

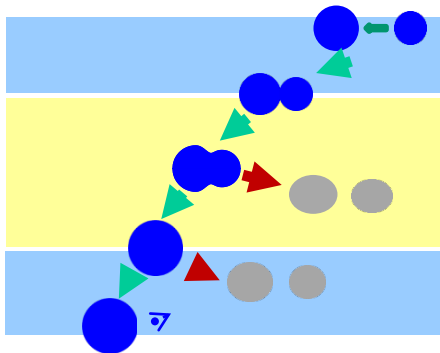
# Theoretical challenges in fusion reactions relevant to superheavy elements

Kouichi Hagino

*Tohoku University, Sendai, Japan*

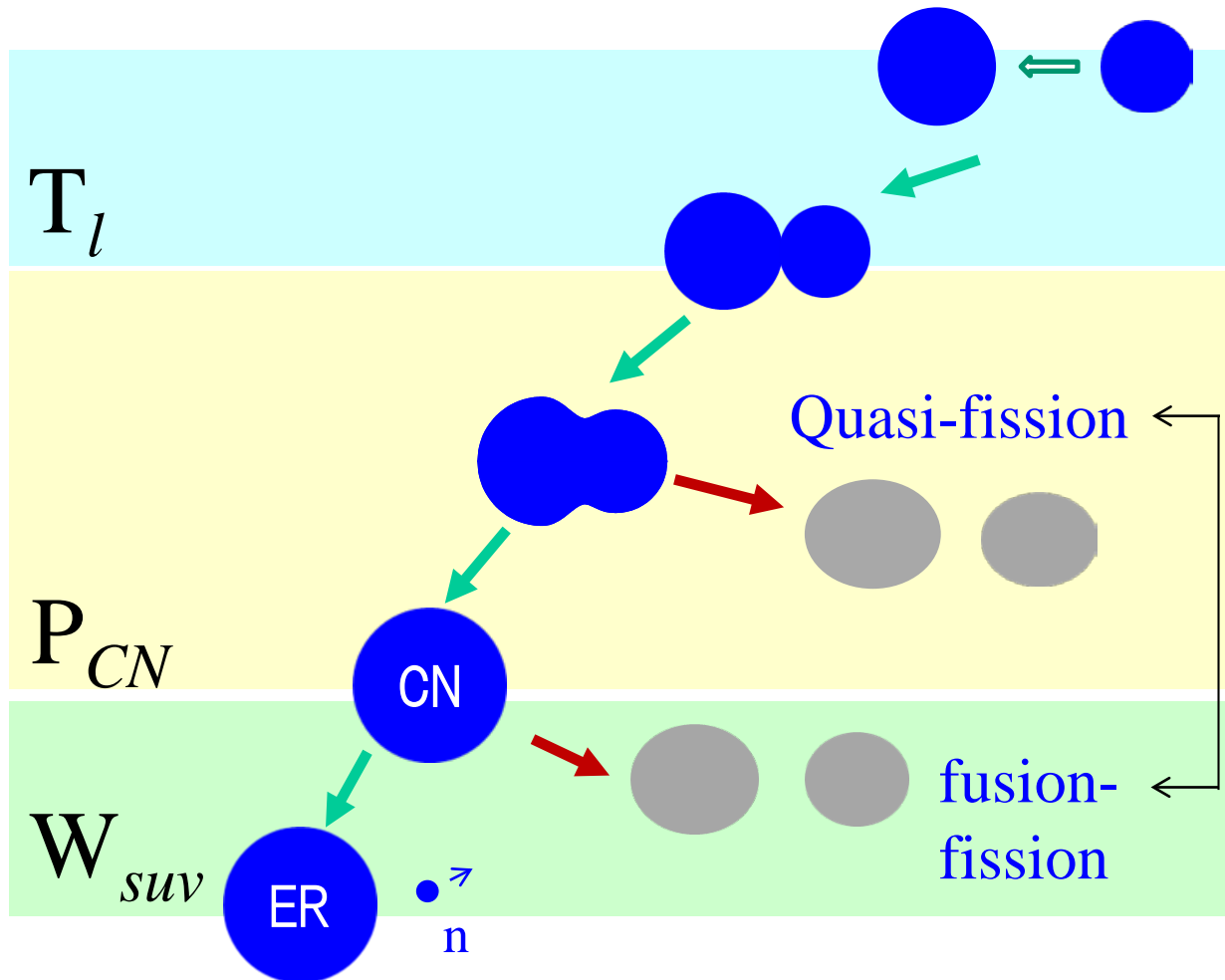


TOHOKU  
UNIVERSITY



1. Introduction: Fusion reactions for SHE
2. Approaching phase: coupled-channels method
3. Formation phase: Langevin method
4. Survival phase: statistical model
5. Towards  $Z=119-120$  and island of stability

