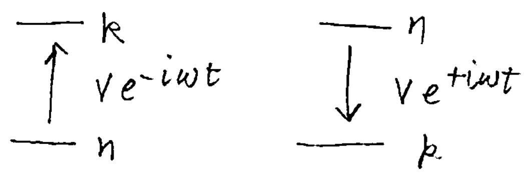
(復習) $V(t)=Ve^{\mp i\omega t}$



単位時間当たりの遷移確率：

$$\Gamma_k = \sum_{\text{終状態}} \frac{2\pi}{\hbar}|V_{kn}|^2\,\delta(\varepsilon_k-\varepsilon_n\mp\hbar\omega)$$

$$=\frac{2\pi}{\hbar}|V_{kn}|^2\,\rho(\varepsilon_n+\hbar\omega)$$

(Fermi の Golden Rule)

$$\rho(E)=\sum_\nu \delta(E-E_\nu)$$

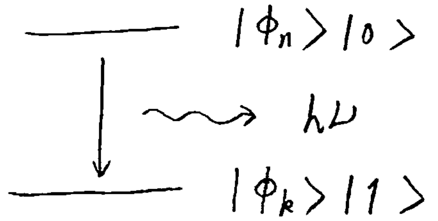
(状態密度)

phase space : $d^3n = \dfrac{V\,d^3p}{(2\pi\hbar)^3}$

・電磁場との相互作用

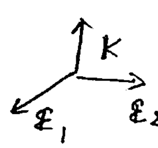
$$H=\frac{1}{2m}\left(\mathbb{p}+\frac{e}{c}\mathbb{A}(\mathrm{r},t)\right)^2+V(r)+H_{em}$$

$$\rightarrow\quad H_{int}\sim\frac{e}{mc}\,\mathbb{A}\cdot\mathbb{p}$$



$$H_{int}=\frac{e}{mc}\sqrt{\frac{2\pi c^2\hbar}{\omega V}}\;\boldsymbol{\varepsilon}\cdot\mathbb{p}\;e^{-i(\mathbb{k}\cdot\mathbb{r}+\omega t)}$$

$\boldsymbol{\varepsilon}$: 偏極ベクトル



$\mathbb{k}$: フォトンの運動量

$\mathbb{p}$: 粒子の運動量

$$\Gamma_k=\int \overset{d\hat{\mathbb{k}}}{d\Omega}\;\frac{1}{2\pi}\cdot\frac{e^2}{\hbar c}\,\frac{\varepsilon_n-\varepsilon_k}{\hbar}\left|\frac{1}{mc}\langle\phi_k|e^{-i\mathbb{k}\cdot\mathbb{r}}\,\boldsymbol{\varepsilon}\cdot\mathbb{p}\,|\phi_n\rangle\right|^2$$

$$\mathbb{P} \to \mathbb{P} - \frac{g}{c} e \mathbb{A}$$

▨ 電磁場との相互作用

$$H = \frac{1}{2m}\mathbb{P}^2 + V(r)$$

$$\to H = \frac{1}{2m}\left(\mathbb{P} + \frac{e}{c}\mathbb{A}(\mathbb{r},t)\right)^2 + V(r) + H_{em}$$

(note)

$$m\ddot{\mathbb{r}} = -e\left[\mathbb{E}(\mathbb{r},t) + \frac{1}{c}\mathbb{v}\times\mathbb{B}(\mathbb{r},t)\right]$$

"minimum principle"

Coulomb gauge $\quad \nabla\cdot\mathbb{A}(\mathbb{r},t) = 0, \ \phi = 0$

$$\mathbb{B} = \nabla\times\mathbb{A}$$

$$\mathbb{E} = -\frac{1}{c}\frac{\partial\mathbb{A}}{\partial t} \ \cancel{(-\nabla\phi)}^{0}$$

