Heavy-ion sub-barrier fusion reactions: a sensitive tool to probe nuclear structure

Kouichi Hagino
Tohoku University, Sendai, Japan

1. *Introduction: heavy-ion fusion reactions*
2. *Fusion and Quasi-elastic barrier distributions*
3. *Semi-microscopic modelling of sub-barrier fusion*
4. *Double octupole phonon excitations in $^{16}O+^{208}Pb$*
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센다이 & 마츠시마

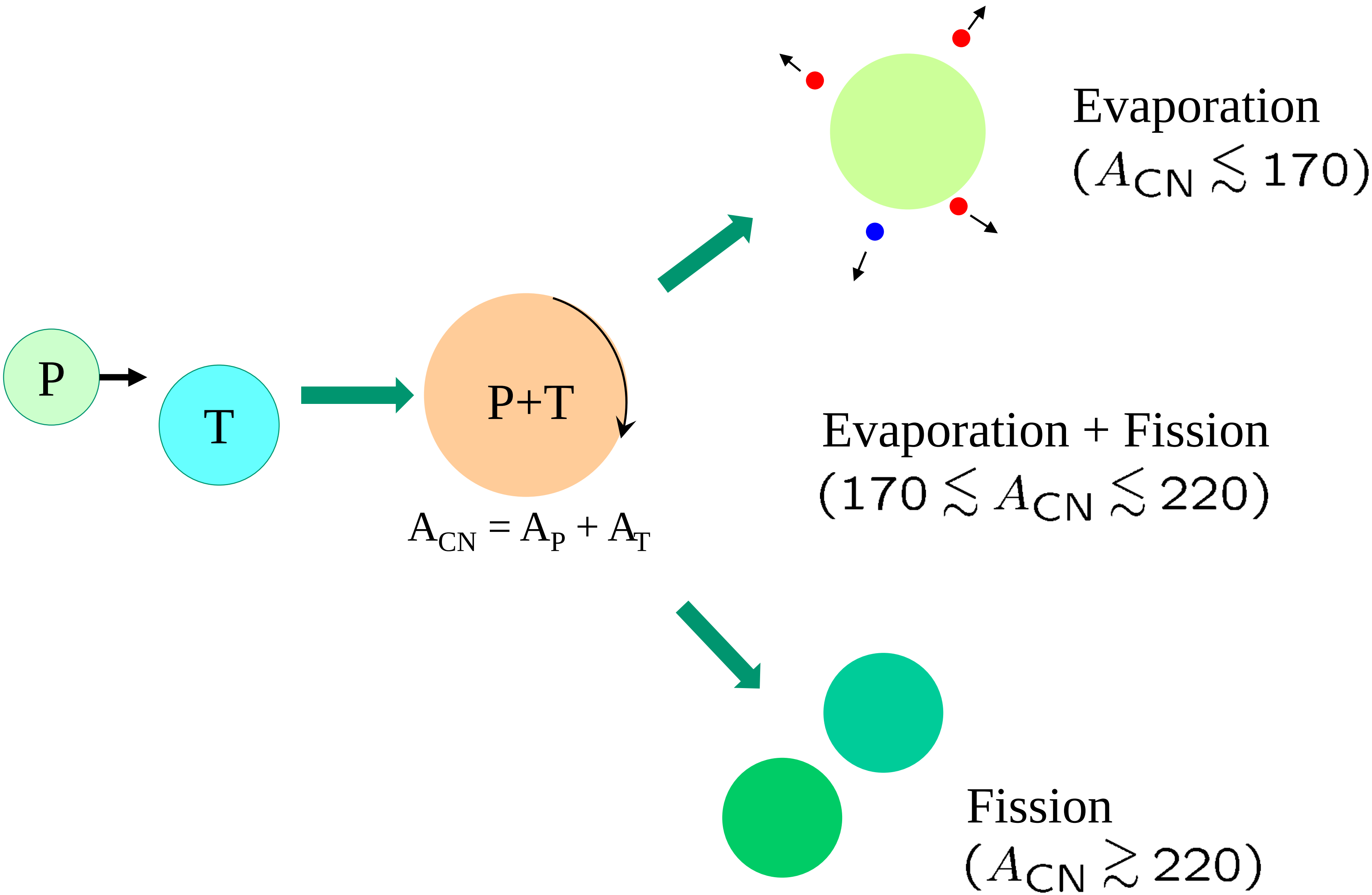
약 1300년 전에 발견됐다고 전해지는, 아키우온천과 같이 오랜 역사의 산골짜기 온천향. 센다이 시가지를 흐르는 히로세강의 원류에서 솟아나, 신록 및 설중탕 등, 사철마다의 강가의 경치를 바라보면서의 입욕을 즐길 수 있다. 또한 카타쿠리노야도 여관 서쪽편에 2008년 10월 관광교류시설이 오픈.

View well-known restaurant information

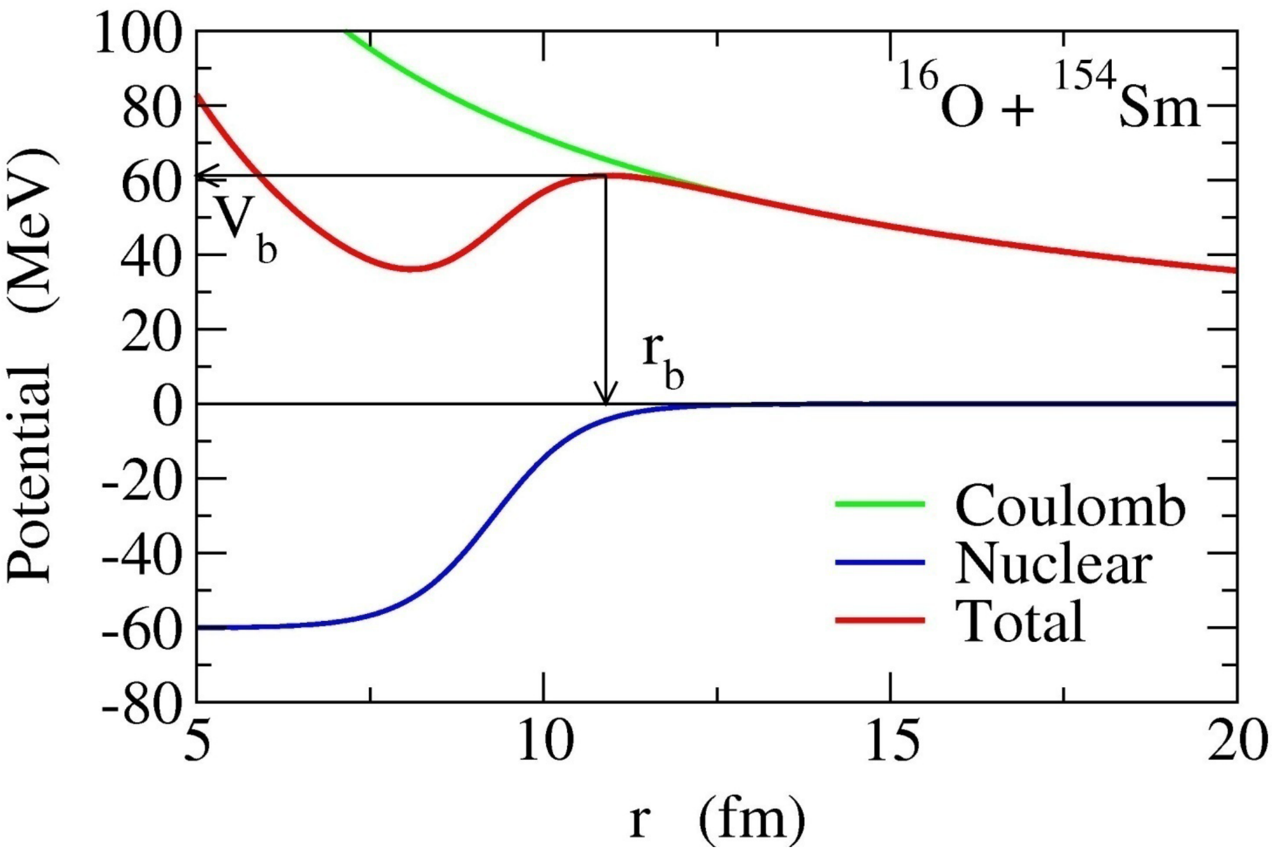
<센다이>라고하면 규탄!
전통의 [키스케]에서 장인의 맛의 조건을 즐기자.

Introduction: heavy-ion fusion reactions

Fusion: compound nucleus formation



Inter-nucleus potential



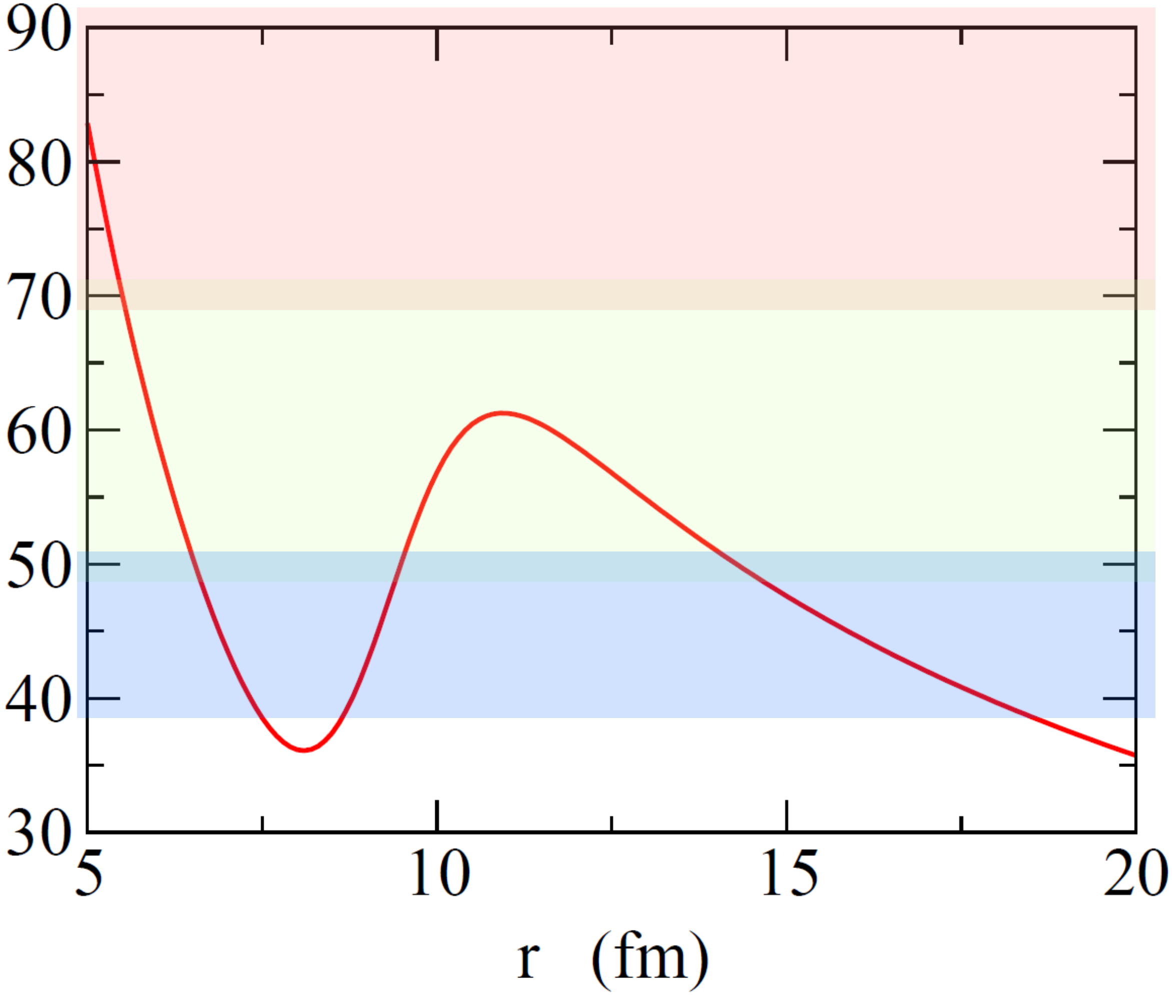
Two forces:

1. Coulomb force
   Long range,
   repulsive
2. Nuclear force
   Short range,
   attractive

Potential barrier
(Coulomb barrier)

- above barrier energies
- sub-barrier energies
- deep subbarrier energies

Energy regions



above barrier region

$(E \gtrsim V_b + 10\text{MeV})$

sub-barrier region ←

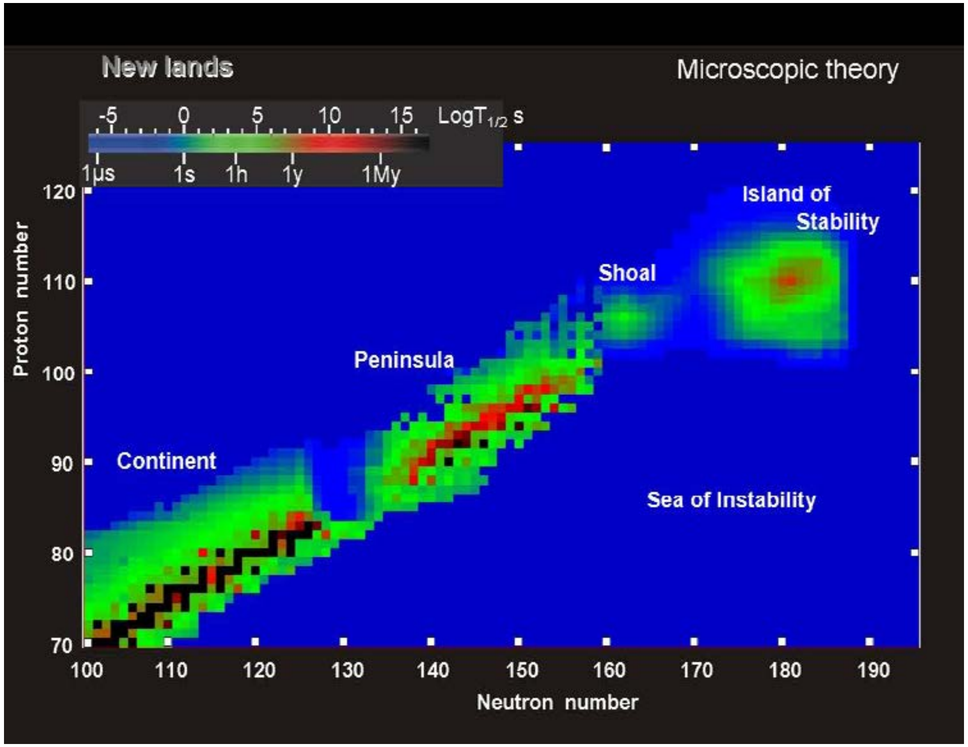
$(|E - V_b| \lesssim 10\text{MeV})$

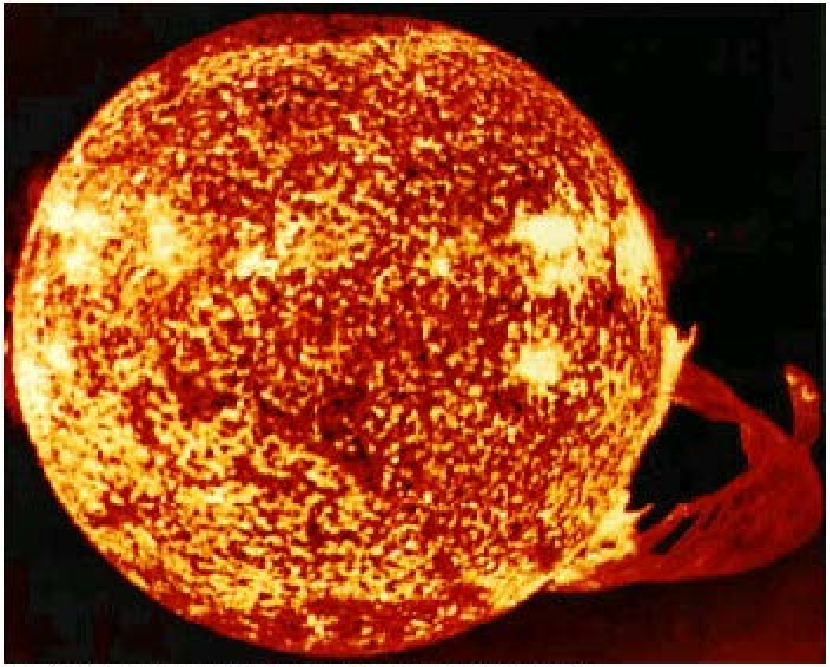
deep sub-barrier region

$(E \lesssim V_b - 10\text{MeV})$

Why (deep) sub-barrier fusion?

