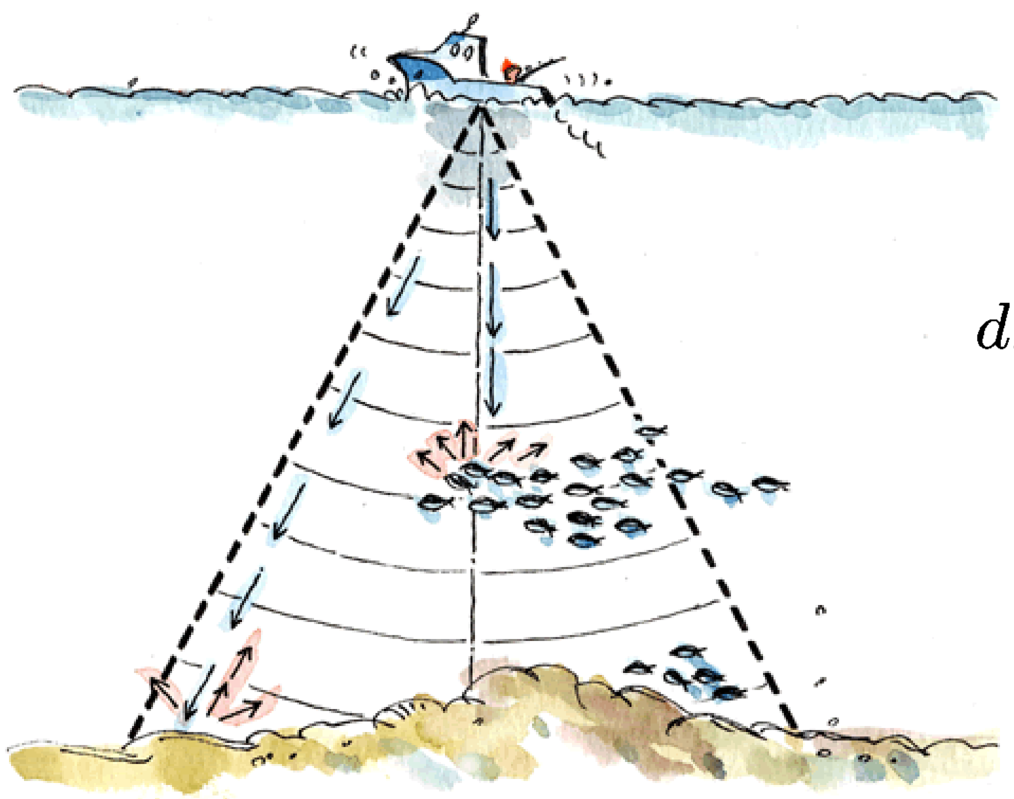


Woods-Saxon + volume &surface imaginary parts

H. Sakaguchi et al.,
PRC26 (1982) 944

Appendix: DWBA in ocean acoustics

Fishfinder 魚群探知機



https://www.furuno.co.jp/technology/about/fishfinder1.html

(backward) scattering of (ultra-)sonic waves due to fish etc.

$$dR(\theta, \phi) = N_{\mathrm{T}} \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega$$

$$\downarrow$$

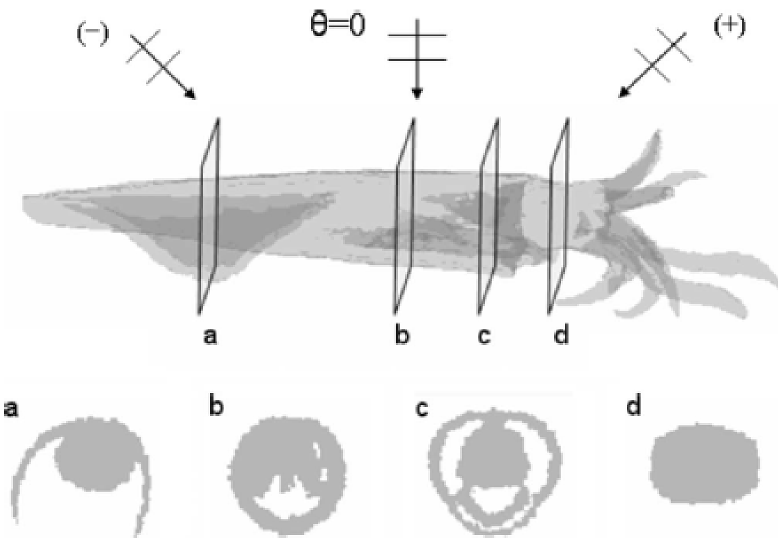
$$N_{\mathrm{T}} = \frac{\frac{dR}{d\Omega}}{j \cdot \frac{d\sigma}{d\Omega}}$$

one can know the number of fish N_{T} if one knows the differential cross sections

Use of the distorted wave Born approximation to predict scattering by inhomogeneous objects: Application to squid

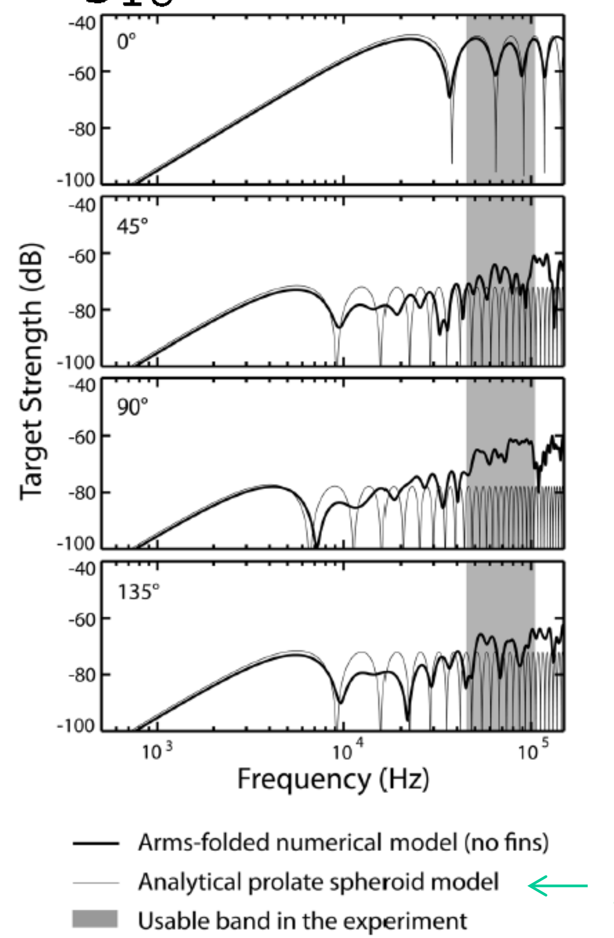
Benjamin A. Jones,[a] Andone C. Lavery, and Timothy K. Stanton
Department of Applied Ocean Physics and Engineering, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts 02543-1053

