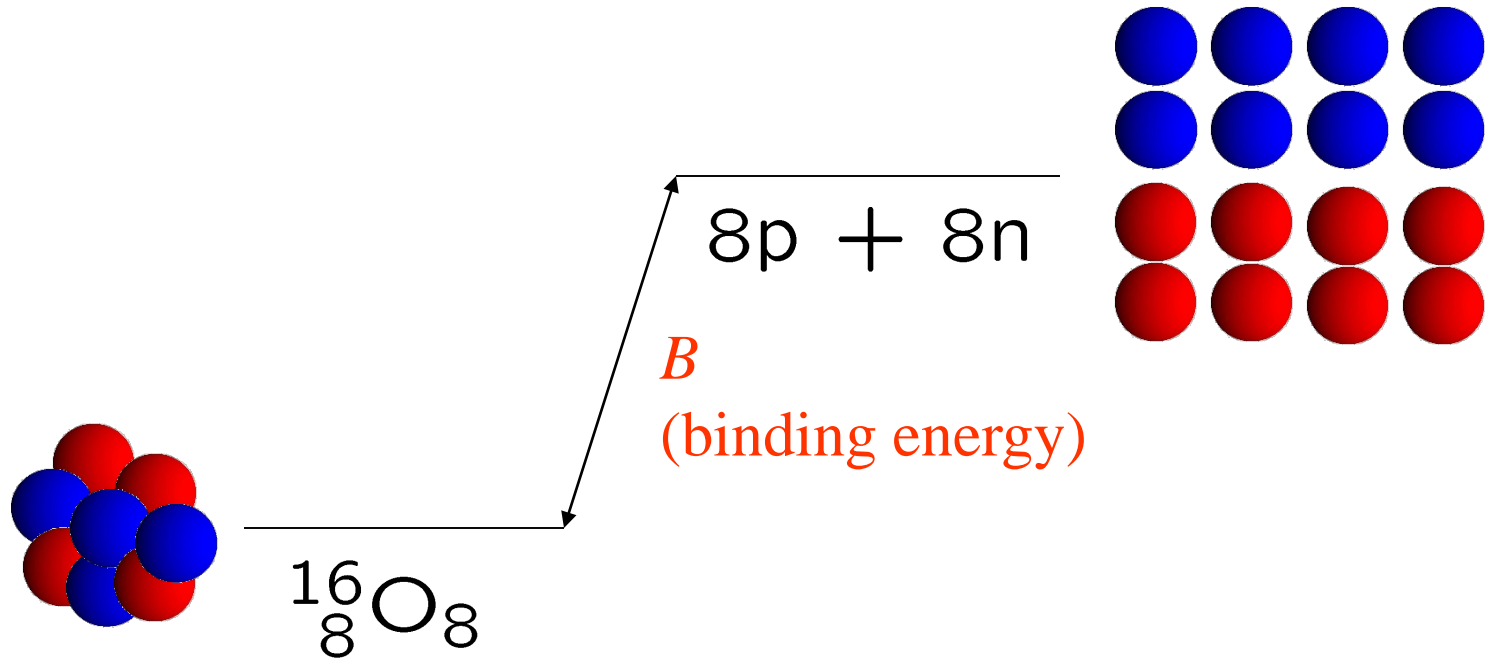
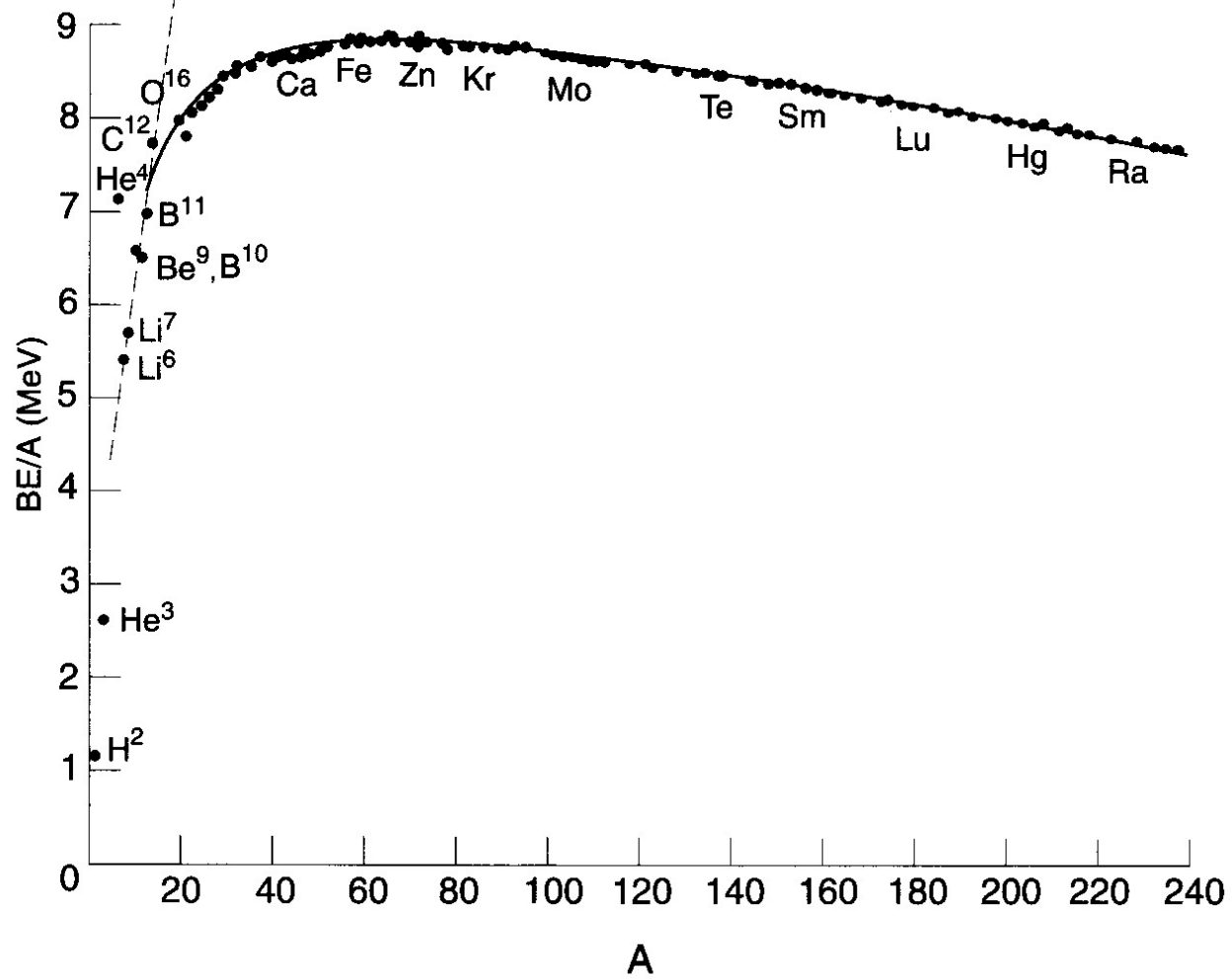
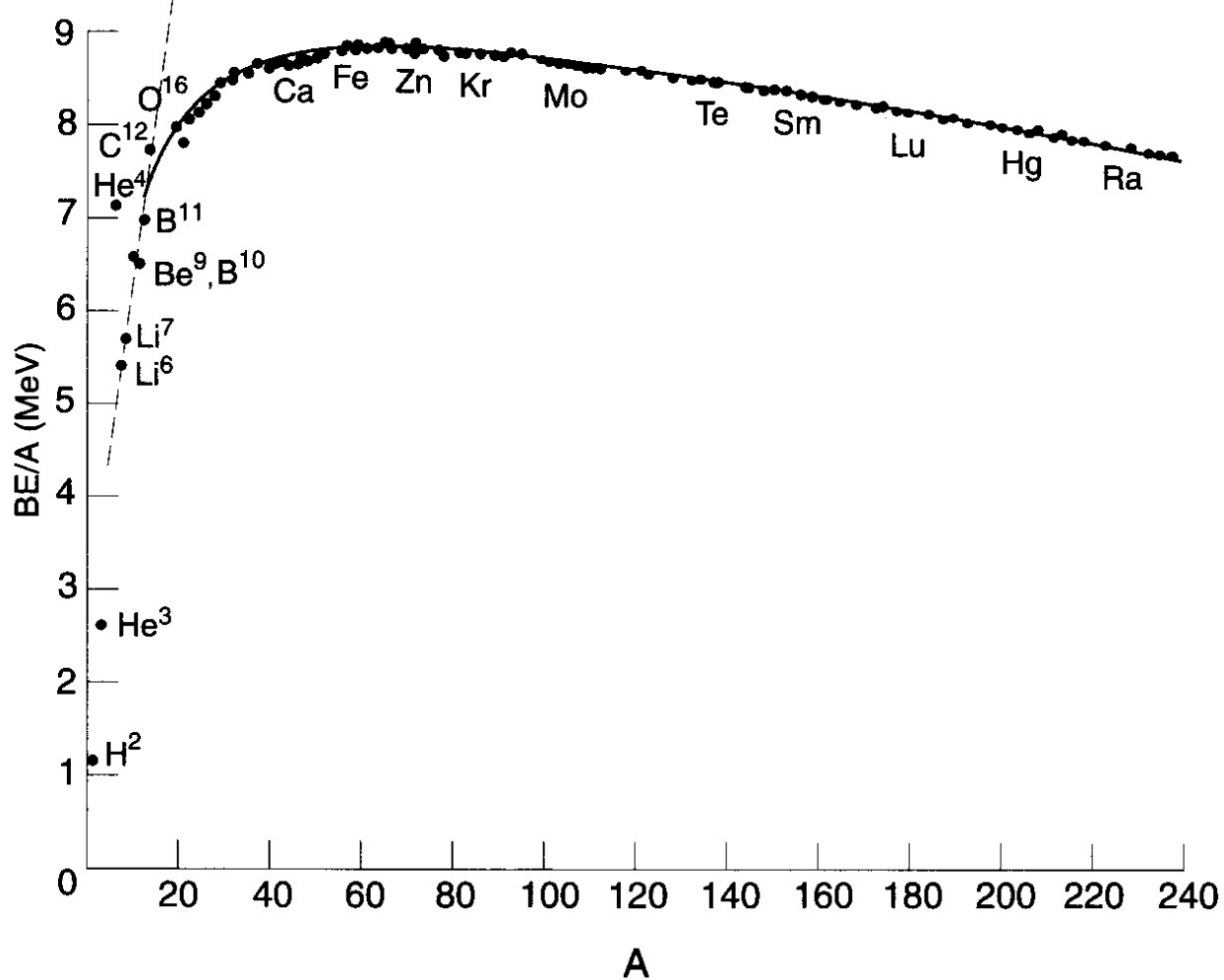


Nuclear Mass



$$m(N, Z)c^2 = Zm_p c^2 + Nm_n c^2 - B$$



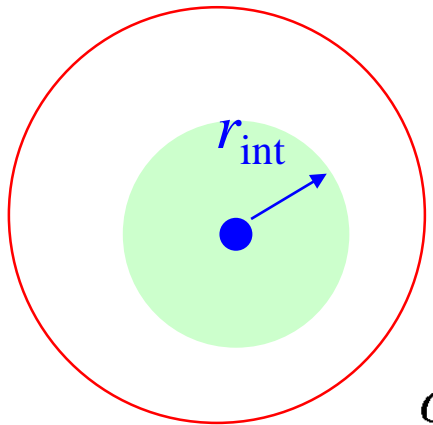


$B(N,Z)/A \sim 8.5 \text{ MeV} (A > 12) \iff$ Short range nuclear force

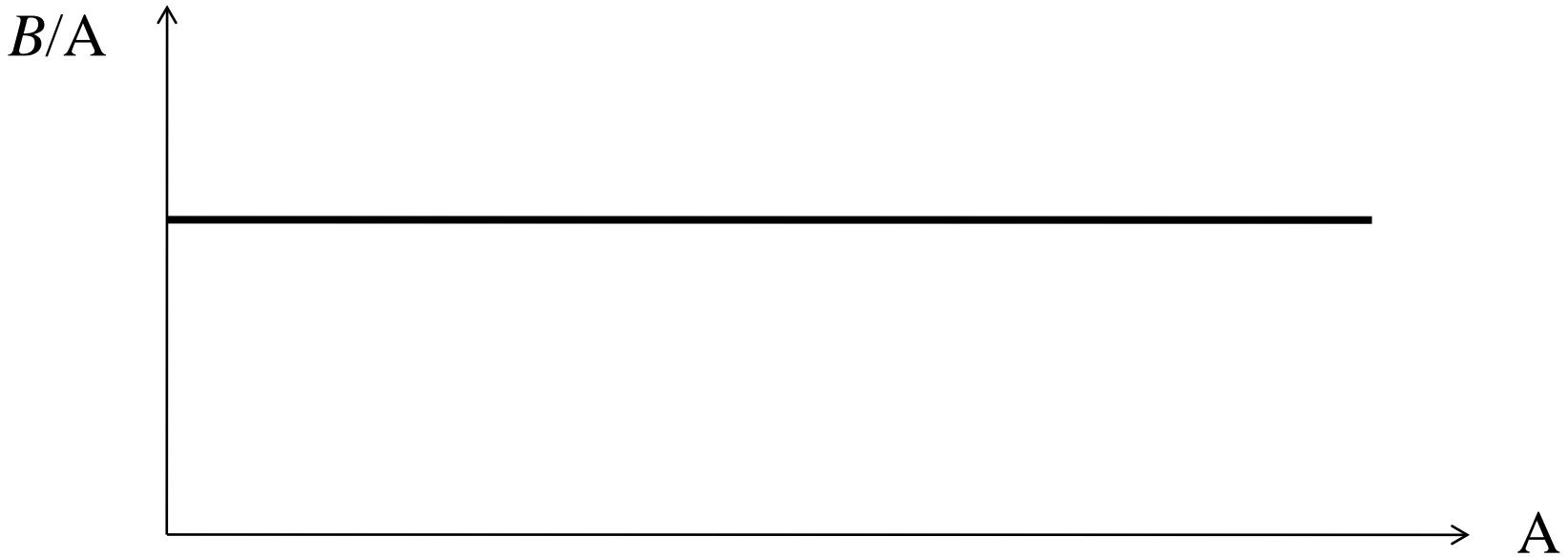
a long range interaction: $B \propto A(A - 1) \sim A^2 \rightarrow B/A \sim A$

