Heavy-ion sub-barrier fusion reactions: a sensitive tool to probe nuclear structure

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1. *Introduction: heavy-ion fusion reactions*
2. *Fusion barrier distributions*
3. *Semi-microscopic modelling of sub-barrier fusion*
4. *Double octupole phonon excitations in $^{16}O+^{208}Pb$*
5. *Quasi-elastic barrier distribution*
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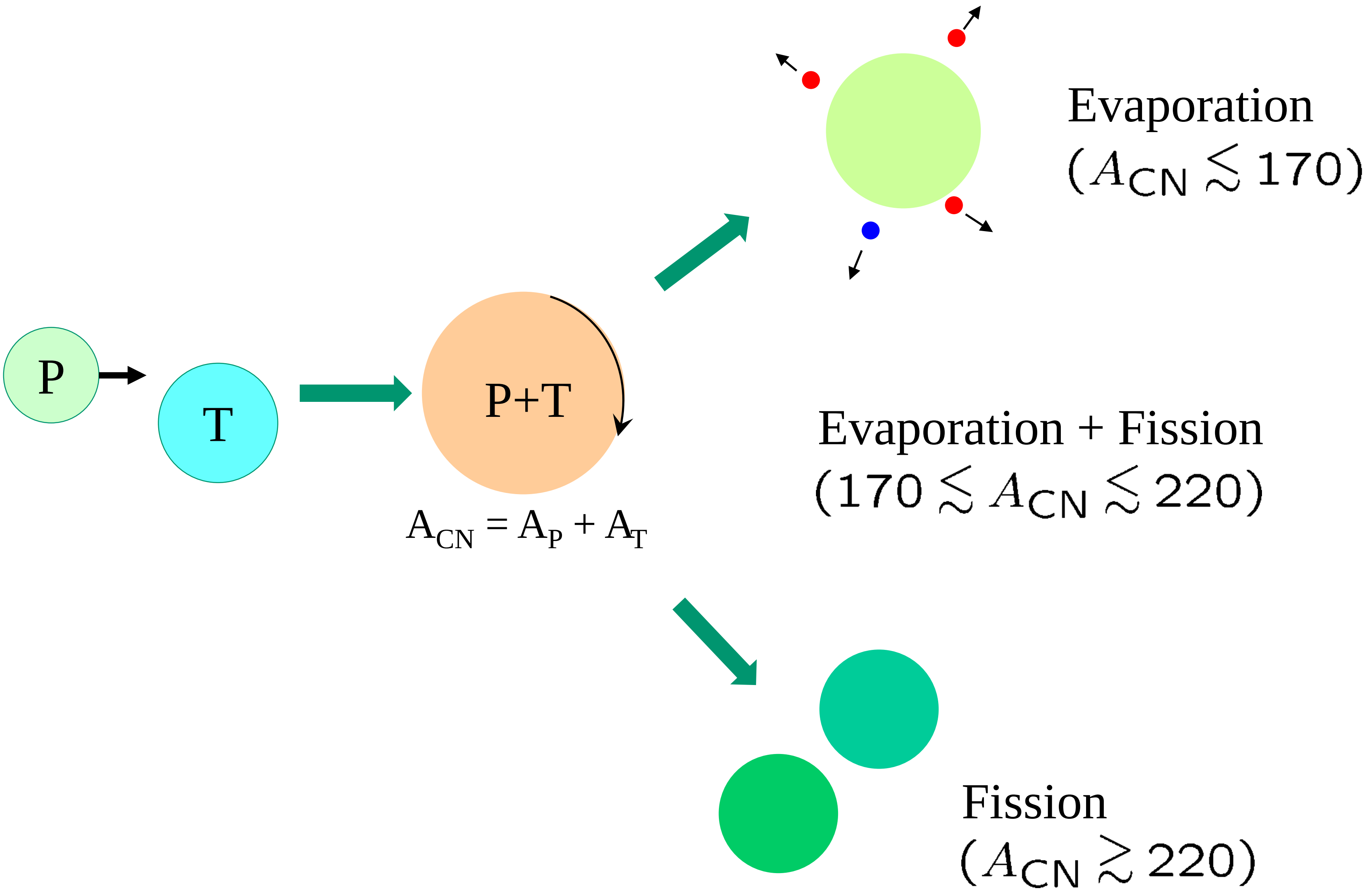
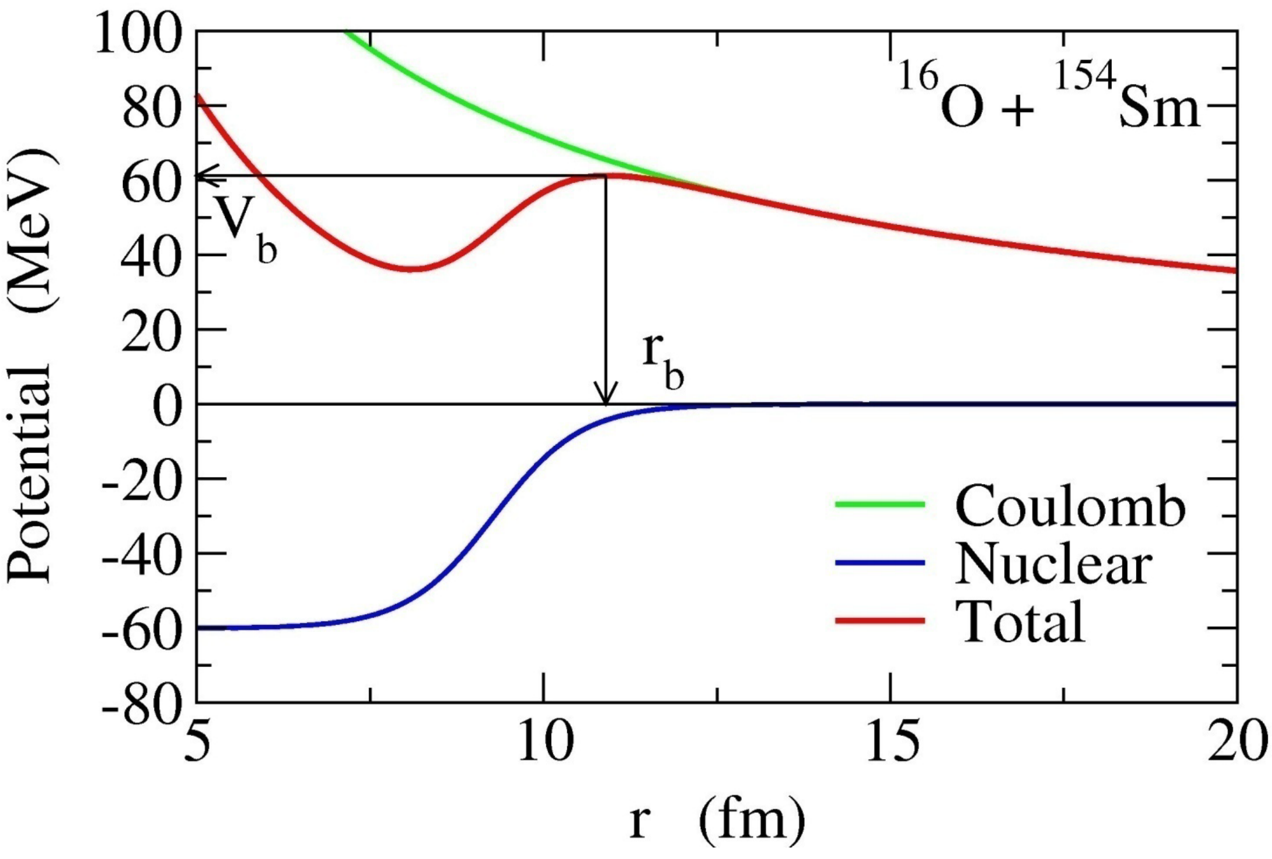
Introduction: heavy-ion fusion reactions

Fusion: compound nucleus formation



Inter-nucleus potential



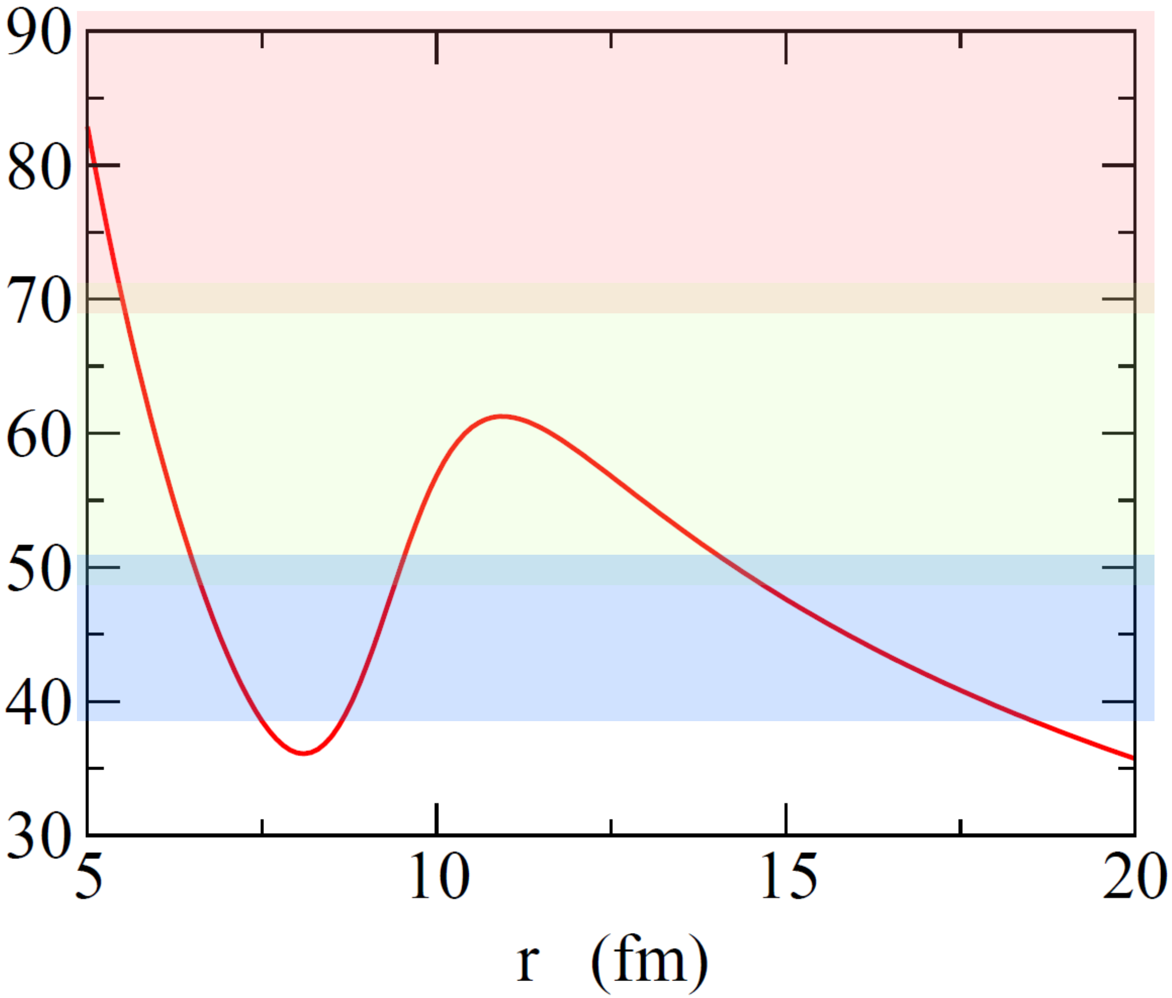
Two forces:

1. Coulomb force
   Long range,
   repulsive
2. Nuclear force
   Short range,
   attractive

Potential barrier
(Coulomb barrier)

- •above barrier energies
- → •sub-barrier energies
- → •deep subbarrier energies

Energy regions



above barrier region

$(E \gtrsim V_b + 10\text{MeV})$

sub-barrier region ←

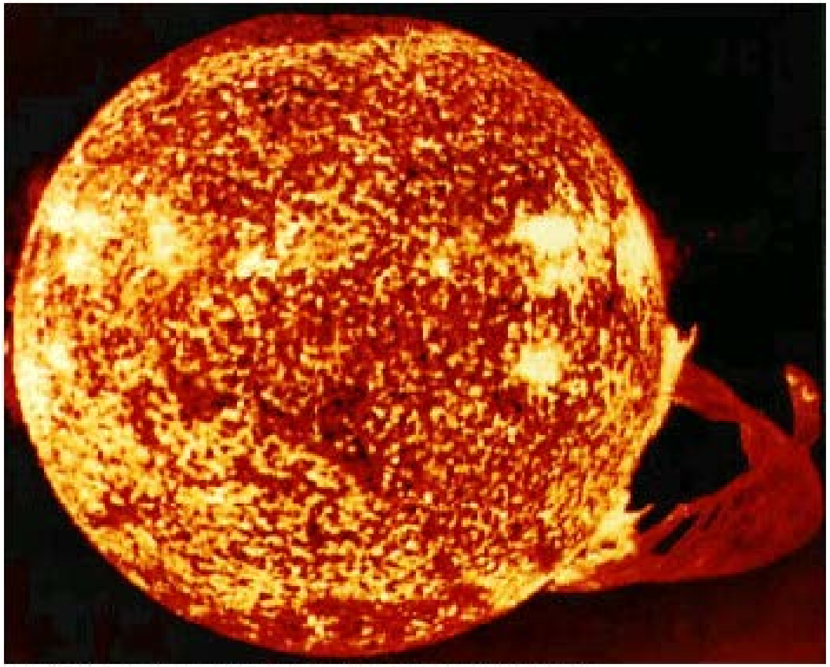
$(|E - V_b| \lesssim 10\text{MeV})$

deep sub-barrier region

$(E \lesssim V_b - 10\text{MeV})$

Why (deep) sub-barrier fusion?

Two obvious reasons:



NASA, Skylab space station December 19, 1973, solar flare reaching 588 000 km off solar surface

discovering new elements
(SHE by cold fusion reactions)

nuclear astrophysics
(fusion in stars)

cf. ^{209}Bi(^{70}Zn,n)

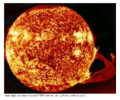
$V_{\mathrm{Bass}} = 260.4$ MeV

$E_{\mathrm{cm}}^{(\mathrm{exp})} = 261.4$ (1st, 2nd), 262.9 MeV (3rd)

Why subbarrier fusion?