Two obvious reasons:





NASA, Skylab space station December 19. 1973, solar flare reaching 588 000 km off solar surface

discovering new elements
(SHE by cold fusion reactions)

nuclear astrophysics
(fusion in stars)

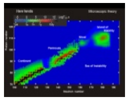
cf. ^{209}Bi(^{70}Zn,n)

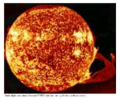
V_{Bass} = 260.4 MeV

$E_{cm}^{(exp)}$ = 261.4 (1st, 2nd), 262.9 MeV (3rd)

Why subbarrier fusion?

Two obvious reasons:

✓discovering new elements (SHE) 

✓nuclear astrophysics (fusion in stars) 

Other reasons:

◆ reaction mechanism

strong interplay between reaction and structure

(channel coupling effects)

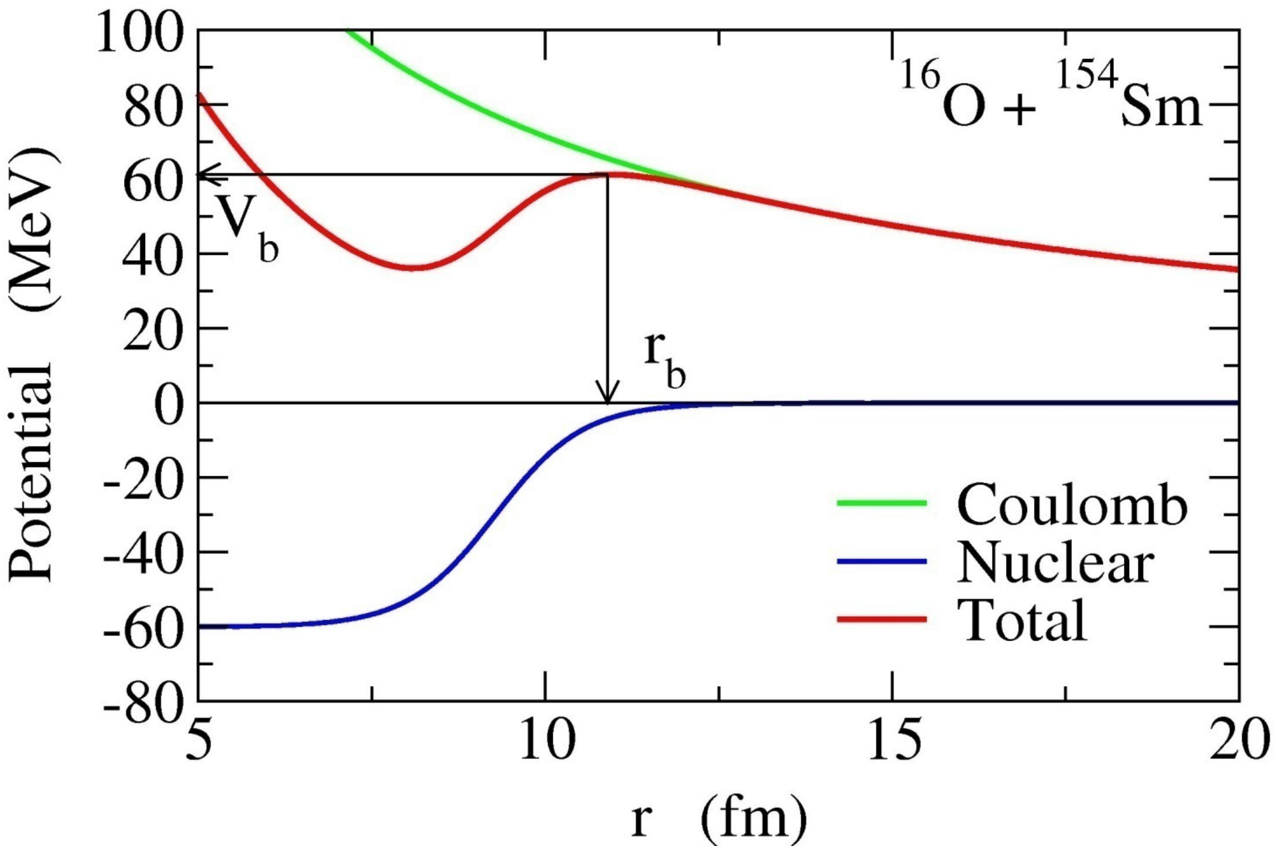
cf. high *E* reactions: much simpler reaction mechanism

◆ many-particle tunneling

cf. alpha decay: fixed energy

tunneling in atomic collision: less variety of intrinsic motions

Potential model for fusion

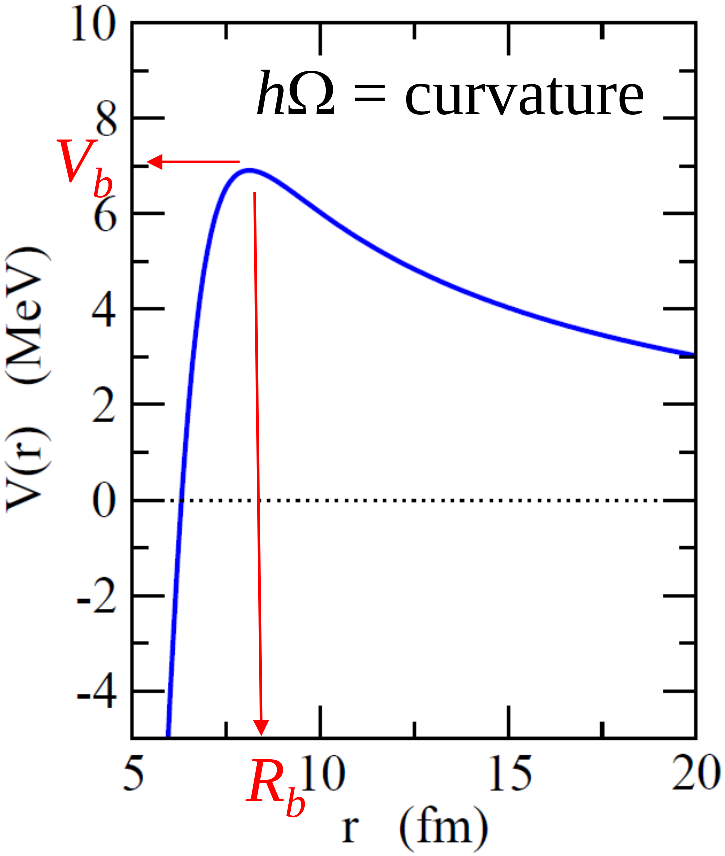


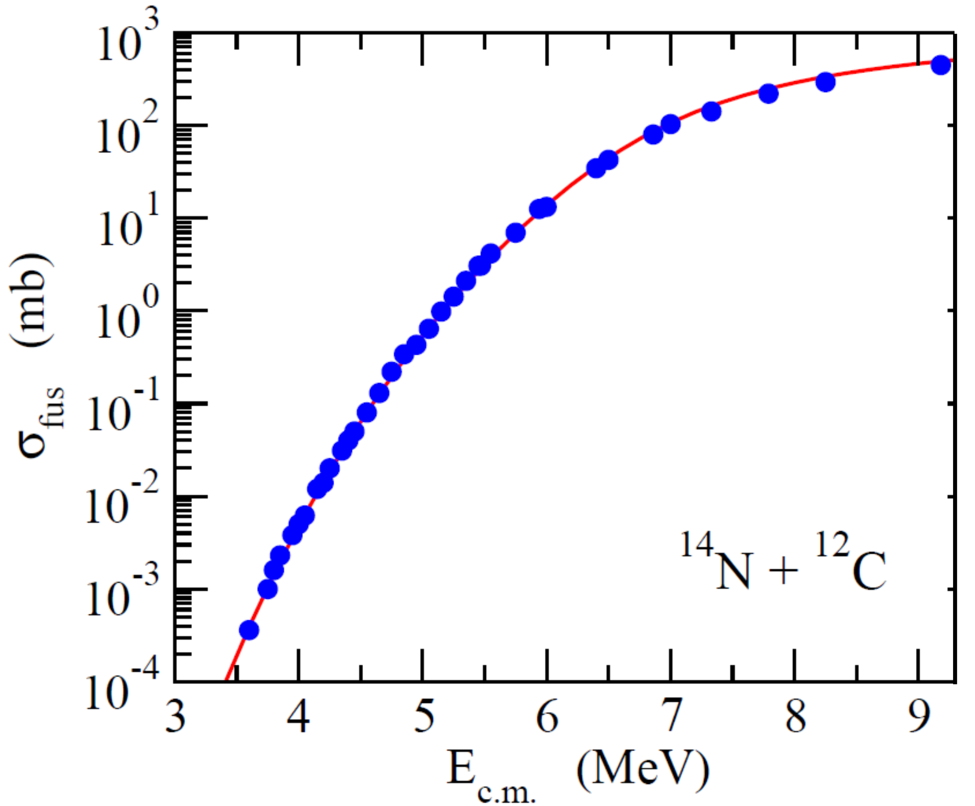
the simplest approach to fusion cross sections: potential model

$$\sigma_{\mathrm{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l+1) P_l(E)$$

the simplest approach: potential model with $V(r)$ + absorption

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l+1) P_l(E)$$

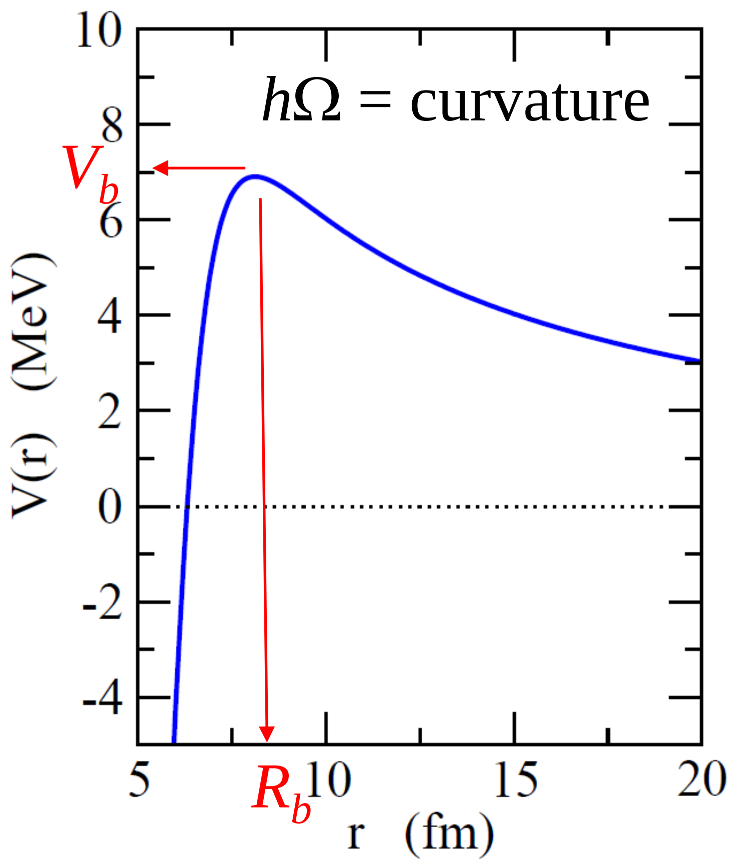


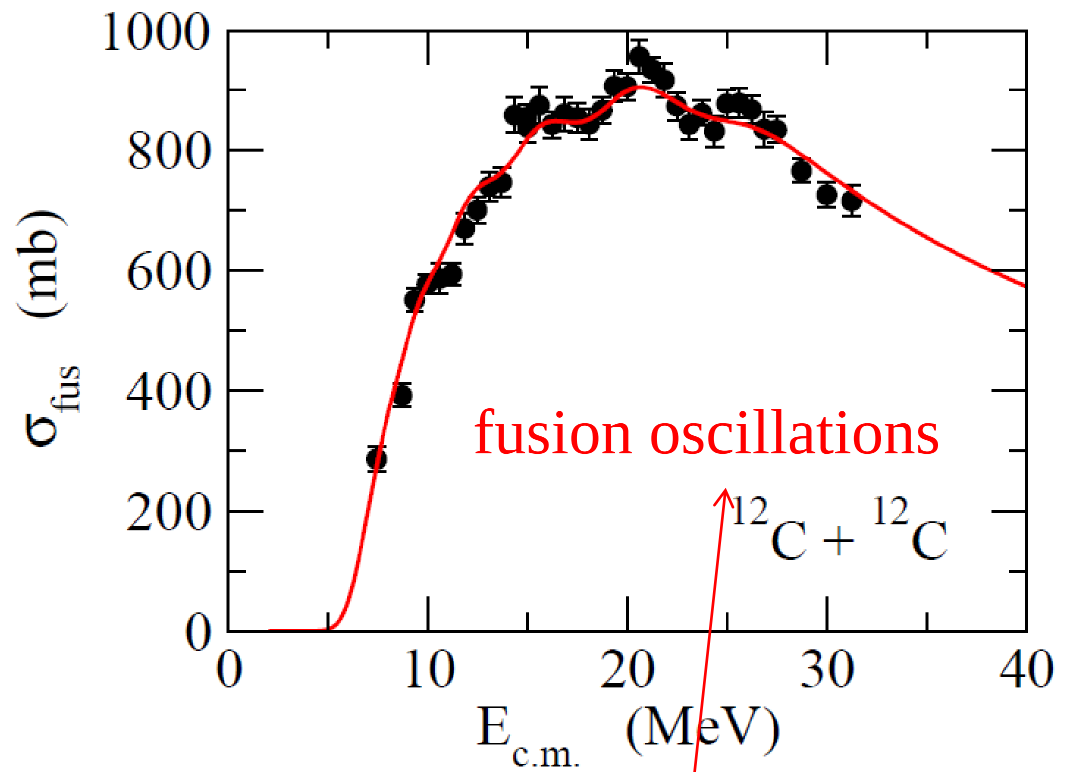


➢ Wong formula [C.Y. Wong, PRL31 ('73)766]

$$\sigma_{\text{fus}}(E) \sim \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

