New approach to coupled-channels calculations for heavy-ion fusion reactions around the Coulomb barrier

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1. Introduction
   - H.I. sub-barrier fusion reactions
   - Coupled-channels (C.C.) approach
2. Phenomenological approach: Bayesian statistics
3. C.C. with nuclear structure calculations
4. Summary

How to do C.C. calculations if there is only limited experimental information on intrinsic degrees of freedom?
Introduction: heavy-ion fusion reactions

Fusion: compound nucleus formation

energy production in stars (Bethe ‘39)

nucleosynthesis

superheavy elements

cf. Bohr ‘36
Introduction: heavy-ion fusion reactions

Fusion: compound nucleus formation

1. Coulomb force: long range, repulsive
2. Nuclear force: short range, attractive

Coulomb barrier
Introduction: heavy-ion fusion reactions

Fusion: compound nucleus formation

1. Coulomb force: long range, repulsive
2. Nuclear force: short range, attractive

Coulomb barrier

fusion reactions in the sub-barrier energy region

\(|E - V_b| \lesssim 10\text{MeV}\)

compound nucleus
Discovery of large sub-barrier enhancement of $\sigma_{\text{fus}}$

Potential model: $V(r) + \text{absorption}$

cf. seminal work:
R.G. Stokstad et al., PRL41('78) 465
Effects of nuclear deformation

\(^{154}\text{Sm} : \) a typical deformed nucleus with \(\beta_2 \sim 0.3\)

\[ \sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta) \]

\(^{16}\text{O} + ^{154}\text{Sm} \)

- Spherical
- \(\theta = 0\) deg.
- \(\theta = 90\) deg.

Fusion: strong interplay between nuclear structure and reaction

* Sub-barrier enhancement also in non-deformed systems:
  couplings to low-lying collective excitations \(\rightarrow\) coupling assisted tunneling
Strong target dependence

at \( E < V_b \)
Coupled-Channels method

many-body problem

two-body problem, but with excitations (coupled-channels approach)

still very challenging
Coupled-channels method: a quantal scattering theory with excitations

\[
\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \epsilon_k - E\right] \psi_k(r) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(r) = 0
\]

excitation energy  excitation operator

|0^+\rangle \rightarrow |0^+\rangle

full order treatment of excitation/de-excitation dynamics during reaction
Inputs for C.C. calculations

i) Inter-nuclear potential
   - a fit to experimental data at above barrier energies

ii) Intrinsic degrees of freedom
   - types of collective motions (rotation / vibration) a/o transfer
   - coupling strengths and excitation energies
   - how many states
C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 (‘99) 143)

✓ Eigen-channel representation of C.C.

\[
\sigma_{\text{fus}}(E) = \sum_k w_k \sigma_{\text{fus}}(E; V_k)
\]

many barriers are “distributed” due to the channel coupling effects
C.C. approach: a standard tool for sub-barrier fusion reactions

- Eigen-channel representation of C.C.

\[
\sigma_{\text{fus}}(E) = \sum_{k} \omega_k \sigma_{\text{fus}}(E; V_k)
\]

- Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254(‘91))

\[
D_{\text{fus}}(E) = \frac{d^2\langle E\sigma_{\text{fus}} \rangle}{dE^2}
\]

\[
D_{\text{fus}}^{(cl)}(E) = \sum_{k} \omega_k \delta(E - V_b^{(k)})
\]

\[
\sim \pi R_b^2 P_{l=0}(E)
\]

\[
\sim \pi R_b^2 \cdot \frac{dP_{l=0}}{dE}
\]
sensitive to nuclear structure

- N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254(‘91) 25
- M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48(‘98)401
A Bayesian approach to fusion barrier distributions

Fusion barrier distributions

- **Coupled-channels analyses**
  - a standard approach
  - need to know the nature of collective excitations

- **Direct fit to experimental data**

\[ D_{\text{fus}}(E) = \sum_k w_k D_0(E; B_k, R_k, \hbar \Omega_k) \]

- phenomenological
- no need to know the nature of coll. excitations
- quick and convenient way
- mapping from D to \( T_l \) (cf. SHE)
- the number of barriers? (over-fitting problem)