Two obvious reasons:

✓discovering new elements (SHE) 

✓nuclear astrophysics (fusion in stars) 

Other reasons:

◆ reaction mechamism

strong interplay between reaction and structure

(channel coupling effects)

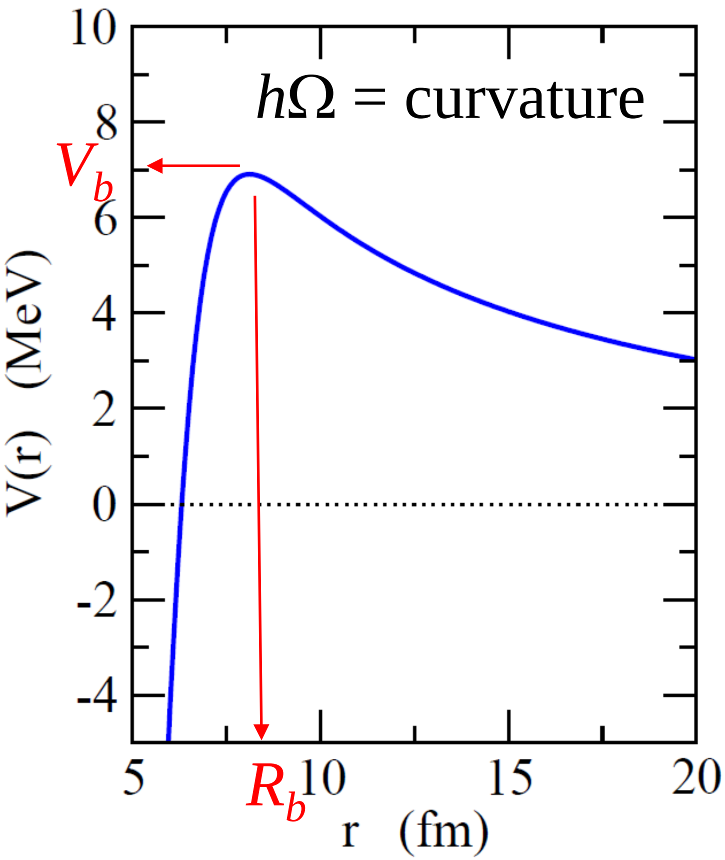
cf. high *E* reactions: much simpler reaction mechanism

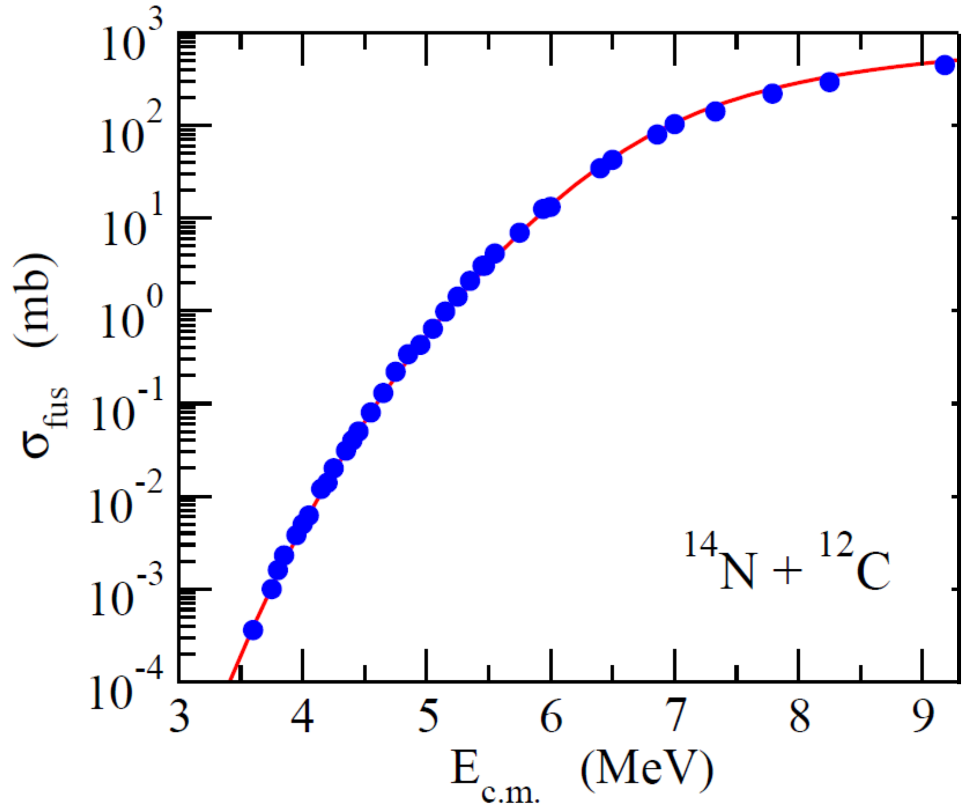
◆ many-particle tunneling

cf. alpha decay: fixed energy

tunneling in atomic collision: less variety of intrinsic motions the simplest approach: potential model with $V(r)$ + absorption

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

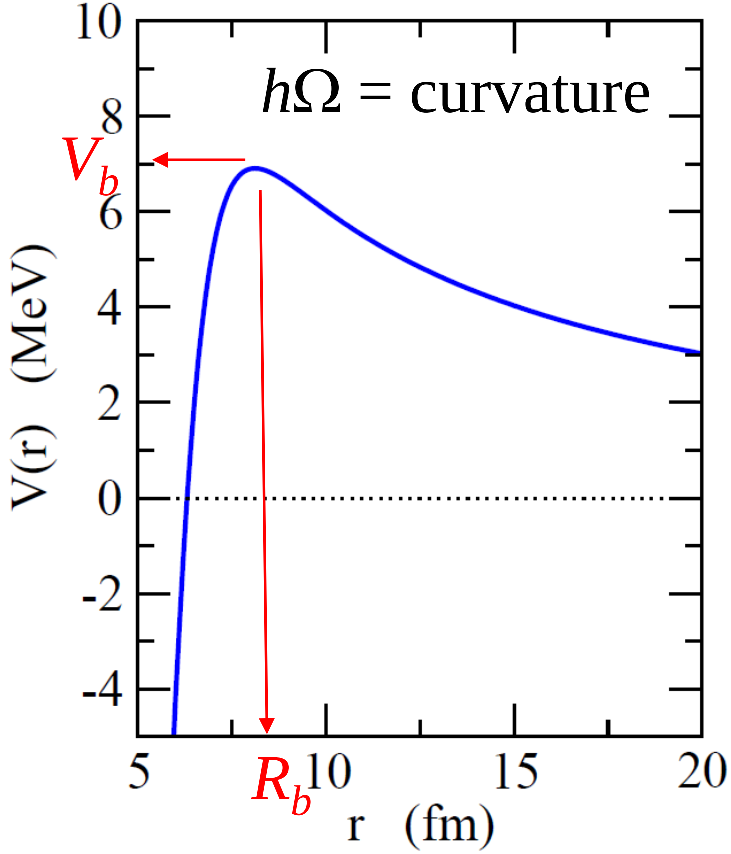


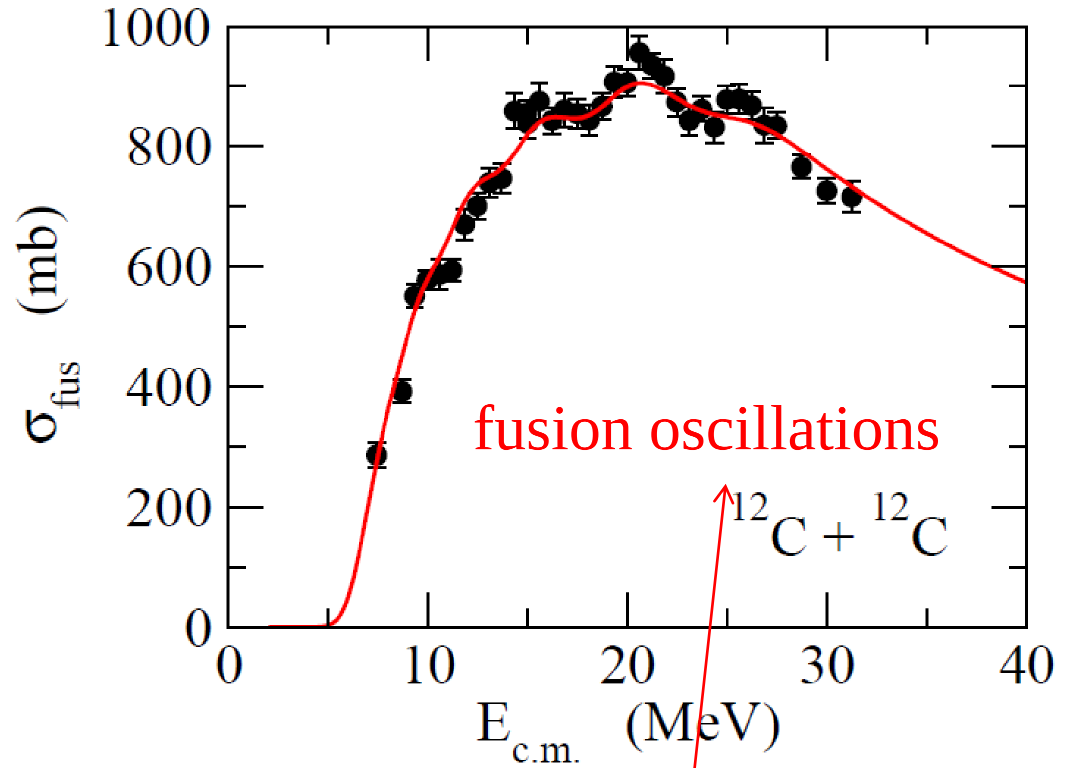


➢ Wong formula [C.Y. Wong, PRL31 ('73)766]

$$\sigma_{\rm fus}(E) \sim \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

potential model: $V(r)$ + absorption



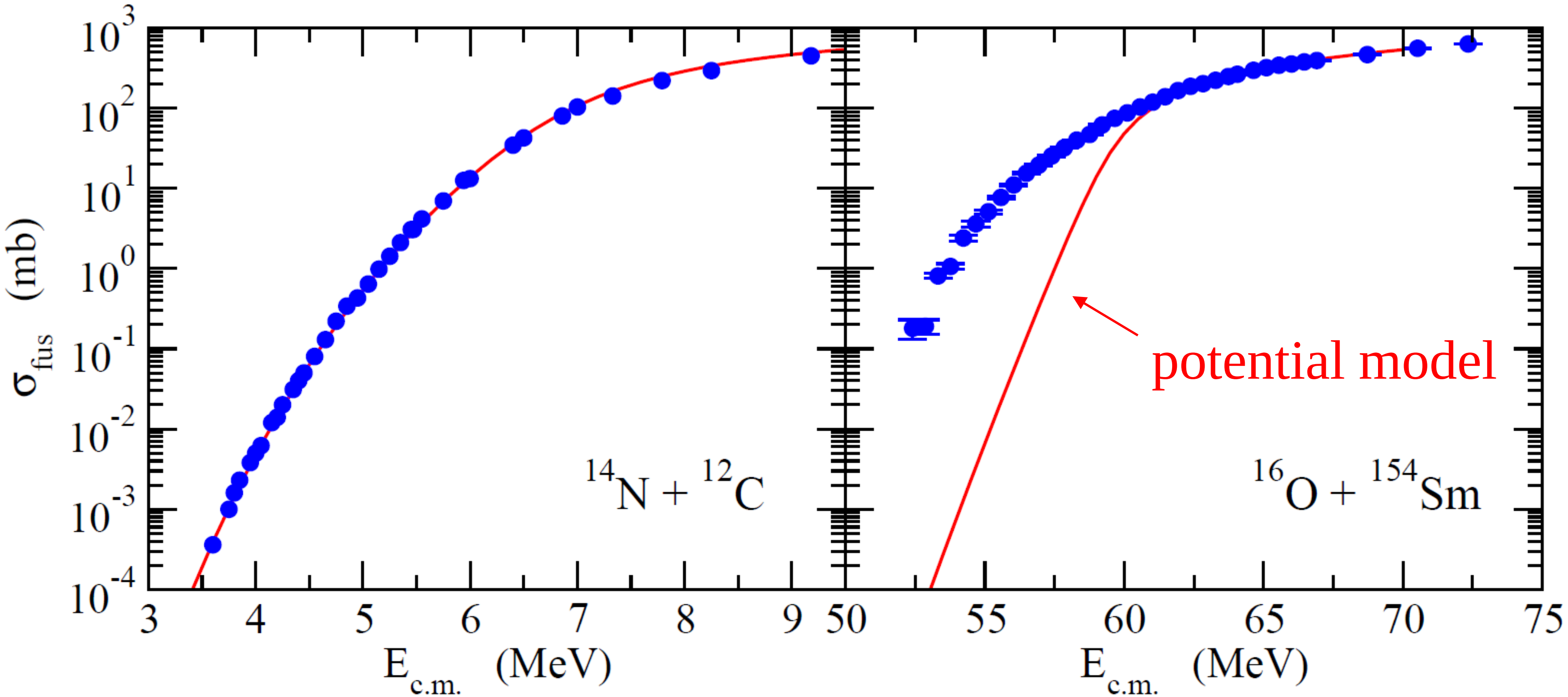


Generalized Wong formula [N. Rowley and K.H., PRC91('15)044617]

$$\sigma_{\text{fus}}(E) \sim \frac{\hbar\Omega_E}{2E} R_E^2 \ln\left[1+\exp\left(\frac{2\pi}{\hbar\Omega_E}(E-V_E)\right)\right] + (osc.)$$

Discovery of large sub-barrier enhancement of σ_{fus} (~ the late 70's)

potential model: $V(r)$ + absorption

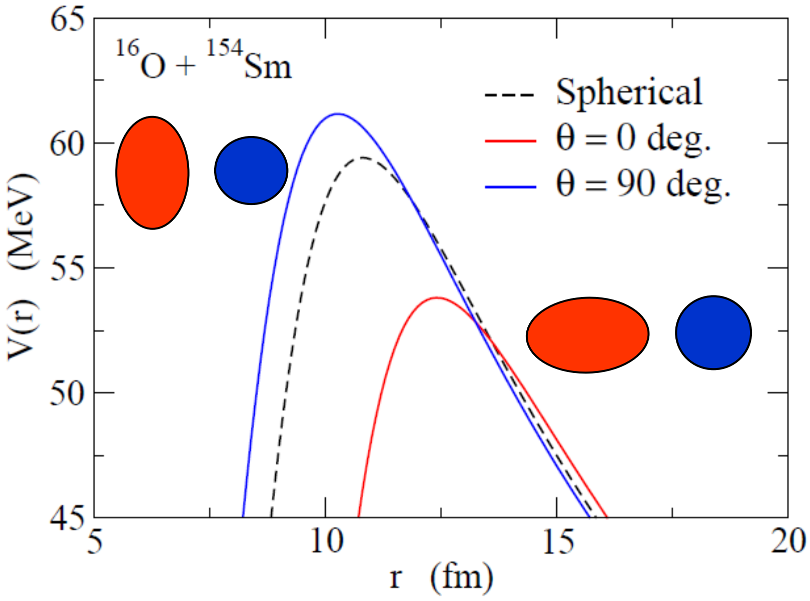


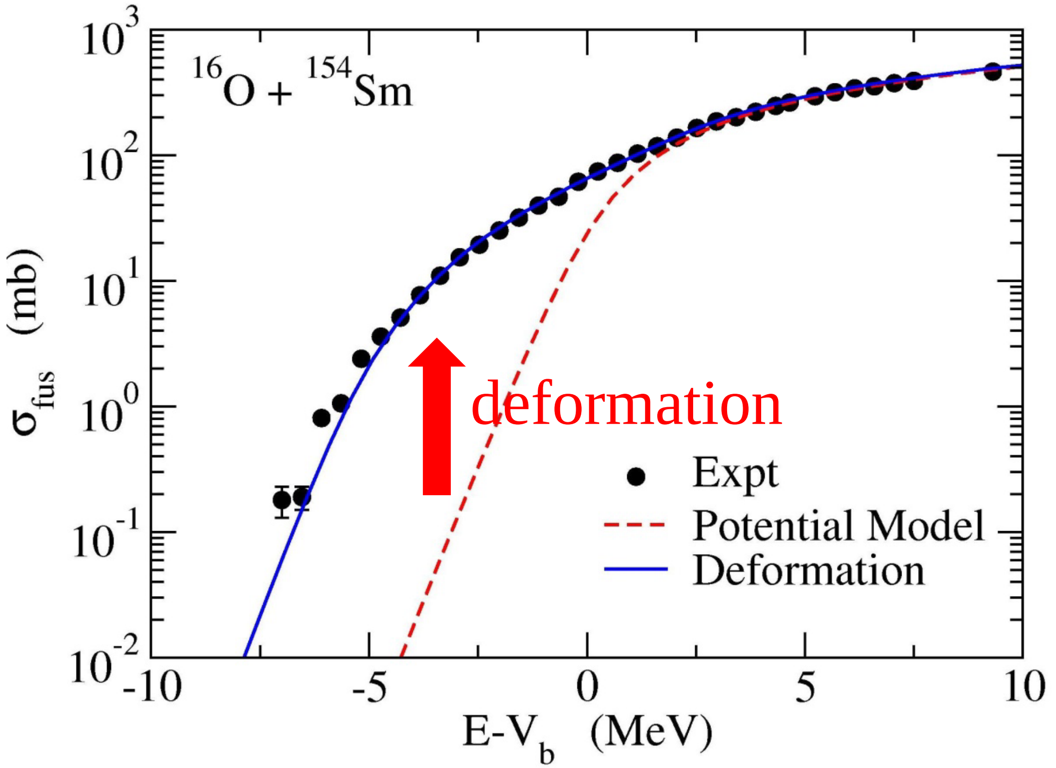
cf. seminal work:

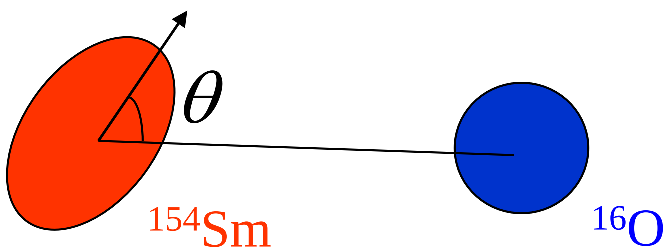
R.G. Stokstad et al., PRL41('78) 465

Effect of nuclear deformation

^{154}Sm : a deformed nucleus with $\beta_2 \sim 0.3$





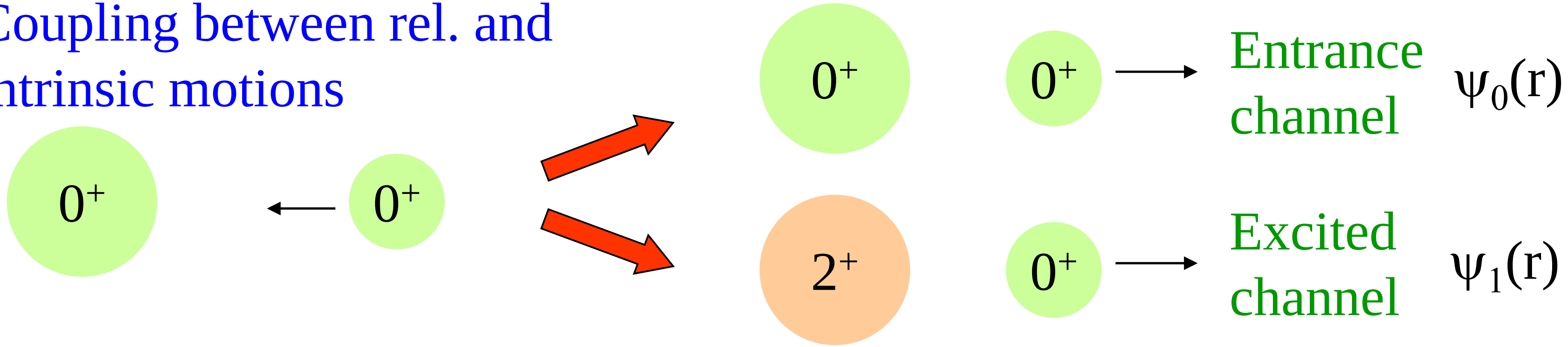


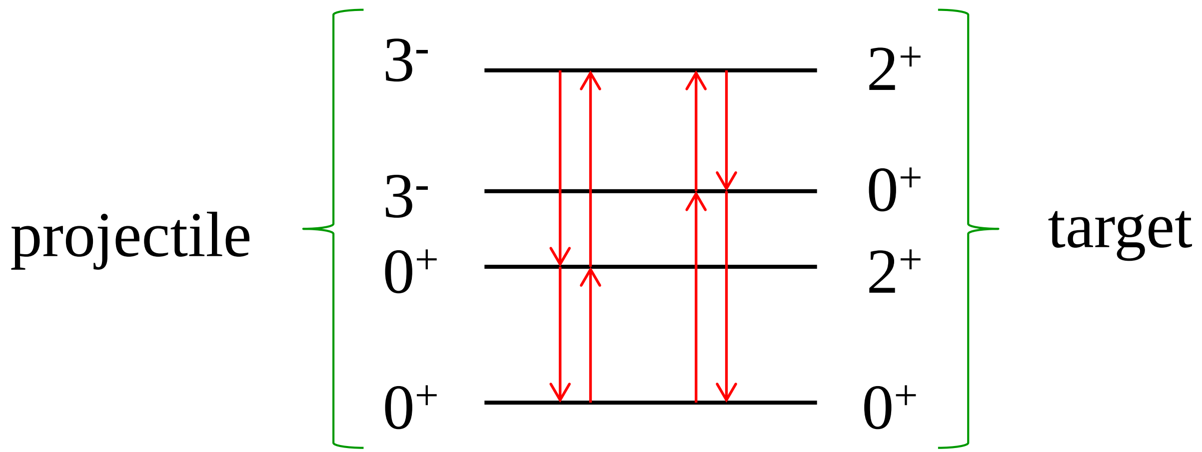
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$

Fusion: strong interplay between nuclear structure and nuclear reaction

Coupled-Channels method

Coupling between rel. and intrinsic motions





$$\Psi(\boldsymbol{r},\xi) = \sum_k \psi_k(\boldsymbol{r})\phi_k(\xi)$$

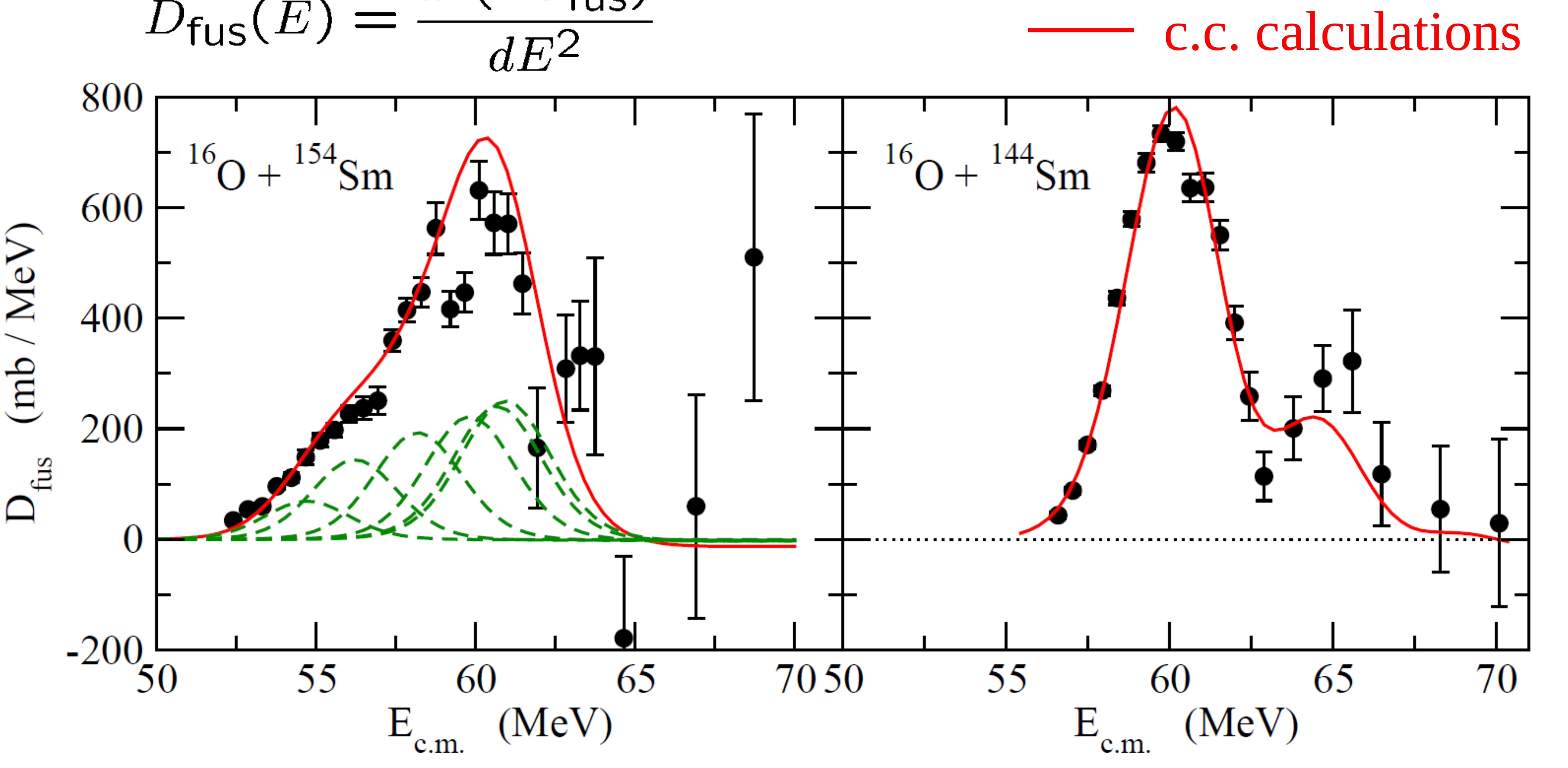
→ coupled Schroedinger equations for $\psi_k(r)$

C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 ('99) 143)

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91))

$$D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2}$$

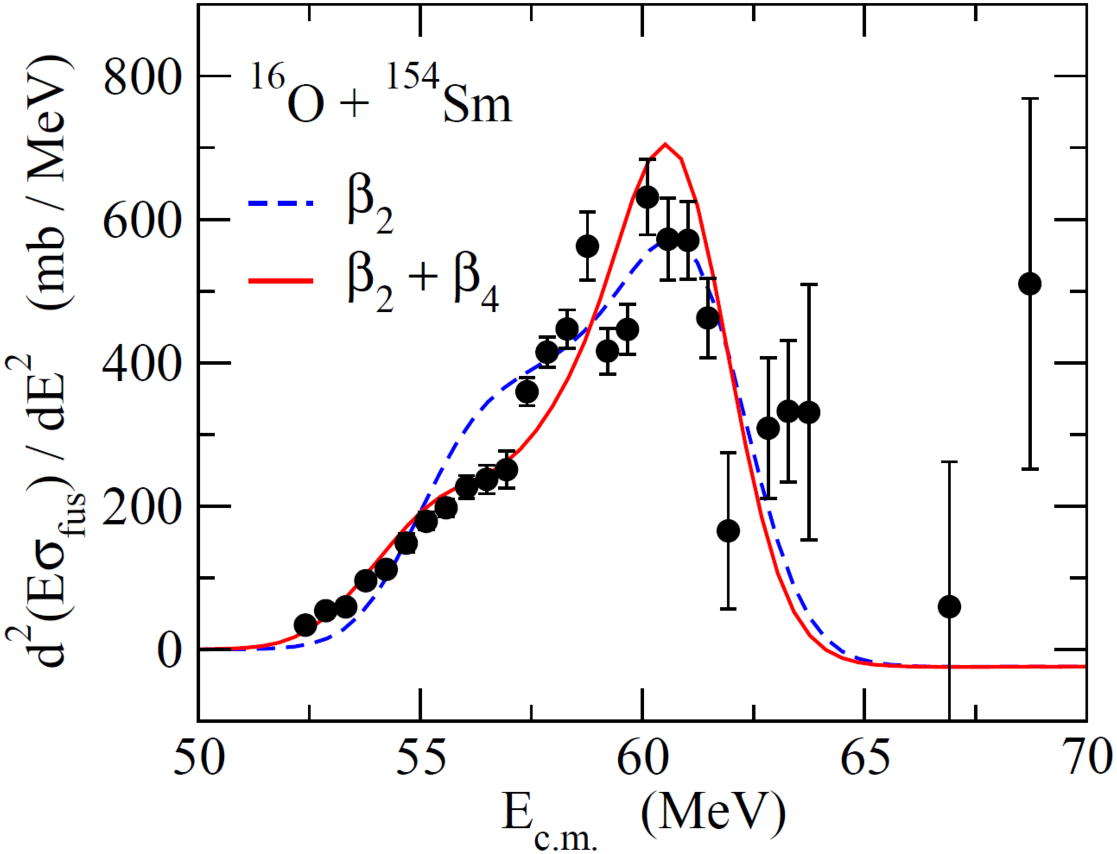


K.H., N. Takigawa, PTP128 ('12) 1061

Fusion barrier distribution

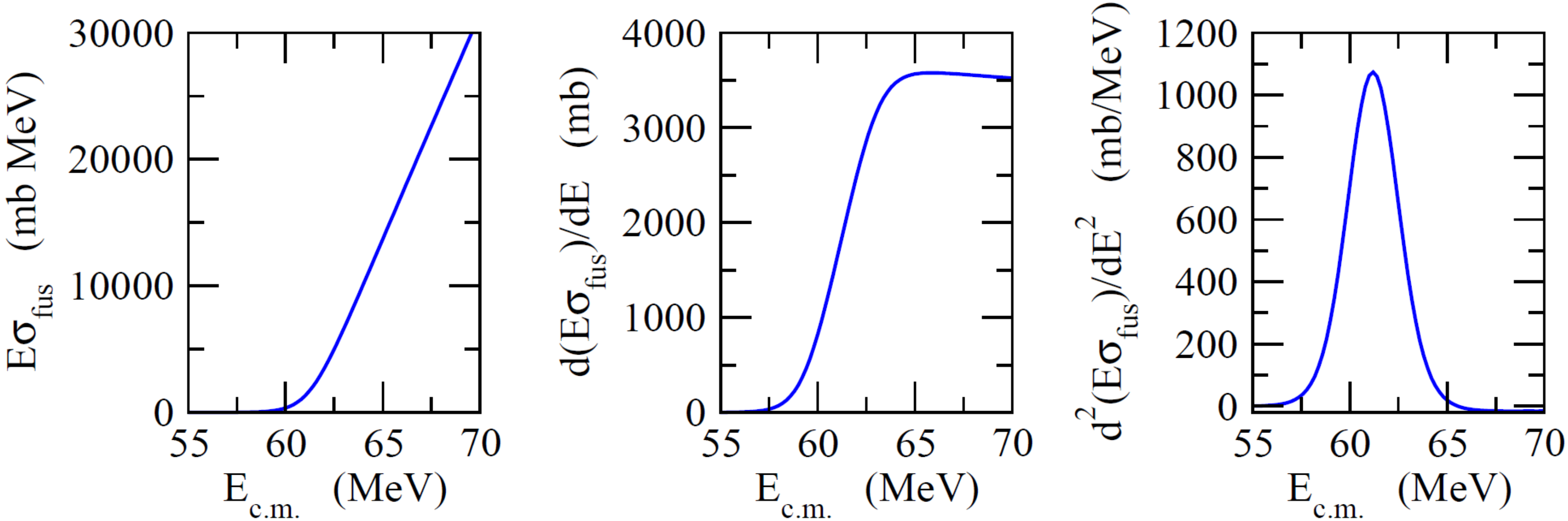
$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

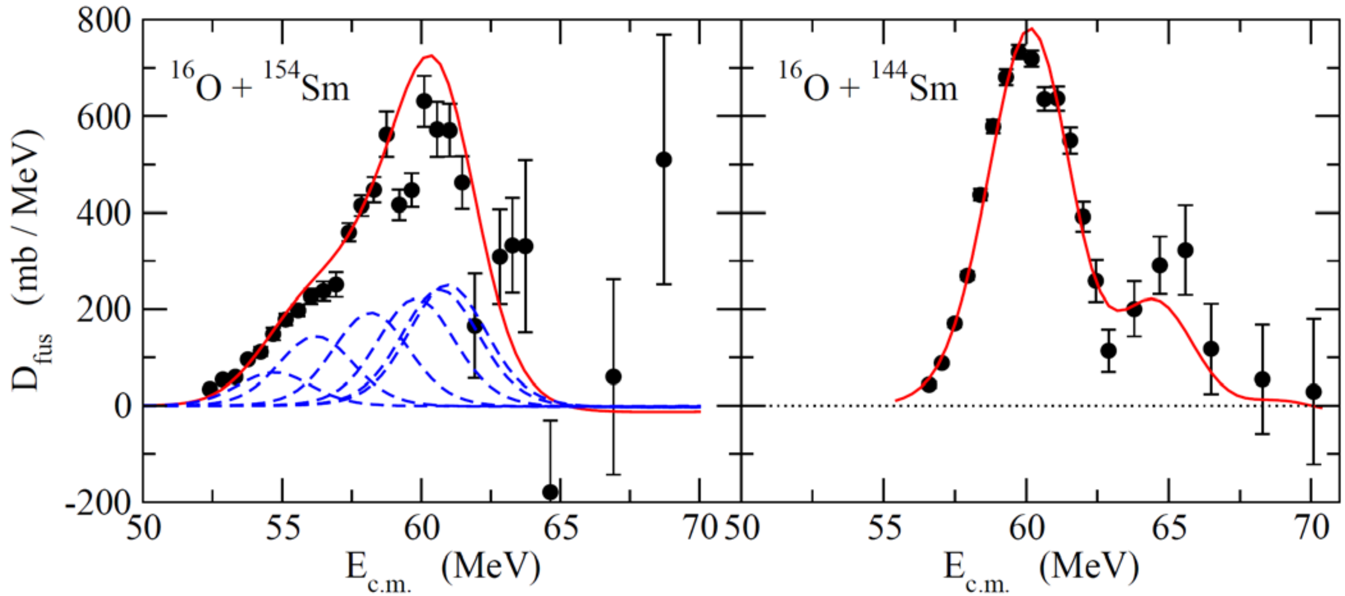
- N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254('91) 25
- J.X. Wei, J.R. Leigh et al., PRL67('91) 3368
- M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48('98)401