potential model: $V(r)$ + absorption



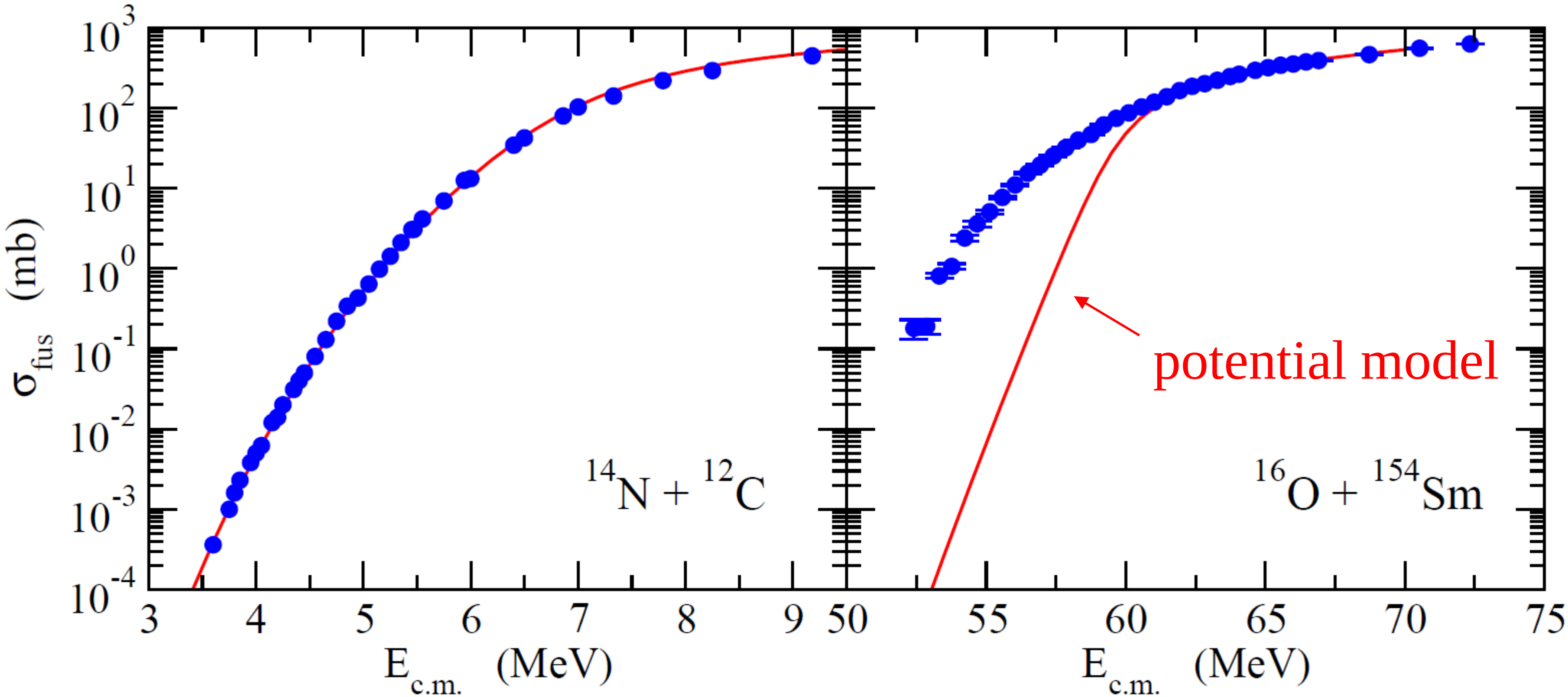


Generalized Wong formula [N. Rowley and K.H., PRC91('15)044617]

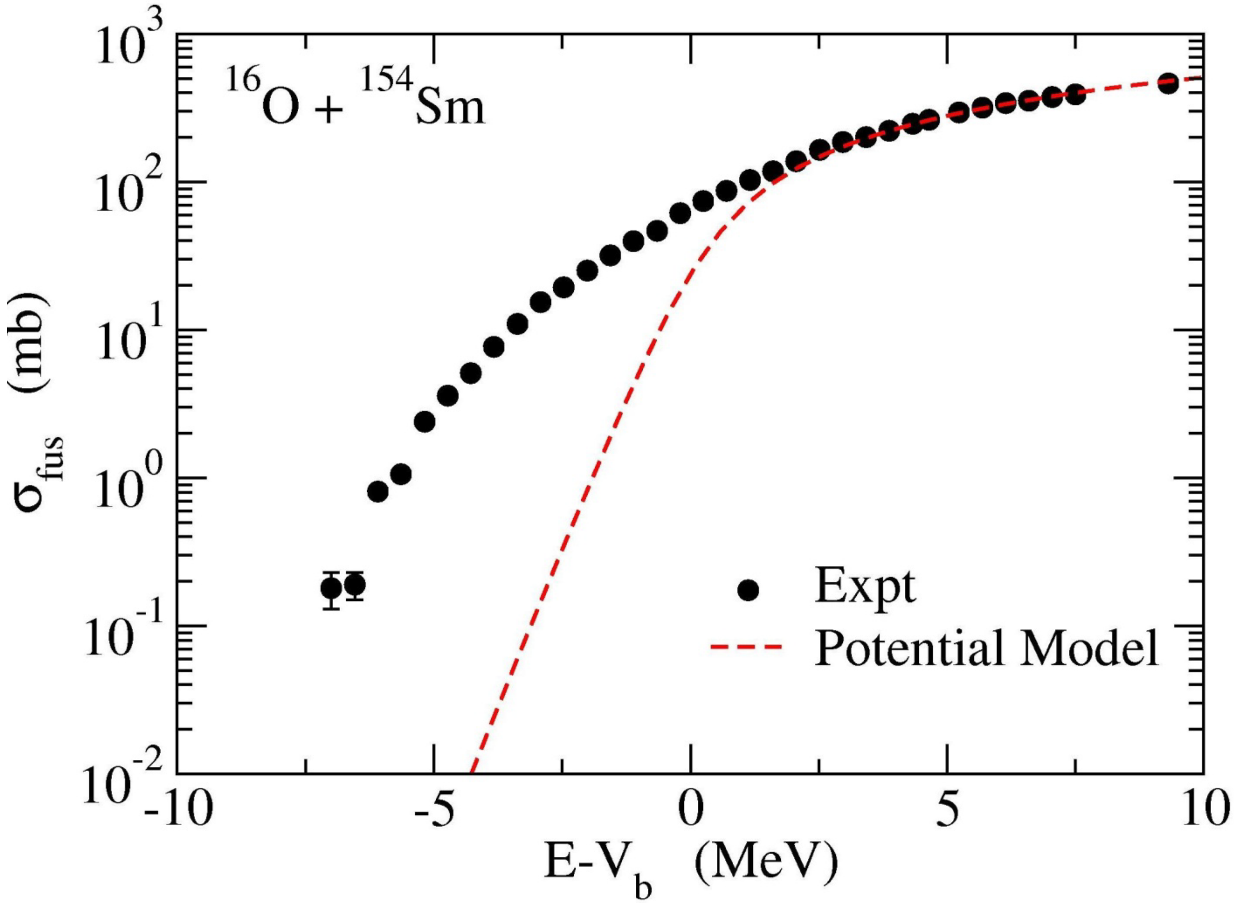
$$\sigma_{\text{fus}}(E) \sim \frac{\hbar\Omega_E}{2E} R_E^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega_E}(E - V_E)\right)\right] + (osc.)$$

Discovery of large sub-barrier enhancement of σ_{fus} (~ the late 70's)

potential model: $V(r)$ + absorption



cf. seminal work:
R.G. Stokstad et al., PRL41('78) 465



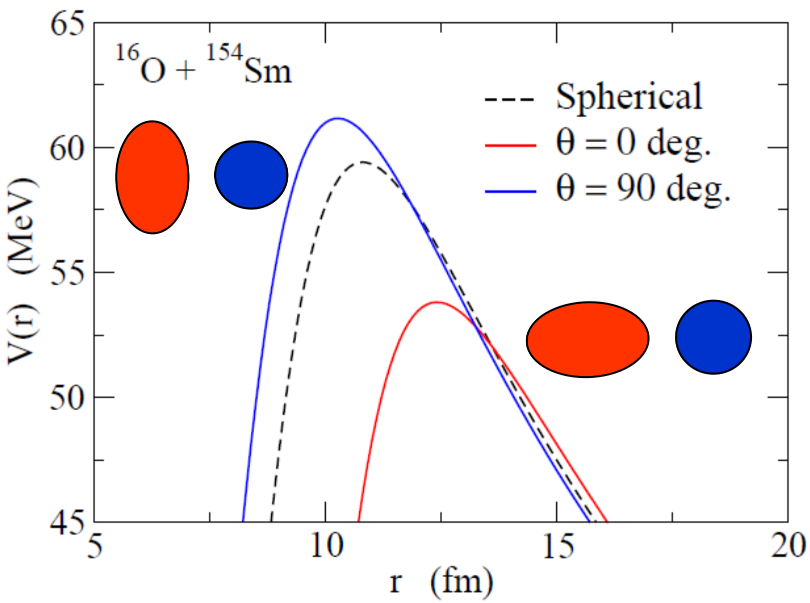
Potential model:
Reproduces the data reasonably well for
$E > V_b$

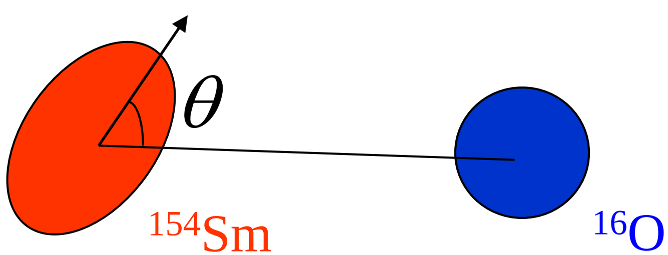
Underpredicts σ_{fus} for
$E < V_b$

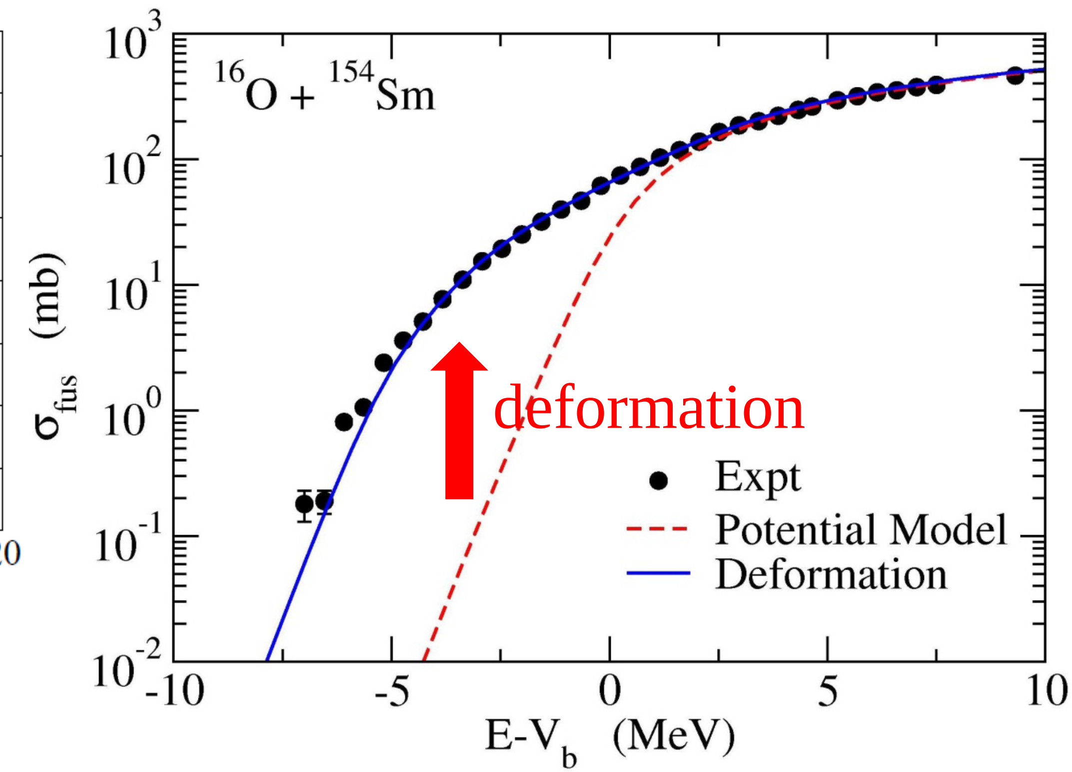
cf. seminal work:
R.G. Stokstad et al., PRL41('78)465
PRC21('80)2427

Effect of nuclear deformation

^{154}Sm : a deformed nucleus with $\beta_2 \sim 0.3$





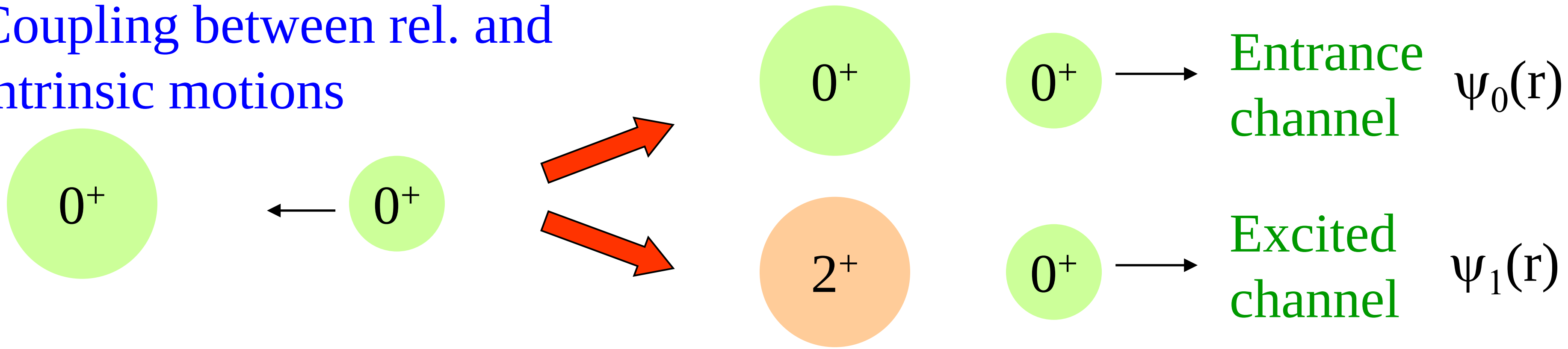


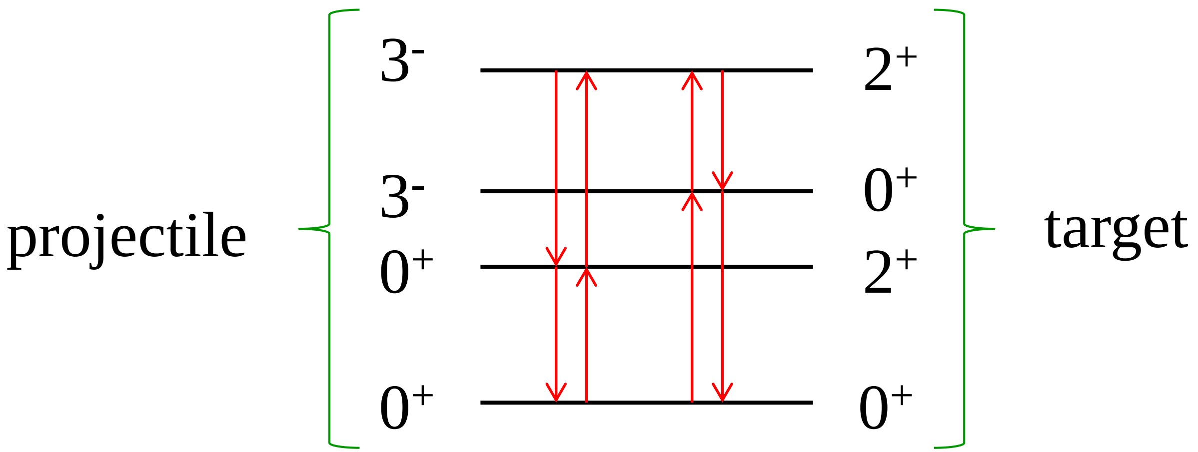
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\text{fus}}(E;\theta)$$

