Perspectives on nuclear reaction theory and superheavy elements

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1. Nuclear Reactions: overview
2. Heavy-ion fusion reactions
3. Fusion for superheavy elements
4. Summary
Introduction: low-energy nuclear physics

- behaviors of atomic nuclei as a quantum many-body systems
  - understanding based on strong interaction

- static properties: nuclear structure
  - ground state properties
    - (mass, size, shape,....)
  - excitations
  - nuclear matter

- dynamics: nuclear reactions

an interplay between these
Quantum Many-body Dynamics (nuclear reactions)

elastic scattering  inel. scattering  fusion
Quantum Many-body Dynamics (nuclear reactions)

- elastic scattering
- inel. scattering
- fusion
1. Coulomb interaction
   long range
   repulsion

2. Nuclear interaction
   short range
   attraction

the barrier height → defines the energy scale of a system

Fusion reactions at energies around the Coulomb barrier
Fusion reactions: compound nucleus formation

Niels Bohr (1936)

Neutron capture of nuclei → compound nucleus

N. Bohr,
Nature 137 (‘36) 351

cf. Experiment of Enrico Fermi (1935)
many very narrow (=long life-time) resonances (width ~ eV)
Fusion reactions: compound nucleus formation

Niels Bohr (1936)

Neutron capture of nuclei $\rightarrow$ **compound nucleus**

N. Bohr, Nature 137 (‘36) 351

forming a compound nucleus with heavy-ion reactions = H.I. fusion
Fusion reactions: compound nucleus formation

energy production in stars (Bethe ‘39)

nucleosynthesis

superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems with strong interaction

← microscopic understanding: an ultimate goal of nuclear physics

cf. Bohr ‘36
Low-energy heavy-ion fusion reactions and quantum tunneling

✓ Reaction dynamics
  strong interplay between reaction and structure
  cf. high $E$ reactions: much simpler reaction mechanisms

✓ Many-particle tunneling
  cf. rich intrinsic motions
    - several nuclear shapes
    - several surface vibrations

several modes
and adiabaticities
Low-energy heavy-ion fusion reactions and quantum tunneling

✓ Reaction dynamics
  strong interplay between reaction and structure
  cf. high $E$ reactions: much simpler reaction mechanisms

✓ Many-particle tunneling
  cf. rich intrinsic motions
  - several nuclear shapes
  - several surface vibrations
  - several types of nucleon transfers
  "environment" can be changed relatively freely
  $E$: variable  cf. $\alpha$ decays: fixed energy

H.I. fusion reaction = an ideal playground to study quantum tunneling with many degrees of freedom
Discovery of large sub-barrier enhancement of $\sigma_{\text{fus}}$ (~80’s)

potential model: inert nuclei (no structure)

$$\sigma_{\text{fus}} = \frac{\pi}{k^2} \sum_l (2l + 1) (1 - |S_l|^2)$$
Discovery of large sub-barrier enhancement of $\sigma_{\text{fus}}$ (~80’s)

$^{154}\text{Sm}$ : a typical deformed nucleus

$^{154}\text{Sm}$

Effects of nuclear deformation

$^{154}\text{Sm}$ : a typical deformed nucleus
Effects of nuclear deformation

\(^{154}\text{Sm} \): a typical deformed nucleus

\[ \sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta) \]

Fusion: strong interplay between nuclear structure and reaction
enhancement of fusion cross sections: a general phenomenon

strong correlation with nuclear spectrum → coupling assisted tunneling

potential model
Coupled-channels method: a quantal scattering theory with excitations

many-body problem

still very challenging
TDHF simulation

$\text{TDHF} = \text{Time Dependent Hartree-Fock}$

(a single Slater determinant)

S. Ebata, T. Nakatsukasa, JPC Conf. Proc. 6 ('15) 020056

$\text{ab-initio, but no tunneling}$

C. Simenel, EPJA48 ('12) 152
Coupled-channels method: a quantal scattering theory with excitations

many-body problem

still very challenging

two-body problem, but with excitations (coupled-channels approach)

coupling

scattering theory with excitations
Coupled-channels method: a quantal scattering theory with excitations

dynamics of excitations/de-excitation during reaction

- Non-perturbative (full order)
- Non-adiabatic (excitation energy)
**Coupled-channels method**: a quantal scattering theory with excitations dynamics of excitations/de-excitations during reaction

- Non-perturbative (full order)
- Non-adiabatic (excitation energy)

In the past, the linear coupling approximation in a Hamiltonian:

\[
H = -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \beta f(r) \hat{O}
\]

full order treatment

\[
V(r, \beta \hat{O})
\]

modellings of coupled-channels calculations

- K.H., N. Rowley, A.T. Kruppa, CPC123 (‘99) 143
- M. Zamrun, K.H., S. Mitsuoka, H. Ikezoe, PRC77 (‘08) 034604
- T. Ichikawa, K.H., A. Iwamoto, PRL103 (‘09) 202701
- S. Yusa, K.H., and N. Rowley, PRC88 (‘13) 044620
- J.M. Yao and K.H., PRC94 (‘16) 11303(R) etc.