# Introduction: Fusion reactions for SHE



$$\sigma_{ER}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) T_l(E) P_{CN}(E, l) W_{suv}(E^*, l)$$

formation of evaporation residues:  
a very rare process

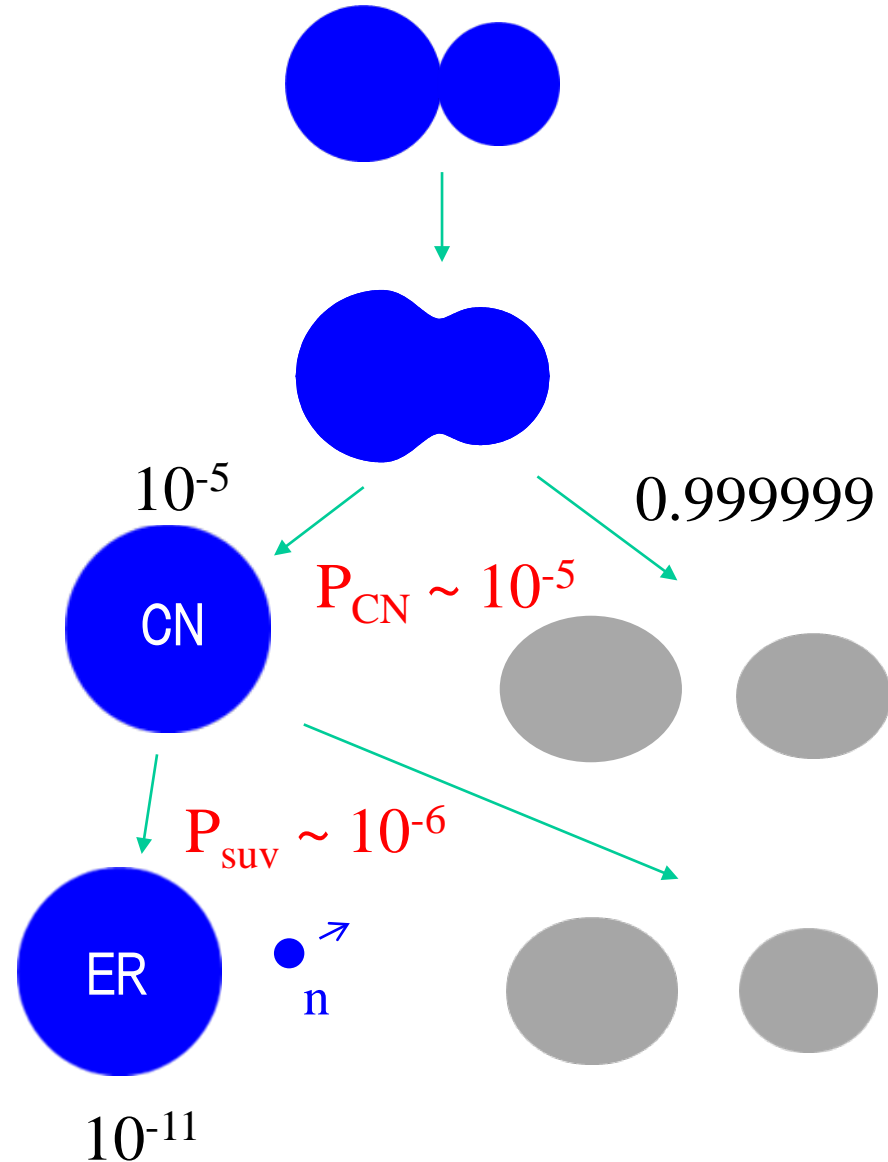


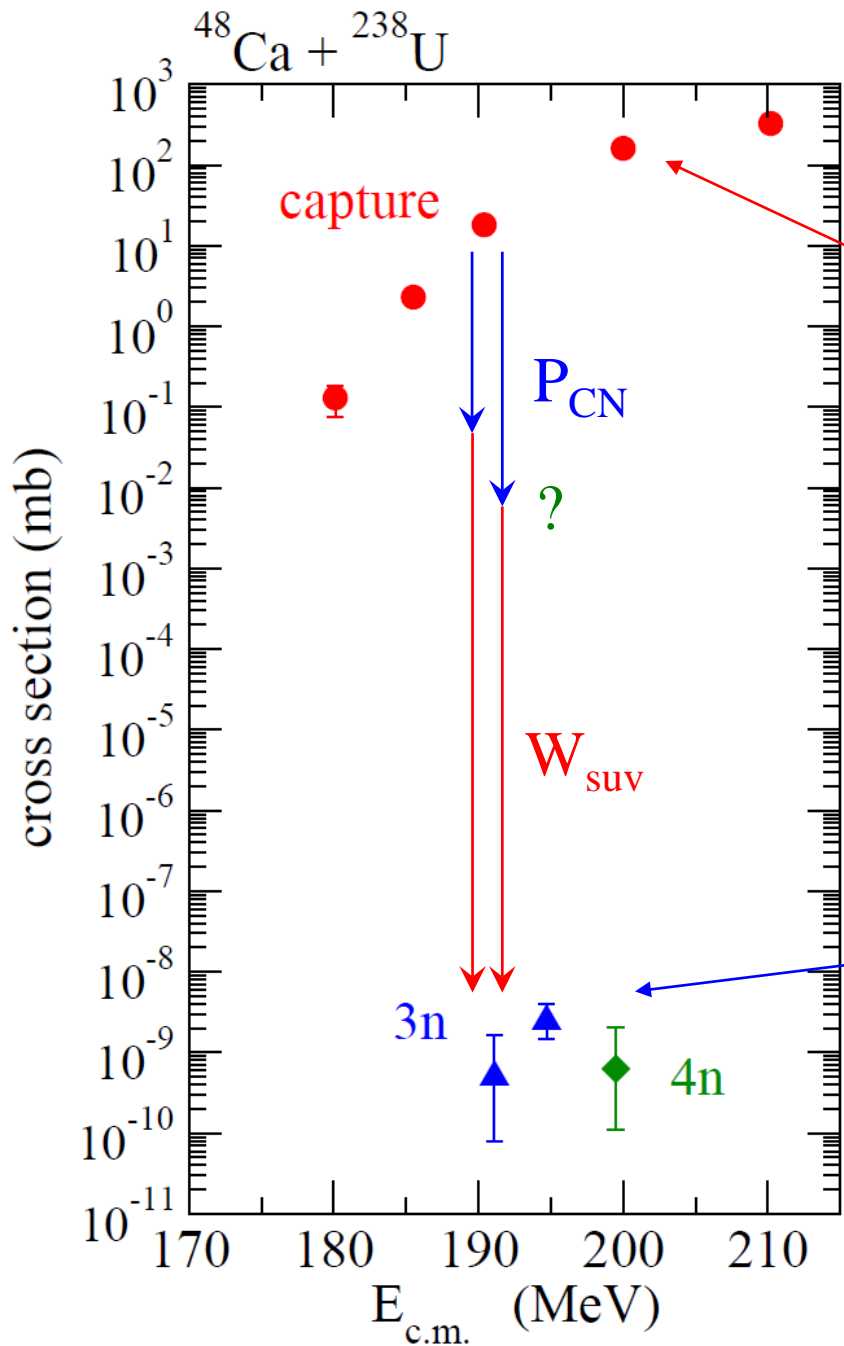
a small uncertainty in each process  
can be largely amplified



challenges: how to reduce theoretical  
uncertainties and make a reliable  
prediction

typical values for  
Ni+Pb reaction





another issue:

no experimental data for  $P_{\text{CN}}$

$$\sigma_{\text{cap}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) T_l(E)$$

~~$$\sigma_{\text{CN}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) T_l(E) \times P_{\text{CN}}$$~~

not available

$$\sigma_{\text{ER}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) T_l(E) \times P_{\text{CN}} \cdot W_{\text{suv}}$$