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = 0.$$

$$H_{em} = \frac{1}{8\pi}\int d\mathbb{r}\left(|\mathbb{B}|^2 + |\mathbb{E}|^2\right)$$

$$= \frac{1}{8\pi}\int d\mathbb{r}\left(\frac{1}{c^2}|\dot{\mathbb{A}}|^2 + |\nabla\times\mathbb{A}|^2\right)$$

Gauge transformation

$$\mathbb{A} \to \mathbb{A} - \nabla g$$

$$\mathbb{B} = \nabla\times A$$
$$= \nabla\times(A - \cancel{\nabla g})$$

$$\phi \to \phi + \dot{g}$$

Maxwell eq.

$$\nabla\cdot\mathbb{B} = 0$$
$$\nabla\cdot\mathbb{E} = \rho$$
$$\nabla\times\mathbb{B} - \frac{1}{c^2}\dot{\mathbb{E}} = \mu_0\vec{j}$$
$$\nabla\times\mathbb{E} + \dot{\mathbb{B}} = 0$$

$$H=\frac{1}{2m}\left(\mathbb{P}+\frac{e}{c}\mathbb{A}\right)^2+V(r)+H_{em}$$

$$=\underbrace{\frac{1}{2m}\mathbb{P}^2+V(r)}_{H_0}+H_{em}+\underbrace{\frac{1}{2m}\cdot\frac{e}{c}\left(\mathbb{P}\cdot\mathbb{A}+\mathbb{A}\cdot\mathbb{P}\right)+\frac{e^2}{2mc^2}\mathbb{A}^2}_{H_{int}}$$

$$H_{int}=\frac{e}{2mc}\left(\cancel{\frac{\hbar}{i}(\nabla\cdot\mathbb{A})}+\frac{\hbar}{i}\mathbb{A}\cdot\nabla+\mathbb{A}\cdot\mathbb{P}\right)+\frac{e^2}{2mc^2}\mathbb{A}^2$$

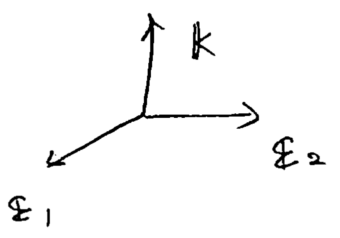
$$=\underbrace{\frac{e}{mc}\mathbb{A}\cdot\mathbb{P}}_{\downarrow\ \text{摂動}}+\underbrace{\frac{e^2}{2mc^2}\mathbb{A}^2}_{\downarrow\ 2\text{次}}\qquad \alpha=\frac{e^2}{\hbar c}=\frac{1}{137}$$

量子電気力学 (QED) : 第2量子化

$$\mathbb{A}(\mathbf{r},t)=\sqrt{\frac{2\pi c^2\hbar}{\omega V}}\sum_{\mathbb{k}}\sum_{\alpha=1,2}\left(a_{\mathbb{k}\alpha}\,\varepsilon_\alpha\, e^{i\mathbb{k}\cdot\mathbf{r}-i\omega_k t}+a^{\dagger}_{\mathbb{k}\alpha}\,\varepsilon_\alpha\, e^{-i\mathbb{k}\cdot\mathbf{r}+i\omega_k t}\right)$$

$a^{\dagger}_{\mathbb{k}\alpha}, a_{\mathbb{k}\alpha}$: photon の生成・消滅演算子

ε_α : 偏極 (polarization) ベクトル



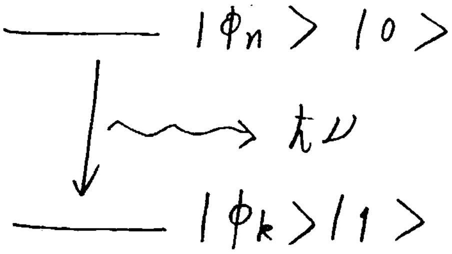
$\nabla\cdot\mathbb{A}=0 \longrightarrow \mathbb{k}\cdot\varepsilon=0$