Fusion: strong interplay between nuclear structure and nuclear reaction

Coupled-Channels method

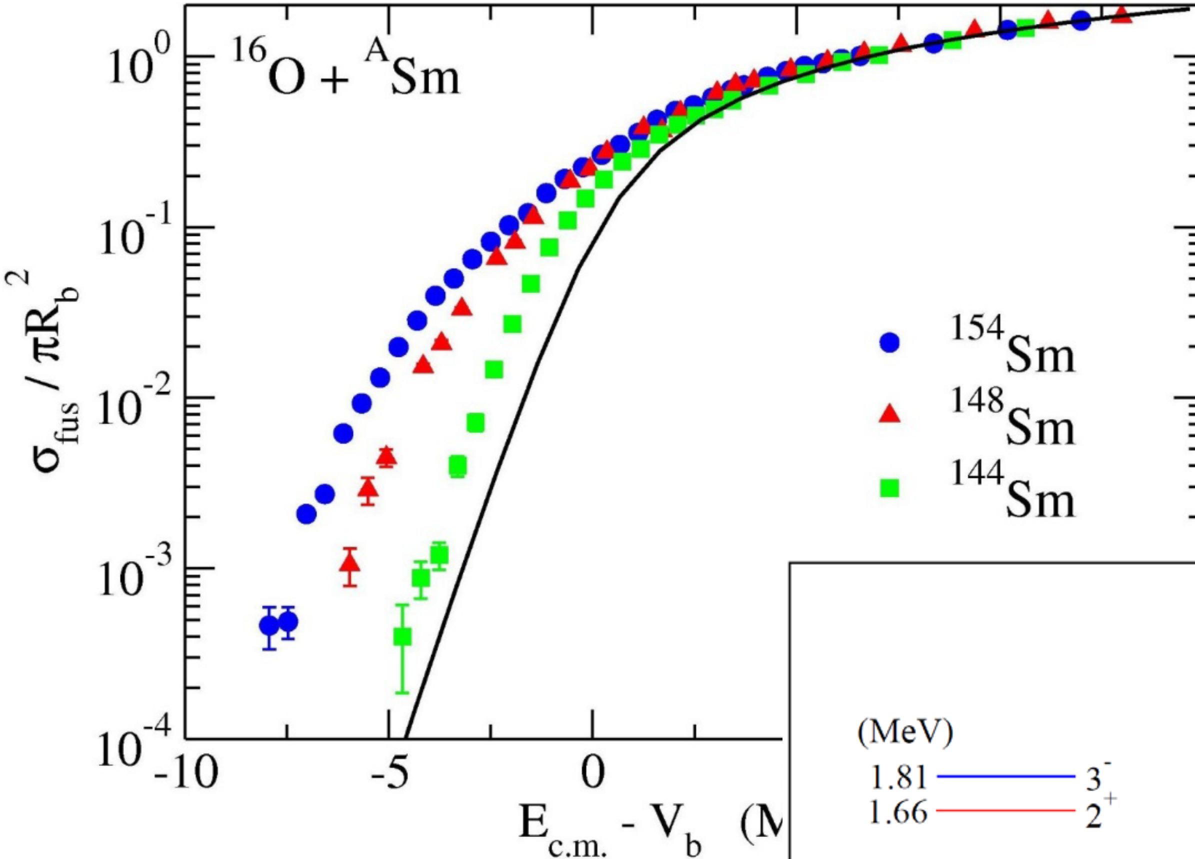
Coupling between rel. and intrinsic motions

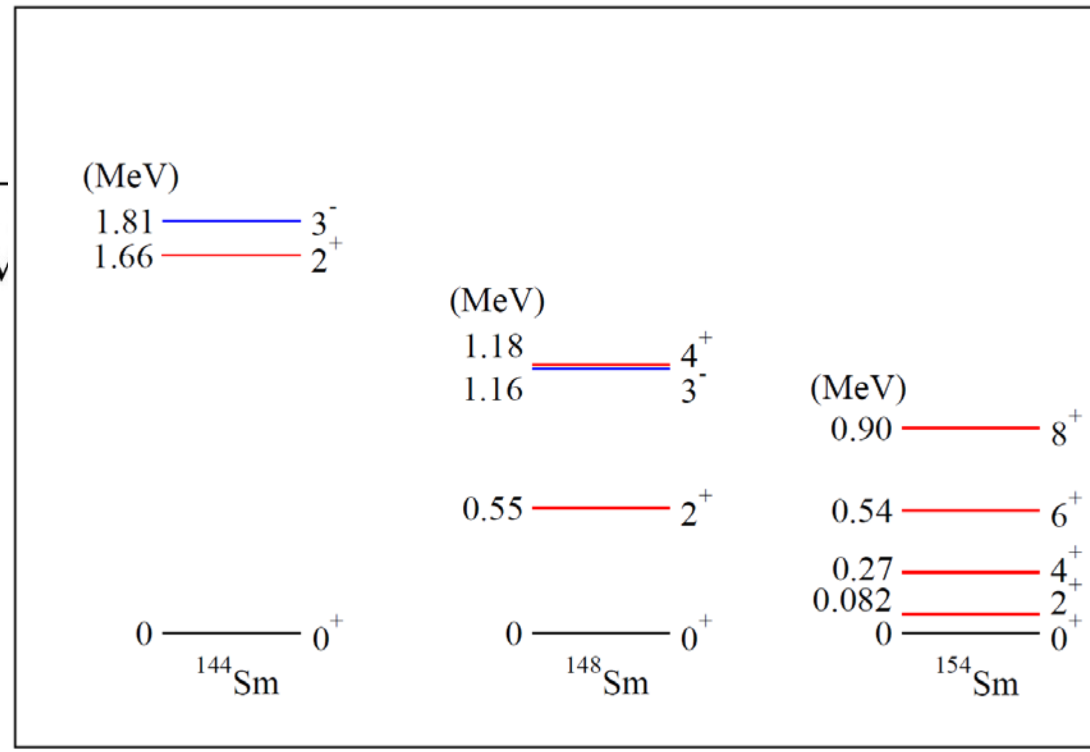




$$\Psi(\boldsymbol{r},\xi) = \sum_k \psi_k(\boldsymbol{r})\phi_k(\xi)$$

→ coupled Schroedinger equations for $\psi_k(r)$





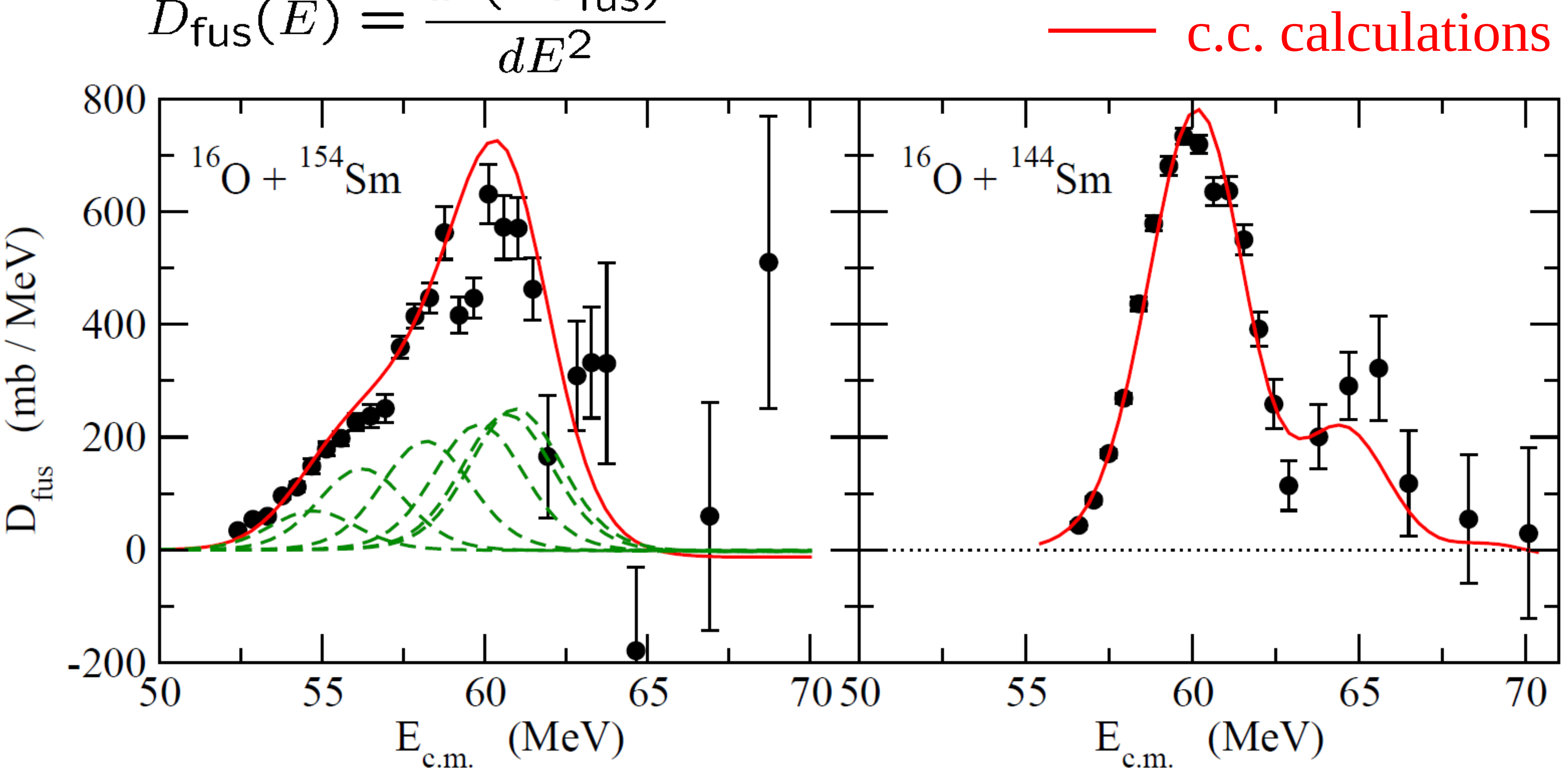
Strong target dependence
at $E < V_b$

C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 (‘99) 143)

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254(‘91))

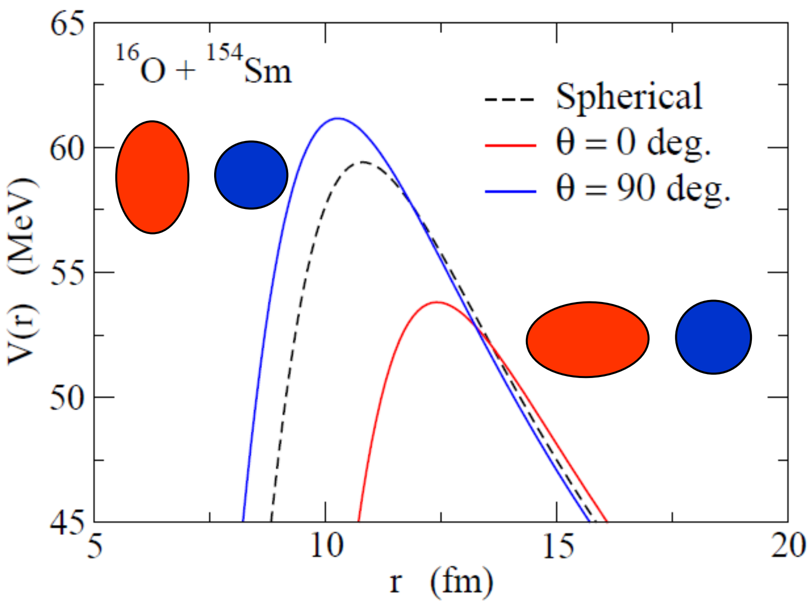
$$D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2}$$



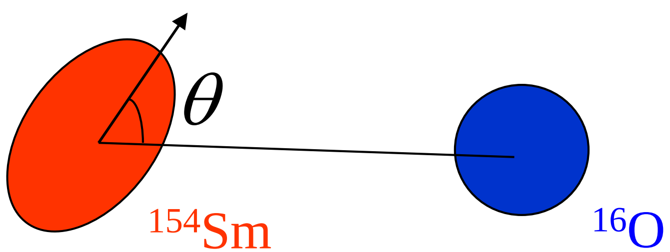
K.H., N. Takigawa, PTP128 (‘12) 1061

Effect of nuclear deformation

^{154}Sm : a deformed nucleus with $\beta_2 \sim 0.3$



deformation:
single barrier → many barriers

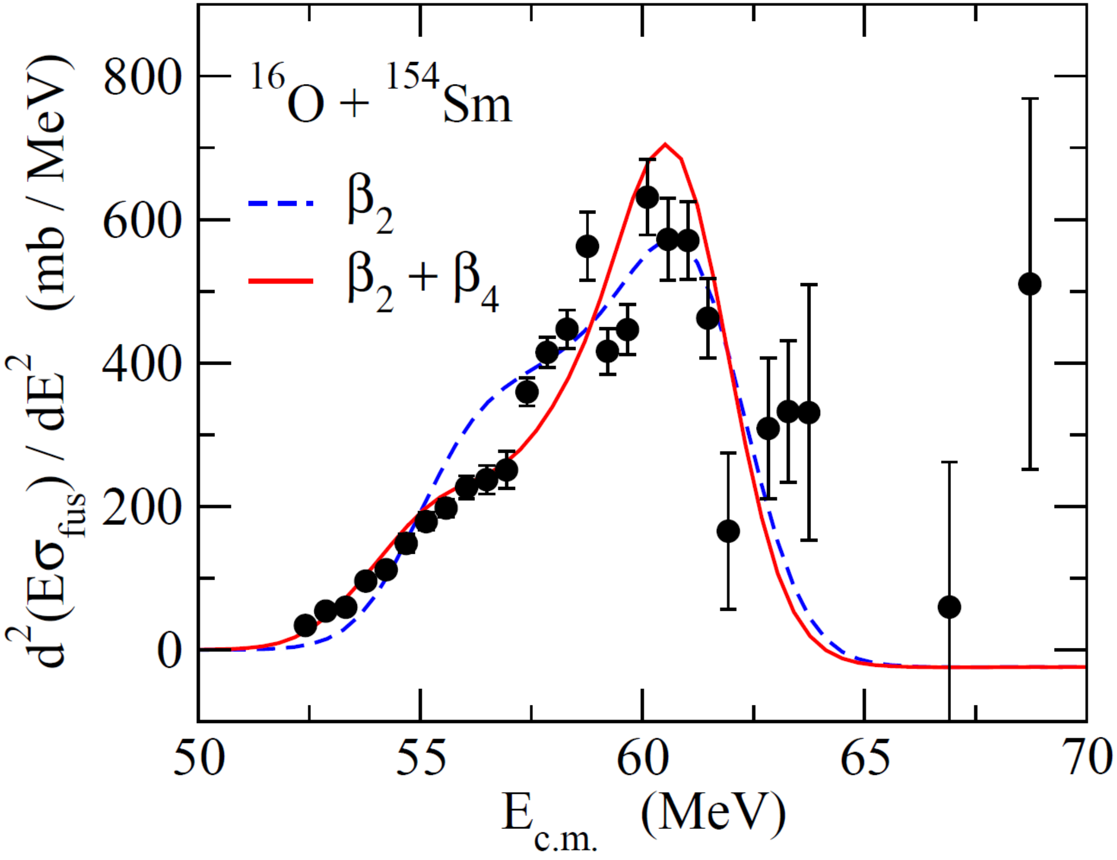


$$\sigma_{\mathrm{fus}}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\mathrm{fus}}(E;\theta)$$

