Towards a microscopic theory for low-energy heavy-ion reactions

Role of internal degrees of freedom in low-energy nuclear reactions

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1. Introduction: Environmental Degrees of Freedom
2. Application of RMT to subbarrier fusion
3. Discussions: Towards a microscopic theory for low-energy heavy-ion reactions
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Introduction

Fusion: compound nucleus formation

Recent review:
K. Hagino and N. Takigawa,

courtesy: Felipe Canto
the simplest approach to fusion cross sections: potential model

\[ \sigma_{\text{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) P_l(E) \]
Subbarrier fusion reactions

Potential model: Reproduces the data reasonably well for $E > V_b$

Underpredicts $\sigma_{\text{fus}}$ for $E < V_b$

cf. seminal work:
R.G. Stokstad et al., PRL41(‘78)465
PRC21(‘80)2427
Strong target dependence at $E < V_b$

low-lying collective excitations
Subbarrier fusion: strong interplay between reaction and structure

\[ \theta \]

\[ ^{154}\text{Sm} \quad ^{16}\text{O} \]

Coupled-channels equations

\[ \sigma_{\text{fus}}(E) = \int_{0}^{1} d(\cos \theta)\sigma_{\text{fus}}(E; \theta) \]

Def. Effect: enhances \( \sigma_{\text{fus}} \) by a factor of 10 ~ 100

Fusion: interesting probe for nuclear structure
Coupled-Channels method

Coupling between rel. and intrinsic motions

\[ H = -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + H_0(\xi) + V_{\text{coup}}(r, \xi) \]

Entrance channel

Excited channel

\[ H_0(\xi) \phi_k(\xi) = \epsilon_k \phi_k(\xi) \]

\[ \psi(r, \xi) = \sum_k \psi_k(r) \phi_k(\xi) \]

coupled Schrödinger equations for \( \psi_k(r) \)
Coupled-channels framework

Coupling between rel. and intrinsic motions

- Quantum theory which incorporates excitations in the colliding nuclei
- A few collective states (vibration and rotation) which couple strongly to the ground state + transfer channel
- Several codes in the market: ECIS, FRESCO, CCFULL……

Has been successful in describing heavy-ion reactions

However, many recent challenges in C.C. calculations
surface diffuseness anomaly

**Scattering processes:**

Double folding potential
Woods-Saxon ($a \sim 0.63$ fm) 

successful

\[ \frac{d\sigma_{el}}{d\sigma_{Ruth}} \]

A. Mukherjee, D.J. Hinde, M. Dasgupta, K.H., et al., PRC75(’07)044608

**Fusion process:** not successful

\[ a \sim 1.0 \text{ fm required (if WS)} \]
Deep subbarrier fusion data

C.L. Jiang et al., PRL93(’04)012701
“steep fall-off of fusion cross section”

M.Dasgupta et al., PRL99(’07)192701

- dynamical effects not included in C.C. calculation?
- energy and angular momentum dissipation?
- weak channels?  this talk
typical excitation spectrum: electron scattering data

low-lying collective excitations

GDR/GQR

low-lying non-collective excitations

- Giant Resonances: high $E_x$, smooth mass number dependence
  $\rightarrow$ adiabatic potential renormalization
- Low-lying collective excitations: barrier distributions, strong isotope dependence
- Non-collective excitations: either neglected completely or implicitly treated through an absorptive potential

$E_{\text{GDR}} \sim 79A^{-1/3}$ MeV
$E_{\text{GQR}} \sim 65A^{-1/3}$ MeV

M. Sasao and Y. Torizuka,
PRC15(‘77)217
IS Octupole response of $^{48}$Ca (Skyrme HF + RPA calculation: SLy4)

Collective state: $|\text{coll}\rangle \sim \sum_{ph} X_{ph} a_p^\dagger a_h |0\rangle$

Strong coupling

Single-particle (non-collective) state

Weak, but many $|s.p.\rangle \sim a_p^\dagger a_h |0\rangle$
Our interest: couplings to (relatively) low-lying single-particle levels e.g., collective levels in $^{116}\text{Sn}$

![Diagram showing energy levels in $^{116}\text{Sn}$](image)

model space in a typical C.C. calculation
Our interest: couplings to (relatively) low-lying single-particle levels

112 levels up to 4.1 MeV (93 single-particle levels) nearly “complete” level scheme

S. Raman et al., PRC43(‘91)521

role of these s.p. levels in reaction dynamics?
Indications of non-collective excitations: a comparison between $^{20}\text{Ne}+^{90}\text{Zr}$ and $^{20}\text{Ne}+^{92}\text{Zr}$

\[
D_{\text{qel}}(E) = -\frac{d}{dE} \left( \frac{\sigma_{\text{qel}}(E, \pi)}{\sigma_{R}(E, \pi)} \right)
\]

QEL = elastic + inelastic + transfer

- C.C. results are almost the same between the two systems
- Yet, quite different barrier distribution and Q-value distribution

E. Piasecki et al., PRC80(‘09)054613

non-collective excitations?
$^90\text{Zr}$ (Z=40 sub-shell closure, N=50 shell closure)

$^92\text{Zr} = ^90\text{Zr} + 2n$

A problem: the nature of non-collective states is poorly known (the energy, spin, parity only) i.e., no information on the coupling strengths
Random Matrix Model