J. Accoust. Soc. Am. 125 ('09) 73

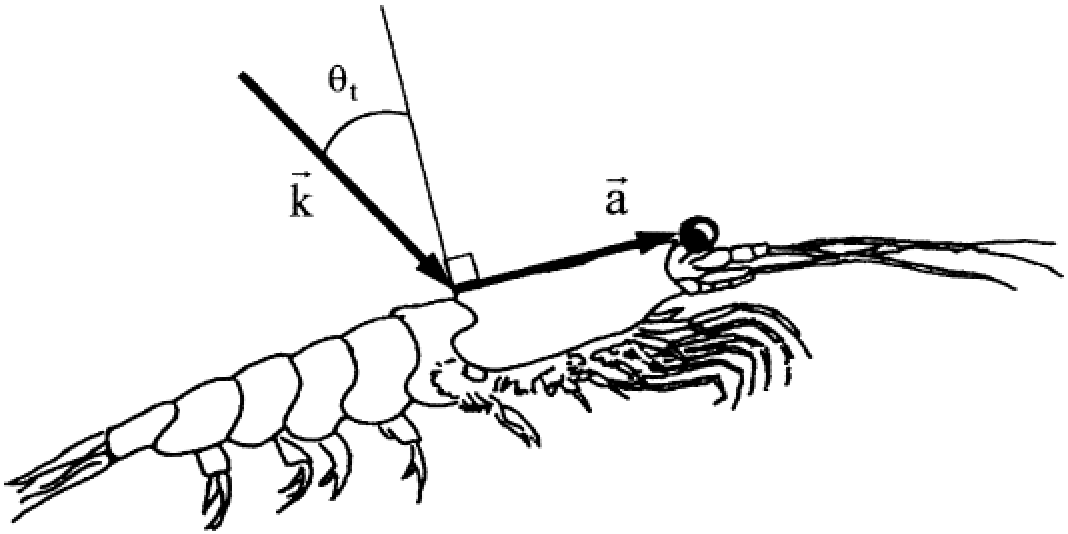


Modeling of squid

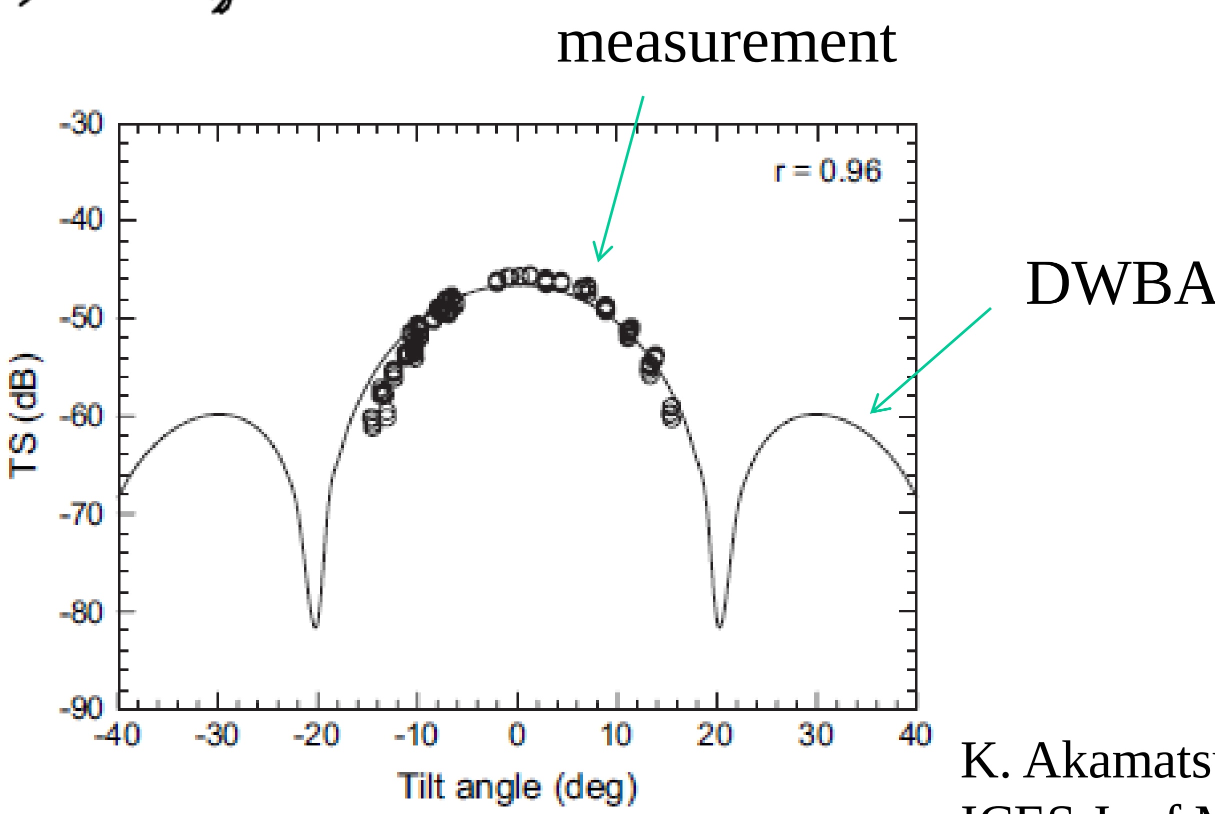
$10 \log_{10} \sigma$



DWBA: local wave number inside a squid



Krill (オキアミ)