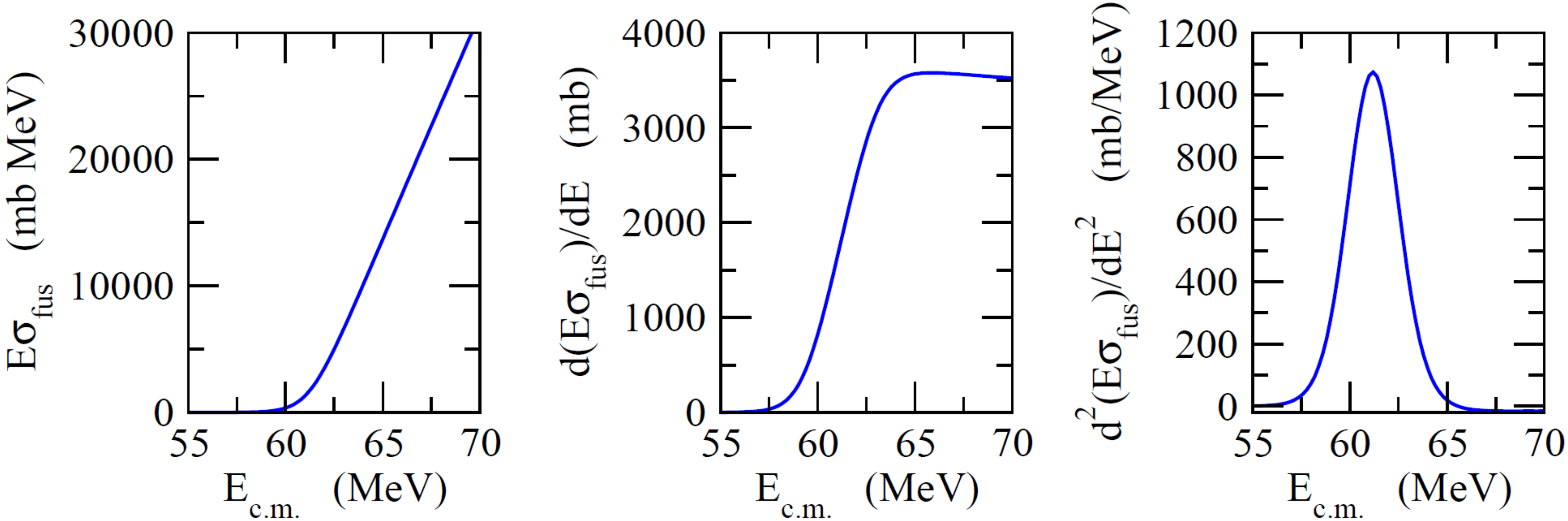
Fusion barrier distribution

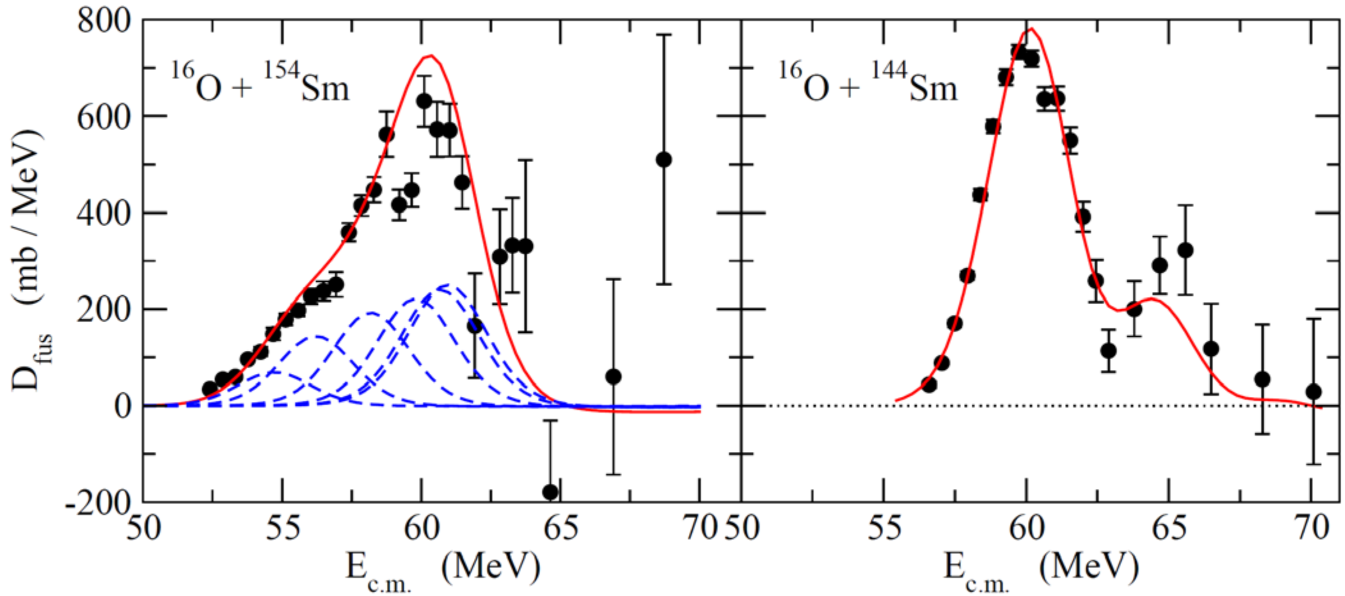
$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

- N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254(‘91) 25
- J.X. Wei, J.R. Leigh et al., PRL67(’91) 3368
- M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48(’98)401

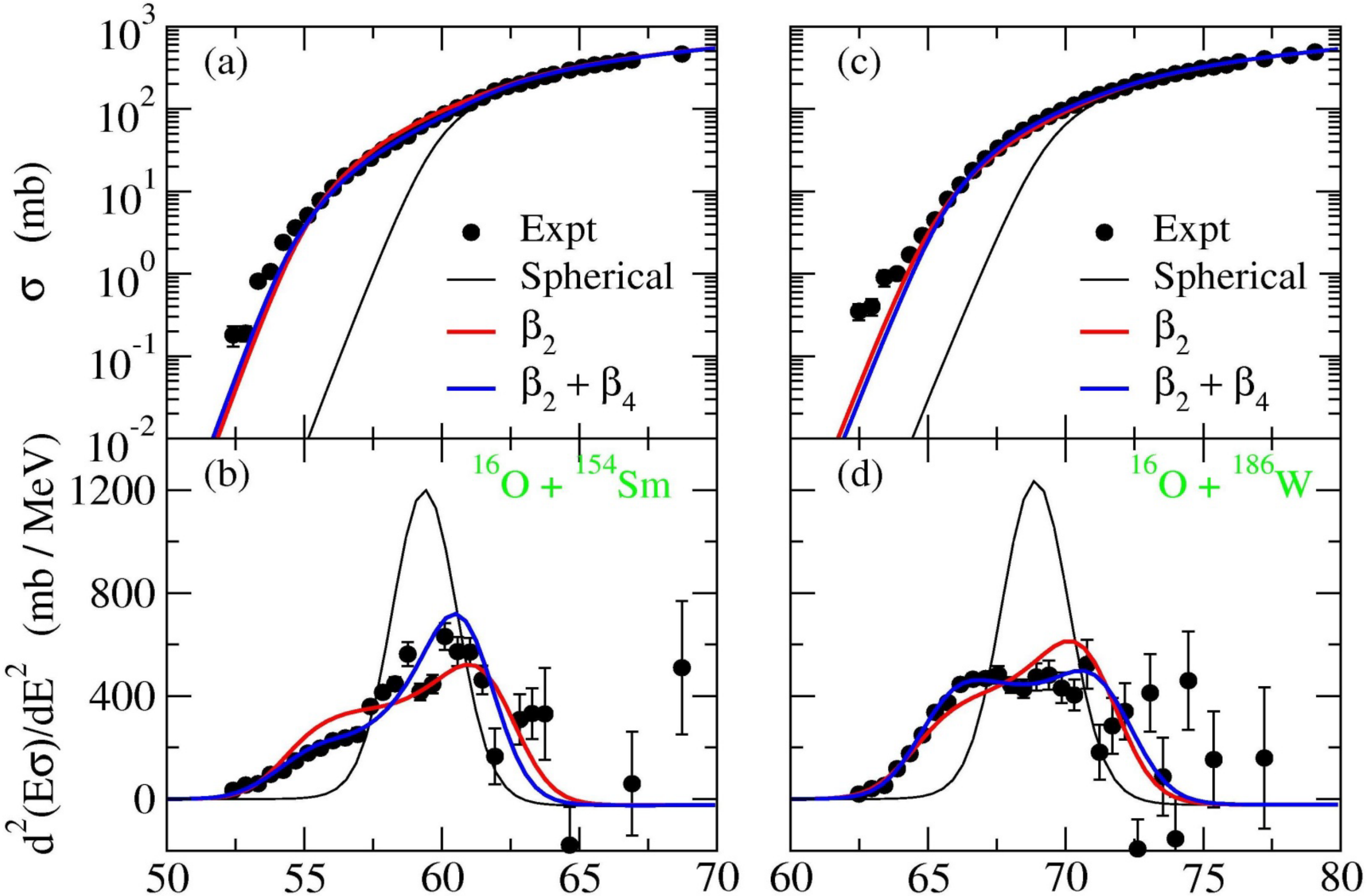


$\sim \pi R_b^2 \, P_{l=0}(E)$ $\sim \pi R_b^2 \cdot dP_{l=0}/dE$





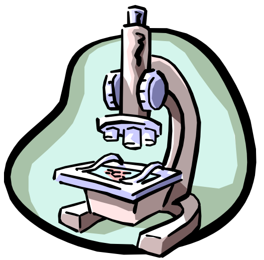
K.H. and N. Takigawa, PTP128 ('12) 1061



$\beta_2 = 0.33$ $\beta_2 = 0.29$

$\beta_4 = +0.05$ $\beta_4 = -0.03$

Fusion barrier distribution:
sensitive to small effects such as β_4



M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48('98)401

Quasi-elastic barrier distributions

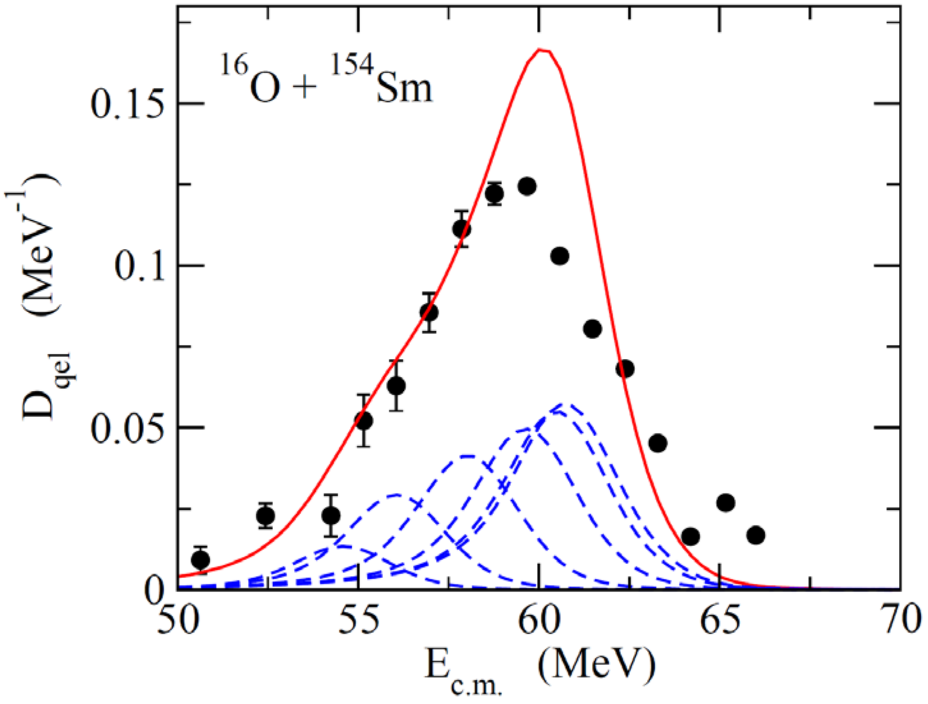
Quasi-elastic scattering:

A sum of all the reaction processes other than fusion
(elastic + inelastic + transfer + ……)

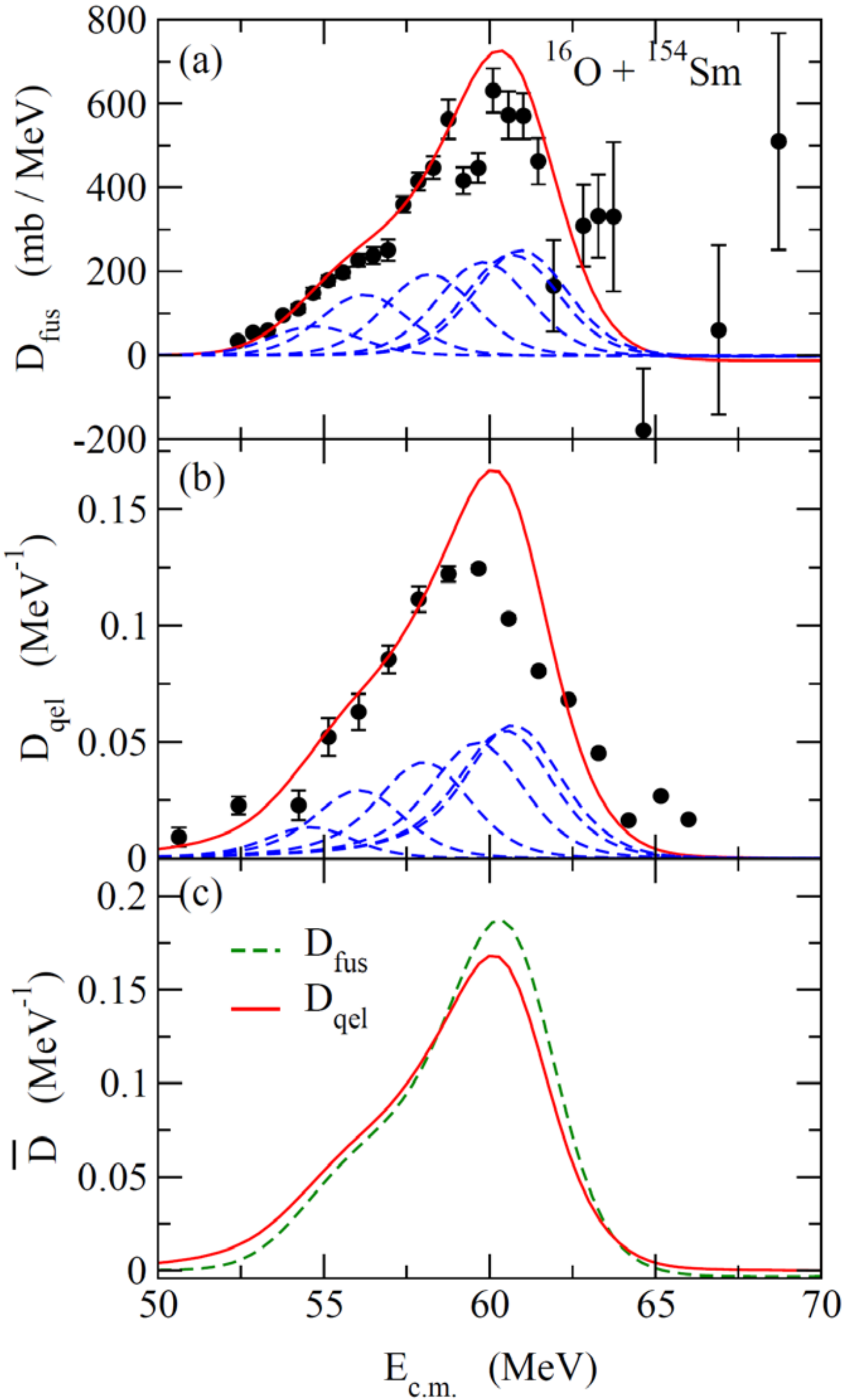
$$P_{l=0}(E) = 1 - R_{l=0}(E) \sim 1 - \frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}$$