Coupled-channels equations:

\[
\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \epsilon_k - E\right] \psi_k(r) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(r) = 0
\]

\[|\phi_k\rangle: \text{complicated single-particle states}\]

coupling matrix elements \( V_{kk'} = \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \) are random numbers generated from a Gaussian distribution:

\[
\overline{V_{ij}(r)} = 0,
\]

\[
\frac{V_{ij}(r)V_{kl}(r')} = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \frac{w_0}{\sqrt{\rho(\epsilon_i)\rho(\epsilon_j)}}
\]

\[
\times e^{-\frac{(\epsilon_i - \epsilon_j)^2}{2\Delta^2}} \cdot e^{-\frac{(r-r')^2}{2\sigma^2}} \cdot h(r)h(r')
\]

D. Agassi, C.M. Ko, and H.A. Weidenmüller, Ann. of Phys. 107('77)140
RMT model for H.I. reactions:

✓ originally developed by Weidenmuller et al. to analyze DIC

✓ similar models have been applied to discuss quantum dissipation
  • M. Wilkinson, PRA41(‘90)4645
  • A. Bulgac, G.D. Dang, and D. Kusnezov, PRE54(‘96)3468
  • S. Mizutori and S. Aberg, PRE56(‘97)6311

D. Agassi, H.A. Weidenmuller, and C.M. Ko, PL 73B(‘78)284
Application to $^{20}$Ne + $^{90,92}$Zr reactions

# of levels

$N(\epsilon)$

$\rho(\epsilon) = \frac{dN(\epsilon)}{d\epsilon}$

$\rho(\epsilon)$ (MeV$^{-1}$)

$\epsilon$ (MeV)
Quasi-elastic cross sections

$E_{\text{eff}} = 2E \frac{\sin(\theta_{\text{c.m.}}/2)}{1 + \sin(\theta_{\text{c.m.}}/2)}$

Q-value distributions

Discussions: towards a microscopic reaction theory

These states are excited during nuclear reactions in a complicated way.

\[ \rho(E) \sim e^{2\sqrt{aE^*}} \]

nuclear intrinsic d.o.f. act as environment for nuclear reaction processes

“intrinsic environment”
How much do we know about “friction”?  

Fusion model → friction free: strong absorption inside the barrier

The topic of energy dissipation in fusion should be re-visited
  - re-analyses of DIC data: maybe helpful
  - Consistent theoretical model (dissipative quantum tunneling)
Non-collective excitations in isolated nuclei

$^{20}_{\text{Ne}} + ^{92}_{\text{Zr}}$

- Random matrix model?

after touching:

- Molecular excitations
- Deep subbarrier fusion

Unified quantum theory for fusion (subbarrier, deep subbarrier) & DIC?

Single-particle (non-collective) excitations in H.I. reactions quantum mechanical model for Wall-Window friction?
(Big) open question:

- Construction of a microscopic nuclear reaction model applicable at low energies?
  - many-particle tunneling

cf. nuclear structure calculations

- 2-body $nn$ interaction $\rightarrow$ mean-field $\rightarrow$ RPA
  - residual interaction $\rightarrow$ TDHF

  advantage: non-empirical
  disadvantage: difficult to control a mean-field

- mean-field pot. $\rightarrow$ residual interaction $\rightarrow$ RPA
  - TDHF

  guiding principle $\leftarrow$ complementary $\rightarrow$ deep understanding of results
many reaction theories correspond to this type

- 2-body nn interaction $\rightarrow$ mean-field $\rightarrow$ RPA
  residual interaction $\rightarrow$ TDHF

- mean-field pot. $\rightarrow$ residual interaction $\rightarrow$ RPA
  $\leftarrow$ TDHF
Microscopic nuclear reaction theories

TDHF, QMD, AMD  not applicable to low-energy fusion (classical nature)

Cluster approach (RGM)

only for light systems

H.O. wave function (separation of cm motion)

Double Folding approach

surface region: OK, but inside?
role of antisymmetrization?
validity of frozen density approximation?

Full microscopic theory:  ATDHF, GCM, ASCC ?
imaginary-time TDHF?

how to understand quantum tunneling from many-particle point of view?
microscopic nuclear reaction theory

Few-body approach
reduce to a few-body Hamiltonian and solve it as accurately as possible

- time-dependent wave packet approach
  
  M. Ito, K. Yabana, T. Nakatsukasa, and M. Ueda,
  PLB637(‘06)53

- (4-body) CDCC

more particles?
  nuclear transfer channel (CDCC)?
Another issue

Is reaction fast or slow?

Many-body (N-particle system) Hamiltonian

\[ H = \sum_i t_i + \sum_{i<j} v_{ij} \]

Large Amplitude Collective Motion

\[ H = H_{rel} + H_{s.p.} + H_{coup} \]

✧ Sudden approach (fast collision)
  
  - Double Folding Model
  - Optical Model
  - Coupled-channels model
  - Resonating Group Method (RGM)

✧ Adiabatic approach (slow collision)
  
  - Liquid-drop model (+ shell correction)
  - Adiabatic TDHF

\[ \text{Coordinate dependent mass } \mu(r) \]
cannot discriminate one of them at present

Sudden approach (frozen density)

Adiabatic approach

need further studies from several perspectives
construction of dynamical model without any assumption on adiabaticity

S. Misic and H. Esbensen, PRL96(’06)112701

T. Ichikawa, K.H., A. Iwamoto, PRC75(‘07)057603
Summary

Heavy-ion subbarrier fusion reactions

✓ strong interplay between reaction and structure
✓ quantum tunneling with several kinds of environment

Open questions

✓ how do we understand many-particle tunneling?
  - related topics: fission, alpha decays, two-proton radioactivities
  Large amplitude collective motions
✓ role of noncollective excitations?
  - dissipation, friction
✓ microscopic understanding of subbarrier fusion?
✓ unified theory of fusion and DIC?