Further development: semi-microscopic modelling

K.H. and J.M. Yao, PRC91('15) 064606

Coupled-channels + microscopic nuclear structure calculations
(GCM, Shell Model, IBM…..)

simple harmonic oscillator

$0^+, 2^+, 4^+ \quad 2\epsilon$

$2^+ \quad \sqrt{2}\beta \quad \epsilon$

$0^+ \quad \beta \quad 0$

anharmonicity in phonon spectra

quantum fluctuation

Mean-field

Transitional

$^{58}\text{Ni}$

relativistic MF + GCM
Relativistic Mean-Field + Quantum fluctuation + coupled-channels

\[ \sigma_{\text{fus}} \]

(a)

\[ \sigma_{\text{fus}} \] (mb)

\[ \log_{10} \sigma_{\text{fus}} \]

\[ ^{16}\text{O} + ^{208}\text{Pb} \]

- No Coupling
- 2 phonon (HO)
- "2 phonon" (GCM)

(b)

\[ \frac{d^2(E\sigma_{\text{fus}})}{dE^2} \] (mb/MeV)

\[ E_{\text{c.m.}} \] (MeV)

J.M. Yao and K.H.,
PRC94 (‘16) 11303(R)
From phenomenological approach to microscopic approach

Macroscopic (phenomenological)

- C.C. with collective model
- C.C. with inputs from microscopic nuclear structure calculations
- TDHF simulations

Microscopic

TDHF = Time Dependent Hartree-Fock

S. Ebata, T. Nakatsukasa, JPC Conf. Proc. 6 (‘15)

ab initio, but no tunneling
From phenomenological approach to microscopic approach

**TDHF simulations**

ab initio, but no tunneling

- “Beyond mean-field” approximations
- Time-dependent GCM?

a single Slater determinant (SD) to multi-SD

\[ |\Psi(t)\rangle = \int dq f(q,t) |\Phi_q(t)\rangle \]

dynamics with a superposition of many

“TDHF trajectories (Slater determinants)”

cf. Stochastic mean-field method
B. Yilmaz et al.,
PRC90 (‘14) 054617

an open problem
K.H., N. Hasegawa, and Y. Tanimura,
a work in progress
Future perspectives: fusion for superheavy elements

nuclei existed in nature around $Z=114$ $N=184$

a prediction of island of stability

(Yuri Oganessian)

(Swiatecki et al., 1966)
Fusion reactions for SHE

the element 113: Nh

November, 2016

Heavy-ion fusion reaction
Future directions of SHE

Superheavy elements synthesized so far

Towards Z=119 and 120 isotopes

Towards the island of stability?

Towards Z=119 and 120 isotopes

Hot fusion reactions with $^{48}\text{Ca}$, $^{50}_{22}\text{Ti}$, $^{51}_{23}\text{V}$, $^{54}_{24}\text{Cr}$ etc.

Towards the island of stability

neutron-rich beams: indispensable → reaction dynamics?
Fusion reactions in the SHE region \((Z_P^*Z_T^* > 1600\sim1800)\)

\[
Z_1^*Z_2^* = 2000
\]

\[
Z_1^*Z_2^* = 1296
\]

C.-C. Sahm et al., Z. Phys. A319(‘84)113

Fusion hindrance

superheavy nuclei

\(\text{Nh}^{113}\)
\(\text{Mc}^{115}\)
\(\text{Ts}^{117}\)
\(\text{Og}^{118}\)

syntheses with heavy-ion fusion reactions

Theoretical issues: understanding the reaction dynamics
Fusion reactions in the SHE region ($Z_P^*Z_T > 1600\sim1800$)

fusion hindrance

C.-C. Sahm et al., Z. Phys. A319(‘84)113

modern interpretation of hindrance

strong Coulomb repulsion → re-separation before the compound nucleus

Fusion reactions in the SHE region ($Z_P^*Z_T > 1600\sim1800$)

fusion hindrance

C.-C. Sahm et al., Z. Phys. A319(‘84)113

strong Coulomb repulsion → re-separation before the compound nucleus
Fusion reactions in the SHE region \((Z_p^*Z_T > 1600\sim1800)\)

**SHE formation: a very rare event**

→ large theoretical uncertainties

- No data for \(P_{CN}\)
- Data: only for \(P_{ER}\)

\(CN=\text{compound nucleus}\)

\(ER = \text{evaporation residues}\)

**Theoretical challenge:**

to reduce theoretical uncertainties and make a reliable prediction

**Modern interpretation of hindrance**

- Quasi-fission
- Compound nucleus
- Evaporation residues

**Strong Coulomb repulsion**

→ re-separation before the compound nucleus
Nuclear friction and heavy-ion fusion reactions

$E^*$

\[ \rho(E) \sim e^{2\sqrt{aE^*}} \]

These states:
are excited in a complicated way.

