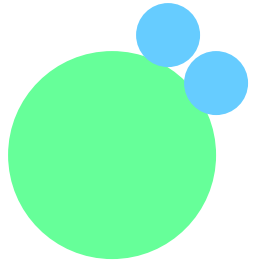
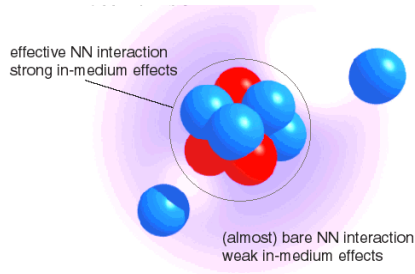


Di-neutron correlation in Borromean nuclei



^{11}Li , ^6He



K. Hagino (Tohoku University)

H. Sagawa (University of Aizu)

What is the spatial structure of valence neutrons?
Compact? Or Extended?

- 1. Introduction: Di-neutron correlation*
- 2. Three-body model for ^{11}Li and ^6He*
 - Spatial structure (geometry) of valence neutrons*
 - E1 strength*
- 3. 1-Dimensional 3-body model*
- 4. Summary*

Borromean nuclei and Di-neutron correlation

Borromean nuclei: unique three-body systems

Three-body model calculations:

strong di-neutron correlation
in ^{11}Li and ^6He

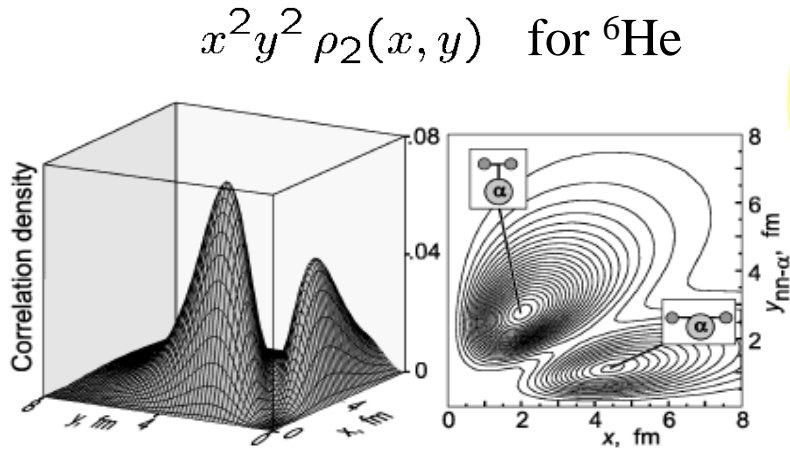
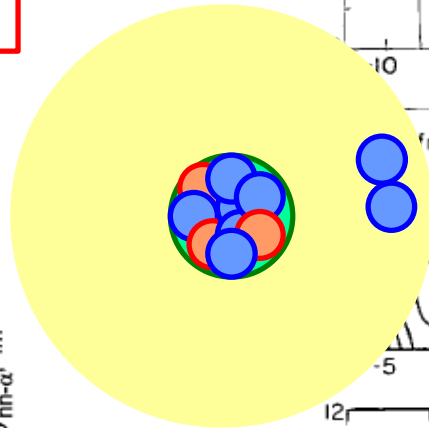
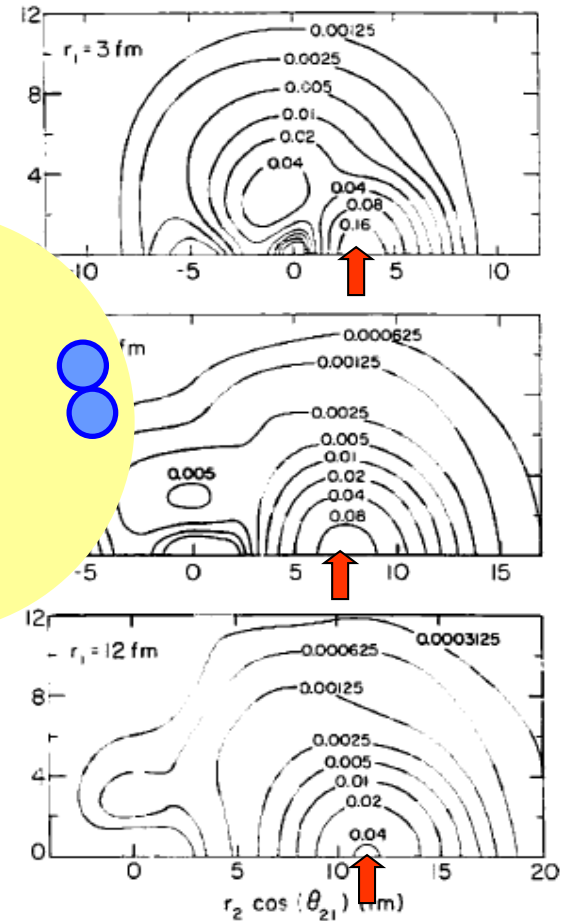


FIG. 1. Spatial correlation density plot for the 0^+ ground state of ^6He . Two components—di-neutron and cigarlike—are shown schematically.

Yu.Ts. Oganessian, V.I. Zagrebaev,
and J.S. Vaagen, *PRL*82('99)4996
M.V. Zhukov et al., *Phys. Rep.* 231('93)151



$\rho_2(r_1, r_2, \theta_{12})$ for ^{11}Li

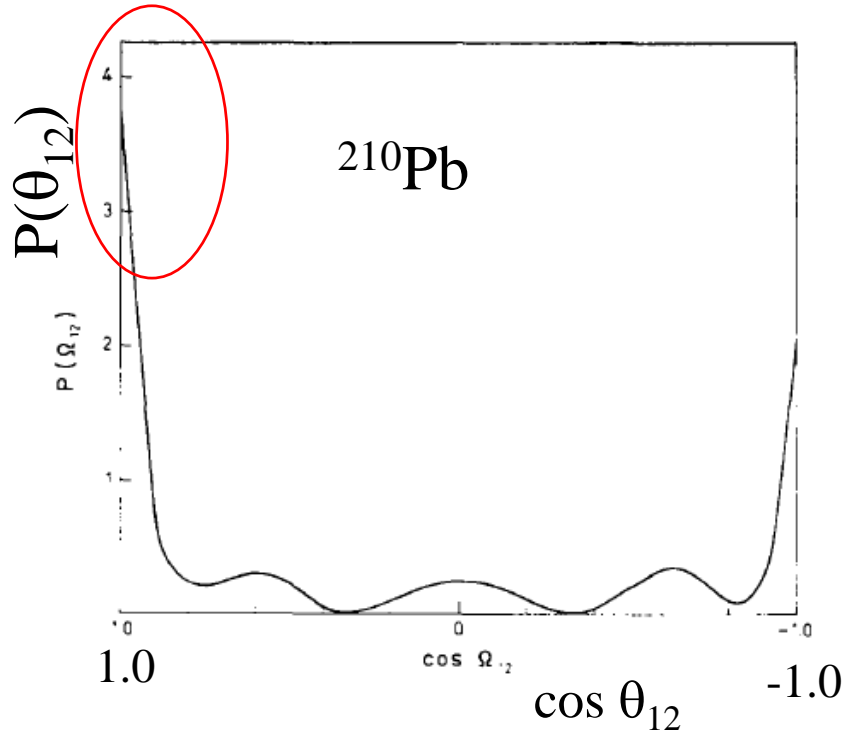


G.F. Bertsch, H. Esbensen,
Ann. of Phys., 209('91)327

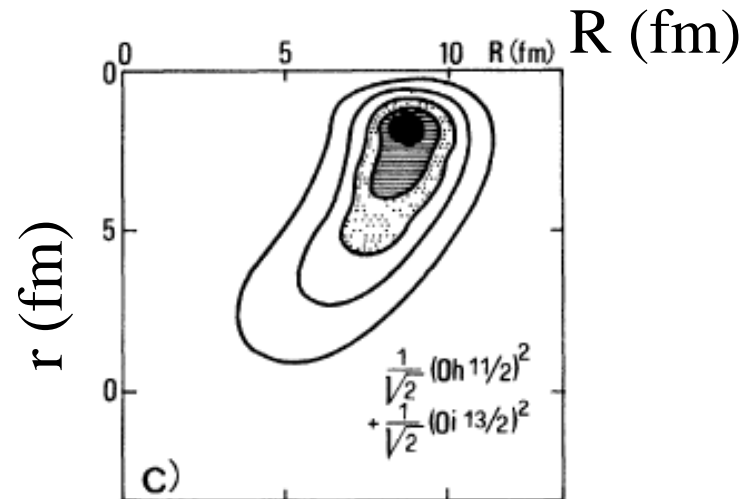
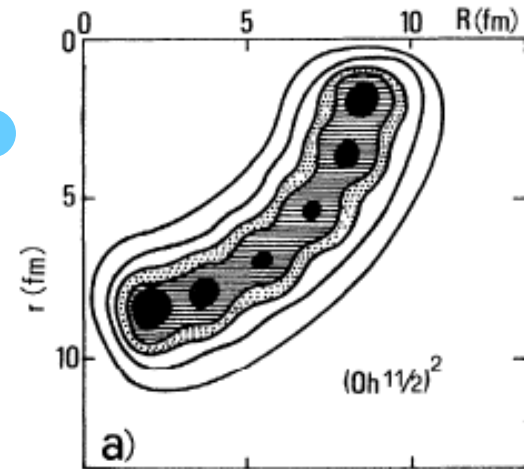
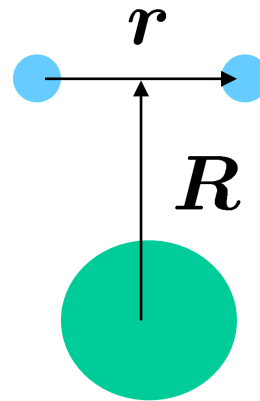
Borromean nuclei and Di-neutron correlation

Three-body model calculations:

strong di-neutron correlation
in ^{11}Li and ^6He



G.F. Bertsch, R.A. Broglia, and C. Riedel,
NPA91('67)123

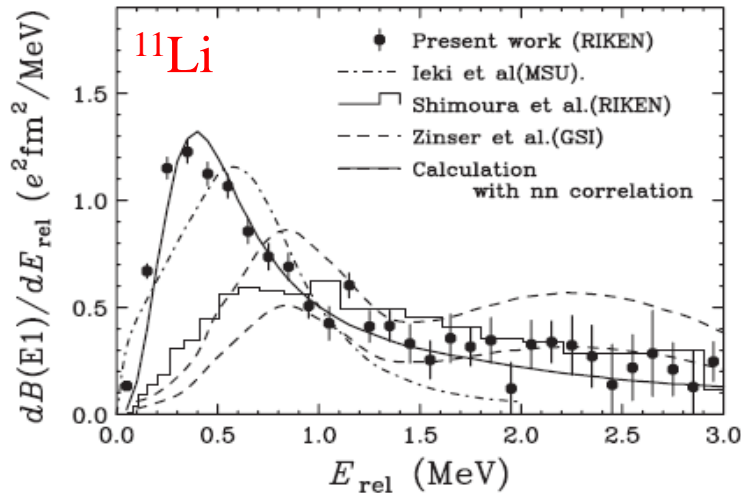


F. Catara, A. Insolia, E. Maglione,
and A. Vitturi, PRC29('84)1091

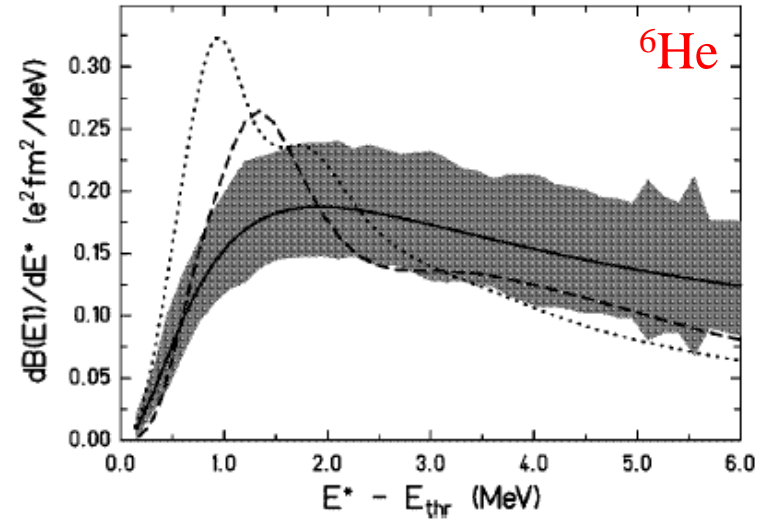
Remaining problem:

How to probe the strong dineutron correlation?

•Coulomb excitations?



T. Nakamura et al., PRL96('06)252502



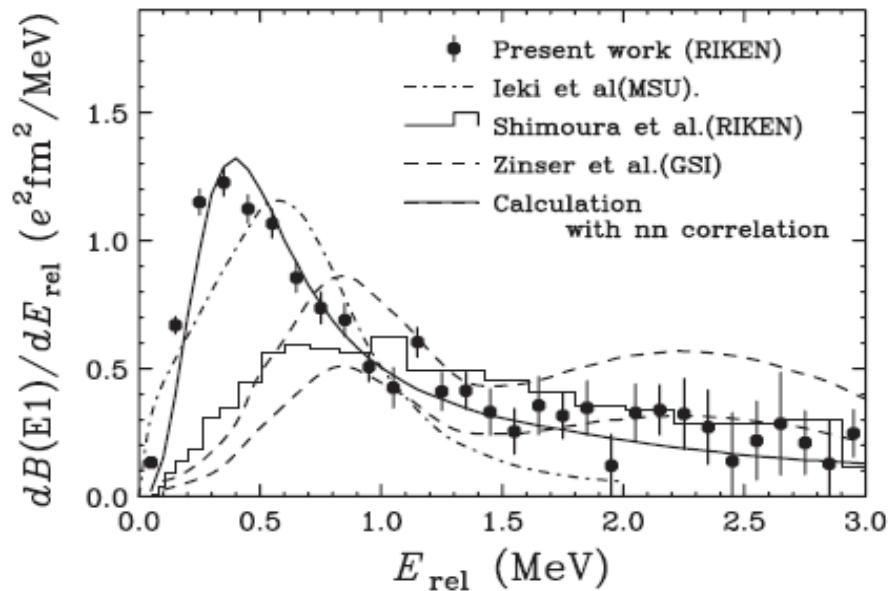
T. Aumann et al., PRC59('99)1252

* (indirect) evidence for dineutron correlation

dineutron correlation in the ground state?

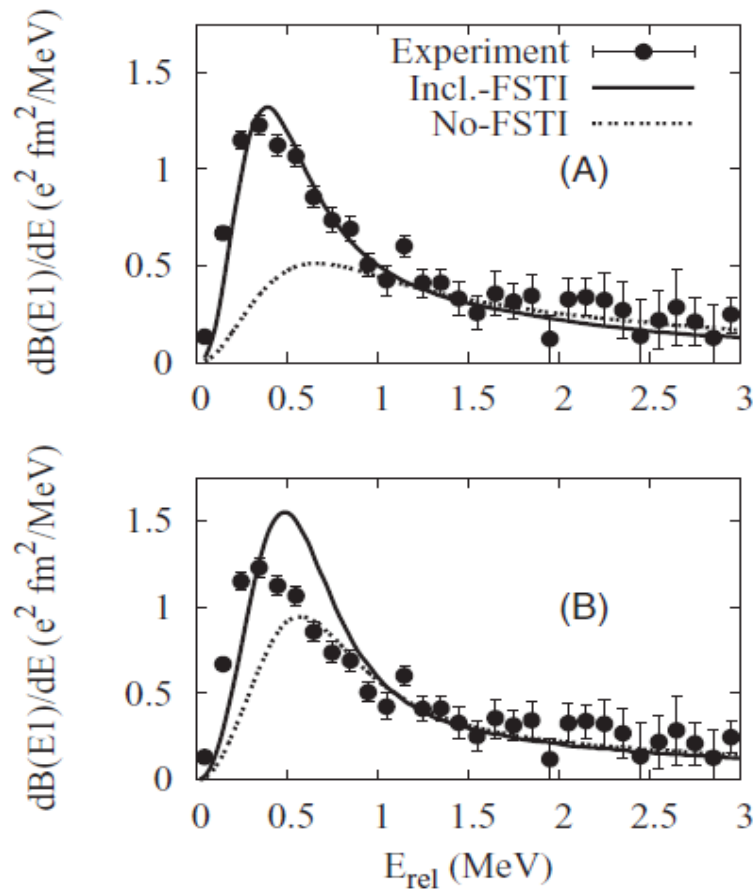
Experimental evidence: T. Nakamura et al., PRL96('06)252502

Recent Coulomb dissociation data of ^{11}Li



renewed interests in dineutron correlations in weakly bound nuclei

c.f. M. Matsuo et al., PRC71('05)064326
PRC73('06)044309



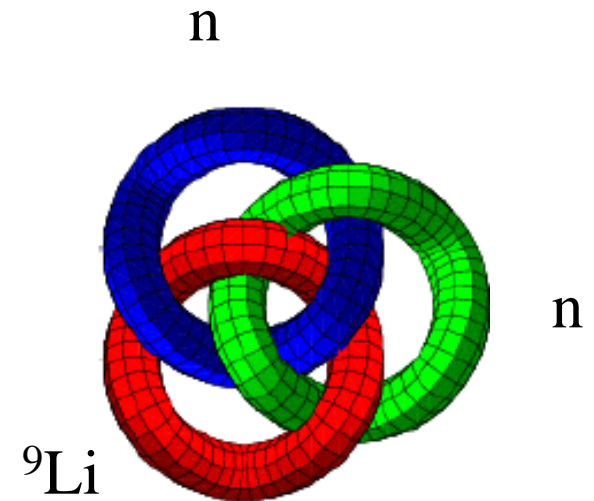
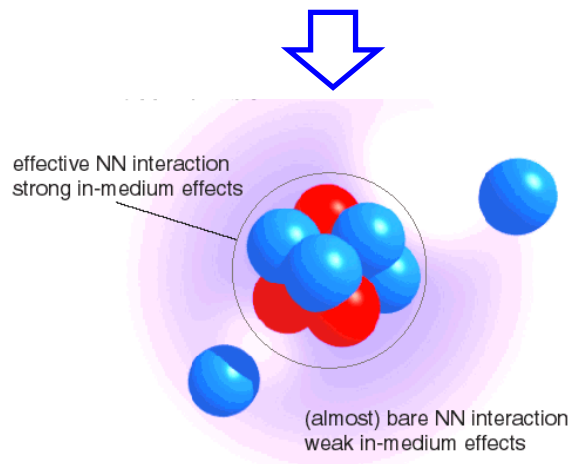
H. Esbensen, K. Hagino,
P. Mueller, and H. Sagawa,
PRC76('07)024302

Remaining problems

- spatial structure of dineutron (cf. a large pair coherence length?)
- dineutron correlation in heavy nuclei?
- E1 excitations?
- Pair transfer?

What is the spatial structure of the valence neutrons?

To what extent is this picture correct?