$\varepsilon_\alpha\cdot\varepsilon_{\alpha'}=\delta_{\alpha,\alpha'}$

$\omega=c|\mathbb{k}|.$

$$\omega = ck = \frac{c}{\hbar} p$$
$$p = \frac{\hbar}{c}\omega$$

∘ photon emission　(bound → bound)



$$H_{int} = \frac{e}{mc}\sqrt{\frac{2\pi c^2\hbar}{\omega V}}\, \boldsymbol{\varepsilon}\cdot\mathbf{p}\, e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$
$$\uparrow$$
$$\frac{e}{mc}\mathbf{A}\cdot\mathbf{p}$$

終状態における状態の数 = photon の状態数

$$d^3n = \frac{V d^3p}{(2\pi\hbar)^3} = \frac{V}{(2\pi\hbar)^3} p^2 dp\, d\Omega_p$$
$$= \frac{V}{(2\pi\hbar)^3}\left(\frac{\hbar\omega}{c}\right)^2 \frac{1}{c} d(\hbar\omega)\, d\Omega_p$$

$$\downarrow$$

$$T = \sum_\alpha \int \frac{\cancel{V}}{(2\pi\hbar)^3}\left(\frac{\hbar\omega}{c}\right)^2 \frac{1}{c} d(\hbar\omega)\, d\Omega_p$$
$$\times \frac{2\pi}{\hbar}\left(\frac{e}{mc}\right)^2 \frac{2\pi c^2\hbar}{\omega \cancel{V}} \left|\langle\phi_k|\boldsymbol{\varepsilon}_\alpha\cdot\mathbf{p}\, e^{-i\mathbf{k}\cdot\mathbf{r}}|\phi_n\rangle\right|^2$$
$$\times \delta(\mathcal{E}_k - \mathcal{E}_n + \hbar\omega)$$

$$係数 = \frac{1}{\cancel{(2\pi\hbar)^3}_{2\pi}} \cdot \frac{(\cancel{\hbar}\omega)^{\cancel{2}}}{c^3} \cdot \frac{\cancel{2\pi}}{\hbar} \cdot \frac{e^2}{m^2\cancel{c^2}} \cdot \frac{\cancel{2\pi c^2\hbar}}{\cancel{\omega}}$$
$$= \frac{\omega e^2}{2\pi c^3 m^2\hbar} = \frac{\omega}{2\pi}\cdot\frac{e^2}{\hbar c}\cdot\frac{1}{m^2c^2}$$

$$= \sum_\alpha \int_{d\Omega_p} d(\hbar\omega)\, \frac{\omega}{2\pi}\cdot\frac{e^2}{\hbar c}\,\frac{1}{m^2c^2}\left|\langle\phi_k|\boldsymbol{\varepsilon}_\alpha\cdot\mathbf{p}\, e^{-i\mathbf{k}\cdot\mathbf{r}}|\phi_n\rangle\right|^2$$
$$\times \delta(\mathcal{E}_k - \mathcal{E}_n + \hbar\omega) \rightarrow$$

$$= \sum_{\alpha} \int d\Omega_p \ \frac{1}{2\pi} \cdot \frac{e^2}{\hbar c} \ \frac{\varepsilon_n - \varepsilon_k}{\hbar} \ \left| \frac{1}{mc} \langle \phi_k | e^{-i\mathbf{k}\cdot\mathbf{r}} \ \boldsymbol{\varepsilon}_{\alpha}\cdot \mathbf{p} \ | \phi_n \rangle \right|^2$$