sensitive to nuclear structure

$\sim \pi R_b^2 \, P_{l=0}(E)$ $\sim \pi R_b^2 \cdot dP_{l=0}/dE$



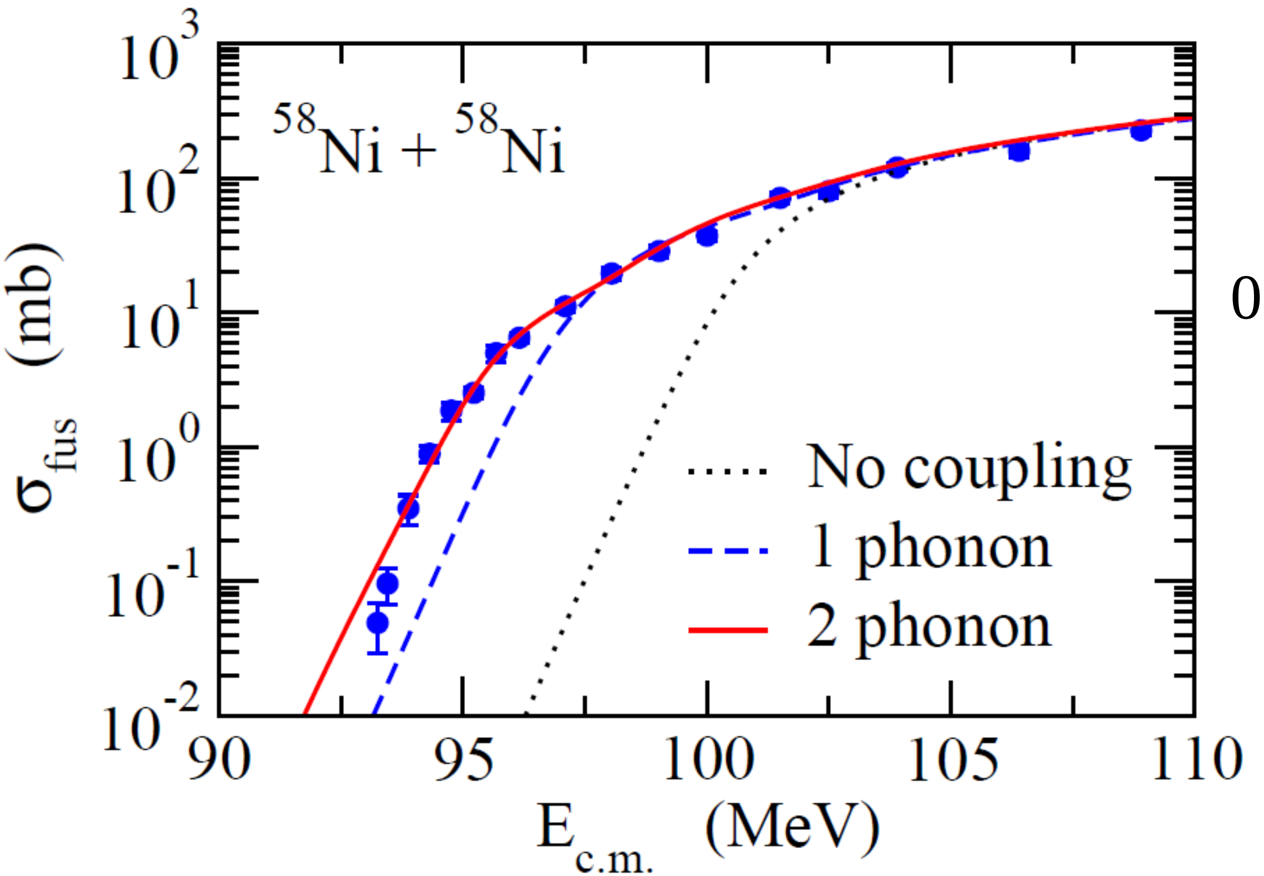


K.H. and N. Takigawa, PTP128 ('12) 1061

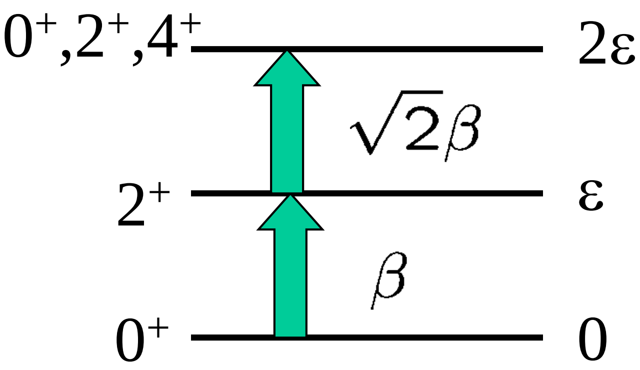
Semi-microscopic modeling of sub-barrier fusion

K.H. and J.M. Yao, PRC91('15) 064606

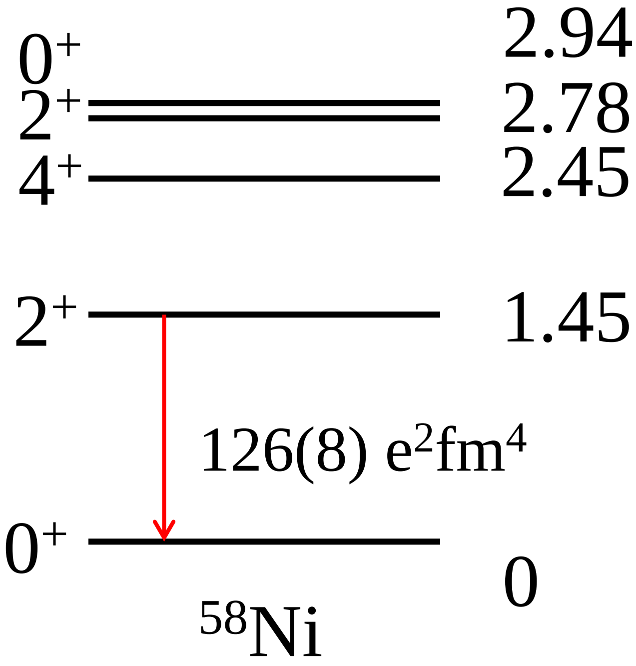
multi-phonon excitations



simple harmonic oscillator



Anharmonic vibrations



$Q(2_1^+) = -10 +/- 6\ e\mathrm{fm}^2$

- Boson expansion
- Quasi-particle phonon model
- Shell model
- Interacting boson model
- Beyond-mean-field method

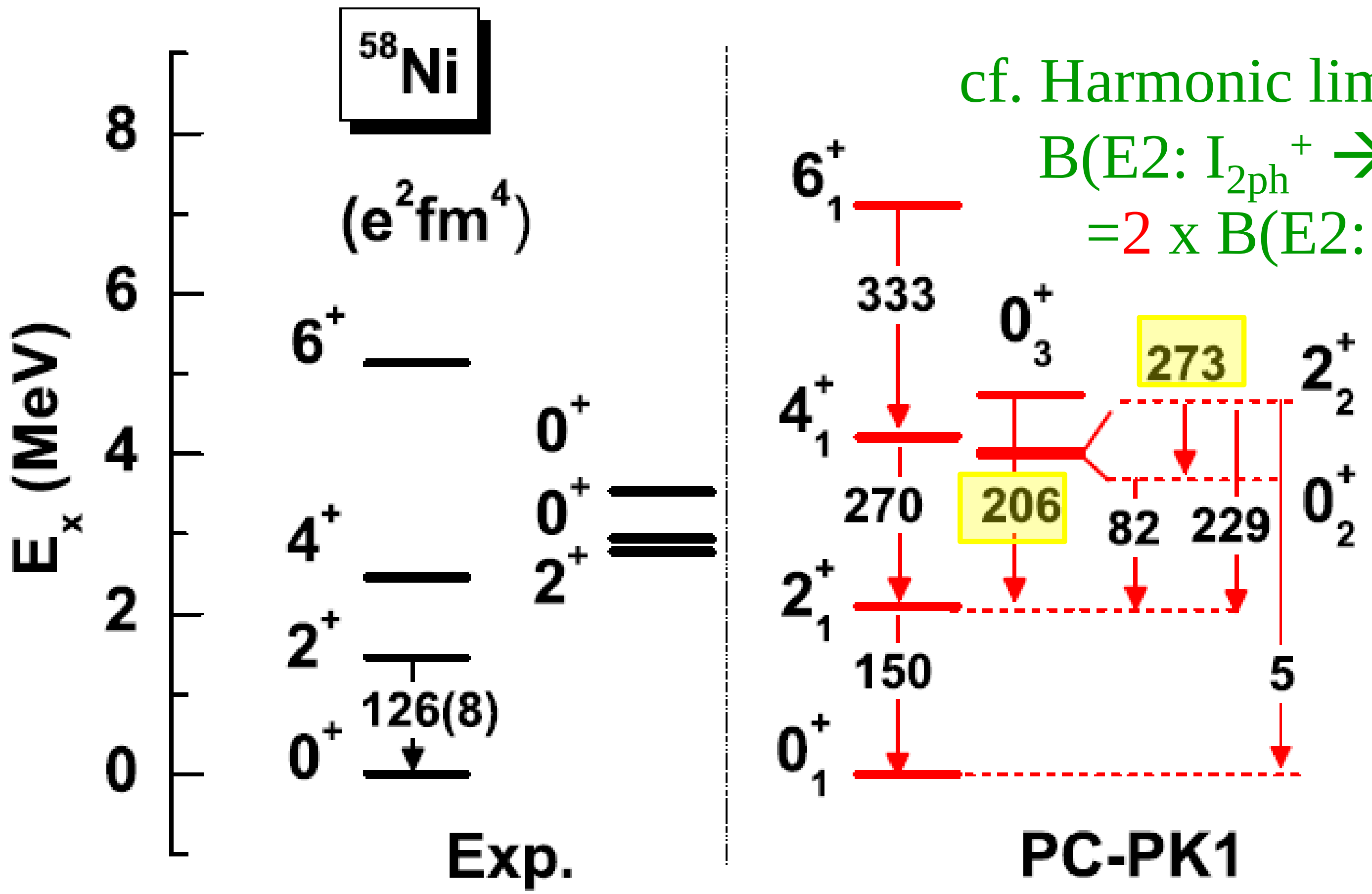
$$|JM\rangle = \int d\beta\, f_J(\beta)\hat{P}^J_{M0}|\Phi(\beta)\rangle$$

✓ MF + ang. mom. projection
+ particle number projection
+ generator coordinate method (GCM)

M. Bender, P.H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 (‘03) 121
J.M. Yao et al., PRC89 (‘14) 054306

Recent beyond-MF (MR-DFT) calculations for ^{58}Ni

K.H. and J.M. Yao, PRC91 ('15) 064606
J.M. Yao, M. Bender, and P.-H. Heenen, PRC91 ('15) 024301



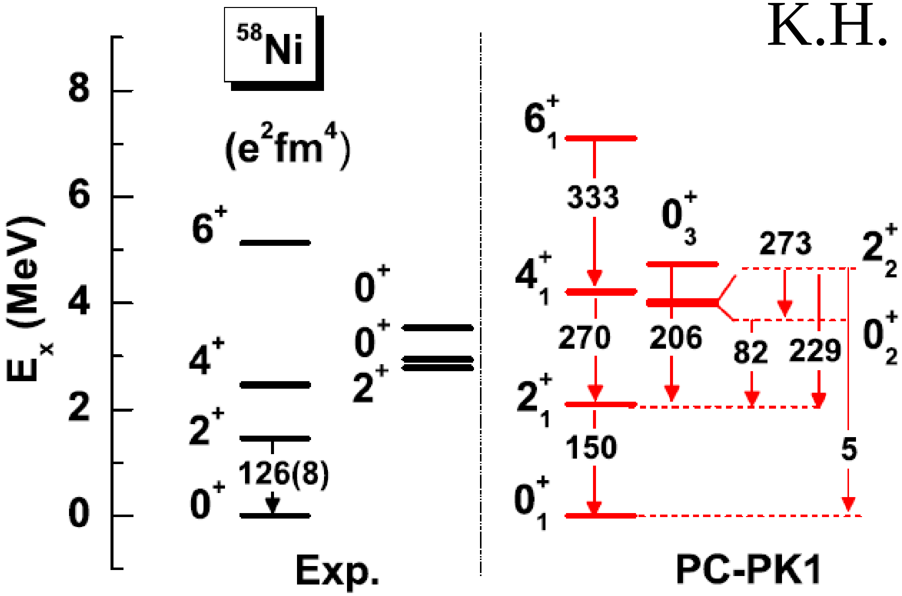
cf. Harmonic limit:
$B(E2: I_{2ph}^+ \rightarrow 2_1^+)$
$= 2 \times B(E2: 2_1^+ \rightarrow 0_1^+)$

- ✓ A large fragmentation of $(2^+ \times 2^+)_{J=0}$
- ✓ A strong transition from 2_2^+ to 0_2^+

→ effects on sub-barrier fusion?

Semi-microscopic coupled-channels model for sub-barrier fusion

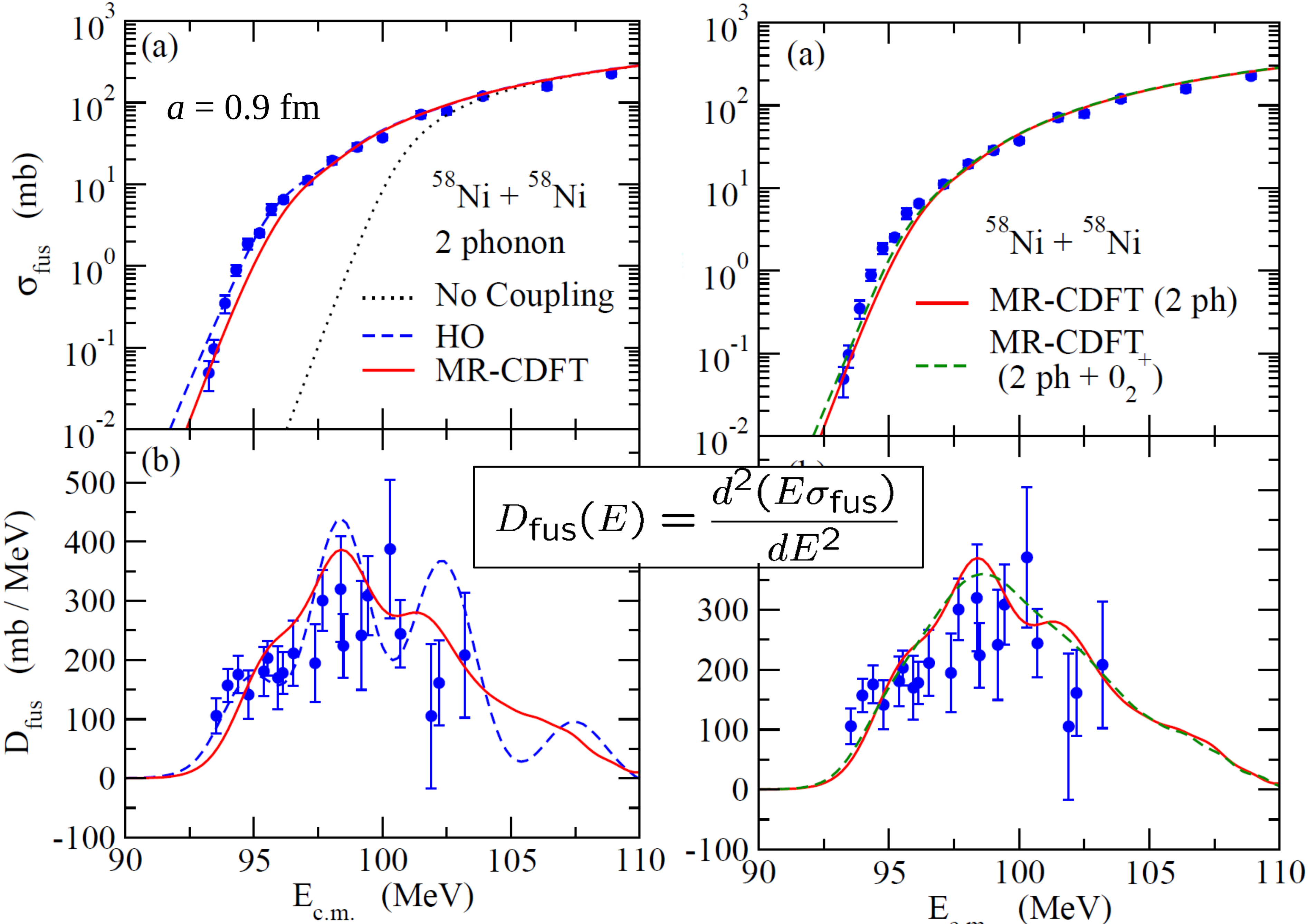
K.H. and J.M. Yao, PRC91 (‘15) 064606



microscopic
multi-pole operator

✓ $$V_{\mathrm{coup}} \sim -R_T \frac{dV_N}{dr} \alpha_\lambda \cdot Y_\lambda(\hat{\boldsymbol{r}}) \rightarrow -R_T \frac{dV_N}{dr} Q_\lambda \cdot Y_\lambda(\hat{\boldsymbol{r}})$$