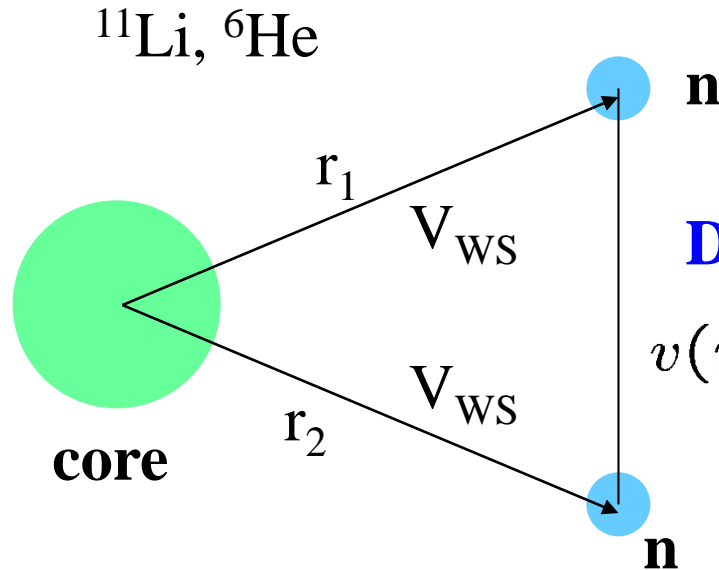
Three-body model with density-dependent delta force

G.F. Bertsch and H. Esbensen,

Ann. of Phys. 209('91)327

H. Esbensen, G.F. Bertsch, K. Hencken,

Phys. Rev. C 56('99)3054

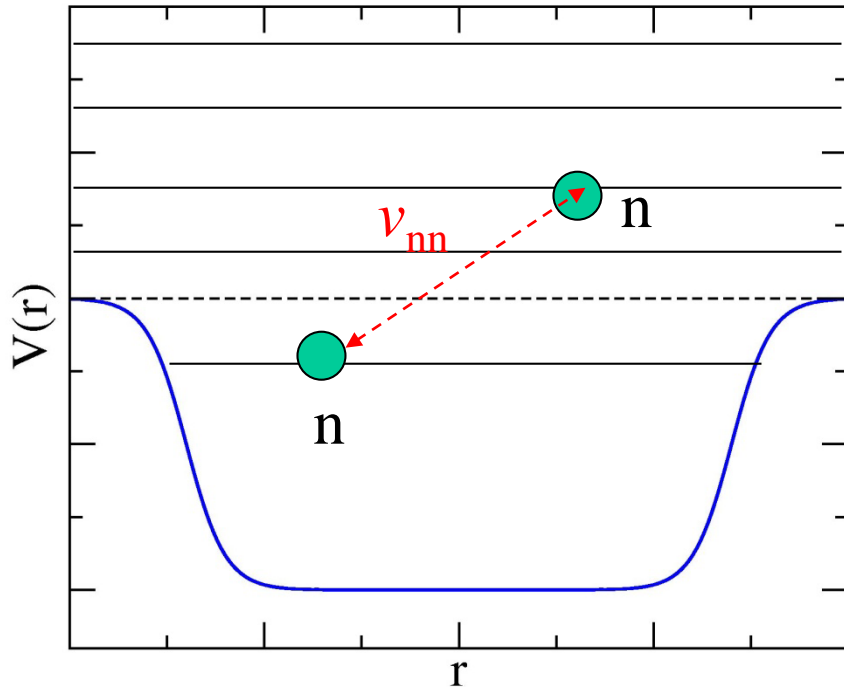


Density-dependent delta-force

$$v(\mathbf{r}_1, \mathbf{r}_2) = v_0(1 + \alpha\rho(r)) \times \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(p_1 + p_2)^2}{2A_c m}$$

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(p_1 + p_2)^2}{2A_C m}$$



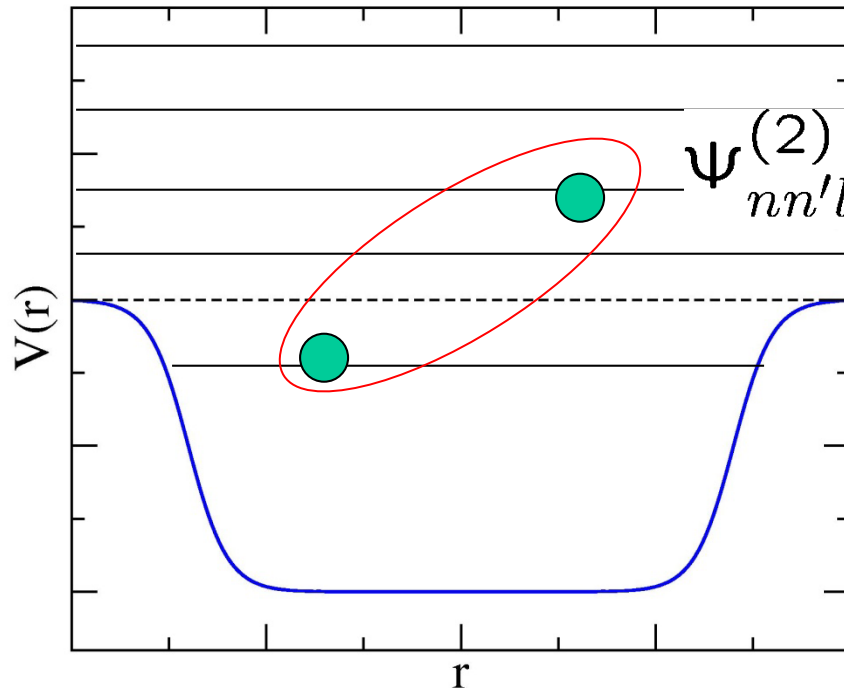
continuum states:
discretized in a large box

$$V_{nn}(\mathbf{r}_1, \mathbf{r}_2) = \delta(\mathbf{r}_1 - \mathbf{r}_2) \left(v_0 + \frac{v_\rho}{1 + \exp[(r_1 - R_\rho)/a_\rho]} \right)$$

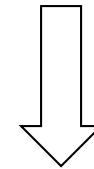
- ✓ contact interaction
- ✓ v_0 : free n-n
- ✓ density dependent term: medium many-body effects

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(p_1 + p_2)^2}{2A_cm}$$

$$\Psi_{gs}(\mathbf{r}, \mathbf{r}') = \mathcal{A} \sum_{nn'lj} \alpha_{nn'lj} \Psi_{nn'lj}^{(2)}(\mathbf{r}, \mathbf{r}')$$



uncorrelated basis



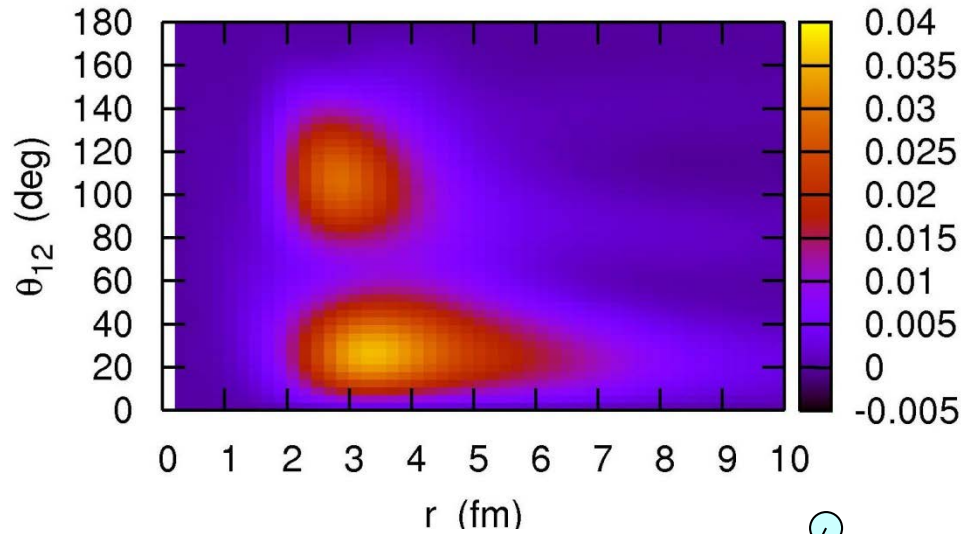
diagonalization of Hamiltonian matrix

(~ 1500 dimensions)

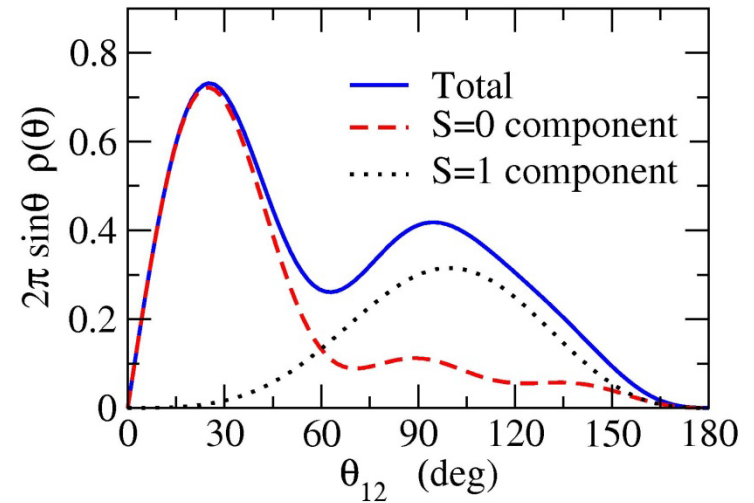
Two-particle density for the ground state of ^{11}Li and ^6He

^{11}Li

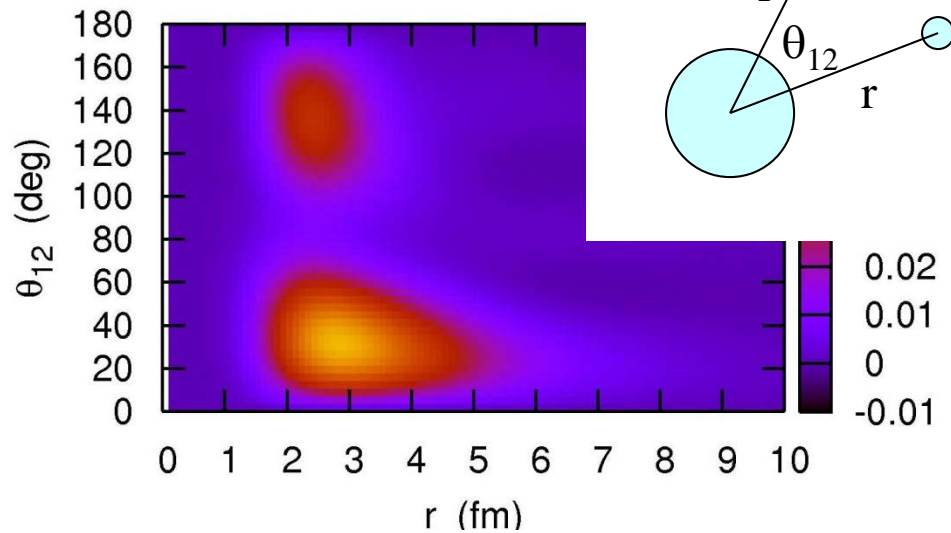
$$8\pi^2 r^4 \sin \theta_{12} \cdot \rho_2(r, r, \theta_{12})$$



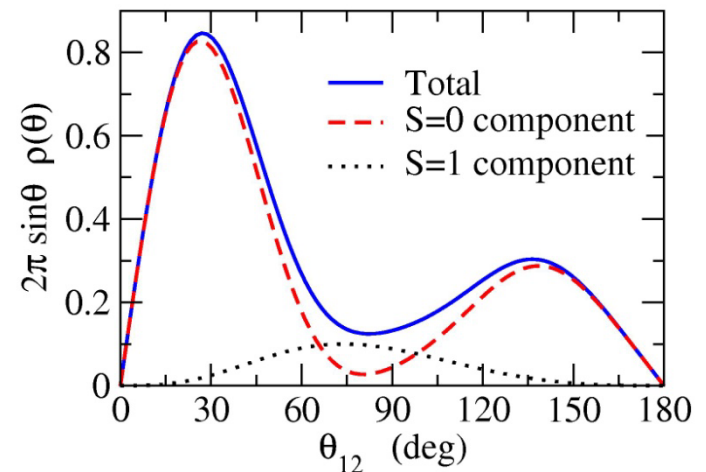
→ strong di-neutron correlation



^6He



→ $\langle \theta_{12} \rangle = 65.29$ deg.



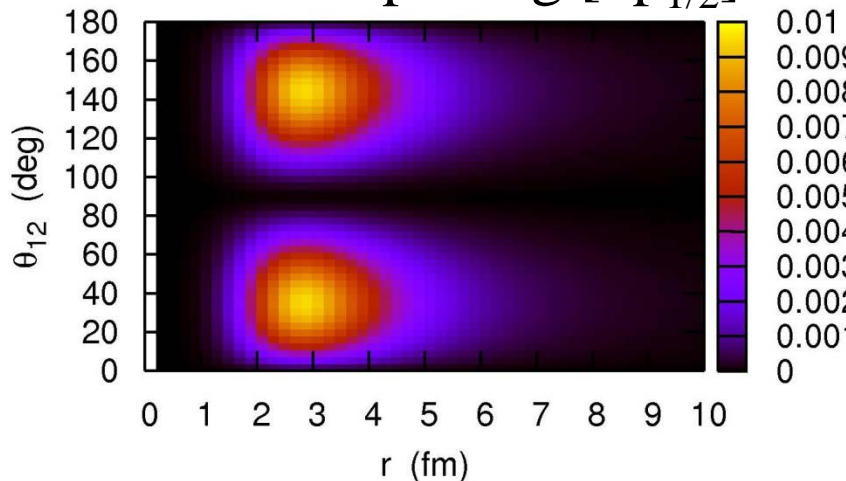
→ $\langle \theta_{12} \rangle = 66.33$ deg.

✧ Role of pairing correlation

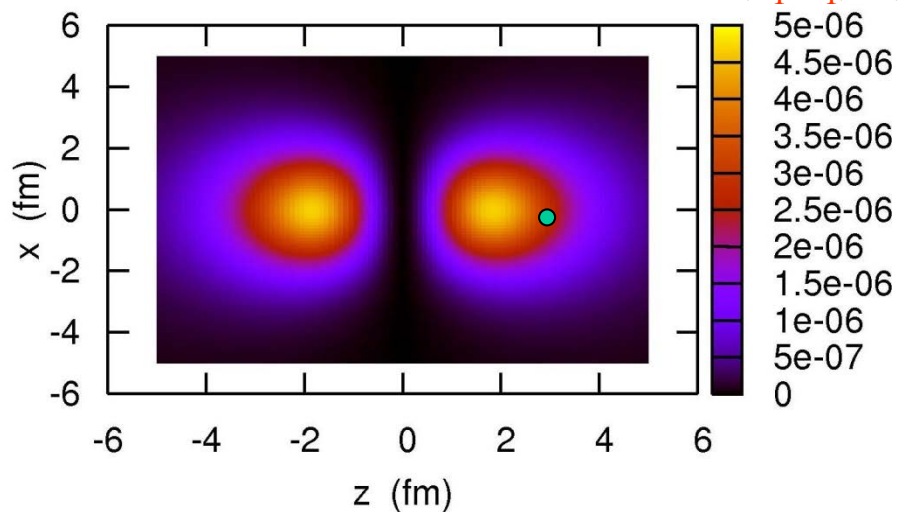
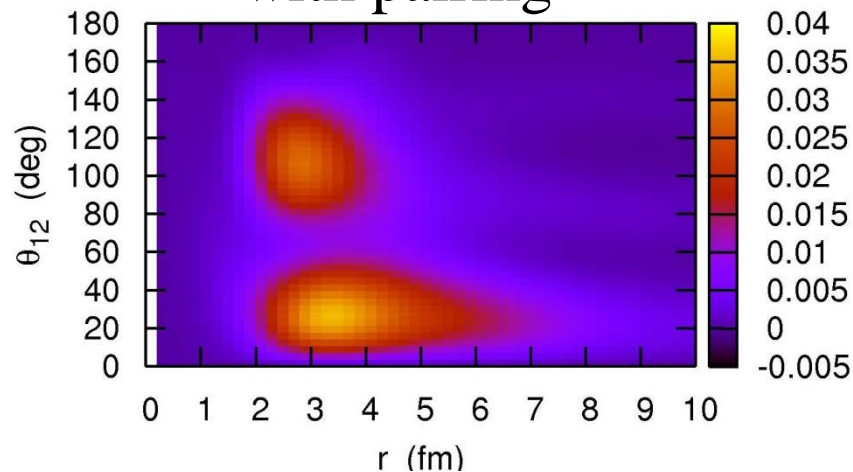
→ configuration mixing of different parity states

^{11}Li

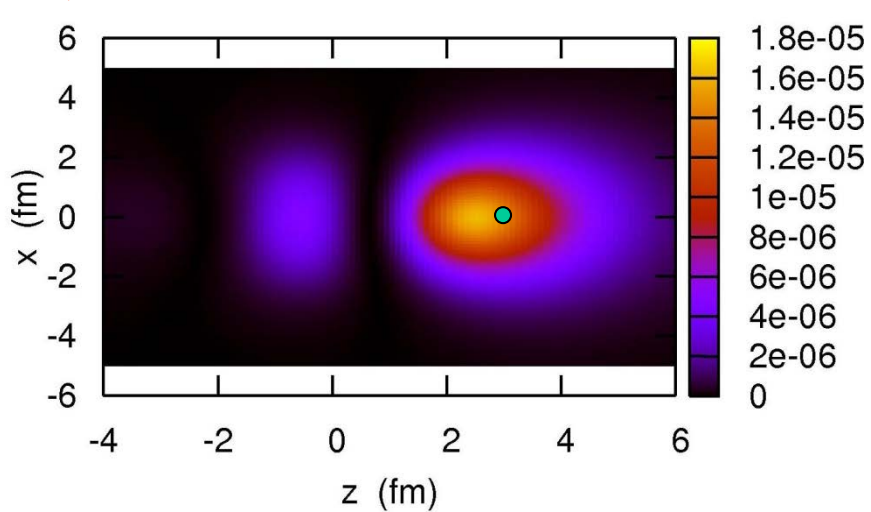
without pairing $[1p_{1/2}]^2$



with pairing

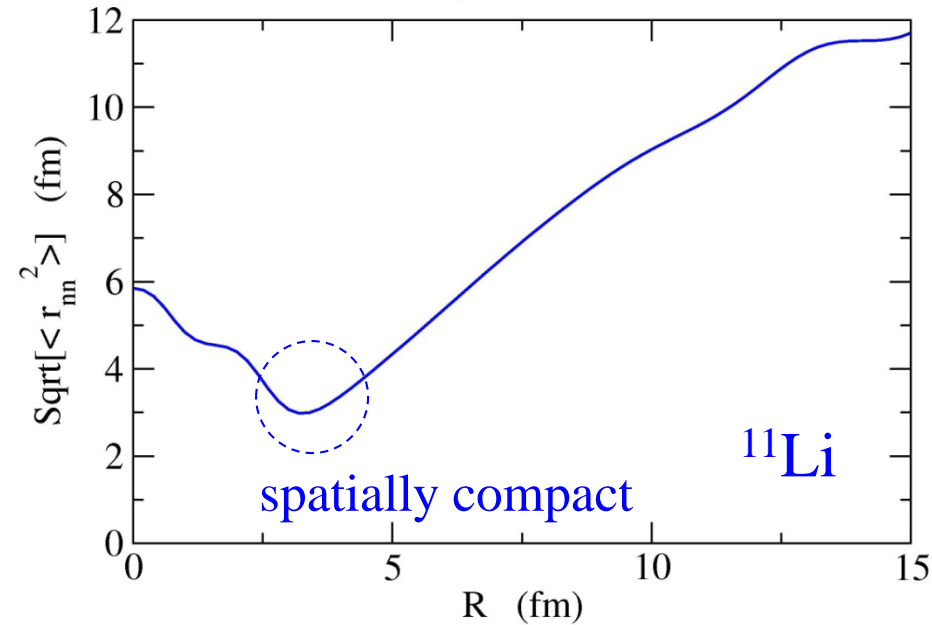


$(z_1, x_1) = (3.4 \text{ fm}, 0)$

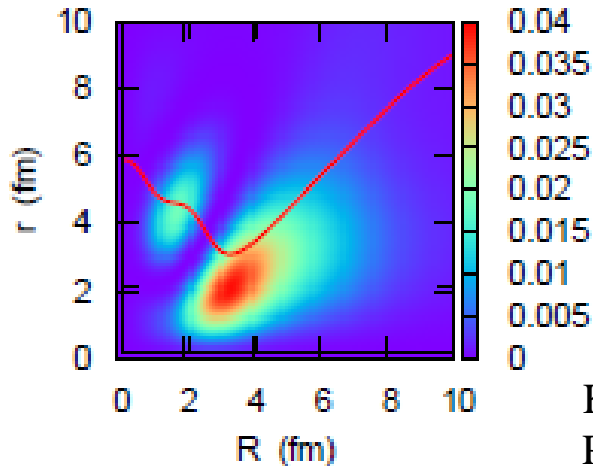


2n-rms distance

$$\sqrt{\langle r_{nn}^2 \rangle}(R) = \sqrt{\frac{\int r^4 dr |f_{L=0}(r, R)|^2}{\int r^2 dr |f_{L=0}(r, R)|^2}}$$

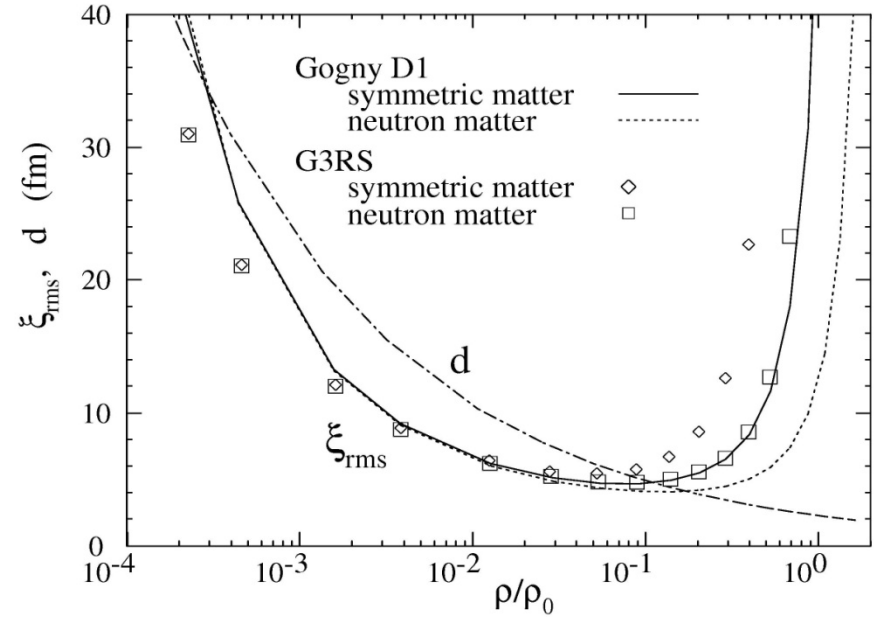


K.Hagino, H. Sagawa, J. Carbonell, and P. Schuck, PRL99('07)022506

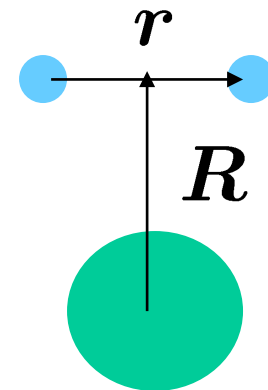


K.H., H. Sagawa, and
P. Schuck, J. of Phys. G37('10)064040

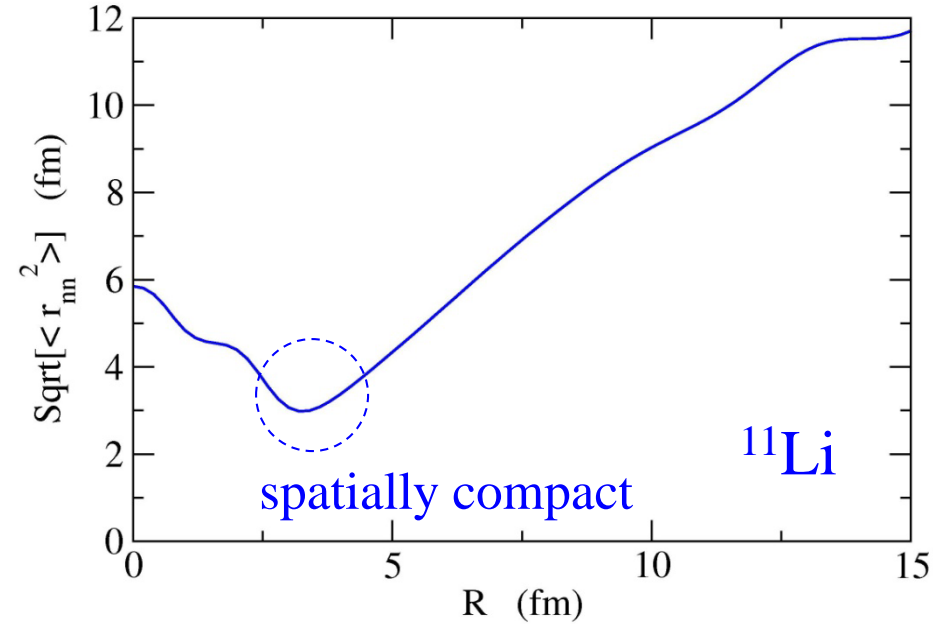
Matter Calc.