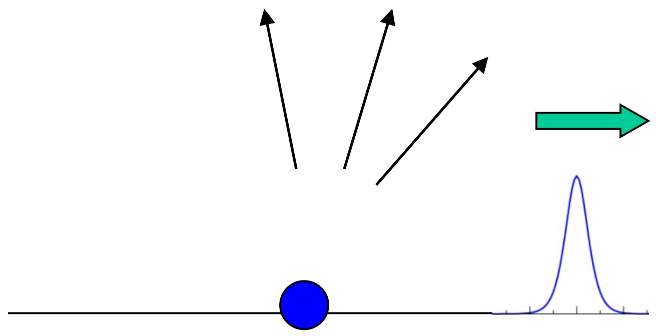


K. Akamatsu and M. Furusawa,
ICES J. of Marine Science 63 (‘06) 36

Absorption cross sections

Reaction processes

- ➢Elastic scatt.
- ➢Inelastic scatt.
- ➢Transfer reaction
- ➢Compound nucleus formation (fusion)



Loss of incident flux (absorption)

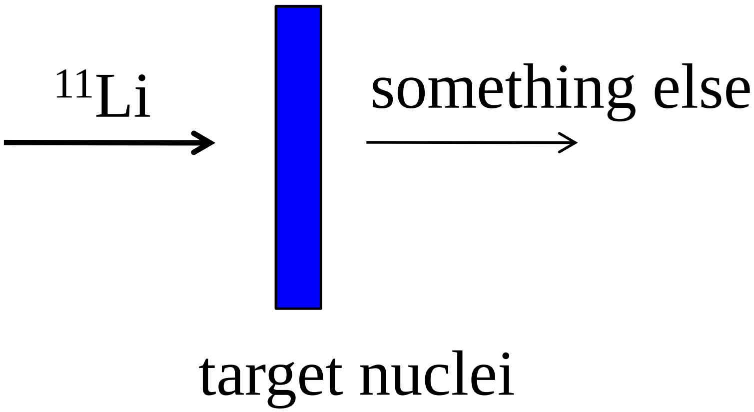
reaction cross sections

total scattering cross section minus elastic cross section

$$\sigma_R = \sigma_{\text{tot}} - \sigma_{\text{el}}$$

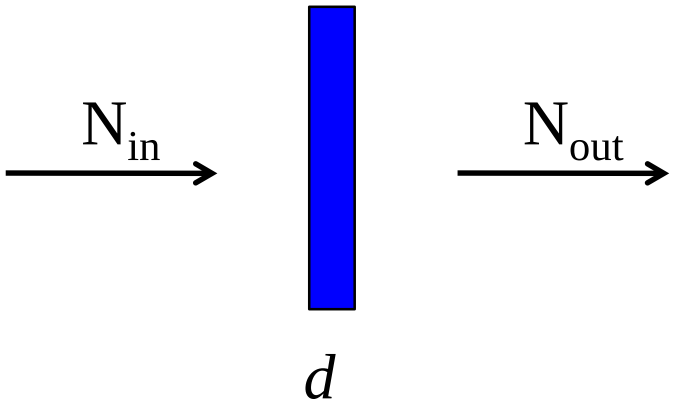
- fusion
- inelastic
- transfer

Interaction cross sections and halo nuclei



interaction cross section σ_I
= cross section for the change of Z a/o N in the incident nucleus

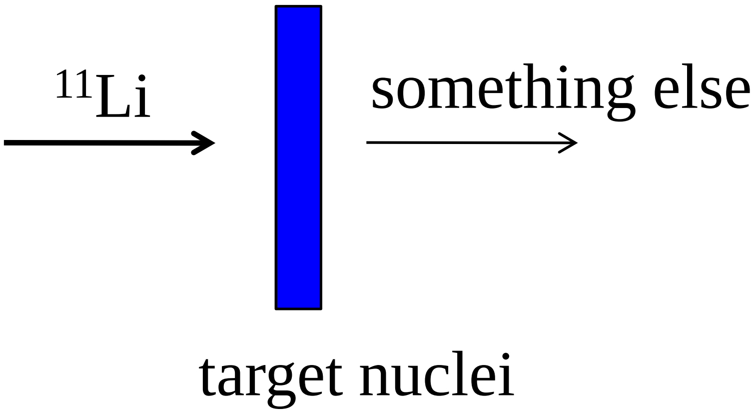
transmission method



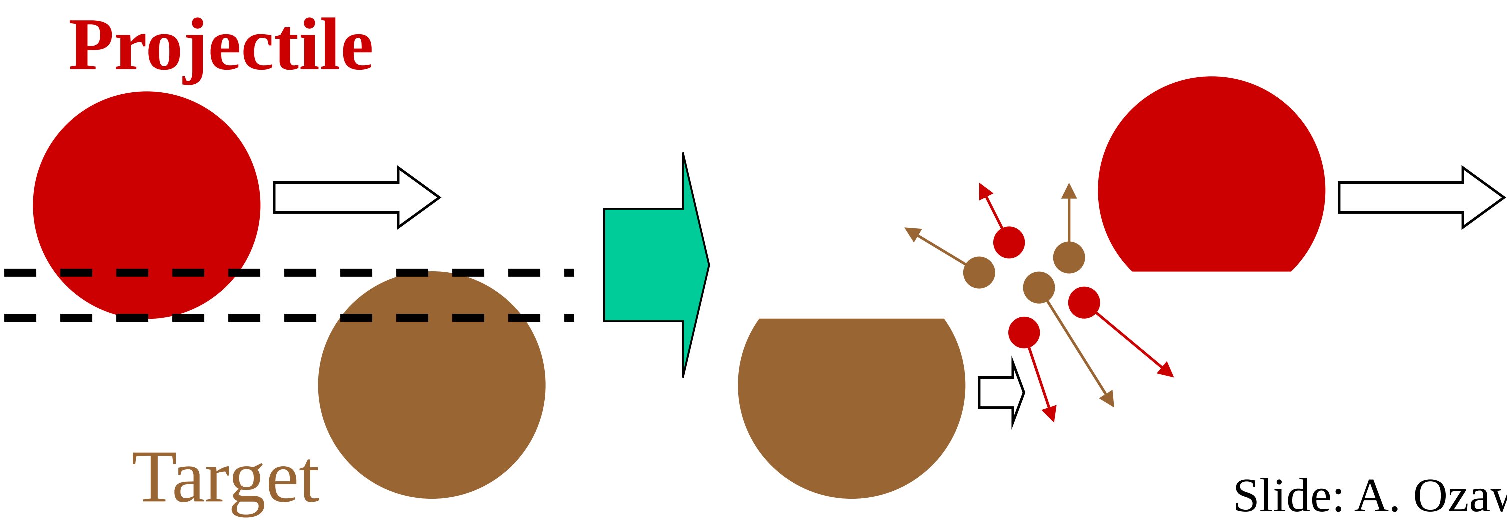
$$\sigma_R = -\frac{1}{t} \ln \left(\frac{N_{\text{out}}}{N_{\text{in}}} \right)$$