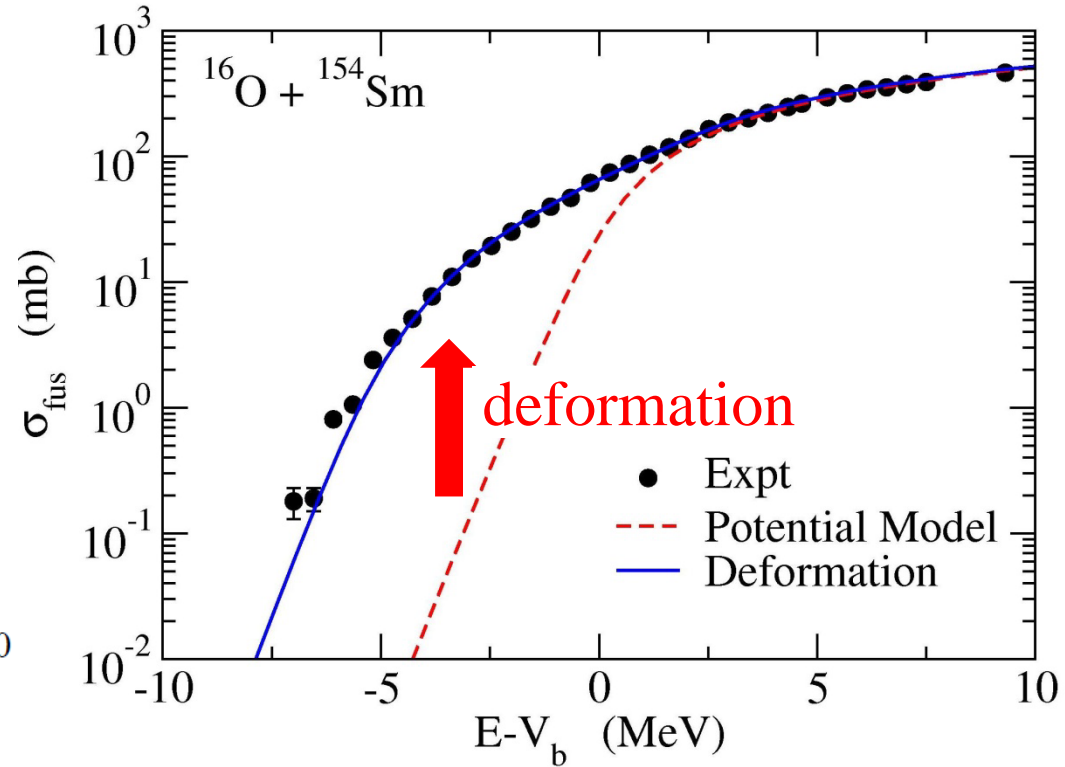
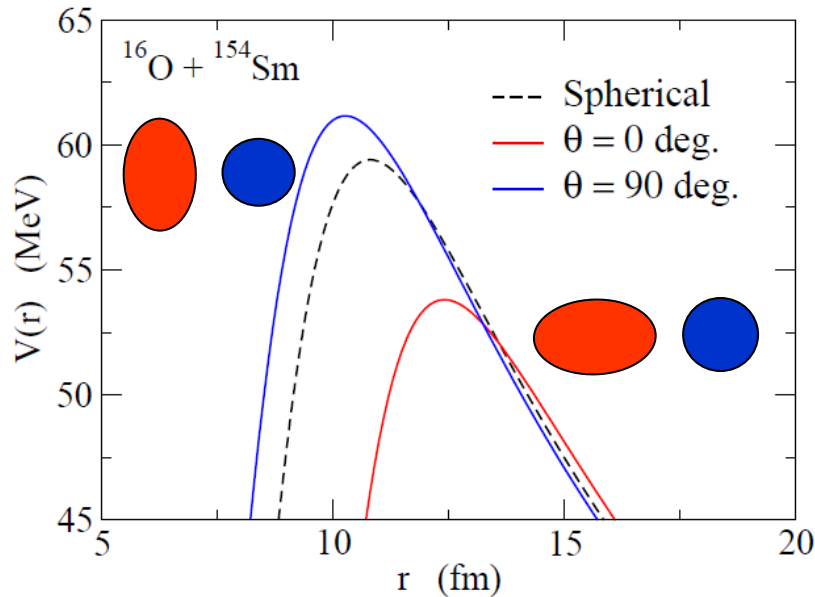
large uncertainties