M. Matsuo, PRC73('06)044309

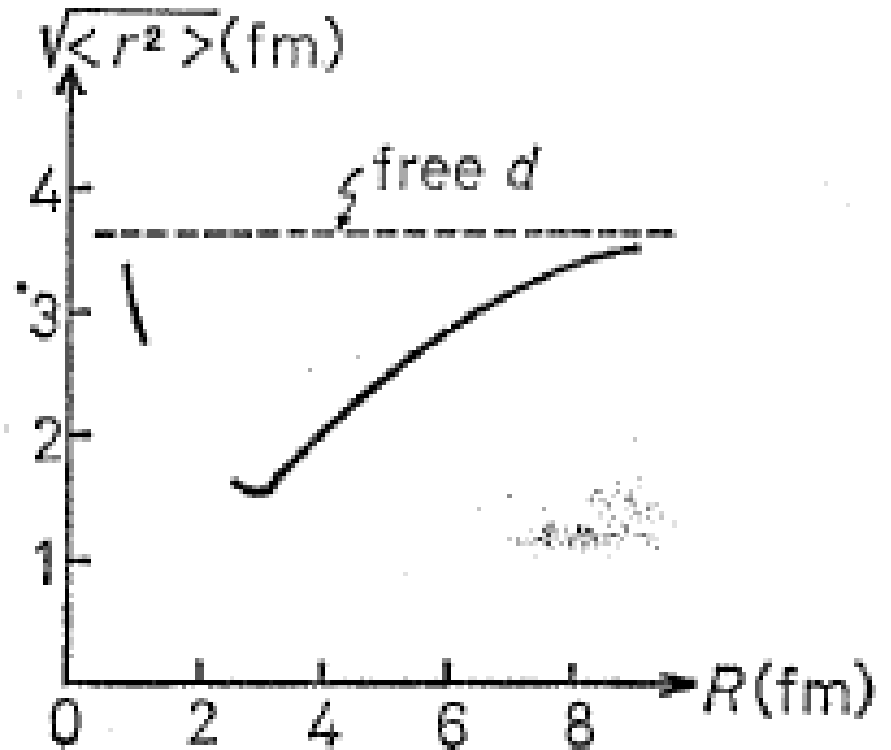


2n-rms distance



K.H., H. Sagawa, J. Carbonell, and P. Schuck,
PRL99('07)022506

Size of deuteron in ^6Li



K. Itonaga and H. Bando,
PTP44('70)1232

Study of Deuteron-Cluster Deformation Using the Reaction ${}^6\text{Li}(d, tp){}^4\text{He}$

J. Y. Grossiord, C. Coste, A. Guichard, M. Gusakow, A. K. Jain,* and J. R. Pizzi

Institut de Physique Nucléaire, Université Claude Bernard Lyon-I,

and Institut National de Physique Nucléaire et de Physique des Particules, 69621 Villeurbanne, France

and

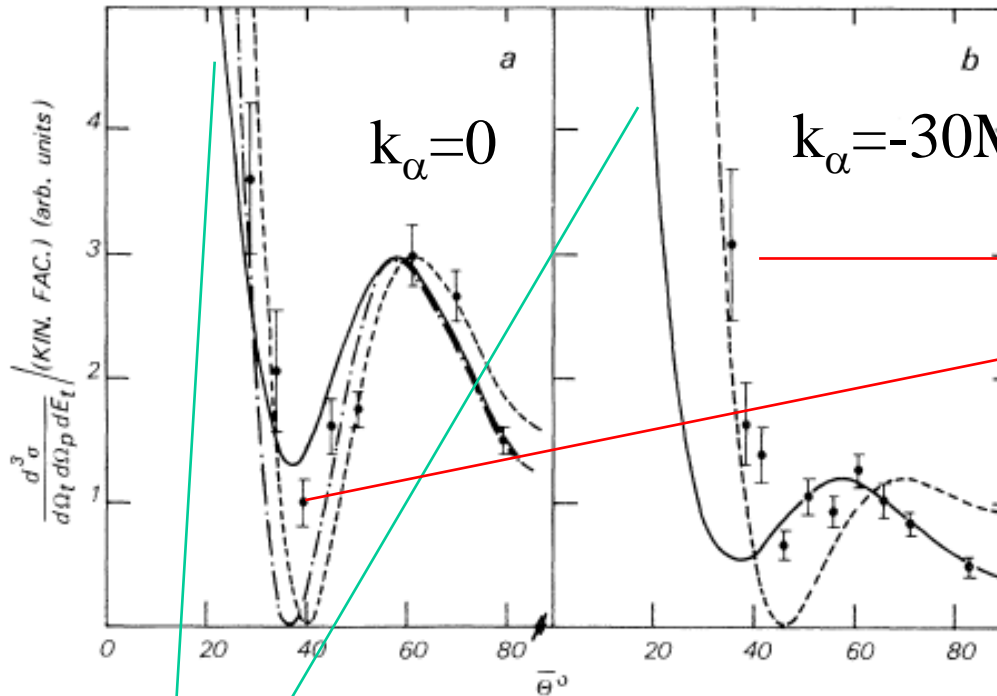
G. Bagieu and R. de Swiniarski

Institut des Sciences Nucléaires de Grenoble

and Institut National de Physique Nucléaire et de Physique des Particules, 38044 Grenoble, France

(Received 20 November 1973)

PRL32('74)173



${}^6\text{Li}(d, tp){}^4\text{He}$

Contraction of the deuteron cluster in ${}^6\text{Li}$

L. Végh

Institute of Nuclear Physics, Debrecen, Hungary

J. Erö

Central Research Institute for Physics, Budapest, Hungary

(Received 15 August 1980)

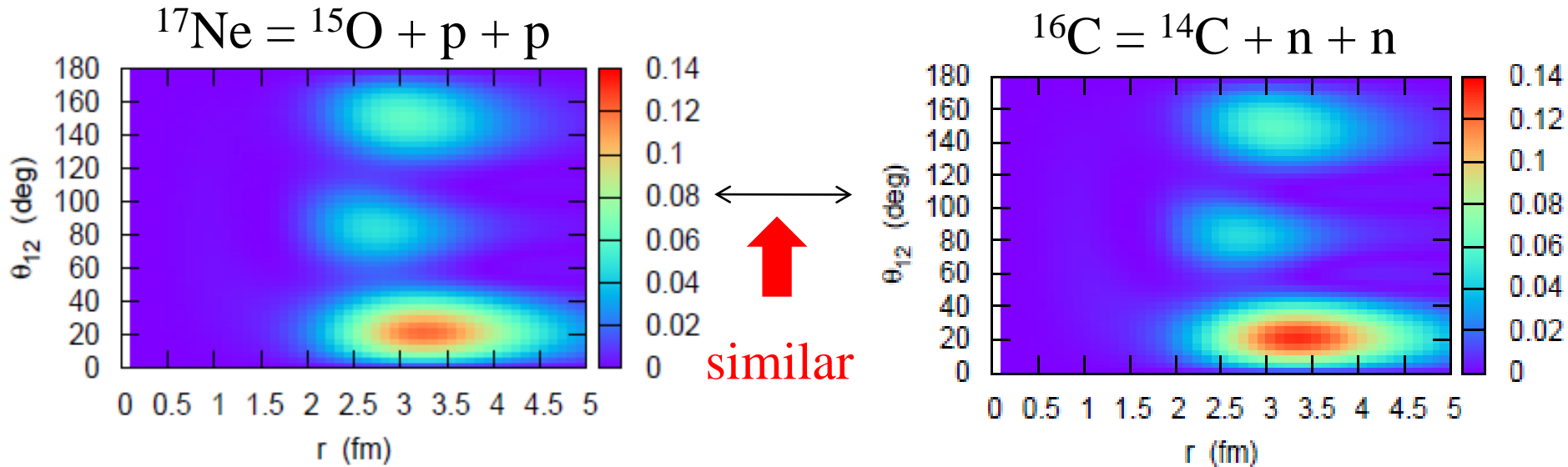
$d(d, t)p$

PRC23('81)2371

cf. “di-proton” correlation

$$^{17}\text{Ne} = ^{15}\text{O} + \text{p} + \text{p} \quad (S_{2\text{p}} = 0.944 \text{ MeV})$$

v_{pp} = density-dep. contact interaction + Coulomb



$$\langle v_{\text{pp}}^{(\text{nucl})} \rangle = -3.26 \text{ MeV}$$

$$\langle v_{\text{pp}}^{(\text{Coul})} \rangle = 0.448 \text{ MeV} \quad \leftarrow \text{about 14\% contribution}$$

cf. “di-proton” correlation

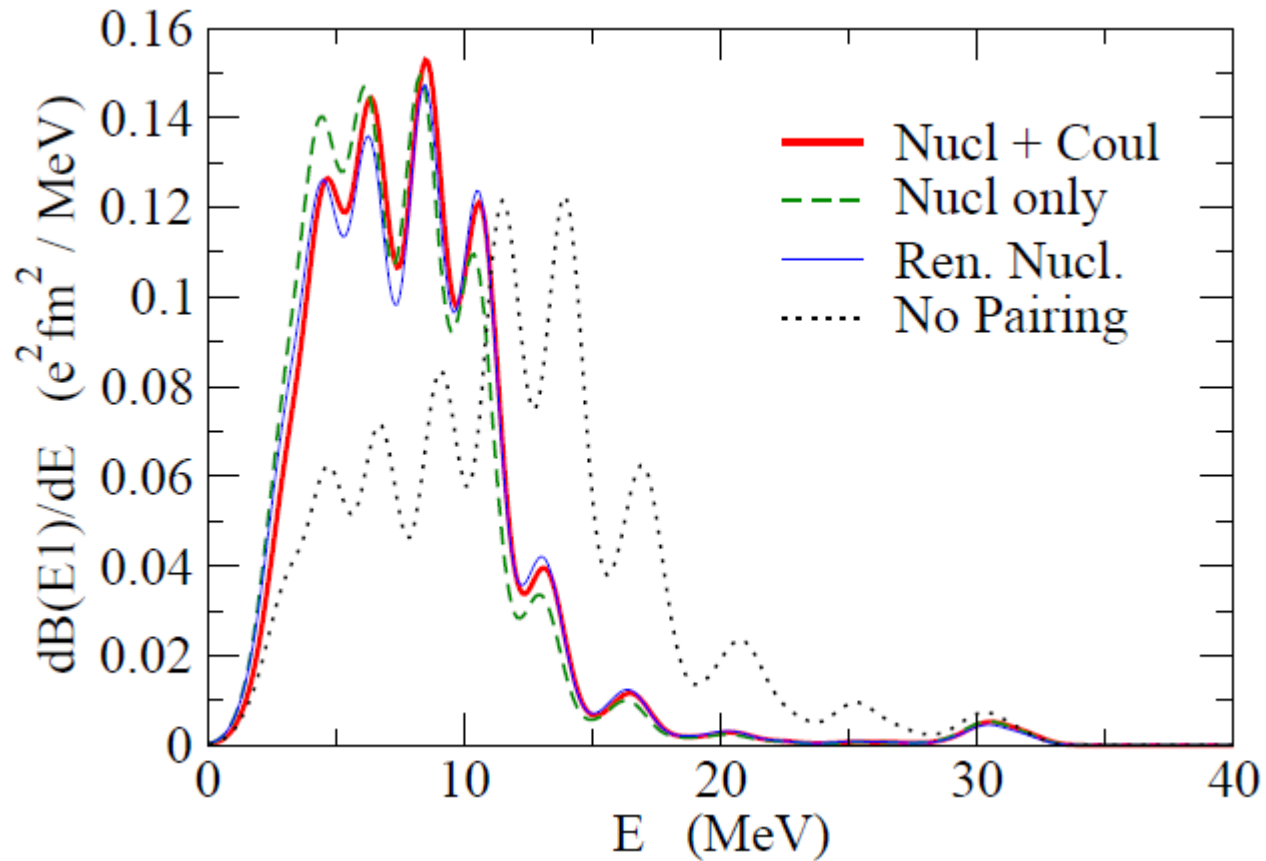
	^{17}Ne	^{17}Ne (No Coulomb)	^{16}C
$\langle V_{\text{pp}}^{(N)} \rangle$ (MeV)	-3.26	-2.76	-3.88
$\langle V_{\text{pp}}^{(C)} \rangle$ (MeV)	0.448	0	0
$P([s_{1/2}]^2)$ (%)	15.16	15.91	20.69
$P([d_{5/2}]^2)$ (%)	75.19	75.68	64.97
$P([d_{3/2}]^2)$ (%)	3.83	3.26	6.63
$\langle r_{\text{NN}}^2 \rangle^{1/2}$ (fm)	4.688	4.749	4.579
$\langle r_{\text{C}-2\text{N}}^2 \rangle^{1/2}$ (fm)	3.037	3.037	3.099
$\delta \langle r^2 \rangle^{1/2}$ (fm)	1.267	1.273	1.306
$\langle \theta_{12} \rangle$ (deg)	76.64	76.03	74.35



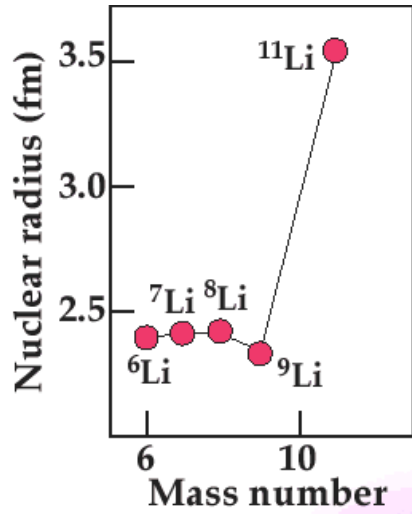
exact treatment
for Coulomb



switch off the Coulomb,
but renormalize the nuclear
force



T. Oishi, K. Hagino, and H. Sagawa,
arXiv:1109.2994 [nucl-th]
Phys. Rev. C, in press.

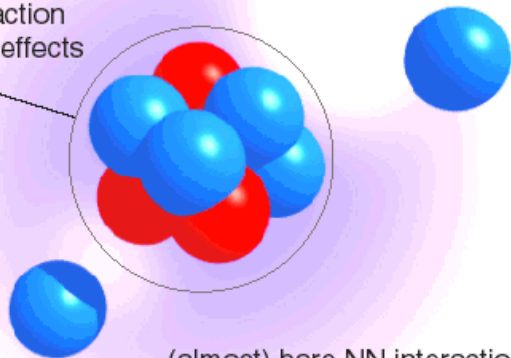


I. Tanihata et al.
 Phys. Rev. Lett. 55, 2676 (1985)

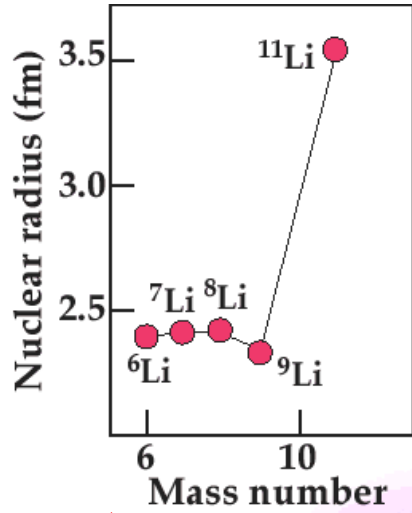
Interaction cross section
 measurements at Bevalac
 (790 MeV/u)

11Li

effective NN interaction
 strong in-medium effects

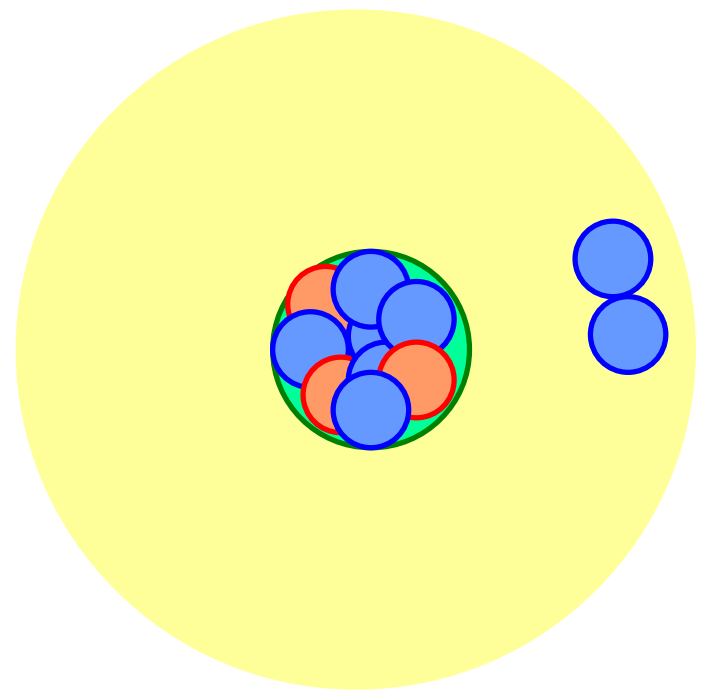
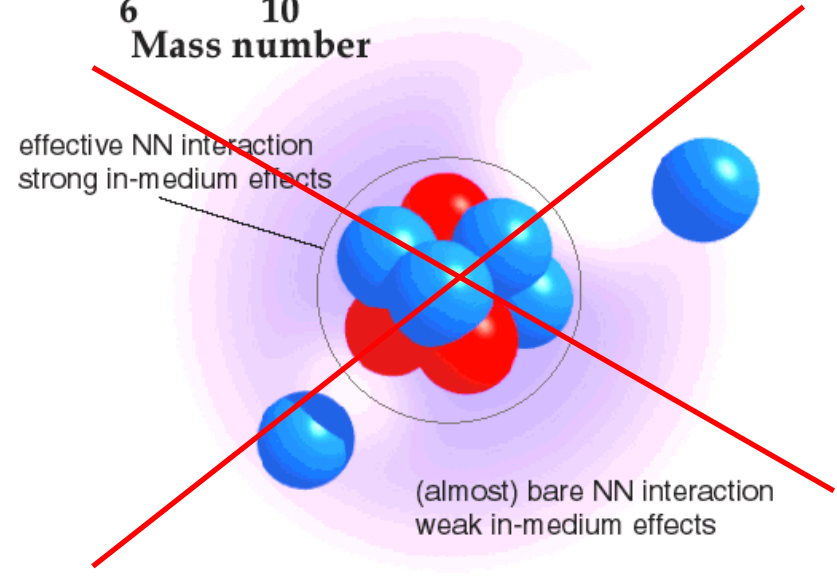


(almost) bare NN interaction
 weak in-medium effects



I. Tanihata et al.
 Phys. Rev. Lett. 55, 2676 (1985)

Interaction cross section
 measurements at Bevalac
 (790 MeV/u)



Dipole excitations

Response to the dipole field:

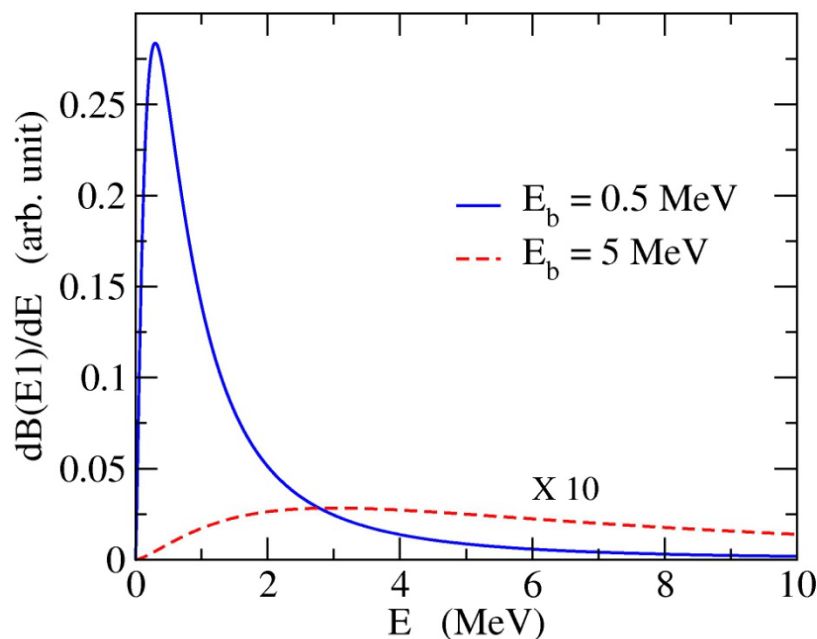
$$B_k(E1) = 3 |\langle \Psi_{1-}^k | \hat{D}_0 | \Psi_{gs} \rangle|^2$$

$$\hat{D} = -\frac{Ze}{A} (r_1 + r_2)$$

excited states ground state

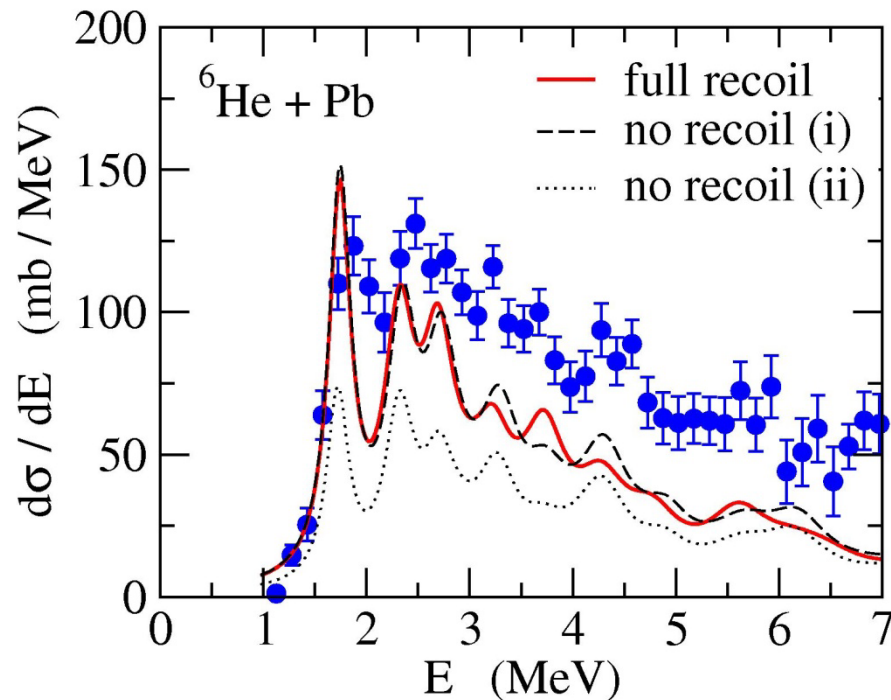
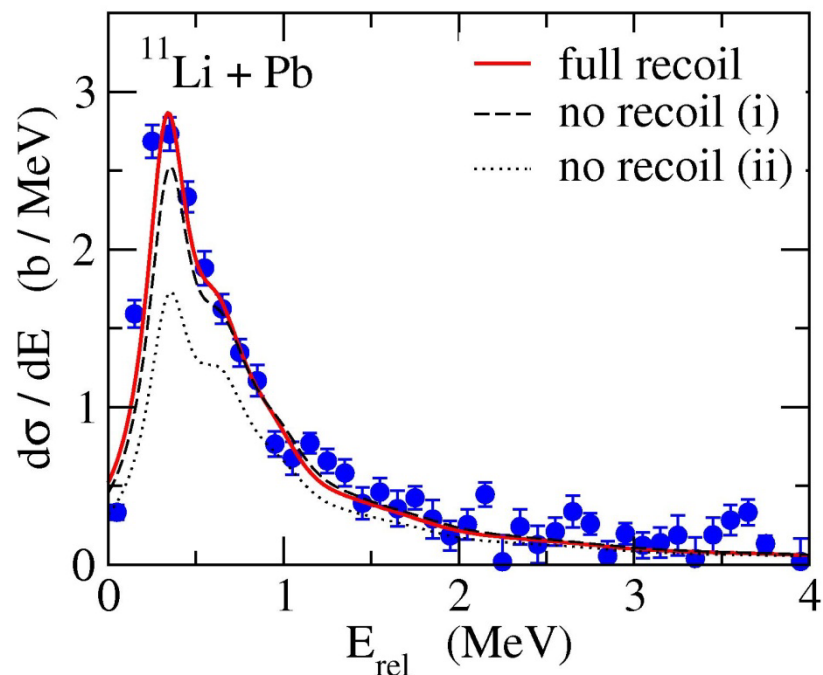
(note) analytic expression for a single-particle potential model

$$\psi_b(r) \sim h_l(ik_b r), \quad \psi_c(r) \sim j_l(k_c r)$$



$$\frac{dB(E1)}{dE} \propto \frac{\sqrt{E_b} E_c^{3/2}}{(E_c + E_b)^4}$$

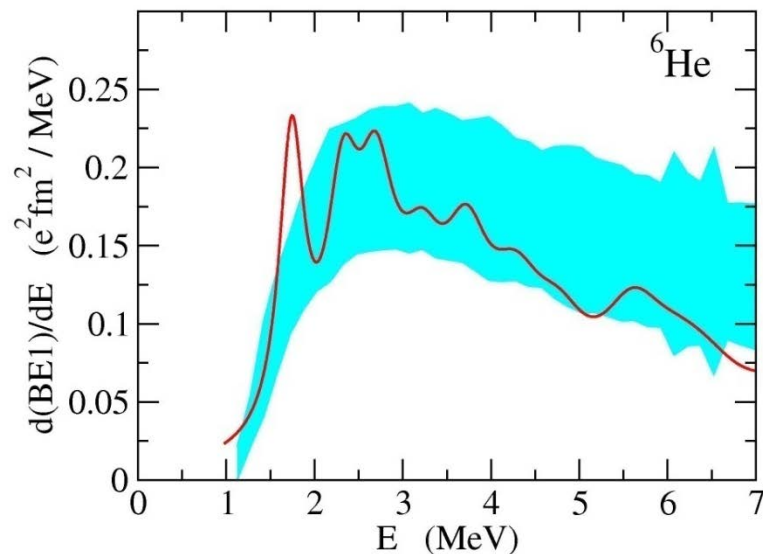
Dipole excitations



Response to the dipole field:

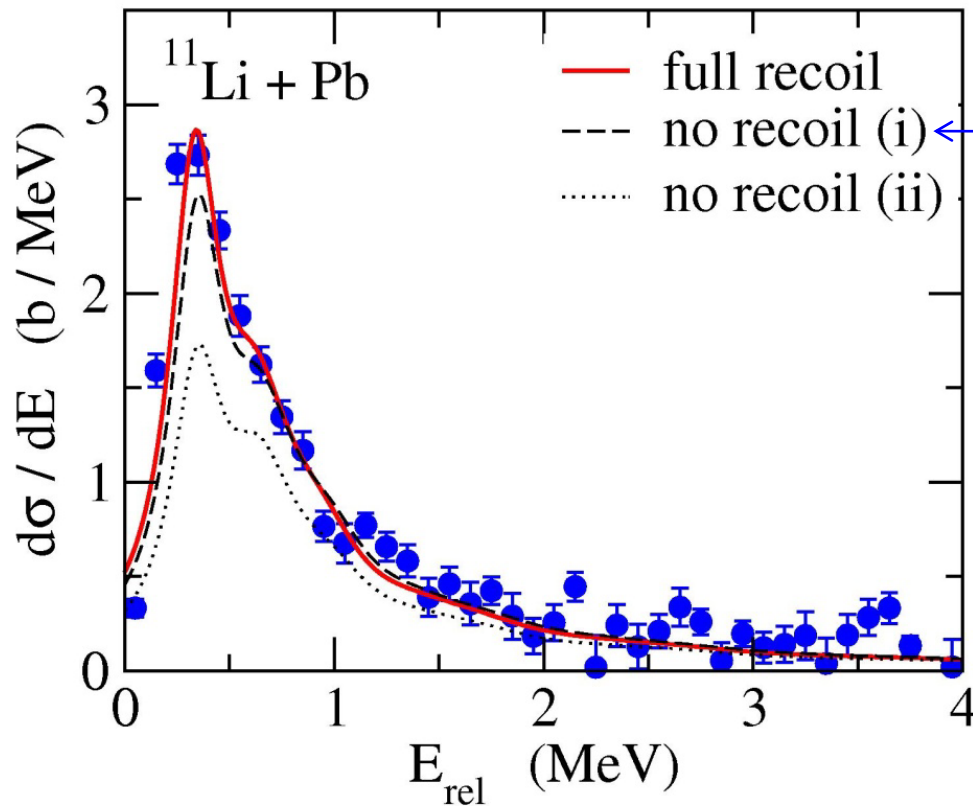
$$B_k(E1) = 3 |\langle \Psi_{1-}^k | \hat{D}_0 | \Psi_{gs} \rangle|^2$$

$$\hat{D} = -\frac{Ze}{A} (r_1 + r_2)$$



recoil term

$$\begin{aligned} H &= \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn}(\mathbf{r}_1, \mathbf{r}_2) + \frac{(p_1 + p_2)^2}{2A_c m} \\ &= \frac{p_1^2}{2\mu} + \frac{p_2^2}{2\mu} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn}(\mathbf{r}_1, \mathbf{r}_2) + \frac{p_1 \cdot p_2}{A_c m} \end{aligned}$$



with recoil for the g.s.
but without for 1^-

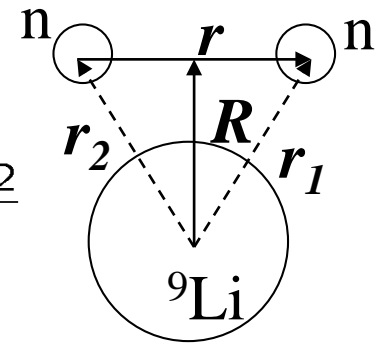


continuum calculations

More direct information on the correlation?

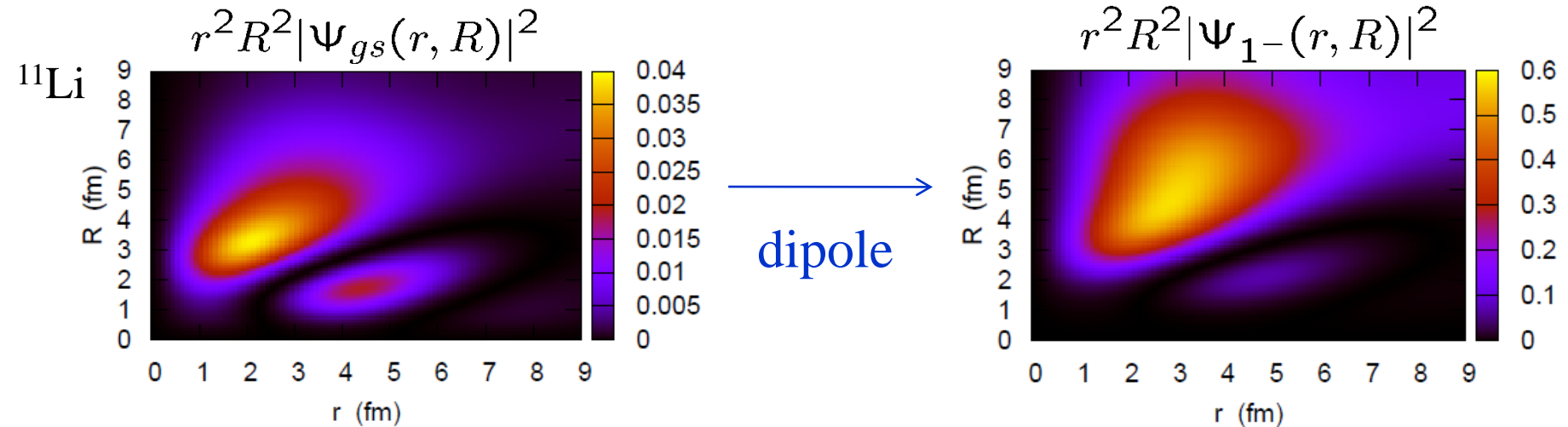
$$|\Psi_{1-}\rangle = \hat{D}|\Psi_{gs}\rangle$$

$$R = \frac{r_1 + r_2}{2}$$



dipole
operator

$$\hat{D} = -\frac{Ze}{A}(r_1 + r_2) = -\frac{2Ze}{A}R$$



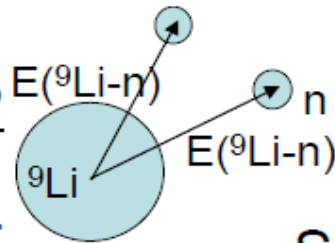
The relative motion $r = r_1 - r_2$ is not affected by the dipole operator.

→ Probing dineutron correlation with E1 excitation?

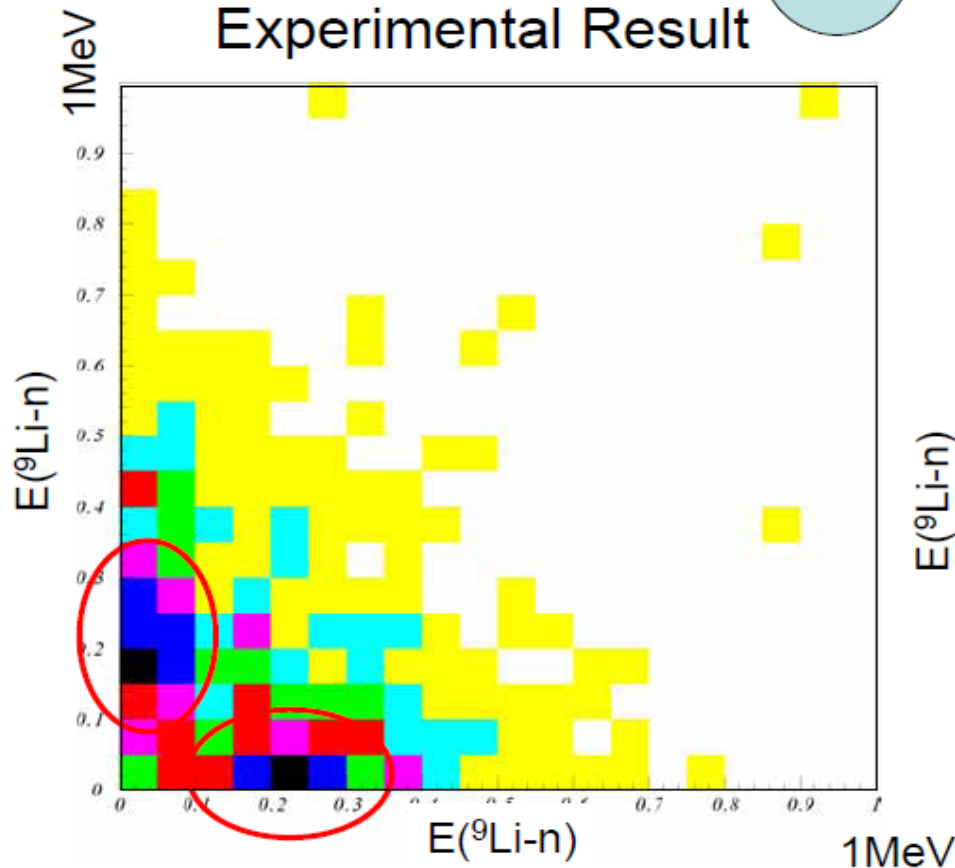
especially with energy (and angular) distribution(s)?

New 2n correlation experiment (Nakamura et al.)

Further Correlation?

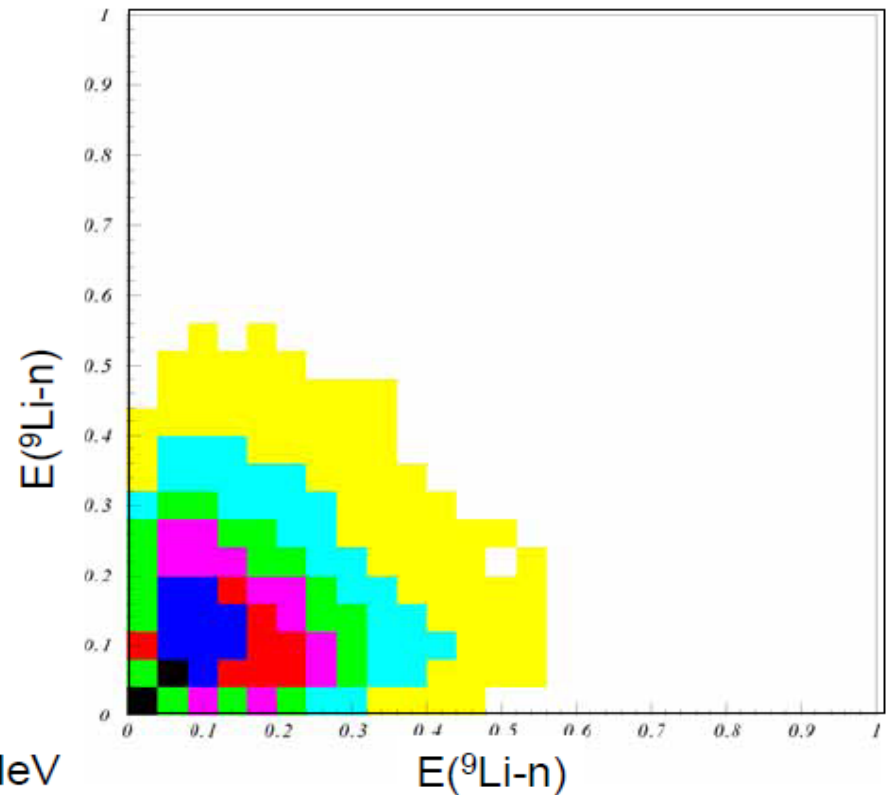


Experimental Result



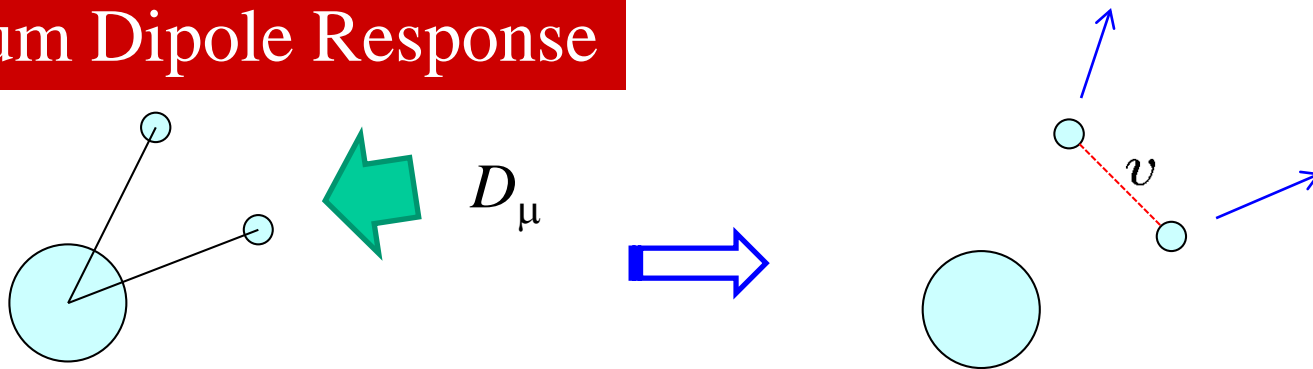
preliminary

Simulation (Phase Space)



T. Nakamura et al., unpublished

Continuum Dipole Response



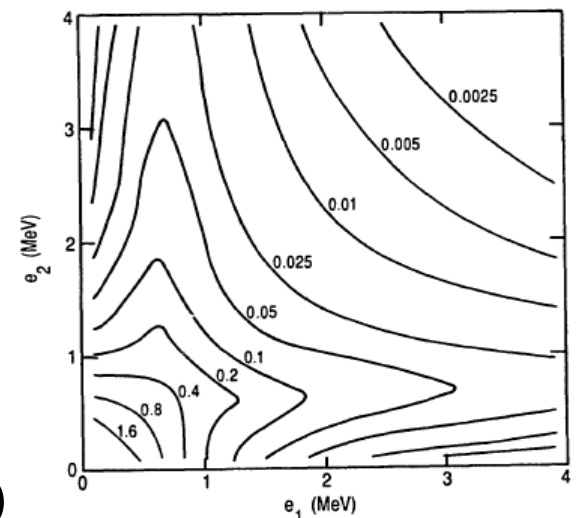
$$\begin{aligned}
 M(E1) &= \langle (j_1 j_2)_{\mu}^1 | (1 - vG_0 + vG_0 vG_0 - \dots) D_{\mu} | \Psi_{gs} \rangle \\
 &= \langle (j_1 j_2)_{\mu}^1 | \underbrace{(1 + vG_0)^{-1}}_{\text{FSI}} D_{\mu} | \Psi_{gs} \rangle
 \end{aligned}$$

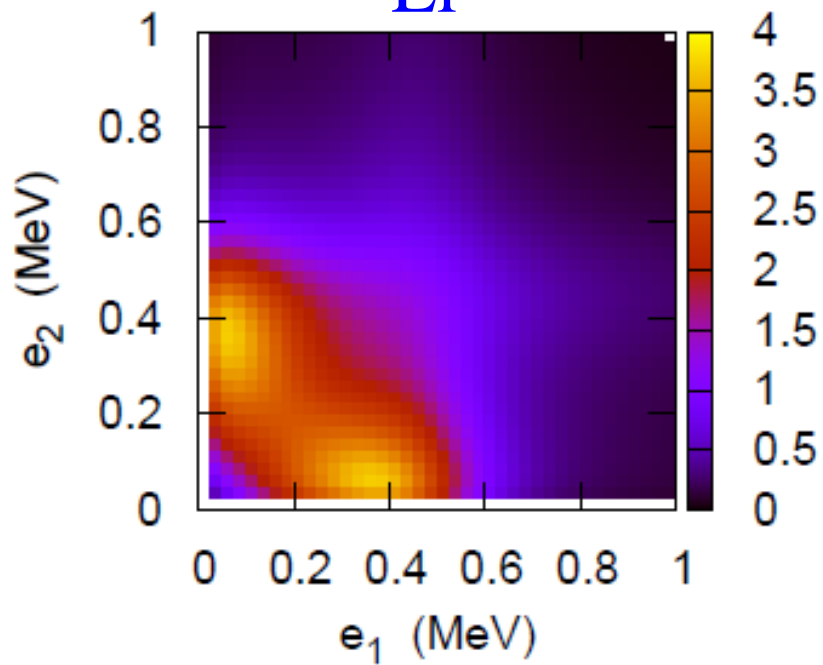
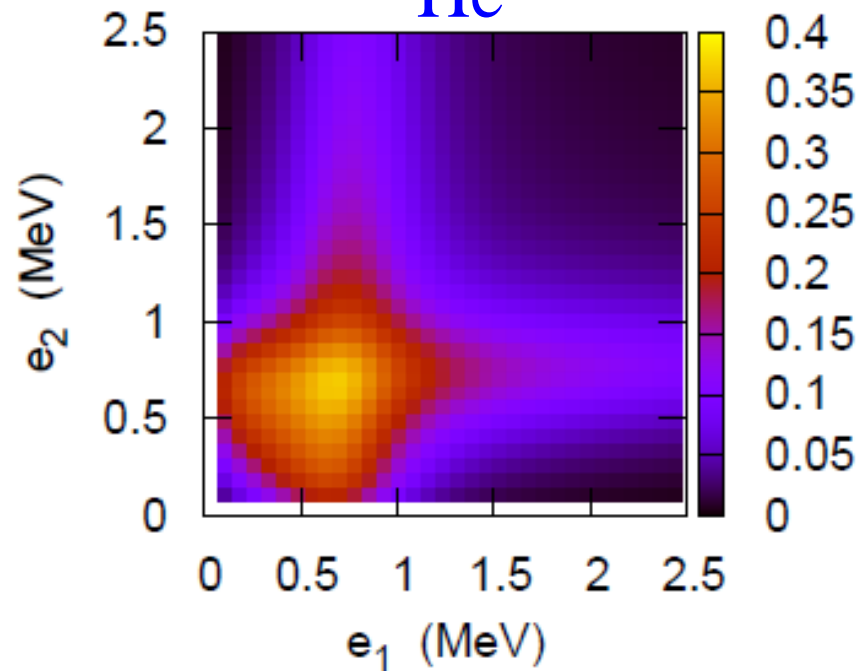
↑ unperturbed continuum wf