$$t = \rho_T \cdot d \cdot \epsilon$$

Interaction cross sections and halo nuclei



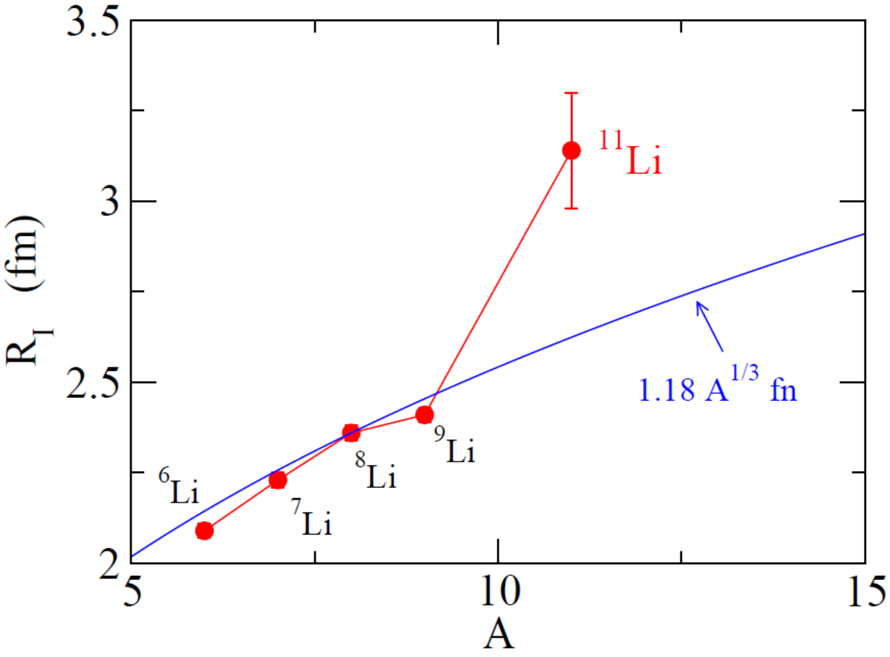
interaction cross section σ_I
= cross section for the change
of Z a/o N in the incident nucleus