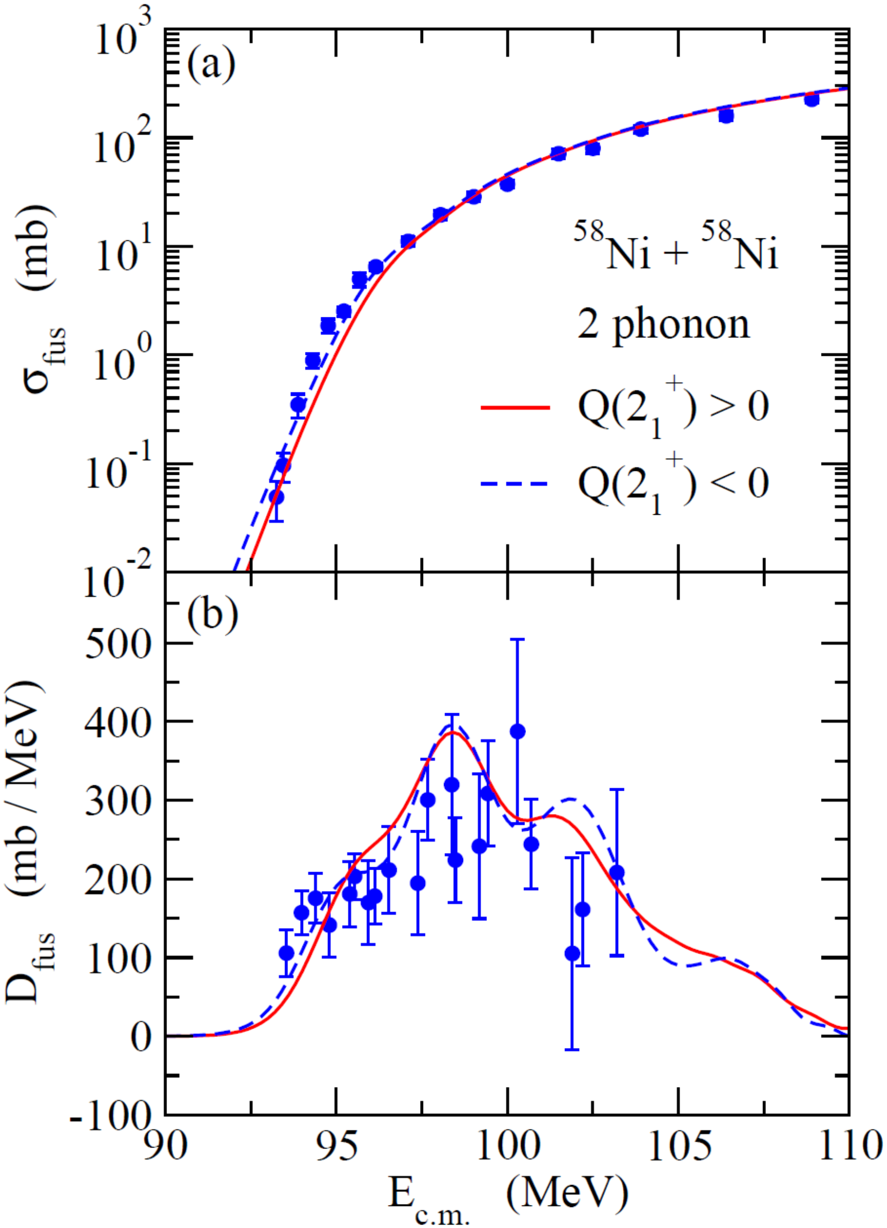
- ✓ *M*(E2) from MR-DFT calculation ← among higher members of phonon states
- ✓ scale to the empirical B(E2; $2_1^+ \rightarrow 0_1^+$)
- ✓ still use a phenomenological potential
- ✓ use the experimental values for E_x
- ✓ β_N and β_C from M_n/M_p for each transition
- ✓ axial symmetry (no 3^+ state)



$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

K.H. and J.M. Yao, PRC91 ('15) 064606

Role of Q-moment of the first 2^+ state

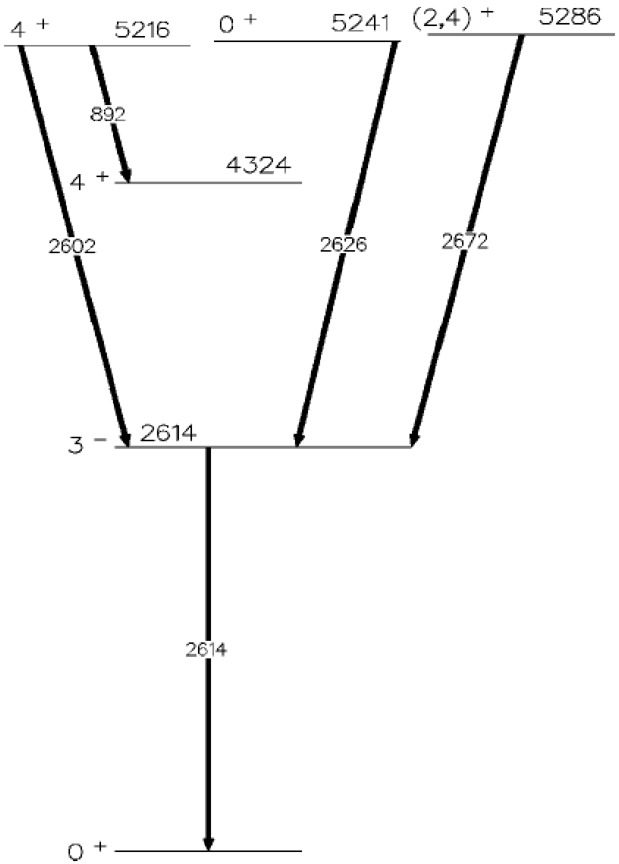


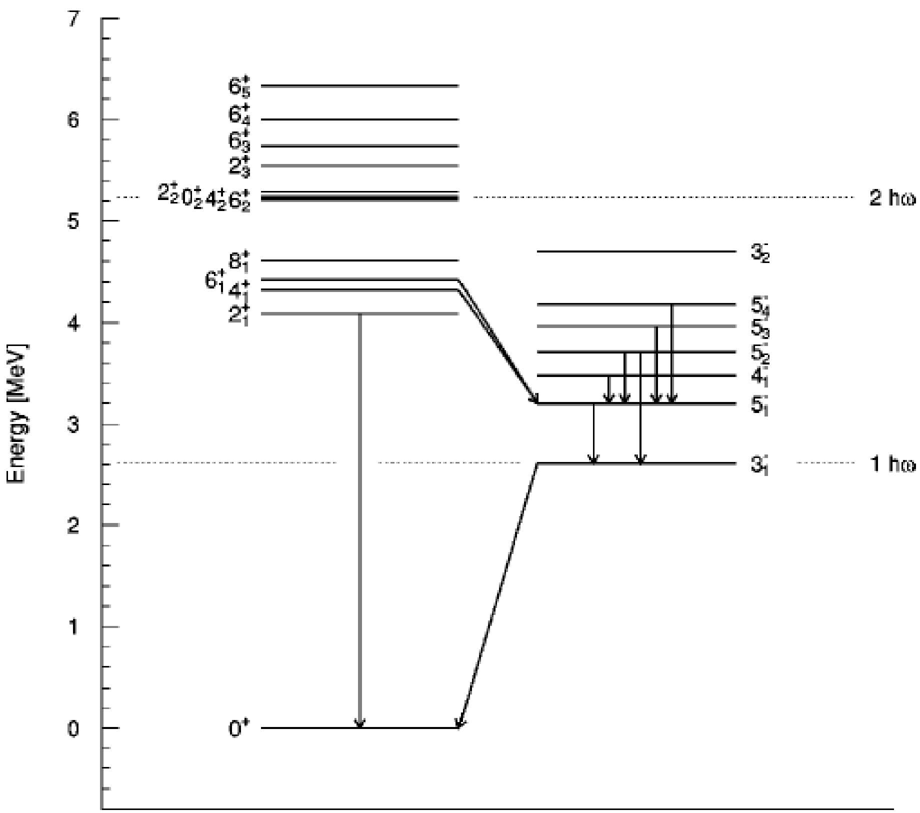
cf. $Q_{exp}(2_1^+)$ = -10 +/- 6 $e$fm^2

P.M.S. Lesser et al., NPA223 ('74) 563.

Application to $^{16}O + ^{208}Pb$ fusion reaction

double-octupole phonon states in ^{208}Pb

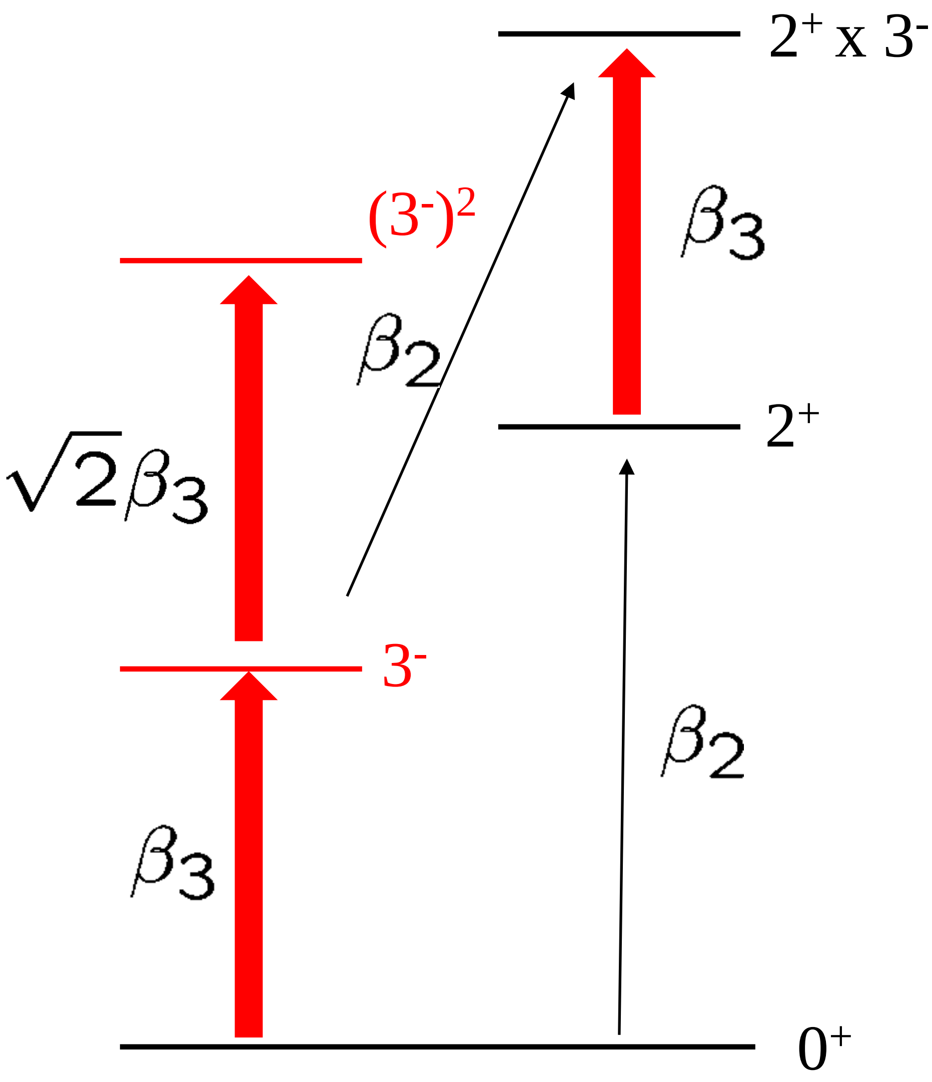


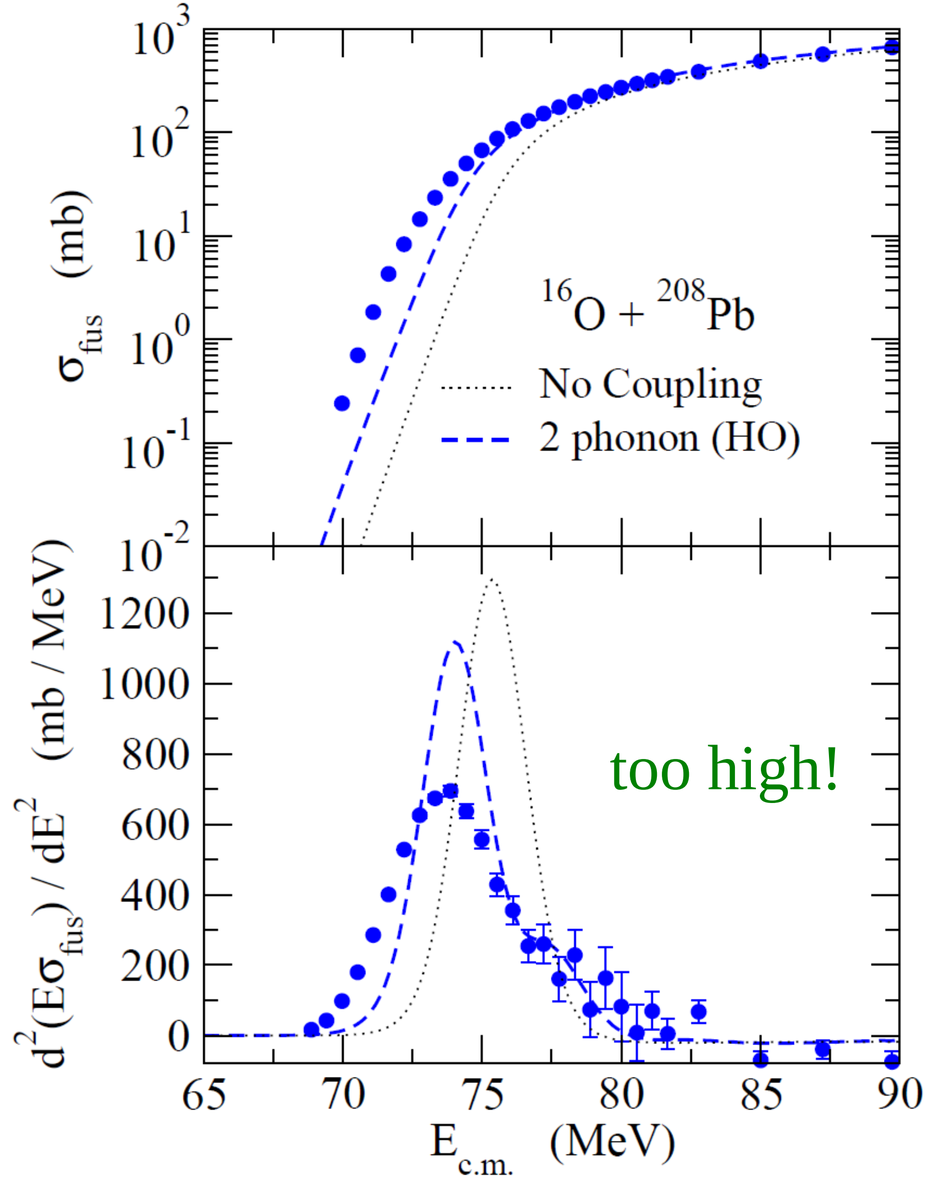


M. Yeh, M. Kadi, P.E. Garrett et al., PRC57 (‘98) R2085

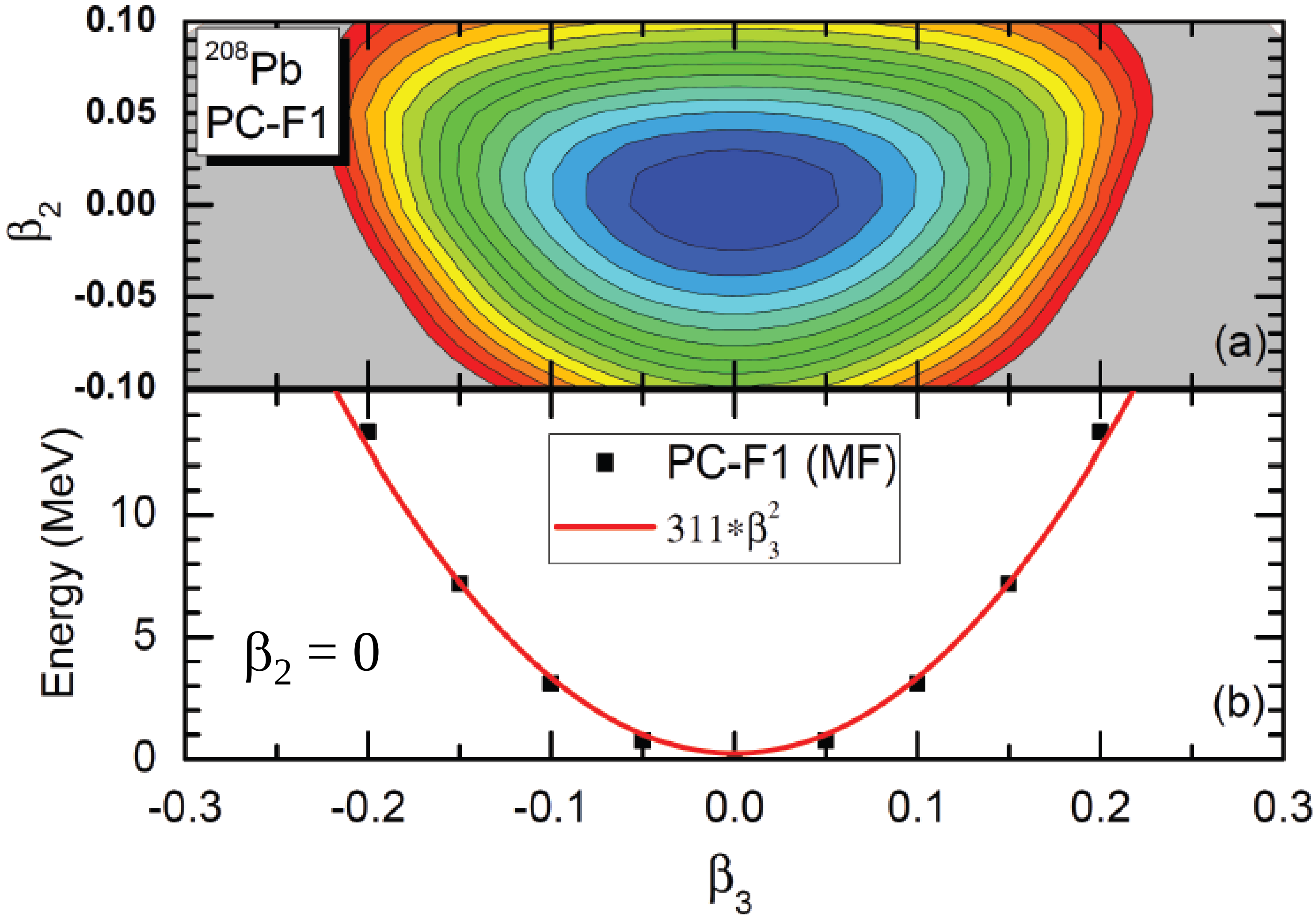
K. Vetter, A.O. Macchiavelli et al., PRC58 (‘98) R2631