↑ dipole operator

$$G_0(E) = \sum_{\mu, f.st.} \frac{|(j_1 j_2)_{\mu}^1\rangle \langle (j_1 j_2)_{\mu}^1|}{e_1 + e_2 - E - i\eta}$$

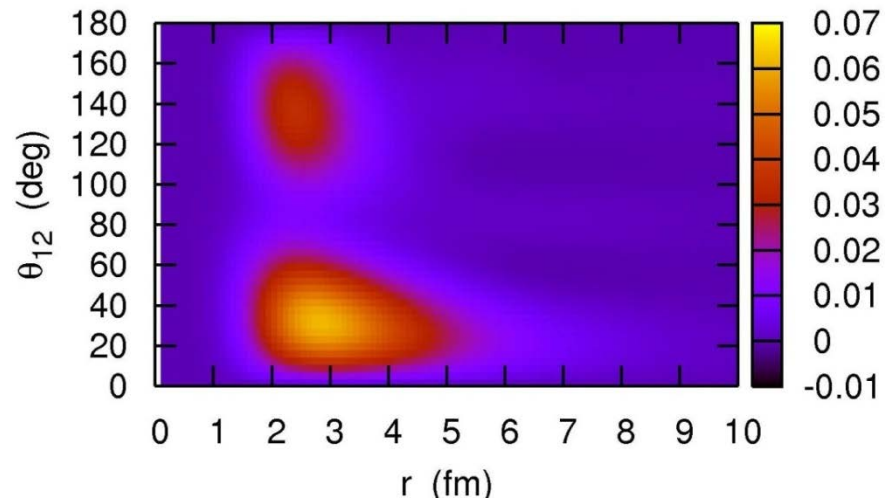
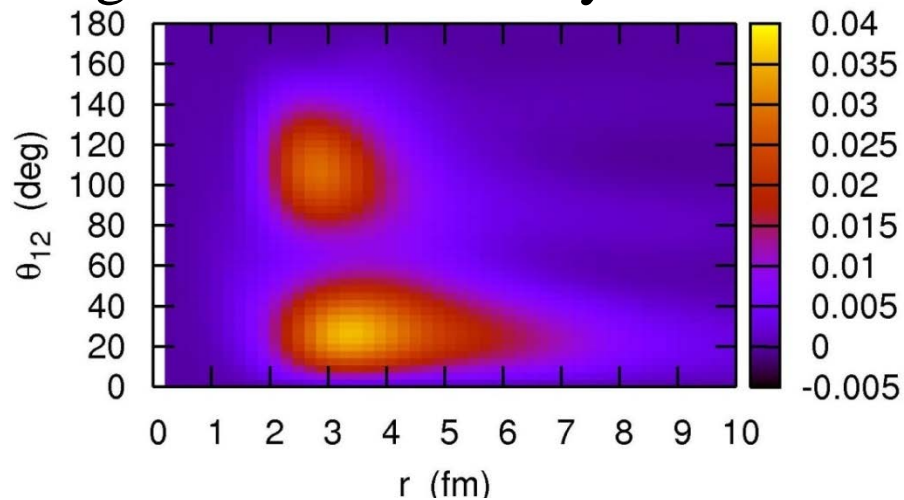
$$\frac{d^2 B(E1)}{de_1 de_2} = 3 \sum_{l_1 j_2 l_2 j_2} |M(E1)|^2 \frac{dk_1}{de_1} \frac{dk_2}{de_2}$$



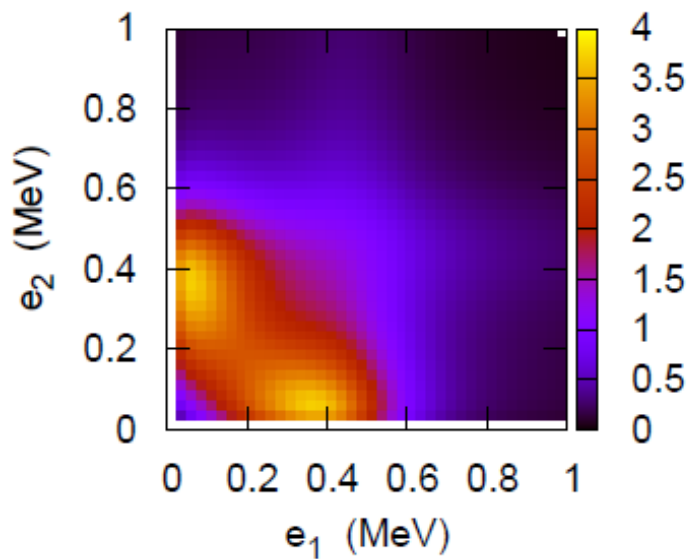
^{11}Li  ^6He 

K.H., H. Sagawa, T. Nakamura, S. Shimoura, PRC80('09)031301(R)

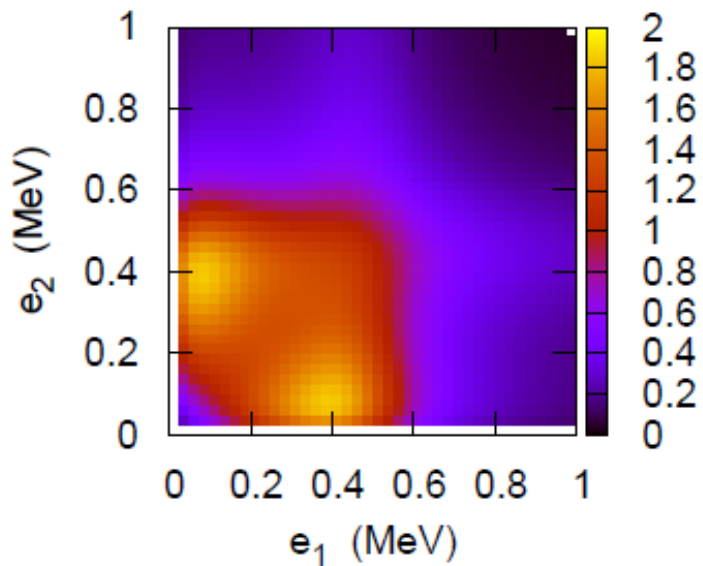
cf. ground state density



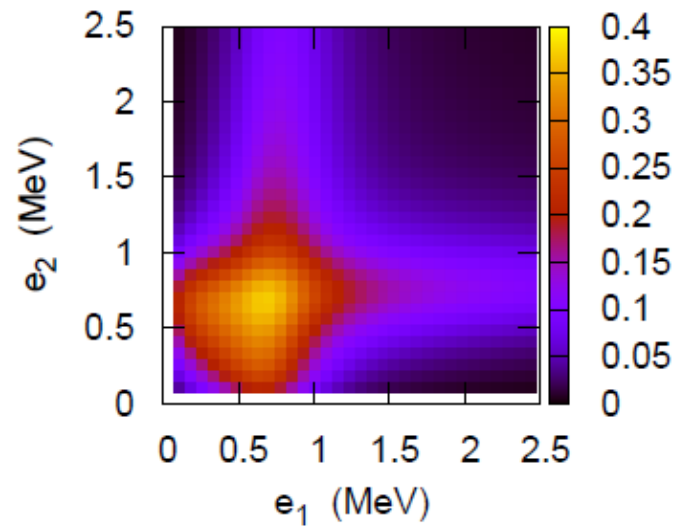
^{11}Li



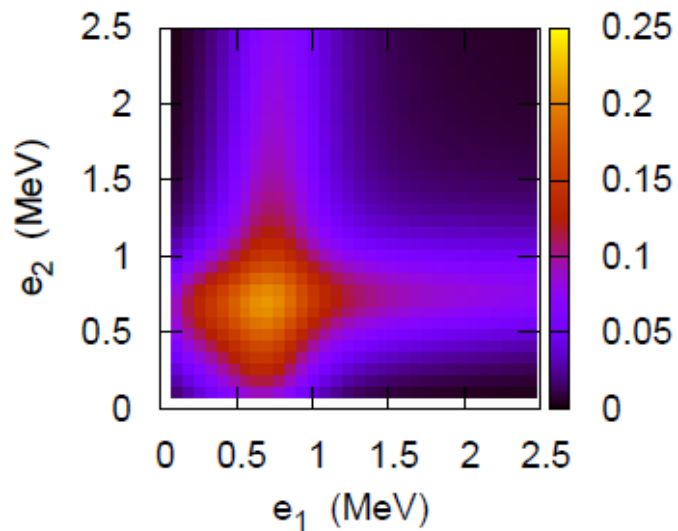
$v_{nn} = 0$



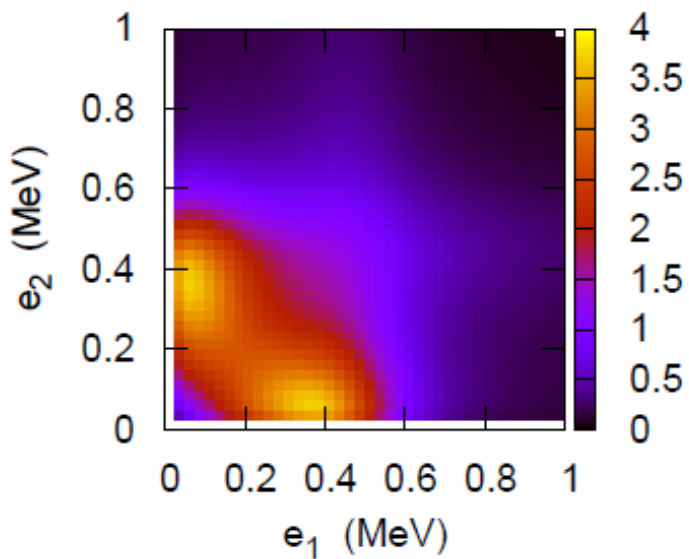
^6He



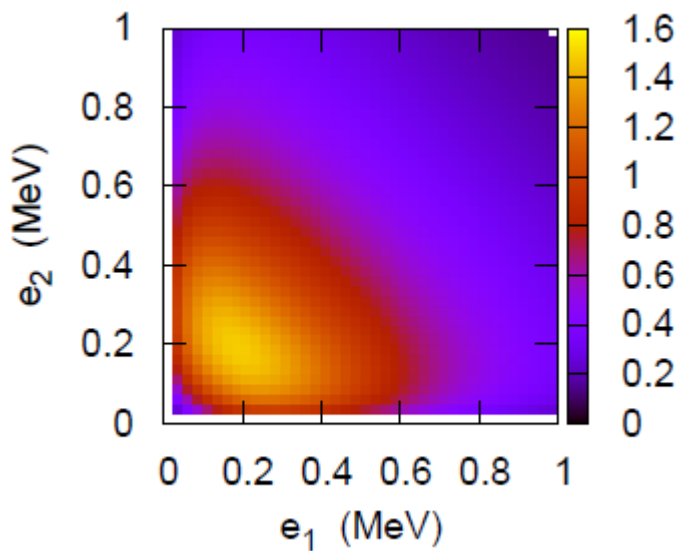
$v_{nn} = 0$



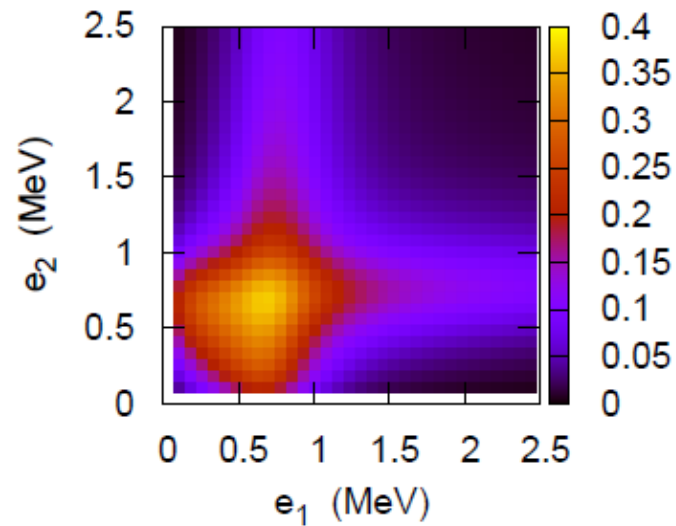
^{11}Li



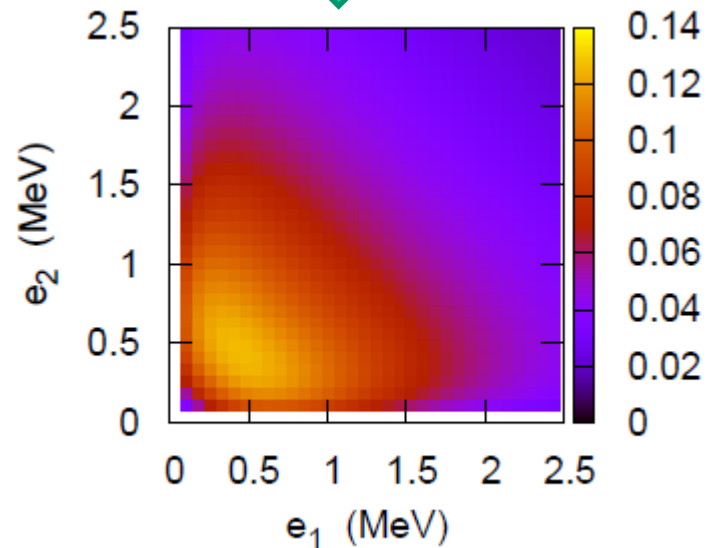
$V_{nC} = 0$



^6He

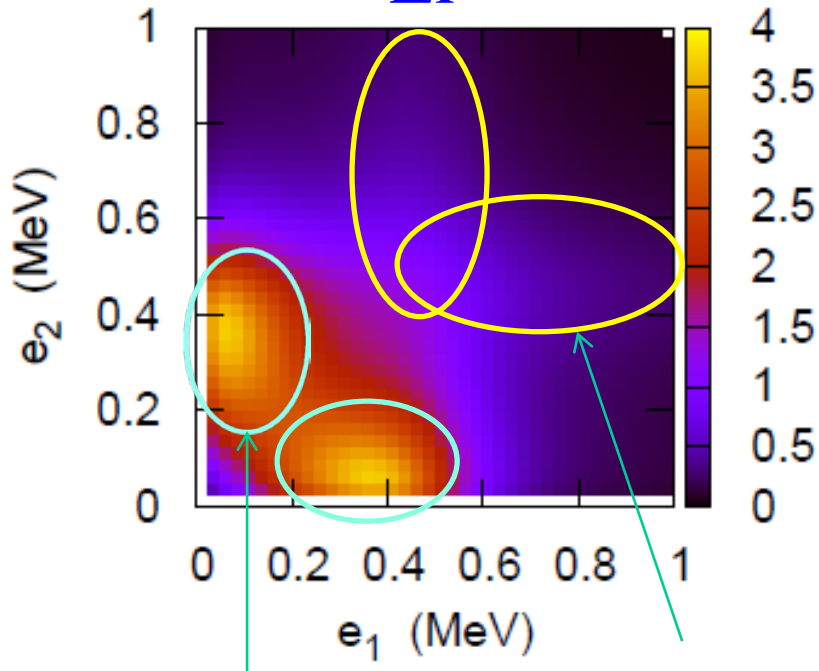


$V_{nC} = 0$



similar

^{11}Li

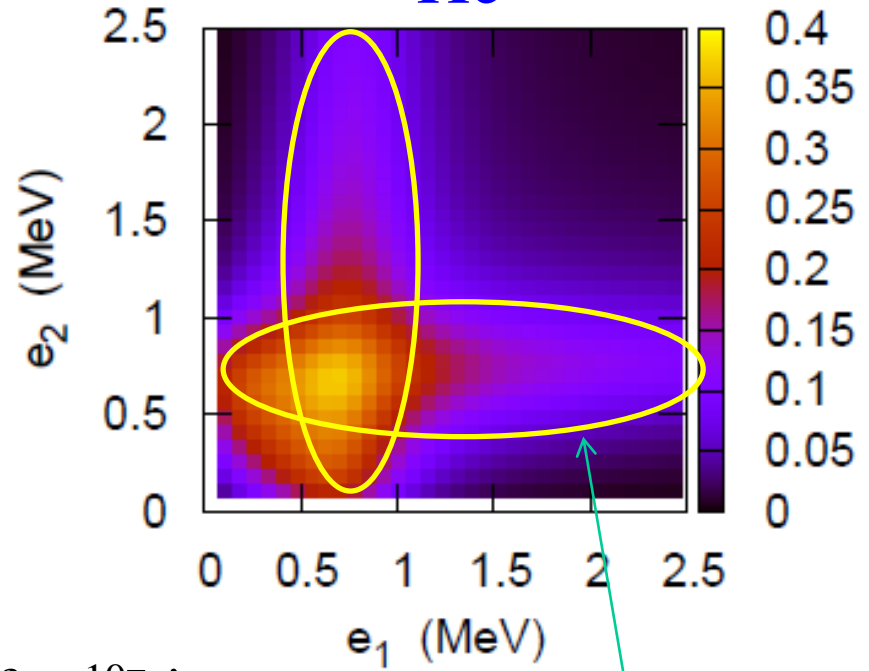


s-wave **virtual state** in ^{10}Li

(scattering length:
 $a = -30^{+12}_{-31}$ fm)

$p_{1/2}$ resonance for ^{10}Li
 at 0.54 MeV

^6He



$p_{3/2}$ resonance for ^5He
 at 0.91 MeV

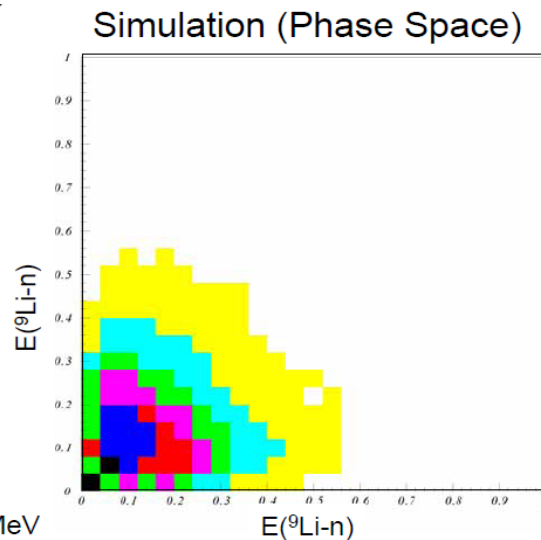
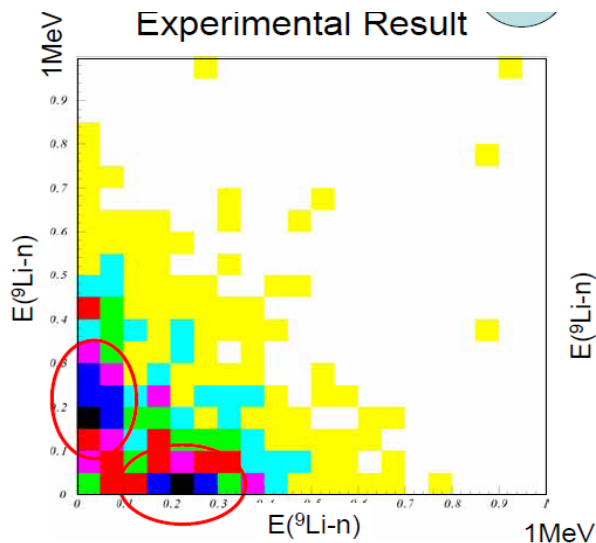
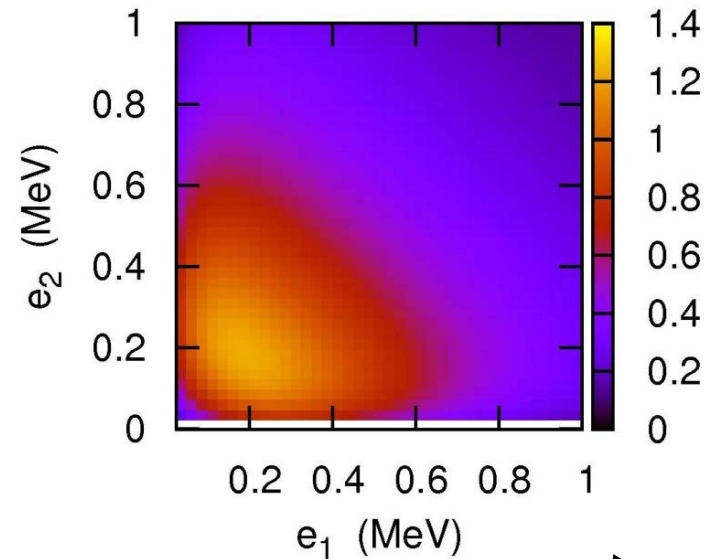
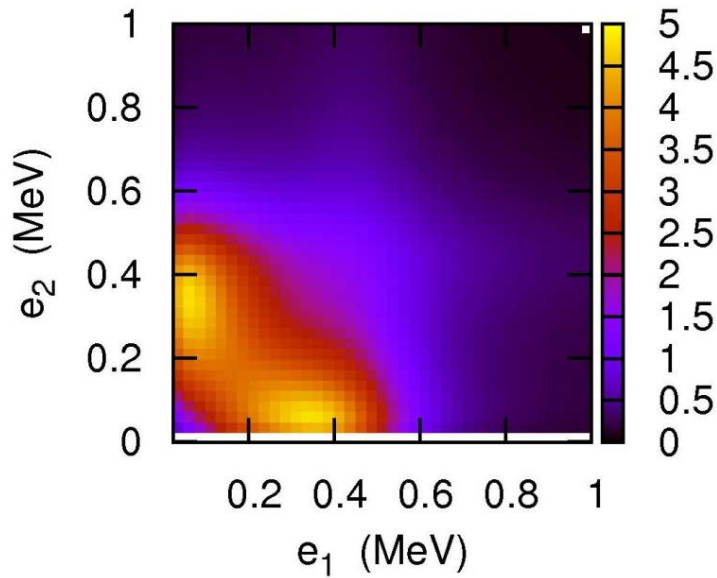
(cf. s-wave scattering length:
 $a = +4.97 \pm 0.12$ fm)

◆ distribution for ^{11}Li : consistent with preliminary expt. data (T. Nakamura et al.)