10/20

∘ dipole approximation

$$e^{-i\mathbf{k}\cdot\mathbf{r}} \sim 1 \qquad \mathbf{k}\cdot\mathbf{r} \ll 1$$

$$k \ll \frac{1}{r} \quad (長波長近似)$$

$$\hbar\omega = pc \sim 10 \ \mathrm{eV}$$

$$k = \frac{p}{\hbar} \sim \frac{10\,\mathrm{eV}}{\hbar c} \sim \frac{1}{200\,\text{Å}}, \quad \lambda \sim \frac{\hbar c}{10\,\mathrm{eV}} \sim 200 \ \text{Å}$$

$$\left(\begin{aligned} \hbar c &\sim 200 \ \mathrm{MeV}\cdot\mathrm{fm} \\ &= 2000 \ \mathrm{eV}\cdot\text{Å} \end{aligned} \right)$$

$$\downarrow$$

$$\langle \phi_k | e^{-i\mathbf{k}\cdot\mathbf{r}} \ \boldsymbol{\varepsilon}\cdot\mathbf{p} \ | \phi_n \rangle \sim \langle \phi_k | \boldsymbol{\varepsilon}\cdot\mathbf{p} \ | \phi_n \rangle$$

(note) Hydrogen-like atom

$$R_{10}(r) = 2 \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-Zr/a_0}$$

-3.4
$\downarrow$ $10.2\,\mathrm{eV}$
-13.6

$$a_0 = \frac{\hbar}{mc\alpha} = 0.53 \ \text{Å}$$

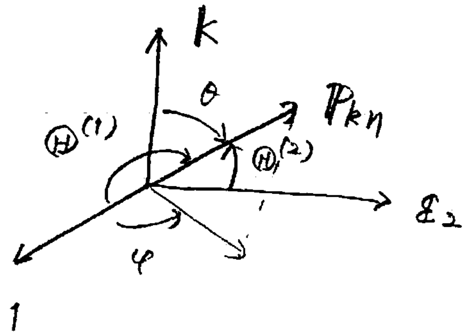
$$E_n = -\frac{1}{2} mc^2 \frac{Z^2\alpha^2}{n^2} \quad \rightsquigarrow \quad E_2 - E_1 = -\frac{mc^2}{2}\cdot Z^2\alpha^2 \left(\frac{1}{4} - 1\right) \sim 10.2 \ \mathrm{eV}$$

$$\Gamma \sim \sum_{\alpha=1,2} \int d\Omega_p \, \frac{e^2 \omega_{nk}}{2\pi m^2 \hbar c^3} \, |\langle \phi_k | \mathbb{e}_\alpha \cdot \mathbb{P} | \phi_n \rangle|^2$$

$$\omega_{nk} = (\varepsilon_n - \varepsilon_k)/\hbar$$

$$= \sum_{\alpha=1,2} \int d\Omega_p \, \frac{e^2 \omega_{nk}}{2\pi m^2 \hbar c^3} \, |\langle \phi_k | \mathbb{P} | \phi_n \rangle|^2 \cos^2 \Theta^{(\alpha)}$$

$\Theta^{(\alpha)}$: angle between $\mathbb{P}_{kn}$ and $\mathbb{e}_\alpha$



$$\cos \Theta^{(1)} = \sin\theta \cos\varphi$$
$$\cos \Theta^{(2)} = \sin\theta \sin\varphi$$

↓

$$\sum_{\alpha=1,2} \int d\Omega_p \cos^2 \Theta^{(\alpha)} = \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\varphi \, \sin^2\theta$$

$$= 2\pi \int_{-1}^{1} d(\cos\theta) \, (1 - \cos^2\theta)$$

$$= 2\pi \cdot \left[x - \frac{x^3}{3} \right]_{-1}^{1} = \frac{8\pi}{3}$$

↓

$$\boxed{\Gamma = \frac{4}{3} \cdot \frac{e^2}{\hbar c} \cdot \frac{\omega_{nk}}{m^2 c^2} \, |\langle \phi_k | \mathbb{P} | \phi_n \rangle|^2}$$

• r - representation

$$[\mathbb{P}^2, \mathbb{r}] = -[\mathbb{r}, \mathbb{P}^2] = -[i\hbar\nabla_p, \mathbb{P}^2] = -2i\hbar\,\mathbb{P}.$$

