Application of beyond-mean-field approach to hypernuclei

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堯江明 J.M. Yao (MSU)
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元場俊雄（大阪電気通信大学）

1. *Introduction: hypernuclei*
2. *Microscopic particle-core coupling schemes for hypernuclei*
3. *Spectrum of Λ hypernuclei*
4. *Summary*

Introduction of Tohoku University and Sendai

Sendai:

✓ the largest town in the Tohoku region

✓ population: about 1 million

city of trees

Introduction of Tohoku University and Sendai

TOHOKU UNIVERSITY

東北大学

Matsushima (one of the “3 most beautiful places” in Japan)

Sendai castle

nice sea-foods

Introduction of Tohoku University and Sendai

Tohoku University

TOHOKU
UNIVERSITY

東北大学

- Established in 1907 (110 years ago)
- the third oldest university in Japan
- the first university in Japan which accepted female students (in 1913)

First
Women
Students
in 1913
Tohoku
University

MIYAGI HISTORY

摄于1906年3月,左第一人为鲁迅,
即将离开仙台时与同班同学的合影。

历史和鲁迅

史迹,鲁迅生活过的地方

约400年前,作为伊达六十万石的城邑而发展起来,
与中国著名文学家鲁迅有深缘的仙台,
还有受伊达政宗藩主之命支仓常长一行
罗马旅行的出发地石卷。
向您介绍宫城县各地的历史风情。

回忆仙台时代生活的段落,
写于1926年,引自《朝花夕拾》。

鲁迅最初寄宿的
“佐藤屋”旧址,
现在的米袋一丁目。

· 鲁迅和东北大学

鲁迅(原名:周树人)1881年9月25日出生于清朝(现在的中华人民共和国)的长江下游浙江省绍兴县。1902年1月毕业南京的江南陆师学堂附属矿务铁路学堂之后，同年4月作为清朝留学生来我国留学,先就读于东京的弘文学院普通速成科。在此学院鲁迅学习了日语和基础科目。

应鲁迅的要求,1904年5月20日当时的清朝·杨公使向仙台医学专门学校(现在的东北大学医学部)提出了就鲁迅的入学要求进行妥善处理的照会信。

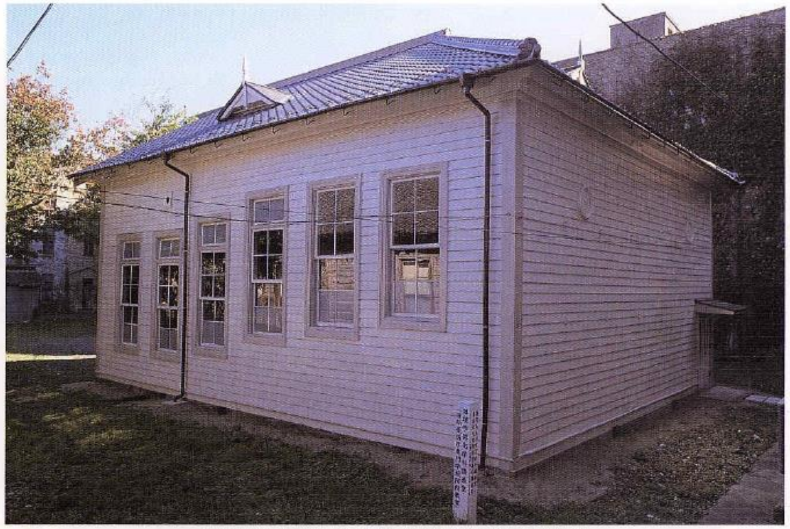
仙台医学专门学校对此以文部省有关入学规则为依据进行探讨之后,决定允许免试入学。并于5月23日给杨 公使寄送了入学许可通知书。

同年9月,鲁迅进入了仙台医学专门学校。

東北大学

藤野厳九郎教授
藤野严九郎教授

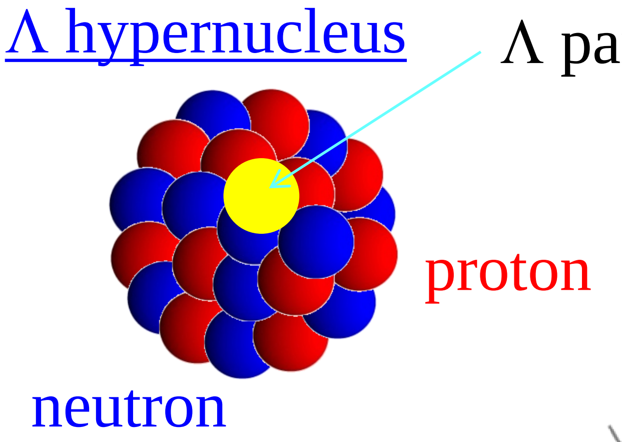
藤野教授



鲁迅が学んだ仙台医学専門学校階段教室外景
(鲁迅曾就读的仙台医学专门学校教学楼外景)

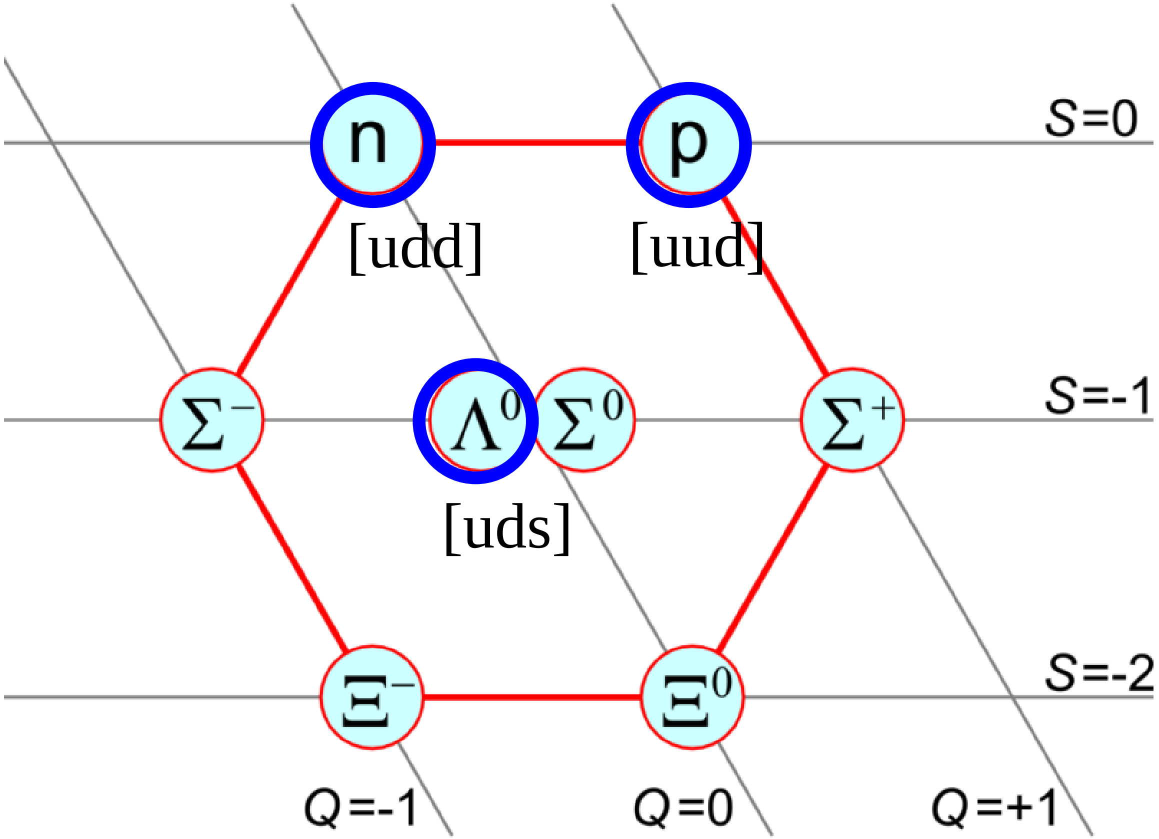
鲁迅阶梯教室

Introduction



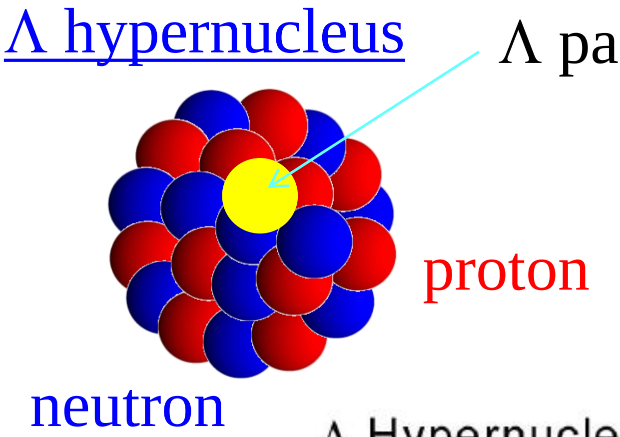
Λ particle: the lightest hyperon
(no charge, no isospin)

*no Pauli principle between nucleons and a Λ particle



(wikipedia)

Introduction



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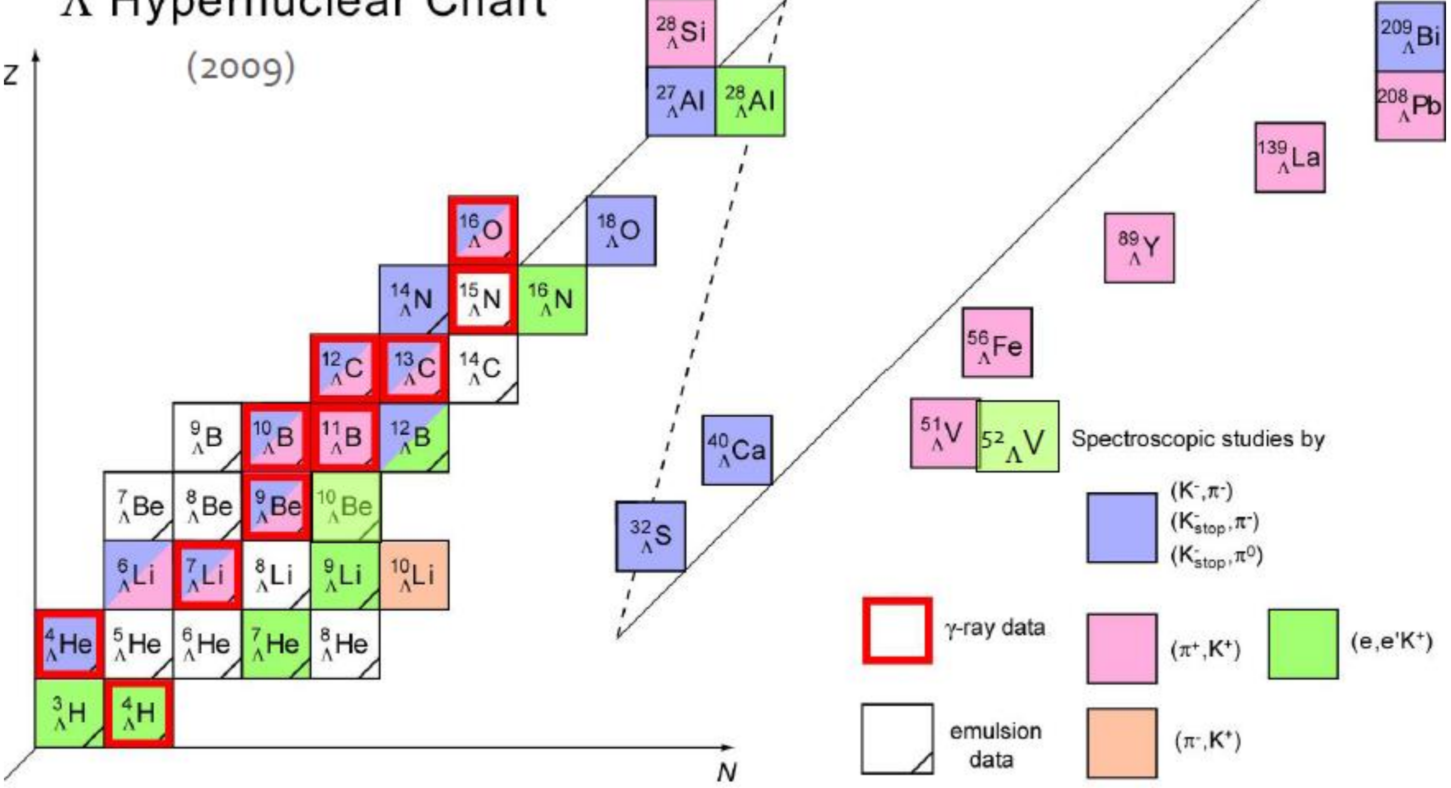
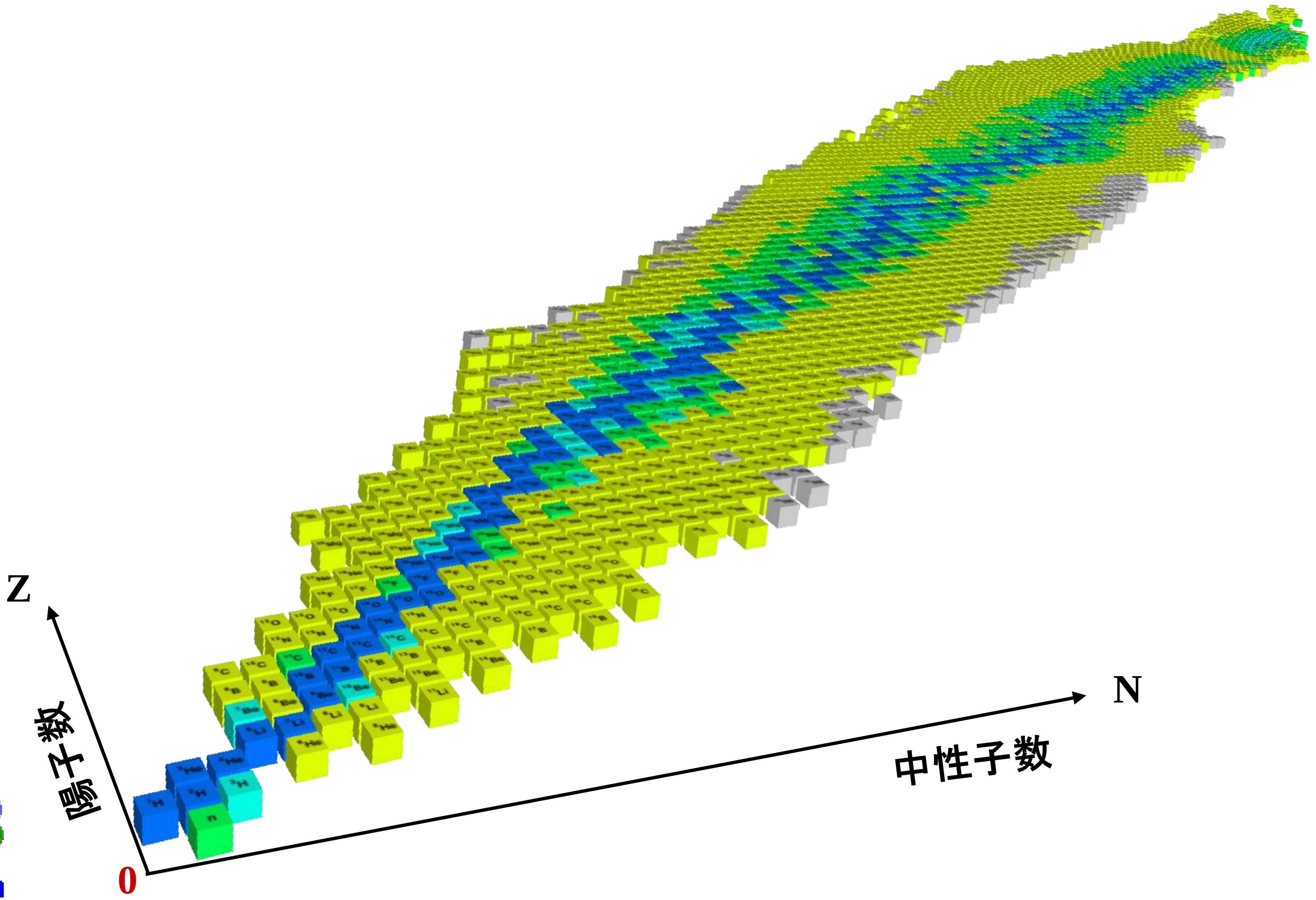
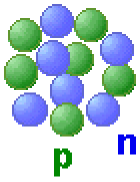
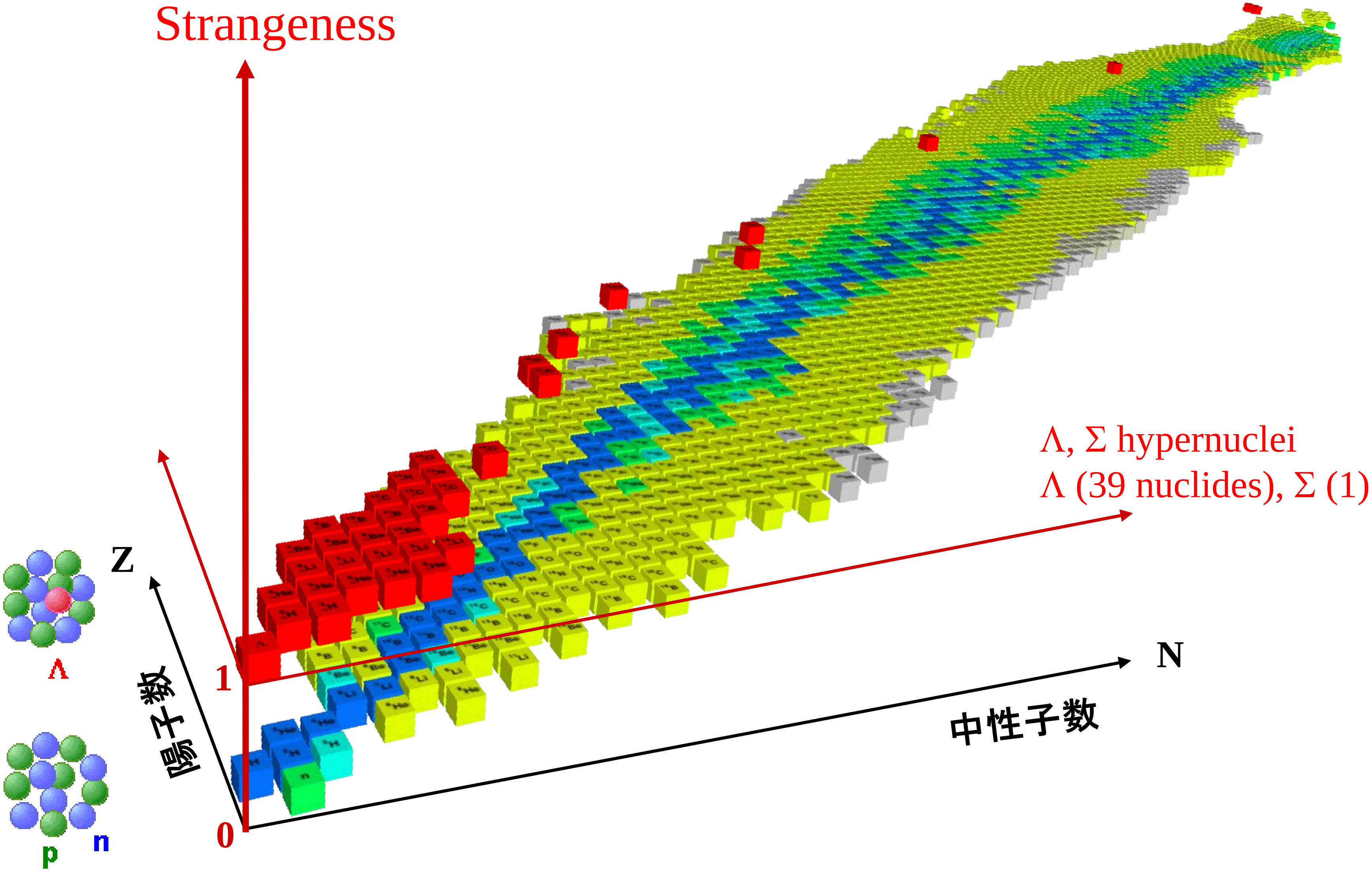