if each nucleon can interact only α -nucleons close by:

$$B \sim \alpha A/2 \longrightarrow B/A \sim \alpha/2 \text{ (const.)}$$

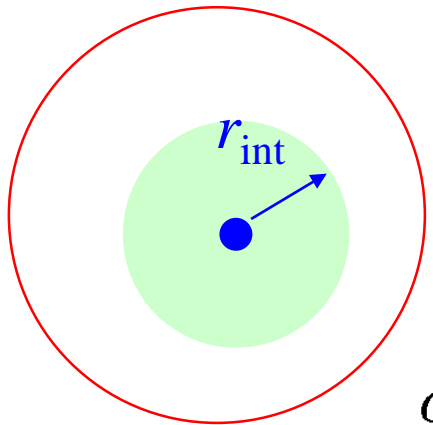


$$\alpha = \frac{4\pi}{3} r_{\text{int}}^3 \cdot \rho$$



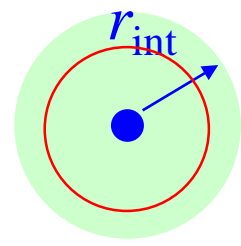
if each nucleon can interact only α -nucleons close by:

$$B \sim \alpha A/2 \longrightarrow B/A \sim \alpha/2 \text{ (const.)}$$

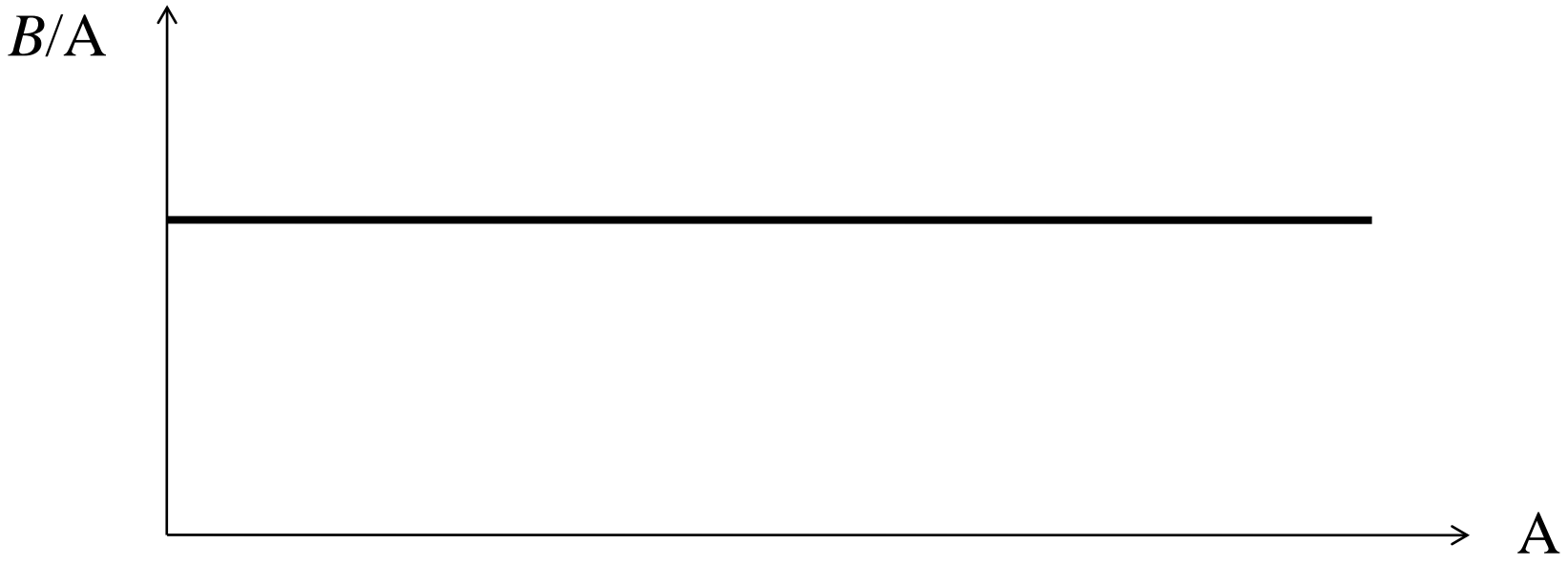


$$\alpha = \frac{4\pi}{3} r_{\text{int}}^3 \cdot \rho$$

a small nucleus

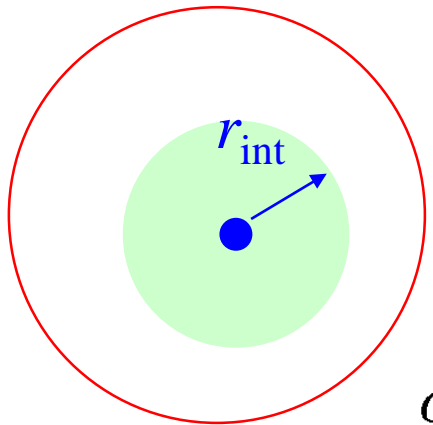


$$\rightarrow B/A \propto A - 1$$

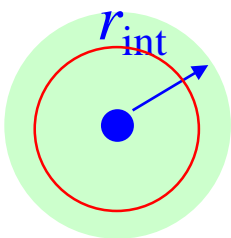


if each nucleon can interact only α -nucleons close by:

$$B \sim \alpha A/2 \longrightarrow B/A \sim \alpha/2 \text{ (const.)}$$

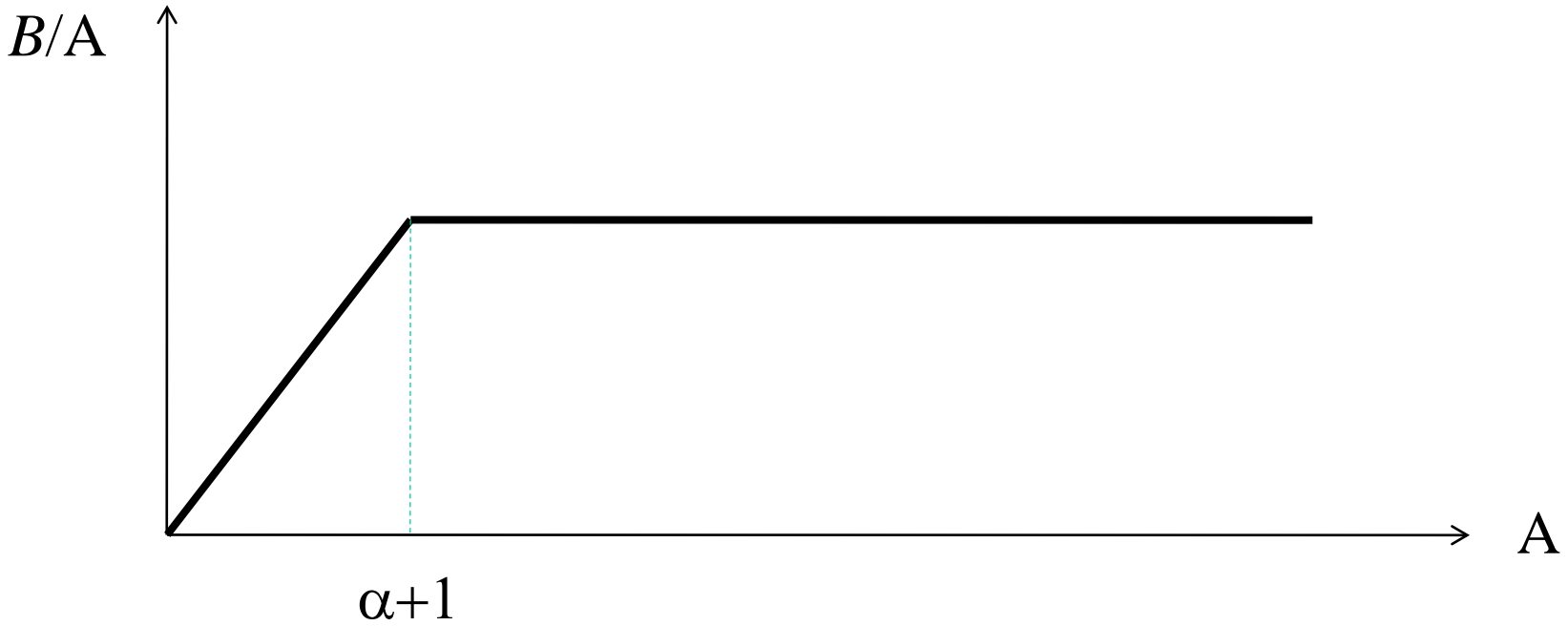


a small nucleus

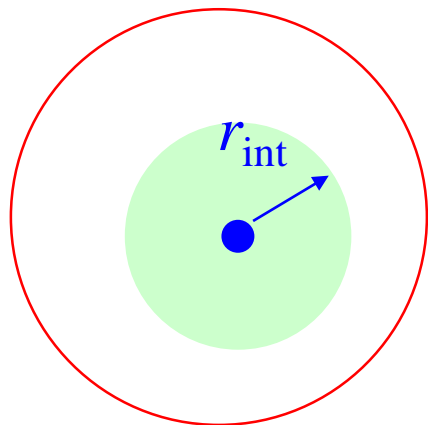


$$\alpha = \frac{4\pi}{3} r_{\text{int}}^3 \cdot \rho$$

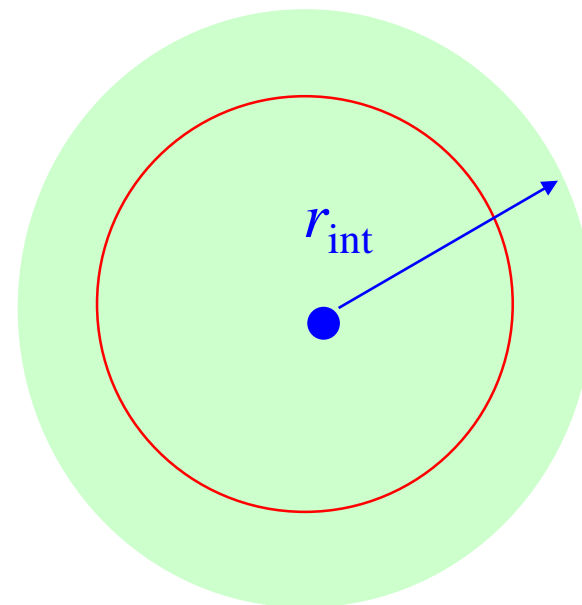
$$\rightarrow B/A \propto A - 1$$



nuclear interaction



Coulomb interaction



B/A

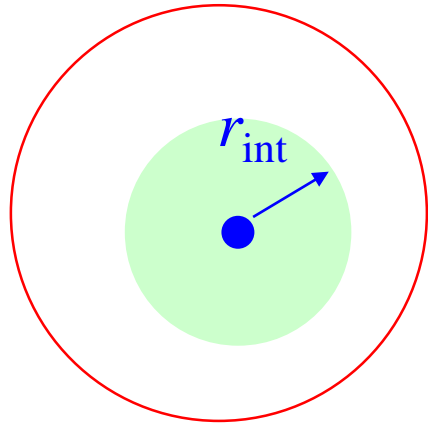


$\alpha+1$

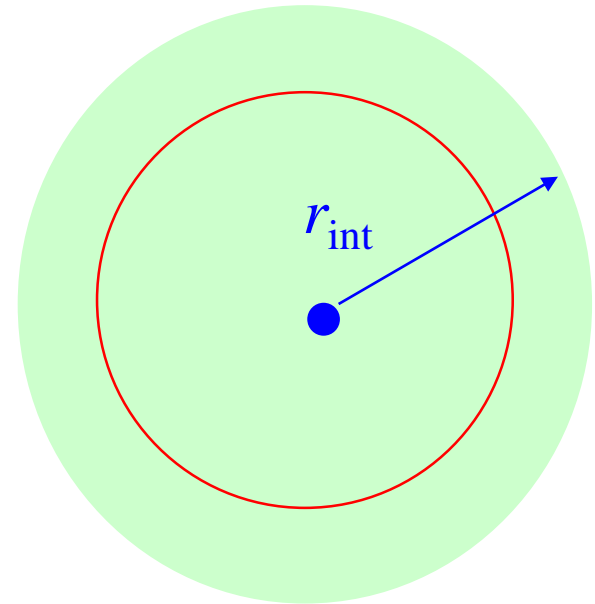
$\rightarrow B/A \propto A - 1$

A

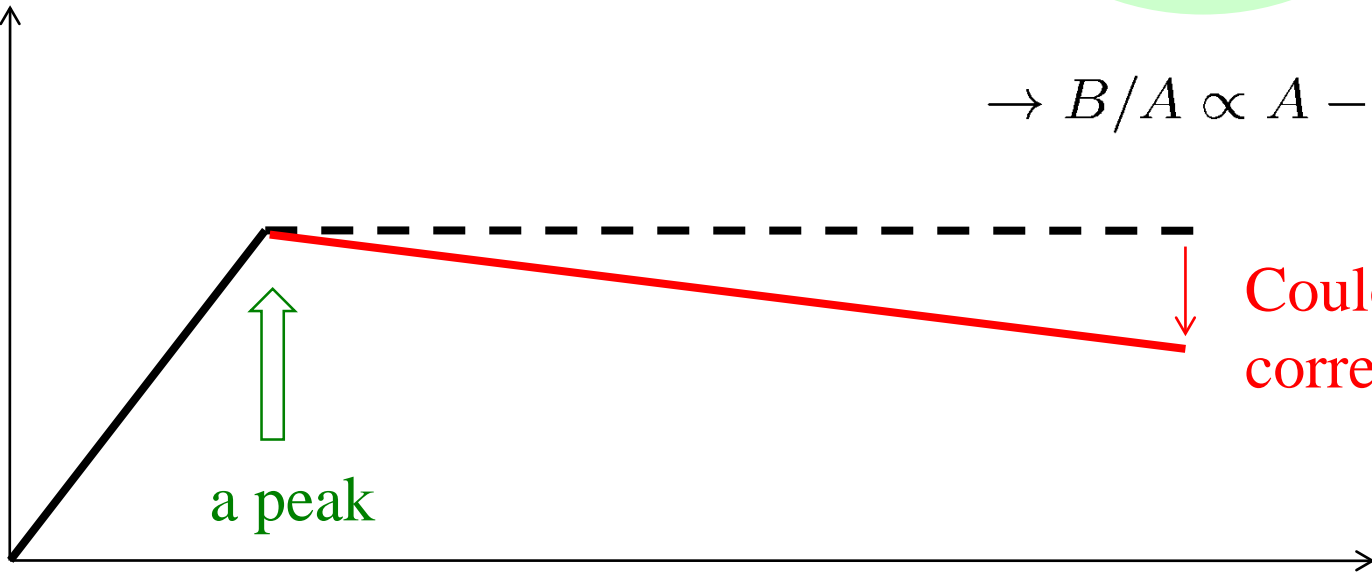
nuclear interaction



Coulomb interaction



B/A

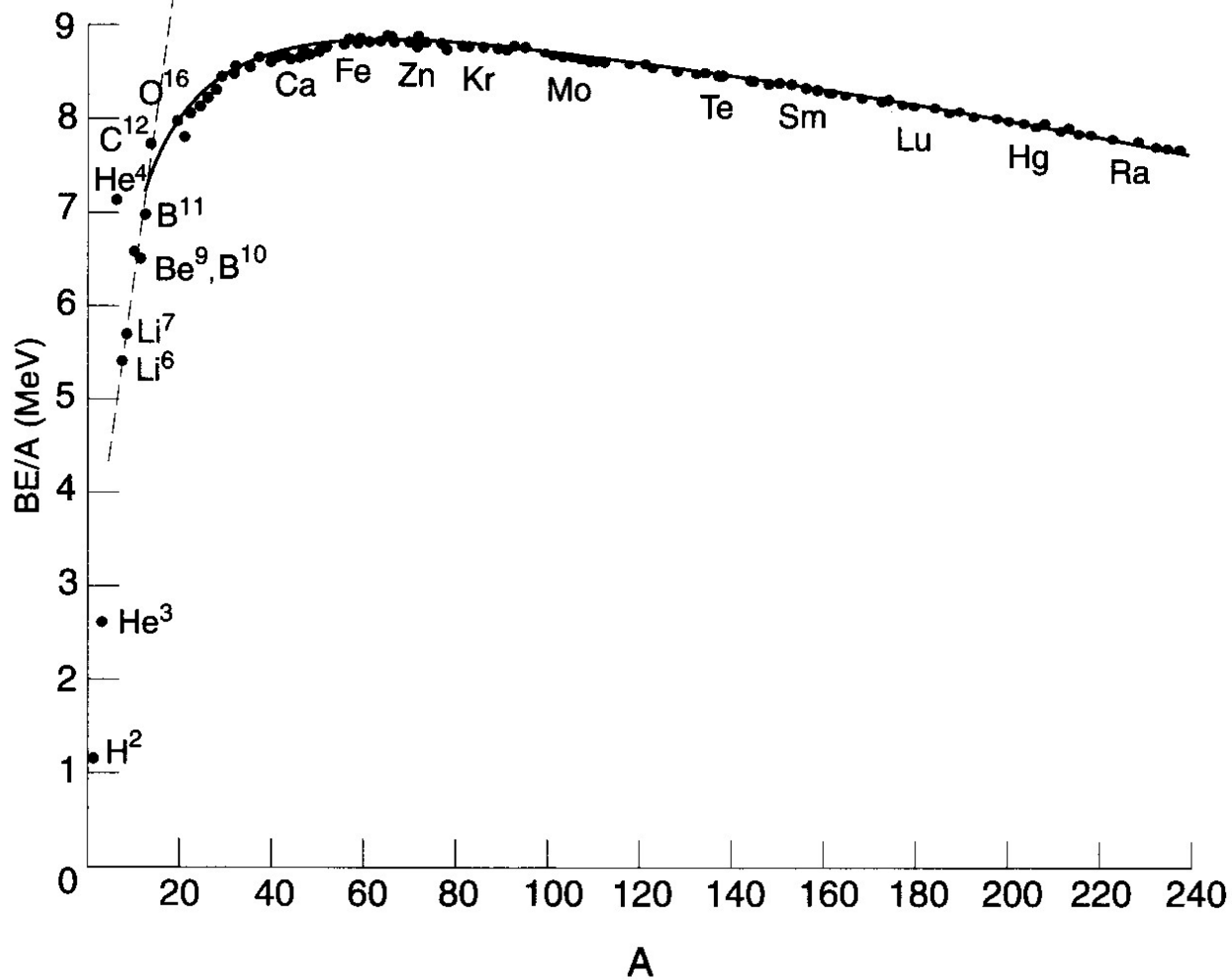


$\rightarrow B/A \propto A - 1$

Coulomb correction

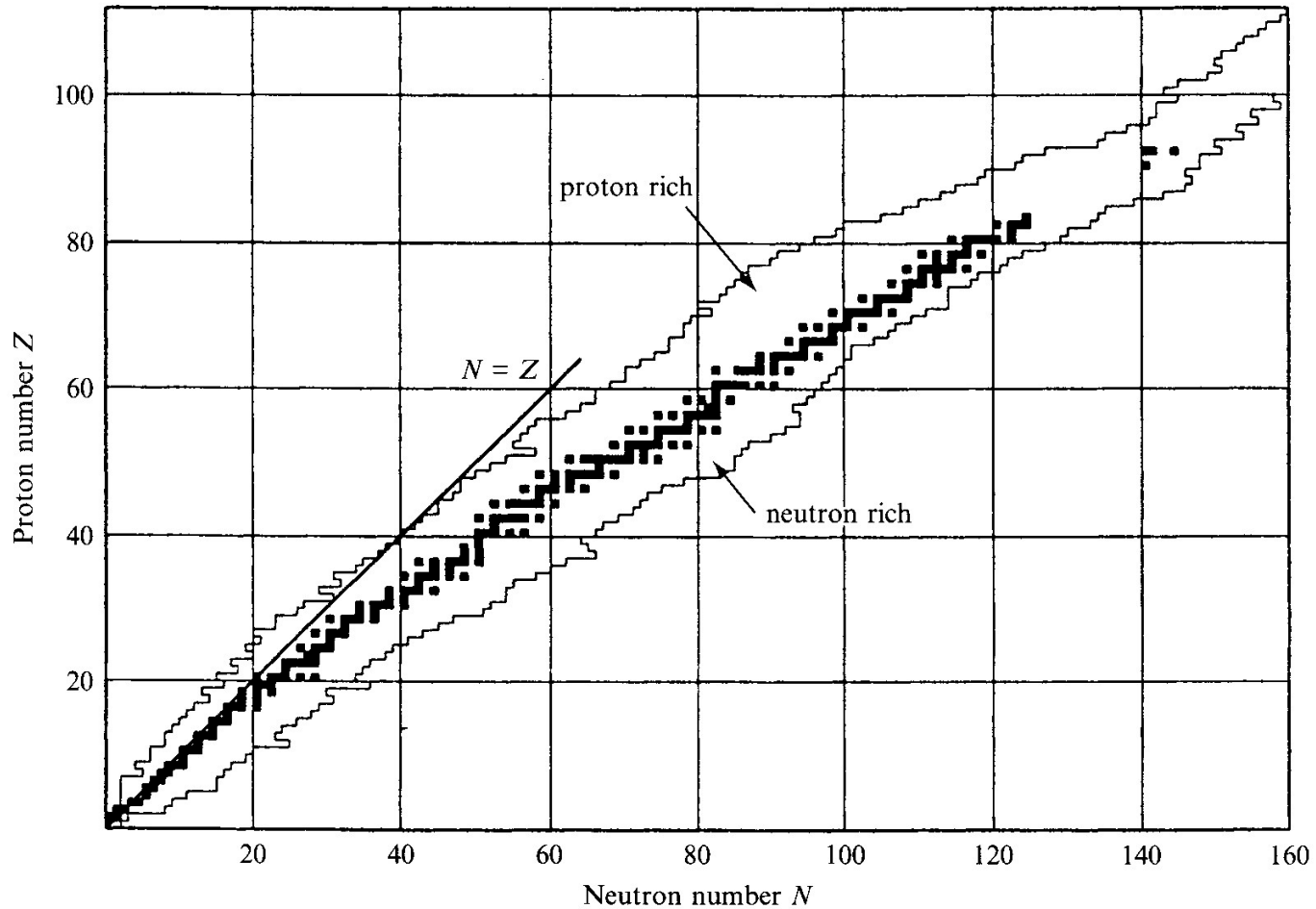
a peak

A

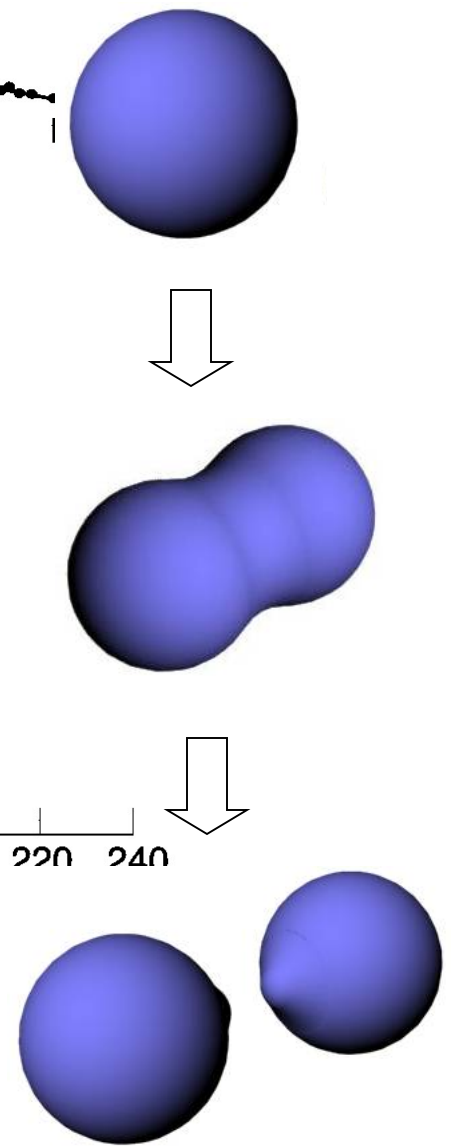
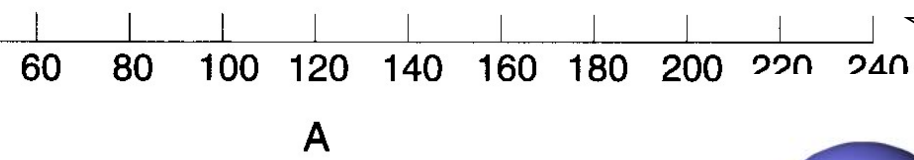
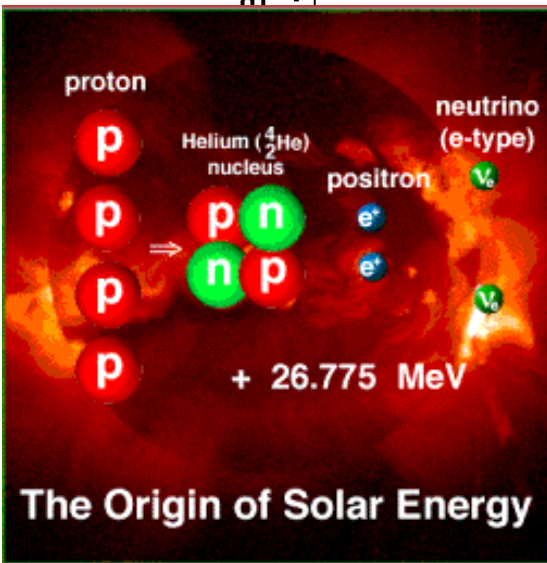
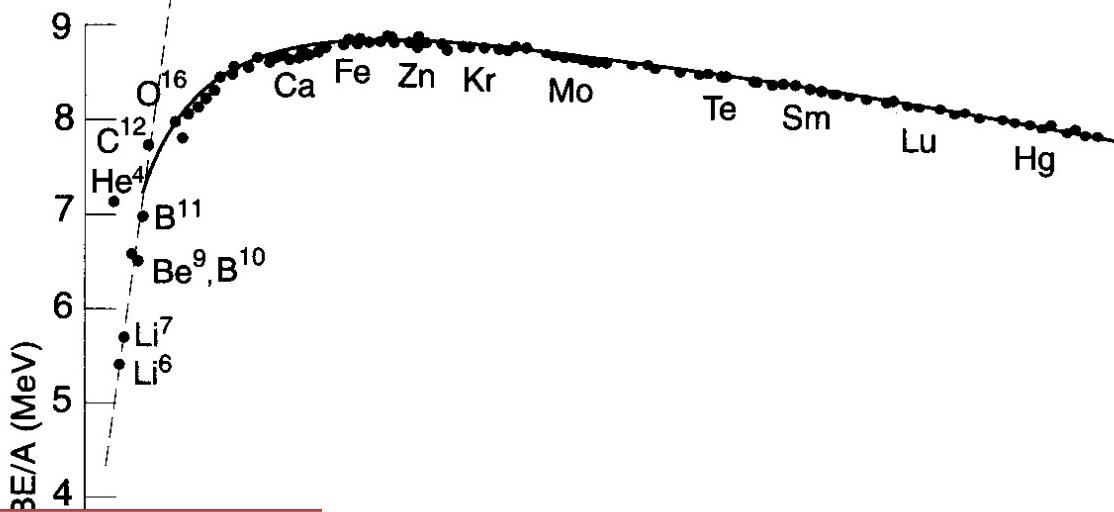