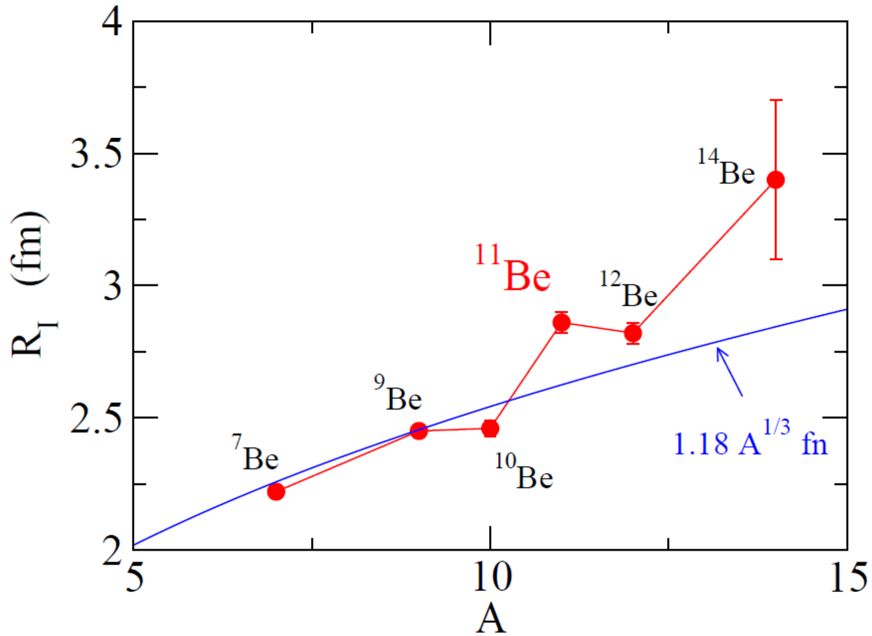


Slide: A. Ozawa

$$\sigma_I \sim \pi[R_I(P) + R_I(T)]^2 \longrightarrow R_I(P)$$

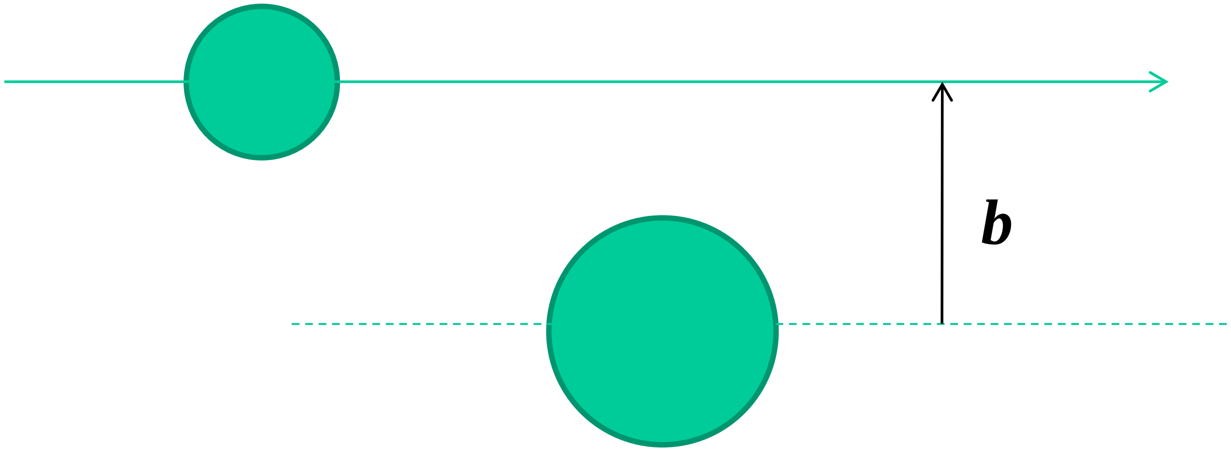
Discovery of **halo nuclei**





I. Tanihata, T. Kobayashi, O. Hashimoto et al., PRL55('85)2676; PLB206('88)592

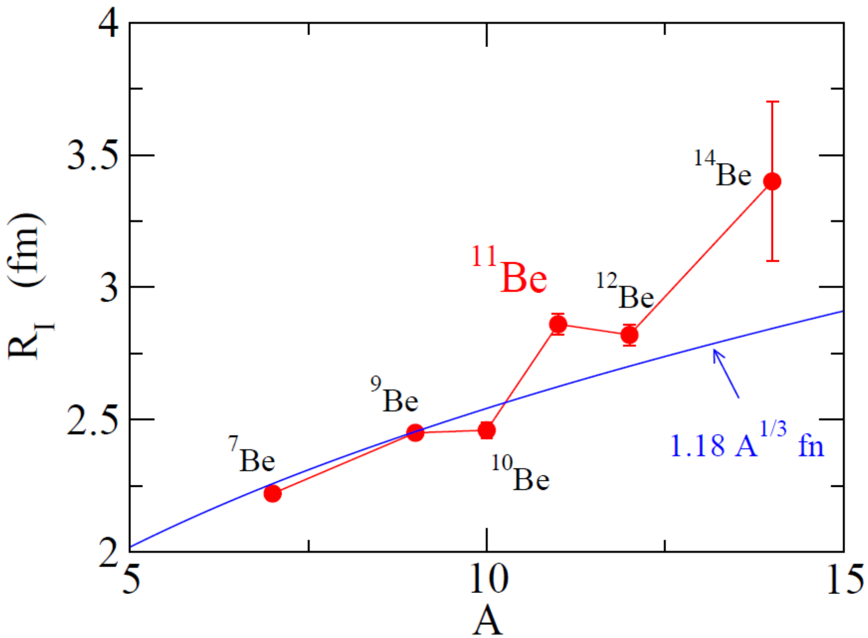
Reaction cross sections



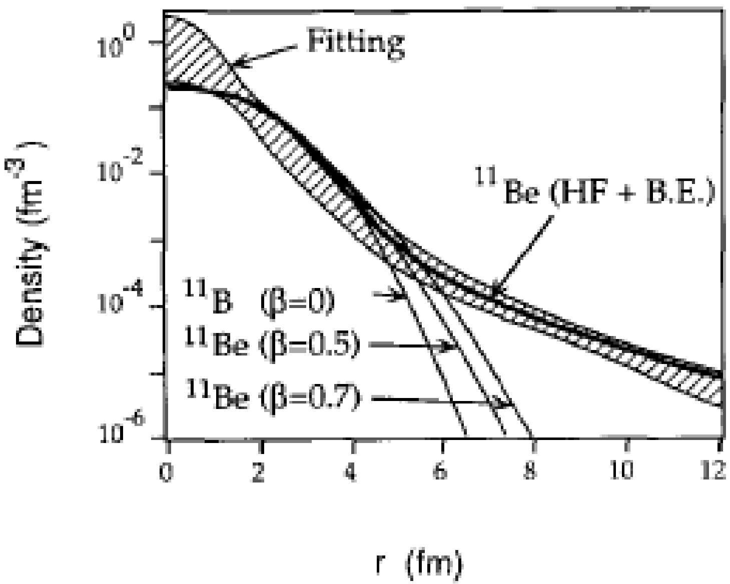
Glauber theory (optical limit approximation : OLA)

$$\sigma_R \sim 2\pi \int_0^\infty b\,db \left[1 - \exp\left(-\sigma_{NN} \int d^2s\,\rho_P^{(z)}(\boldsymbol{s})\rho_T^{(z)}(\boldsymbol{s}-\boldsymbol{b})\right)\right]$$

- straight-line trajectory (high energy scattering)
- adiabatic approximation
- simplified treatment for multiple scattering: $(1-x)^N \to e^{-Nx}$



Density distribution which explains the experimental σ_R

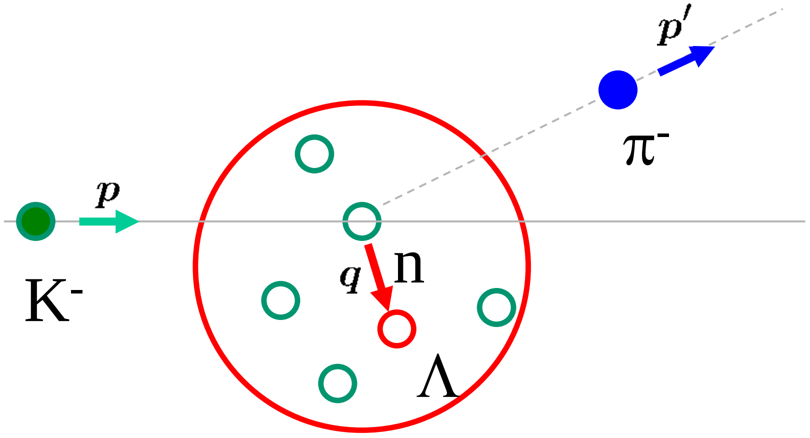


M. Fukuda et al., PLB268('91)339

$$\sigma_R \sim 2\pi \int_0^\infty bdb \left[1 - \exp\left(-\sigma_{NN} \int d^2 s \rho_P^{(z)}(\boldsymbol{s}) \rho_T^{(z)}(\boldsymbol{s}-\boldsymbol{b})\right)\right]$$

Impulse approximation

example: ${}^{A}Z(K^{-},\pi^{-}){}^{A}_{\Lambda}Z$ reaction



✓ high energy
✓ single scattering approximation

$$T_{fi} \sim \left\langle \psi_{\pi^-} \left| \left\langle \Psi_{{}^A_\Lambda Z} \left| \sum_j v_{eff}(j) \right| \Psi_{{}^A Z} \right\rangle \right| \psi_{K^-} \right\rangle$$