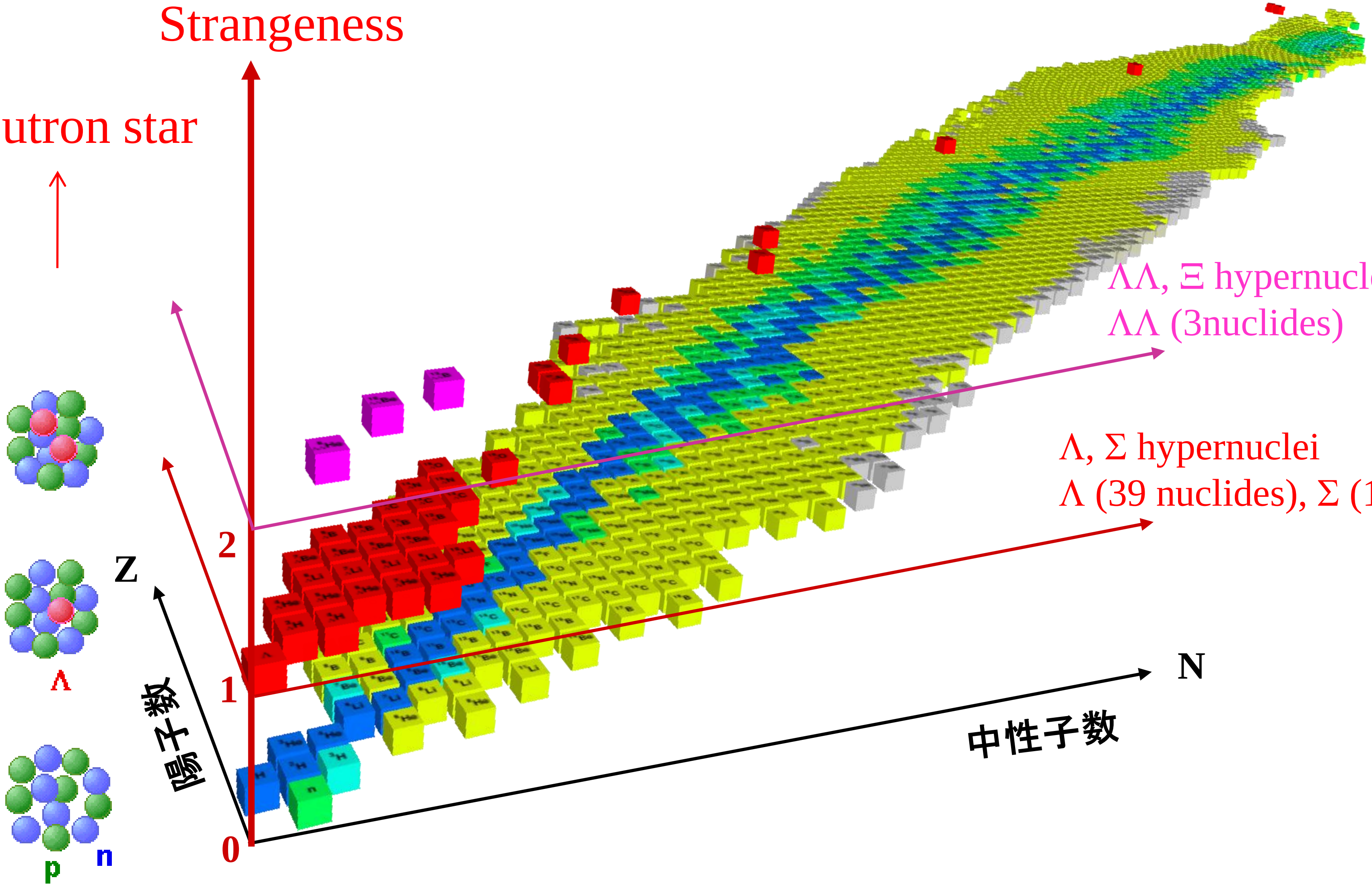
O. Hashimoto and H. Tamura, Prog. Part. Nucl. Phys. 57('06)564


Z
陽子数
N
中性子数
0


p
n



Λ, Σ hypernuclei
Λ (39 nuclides), Σ (1)



ΛΛ, Ξ hypernuclei
ΛΛ (3nuclides)

Λ, Σ hypernuclei
Λ (39 nuclides), Σ (1)

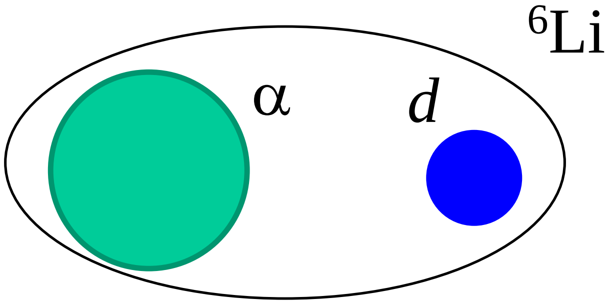
Introduction

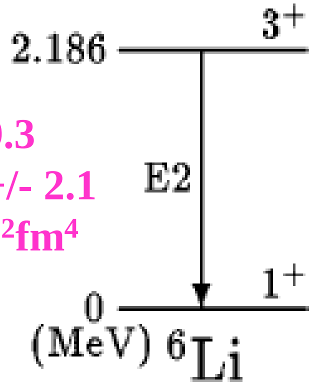
Impurity effects: one of the main interests of hypernuclear physics

how does a Λ particle affect several properties of atomic nuclei?

- size, shape, density distribution, single-particle energy, shell structure, fission barrier……

the most prominent example:
the reduction of B(E2) in $^{7}_{\Lambda}$Li





9.3 +/- 2.1 e^2fm^4

Introduction

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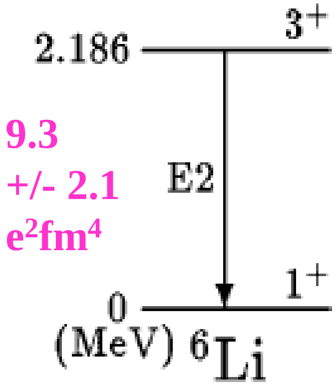
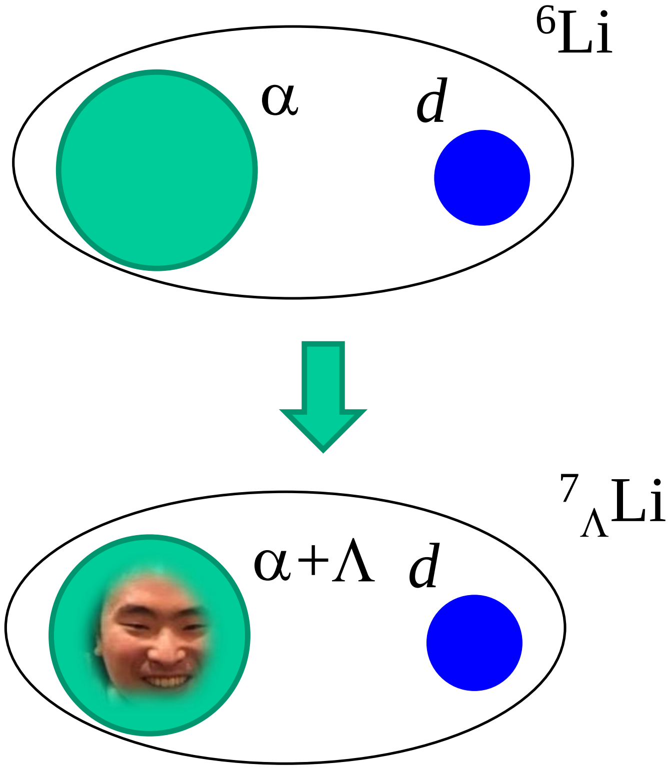


9.3
+/- 2.1
e^2fm^4



K. Tanida et al., PRL86 (‘01) 1982

Introduction

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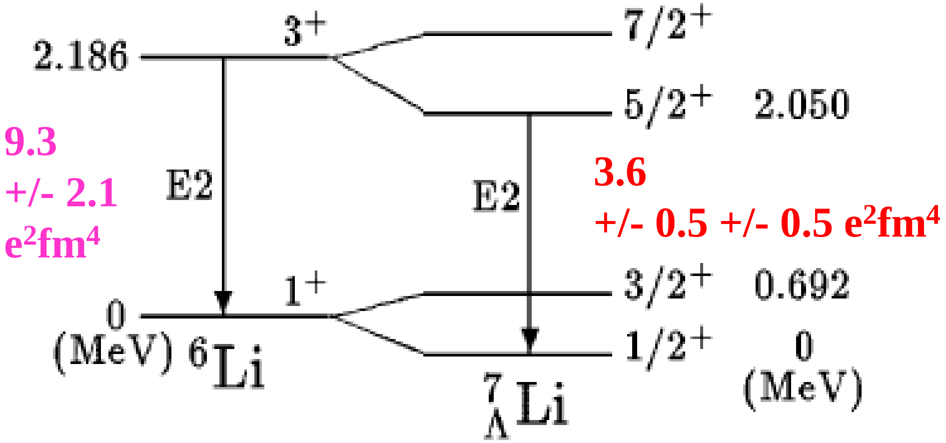
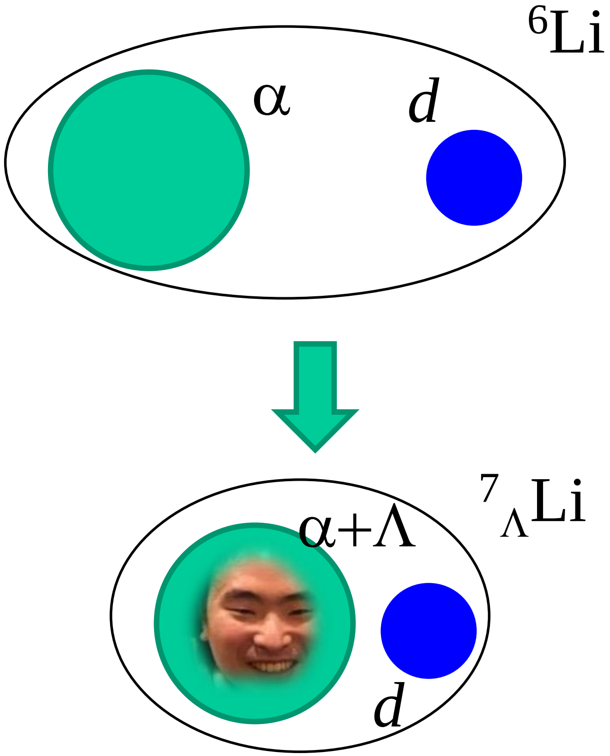


K. Tanida et al., PRL86 ('01) 1982



about 19% reduction of size
(shrinkage effect)

Introduction

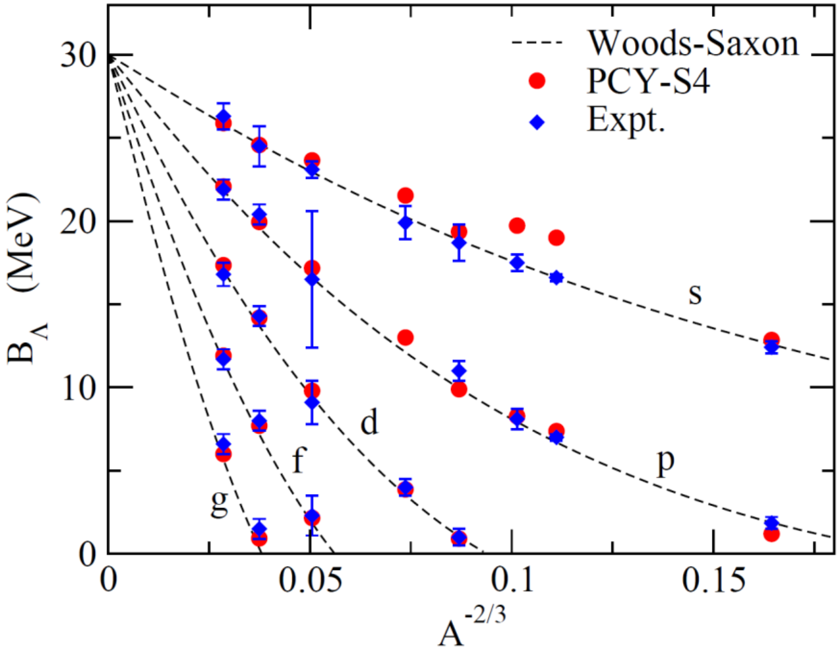
Impurity effects: one of the main interests of hypernuclear physics

how does a Λ particle affect several properties of atomic nuclei?