J.R. Leigh et al., PRC52 (‘95) 3151
over-fitting problem

\[ y = ax^3 + bx^2 + cx + d \]
over-fitting problem

\[ y = ax^3 + bx^2 + cx + d \]
over-fitting problem

\[ y = ax^3 + bx^2 + cx + d \]

J.R. Leigh et al., PRC52 (‘95) 3151

one can make \( \chi^2 \) small by increasing the number of barriers

how many barriers?
Bayesian statistics

✓ data set: $D_{\text{exp}} = \{E_i, d_i, \delta d_i\}$ (i = 1 ~ M)
✓ fit with $d_{\text{model}} (E; a)$  $a$: a model parameter

**Bayes theorem**

$$P(a|D_{\text{exp}}) \propto P(D_{\text{exp}}|a)P(a)$$

$P(a)$: a prior probability of $a$  
(a guess distribution before experiment)

$P(D_{\text{exp}}|a)$: a probability to realize $D_{\text{exp}}$ when $a$ is given

$$P(D_{\text{exp}}|a) \propto \exp \left[ -\frac{1}{2} \sum_i \left( \frac{d_i - d_{\text{model}}(E_i; a)}{\delta d_i} \right)^2 \right]$$

$P(a|D_{\text{exp}})$: a posterior probability of $a$  
(an updated distribution after knowing the data)
Bayesian statistics

Bayes theorem

\[ P(a|D_{\text{exp}}) \propto P(D_{\text{exp}}|a)P(a) \]

\( P(a) \): a prior probability of \( a \)
(a guess distribution before experiment)

\( P(D_{\text{exp}}|a) \): a probability to realize \( D_{\text{exp}} \) when \( a \) is given

\( P(a|D_{\text{exp}}) \): a posterior probability of \( a \)
(an updated distribution after knowing the data)

Before expt.

\[ P(a) \]

After expt.

\[ P(a|D_{\text{exp}}) \]
Bayesian spectrum deconvolution

K. Nagata, S. Sugita, and M. Okada,
Neural Networks 28 (‘12) 82

✓ data set: \( D_{\text{exp}} = \{ E_i, d_i, \delta d_i \} \) \hspace{1cm} (i = 1 \sim M)

✓ fitting function: \( D_{\text{fit}}(E; \bar{\theta}, K) = \sum_{k=1}^{K} w_k \phi_k(E; \theta_k), \quad \bar{\theta} \equiv \{ w_k, \theta_k \} \)

\( K \): the number of barriers

Bayes theorem

\[
P(K|D_{\text{exp}}) \propto P(D_{\text{exp}}|K)P(K)
\]

\[\propto P(D_{\text{exp}}|K) = \int d\bar{\theta} \, e^{-\chi^2(\bar{\theta}, K)/2} P(\bar{\theta})\]

\[
\chi^2(\bar{\theta}, K) = \sum_{i=1}^{M} \left( \frac{d_i - D_{\text{fit}}(E_i; \bar{\theta}, K)}{\delta d_i} \right)^2
\]

most probable value of \( K \): maximize \( P(K|D_{\text{exp}}) \)

or, equivalently, minimize \( F = -\ln P(K|D_{\text{exp}}) \)

\( \longrightarrow \) optimize the other parameters for a given value of \( K \)
Future perspective: application to SHE formation reactions

\[ \sigma_{ER}(E) = \frac{\pi}{k^2} \sum_{l} (2l + 1) T_l(E) P_{CN}(E, l) W_{suv}(E^*, l) \]

CN = compound nucleus
ER = evaporation residue

Coupled-channels

Langevin approach

statistical model

CN = compound nucleus
ER = evaporation residue

\( T_l \) from \( \sigma_{\text{cap}} \)?
Bayesian approach to $\sigma_{\text{ER}}$

$$D_{\text{exp}}(E) = \sum_{i=1}^{K} w_k D_0(E; V_k(r))$$

either $D_{\text{fus}}$ or $D_{\text{qel}}$

$$T_l = \sum_{k=1}^{K} w_k T_l(E; V_k(r))$$

* no need to know the details of the couplings

+ Langevin + stat. model calculations

$$\sigma_{\text{ER}}(E) = \frac{\pi}{k^2} \sum_{l} (2l + 1) T_l(E) \times P_{\text{CN}}(E, l) W_{\text{suV}}(E^*, l)$$