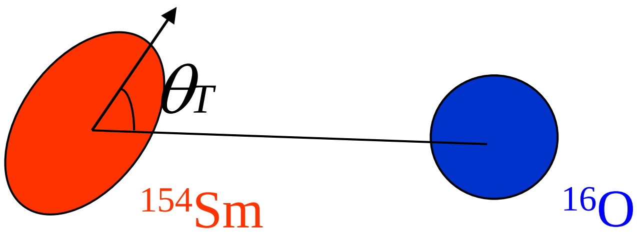
$$D_{\rm qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\rm qel}(E,\pi)}{\sigma_{\rm Ruth}(E,\pi)}\right)$$

H. Timmers et al.,
NPA584('95)190



D_{fus} and D_{qel}: behave similarly to each other





$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\text{fus}}(E;\theta_T)$$

$$\sigma_{\text{qel}}(E,\theta) = \sum_I \sigma(E,\theta)$$

$$= \int_0^1 d(\cos\theta_T)\sigma_{\text{el}}(E,\theta;\theta_T)$$

K.H. and N. Rowley, PRC69('04)054610

Experimental advantages for D_{qel}

$$D_{qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{qel}(E,\pi)}{\sigma_R(E,\pi)}\right) \qquad D_{fus}(E) = \frac{d^2(E\sigma_{fus})}{dE^2}$$

- less accuracy is required in the data (1st vs. 2nd derivative)
- much easier to be measured

Qel : a sum of everything

⟶ a very simple charged-particle detector

Fusion : requires a specialized recoil separator

to separate ER from the incident beam

ER + fission for heavy systems

- several effective energies can be measured at a single-beam energy $\leftrightarrow$ $E_{\rm eff} \sim 2E\,\frac{\sin(\theta/2)}{1+\sin(\theta/2)}$

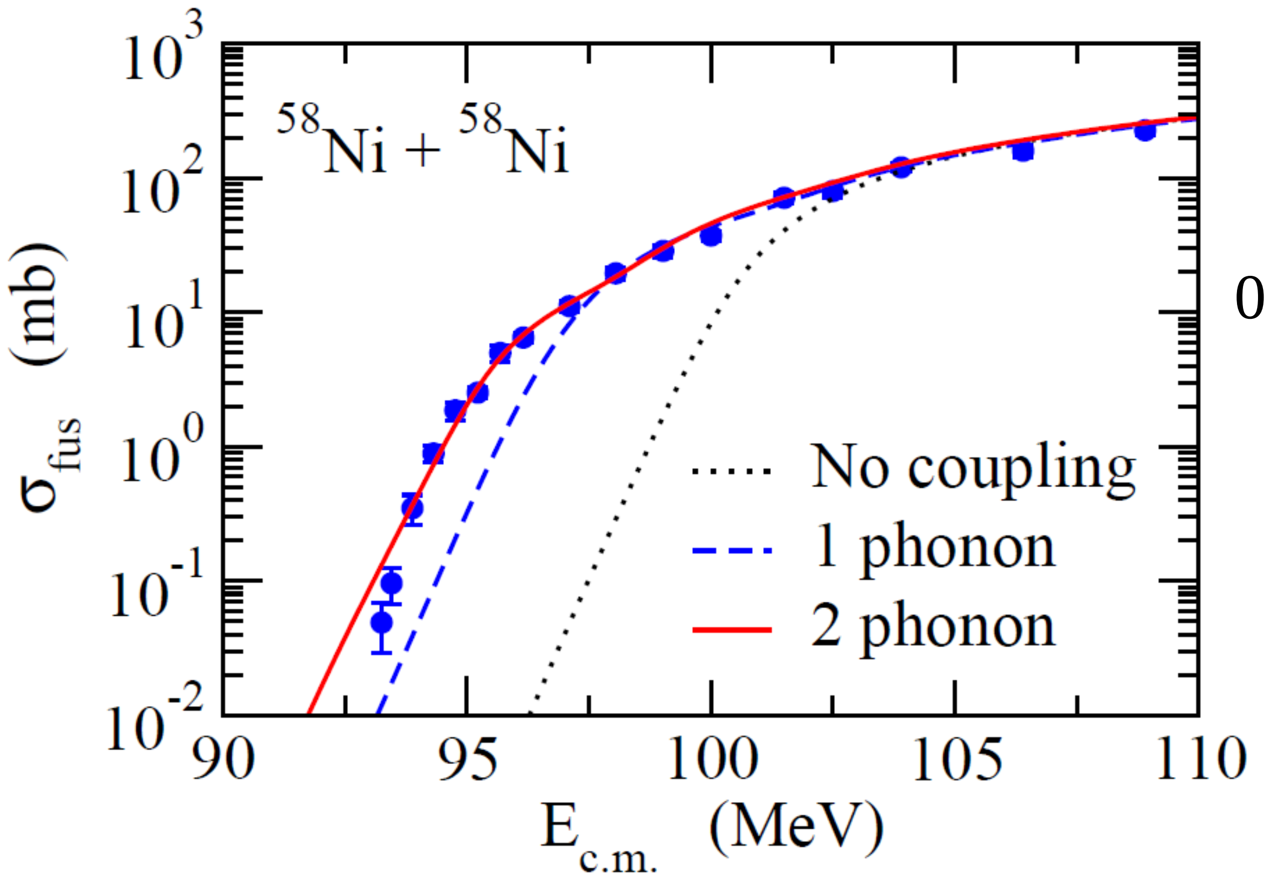
⇨ measurements with a cyclotron accelerator: possible

➡ Suitable for low intensity RI beams

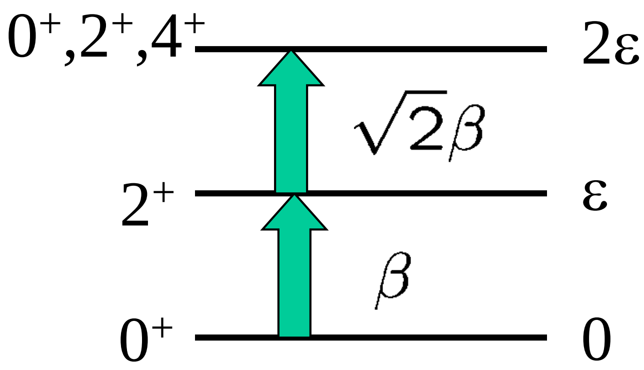
Semi-microscopic modeling of sub-barrier fusion

K.H. and J.M. Yao, PRC91('15) 064606

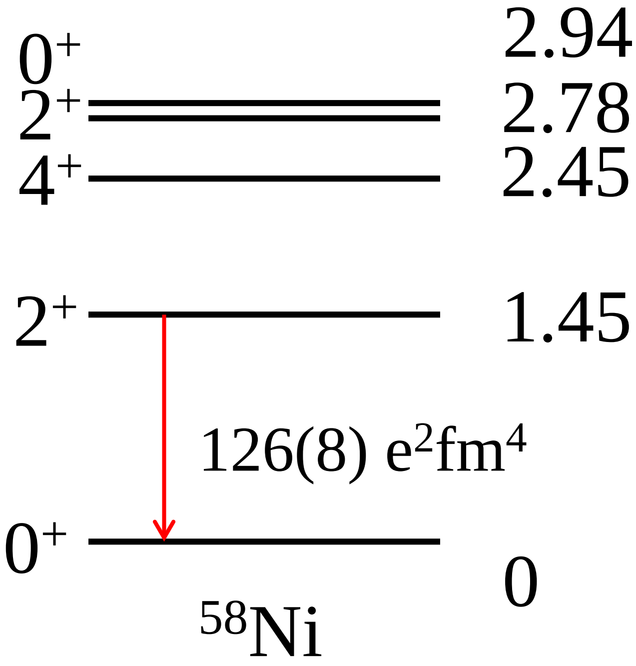
multi-phonon excitations



simple harmonic oscillator



Anharmonic vibrations



$Q(2_1^+) = -10 +/- 6\ e\text{fm}^2$

- Boson expansion
- Quasi-particle phonon model
- Shell model
- Interacting boson model
- Beyond-mean-field method

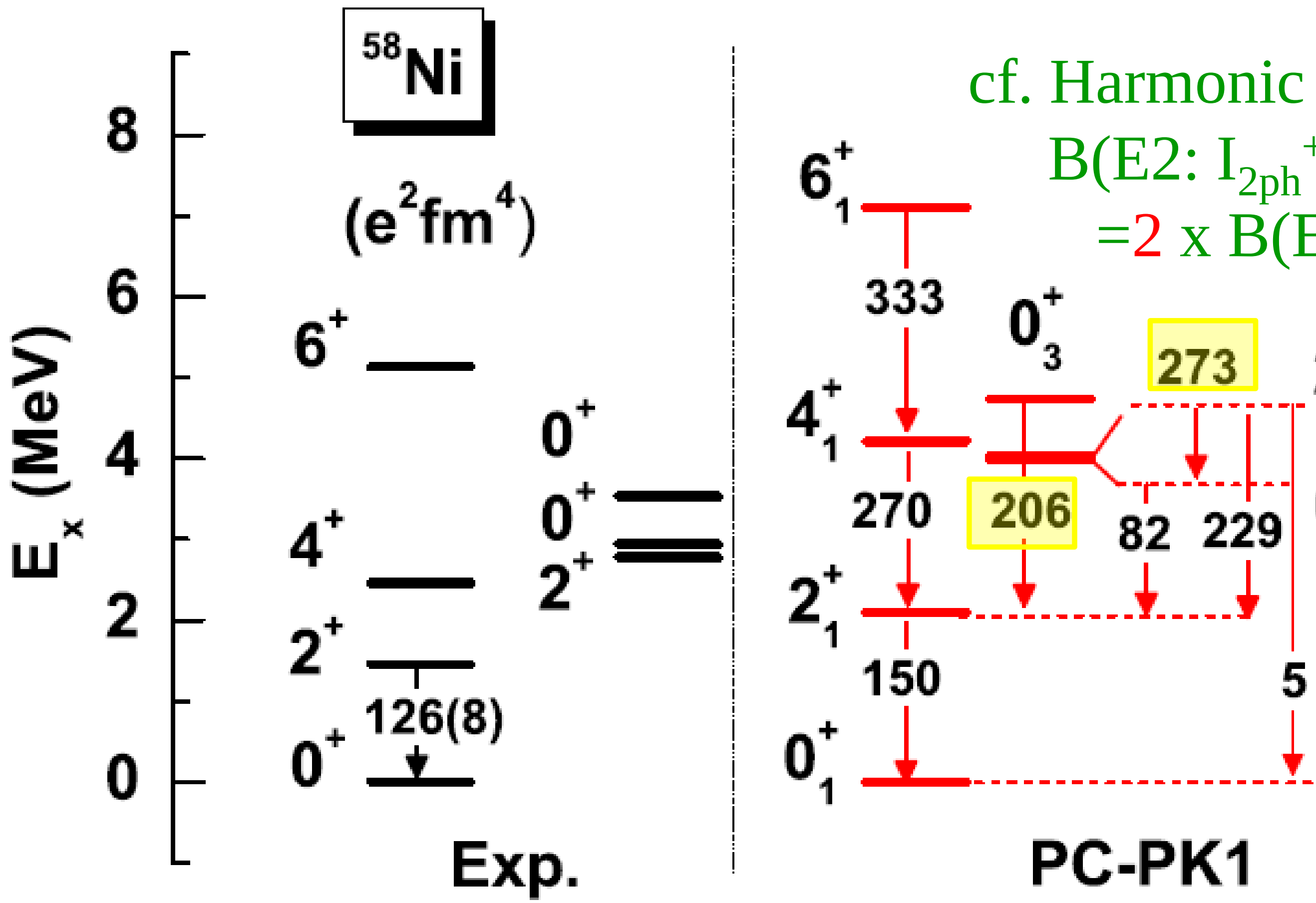
$$|JM\rangle = \int d\beta\, f_J(\beta) \hat{P}^J_{M0} |\Phi(\beta)\rangle$$

✓ MF + ang. mom. projection
+ particle number projection
+ generator coordinate method (GCM)

M. Bender, P.H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 ('03) 121
J.M. Yao et al., PRC89 ('14) 054306

Recent beyond-MF (MR-DFT) calculations for ^{58}Ni

K.H. and J.M. Yao, PRC91 ('15) 064606
J.M. Yao, M. Bender, and P.-H. Heenen, PRC91 ('15) 024301



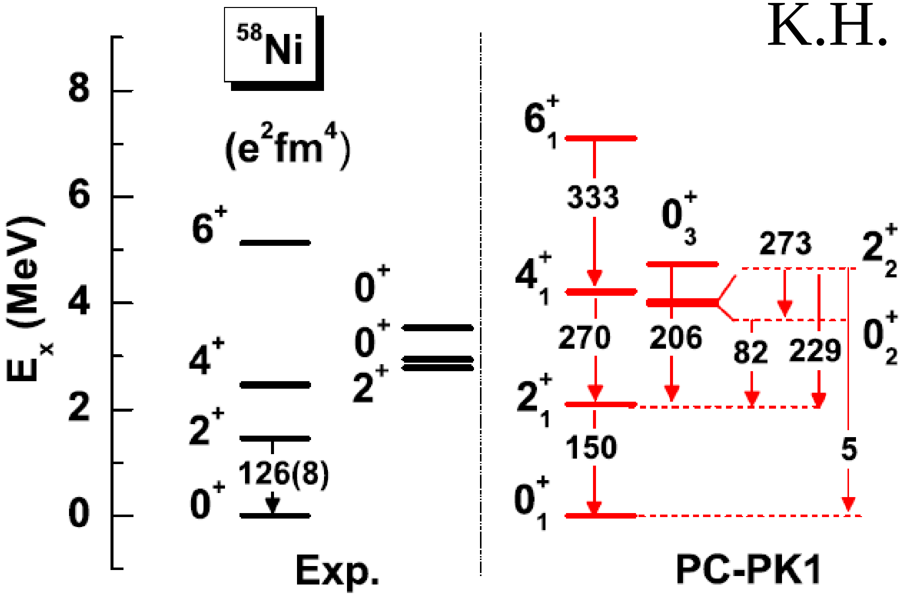
cf. Harmonic limit:
$B(E2: I_{2ph}^+ \rightarrow 2_1^+)$
$= 2 \times B(E2: 2_1^+ \rightarrow 0_1^+)$

- ✓ A large fragmentation of $(2^+ \times 2^+)_{J=0}$
- ✓ A strong transition from 2_2^+ to 0_2^+

→ effects on sub-barrier fusion?