$\downarrow$

$$\langle\phi_k|\mathbb{P}|\phi_n\rangle = \langle\phi_k|\frac{1}{-2i\hbar}[\mathbb{P}^2, \mathbb{r}]|\phi_n\rangle$$

$$= \frac{im}{\hbar}\langle\phi_k|[H_0, \mathbb{r}]|\phi_n\rangle$$

$$H_0 = \frac{\mathbb{P}^2}{2m} + V_0(r) \qquad = \frac{im}{\hbar}\langle\phi_k|(H_0\mathbb{r} - \mathbb{r}H_0)|\phi_n\rangle$$

$$= \frac{im}{\hbar}(\varepsilon_k - \varepsilon_n)\langle\phi_k|\mathbb{r}|\phi_n\rangle.$$

E1 遷移

(note) $x = r\sin\theta\cos\varphi = r\cdot\frac{1}{2}\sqrt{\frac{8\pi}{3}}(-Y_{11} + Y_{1-1})$

$y = r\sin\theta\sin\varphi = r\cdot\frac{i}{2}\sqrt{\frac{8\pi}{3}}(Y_{11} + Y_{1-1})$

$z = r\cos\theta = r\cdot\sqrt{\frac{4\pi}{3}}\,Y_{10}$

◦ 選択則

角度成分 $\langle Y_{l_k m_k}|Y_{1m}|Y_{l_n m_n}\rangle$

$$= \langle Y_{l_k m_k}|\left(Y_{1m}|Y_{l_n m_n}\rangle\right)$$

(note)

$$Y_{1m}(\Omega)\,Y_{l_n m_n}(\Omega) = \sum_{L=|l_n-1|}^{l_n+1}\sqrt{\frac{3\cdot(2l_n+1)}{4\pi(2L+1)}}\langle 1\,0\,l_n\,0|L\,0\rangle \times \langle 1\,m\,l_n\,m_n|L\;\;m+m_n\rangle \times Y_{L\,m+m_n}(\Omega)$$

$\downarrow$

$$\langle Y_{l_k m_k} | Y_{1m} | Y_{l_n m_n} \rangle$$

$$= \sum_{L=|l_n-1|}^{l_n+1} \langle 1m\ l_n m_n | l_k\ m+m_n \rangle \underbrace{\int d\Omega\, Y^*_{l_k m_k}(\Omega) Y_{L\, m+m_n}}_{\delta_{L,l_k}\delta_{m_k,m+m_n}}$$

$$\times \frac{\sqrt{3}\,\hat{l}_n}{\sqrt{4\pi}\,\hat{L}} \langle 10\, l_n 0 | L 0 \rangle$$

$$\rightarrow \quad l_k = \cancel{l_n},\ l_n \pm 1$$

$$m_k = m_n,\ m_n \pm 1$$

パリティ変化: +

$\Delta S = 0.$

$2p \rightarrow 1s$

(note)

- $\ddot{r}$ - representation

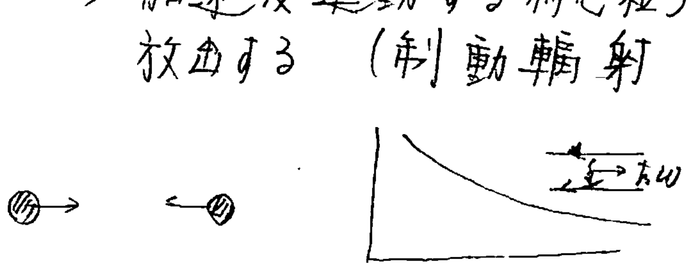
$$[H_0, \mathbb{P}] = [V_0(r), \mathbb{P}] = i\hbar (\nabla V_0)$$

$\downarrow$

$$\langle \phi_k | \mathbb{P} | \phi_n \rangle = \frac{1}{\varepsilon_k - \varepsilon_n} \langle \phi_k | \underbrace{\varepsilon_k \mathbb{P} - \mathbb{P}\varepsilon_n}_{=[H_0, \mathbb{P}]} | \phi_n \rangle$$

$$= \frac{i\hbar}{\varepsilon_k - \varepsilon_n} \langle \phi_k | \underbrace{\nabla V_0}_{\downarrow\ -加速度/m} | \phi_n \rangle$$