- ➢size, shape, density distribution, single-particle energy, shell structure, fission barrier……

Theoretical approaches:

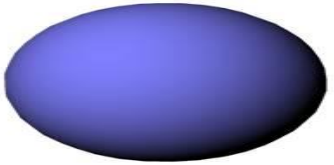
- ✓ cluster model
- ✓ shell model
- ✓ AMD
- ✓ self-consistent mean-field models



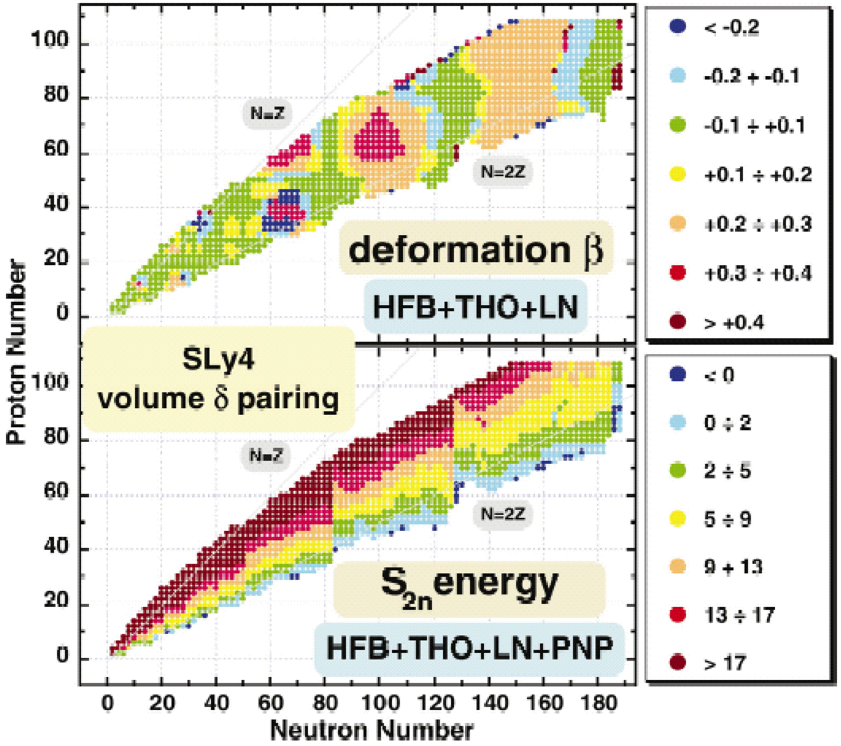
K.H. and J.M. Yao,
Int. Rev. Nucl. Phys. 10 (‘16) 263

Mean-field approximation and beyond

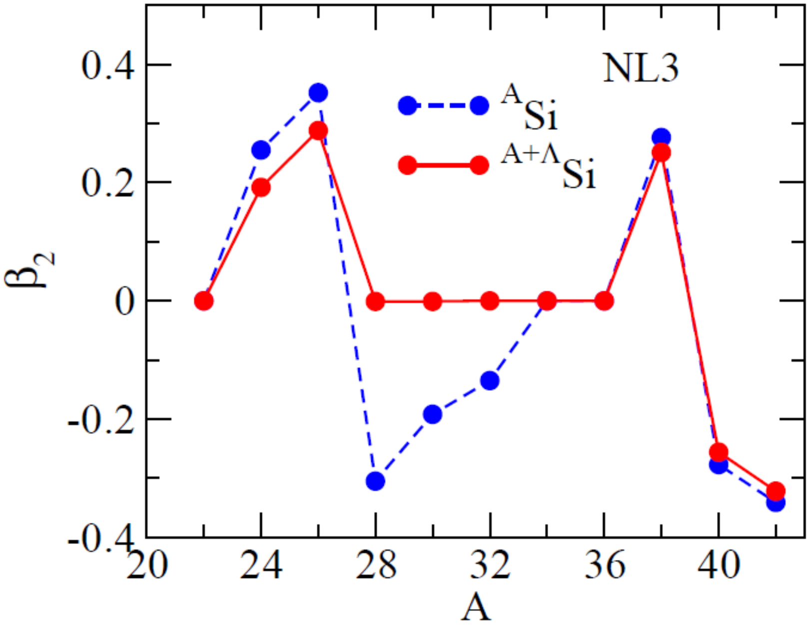
Self-consistent mean-field (Hartree-Fock) method:



- independent particles in a mean-field potential
- global theory for **the whole nuclear chart**
- body-fixed frame → intuitive picture for nuclear deformation
- optimized shape can be automatically determined



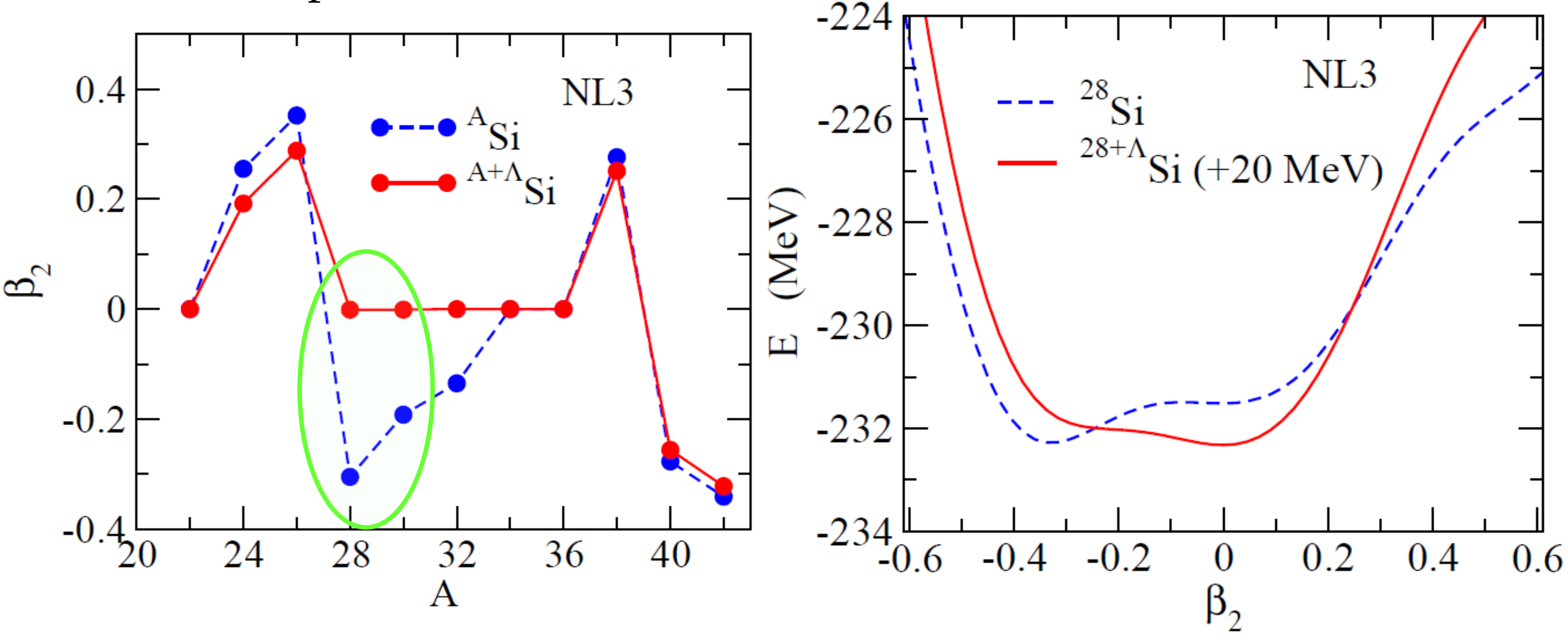
M.V. Stoitsov et al., PRC68('03)054312



Myaing Thi Win and K.H.,
PRC78('08)054311

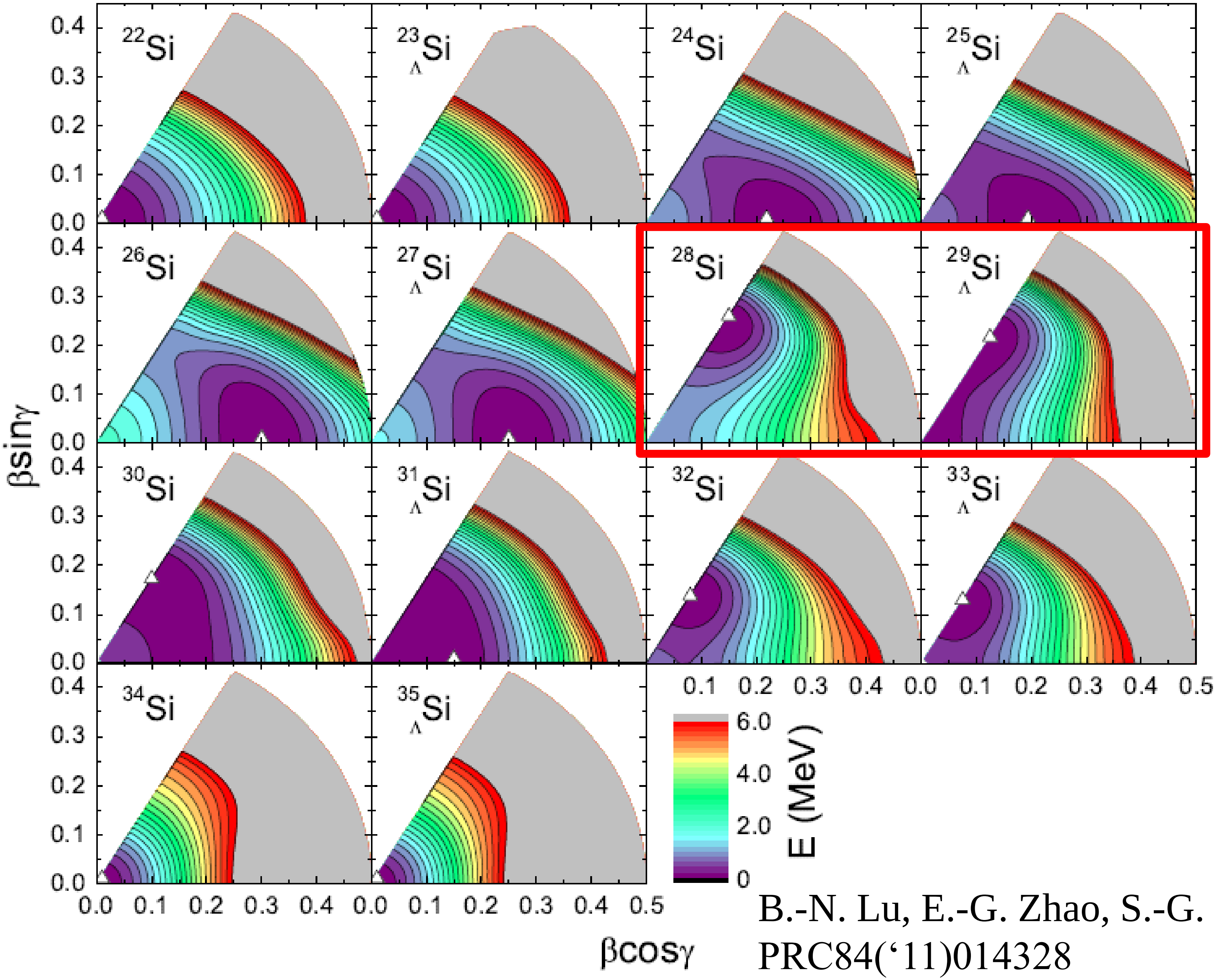
The previous Mean-field calculations for hypernuclei

Si isotopes



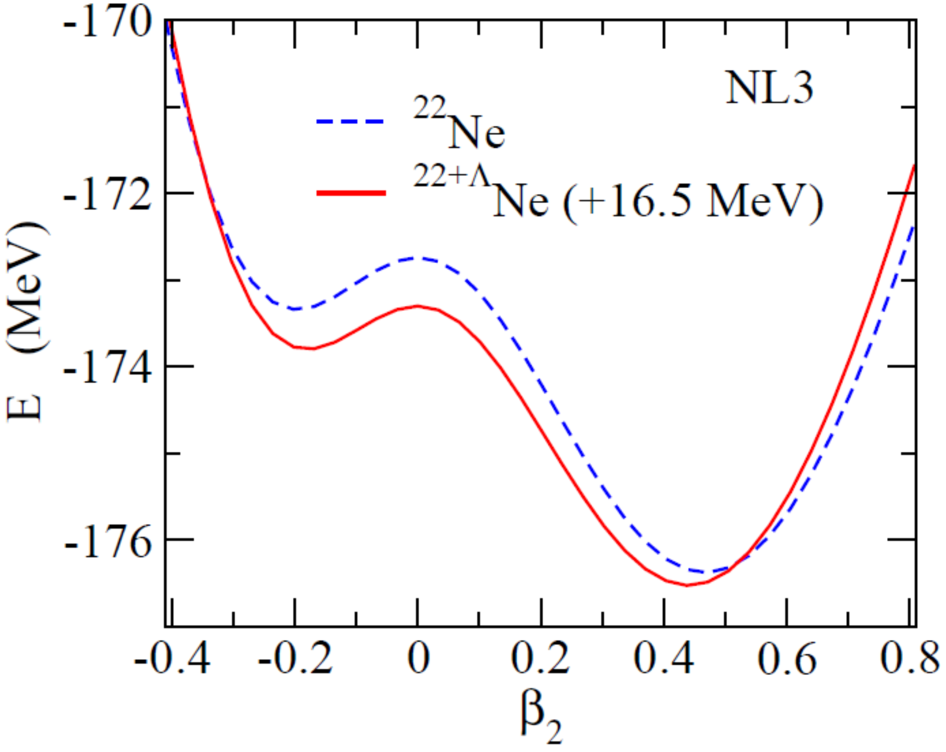
Myaing Thi Win and K.Hagino, PRC78(‘08)054311