Semi-microscopic coupled-channels model for sub-barrier fusion

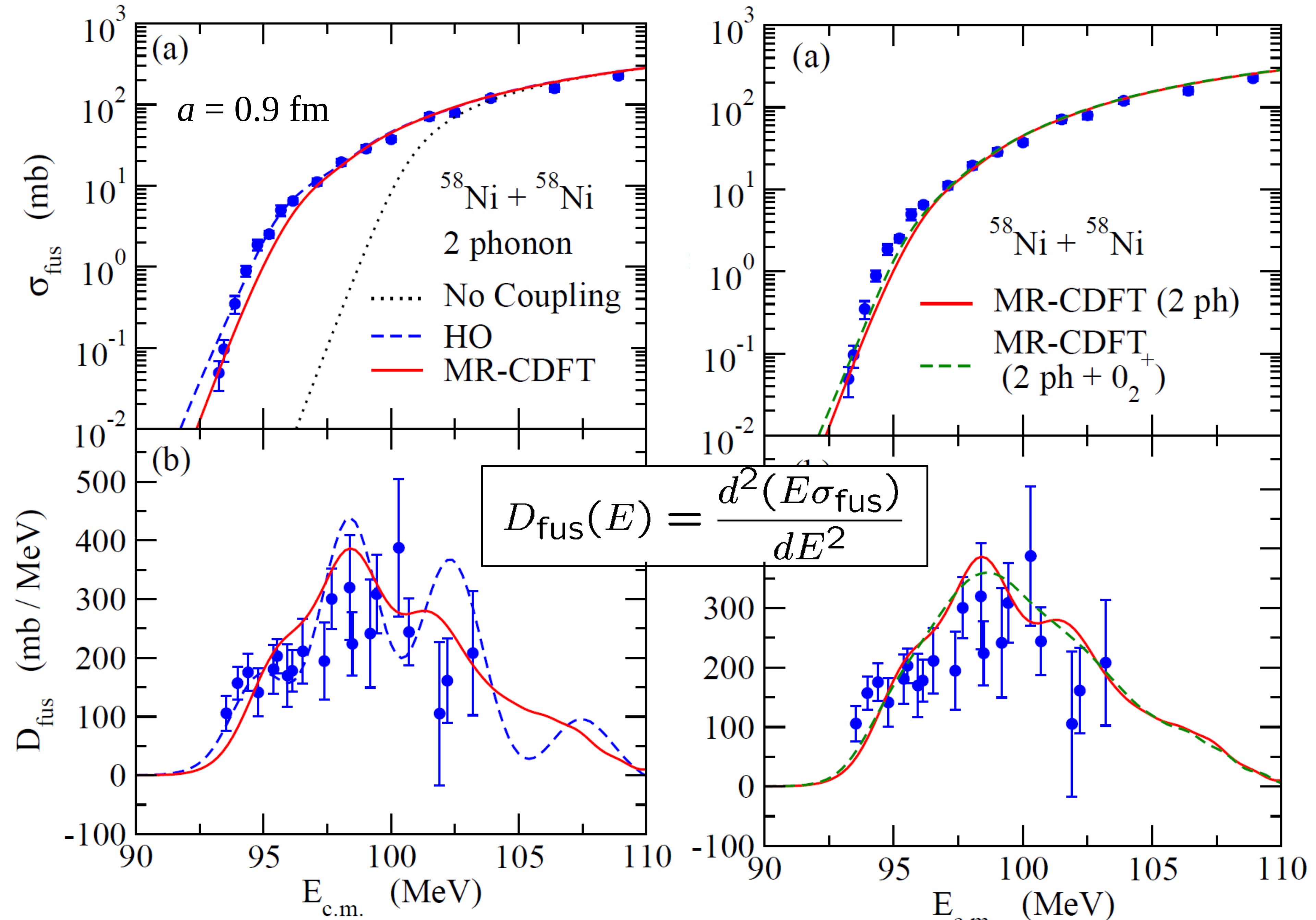
K.H. and J.M. Yao, PRC91 ('15) 064606



microscopic multi-pole operator

✓ $$V_{\text{coup}} \sim -R_T\frac{dV_N}{dr}\alpha_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}}) \rightarrow -R_T\frac{dV_N}{dr}Q_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}})$$

- ✓ *M*(E2) from MR-DFT calculation ← among higher members of phonon states
- ✓ scale to the empirical B(E2; $2_1^+ \rightarrow 0_1^+$)
- ✓ still use a phenomenological potential
- ✓ use the experimental values for E_x
- ✓ β_N and β_C from M_n/M_p for each transition
- ✓ axial symmetry (no 3^+ state)

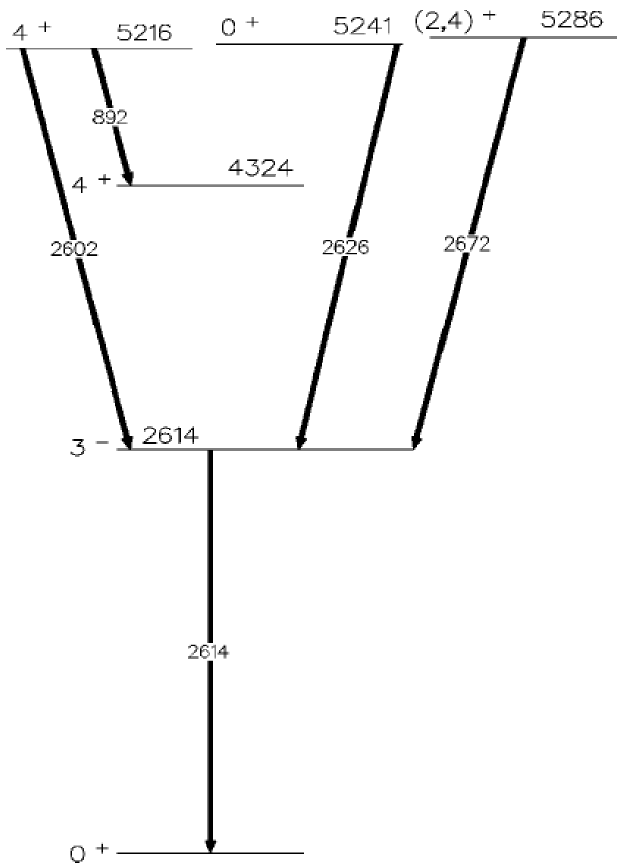


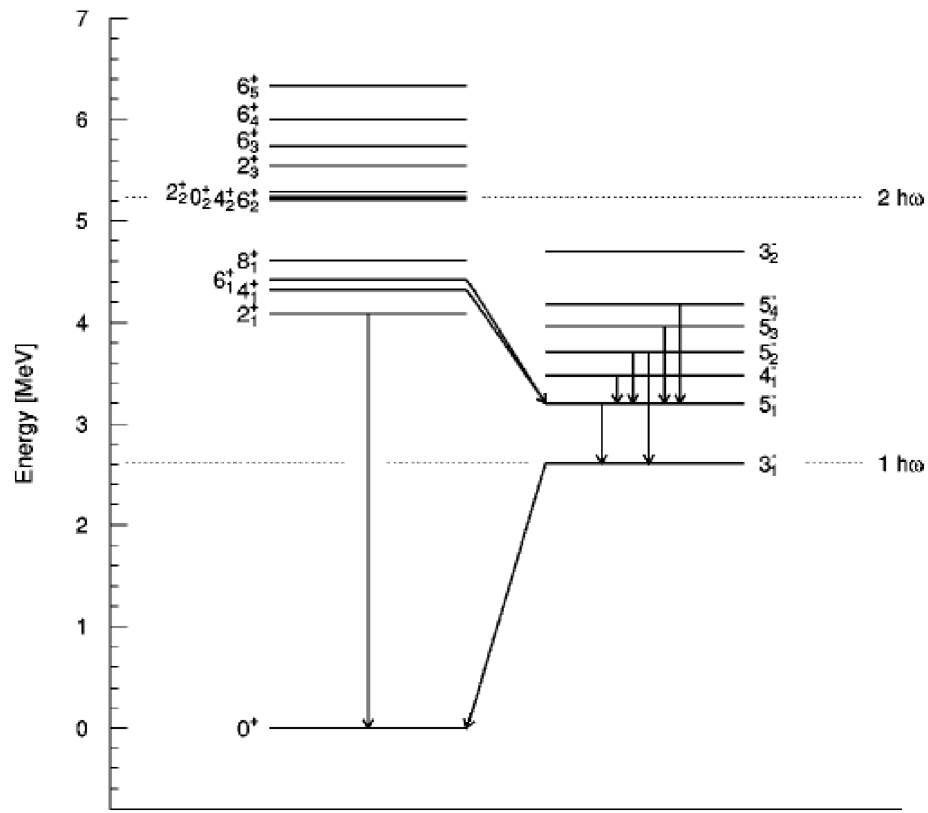
$$D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}$$