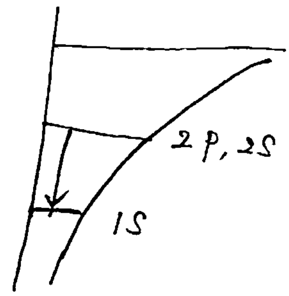
→ 加速度運動する荷電粒子は フォトンを自発的に放出する（制動輻射：bremsstrahlung）



◦ $2p \to 1s$ transition

$$T = \frac{4}{3} \cdot \frac{e^2}{\hbar c} \cdot \frac{\omega_{nk}^3}{c^2} \quad |\langle \phi_k | \mathbf{r} | \phi_n \rangle|^2 .$$

$$n : 2p , \qquad k : 1s$$



$$\sum_{m'} T_{nlm \to n'l'm'}$$

$$= \sum_{m'} \frac{4}{3} \cdot \frac{e^2}{\hbar c} \cdot \frac{\omega_{nk}^3}{c^2} \quad \langle \phi_{n'l'm'} | \mathbf{r} | \phi_{nlm} \rangle \cdot \langle \phi_{nlm} | \mathbf{r} | \phi_{n'l'm'} \rangle$$

(note)
$$x = r \cdot \frac{1}{2} \sqrt{\frac{8\pi}{3}} \left(-Y_{11} + Y_{1-1} \right)$$
$$y = r \cdot \frac{i}{2} \sqrt{\frac{8\pi}{3}} \left(Y_{11} + Y_{1-1} \right)$$
$$z = r \cdot \sqrt{\frac{4\pi}{3}} \, Y_{10}$$

$\downarrow$

$$xx + yy + zz = r^2 \cdot \frac{4\pi}{3} \left\{ \frac{1}{2} \left(-Y_{11} + Y_{1-1} \right)^2 - \frac{1}{2} \left(Y_{11} + Y_{1-1} \right)^2 + Y_{10}^2 \right\}$$
$$= r^2 \cdot \frac{4\pi}{3} \left(-2 \, Y_{11} Y_{1-1} + Y_{10}^2 \right)$$

$$= \overset{m'=0}{\cancel{\sum_{m'}}} \frac{4}{3} \cdot \frac{e^2}{\hbar c} \cdot \frac{\omega_{nk}^3}{c^2} \left(\int_0^\infty r^2 dr \; r \; R_{nl}(r) R_{n'l'}(r) \right)^2 \cdot \frac{4\pi}{3}$$

$$\times \Big\{ - \langle Y_{00} | Y_{11} | Y_{1m} \rangle \langle Y_{1m} | Y_{1-1} | Y_{00} \rangle$$
$$- \langle Y_{00} | Y_{1-1} | Y_{1m} \rangle \langle Y_{1m} | Y_{11} | Y_{00} \rangle$$
$$+ \langle Y_{00} | Y_{10} | Y_{1m} \rangle \langle Y_{1m} | Y_{10} | Y_{00} \rangle \Big\}$$

(note)
$$\langle Y_{00} | Y_{1k} | Y_{1m} \rangle = \int d\hat{r} \; Y_{00}^* \, Y_{1k} \, Y_{1m}$$
$$= \frac{1}{\sqrt{4\pi}} \int d\hat{r} \; (-)^k \, Y_{1-k}^* \, Y_{1m}$$
$$= \frac{1}{\sqrt{4\pi}} \cdot (-)^k \, \delta_{m,-k}$$

$$\langle Y_{1m} | Y_{1k} | Y_{00} \rangle = \int d\hat{r}\, Y_{1m}^* Y_{1k} Y_{00}$$
$$= \frac{1}{\sqrt{4\pi}} \int d\hat{r}\, Y_{1m}^* Y_{1k}$$
$$= \frac{1}{\sqrt{4\pi}} \delta_{m,k}$$