1. $B(N,Z)/A \sim 8.5 \text{ MeV} (A > 12) \iff$ Short range nuclear force
2. Effect of Coulomb force for heavy nuclei

Nuclear Chart



Stable nuclei: $N \geq Z$



1. $B(A, Z)/A \approx 0.8 \text{ MeV}$ ($A > 12$) \iff Short range
2. Effect of Coulomb force for heavy nuclei
3. Fusion for light nuclei
4. Fission for heavy nuclei

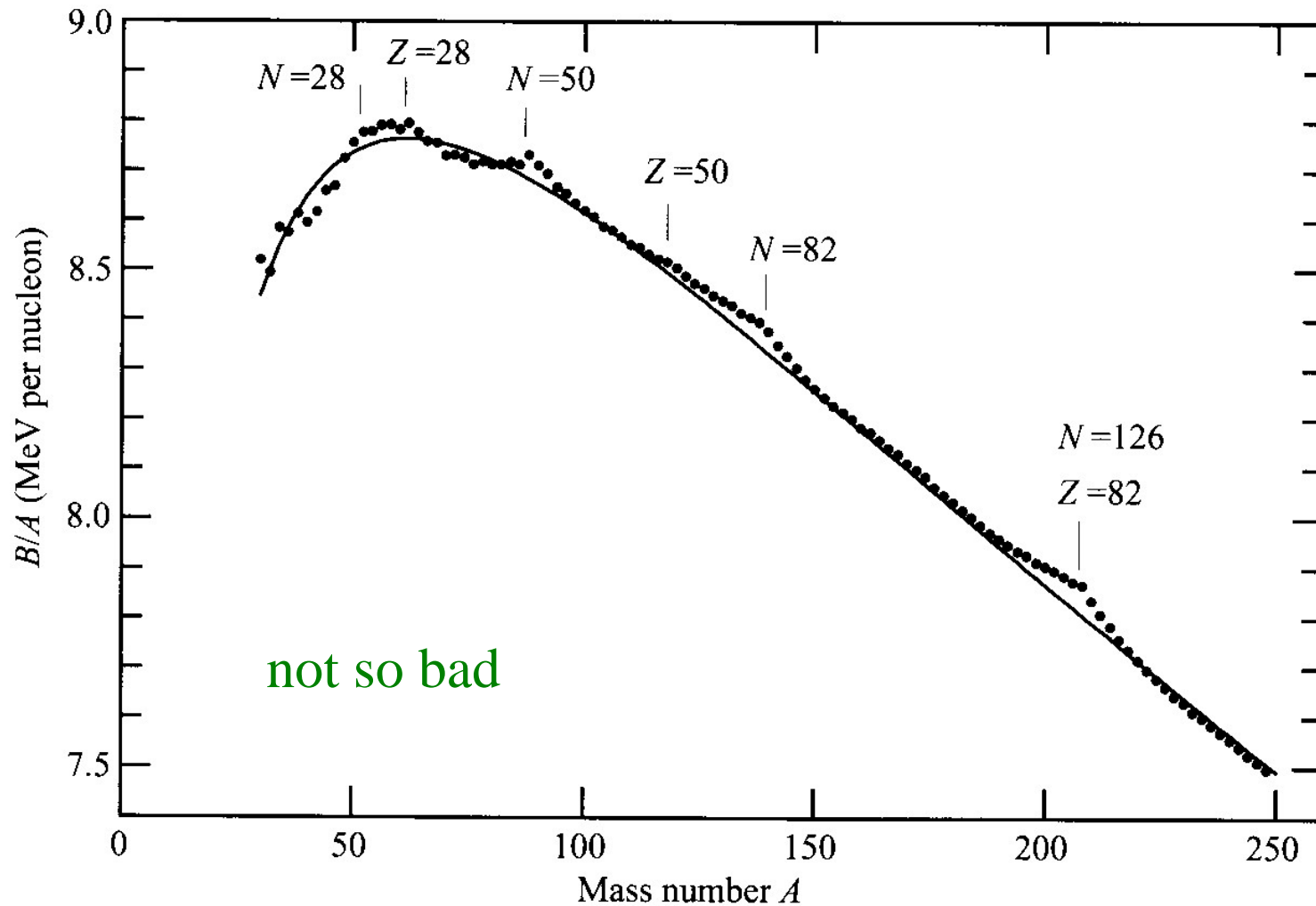
Semi-empirical mass formula

(Bethe-Weizacker formula: Liquid-drop model)

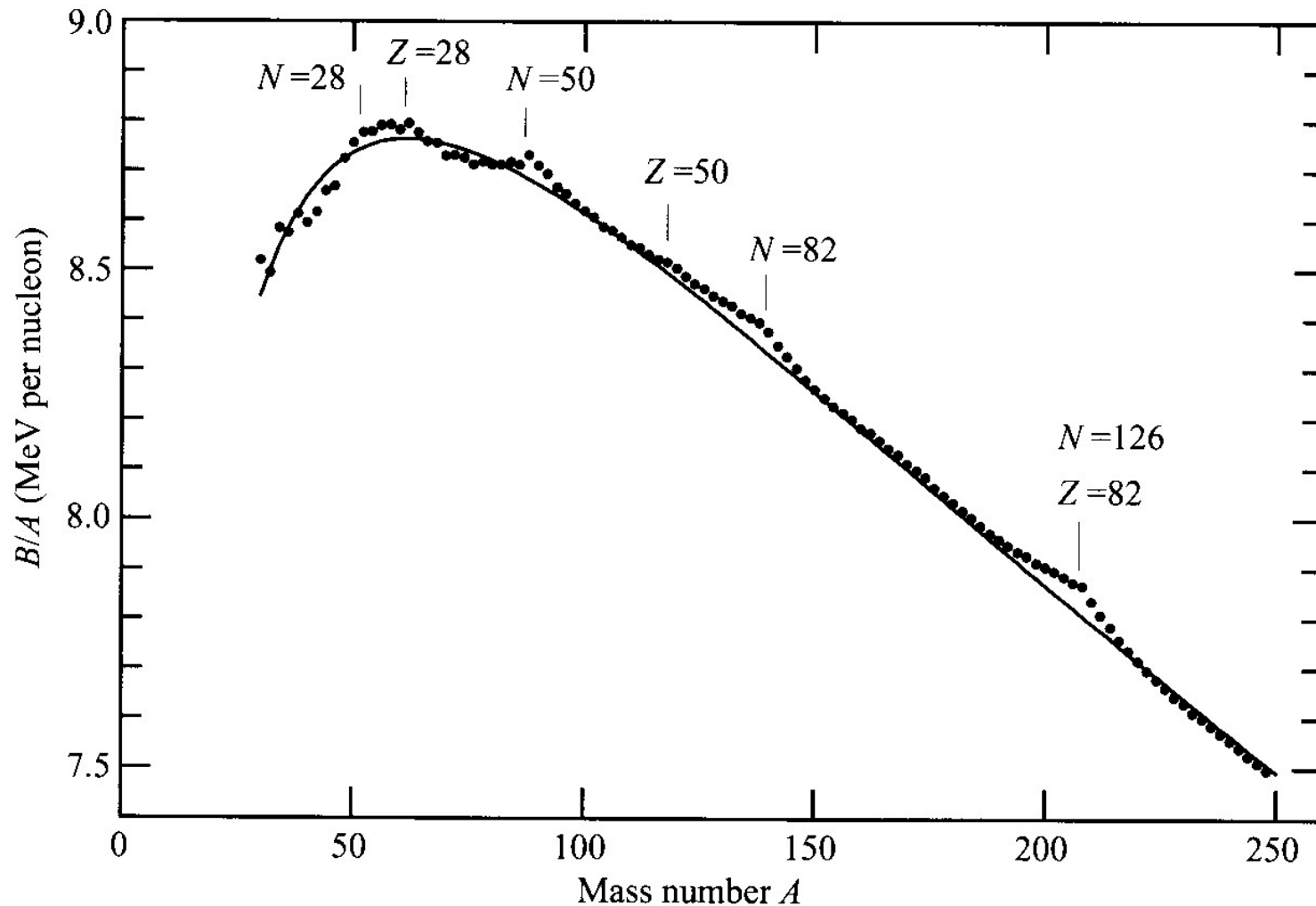
$$B(N, Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N - Z)^2}{A}$$

- Volume energy: $a_v A$
- Surface energy: $-a_s A^{2/3}$
- Coulomb energy: $-a_C Z^2 / A^{1/3}$
- Symmetry energy: $-a_{\text{sym}} (N - Z)^2 / A$

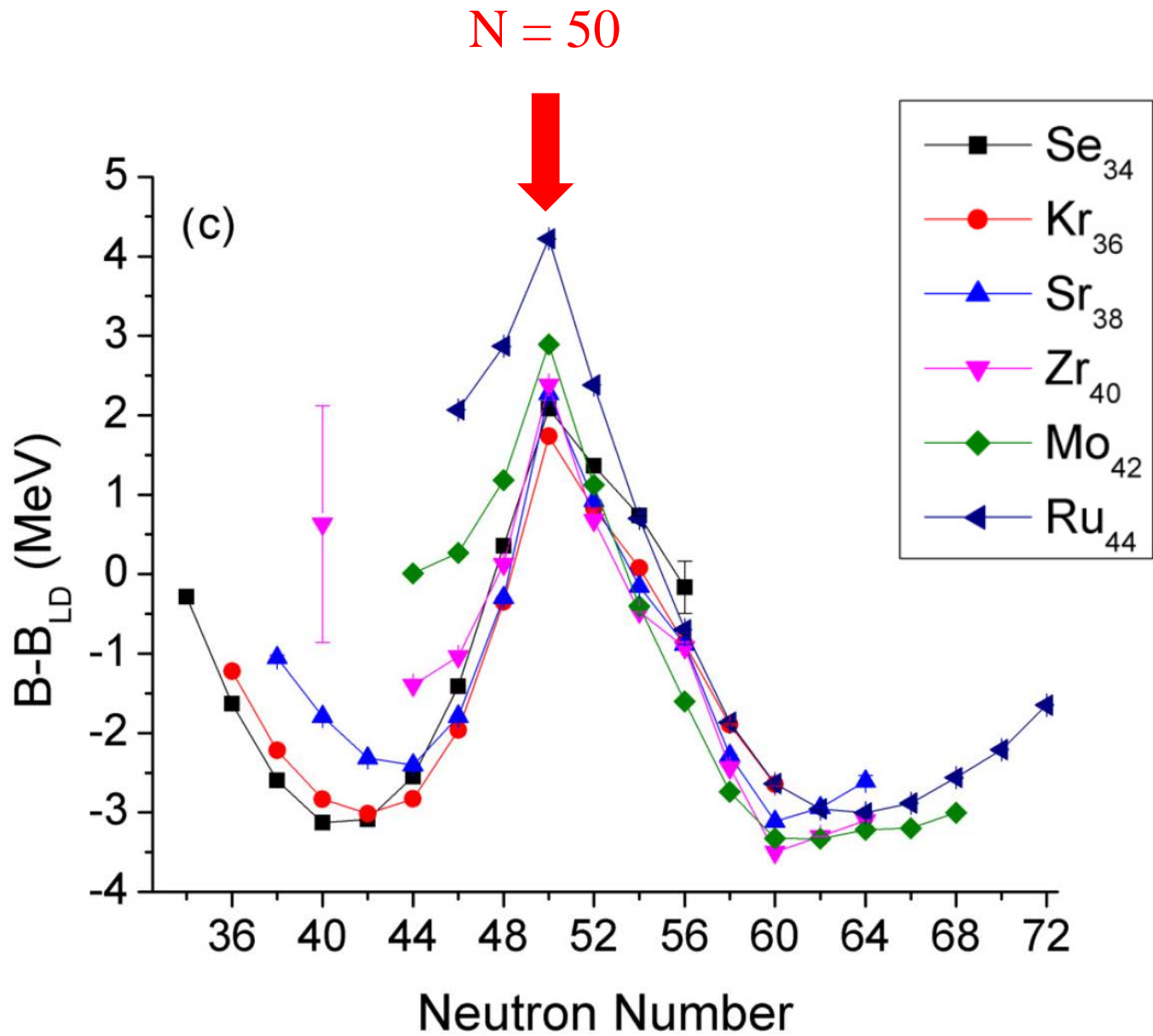
How well does the Bethe-Weizacker formula reproduce the data?