superheavy elements
Semi-microscopic modeling of sub-barrier fusion

K.H. and J.M. Yao, PRC91('15) 064606

multi-phonon excitations

\[ {^{58}\text{Ni} + ^{58}\text{Ni}} \]

\[ \sigma_{\text{fus}} \]

\[ \begin{array}{c}
0^+ \\
2^+ \\
4^+ \\
\end{array} \]

\[ \begin{array}{c}
\varepsilon \\
\sqrt{2}\beta \\
2\varepsilon \\
\end{array} \]

\[ 126(8) \text{ e}^2\text{fm}^4 \]

\[ ^{58}\text{Ni} \]

\[ 0 \]

Simple harmonic oscillator → justifiable?

Often data available only for the 1st excited state
Anharmonic vibrations

- Boson expansion
- Quasi-particle phonon model
- Shell model
- Interacting boson model
- Beyond-mean-field method

\[ \left| JM \right\rangle = \int d\beta f_J(\beta) \hat{P}^J_{M0} \left| \Phi(\beta) \right\rangle \]

- MF + ang. mom. projection
- + particle number projection
- + generator coordinate method (GCM)

M. Bender, P.H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 (‘03) 121
J.M. Yao et al., PRC89 (‘14) 054306
Beyond mean-field approximation

- Angular momentum + particle number projections
- Quantum fluctuation (GCM)

J.M. Yao, K.H., Z.P. Li, J. Meng, and P. Ring, PRC89 (‘14) 054306
Semi-microscopic coupled-channels model for sub-barrier fusion

K.H. and J.M. Yao, PRC91 (‘15) 064606
Semi-microscopic coupled-channels model for sub-barrier fusion

K.H. and J.M. Yao, PRC91 ('15) 064606

\[ \begin{align*}
0^+, 2^+, 4^+ & \quad 2\varepsilon \\
2^+ & \quad \varepsilon \\
0^+ & \quad 0
\end{align*} \]

\[ \sqrt{2}\beta \]

M\((E^2)\) from MR-DFT calculation
scale to the empirical B(E2; 2^+_1 \rightarrow 0^+_1)
still use a phenomenological potential
use the experimental values for \(E_x\)

* axial symmetry (no 3^+ state)
Application to $^{16}\text{O} + ^{208}\text{Pb}$ fusion reaction
double-octupole phonon states in $^{208}\text{Pb}$

\[
\begin{array}{c}
\text{2.61} \\
\text{0}^+
\end{array}
\]

\[
\begin{array}{c}
\sim 5.2 \\
\text{0}^+,2^+,4^+,6^+
\end{array}
\]

M. Yeh, M. Kadi, P.E. Garrett et al., PRC57 (‘98) R2085
K. Vetter, A.O. Macchiavelli et al., PRC58 (‘98) R2631
V. Yu. Pnomarev and P. von Neumann-Cosel, PRL82 (‘99) 501
B.A. Brown, PRL85 (‘00) 5300

large fragmentations, especially 6$^+$ state
Application to $^{16}\text{O} + ^{208}\text{Pb}$ fusion reaction

cf. C.R. Morton et al., PRC60(‘99) 044608
fluctuation both in $\beta_3$ and $\beta_2$

2$_1^+$ state: strong coupling both to g.s. and 3$_1^-$

$$\longrightarrow |2_1^+\rangle = \alpha|2^+\rangle_{HO} + \beta|3^- \otimes 3^-\rangle_{HO}^{(I=2)} + \cdots$$
Summary

Heavy-ion subbarrier fusion reactions

- strong interplay between reaction and structure
  cf. fusion barrier distributions

- **A Bayesian approach to fusion barrier distributions**
  - a quick and convenient way to analyze data
  - determination of the number of barriers

- **C.C. calculations with rel. beyond MF method**
  - anharmonicity
  - truncation of phonon states
  - octupole vibrations: $^{16}\text{O} + ^{208}\text{Pb}$

more flexibility:
  - application to transitional nuclei

C.C. with shell model?
Why not full microscopic treatment?

microscopic potential (e.g., double folding potential)

\[ a \sim 0.63 \text{ fm} \]

does not work for fusion