$\Downarrow$

$$\text{angular part} = \frac{1}{4\pi} \quad \text{for all } m$$

$\Downarrow$

$$\Gamma = \frac{4}{9} \cdot \frac{e^2}{\hbar c} \cdot \frac{\omega^3}{c^2} \left(\int_0^\infty r^3 dr\, R_{n\ell}(r) R_{n'\ell'}(r) \right)^2$$

(note) in general $\quad \sum_{m'} \rightarrow \frac{1}{2J+1} \sum_m \sum_{m'}$

(終状態で和, 始状態で平均)

- radial integral :

$$R_{1s}(r) = 2 \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-Zr/a_0} \quad \leftarrow \text{ch. 12}$$
$$R_{2p}(r) = \frac{1}{\sqrt{3}} \left(\frac{Z}{2a_0}\right)^{\frac{3}{2}} \frac{Zr}{a_0} e^{-Zr/2a_0}$$

$$a_0 = \frac{\hbar}{mc\alpha}$$

$\Downarrow$

$$\int_0^\infty r^3 dr\, R_{1s}(r) R_{2p}(r) = \frac{1}{\sqrt{6}} \left(\frac{Z}{a_0}\right)^4 \int_0^\infty r^4 e^{-3Zr/2a_0} dr.$$

$$x = \frac{3Z}{2a_0} r$$

$$= \frac{1}{\sqrt{6}} \left(\frac{Z}{a_0}\right)^4 \left(\frac{2a_0}{3Z}\right)^5 \underbrace{\int_0^\infty x^4 e^{-x} dx}_{=4!} = \frac{24}{\sqrt{6}} \left(\frac{2}{3}\right)^5 \frac{a_0}{Z}$$

$$E_n = -\frac{1}{2} mc^2 \frac{Z^2 \alpha^2}{n^2} \qquad c = 3 \times 10^{10}\ \mathrm{cm/sec}$$

$$\hbar\omega = -\frac{1}{2} mc^2 Z^2 \alpha^2 \cdot \left(\frac{1}{4} - 1\right) = \frac{3}{8} mc^2 \cdot Z^2 \alpha^2$$

$\Downarrow$

$$\Gamma = \frac{4}{9} \cdot \frac{e^2}{\hbar c} \cdot \frac{1}{c^2} \left(\frac{\frac{3}{8} mc^2 \cdot Z^2 \alpha^2}{\hbar} \right)^3 \cdot \frac{24^2}{6} \left(\frac{2}{3}\right)^{10} \left(\frac{a_0}{Z}\right)^2$$

$$= \frac{4}{9} \cdot \frac{\alpha}{\cancel{c^2}} \cdot \frac{3^3}{8^3} (mc^2)^3 Z^6 \alpha^6 \cdot \frac{1}{\hbar^3} \cdot \frac{24^2}{6} \left(\frac{2}{3}\right)^{10} Z^{-2} \frac{\hbar^2}{\cancel{m^2 c^2 \alpha^2}}$$

$$= mc^2 \cdot \alpha^5 \cdot Z^4 \cdot \frac{1}{\hbar} \cdot \left(\frac{4}{9} \cdot \frac{3^3}{8^3} \cdot \frac{24^2}{6} \cdot \frac{4}{9} \left(\frac{2}{3}\right)^8 \right)$$

$$\parallel$$

$$\frac{4 \cdot 4 \cdot \cancel{9} \cdot \cancel{3} \cdot \cancel{24} \cdot 24}{\cancel{9} \cdot \cancel{9} \cdot \cancel{8} \cdot 8 \cdot 8 \cdot 6} = 1$$

$$= \left(\frac{2}{3}\right)^8 \cdot \frac{mc^2}{\hbar} \cdot \alpha^5 \cdot Z^4$$

$$\sim 0.627 \times 10^9\ Z^4\ \mathrm{sec}^{-1} \qquad P = 1 - \Gamma t \sim e^{-\Gamma t}$$

$$\tau \equiv 1/\Gamma = 1.59 \times 10^{-4} \cdot Z^{-4}\ \mathrm{sec} \quad (平均寿命)$$