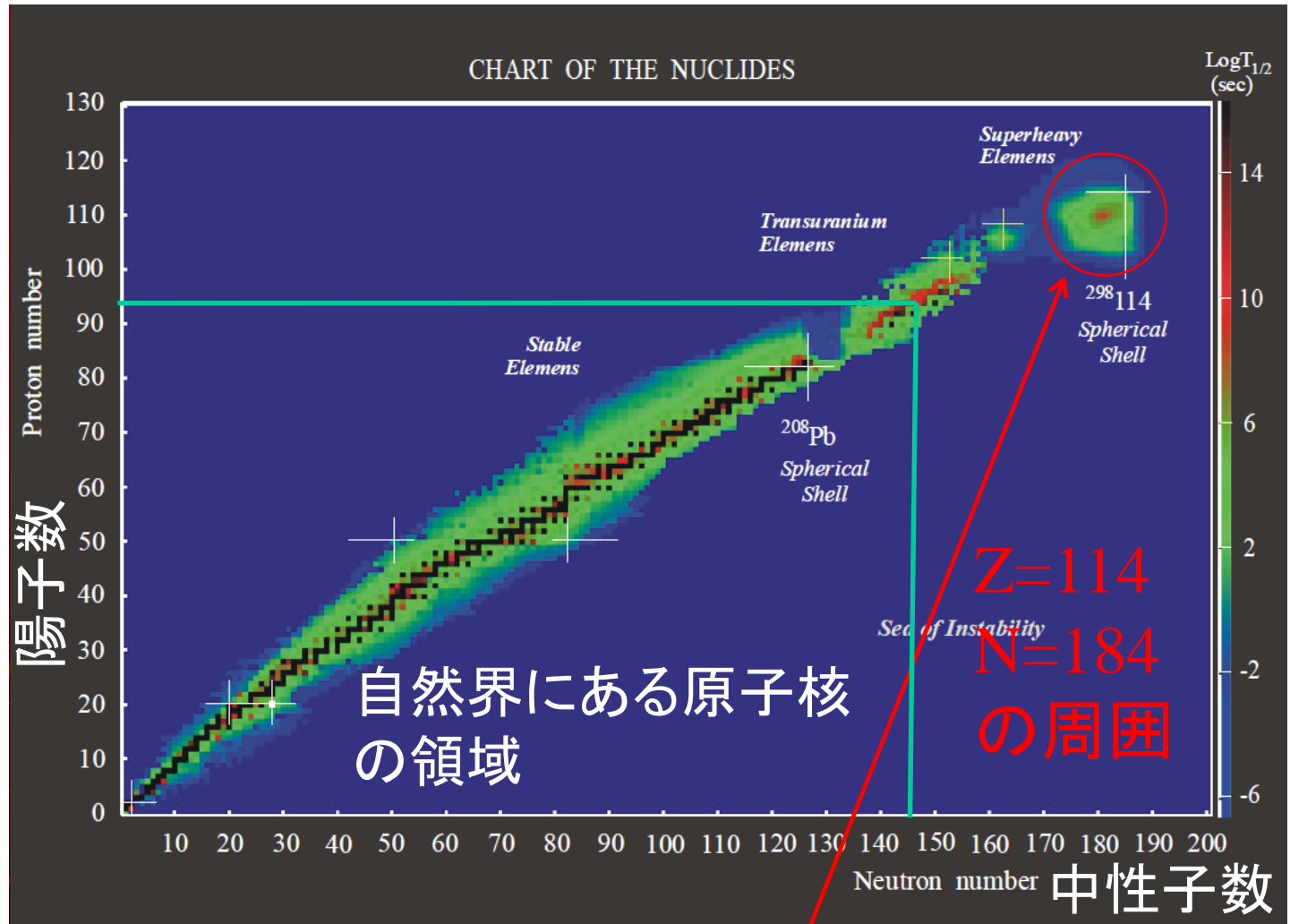
How well does the Bethe-Weizacker formula reproduce the data?



cf. $N, Z = 2, 8, 20, 28, 50, 82, 126$: large binding energy
“magic numbers”



超重元素(超重原子核)

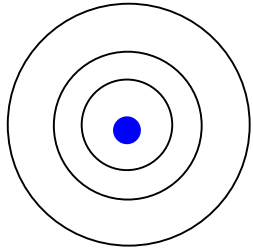


Yuri Oganessian

原子核の安定領域の理論的予言
「安定の島」

(note) 原子の魔法数 (貴ガス・希ガス)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)



電子の殻構造

→閉殻になると安定

元素の周期表

	1A	2A	3A	4A	5A	6A	7A	8	1B	2B	3B	4B	5B	6B	7B	0		
1	H															He		
2	Li	Be									B	C	N	O	F	Ne		
3	Na	Mg									Al	Si	P	S	Cl	Ar		
4	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
5	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
6	Cs	Ba	L	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
7	Fr	Ra	A															
	L	La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu		
	A	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

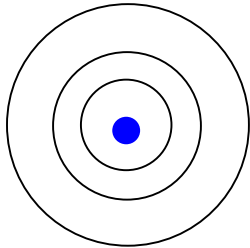
Legend:

- 典型金属元素 (Orange)
- 半金属元素 (Light Green)
- 非金属元素 (Cyan)
- 遷移金属元素 (Yellow)
- 希ガス (Pink)

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(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)

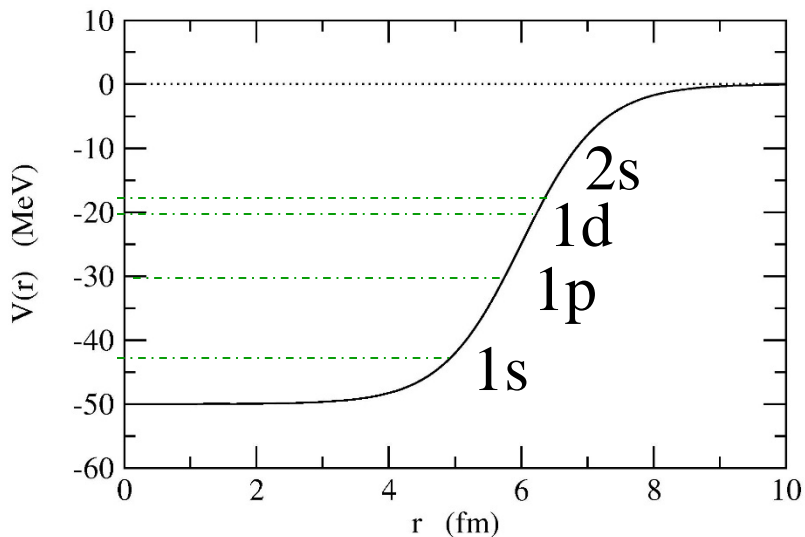


Shell structure

Similar attempt in nuclear physics: independent particle motion in a potential well

Woods-Saxon potential

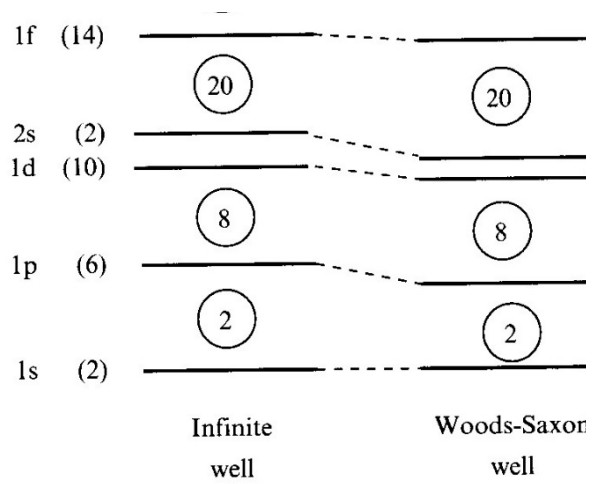
$$V(r) = -V_0/[1 + \exp((r - R_0)/a)]$$



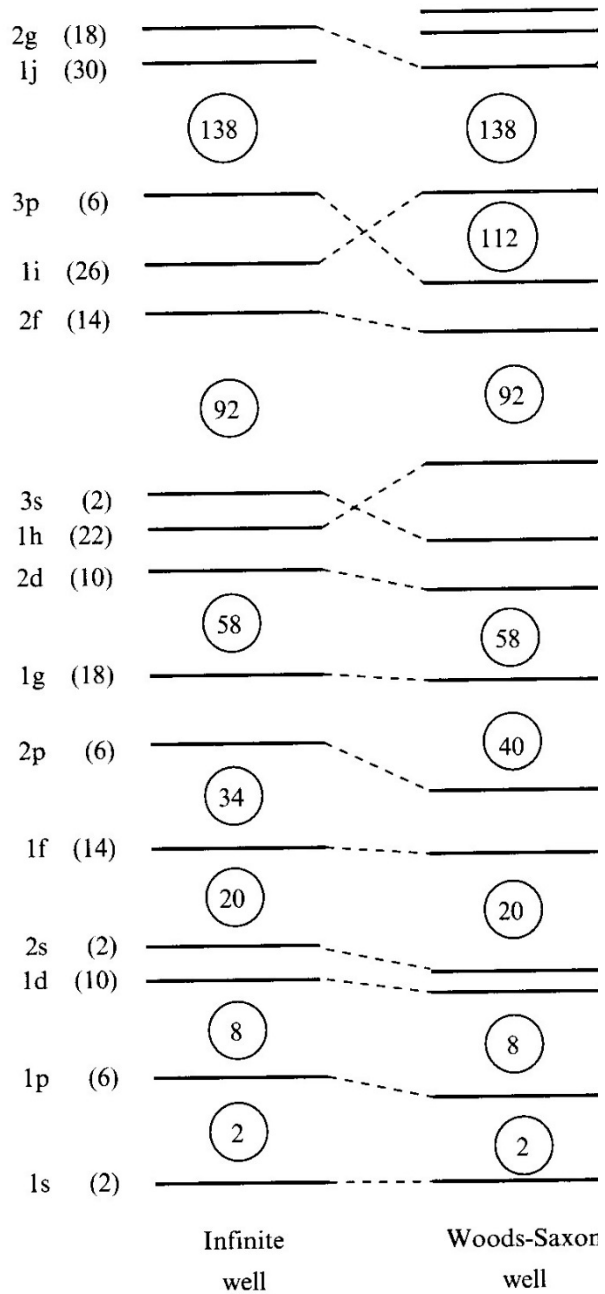
$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) - \epsilon \right] \psi(\mathbf{r}) = 0$$

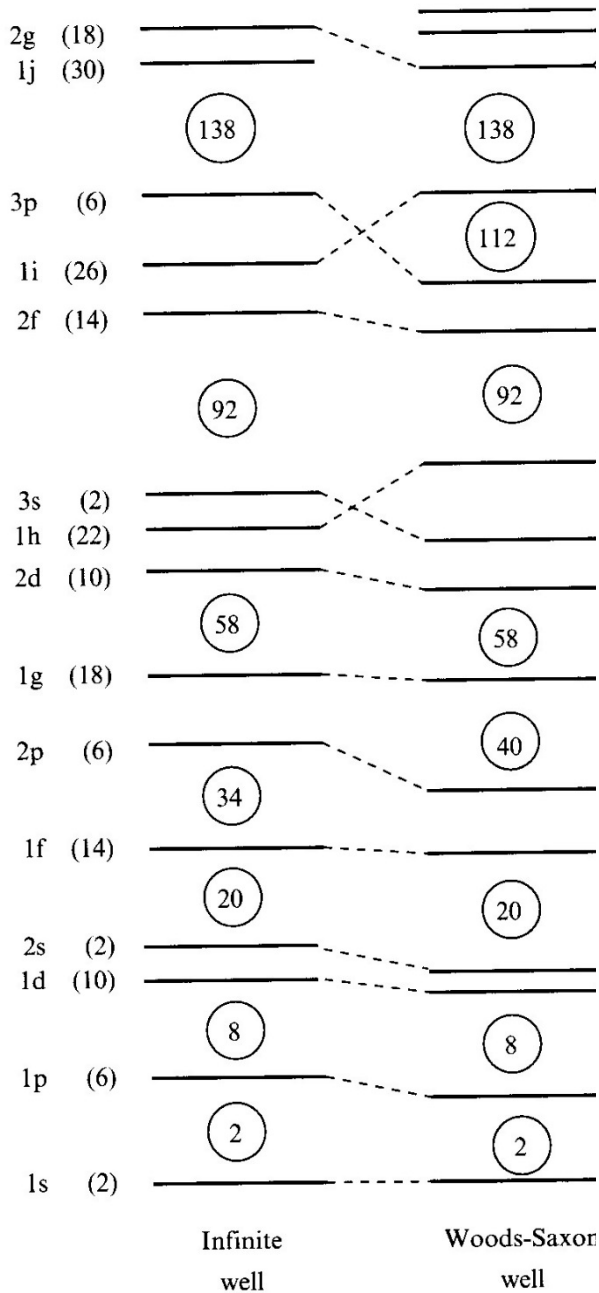
$$\psi(\mathbf{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\mathbf{r}}) \cdot \chi_{m_s}$$

Nuclear magic numbers: 2, 8, 20, 28, 50, 82, 126

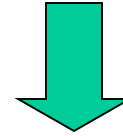


Nuclear magic numbers:
2, 8, 20, 28, 50, 82, 126





Woods-Saxon itself does not provide the correct magic numbers (2,8,20,28, 50,82,126).