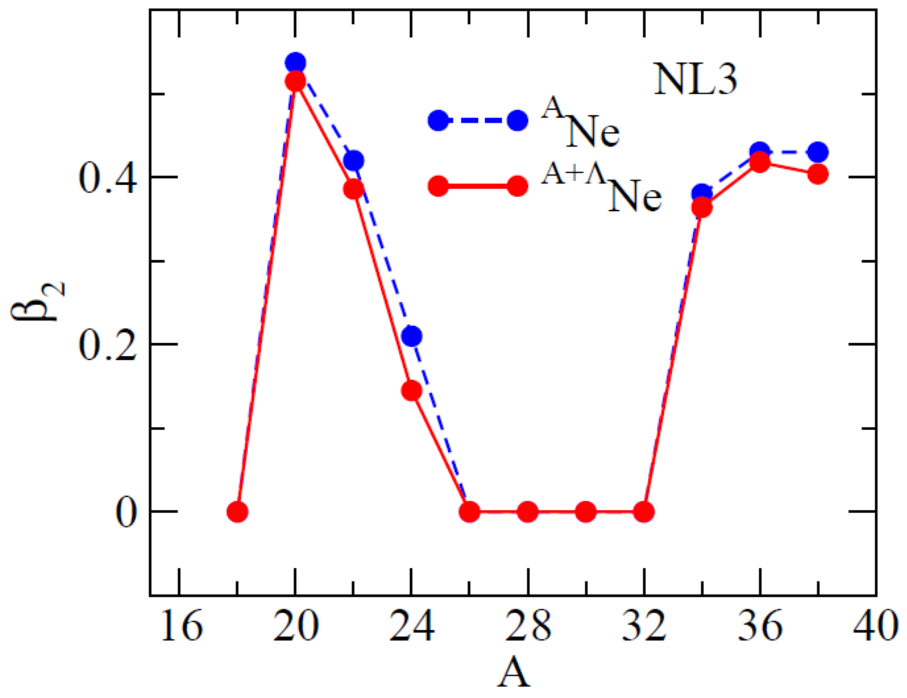
c.f. 3D RMF calculations



B.-N. Lu, E.-G. Zhao, S.-G. Zhou, PRC84('11)014328

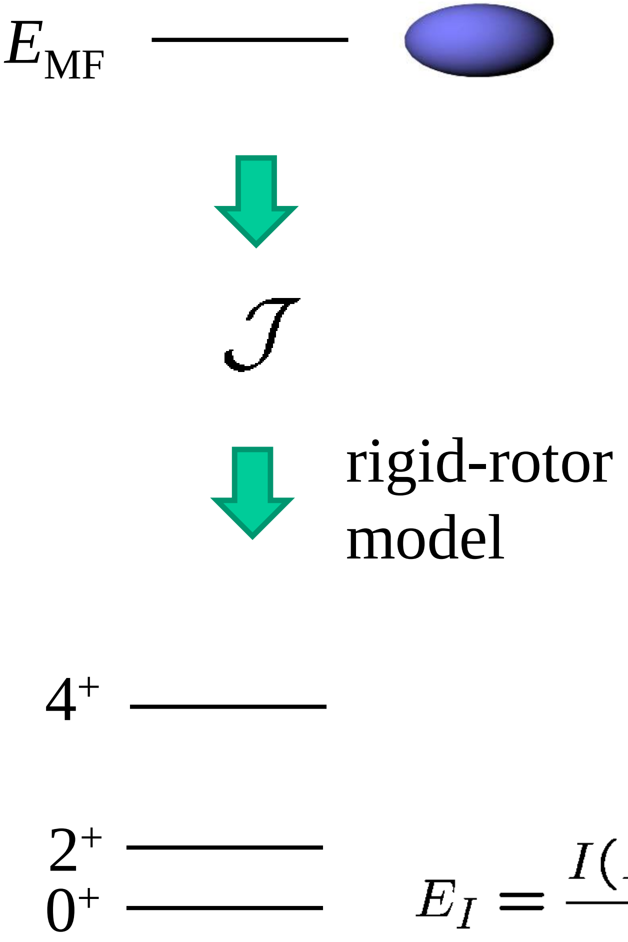
cf. Ne isotopes





Myaing Thi Win and K.Hagino, PRC78(‘08)054311

spectrum based on the mean-field approximation?



E_{MF}

$\mathcal{J}$

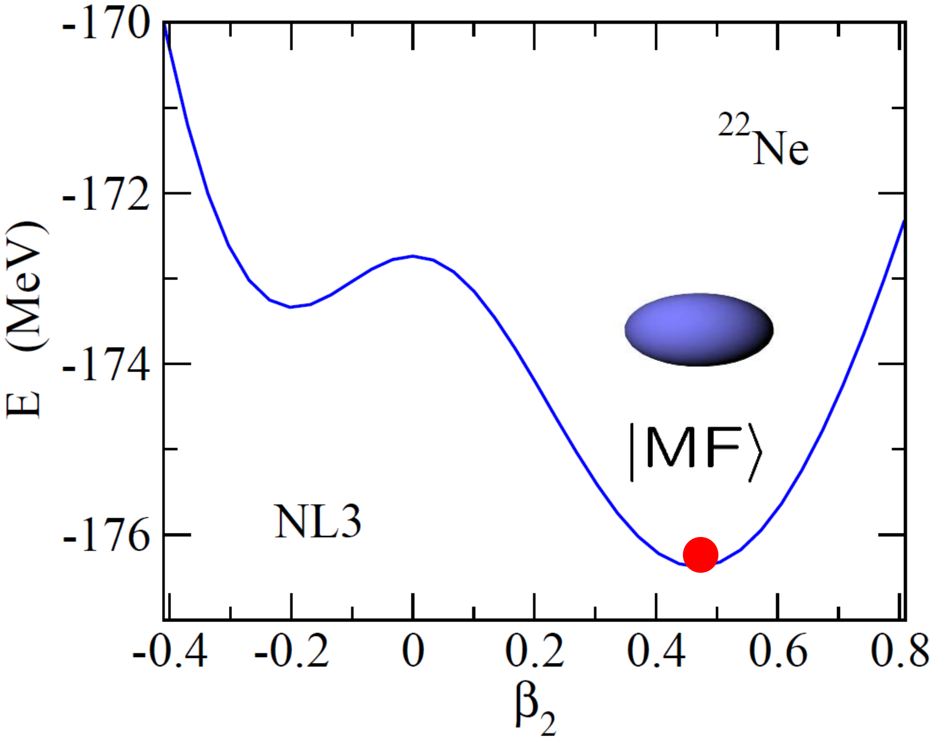
rigid-rotor model

4^+

2^+

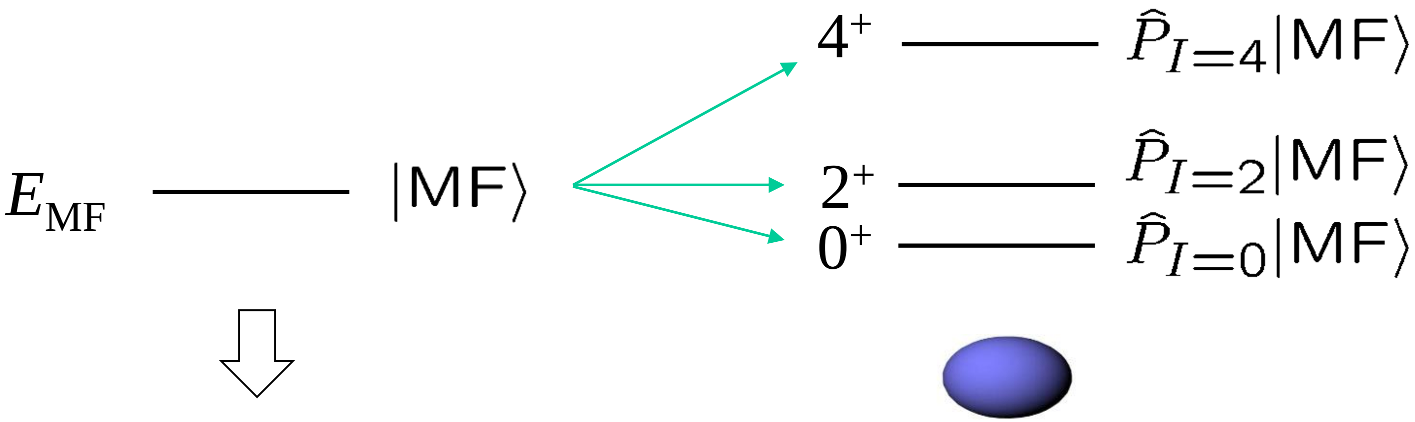
0^+

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$



spectrum based on the mean-field approximation?

angular mom.
projection



4^+ $\hat{P}_{I=4}|\mathrm{MF}\rangle$

E_{MF} $|\mathrm{MF}\rangle$

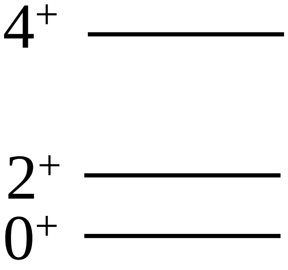
2^+ $\hat{P}_{I=2}|\mathrm{MF}\rangle$

0^+ $\hat{P}_{I=0}|\mathrm{MF}\rangle$

⇩

$\mathcal{J}$

⇩ rigid-rotor
model



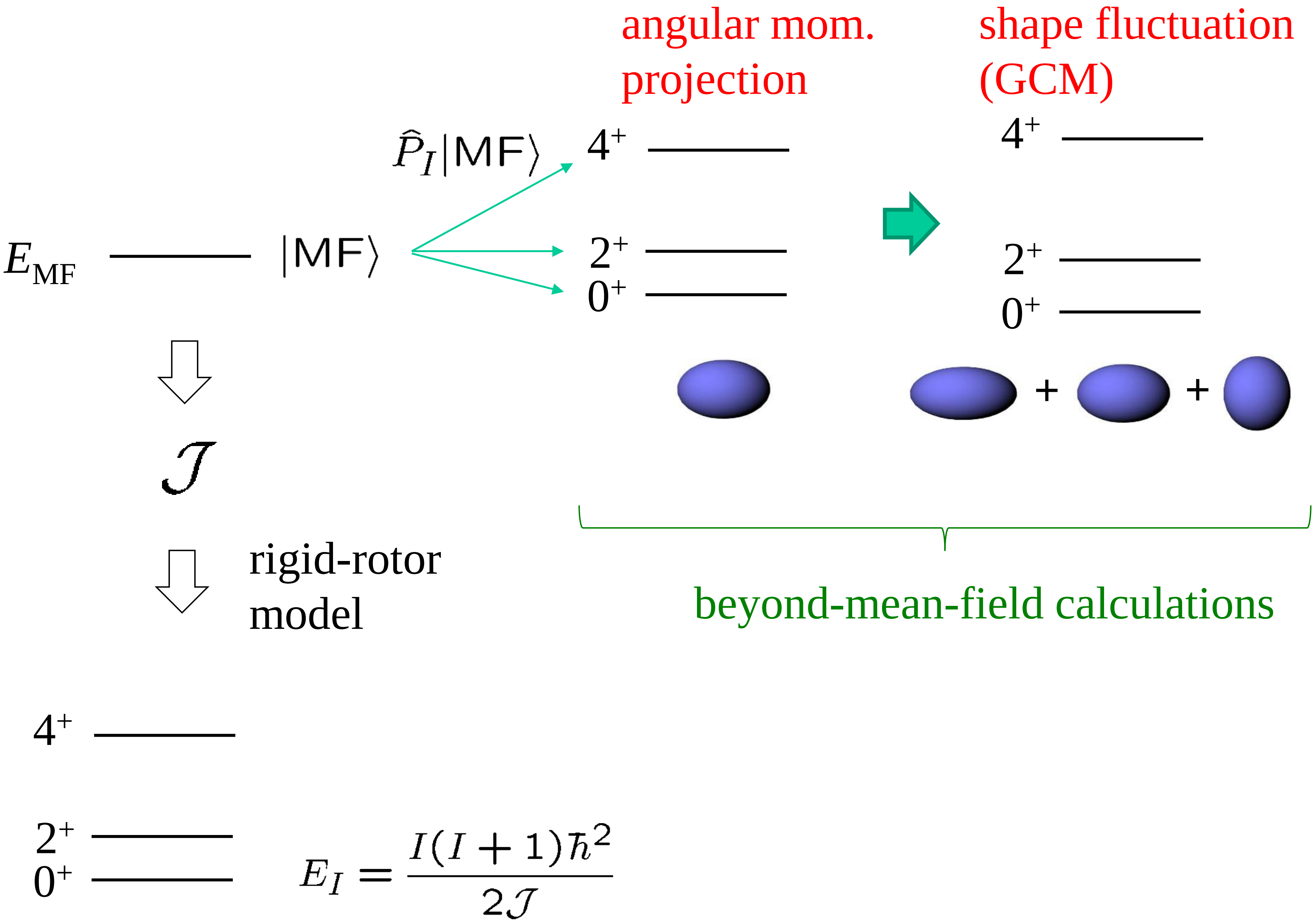
4^+

2^+

0^+

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

spectrum based on the mean-field approximation?



angular mom. projection

shape fluctuation (GCM)