(note)

$$\left(\frac{2}{3}\right)^8 \sim 0.039$$

$$\Gamma \sim 0.039 \times \frac{0.51\ \mathrm{MeV}}{197.1 \times 10^{-13}\ \mathrm{MeV \cdot cm}} \cdot \left(\frac{1}{137}\right)^5 \cdot 3 \times 10^{10}\ \mathrm{cm/sec} \times Z^4$$

$$\sim 6.27 \times 10^8\ Z^4\ \mathrm{sec}^{-1}$$

∘ Quadrupole transition

$$e^{-i\mathbb{k}\cdot\mathbb{r}} \sim 1 - \underline{i\mathbb{k}\cdot\mathbb{r}} + \cdots$$

$$(\mathbb{k}\cdot\mathbb{r})(\boldsymbol{\varepsilon}\cdot\mathbb{p}) = \frac{1}{2}\left\{(\mathbb{k}\cdot\mathbb{r})(\boldsymbol{\varepsilon}\cdot\mathbb{p}) + (\mathbb{k}\cdot\mathbb{p})(\boldsymbol{\varepsilon}\cdot\mathbb{r})\right\}$$
$$+ \frac{1}{2}\left\{(\mathbb{k}\cdot\mathbb{r})(\boldsymbol{\varepsilon}\cdot\mathbb{p}) - (\mathbb{k}\cdot\mathbb{p})(\boldsymbol{\varepsilon}\cdot\mathbb{r})\right\}$$

the first term

$$= \frac{1}{2}\mathbb{k}\cdot(\mathbb{r}\mathbb{p} + \mathbb{p}\mathbb{r})\cdot\boldsymbol{\varepsilon}$$

(note)

$$[H_0, \mathbb{r}\mathbb{r}] = -\frac{\hbar^2}{2m}[\nabla^2, \mathbb{r}\mathbb{r}] = -\frac{\hbar^2}{m}(\nabla\mathbb{r} + \mathbb{r}\nabla)$$

$$= -\frac{i\hbar}{m}(\mathbb{p}\mathbb{r} + \mathbb{r}\mathbb{p})$$

$$\rightarrow \frac{\mathbb{k}}{2}\cdot\langle\phi_k|\mathbb{r}\mathbb{p} + \mathbb{p}\mathbb{r}|\phi_n\rangle\cdot\boldsymbol{\varepsilon}$$

$$= \frac{im}{\hbar}\cdot\frac{\mathbb{k}}{2}\cdot\langle\phi_k|[H_0, \mathbb{r}\mathbb{r}]|\phi_n\rangle\cdot\boldsymbol{\varepsilon}$$

$$= \frac{im}{2\hbar}(\varepsilon_k - \varepsilon_n)\,\mathbb{k}\cdot\langle\phi_k|\mathbb{r}\mathbb{r}|\phi_n\rangle\cdot\boldsymbol{\varepsilon}$$

E2 遷移
$\Delta l = 2$
no spin change
$(3d \rightarrow 1s)$

the second term :

$$(\mathbb{k}\cdot\mathbb{r})(\boldsymbol{\varepsilon}\cdot\mathbb{p}) - (\mathbb{k}\cdot\mathbb{p})(\boldsymbol{\varepsilon}\cdot\mathbb{r}) = \underbrace{(\mathbb{k}\times\boldsymbol{\varepsilon})}_{=\,\vec{B}}\cdot\underbrace{(\mathbb{r}\times\mathbb{p})}_{=\,\vec{\ell}}$$

M1 遷移

$\Delta \ell - 1$

no parity change