Mayer and Jensen (1949):
Strong spin-orbit interaction



$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) + V_{ls}(r) \mathbf{l} \cdot \mathbf{s} - \epsilon \right] \psi(\mathbf{r}) = 0$$

$$V_{ls}(r) \sim -\lambda \frac{1}{r} \frac{dV}{dr} \quad (\lambda > 0)$$

Infinite well Woods-Saxon well

jj 結合殻模型

軌道運動とスピンは独立の自由度

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) - \epsilon \right] \psi(\mathbf{r}) = 0 \implies \psi_{l m m_s}(\mathbf{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\mathbf{r}}) \cdot \chi_{m_s}$$

スピン・軌道力

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) + V_{ls}(r) \mathbf{l} \cdot \mathbf{s} - \epsilon \right] \psi(\mathbf{r}) = 0$$

$$\text{(note) } \mathbf{j} = \mathbf{l} + \mathbf{s} \implies \mathbf{l} \cdot \mathbf{s} = (j^2 - l^2 - s^2)/2$$

l と s を結合して j を組む。

$$\rightarrow j = l \pm 1/2$$

jj 結合殻模型


$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) - \epsilon \right] \psi(\mathbf{r}) = 0 \implies \psi_{lmm_s}(\mathbf{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\mathbf{r}}) \cdot \chi_{m_s}$$

スピン・軌道力

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) + V_{ls}(r) \mathbf{l} \cdot \mathbf{s} - \epsilon \right] \psi(\mathbf{r}) = 0$$

(note) $\mathbf{j} = \mathbf{l} + \mathbf{s} \implies \mathbf{l} \cdot \mathbf{s} = (j^2 - l^2 - s^2)/2$

l と s を結合して j を組む。


$$\begin{aligned} \psi_{jlm}(\mathbf{r}) &= \frac{u_{jl}(r)}{r} \mathcal{Y}_{jlm}(\hat{\mathbf{r}}) \\ \mathcal{Y}_{jlm}(\hat{\mathbf{r}}) &= \sum_{m_l, m_s} \langle l \ m_l \ 1/2 \ m_s | j \ m \rangle Y_{lm_l}(\hat{\mathbf{r}}) \chi_{m_s} \end{aligned}$$

$$j^2 |\mathcal{Y}_{jlm}\rangle = j(j+1) |\mathcal{Y}_{jlm}\rangle$$

$$j_z |\mathcal{Y}_{jlm}\rangle = m |\mathcal{Y}_{jlm}\rangle$$

$$l^2 |\mathcal{Y}_{jlm}\rangle = l(l+1) |\mathcal{Y}_{jlm}\rangle$$

$$s^2 |\mathcal{Y}_{jlm}\rangle = 3/4 |\mathcal{Y}_{jlm}\rangle$$

jj 結合殻模型

l と s を結合して j を組む。

$$\psi_{jlm}(\mathbf{r}) = \frac{u_{jl}(r)}{r} \mathcal{Y}_{jlm}(\hat{\mathbf{r}})$$

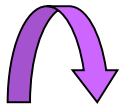
$$j^2 |\mathcal{Y}_{jlm}\rangle = j(j+1) |\mathcal{Y}_{jlm}\rangle$$

$$j_z |\mathcal{Y}_{jlm}\rangle = m |\mathcal{Y}_{jlm}\rangle$$

$$l^2 |\mathcal{Y}_{jlm}\rangle = l(l+1) |\mathcal{Y}_{jlm}\rangle$$

$$s^2 |\mathcal{Y}_{jlm}\rangle = \frac{3}{4} |\mathcal{Y}_{jlm}\rangle$$

(note) $j = l + s \implies l \cdot s = (j^2 - l^2 - s^2)/2$



$$l \cdot s |\mathcal{Y}_{jlm}\rangle = \frac{1}{2} \left(j(j+1) - l(l+1) - \frac{3}{4} \right) |\mathcal{Y}_{jlm}\rangle$$

$$l \cdot s |\mathcal{Y}_{jlm}\rangle = \frac{l}{2} |\mathcal{Y}_{jlm}\rangle \quad (j = l + 1/2)$$

$$l \cdot s |\mathcal{Y}_{jlm}\rangle = -\frac{l+1}{2} |\mathcal{Y}_{jlm}\rangle \quad (j = l - 1/2)$$

符号が逆！

jj 結合殻模型

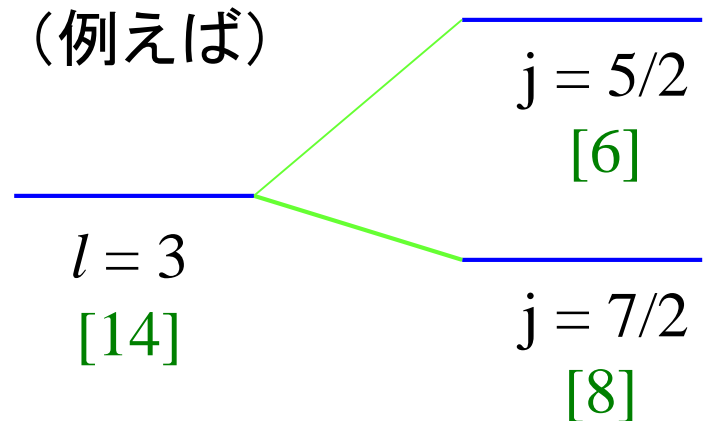
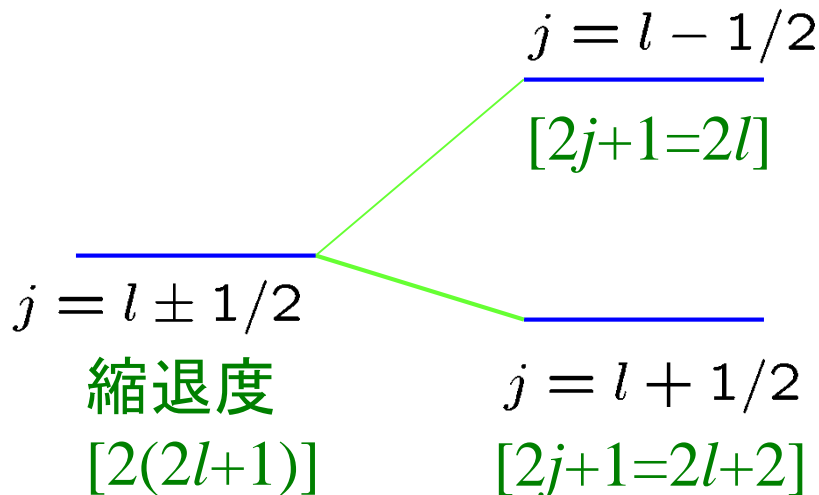
$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) + V_{ls}(r) \mathbf{l} \cdot \mathbf{s} - \epsilon \right] \psi(\mathbf{r}) = 0$$

$$\mathbf{l} \cdot \mathbf{s} |\mathcal{Y}_{jlm}\rangle = \frac{l}{2} |\mathcal{Y}_{jlm}\rangle \quad (j = l + 1/2)$$

$$\mathbf{l} \cdot \mathbf{s} |\mathcal{Y}_{jlm}\rangle = -\frac{l+1}{2} |\mathcal{Y}_{jlm}\rangle \quad (j = l - 1/2)$$

符号が逆！

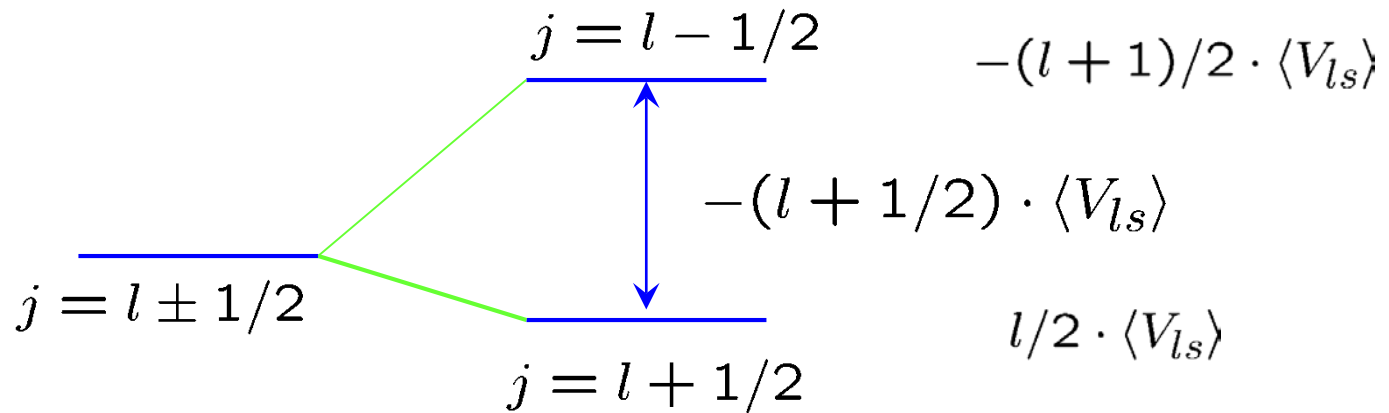
$j = l \pm 1/2$ で準位が分離



jj 結合殻模型

$$l \cdot s |\mathcal{Y}_{jlm}\rangle = \frac{l}{2} |\mathcal{Y}_{jlm}\rangle \quad (j = l + 1/2)$$

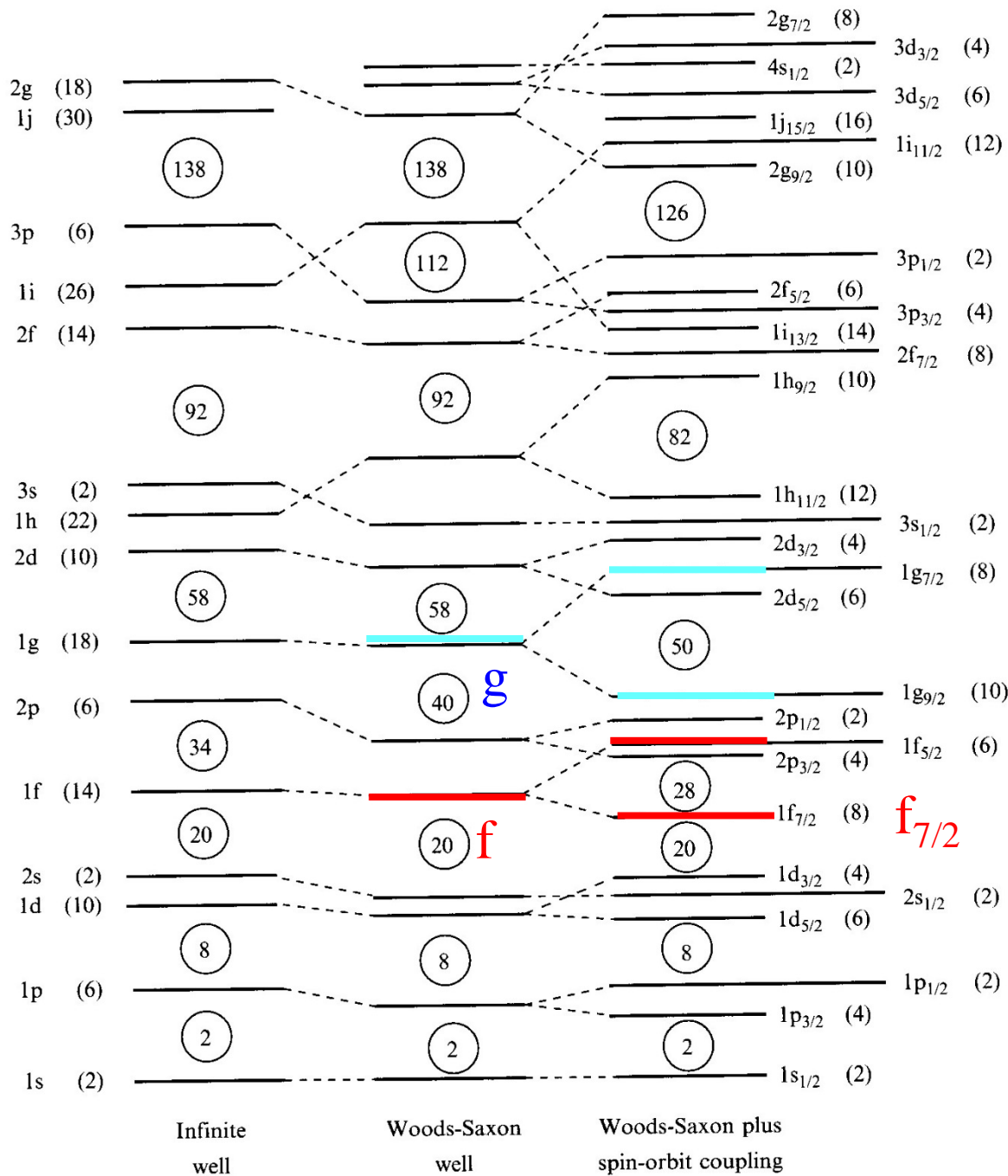
$$l \cdot s |\mathcal{Y}_{jlm}\rangle = -\frac{l+1}{2} |\mathcal{Y}_{jlm}\rangle \quad (j = l - 1/2)$$



$j = l \pm 1/2$ で準位が分離: l が大きくなればなるほど
分離は大

* ただし、スピン平均はゼロ:

$$+\frac{l}{2} (2(l+1/2) + 1) - \frac{l+1}{2} (2(l-1/2) + 1) = 0$$



g [18] \rightarrow $g_{7/2}$ [8]
 g [18] \rightarrow $g_{9/2}$ [10]

f [14] \rightarrow $f_{5/2}$ [6]
 f [14] \rightarrow $f_{7/2}$ [8]