Application to $^{16}O + {}^{208}Pb$ fusion reaction





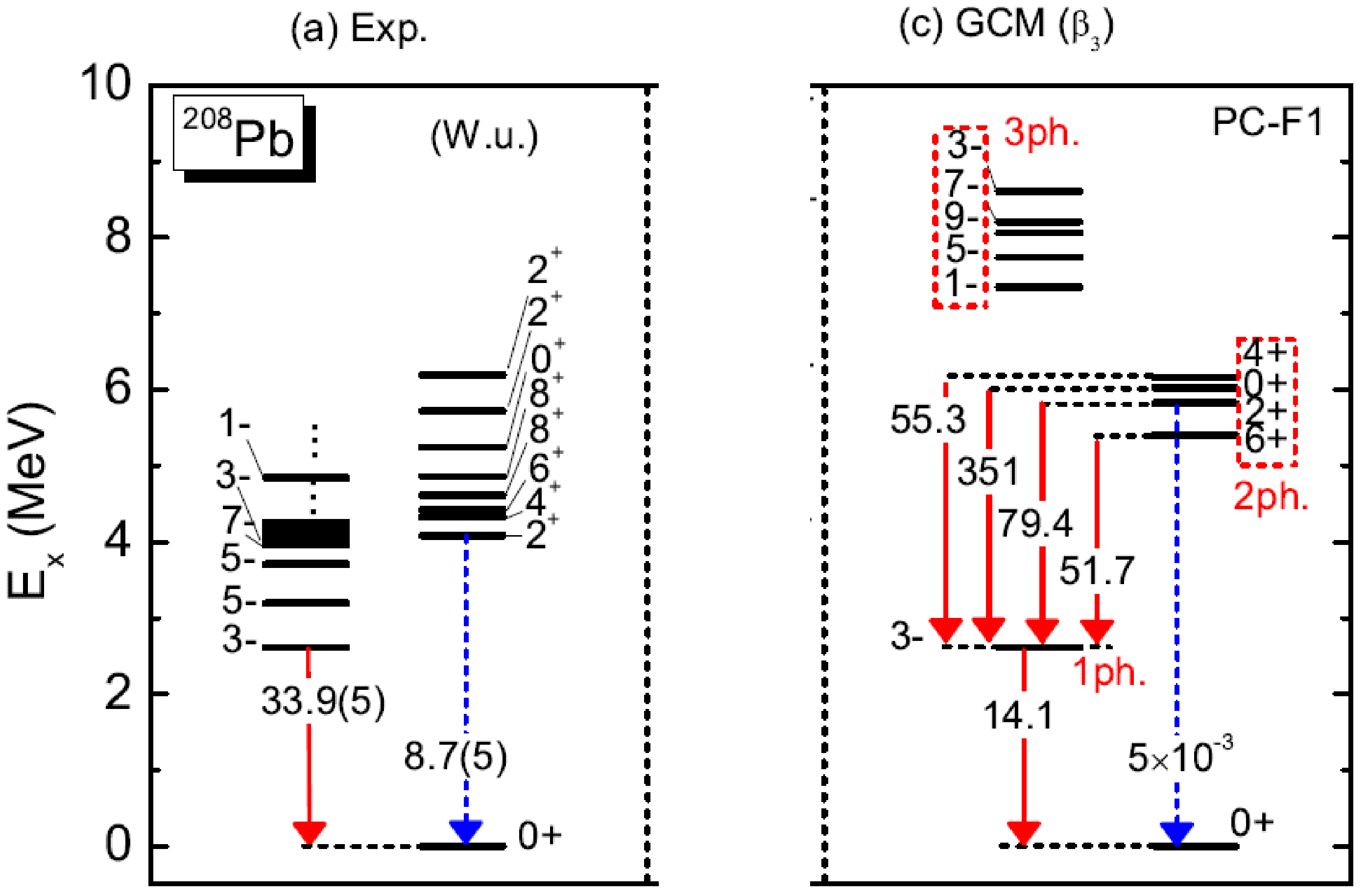
cf. C.R. Morton et al., PRC60('99) 044608 potential energy surface of ^{208}Pb (RMF with PC-F1)



J.M. Yao and K.H., submitted. (2016)

Expt. data

β_2=0, fluctuation in β_3



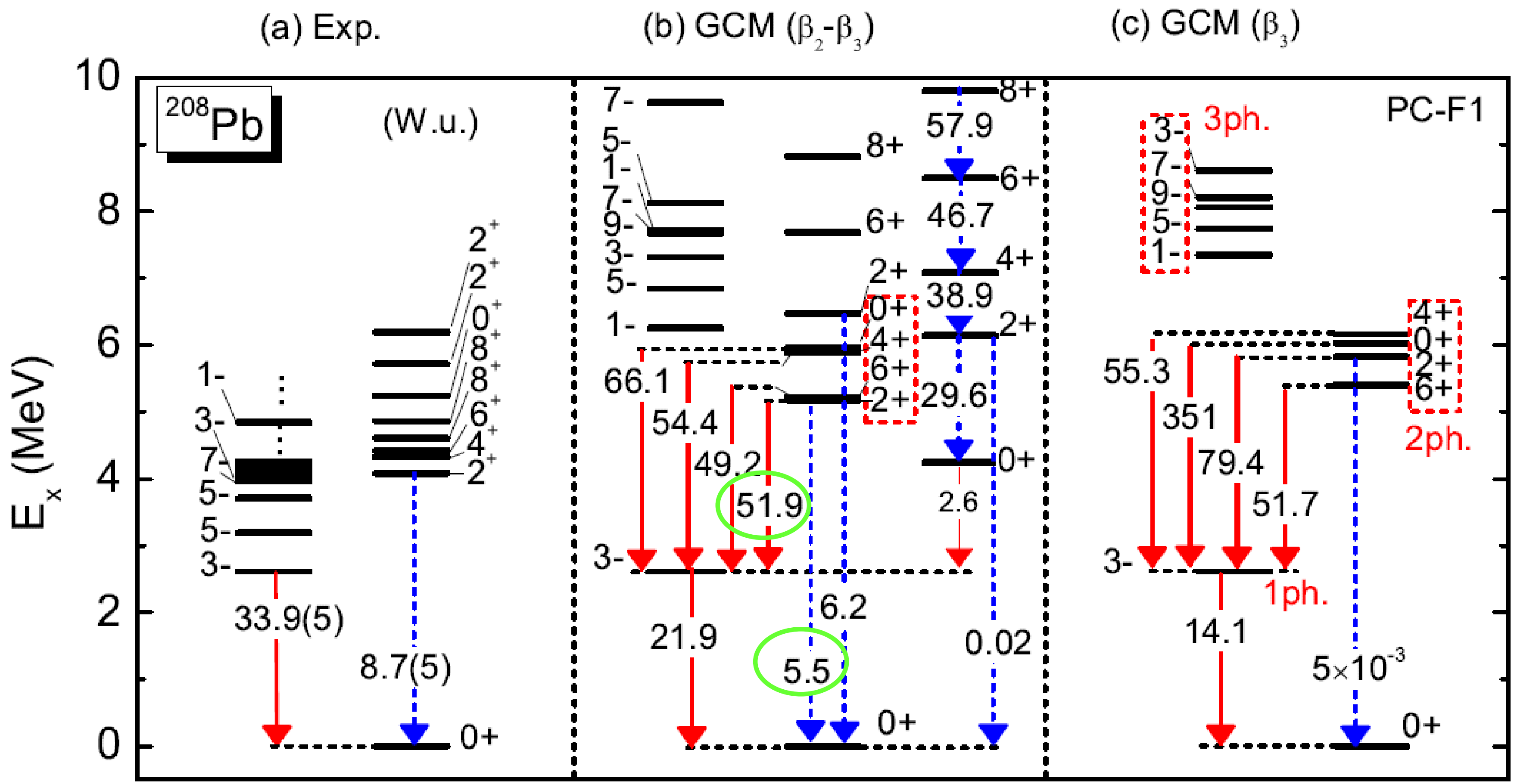
- $E_{2ph} \sim E_{1ph}$
- large anharmonicity in B(E3);

  cf. H.O.: $B(E3: I_{2ph} \rightarrow 3_1^-) = 2\,B(E3: 3_1^- \rightarrow \text{g.s.})$
- underestimate B(E3) (and B(E2))