nuclear intrinsic d.o.f.:
act as environment

"intrinsic environment"

\[ \rightarrow \text{friction} \]
Langevin approach

Multi-dimensional space
- internuclear separation
- deformation
- mass asymmetry of the two fragments

thermal diffusion

\[ m \frac{d^2 q}{dt^2} = -\frac{dV(q)}{dq} - \gamma \frac{dq}{dt} + R(t) \]
Analysis with an extended fusion-by-diffusion approach
New hybrid model: TDHF + Langevin approach

K. Sekizawa and K.H., PRC99 (2019) 051602(R)

TDHF+Langevin: a new hybrid model of fusion reactions for superheavy elements

1st stage: TDHF

2nd stage: Langevin model

3rd stage: statistical model

<table>
<thead>
<tr>
<th>System</th>
<th>CN</th>
<th>$R_{\text{min}}$ (fm)</th>
<th>$P_{\text{fus}}$ ($\times 10^{13}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}\text{Ca} + ^{254}\text{Fm}$</td>
<td>302</td>
<td>12.93</td>
<td>302</td>
</tr>
<tr>
<td>$^{54}\text{Cr} + ^{248}\text{Cm}$</td>
<td>302</td>
<td>13.09</td>
<td>2.47</td>
</tr>
<tr>
<td>$^{51}\text{V} + ^{249}\text{Bk}$</td>
<td>300</td>
<td>12.94</td>
<td>0.461</td>
</tr>
<tr>
<td>$^{48}\text{Ca} + ^{257}\text{Fm}$</td>
<td>305</td>
<td>12.94</td>
<td>1.82</td>
</tr>
</tbody>
</table>

a special role of $^{48}\text{Ca}$?
issues from the theoretical physics point of view

✓ how to thermalize?
Quantum friction theory
c.f. tunneling with quantum friction

M. Tokieda and K.H., PRC95 (‘17) 054604

✓ non-Markov effect?
✓ quantum correction for diffusion over the barrier?

fusion → re-separation

V(ε) (MeV)

thermal diffusion

Langevin approach

ε (deformation)
Quantum friction

classical eq. of motion \[ \dot{p} = -V'(x) - \gamma p \]
a quantization: Kanai model  
E. Kanai, PTP 3 (1948) 440

\[
H = \frac{p^2}{2m} + V(x) \rightarrow \frac{\pi^2}{2m} e^{-\gamma t} + e^{\gamma t} V(x) \quad (\pi = e^{\gamma t} p)
\]

\[
\frac{d}{dt} \langle p \rangle = -\langle V'(x) \rangle - \gamma \langle p \rangle
\]

time-dep. wave packet approach

M. Tokieda and K.H., PRC95 ('17) 054604
Fusion reactions and non-equilibrium statistical mechanics: Langevin dynamics under a temperature gradient

- Superheavy elements
- a math model for molecular motors

\[ E_{\text{int}} = E^* - E_{\text{kin}} - V(\epsilon) = aT^2 \]

\( \rightarrow \) coordinate dependent temperature

\( \rightarrow \) one-way dynamics

SHE formation reactions as a general problem of non-eq. stat. mechanics?
Fusion of unstable nuclei

neutron-rich beams: indispensable → reaction dynamics?


K.H. and H. Sagawa, PRC72('05)044321

need further improvements

good understandings of the structure of neutron-rich nuclei is also important
reactions of neutron-rich nuclei

- fusion
- transfer

- development of microscopic nuclear reaction theory
- nuclear reactions in neutron stars

Accreting neutron

fusion of neutron-rich nuclei

$^{24}\text{O} + ^{24}\text{O}, ^{28}\text{Ne} + ^{28}\text{Ne}$ etc.
Physics of SHE with n-rich nuclei as important ingredient

- reactions of neutron-rich nuclei
  - fusion
  - transfer

- development of microscopic nuclear reaction theory
- nuclear reactions in neutron stars

- structure of neutron-rich nuclei
  - nucleon correlations
  - collective motions
  - fission

From few-body to many-body
Summary

SHE: quantum many-body systems with a strong Coulomb field

**Physics**
- Reaction dynamics
- Quantum friction
- Neutron-rich nuclei

**Chemistry**
- Origin of elements
- R-process
- Kilonova

**Astronomy**
- Interdisciplinary SHE science

Interdisciplinary SHE science

Periodic Table of Elements

- Nh: Nihonium
- Mc: Moscovium
- Ts: Tennessine
- Og: Oganesson

International Year of the Periodic Table of Chemical Elements 2019 (IYPT)
\[ H = -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \beta f(r) \hat{O} \]

Full order treatment

\[ V(r, \beta \hat{O}) \]

\[ \hat{O} | \phi_k \rangle = \lambda_k | \phi_k \rangle \]

Diagonalize \[ \langle n | \hat{O} | m \rangle \]

\[ \langle n | V(r, \beta \hat{O}) | m \rangle = \sum_k \langle n | \phi_k \rangle \langle \phi_k | m \rangle V(r, \beta \lambda_k) \]

K.H., N. Rowley, and A.T. Kruppa,