Single particle spectra

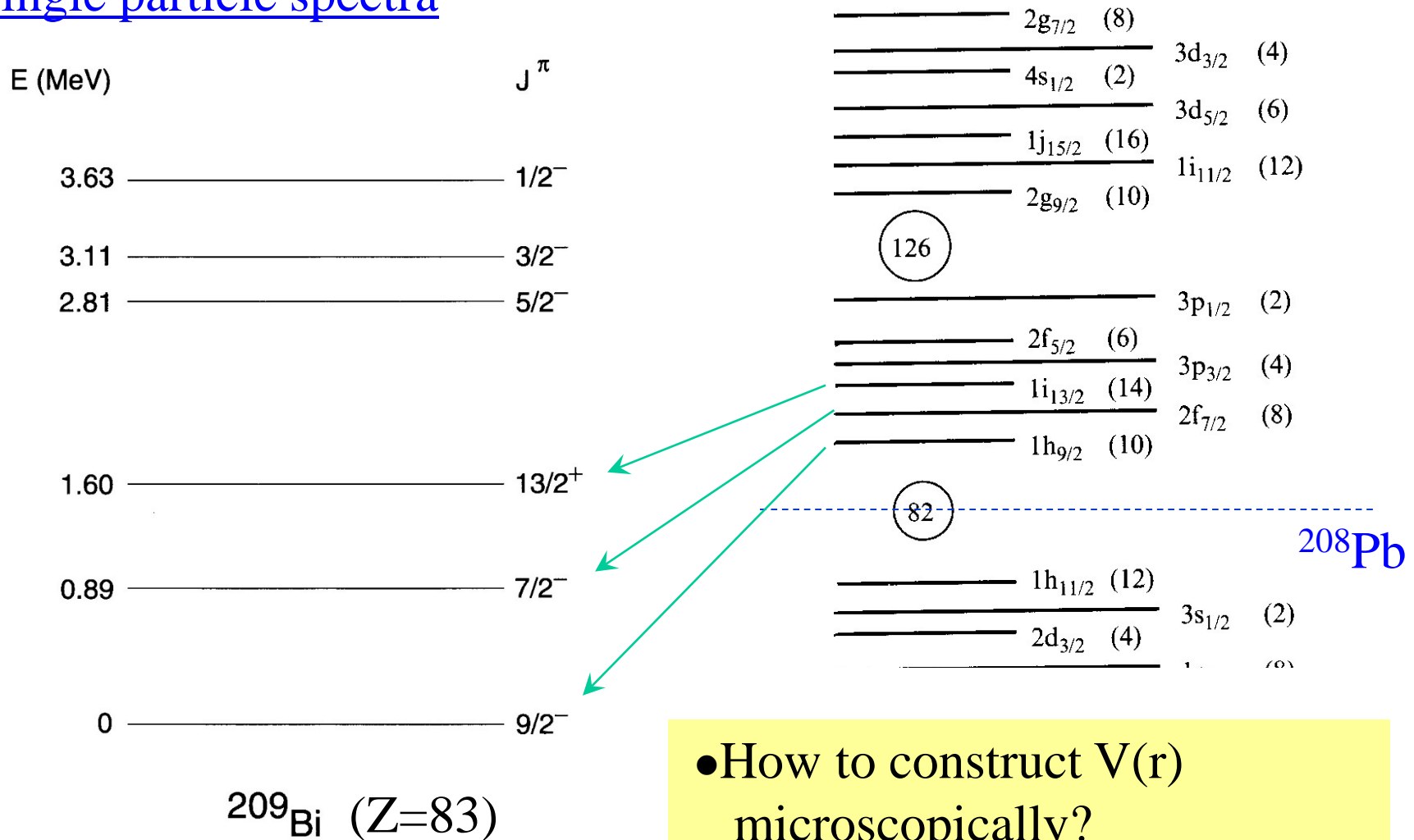


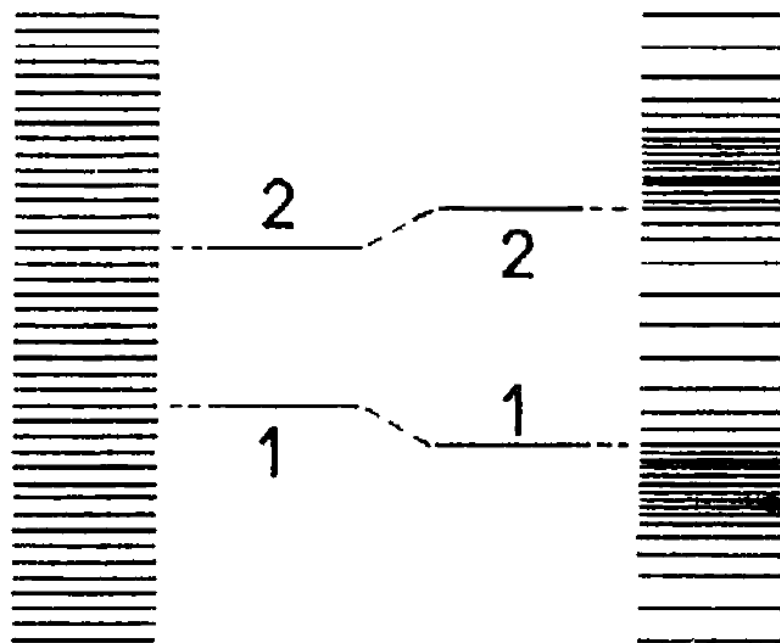
FIG. 3.6. Low-lying single-particle levels of ^{209}Bi .

- How to construct $V(r)$ microscopically?
- Does the independent particle picture really hold?

➡ Later in this course

何故、閉殻の原子核は安定になるのか？

準位密度



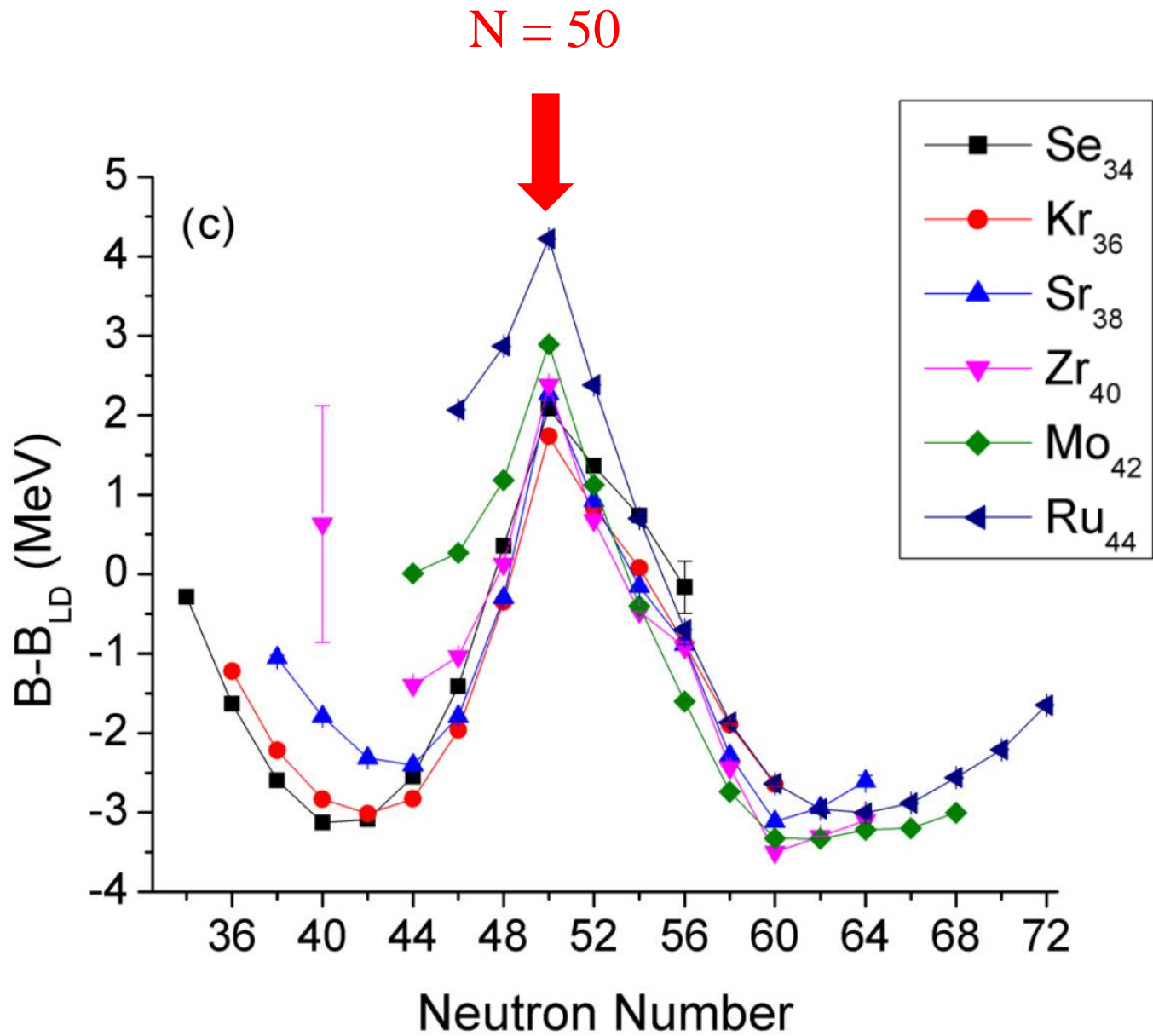
(a)

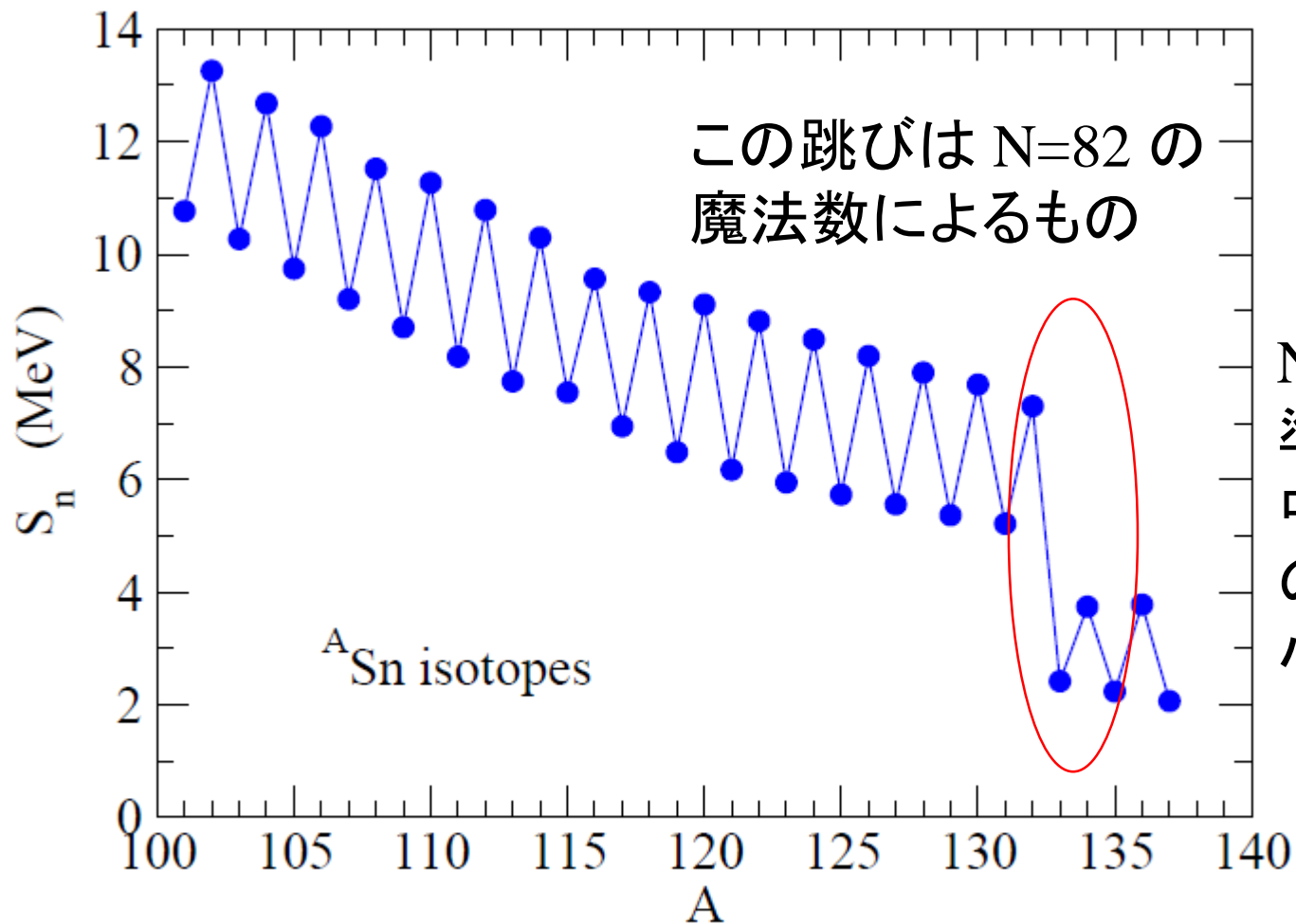
(b)

均一の場合

濃淡がある場合

準位密度に濃淡があれば、下から数えて濃淡の終わりまで準位が
つまると(図の1の場合)、均一の場合に比べてエネルギーが小さい





1n separation energy: $S_n(A,Z) = B(A,Z) - B(A-1,Z)$

生命誕生のための幸運な偶然

原子の魔法数

電子の数が 2, 10, 18, 36, 54, 86

元素の周期表

二重閉殻核

1A	2A	3A	4A	5A	6A	7A	8	1B	2B	3B	4B	5B	6B	7B	8		
1 H															2 He		
2 Li	4 Be														10 Ne		
3 Na	12 Mg														18 Ar		
4 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6 Cs	56 Ba	57 L	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7 Fr	88 Ra	89 A															
		57 L	58 La	59 Ce	60 Pr	61 Nd	62 Pm	63 Sm	64 Eu	65 Gd	66 Tb	67 Dy	68 Ho	69 Er	70 Tm	71 Yb	72 Lu
		89 A	90 Ac	91 Th	92 Pa	93 U	94 Np	95 Pu	96 Am	97 Cm	98 Bk	99 Cf	100 Es	101 Fm	102 Md	103 No	104 Lr

● 典型金属元素
● 半金属元素
● 非金属元素
● 遷移金属元素
● 希ガス

不活性ガス: He, Ne, Ar, Kr, Xe, Rn

原子核の魔法数

陽子または中性子の数が
2, 8, 20, 28, 50, 82, 126 の時安定

→ 例えば $^{16}_8\text{O}_8$ (二重閉殻核)

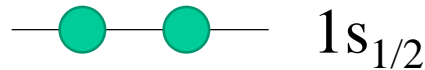
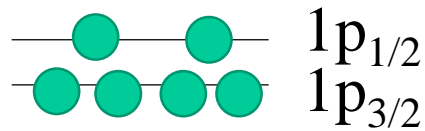
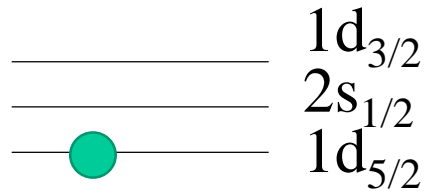
→ 酸素元素は元素合成の過程で数多く生成された

→ しかし、酸素は化学的には「活性」

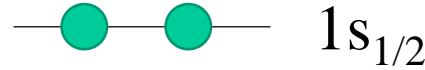
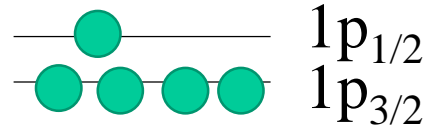
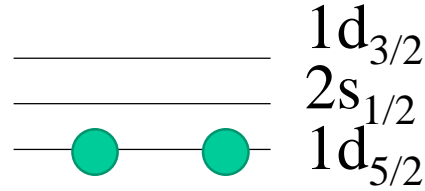
→ 化学反応により様々な複雑な物質をつくり生命に至った

single-j model

shell model



configuration 1



configuration 2

..... several others

angular momentum (spin) and parity for each configuration?