Analysis with three-body model

$$\frac{d^2B(E1)}{de_1 de_2} = \sum_{l_1 j_1 l_2 j_2} | \langle [(e_1 j_1 l_1)(e_2 j_2 l_2)]^{(J=1)} | \hat{O}_{E1} | \Psi_{gs} \rangle |^2$$

continuum response

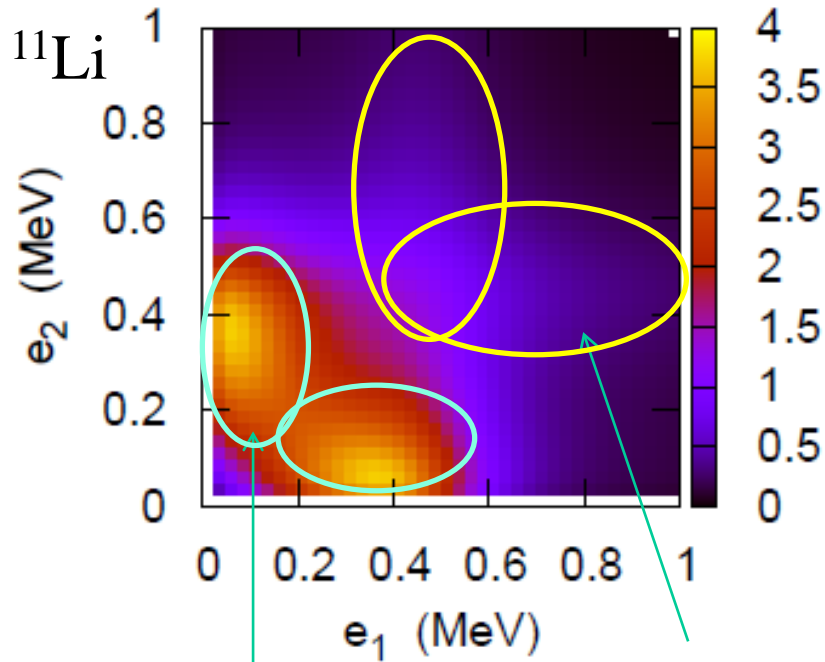


plane wave
final state wf
(No FSI):
($v_{nn}=0, V_{nc}=0$)

Remaining problem:

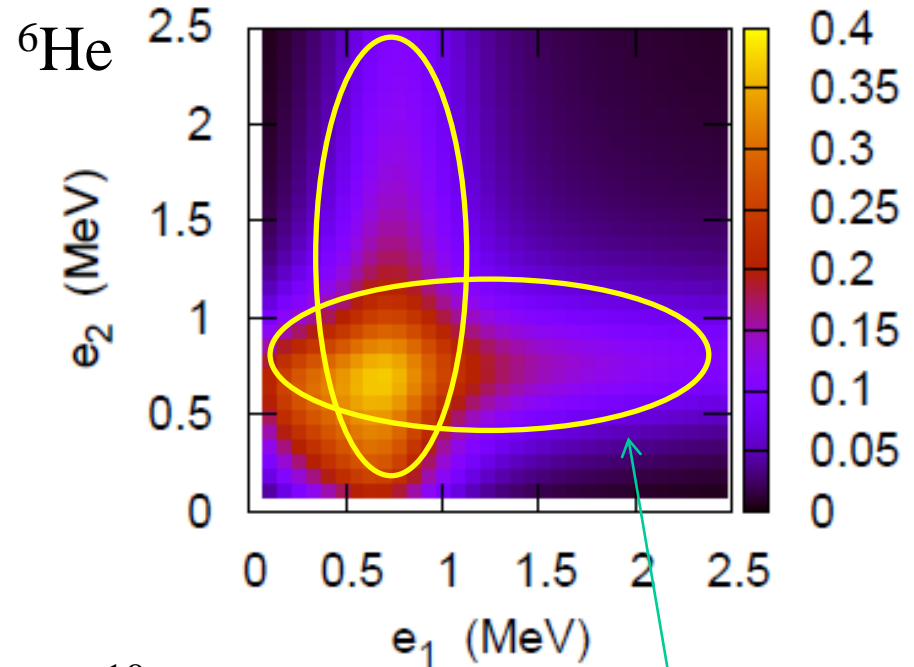
How to probe the strong dineutron correlation?

- Coulomb excitations? → A problem: an external field is too weak



s-wave **virtual state** in ^{10}Li

$p_{1/2}$ resonance in ^{10}Li at 0.54 MeV

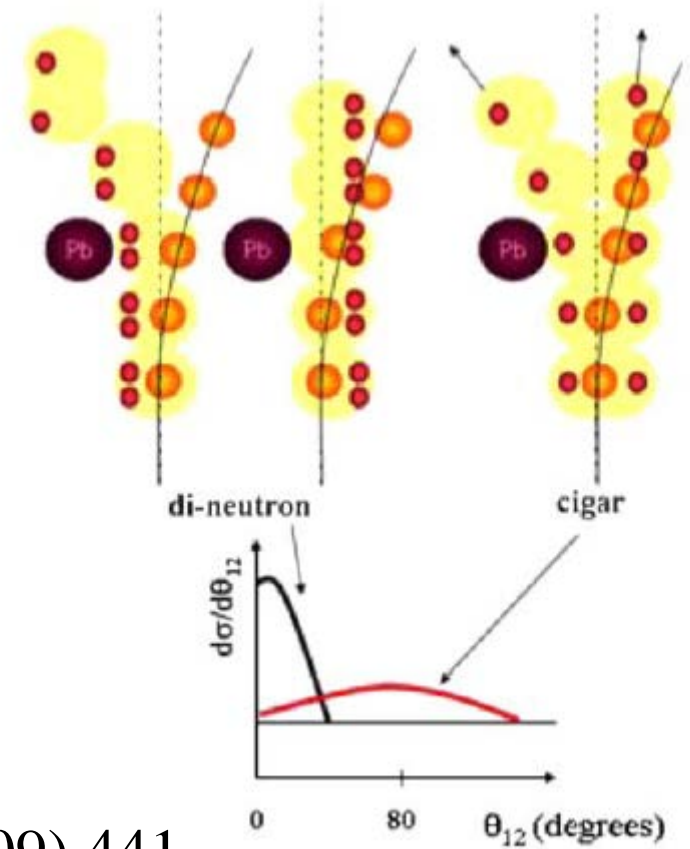
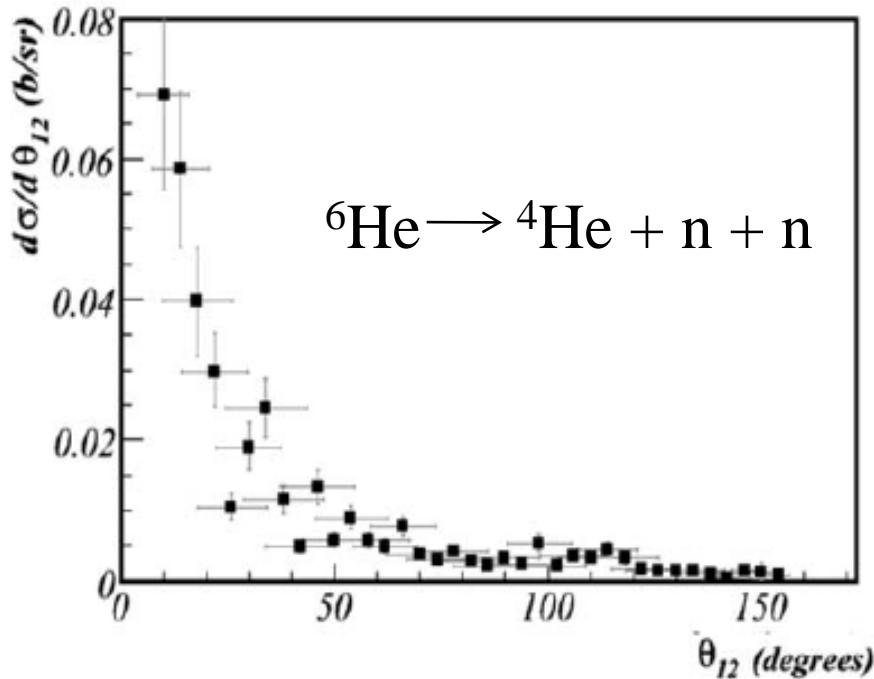


$p_{3/2}$ resonance for ^5He at 0.91 MeV

Remaining problem:

How to probe the strong dineutron correlation?

- Coulomb excitations?
- Nuclear breakup?



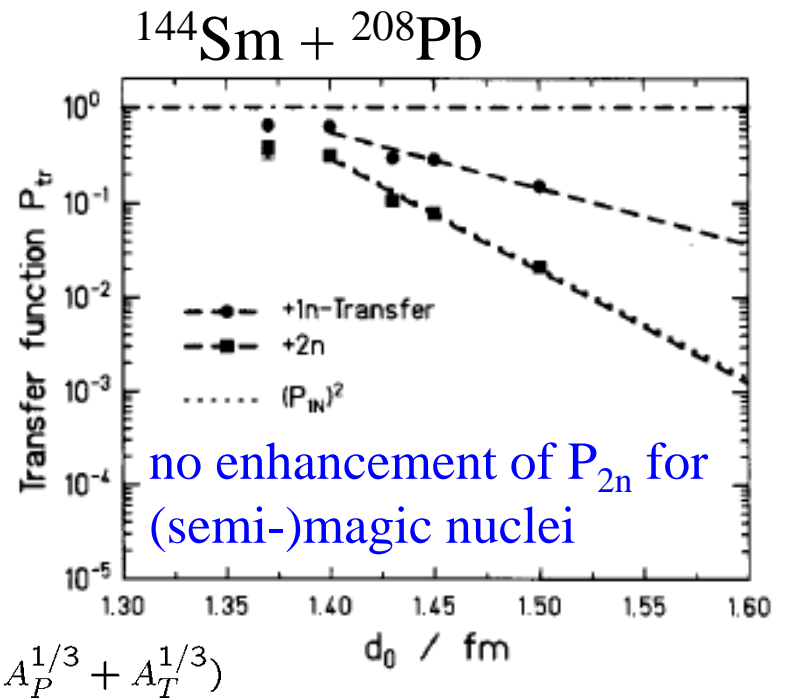
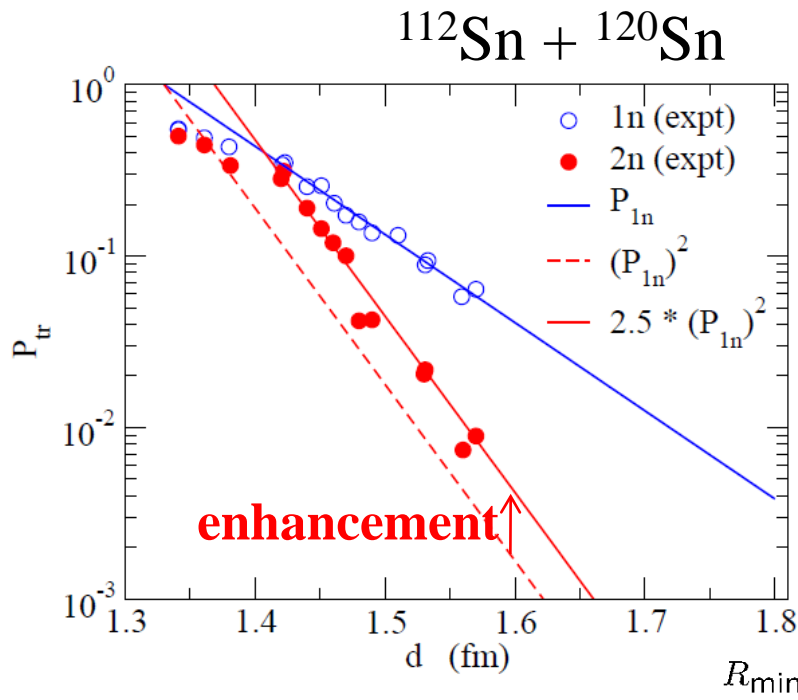
M. Assie et al., Eur. Phys. J. A42 ('09) 441

cf. 4-body CDCC for exclusive cross sections?

Remaining problem:

How to probe the strong dineutron correlation?

- Coulomb excitations?
- Nuclear breakup?
- Pair transfer?



Remaining problem:

How to probe the strong dineutron correlation?

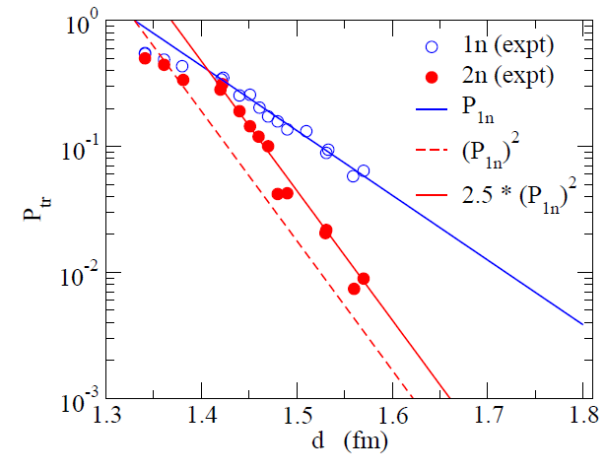
- Coulomb excitations?
- Nuclear breakup?
- Pair transfer?

✓ **Reaction mechanism?**

- sequential vs simultaneous
- Q-value, angular momentum matchings

✓ Role of dineutron correlation (on the surface)?

✓ Influence to other reaction processes (e.g., subbarrier fusion)?



have not yet been fully clarified

→ to be discussed on the next Monday

example of 2n transfer calculation

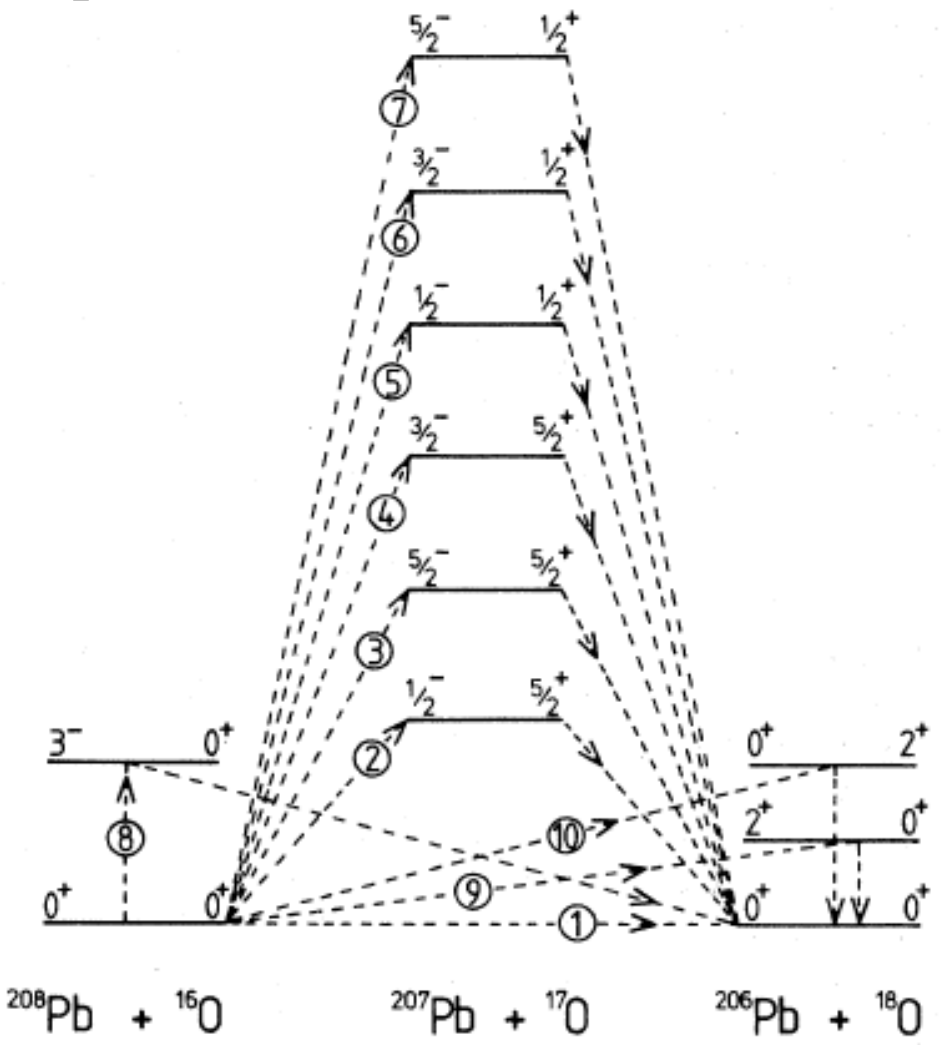
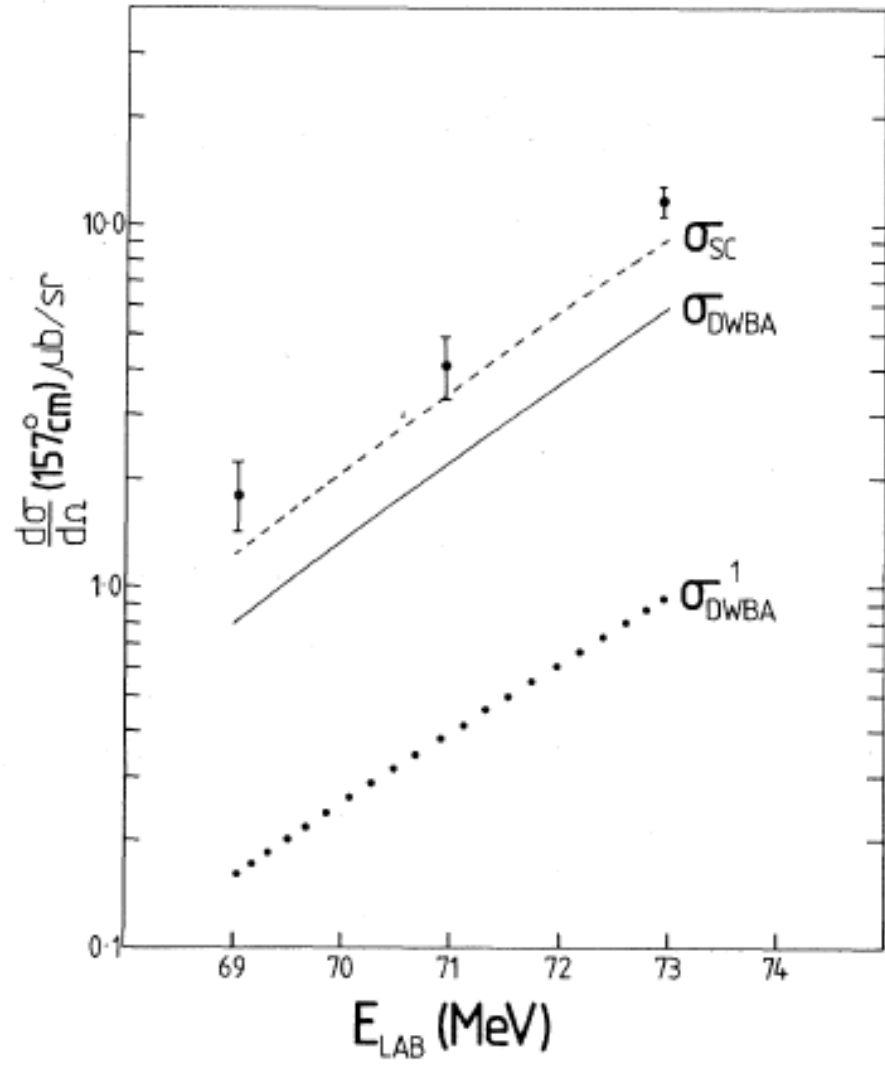


FIG. 2. The direct and two-step paths considered in the predictions of the cross section for the reaction $^{208}\text{Pb}(^{16}\text{O}, ^{18}\text{O})^{206}\text{Pb}$. The levels in ^{207}Pb and ^{17}O concerned in the sequential transfer paths are the $p_{1/2}$, $f_{5/2}$, and $p_{3/2}$ single-hole states and the $d_{5/2}$ and $s_{1/2}$ single-particle states, respectively.

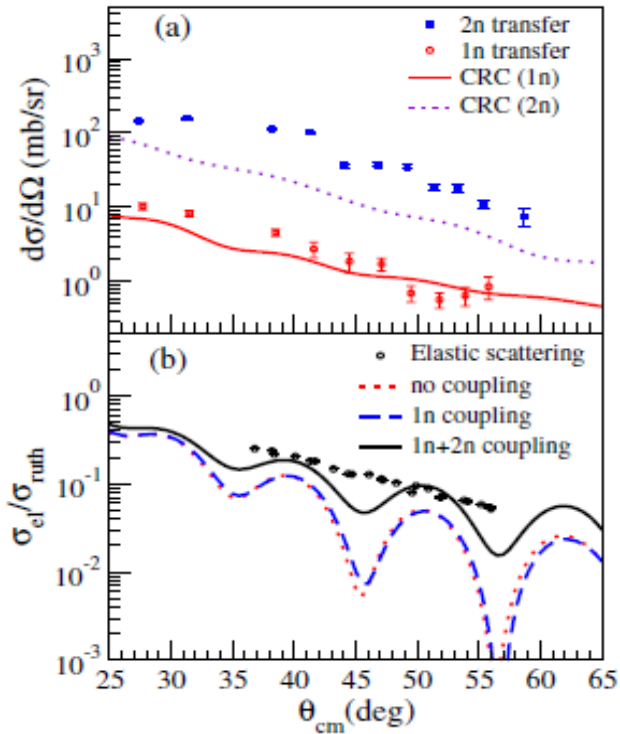


M.A. Franey et al.,
PRL41('78)837

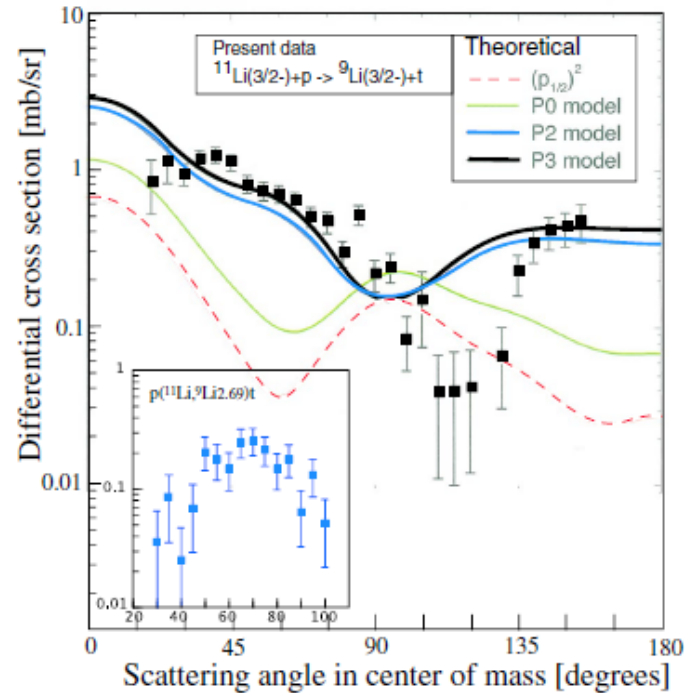
effects of unbound intermediate states?