$\hat{P}_I|\mathrm{MF}\rangle$

E_{MF} $|\mathrm{MF}\rangle$

4^+

2^+

0^+

4^+

2^+

0^+

$+$ $+$

beyond-mean-field calculations

$\mathcal{J}$

rigid-rotor model

4^+

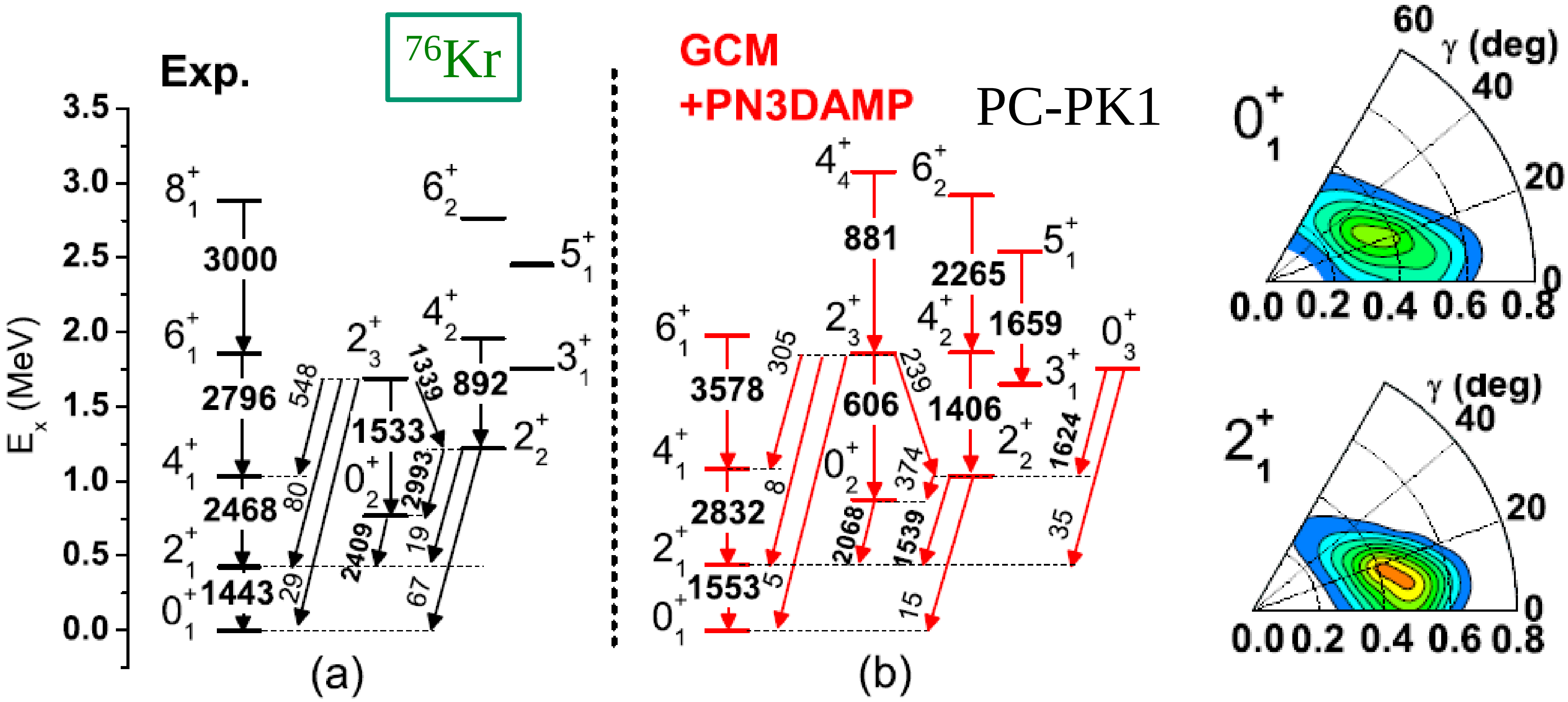
2^+

0^+

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

beyond mean-field calculations

- ✓ angular momentum + particle number projections
- ✓ quantum fluctuation (GCM)

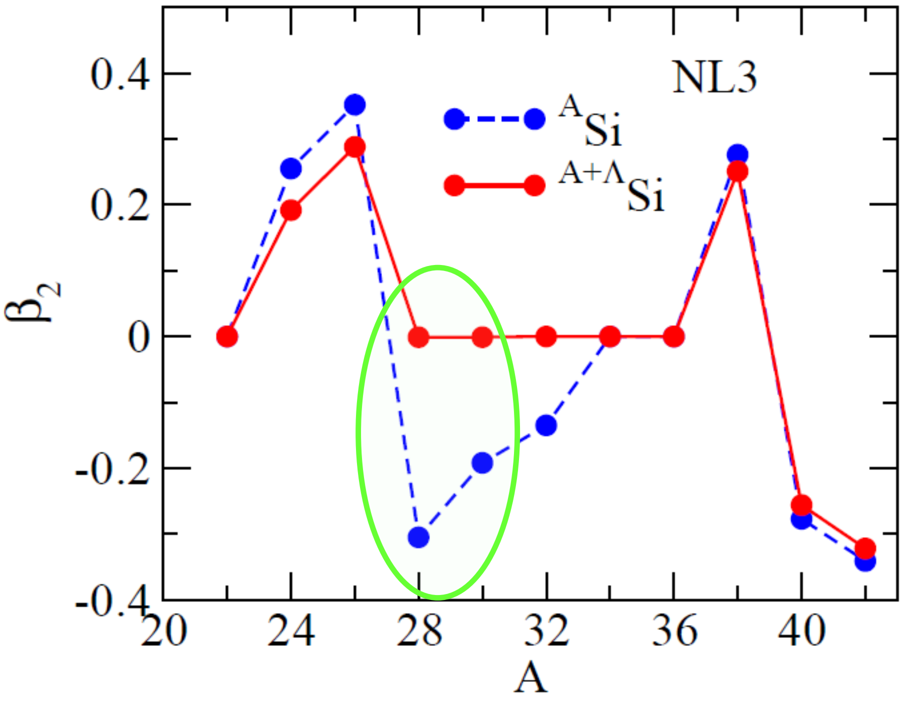


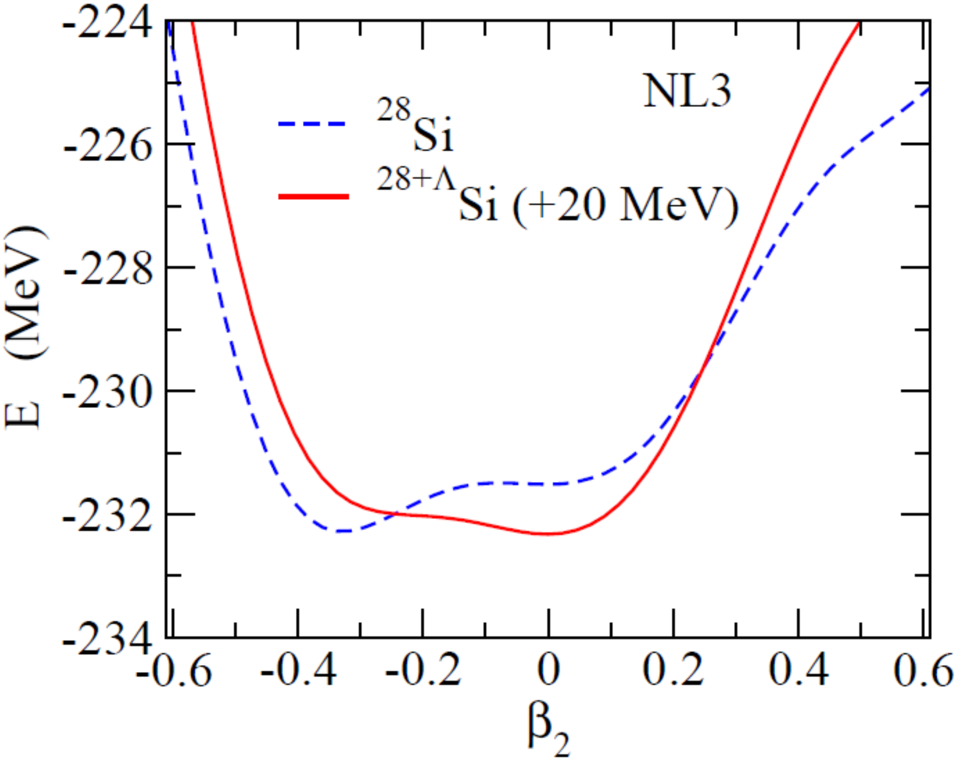
J.M. Yao, K.H., Z.P. Li, J. Meng, and P. Ring, PRC89 (‘14) 054306

cf. collective coordinates: Quadrupole moments (local operators)

The previous Mean-field calculations for hypernuclei

Si isotopes





Myaing Thi Win and K.Hagino, PRC78(‘08)054311

soft energy surface → role of beyond-mean-field effect
(especially shape fluctuation)?

Beyond Mean-Field calculations for hypernuclei

◆ Projection+GCM for the whole（A_c+1） system

H. Mei, K.H. and J.M. Yao, PRC93 (‘16) 011301(R)

$$|\Psi_{IM}\rangle = \int d\beta f(\beta)\, \hat{P}^I_{MK} \hat{P}^N \hat{P}^Z \left[|\Psi_{\mathsf{A_c}}(\beta)\rangle|\Psi_\Lambda(\beta)\rangle\right]$$

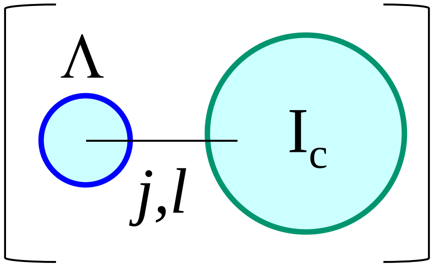
cf. Hill-Wheeler eq. for each *I*

◆ Microscopic particle-rotor model for single-Λ hypernuclei

H. Mei, K.H., J.M. Yao, and T. Motoba,

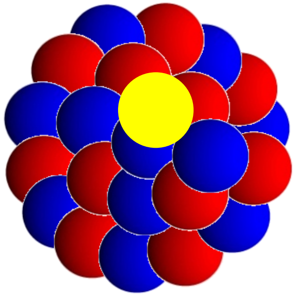
- ✓ PRC90 (‘14) 064302
- ✓ PRC91 (‘15) 064305
- ✓ PRC93 (‘16) 044307
- ✓ PRC96 (‘17) 014308
- ✓ PRC97 (‘18) 064318

$$|\Psi_{IM}\rangle = \sum_{j,l,I_c}$$



(IM)

Microscopic particle-rotor model for hypernuclei

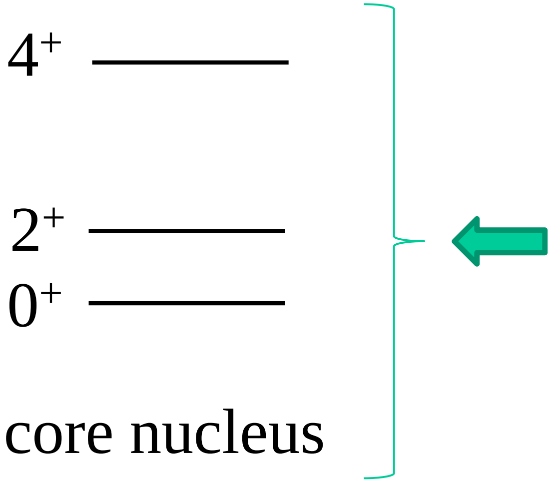


hypernucleus: a clear separation of deg. of freedom

⟶ Λ+core model

$$|\mathrm{g.s.}\rangle \sim \psi_{s_{1/2}}(\Lambda)\otimes\Psi_{\mathrm{core}}(0^+)$$
$$+\psi_{d_{5/2}}(\Lambda)\otimes\Psi_{\mathrm{core}}(2^+)+\cdots$$

Λ+core model with core excitations



beyond MF (proj.+GCM) calculations

- ✓ fully microscopic
- ✓ shape fluctuation

  cf. conventional part.-rot model
- ✓ no Pauli principle
- ✓ rot. and vib. on the equal footing
- ✓ transitional core

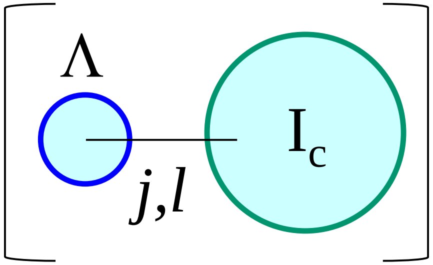
◆ Microscopic particle-rotor model for single-Λ hypernuclei

H. Mei, K.H., J.M. Yao, and T. Motoba, PRC90('14)064302, PRC91('15) 064305
PRC93('16)044307, PRC96('17) 014308

i) beyond mean-field calculations for e-e core: $|\Phi_{0^+}\rangle, |\Phi_{2^+}\rangle, |\Phi_{4^+}\rangle, \cdots$

ii) coupling of Λ to the core states

$$|\Phi_{IM}\rangle = \sum_{j,l,I_c} \left[\quad \right]^{(IM)}$$



Λ+core model with core excitations

$$\begin{cases} \Psi_{JM}(\boldsymbol{r}_\Lambda, \{\boldsymbol{r}_i\}) = \sum_{n,j,\ell,I} R_{j\ell nI}(r_\Lambda)[\mathcal{Y}_{j\ell}(\hat{\boldsymbol{r}}_\Lambda) \otimes \Phi_{nI}(\{\boldsymbol{r}_i\})]^{(JM)} \\ H = H_c + T_\Lambda + \sum_{i=1}^{A_c} v_{N\Lambda}(\boldsymbol{r}_\Lambda, \boldsymbol{r}_i) \end{cases}$$

coupled equations for R_{jlnI}

$v_{N\Lambda}$: point coupling relativistic NΛ interaction

◆ Microscopic particle-rotor model for single-Λ hypernuclei

$$\left\{\begin{aligned} &\Psi_{JM}(\boldsymbol{r}_\Lambda,\{\boldsymbol{r}_i\}) = \sum_{n,j,\ell,I} R_{j\ell nI}(r_\Lambda)[\mathcal{Y}_{j\ell}(\hat{\boldsymbol{r}}_\Lambda)\otimes\Phi_{nI}(\{\boldsymbol{r}_i\})]^{(JM)} \\ &H = H_c + T_\Lambda + \sum_{i=1}^{A_c} v_{N\Lambda}(\boldsymbol{r}_\Lambda,\boldsymbol{r}_i) \end{aligned}\right.$$

$$\langle[\mathcal{Y}_{jl}(\hat{\boldsymbol{r}}_\Lambda)\otimes\Phi_{nI}(\{\boldsymbol{r}_i\})]^{(JM)}|\underline{H - E_J|\Psi_{JM}\rangle = 0}$$

$$\Rightarrow\quad \boxed{\begin{aligned} &[T_\Lambda(jl) + \epsilon_{nI} - E_J]\,\mathcal{R}_{jlnI}(r_\Lambda) \\ &\qquad + \sum_{j',l'}\sum_{n',I'} V_{jlnI,j'l'n'I'}(r_\Lambda)\,\mathcal{R}_{j'l'n'I'}(r_\Lambda) = 0 \end{aligned}}$$

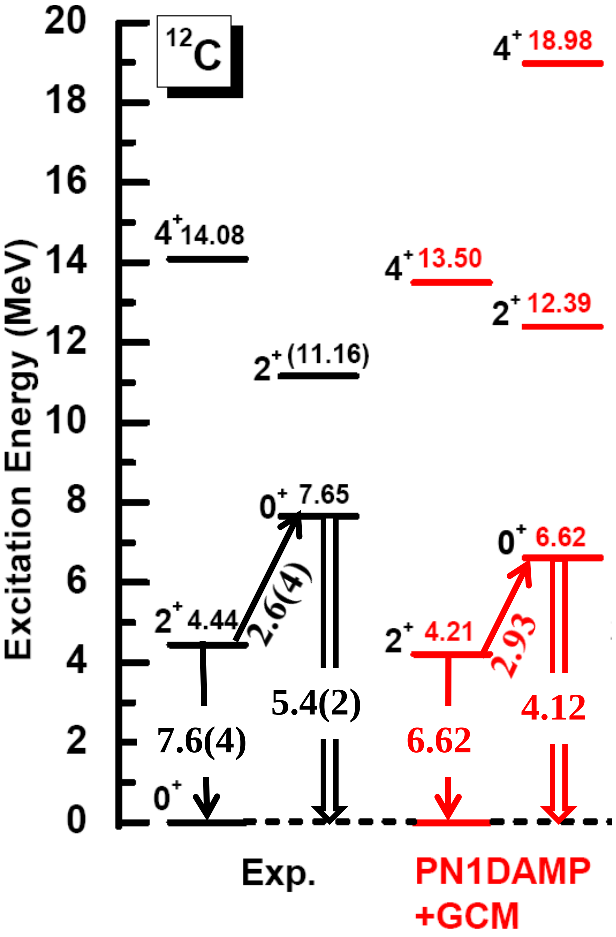
$$V_{jlnI,j'l'n'I'}(r_\Lambda) = \left\langle jlnI \left| \sum_{i=1}^{A_C} v_{N\Lambda}(\boldsymbol{r}_\Lambda,\boldsymbol{r}_i) \right| j'l'n'I' \right\rangle$$