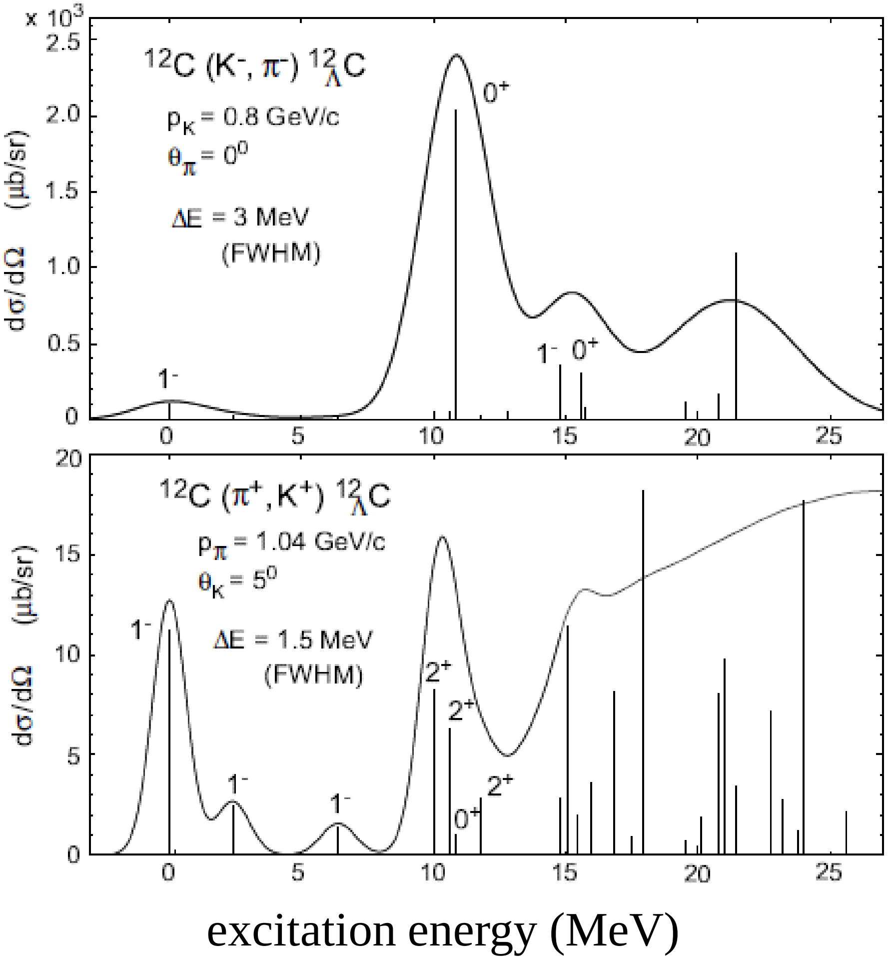
$$\frac{d\sigma}{d\Omega} \sim \underline{\alpha_{\mathrm{kin}}} \underline{\left(\frac{d\sigma}{d\Omega}\right)_{K^- n \to \pi^- \Lambda}} N_{\mathrm{eff}}(\theta; i \to f)$$

kinematical factor

elementary process

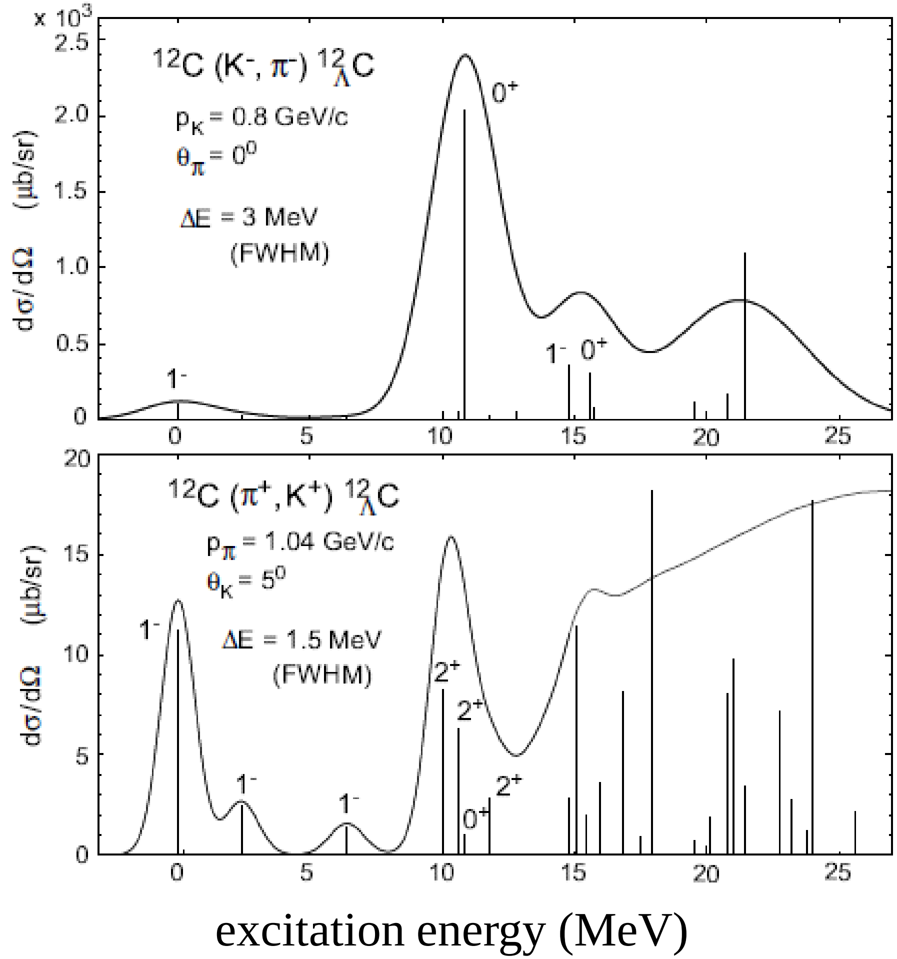
$$N_{\mathrm{eff}}(\theta; i \to f) \sim \left| \int d\boldsymbol{r}\, \psi^*_{\pi^-}(\boldsymbol{r})\, \underline{\varphi^{(\Lambda)*}_{j_\Lambda l_\Lambda m_\Lambda}(\boldsymbol{r}) \varphi^{(n)}_{j_n l_n m_n}(\boldsymbol{r})}\, \psi_{K^-}(\boldsymbol{r}) \right|^2$$