expt. data

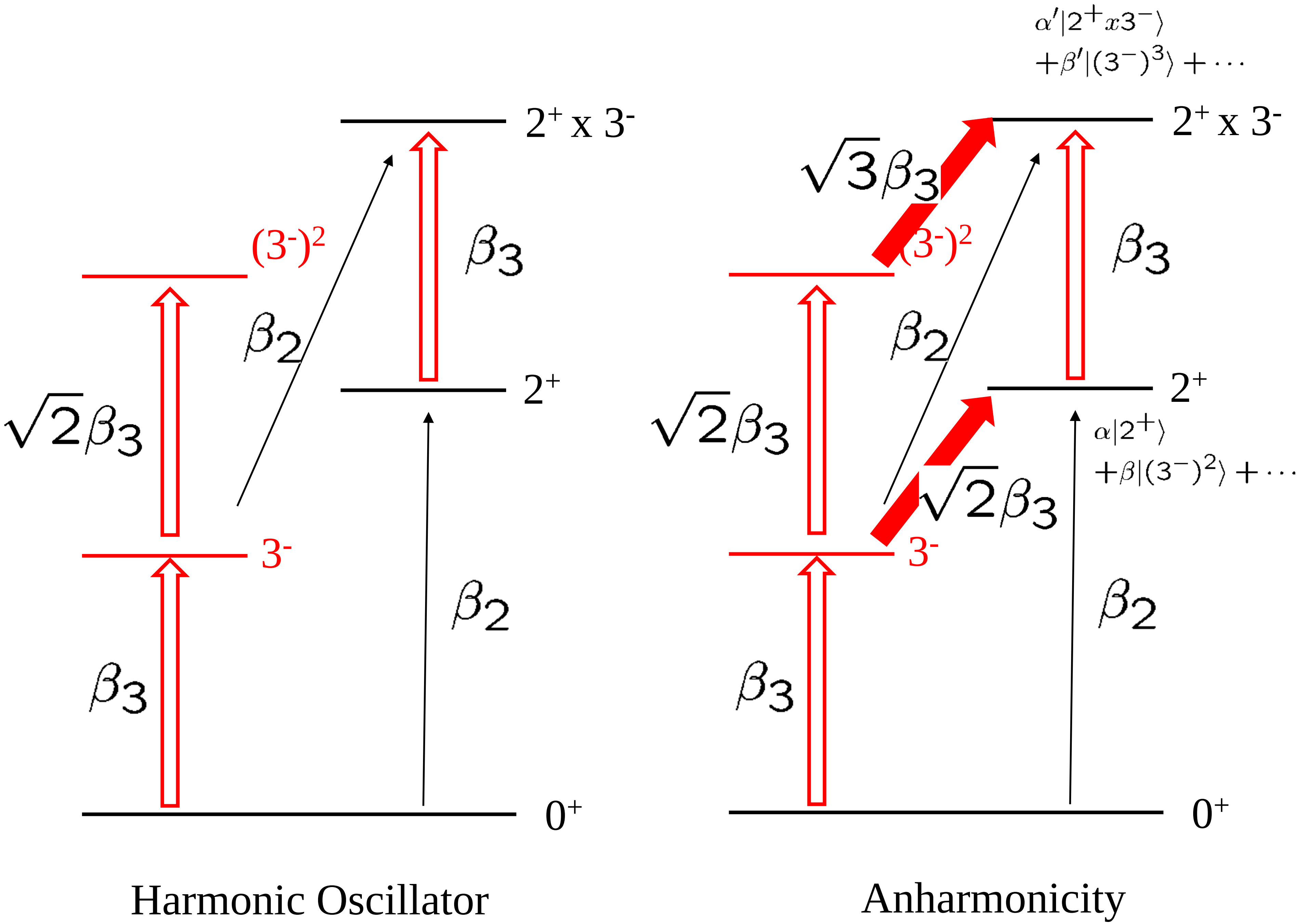
fluctuation both in β_3 and β_2

fluctuation in β_3 frozen at $\beta_2=0$

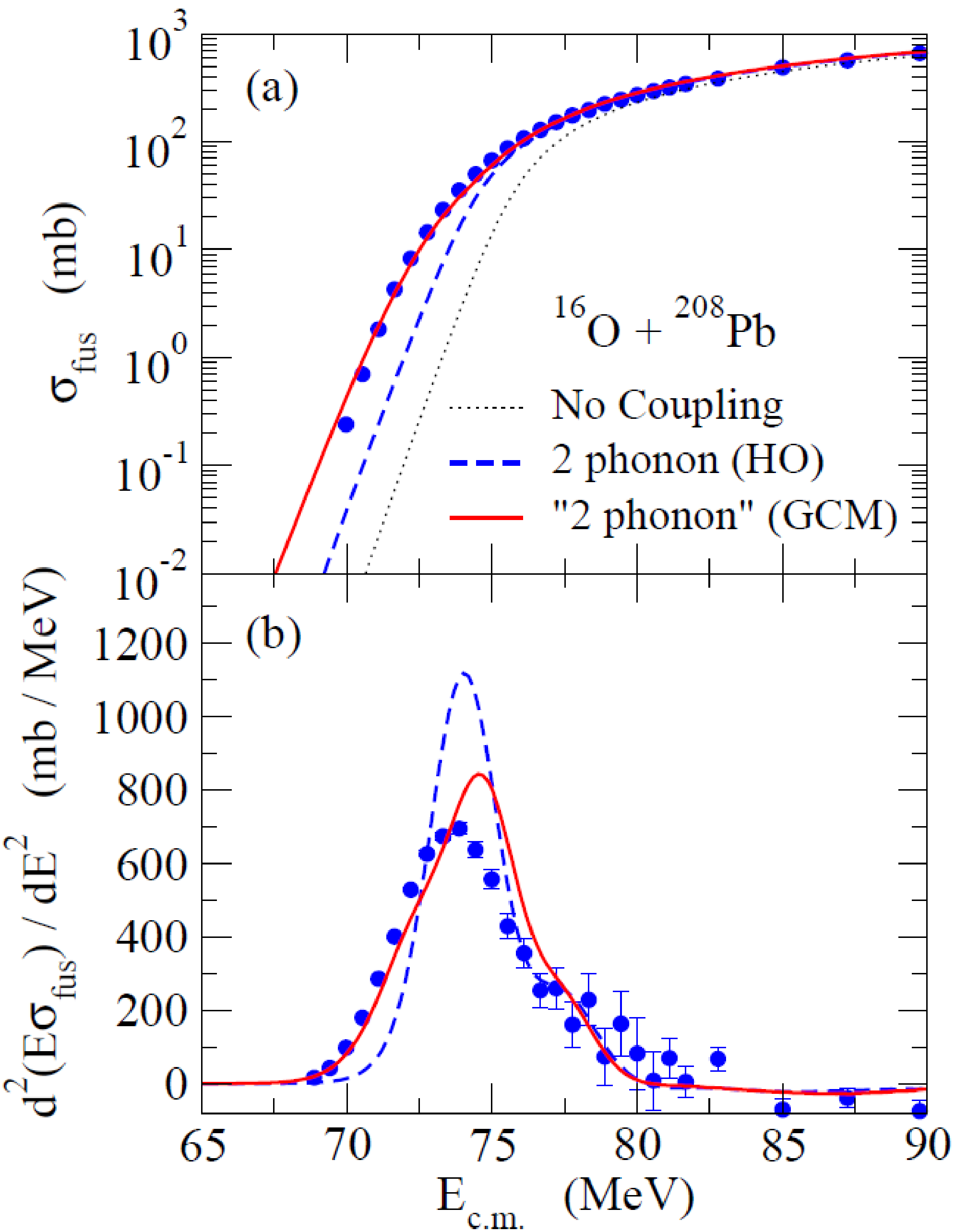


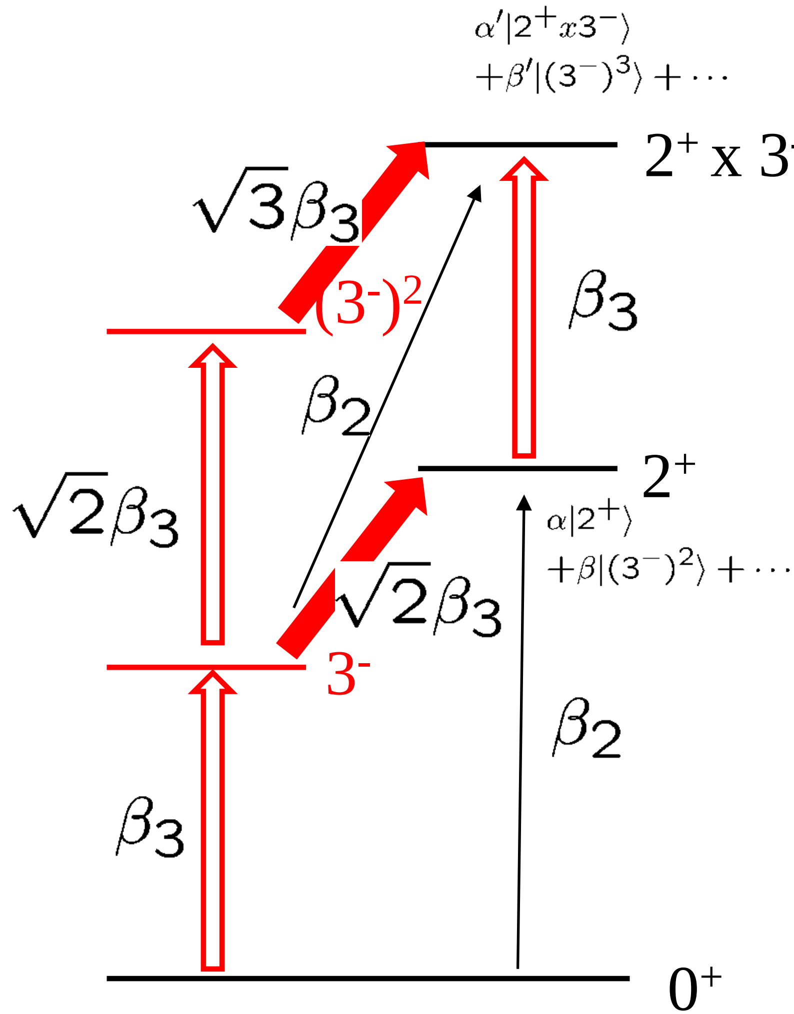
2_1^+ state: strong coupling both to g.s. and 3_1^-

$$\longrightarrow \quad |2_1^+\rangle = \alpha|2^+\rangle_{HO} + \beta|[3^- \otimes 3^-]^{(I=2)}\rangle_{HO} + \cdots$$



$2^+ \times 3^-$
$(3^-)^2$
β_3
β_2
$\sqrt{2}\beta_3$
2^+
3^-
β_2
β_3
0^+
Harmonic Oscillator
$\alpha'|2^+ x 3^-\rangle$
$+\beta'|(3^-)^3\rangle + \cdots$
$2^+ \times 3^-$
$\sqrt{3}\beta_3$
$(3^-)^2$
β_3
β_2
$\sqrt{2}\beta_3$
2^+
$\alpha|2^+\rangle$
$+\beta|(3^-)^2\rangle + \cdots$
$\sqrt{2}\beta_3$
3^-
β_2
β_3
0^+
Anharmonicity





J.M. Yao and K.H.,
submitted (2016)

Quasi-elastic barrier distributions

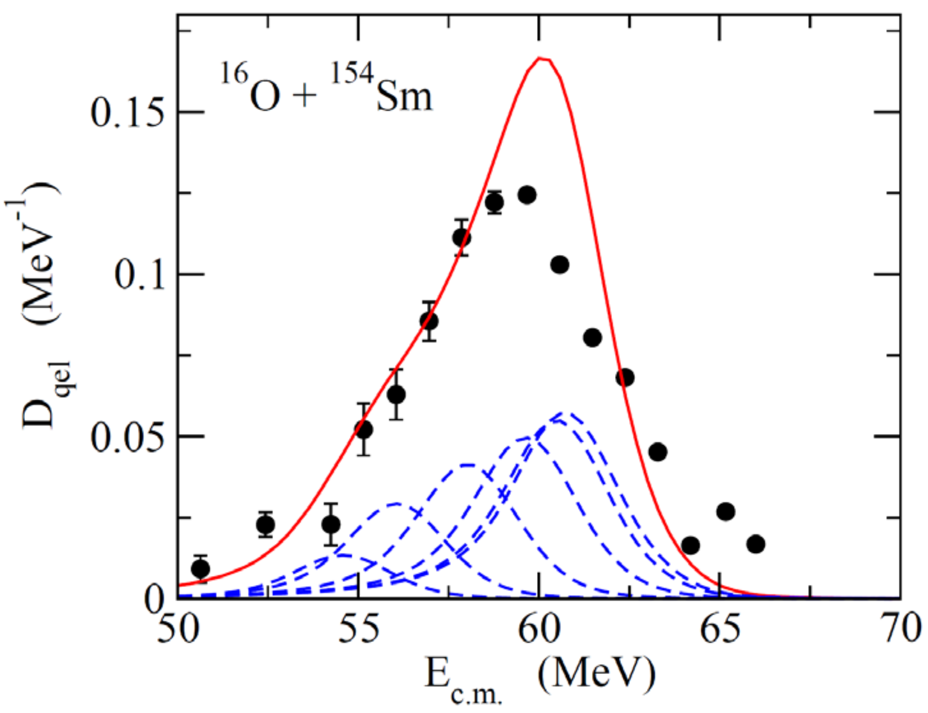
Quasi-elastic scattering:

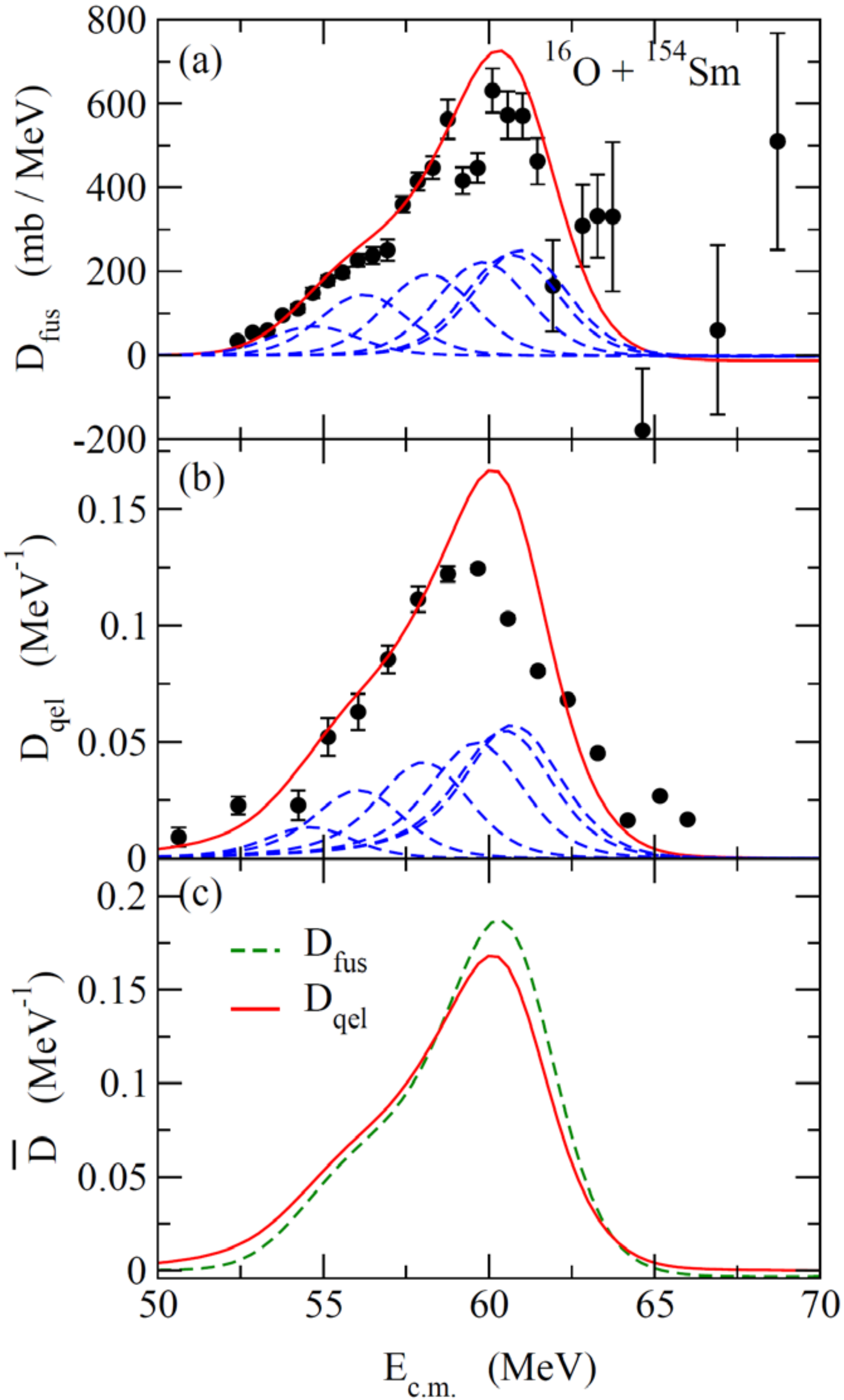
A sum of all the reaction processes other than fusion
(elastic + inelastic + transfer + ……)

$$P_{l=0}(E) = 1 - R_{l=0}(E) \sim 1 - \frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}$$

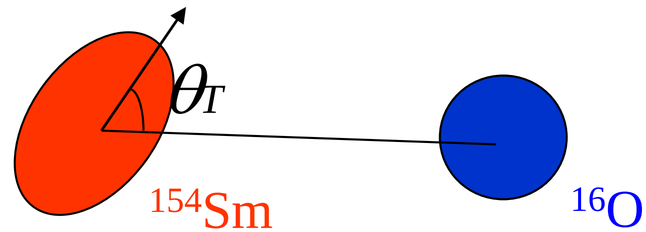
$$D_{\rm qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}\right)$$

H. Timmers et al.,
NPA584('95)190





D_{fus} and D_{qel}: behave similarly to each other



$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\rm fus}(E;\theta_T)$$

$$\sigma_{\rm qel}(E,\theta) = \sum_I \sigma(E,\theta)$$

$$= \int_0^1 d(\cos\theta_T)\sigma_{\rm el}(E,\theta;\theta_T)$$

K.H. and N. Rowley, PRC69('04)054610

Experimental advantages for D_{qel}

$$D_{qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{qel}(E,\pi)}{\sigma_R(E,\pi)}\right) \qquad D_{fus}(E) = \frac{d^2(E\sigma_{fus})}{dE^2}$$

- less accuracy is required in the data (1st vs. 2nd derivative)
- much easier to be measured

  Qel : a sum of everything

  → a very simple charged-particle detector

  Fusion : requires a specialized recoil separator

  to separate ER from the incident beam

  ER + fission for heavy systems

- several effective energies can be measured at a single-beam energy $\leftrightarrow$ $E_{eff} \sim 2E\,\frac{\sin(\theta/2)}{1+\sin(\theta/2)}$

⇒ measurements with a cyclotron accelerator: possible

➡ Suitable for low intensity RI beams

Theoretical justification: Sum-of-differences (SOD) method

J.T. Holdeman and R.M. Thaler, PRL14('65)81, PR139('65)B1186
C. Marty, Z. Phys. A309('83)261, A322('85)499

$$\sigma_R \sim 2\pi \int_{\theta_{\rm min}}^{\pi} \sin\theta\, d\theta \left(\sigma_{\rm Ruth}(\theta) - \sigma_{\rm el}(\theta)\right)$$

expt.: H. Wojciechowski et al., PRC16('77)1767
T. Yamaya et al., PLB417('98)7 etc.

generalization (K.H. and N. Rowley, EPJ Web of Conf. 86 ('15) 00014)

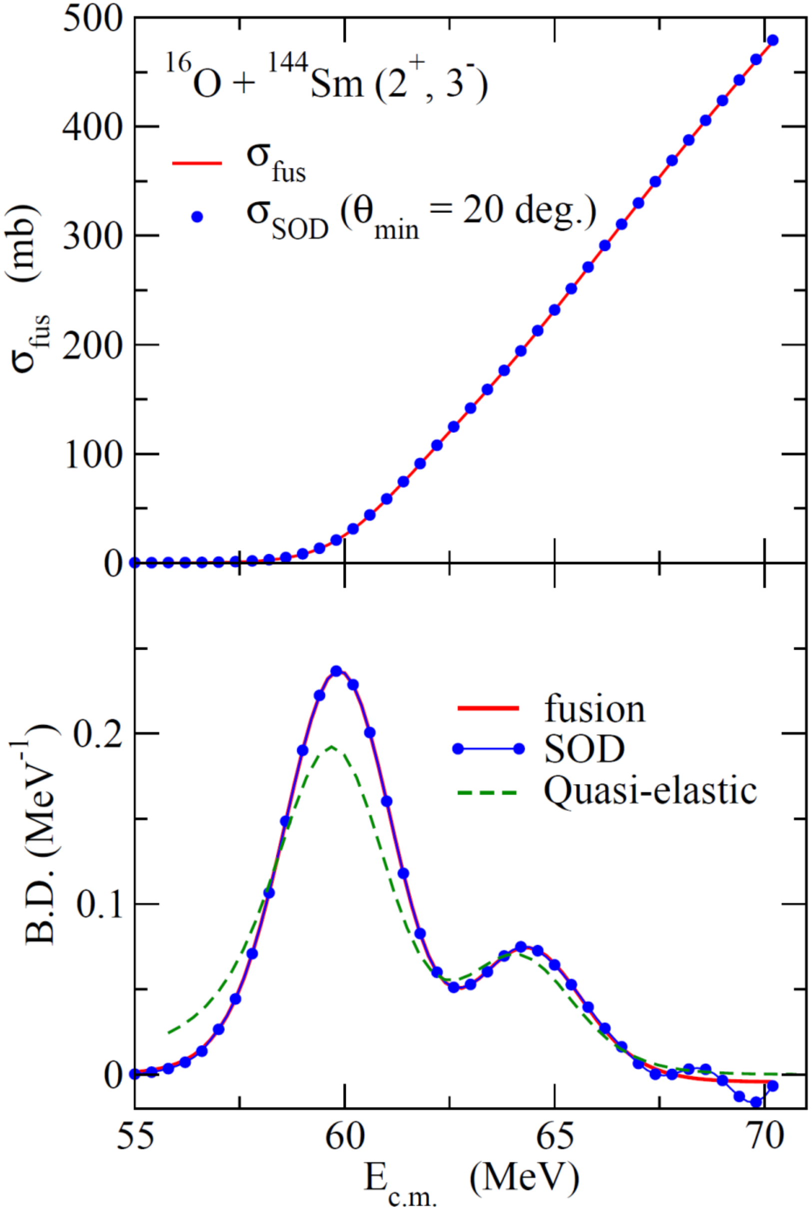
$$\sigma_R = \sigma_{\rm fus} + \sigma_{\rm inel} + \sigma_{\rm tr}$$