Microscopic Particle-Rotor Model for Λ hypernuclei

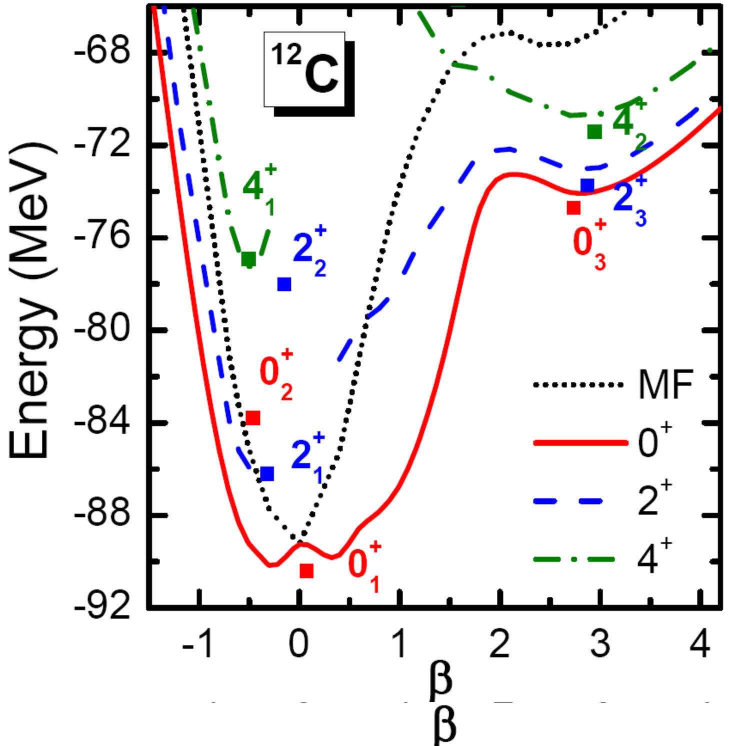
Example: $^{13}{}_{\Lambda}\mathrm{C}$

i) beyond mean-field calculations for e-e core (^{12}C) : GCM + projections

$$|\Phi_{I_cM_c}\rangle = \int d\beta\, f(\beta)|\Psi_{I_cM_c}(\beta)\rangle = \int d\beta\, f(\beta)\hat{P}^{I_c}_{M_cK_c}\hat{P}^{N}\hat{P}^{Z}|\Psi_{\mathsf{MF}}(\beta)\rangle$$



- ✓ axial symmetry
- ✓ relativistic PC-F1



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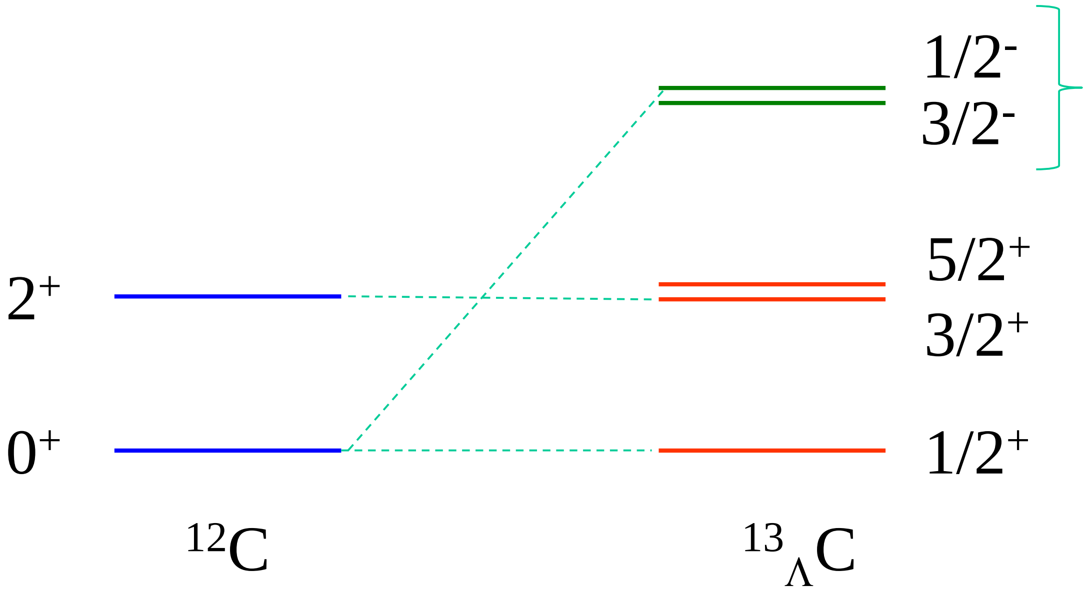
Example: $^{13}{}_{\Lambda}\mathrm{C}$

(i) beyond mean-field calculations for e-e core (^{12}C)

(ii) coupling of Λ to the core states

$$|\Phi_{IM}\rangle = \sum_{j,l,I_c} \Big[\underset{\underset{\Lambda}{\uparrow}}{\psi_{jl}(\boldsymbol{r}_\Lambda)} \otimes \underset{\underset{\text{core}}{\uparrow}}{|\Phi_{I_c}\rangle}\Big]^{(IM)}$$

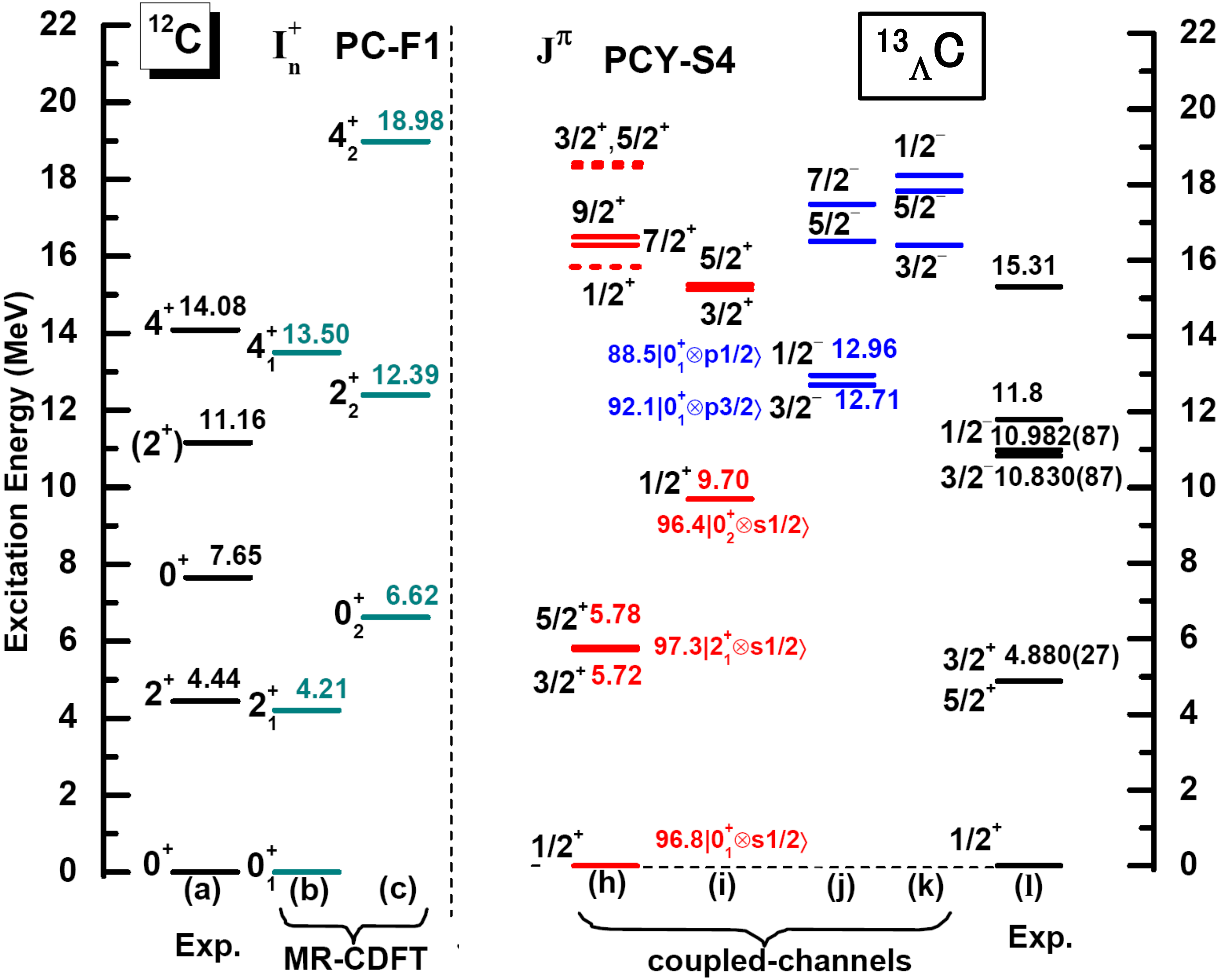


$\sim 88.5\%|p_{1/2}\otimes 0^+\rangle$
$+11\%|p_{3/2}\otimes 2^+\rangle$

$\sim 92.1\%|p_{3/2}\otimes 0^+\rangle$
$+5\%|p_{1/2}\otimes 2^+\rangle$

$\sim 97.3\%|s_{1/2}\otimes 2^+\rangle$

$\sim 96.8\%|s_{1/2}\otimes 0^+\rangle$



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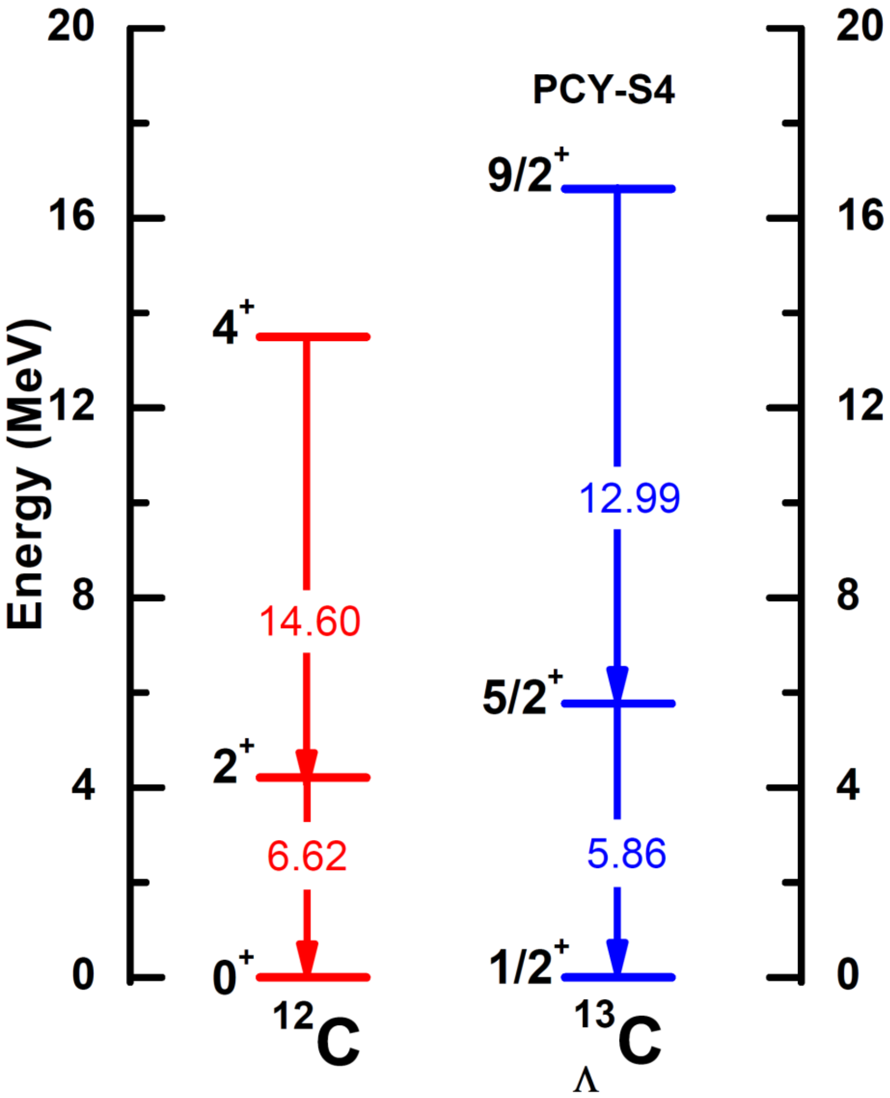
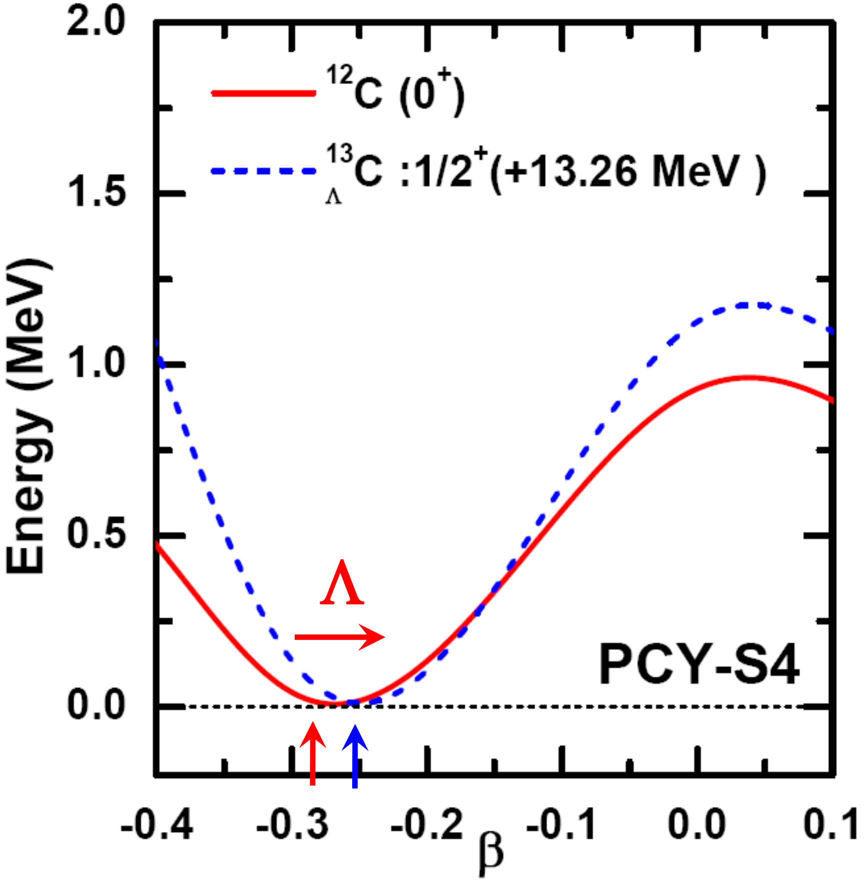
B(E2) transition rates (e^{2}fm^4)