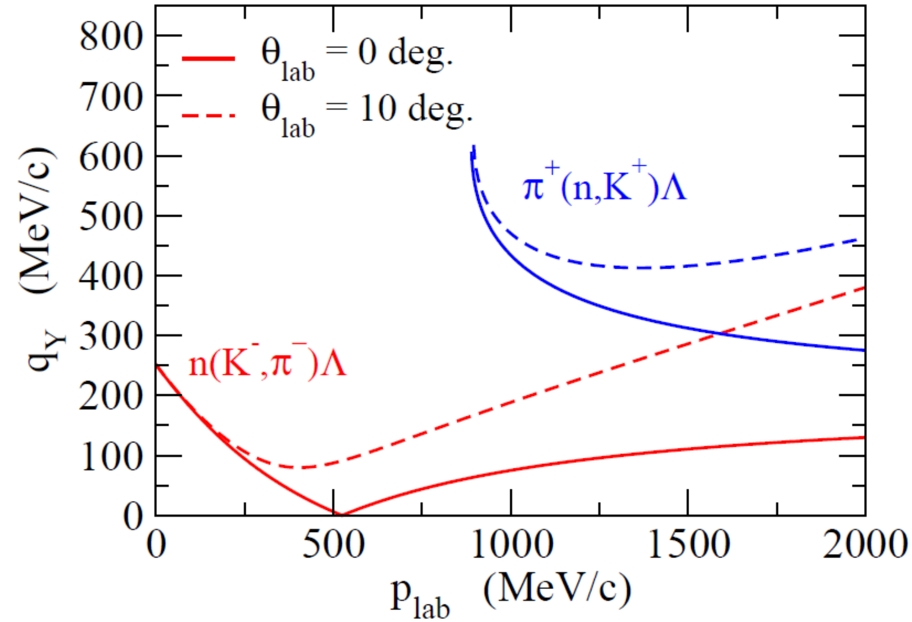
- Plane wave impulse approximation (PWIA)
- Distorted wave impulse approximation (DWIA)