K.H. and J.M. Yao, PRC91 ('15) 064606

Application to ^{16}O + ^{208}Pb fusion reaction

double-octupole phonon states in ^{208}Pb

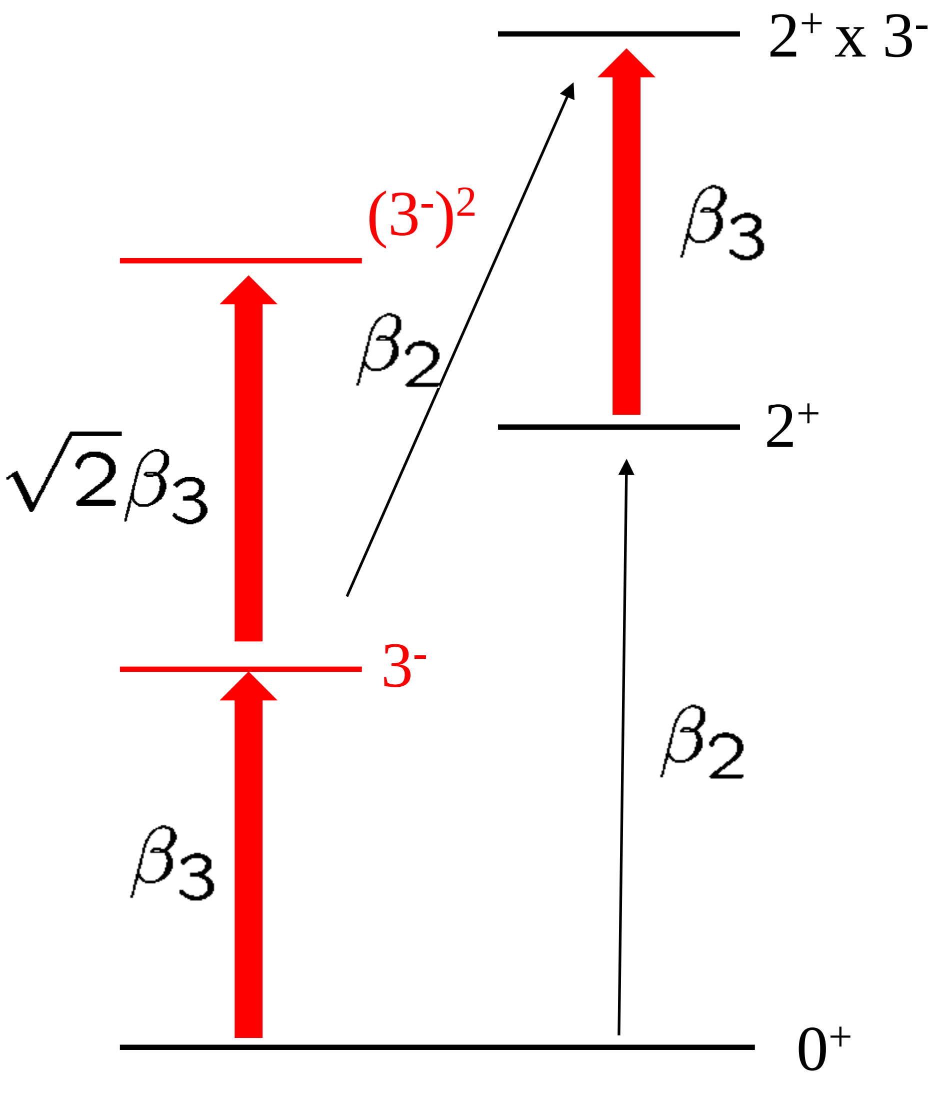


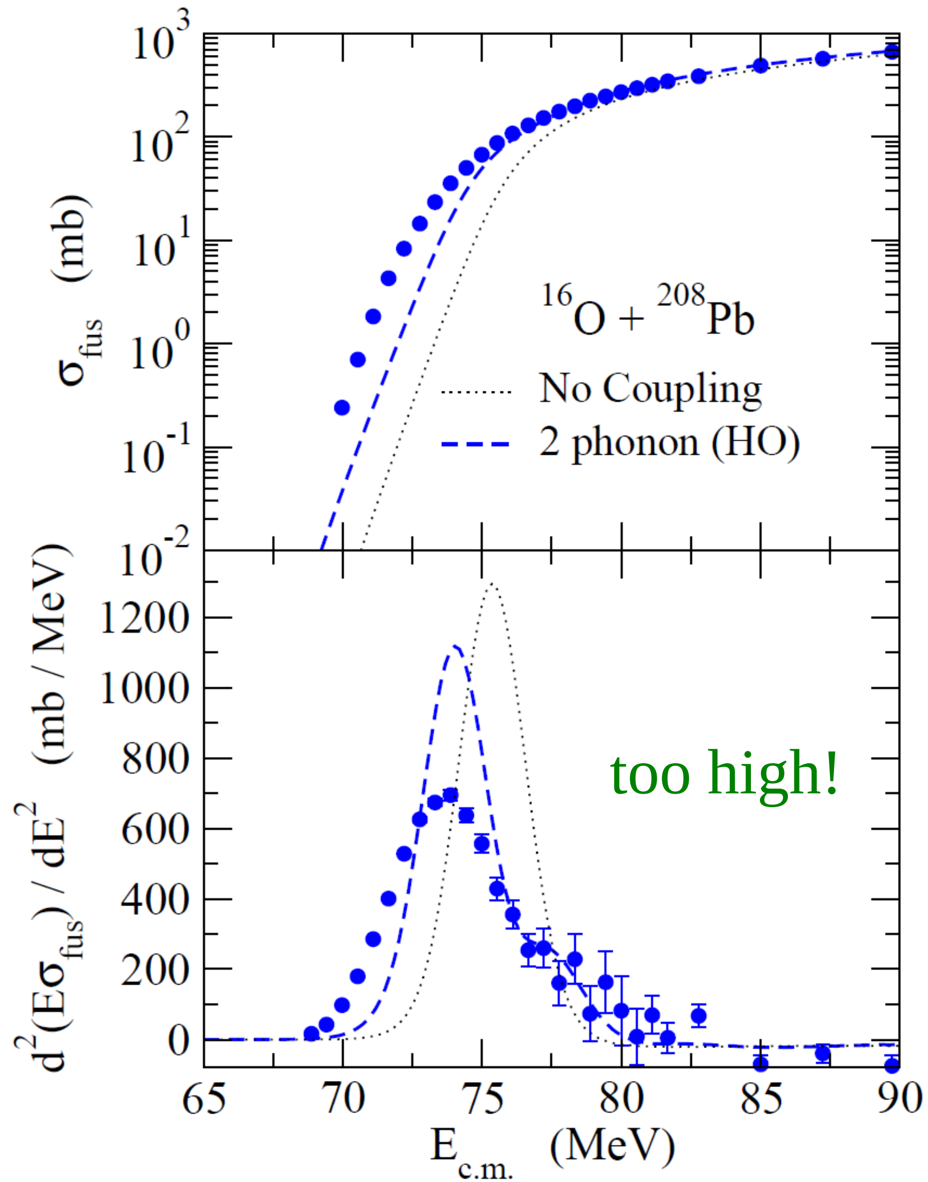


M. Yeh, M. Kadi, P.E. Garrett et al., PRC57 (‘98) R2085

K. Vetter, A.O. Macchiavelli et al., PRC58 (‘98) R2631

Application to $^{16}O + ^{208}Pb$ fusion reaction

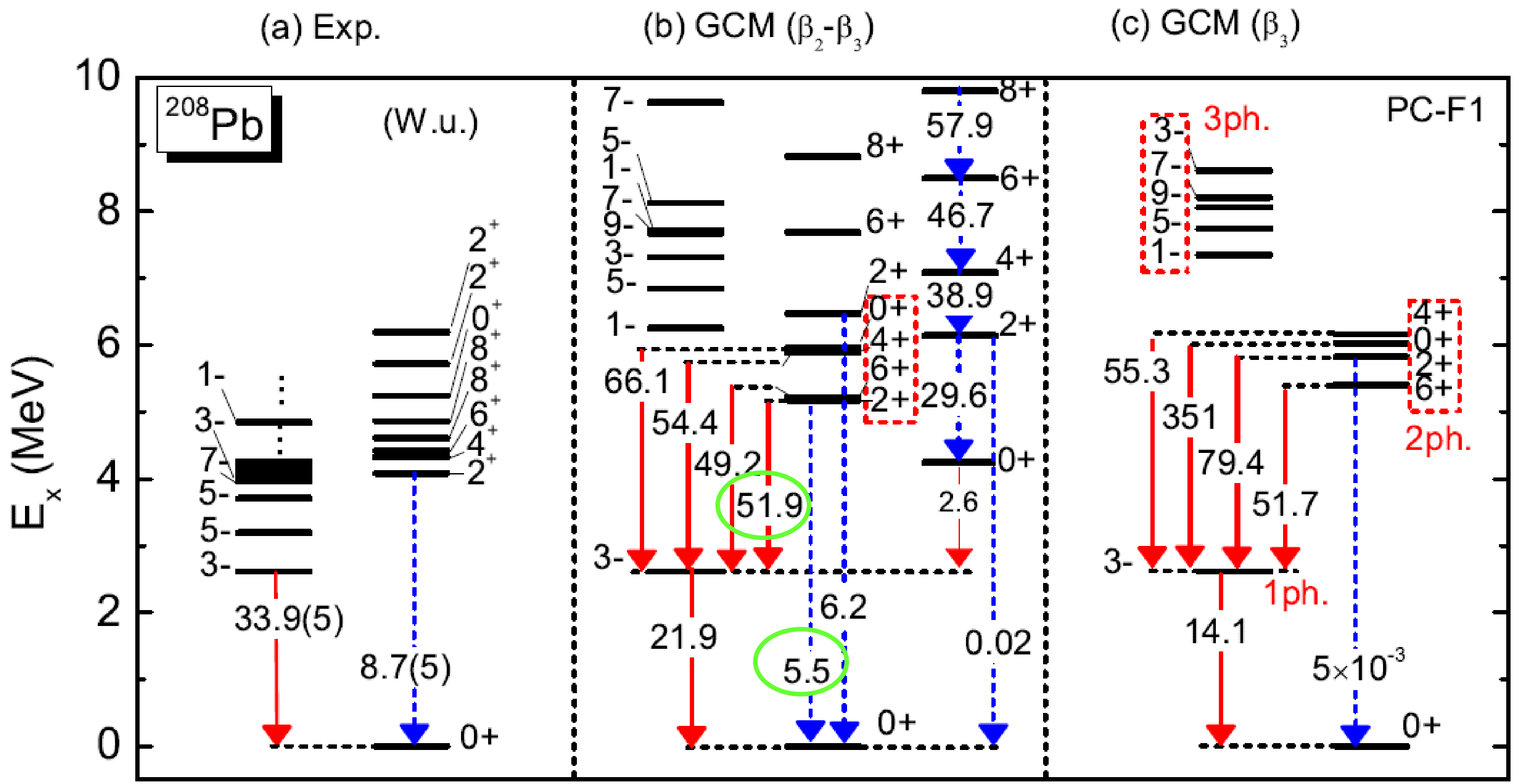




cf. C.R. Morton et al., PRC60('99) 044608 expt. data

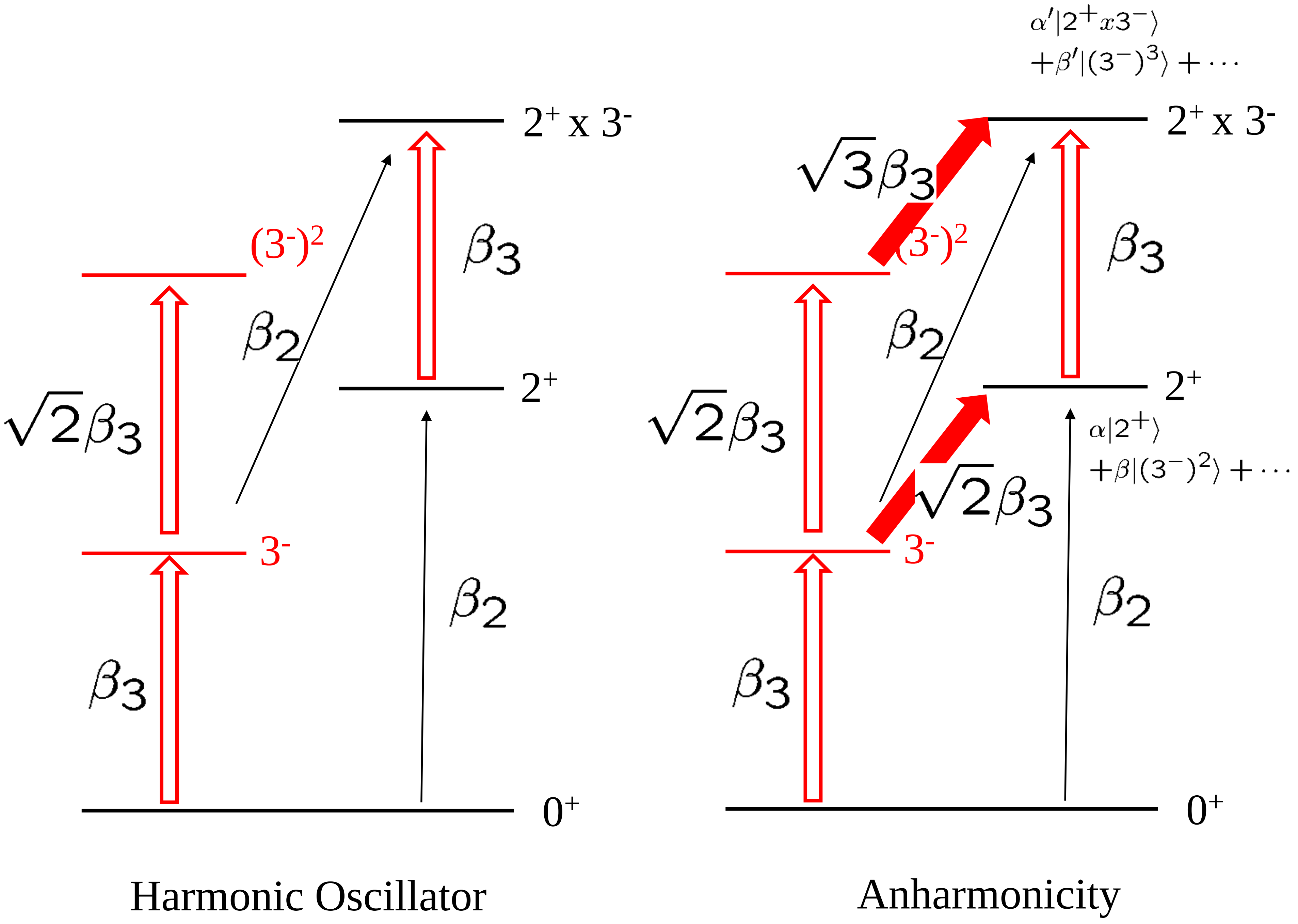
fluctuation both in β_3 and β_2

fluctuation in β_3 frozen at $\beta_2=0$

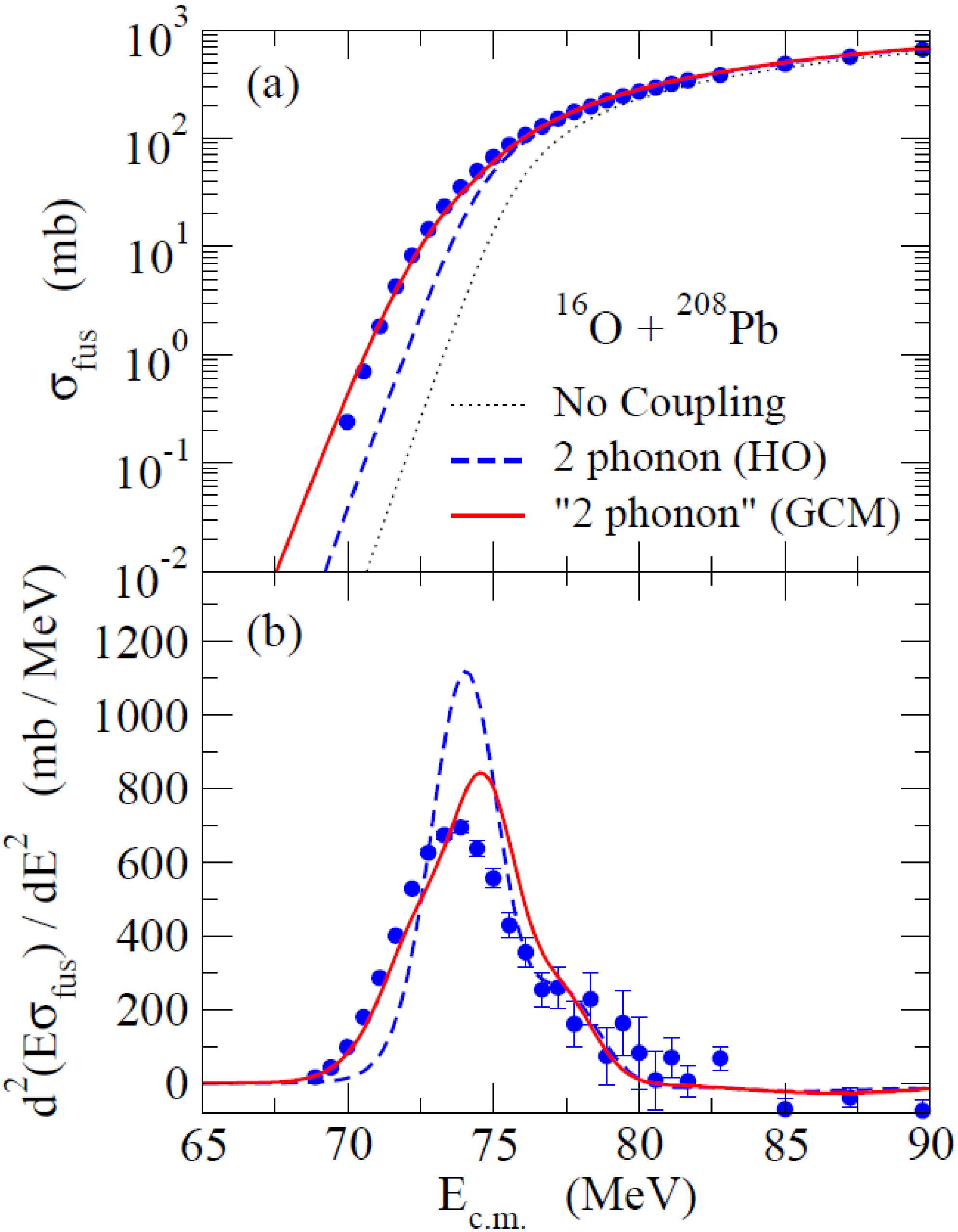


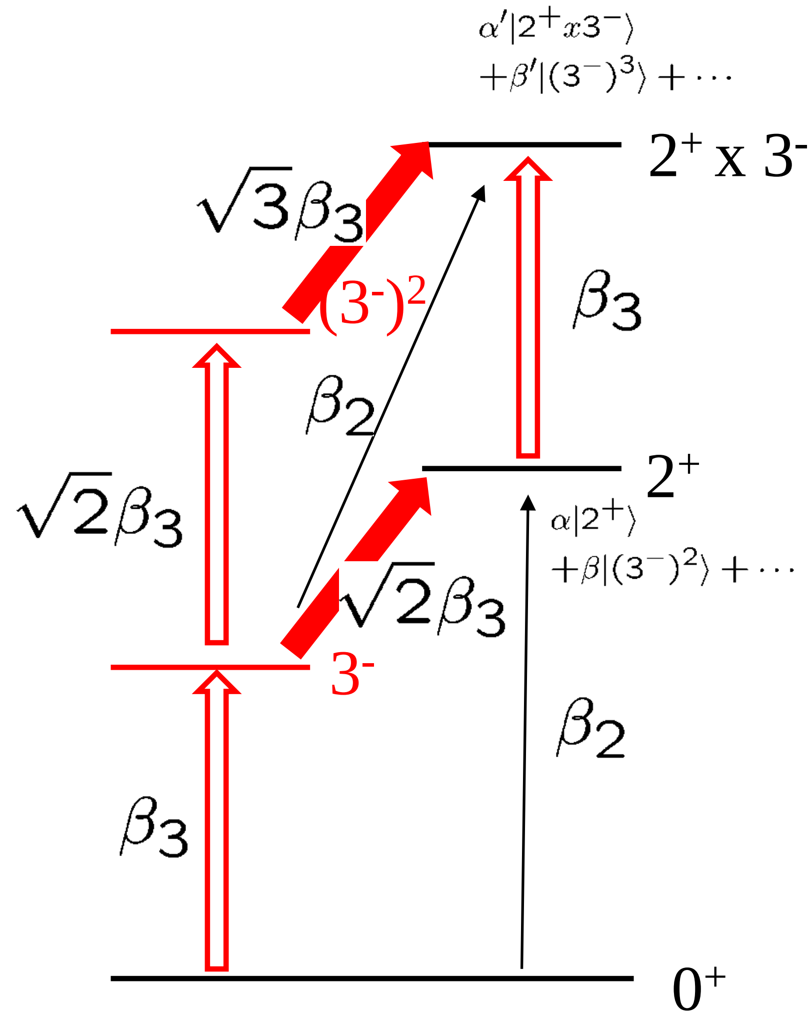
2_1^+ state: strong coupling both to g.s. and 3_1^-

$$\longrightarrow \quad |2_1^+\rangle = \alpha|2^+\rangle_{\rm HO} + \beta|[3^- \otimes 3^-]^{(I=2)}\rangle_{\rm HO} + \cdots$$



$\alpha'|2^+x3^-\rangle$
$+\beta'|(3^-)^3\rangle + \cdots$
2^+ x 3^-
2^+ x 3^-
$\sqrt{3}\beta_3$
$(3^-)^2$
$(3^-)^2$
β_3
β_3
β_2
β_2
2^+
2^+
$\sqrt{2}\beta_3$
$\sqrt{2}\beta_3$
$\sqrt{2}\beta_3$
$\alpha|2^+\rangle$
$+\beta|(3^-)^2\rangle + \cdots$
3^-
3^-
β_2
β_2
β_3
β_3
0^+
0^+
Harmonic Oscillator
Anharmonicity





J.M. Yao and K.H., submitted (2016)

Summary

Heavy-ion subbarrier fusion reactions

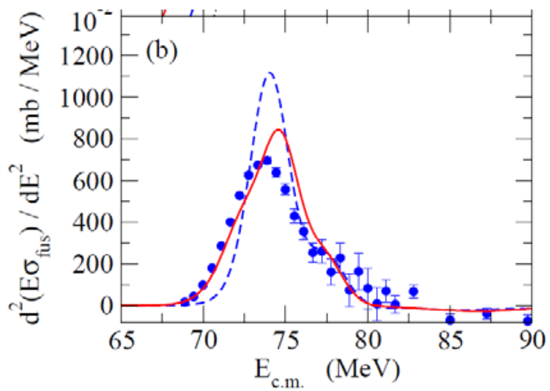
✓strong interplay between reaction and structure
cf. fusion barrier distributions

➢C.C. calculations with MR-DFT method

✓ anharmonicity
✓ truncation of phonon states
✓ octupole vibrations: ^{16}O + ^{208}Pb

more flexibility:

- application to transitional nuclei
- a good guidance to a Q-moment of excited states



➢Quasi-elastic barrier distribution

- an alternative to fusion barrier distribution
- more suitable to RI beams