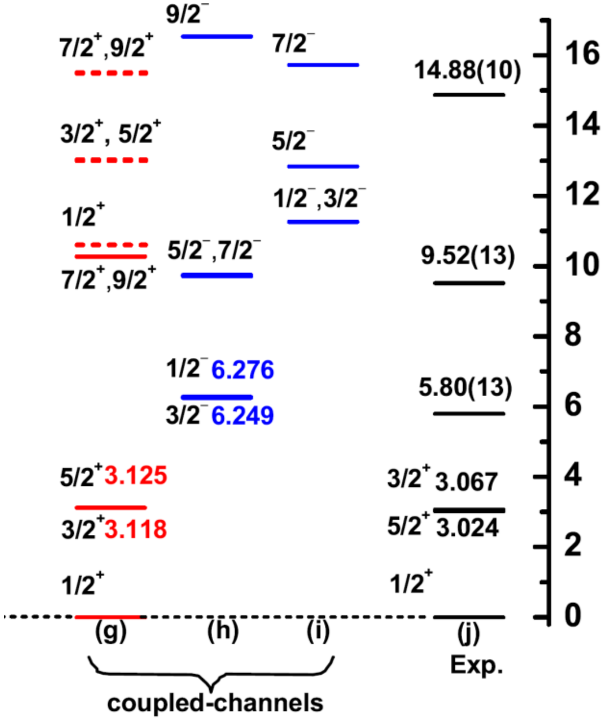


	^{12}C	$^{13}_{\Lambda}$C
β	-0.27	-0.25
r_p (fm)	2.44	2.39

- B(E2) : ~ 11% reduction

other applications:

$^{9}_{\Lambda}$Be



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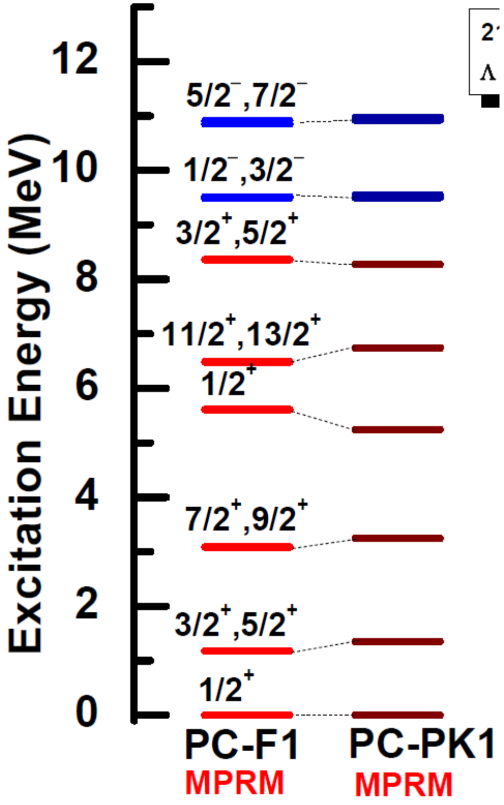
$^{21}_{\Lambda}$Ne



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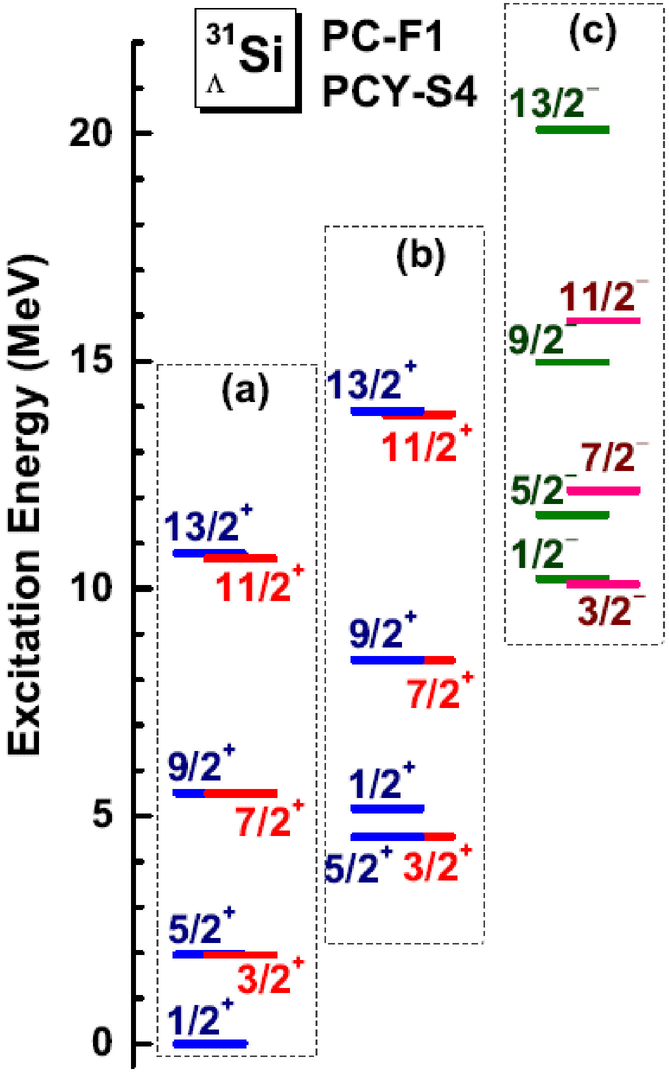

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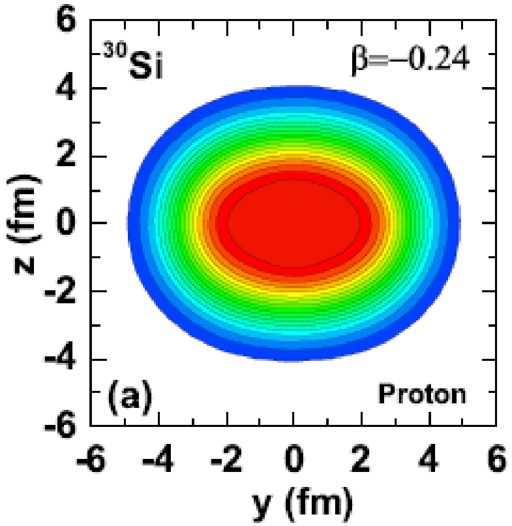
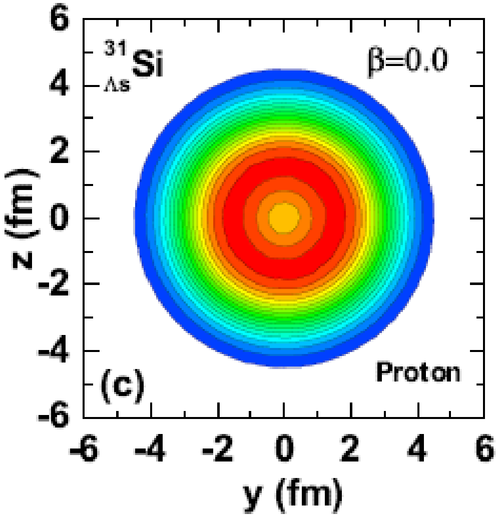
Beyond MF effect on deformation change in $^{31}_{\Lambda}$Si

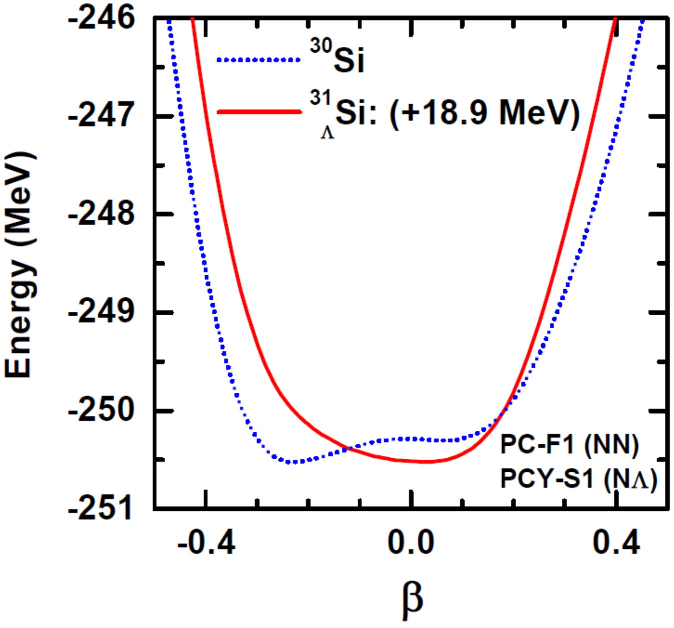
Mean-field calculations:



Λ

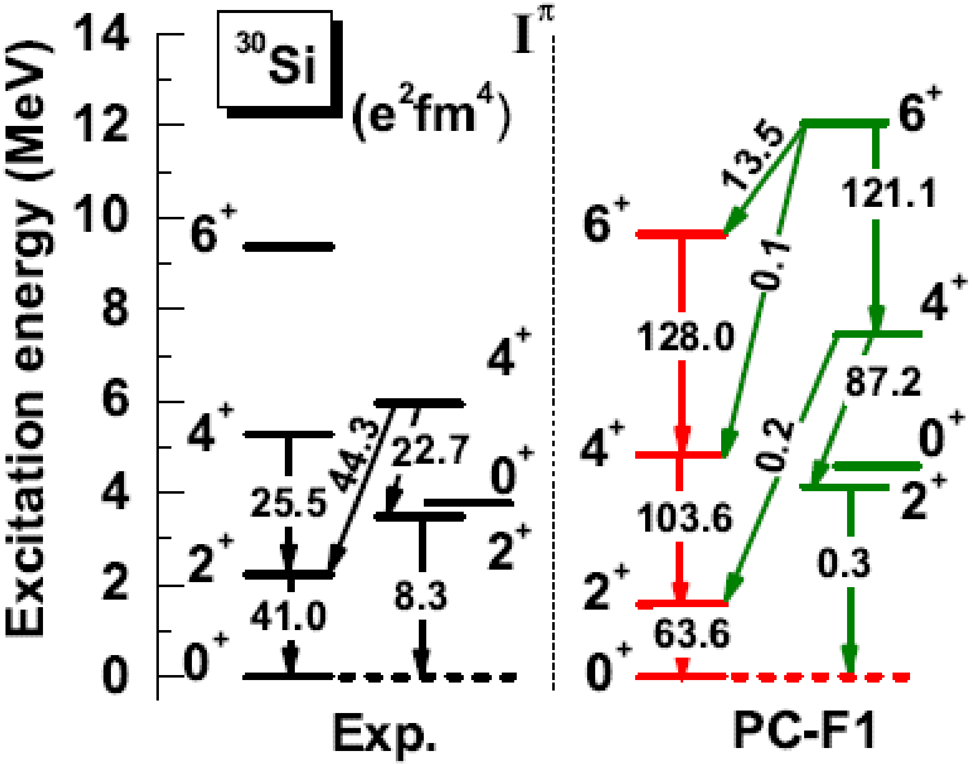
oblate → spherical

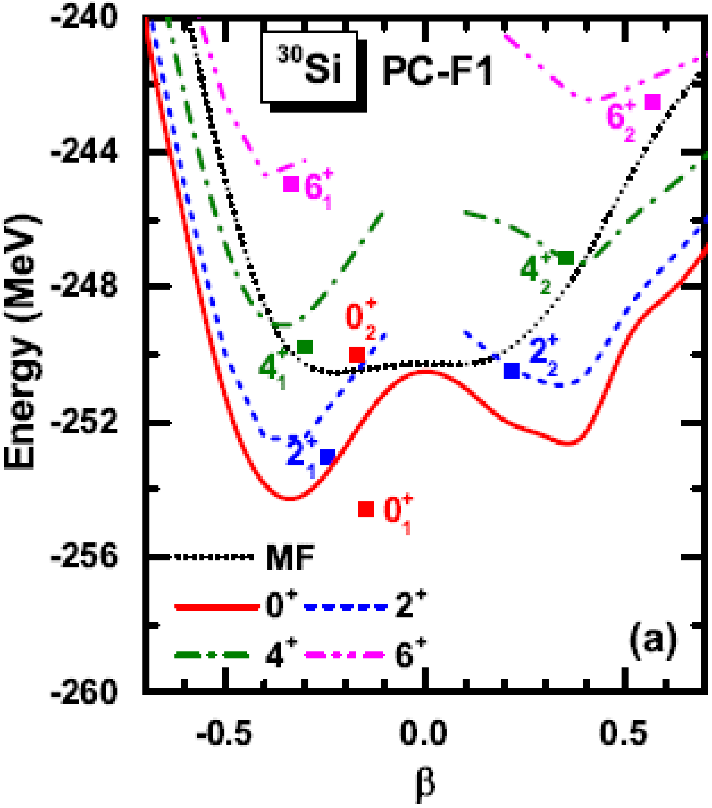




Soft potential energy surface
→ beyond-mean-field effects?