O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 ('06)564

T. Motoba et al., PRC38('88)1322





m_n+m_K = 1432 MeV

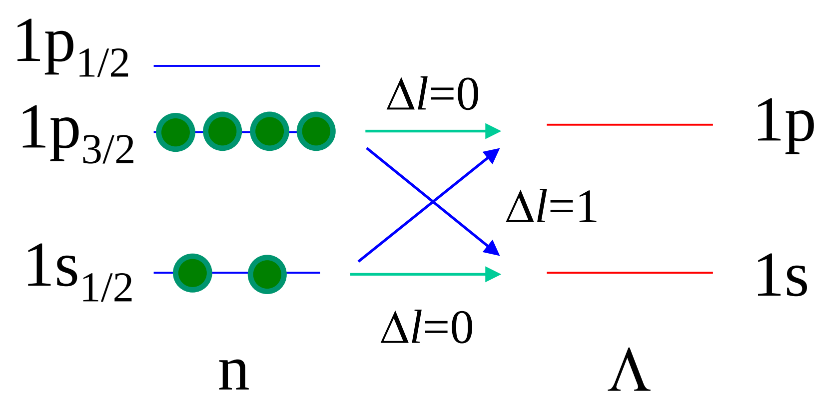
$m_\pi+m_\Lambda$ = 1255.3 MeV

Q > 0

$m_\pi+m_n$ = 1079.2 MeV

m_K+m_Λ = 1609.4 MeV

Q < 0



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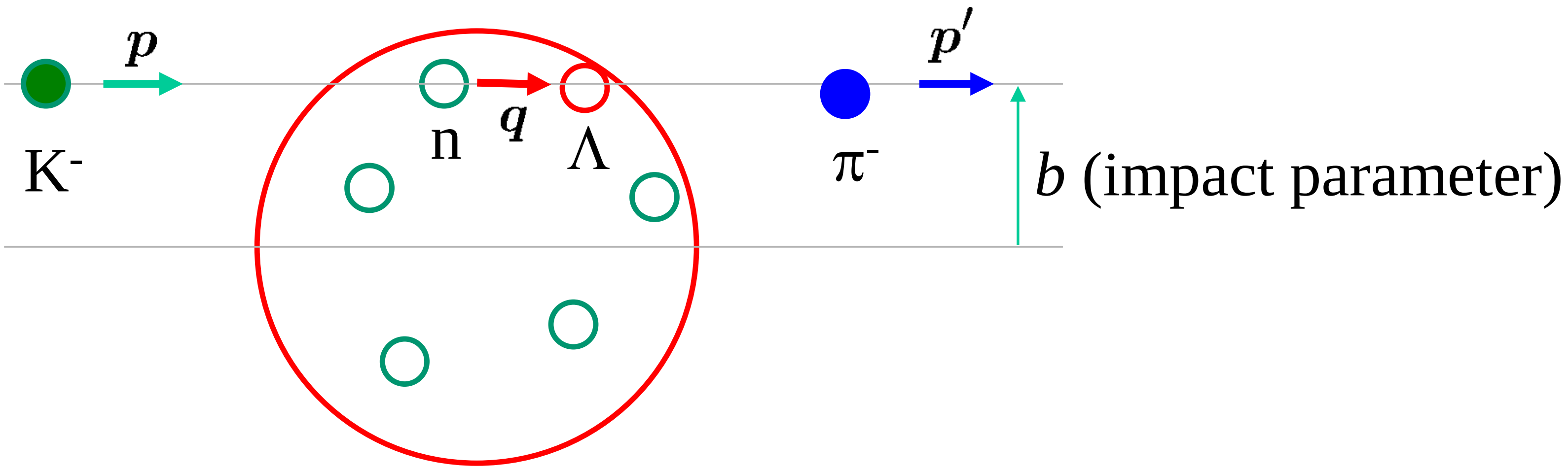
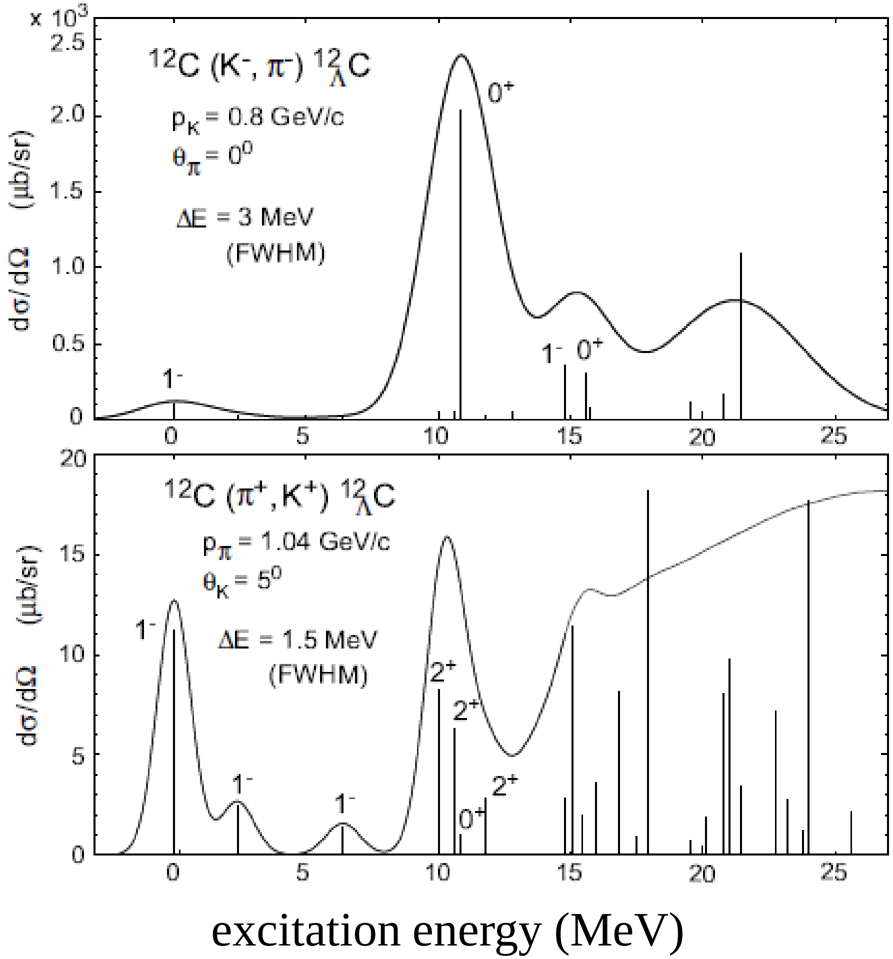
T. Motoba et al., PRC38(‘88)1322

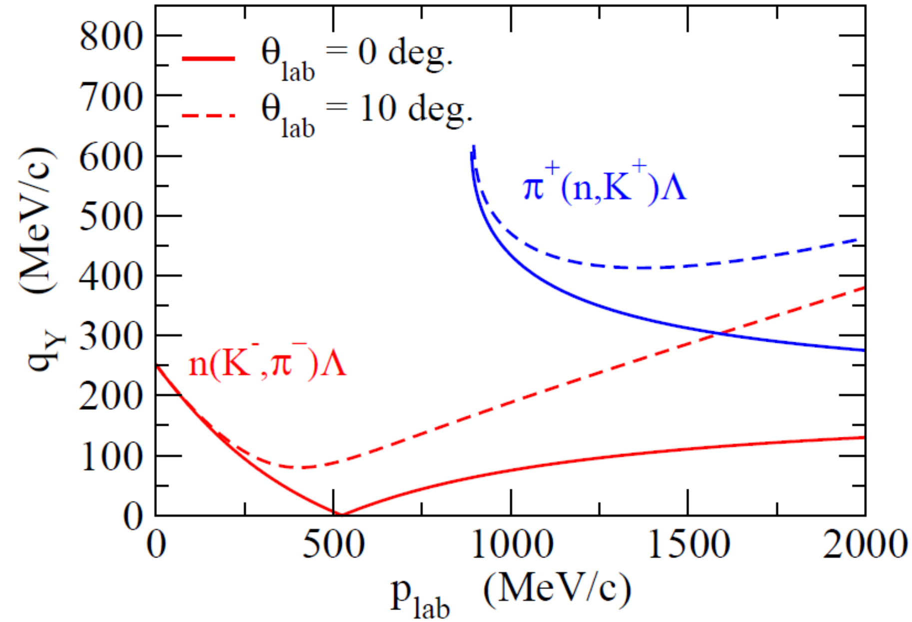
relation between q and Δl

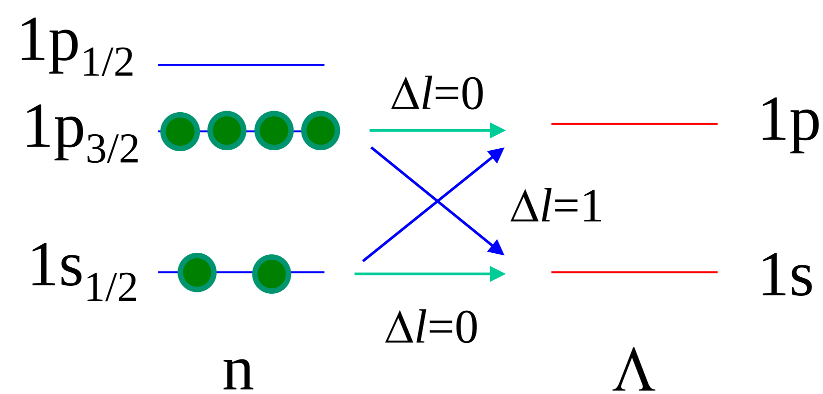


$$l \sim kb \text{ (classically)}$$

$$\Longrightarrow \quad \Delta l \sim b(p' - p) = bq$$







O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 ('06)564

T. Motoba et al., PRC38('88)1322

$$\Delta l \sim b(p'-p) = bq$$