Recent experiments for transfer reaction of neutron-rich nuclei

${}^6\text{He} + {}^{65}\text{Cu}$



${}^1\text{H}({}^{11}\text{Li}, {}^9\text{Li}){}^3\text{H}$



A. Chatterjee et al., PRL101('08)032701

I. Tanihata et al., PRL100('08)192502

It is timely to construct:

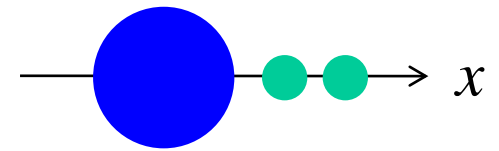
a new theory of pair transfer with dineutron correlation.

→ need a deep understanding of reaction dynamics



a simple and intuitive schematic model

One-dimensional three-body model

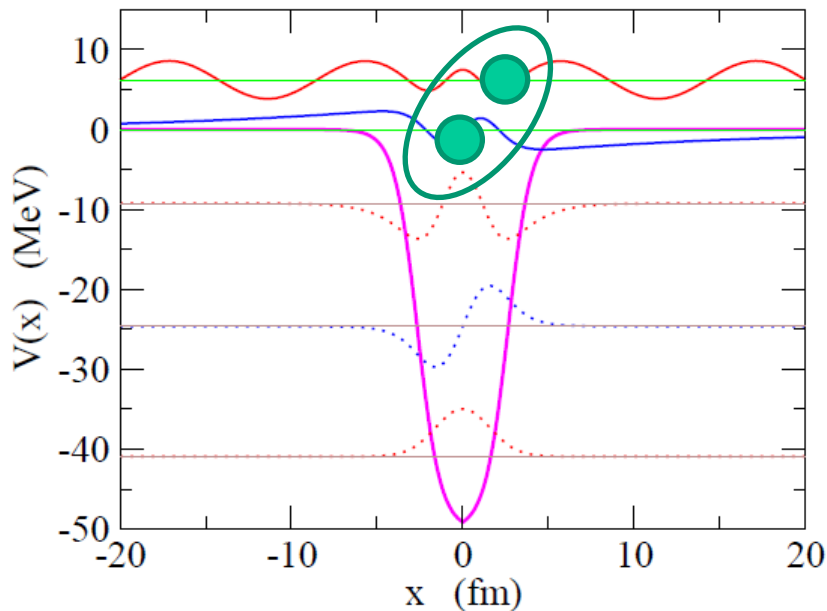


Two interacting neutrons in a one-dimensional potential well:

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx_1^2} + V(x_1) - \frac{\hbar^2}{2m} \frac{d^2}{dx_2^2} + V(x_2) + v_{nn}(x_1, x_2)$$

density-dependent contact interaction:

$$v_{nn}(x, x') = -g \left(1 - \frac{1}{1 + e^{(|x|-R)/a}} \right) \delta(x - x')$$



$$\Psi_{\text{gs}}(x_1, x_2) = \sum_{n \leq n'} \alpha_{nn'} \Psi_{nn'}(x_1, x_2)$$

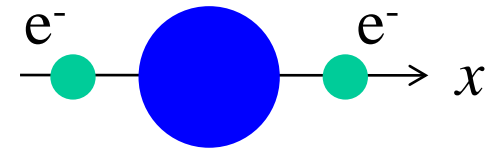
$$\Psi_{nn'}(x_1, x_2) \propto \mathcal{S}[\phi_n(x_1)\phi_{n'}(x_2)] \times |S=0\rangle$$

- **S = 0 state**: symmetric for the spatial part of wf
- **n, n'**: the same parity

Similar one-dimensional model for two-electron systems

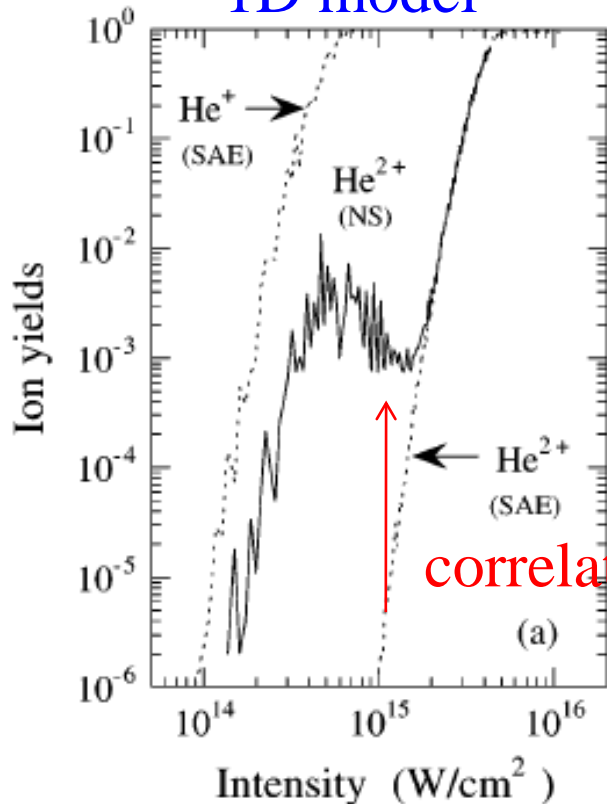
He atom (${}^4\text{He} + e^- + e^-$)

H⁻ atom ($p + e^- + e^-$)

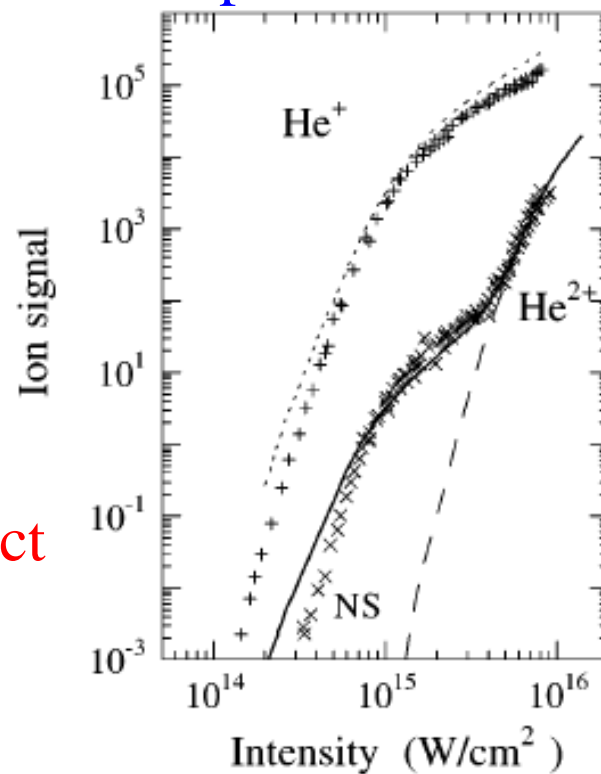


double ionization by intense laser fields

1D model



Experimental Data



SAE: single active electron approximation

NS: non-sequential

J.B. Watson et al., PRL78('97)1884

cf. TDH(F) for a one-dimensional system

B. Yoon and J.W. Negele, PRA16('77) 1451

PHYSICAL REVIEW A

VOLUME 16, NUMBER 4

OCTOBER 1977

Time-dependent Hartree approximation for a one-dimensional system of bosons with attractive δ -function interactions*

B. Yoon and J. W. Negele[†]

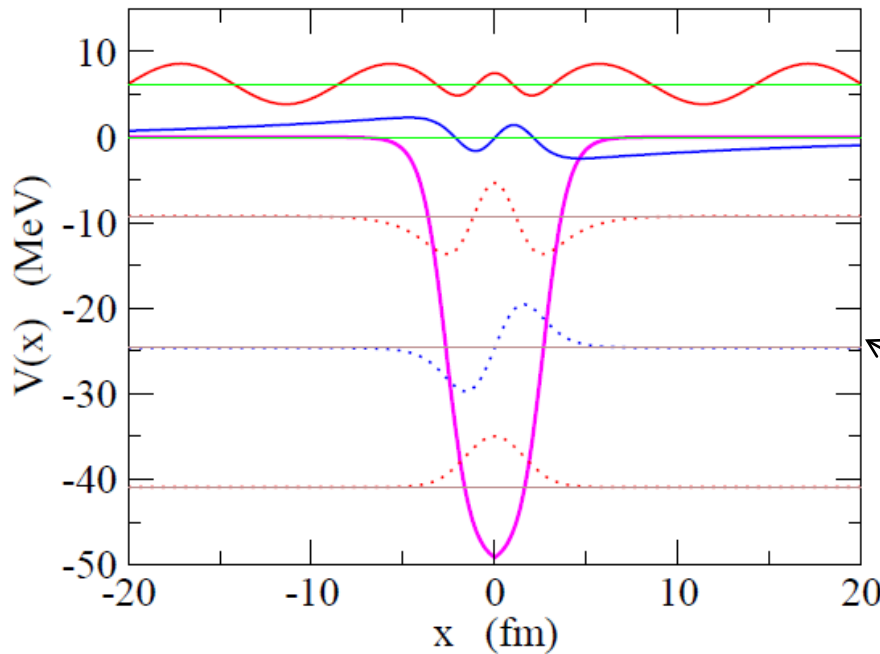
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 29 November 1976)

The time-dependent Hartree approximation is compared with an exact solution for the scattering between two N -particle bound states in the case of a 1-dimensional system of bosons with attractive δ -function interactions. It is shown that to leading order in N , the approximation is exact, and arguments are presented relating this asymptotic agreement to the nonsaturation of the bound states.

$$H = -\frac{1}{2} \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2} - g \sum_{i < j=1}^N \delta(x_i - x_j)$$

Model Setup for core+2n



discretized continuum states

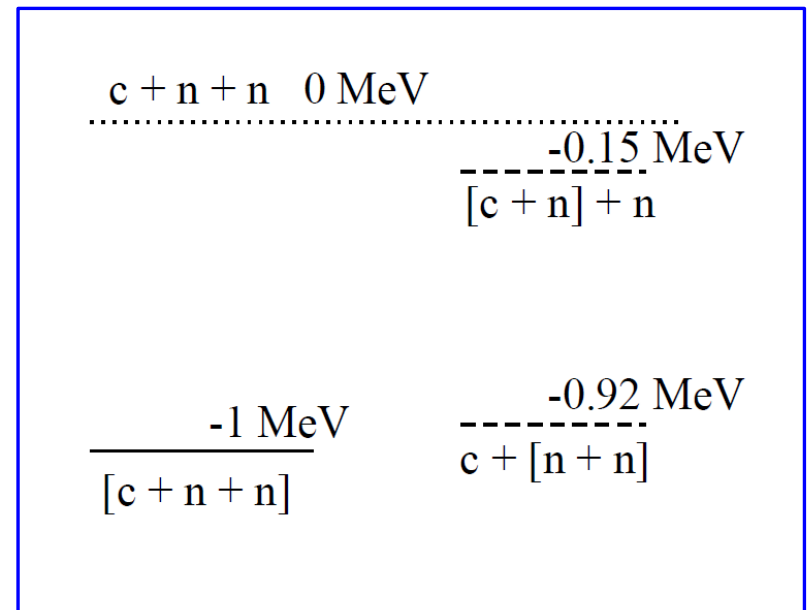
Weakly bound s.p. level at -0.15 MeV

occupied by the core nucleus

the strength of the pairing interaction g :
adjusted so that $E_{gs} = -1 \text{ MeV}$

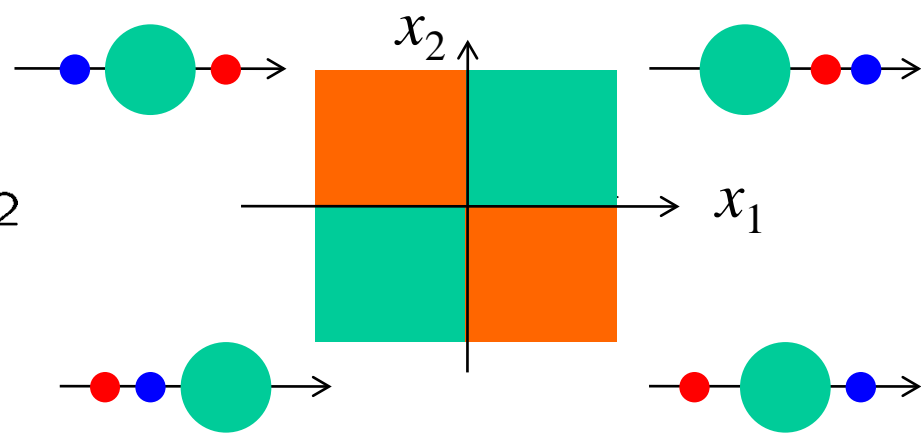
$$E_{\text{cut}} = 30 \text{ MeV}, \quad R_{\text{box}} = 90 \text{ fm}$$

$$P_{\text{bb}}^{(gs)} = 81.2\%$$



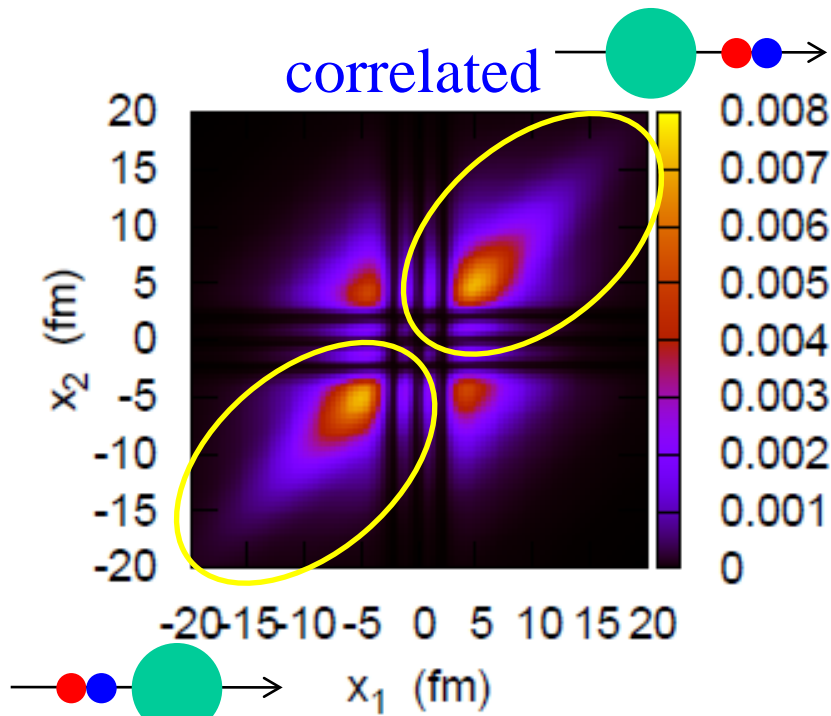
Ground state properties

two-particle density: $|\Psi_{\text{gs}}(x_1, x_2)|^2$

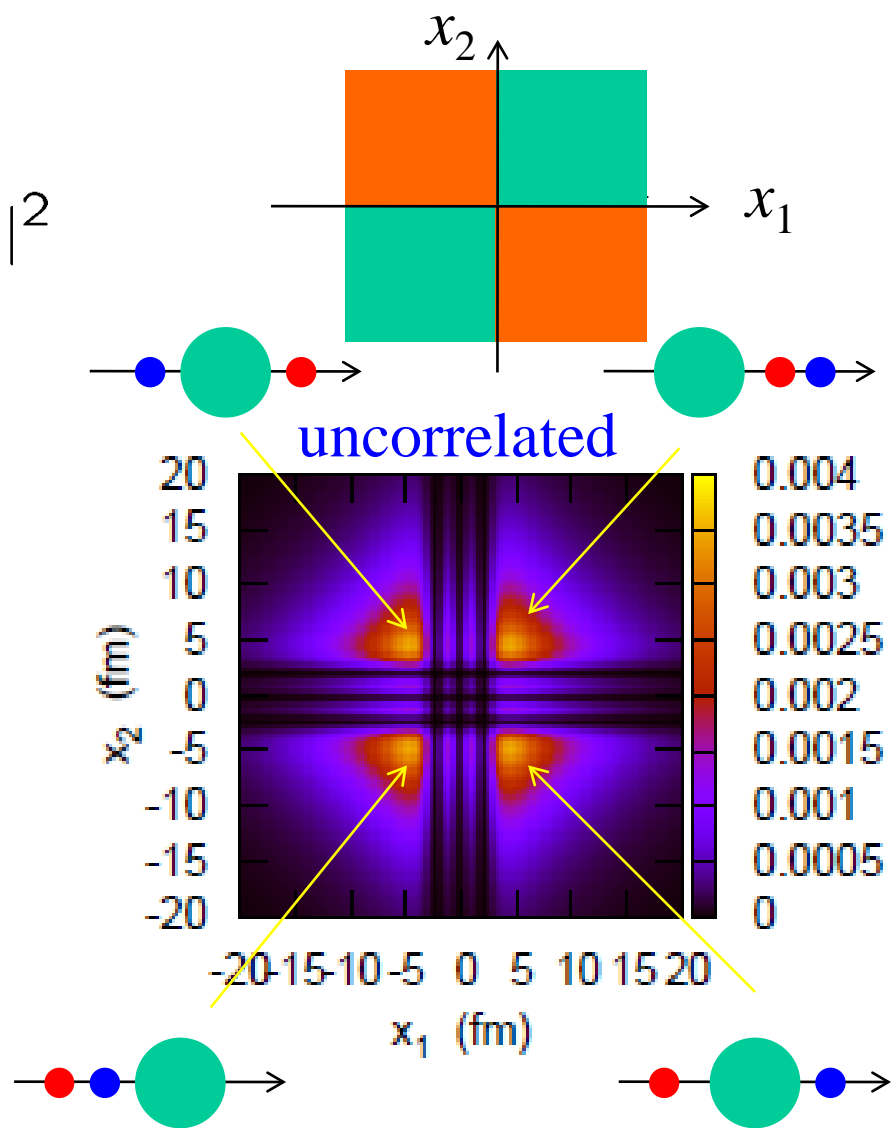
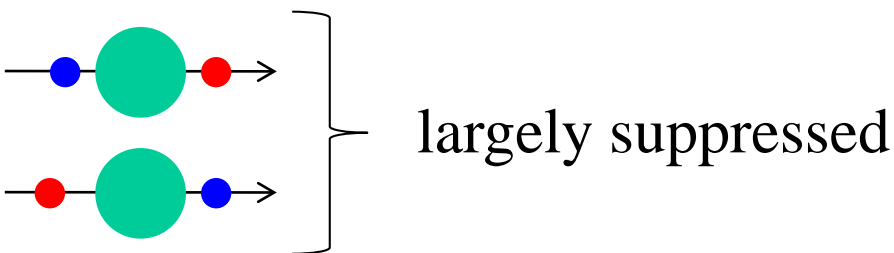


Ground state properties

two-particle density: $|\Psi_{\text{gs}}(x_1, x_2)|^2$



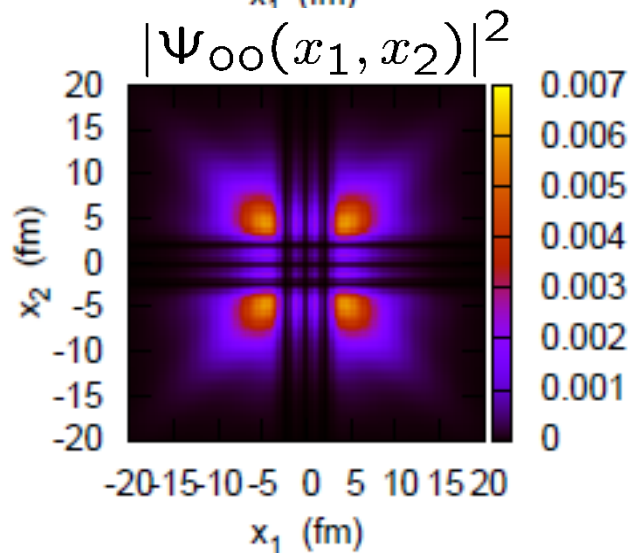
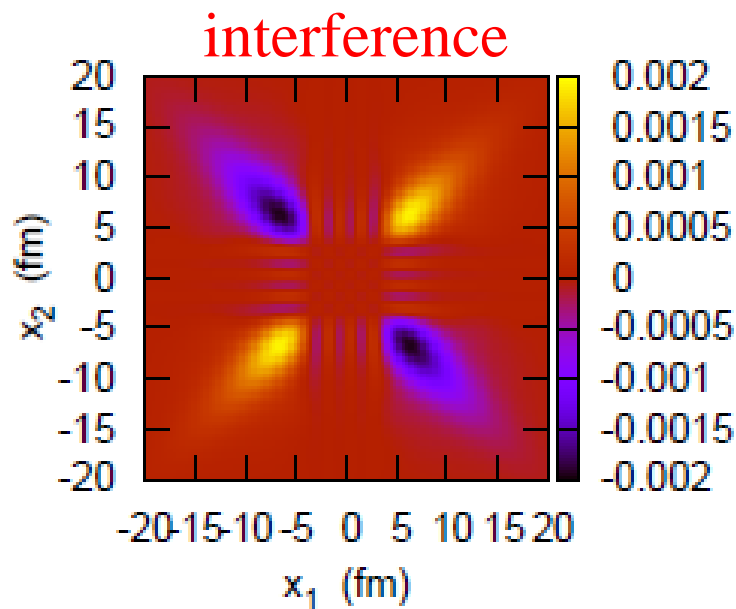
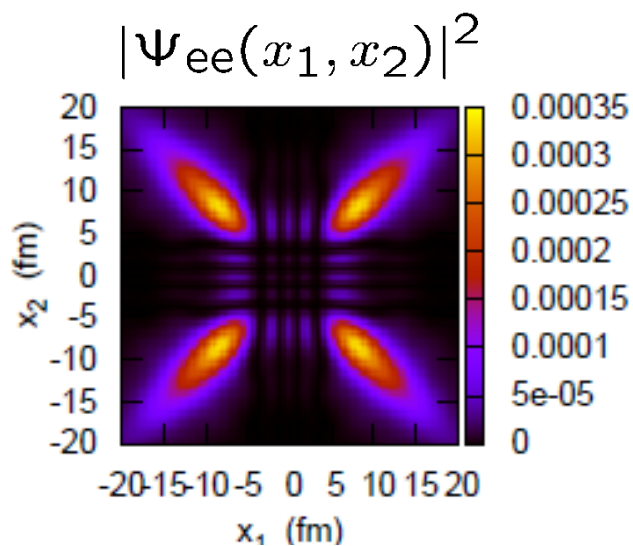
dineutron correlation



four symmetric peaks

$$\Psi_{\text{gs}}(x_1, x_2) = \Psi_{\text{ee}}(x_1, x_2) + \Psi_{\text{oo}}(x_1, x_2)$$

$$\longrightarrow \rho_2(x_1, x_2) = |\Psi_{\text{ee}}(x_1, x_2)|^2 + |\Psi_{\text{oo}}(x_1, x_2)|^2 + 2\Psi_{\text{ee}}(x_1, x_2)\Psi_{\text{oo}}(x_1, x_2)$$