spectrum of ^{30}Si



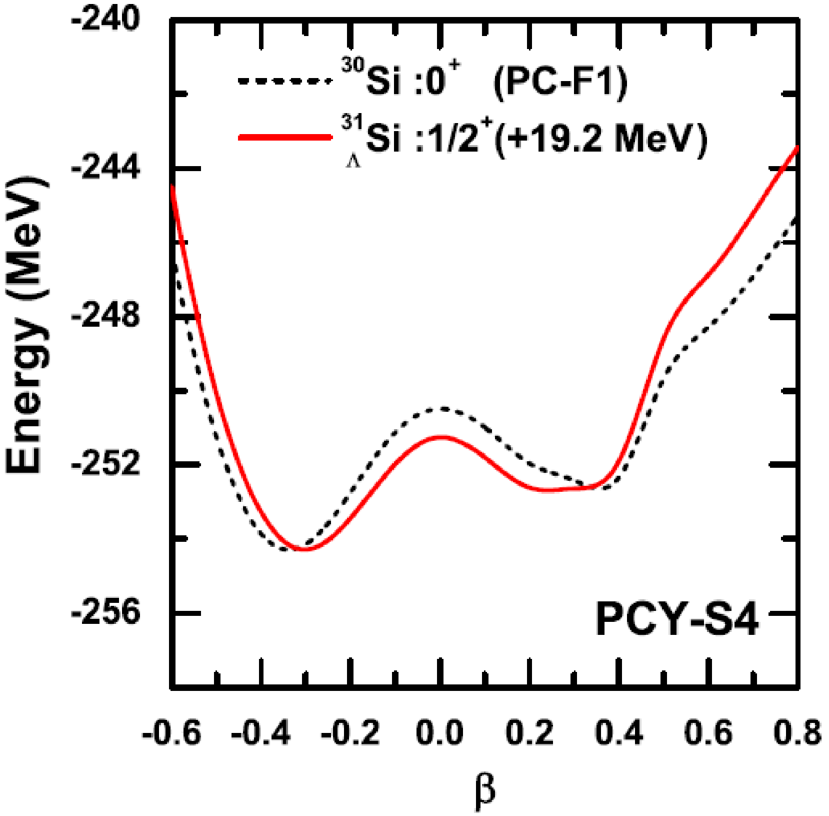


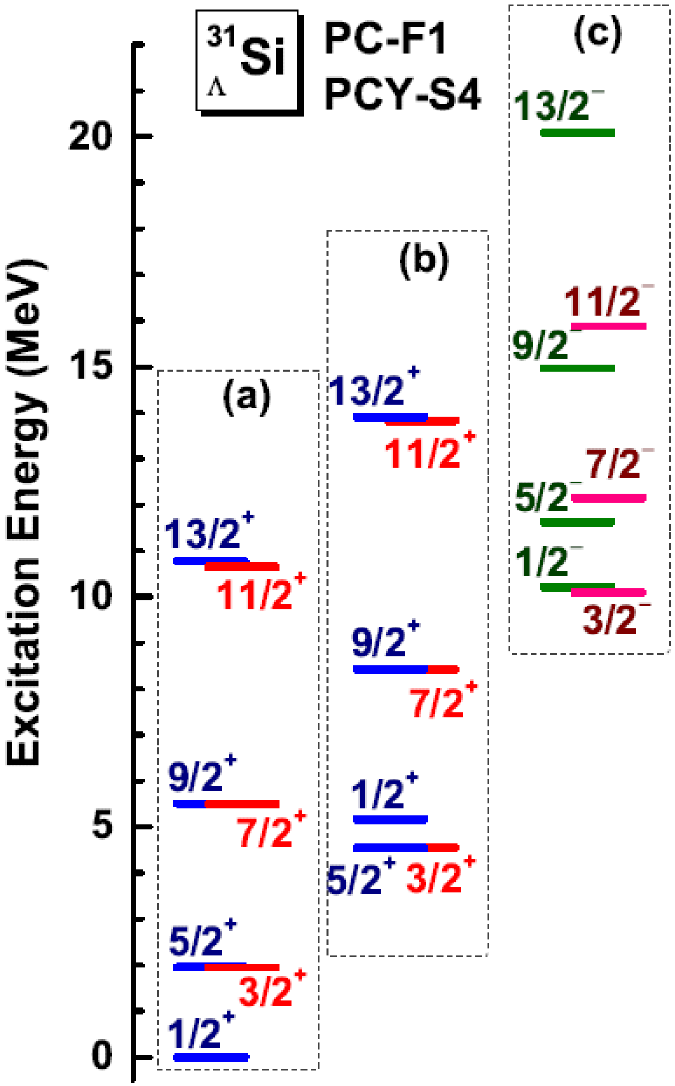
* axial symmetry

$R_{4/2}$ = E(4$^+$) / E(2$^+$) = 3.083

cf. Expt: $R_{4/2}$ = 2.66

spectrum of $^{31}_{\Lambda}$Si





still rotational spectrum

[$R_{4/2} = 3.08 \rightarrow 2.83$]

changes towards spherical, but not quite

→ large BMF effects

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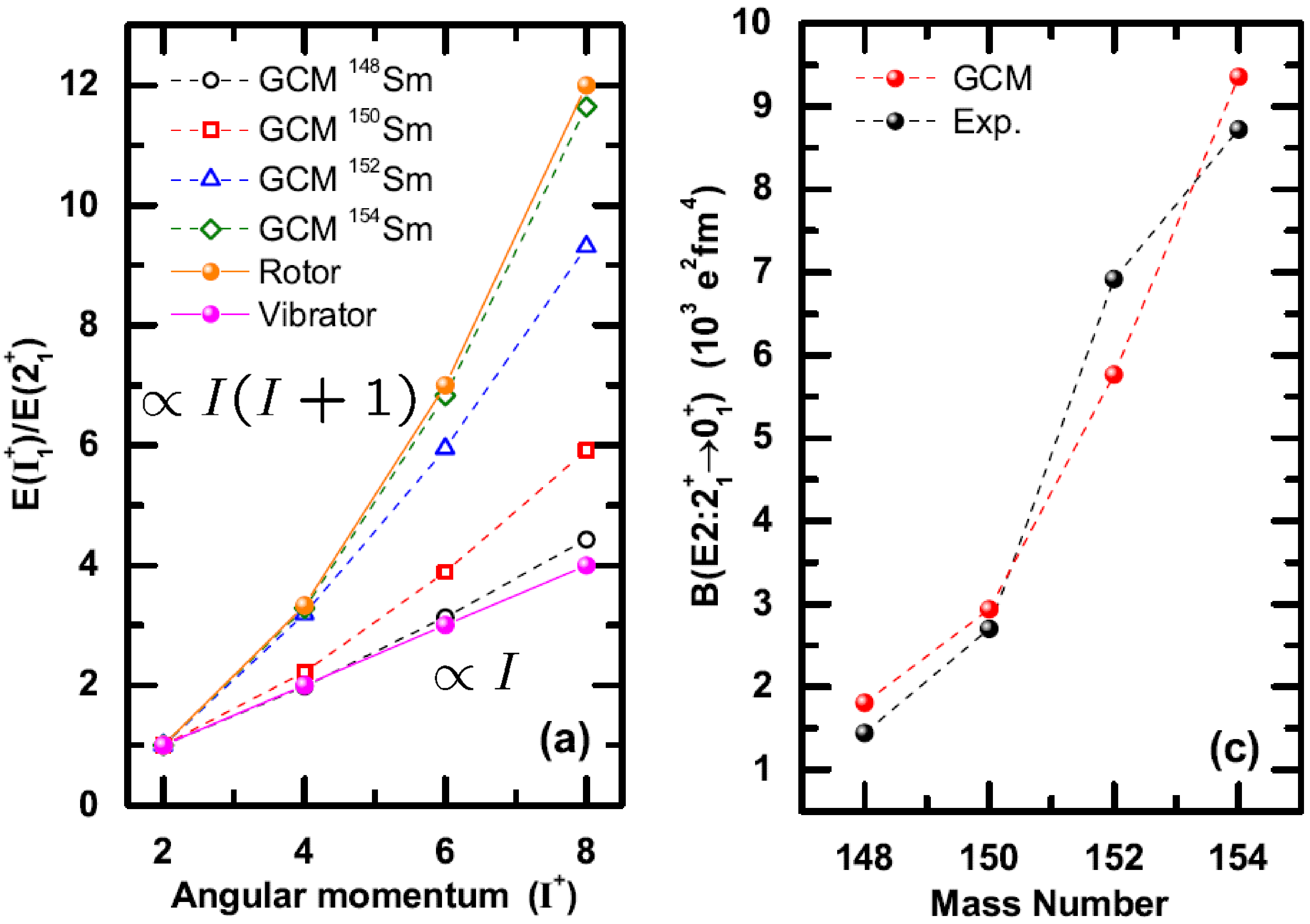
Application to Sm isotopes



Results for $^{155}{}_{\Lambda}$Sm

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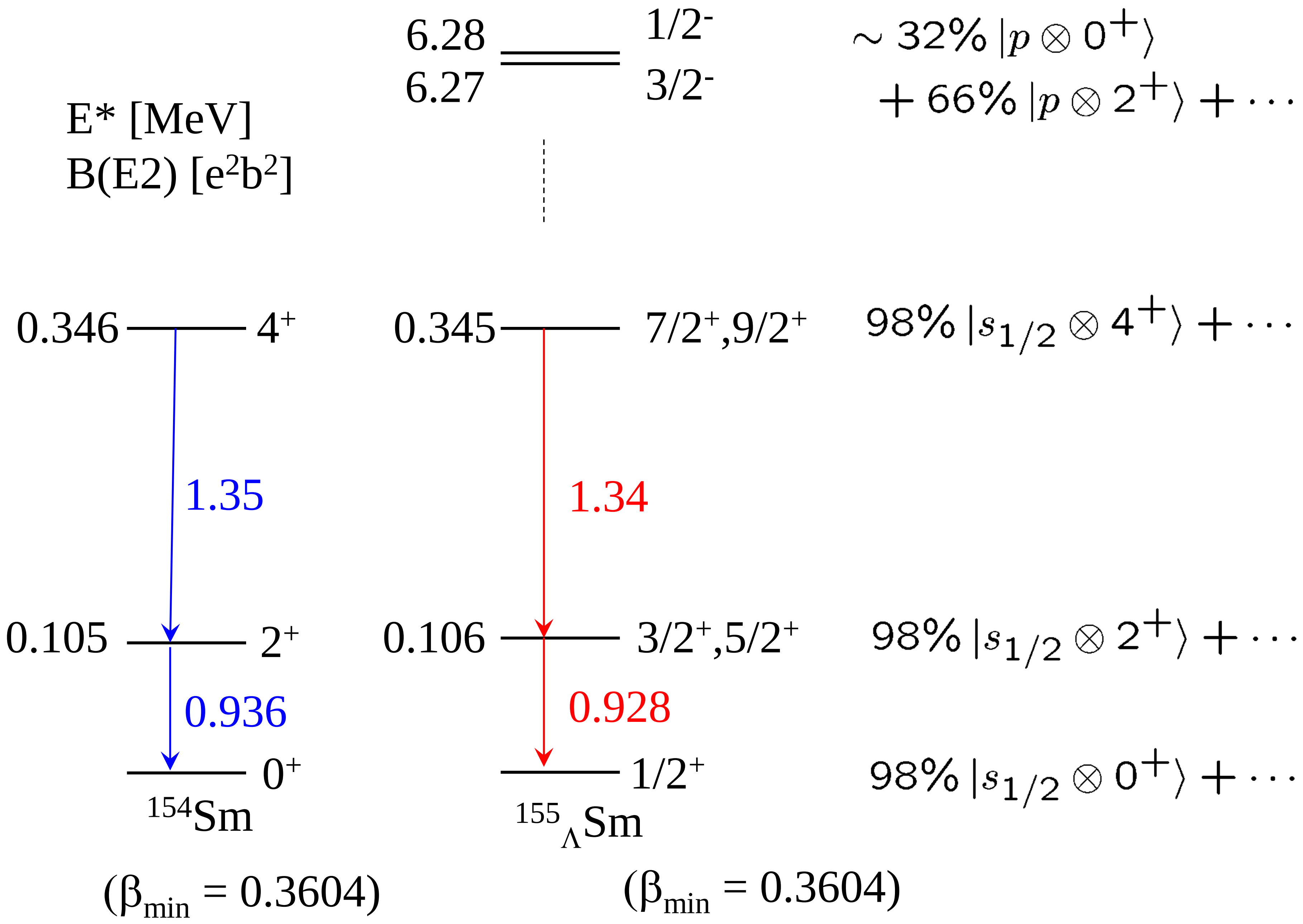


E* [MeV]

B(E2) [e^2b^2]

6.28 $1/2^-$

6.27 $3/2^-$

$\sim 32\% \, |p \otimes 0^+\rangle + 66\% \, |p \otimes 2^+\rangle + \cdots$

0.346 4^+

0.345 $7/2^+, 9/2^+$

$98\% \, |s_{1/2} \otimes 4^+\rangle + \cdots$

1.35

1.34

0.105 2^+

0.106 $3/2^+, 5/2^+$

$98\% \, |s_{1/2} \otimes 2^+\rangle + \cdots$

0.936

0.928

0^+

$1/2^+$

$98\% \, |s_{1/2} \otimes 0^+\rangle + \cdots$

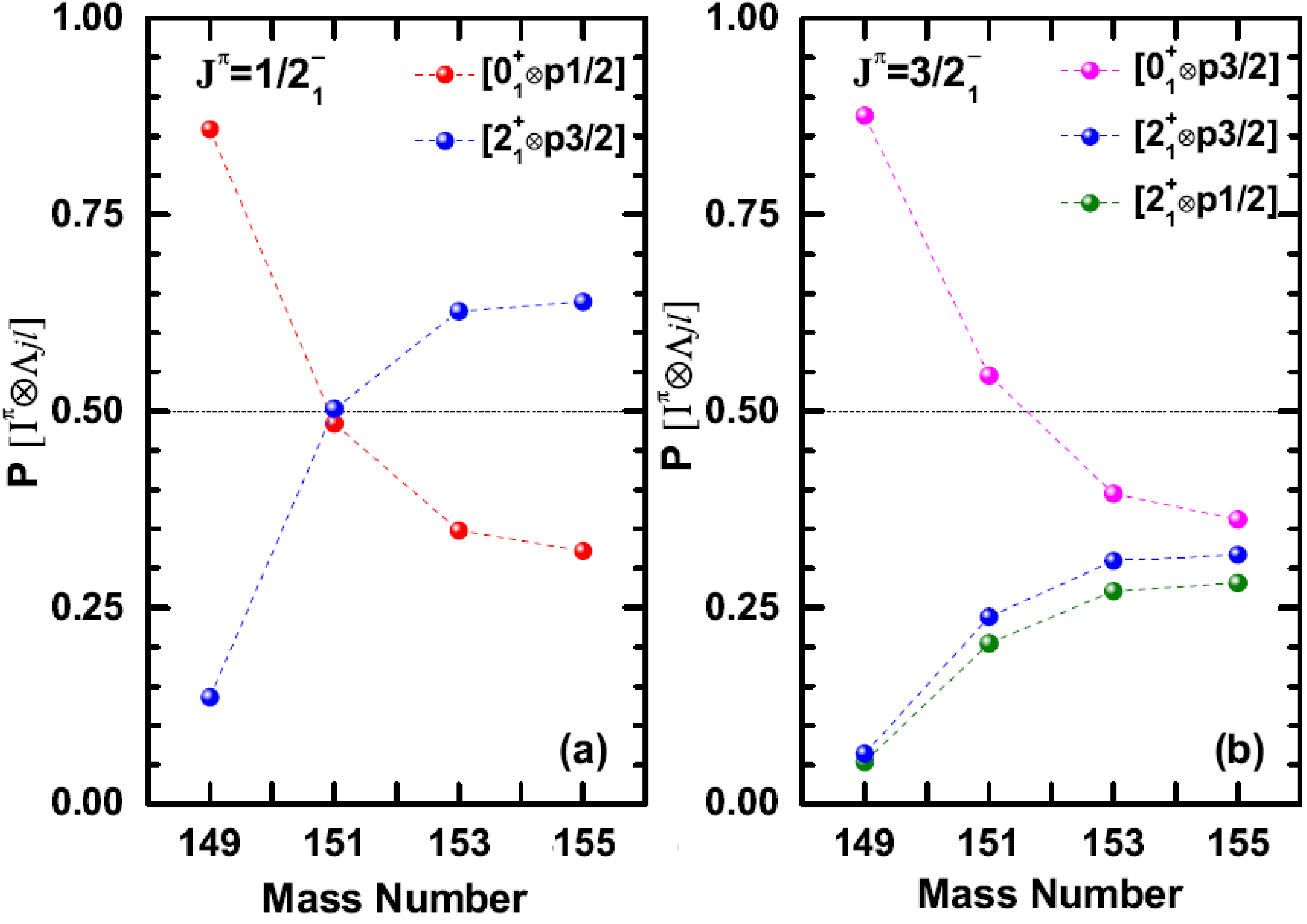
^{154}Sm

$^{155}{}_{\Lambda}$Sm

(β_{min} = 0.3604)

(β_{min} = 0.3604)

weak coupling → strong coupling



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Summary

Applications of Beyond-Mean-Field method to hypernuclei

➢Low-lying spectra of Λ hypernuclei

Microscopic particle-rotor/vibrator model

- ✓Λ + GCM states for core: particle-core model with core excitations
- ✓the first calculations for low-lying spectra for hypernuclei based on mean-field type calculations
- ✓ from C to Sm: both rotor and vibrator on an equal footing
- ✓ transitional nuclei

➢ Future perspectives

- ✓ extension to include triaxiality (cf. $^{25}{}_{\Lambda}$Mg)

謝謝！