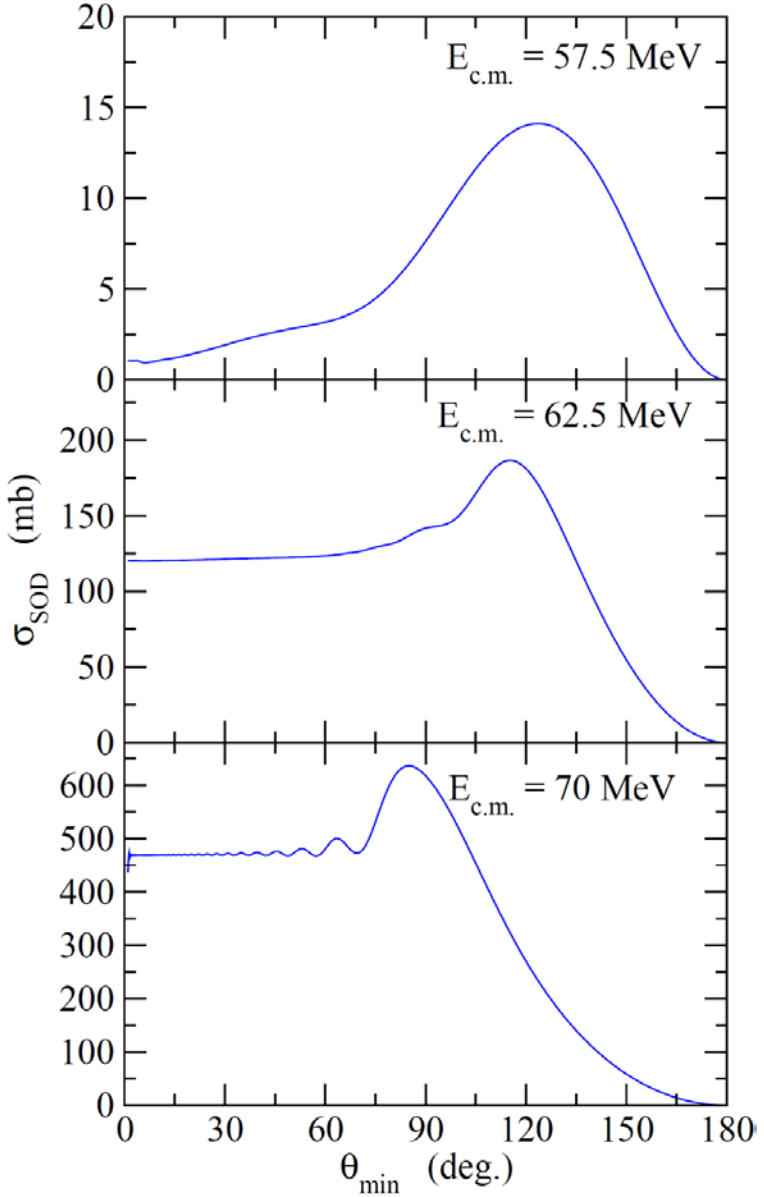
$$\begin{aligned}\sigma_{\rm fus} &\sim 2\pi \int_{\theta_{\rm min}}^{\pi} \sin\theta\, d\theta \left(\sigma_{\rm Ruth}(\theta) - \sigma_{\rm qel}(\theta)\right) \\ &= 2\pi \int_{\theta_{\rm min}}^{\pi} \sin\theta\, d\theta\, \sigma_{\rm Ruth}(\theta) \left(1 - \frac{\sigma_{\rm qel}(\theta)}{\sigma_{\rm Ruth}(\theta)}\right)\end{aligned}$$

$$\longrightarrow \quad P_{\rm fus}^{(l=0)} \approx 1 - \frac{\sigma_{\rm qel}(\theta=\pi)}{\sigma_{\rm Ruth}(\theta=\pi)}$$

Does SOD work for fusion barrier distributions?

$$\sigma_{\rm SOD} = 2\pi \int_{\theta_{\rm min}}^{\pi} \sin\theta\, d\theta\, (\sigma_{\rm Ruth}(\theta) - \sigma_{\rm qel}(\theta))$$



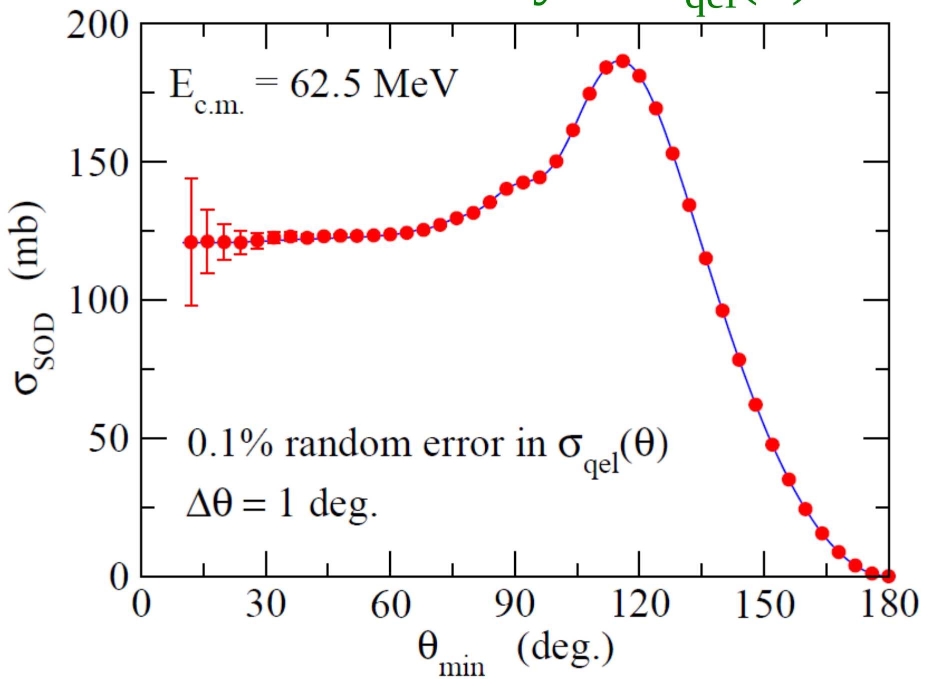


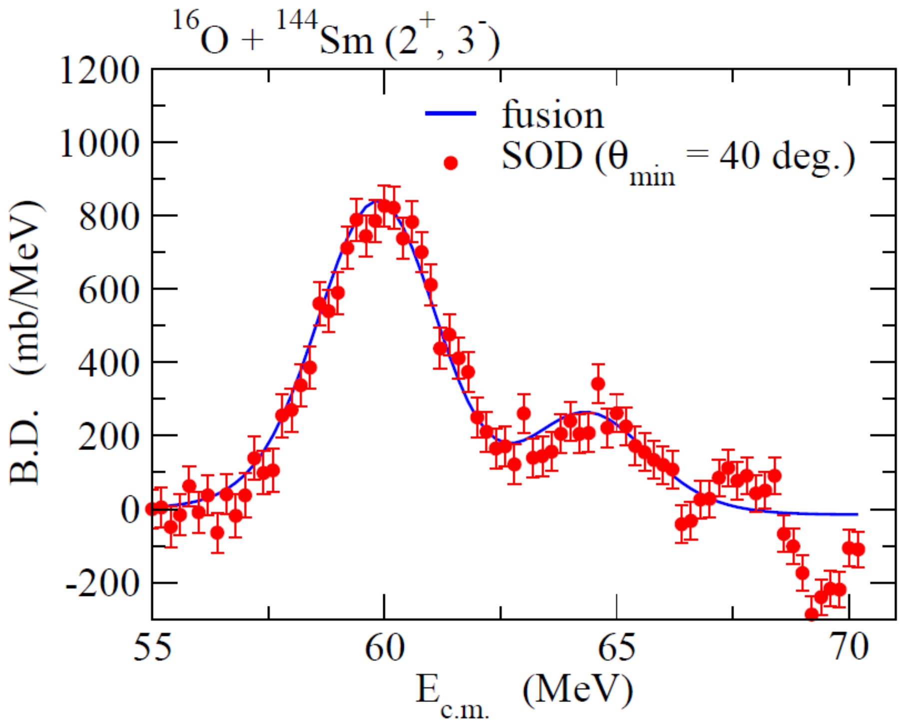
SOD with "experimental" quasi-elastic cross sections

$$\sigma_{qel}^{(exp)}(E,\theta) \sim \sigma_{qel}^{(th)}(E,\theta) + \Delta\sigma_{qel}^{(th)}(E,\theta)$$

← randomly generated

0.1% accuracy in $\sigma_{qel}(\theta)$



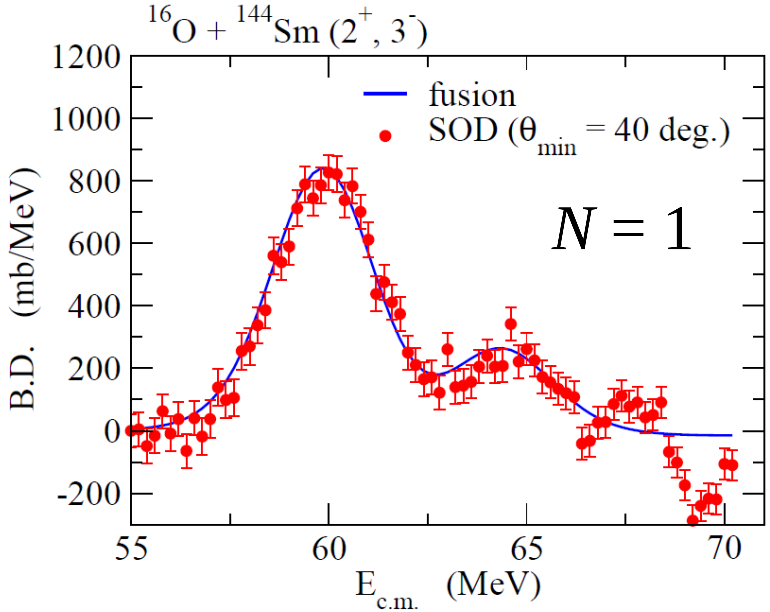


uncertainty in σ_{SOD}

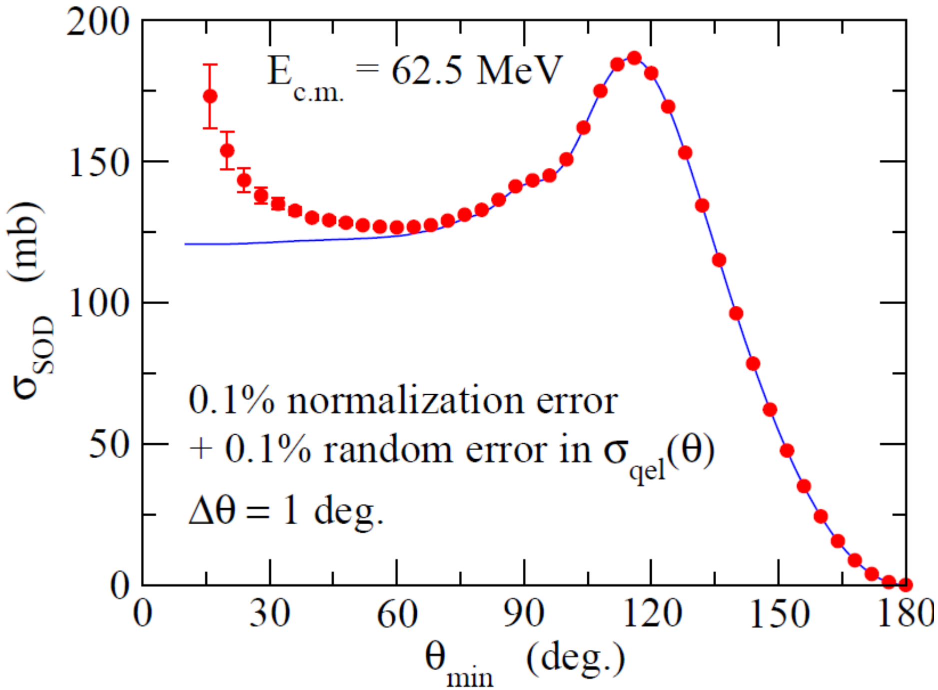
θ_{min} = 40 deg.	0.95%
30 deg.	1.96%
20 deg.	5.41%

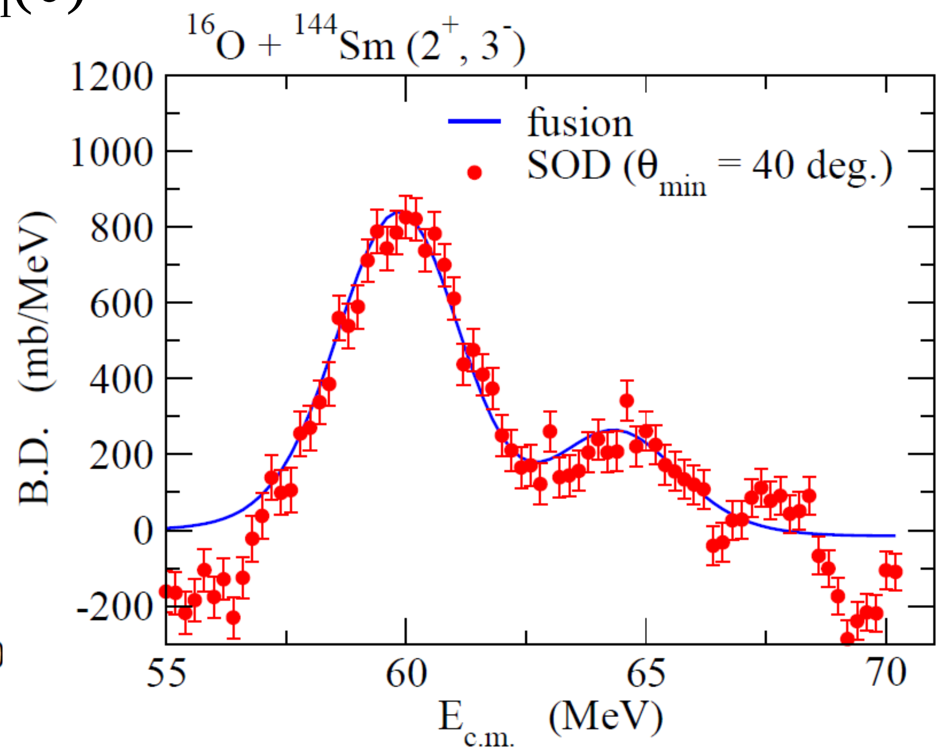
Effect of normalization error

$$\sigma_{\mathrm{qel}}^{(\mathrm{exp})}(E,\theta) \sim N\sigma_{\mathrm{qel}}^{(\mathrm{th})}(E,\theta) + \Delta\sigma_{\mathrm{qel}}^{(\mathrm{th})}(E,\theta)$$



$N = 0.999$ + 0.1% accuracy in $\sigma_{qel}(\theta)$





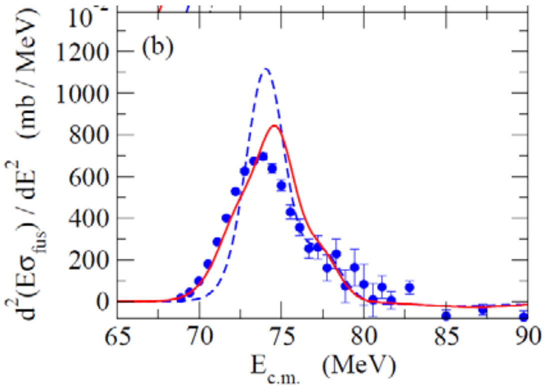
Summary

Heavy-ion subbarrier fusion reactions

✓strong interplay between reaction and structure
cf. fusion barrier distributions

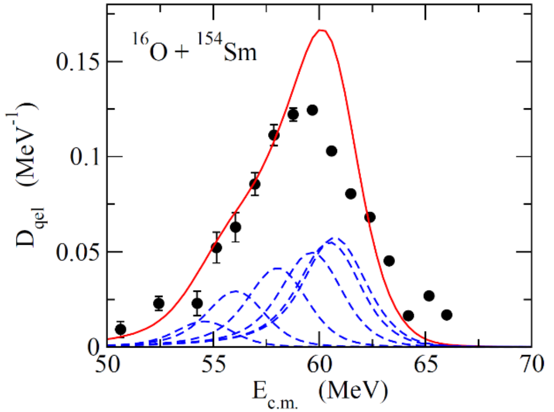
➢C.C. calculations with MR-DFT method

✓ anharmonicity
✓ truncation of phonon states
✓ octupole vibrations: $^{16}O + ^{208}Pb$



more flexibility:

- application to transitional nuclei
- a good guidance to a Q-moment of excited states



➢Quasi-elastic barrier distribution

- an alternative to fusion barrier distribution
- Relation to SOD
- more suitable to RI beams