→ let us first investigate a single-j case

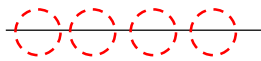
single-j level: one level with an angular momentum j

————— j

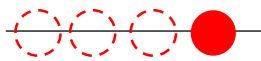

example: $j = p_{3/2}$

⊖ ⊖ ⊖ ⊖ ——— $p_{3/2}$

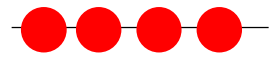

can accommodate 4 nucleons
($j_z = +3/2, +1/2, -1/2, -3/2$)

 $p_{3/2}$ can accommodate 4 nucleons
 ($j_z = +3/2, +1/2, -1/2, -3/2$)

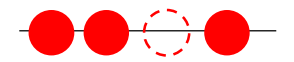

i) 1 nucleon

 $p_{3/2}$  $I^\pi = 3/2^-$
 (there are 4 ways to occupy this level)

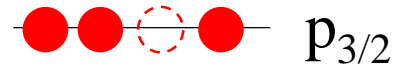
ii) 4 nucleons

 $p_{3/2}$  $I^\pi = 0^+$
 $I = j_1 + j_2 + j_3 + j_4$ (there is only 1 way to occupy this level)
 parity: $(-1) \times (-1) \times (-1) \times (-1) = +1$

iii) 3 nucleons

 $p_{3/2}$  $I^\pi = 3/2^-$
 $I = j_1 + j_2 + j_3$ (there are 4 ways to make a hole)
 parity: $(-1) \times (-1) \times (-1) = -1$

iii) 3 nucleons



$p_{3/2}$



$$I^\pi = 3/2^-$$

(there are 4 ways to make a hole)

$$\text{parity: } (-1) \times (-1) \times (-1) = -1$$

$$I = j_1 + j_2 + j_3$$

iv) 2 nucleons



$p_{3/2}$

there are $4 \times 3/2 = 6$ ways to occupy this level with 2 nucleons.

$$I = j_1 + j_2$$

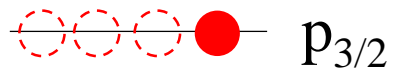


$$I^\pi = 0^+ \text{ or } 2^+ (= 1+5)$$

$$3/2 + 3/2 \rightarrow I = 0, \cancel{1}, \cancel{2}, \cancel{3}$$

anti-symmetrization

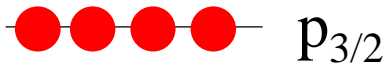
i) 1 nucleon



$$I^\pi = 3/2^-$$

(there are 4 ways to occupy this level)

ii) 4 nucleons

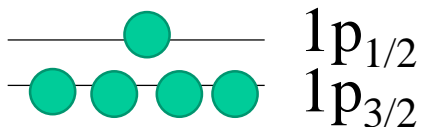


$$I^\pi = 0^+$$

(there is only 1 way to occupy this level)

$$I = j_1 + j_2 + j_3 + j_4$$

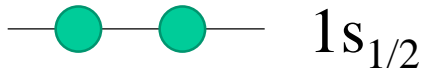
parity: $(-1) \times (-1) \times (-1) \times (-1) = +1$



$$I^\pi = 1/2^-$$



$$I^\pi = 0^+$$



$$I^\pi = 0^+$$

in total,
 $I^\pi = 1/2^-$

example: (main) shell model configurations for $^{11}_5\text{B}_6$

cf. $^{12}\text{C}(e,e'\text{K}^+)^{12}_\Lambda\text{B} (=^{11}\text{B}+\Lambda)$

MeV

5.02 ————— $3/2^-$

4.44 ————— $5/2^-$

2.12 ————— $1/2^-$

0 ————— $3/2^-$

$^{11}_5\text{B}_6$

cf. $^{12}\text{C}(e,e'\text{K}^+)^{12}_{\Lambda}\text{B} (=^{11}\text{B}+\Lambda)$

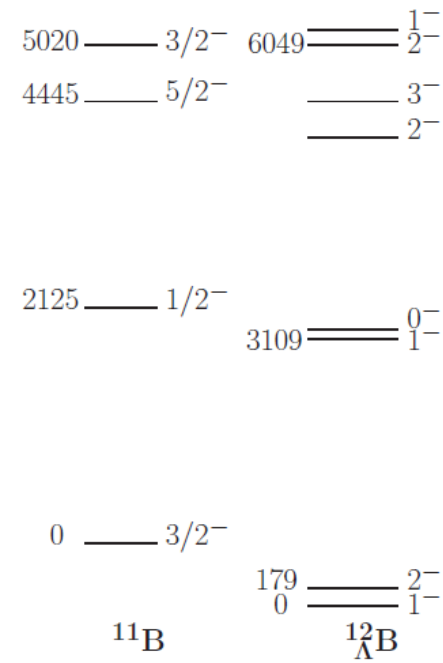
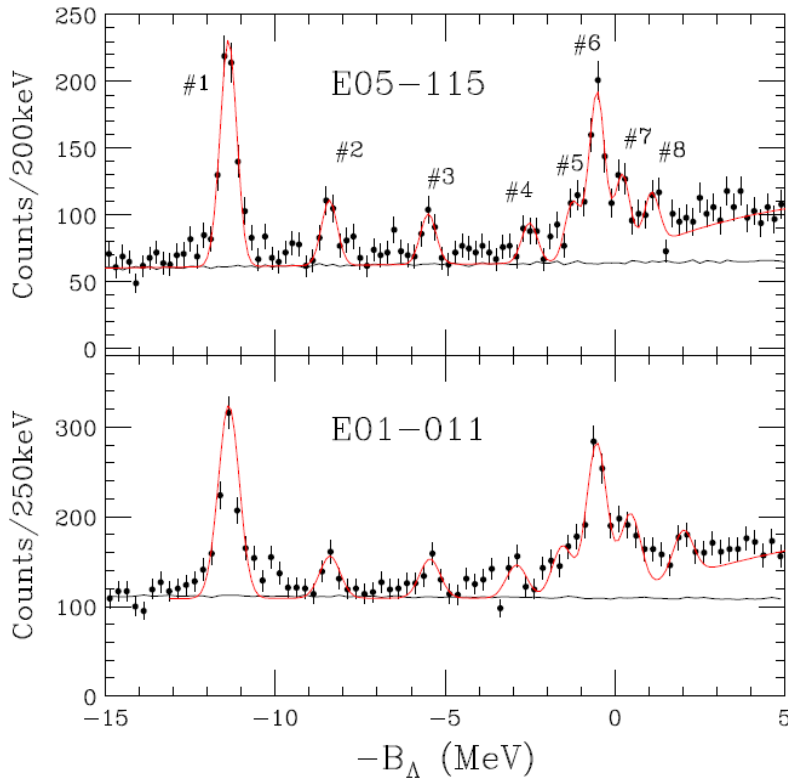
PHYSICAL REVIEW C **90**, 034320 (2014)



Experiments with the High Resolution Kaon Spectrometer at JLab Hall C and the new spectroscopy of $^{12}_{\Lambda}\text{B}$ hypernuclei

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(b)

example: (main) shell model configurations for ^{11}B

cf. $^{12}\text{C}(e,e'\text{K}^+)^{12}_{\Lambda}\text{B} (=^{11}\text{B}+\Lambda)$

MeV

5.02 ————— $3/2^-$

4.44 ————— $5/2^-$

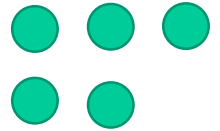
2.12 ————— $1/2^-$

0 ————— $3/2^-$

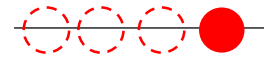
$^{11}_5\text{B}_6$

————— $1p_{1/2}$
 ————— $1p_{3/2}$

————— $1s_{1/2}$



single-j



$p_{3/2}$



$I^\pi = 3/2^-$



$p_{3/2}$



$I^\pi = 0^+$ or 2^+



$p_{3/2}$



$I^\pi = 3/2^-$



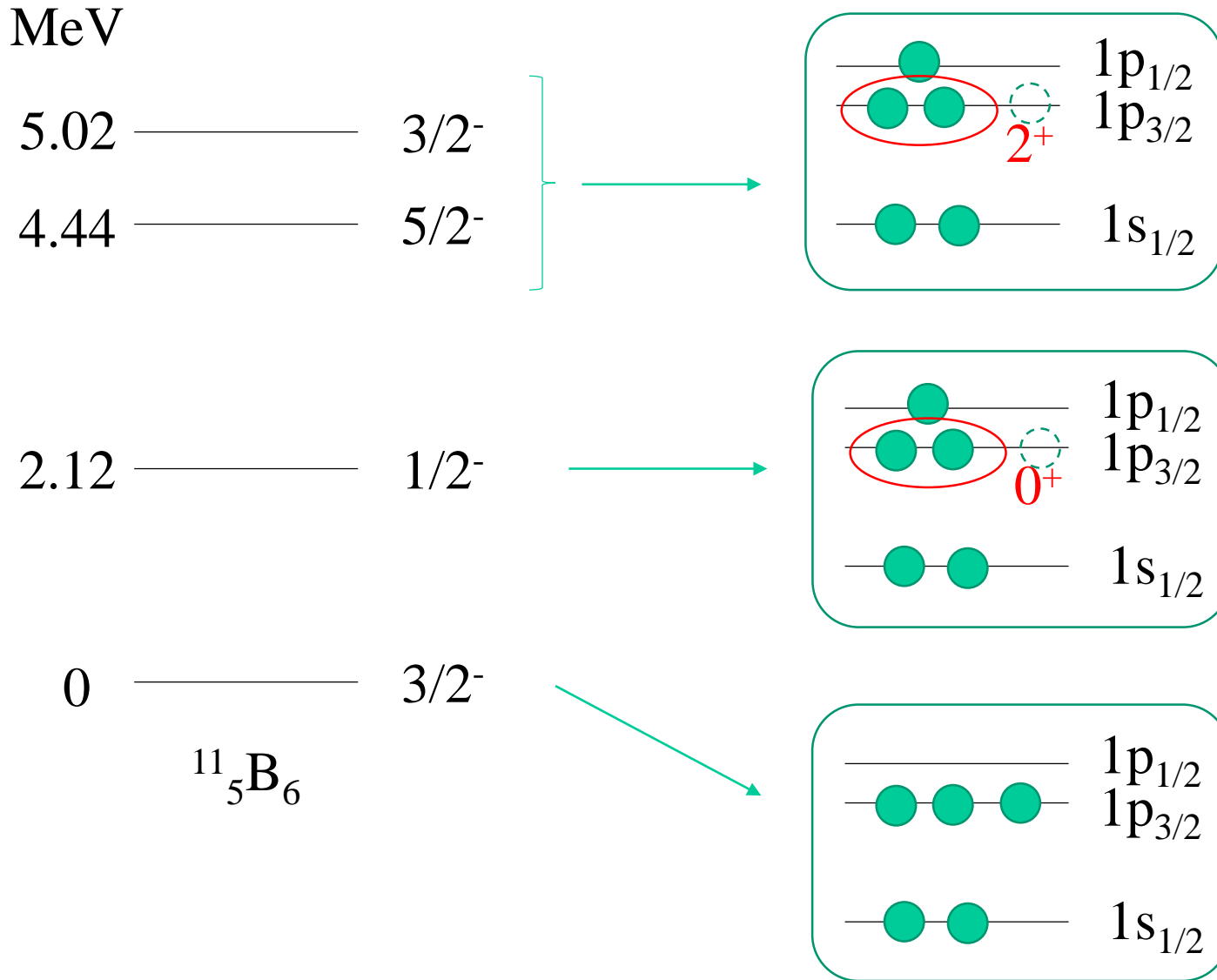
$p_{3/2}$



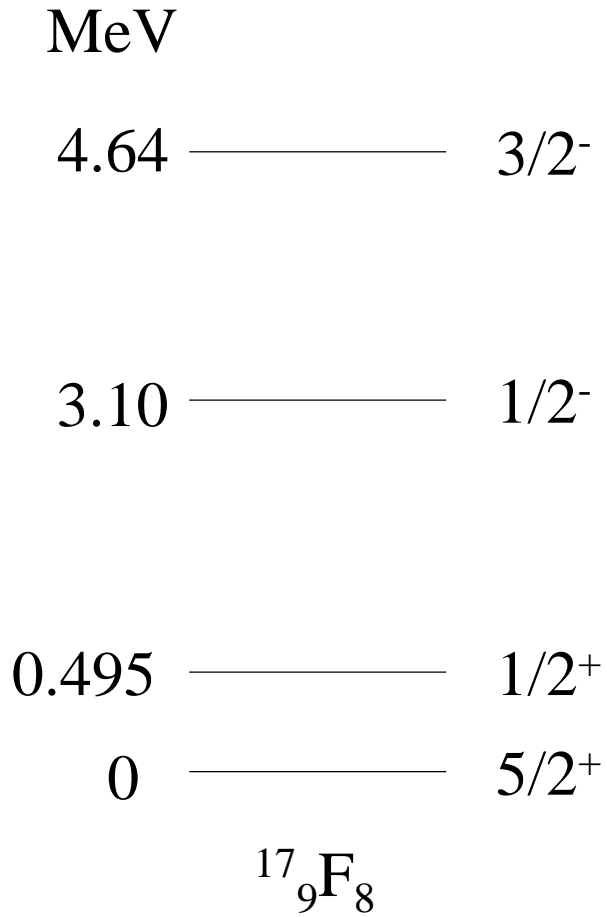
$I^\pi = 0^+$

example: (main) shell model configurations for ^{11}B

cf. $^{12}\text{C}(e,e'\text{K}^+)^{12}_{\Lambda}\text{B} (=^{11}\text{B}+\Lambda)$



another example: (main) shell model configurations for ^{17}F



another example: (main) shell model configurations for ^{17}F