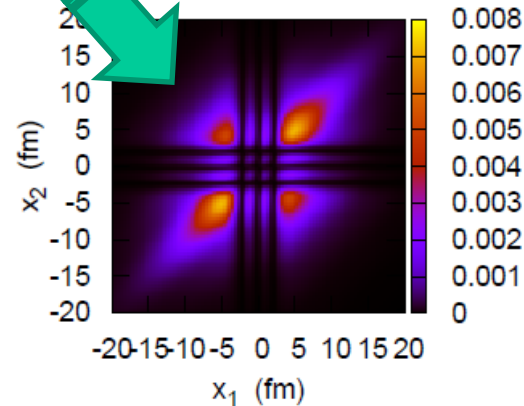


$$\begin{aligned} & \Psi_{\text{ee}}(x_1, x_2)\Psi_{\text{oo}}(x_1, x_2) \\ &= -\Psi_{\text{ee}}(x_1, -x_2)\Psi_{\text{oo}}(x_1, -x_2) \end{aligned}$$

Nuclear Breakup Process



(one-body) external field



$\Psi_{\text{gs}}(x_1, x_2)$

Time-dependent two-particle Schroedinger equation:

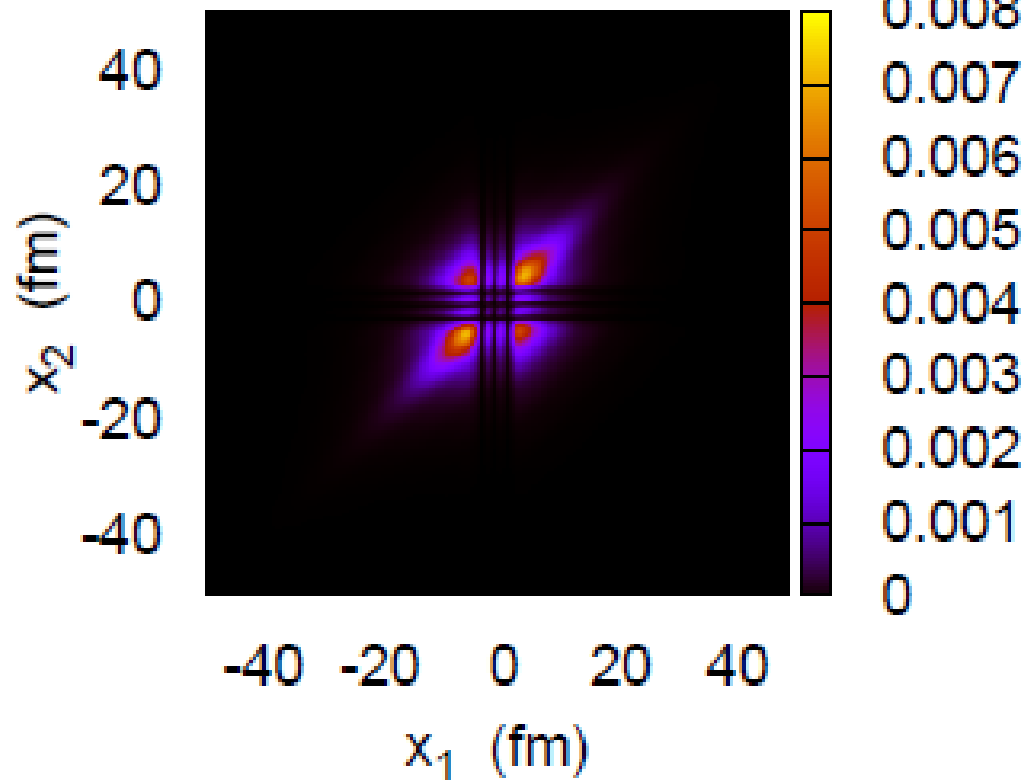
$$i\hbar \frac{\partial}{\partial t} \Psi(x_1, x_2, t) = [H + V_{\text{ext}}(x_1, x_2, t)] \Psi(x_1, x_2, t)$$

$$V_{\text{ext}}(x_1, x_2, t) = \sum_{i=1,2} V_c e^{-t^2/2\sigma_t^2} e^{-(x_i-x_0)^2/2\sigma_x^2}$$

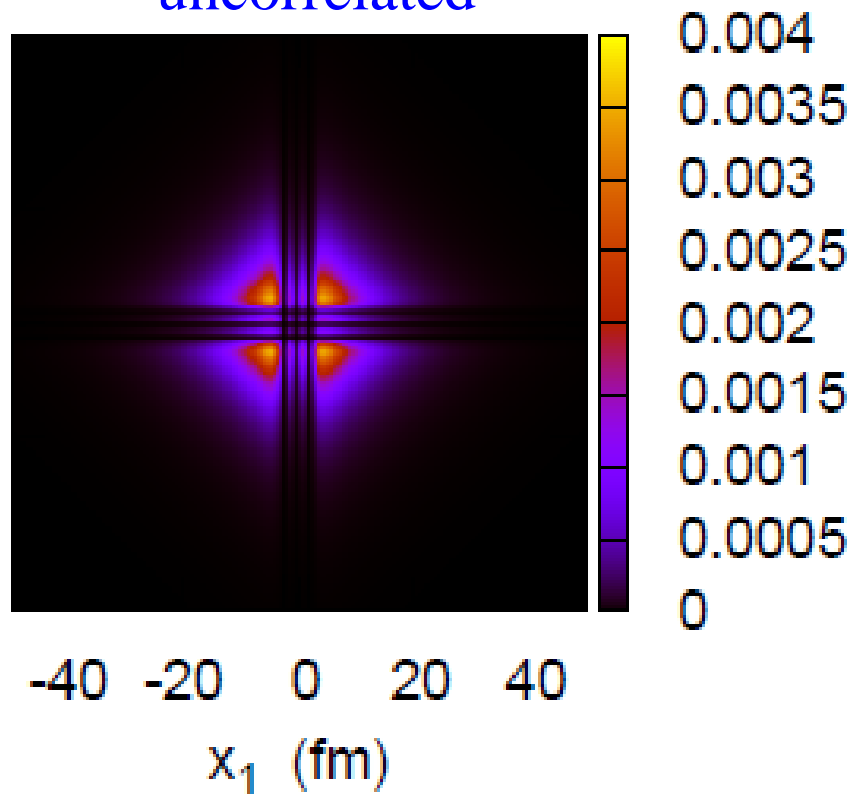
$$V_c = 3 \text{ MeV}, \sigma_t = 2.1 \text{ hbar/MeV}, x_0 = 0$$

two-particle density at $t = t_{\text{ini}}$

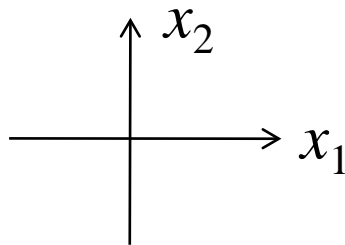
correlated



uncorrelated

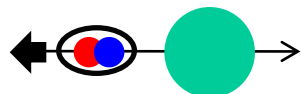
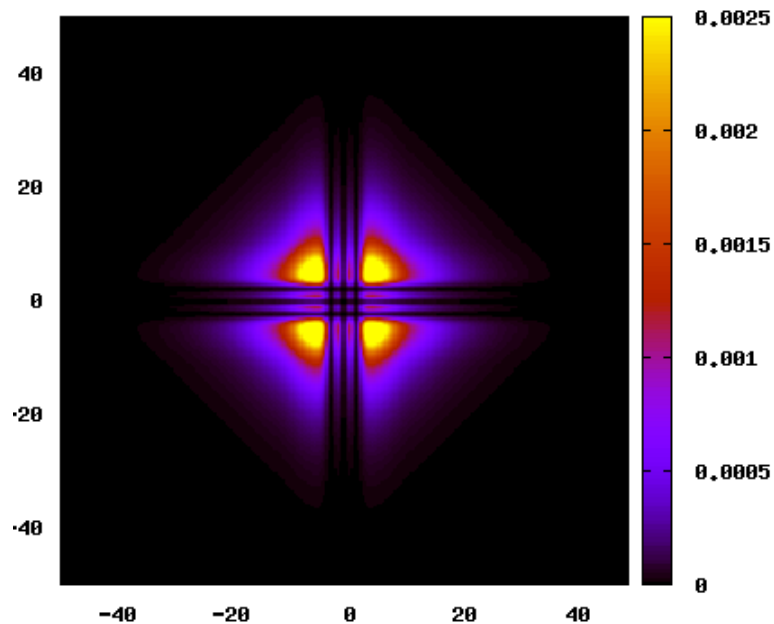
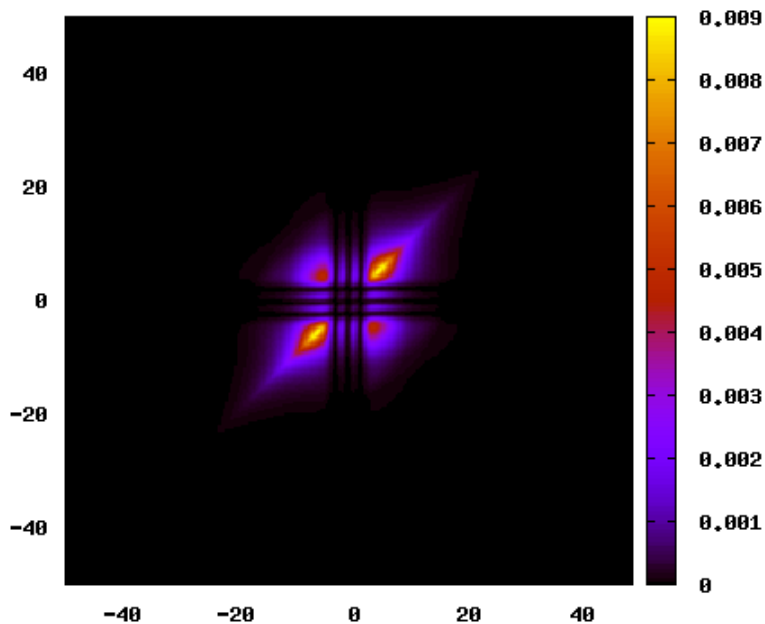


time evolution



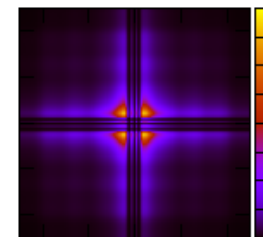
correlated

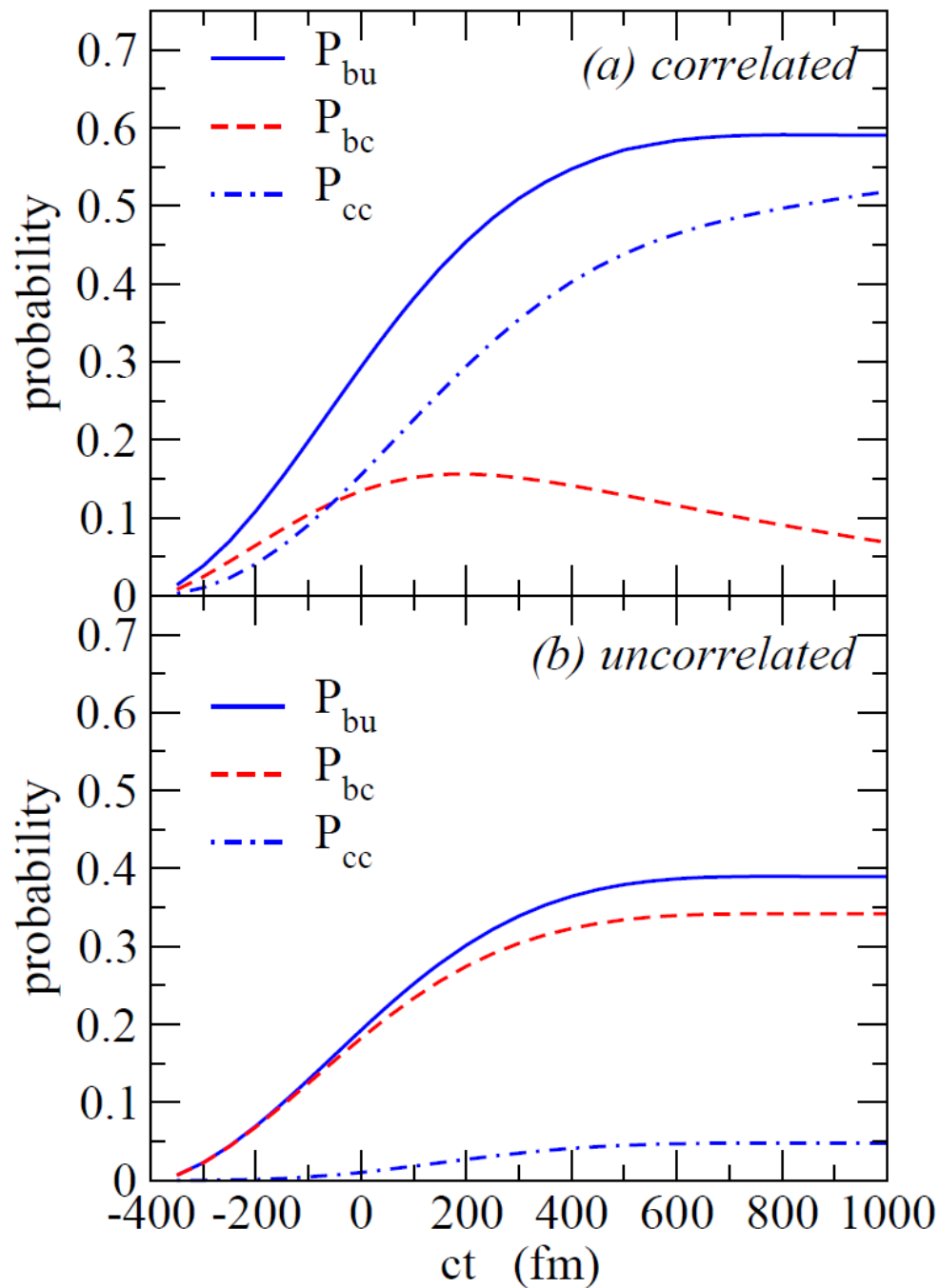
uncorrelated



“dineutron emission”

large (bc) component





➤ Pairing: enhances the breakup

➤ Correlated: (cc) process

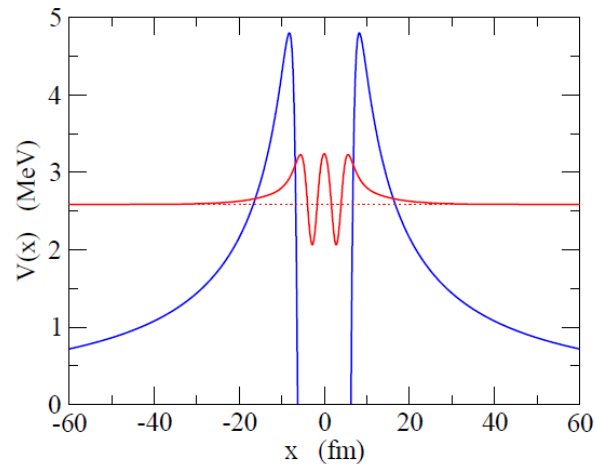
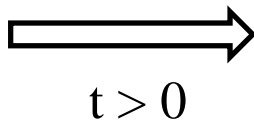
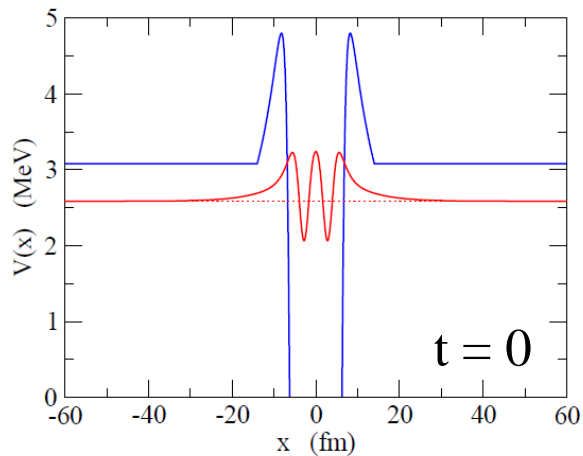
➤ Uncorrelated: (bc) process

P_{cc} : 2 neutron breakup

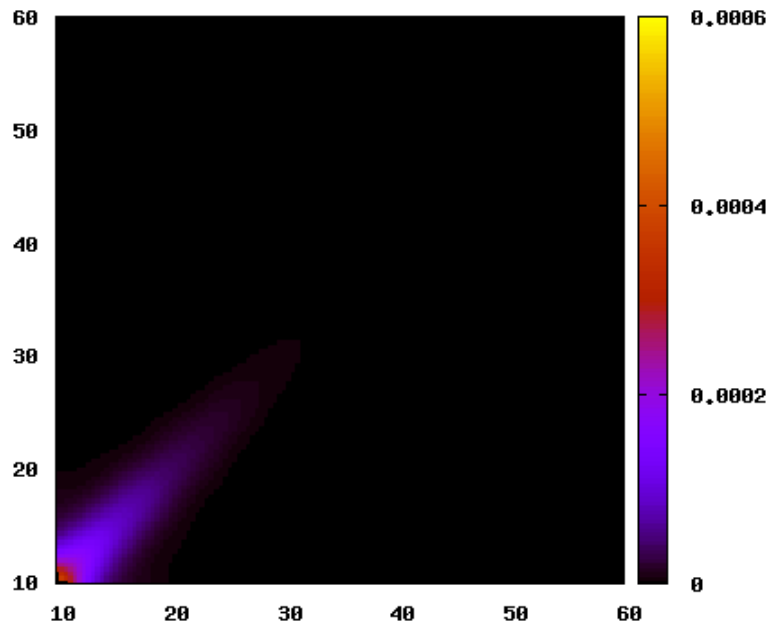
P_{bc} : 1 neutron breakup

Application to 2p radioactivity

T. Maruyama, T. Oishi, K.H., preliminary

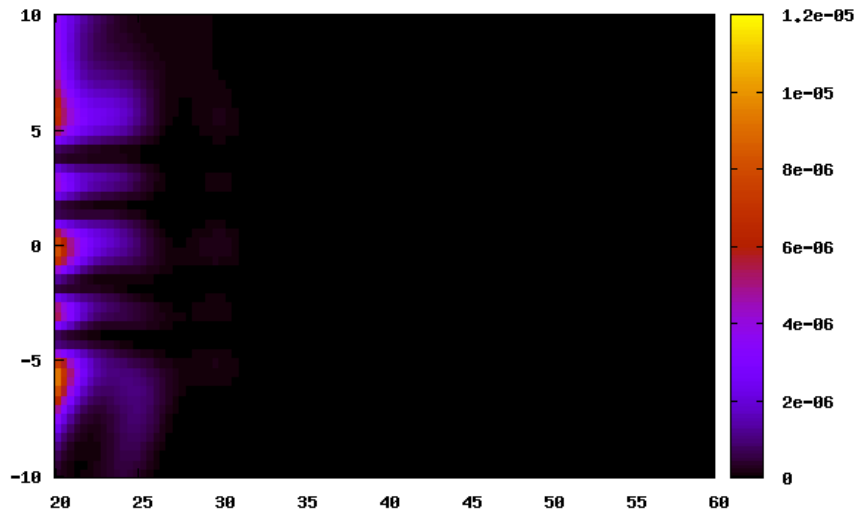


'fort.11' u 1:2:3



di-proton emission ($x_1 = x_2$)

'fort.11' u 1:2:3



sequential 2p emission ($x_2 \sim 0$)

Summary

Three-body model with density-dependent contact interaction

◆ Ground state of ^{11}Li and ^6He

- similar di-neutron correlation

◆ Energy distribution

of neutrons from the E1 excitations in ^{11}Li and ^6He

- continuum responses
- the shape of distributions: primarily determined by n-core dynamics rather than n-n
- energy distribution: very different between ^{11}Li and ^6He
 - s-wave virtual state in ^{11}Li
 - cf. similar dineutron correlations in the g.s.

◆ One dimensional model for 2n halo nuclei

- simple, and schematic model. detailed studies of dynamics