# Approaching phase: coupled-channels method

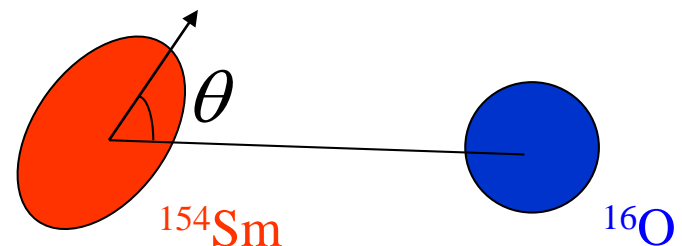
Sub-barrier enhancement of capture cross sections

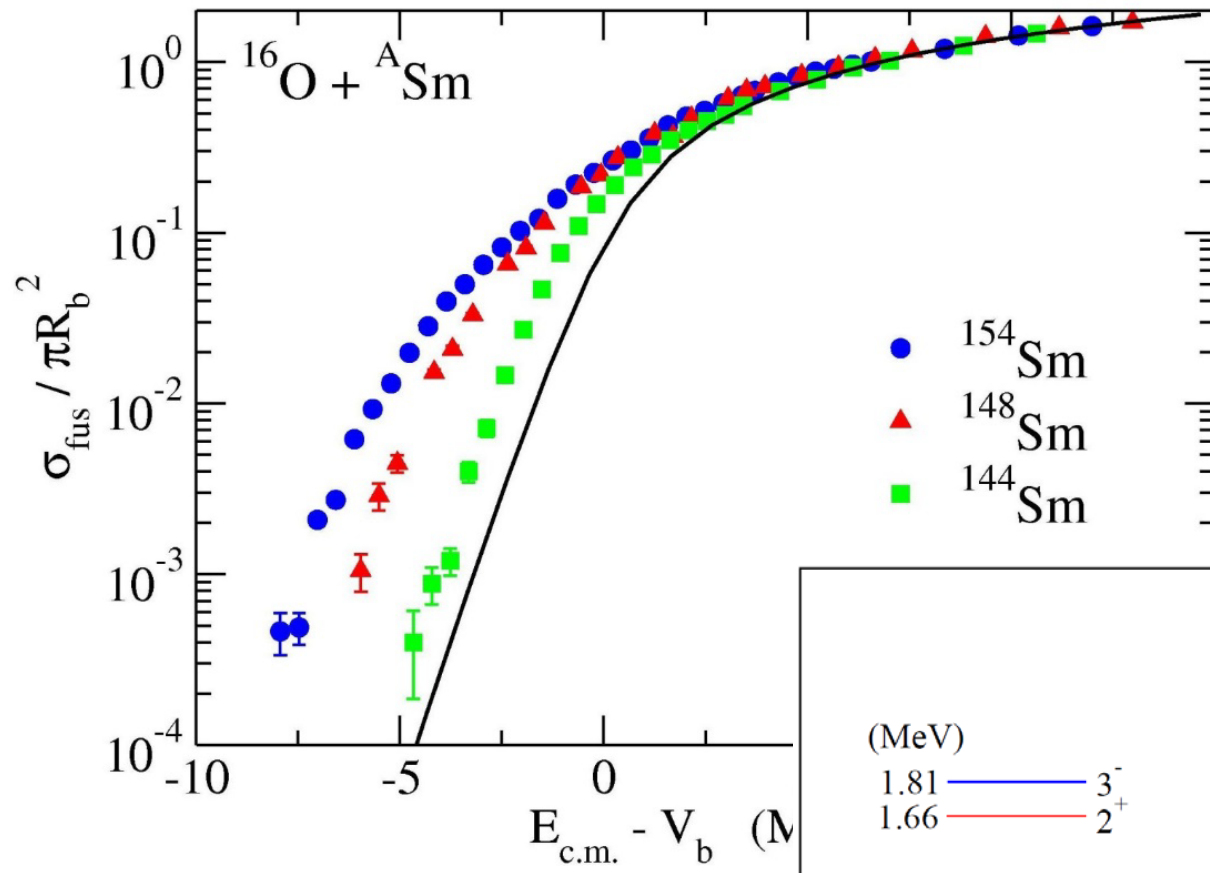
← channel coupling effects

## role of deformation



$$\sigma_{\text{cap}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{cap}}(E; \theta)$$



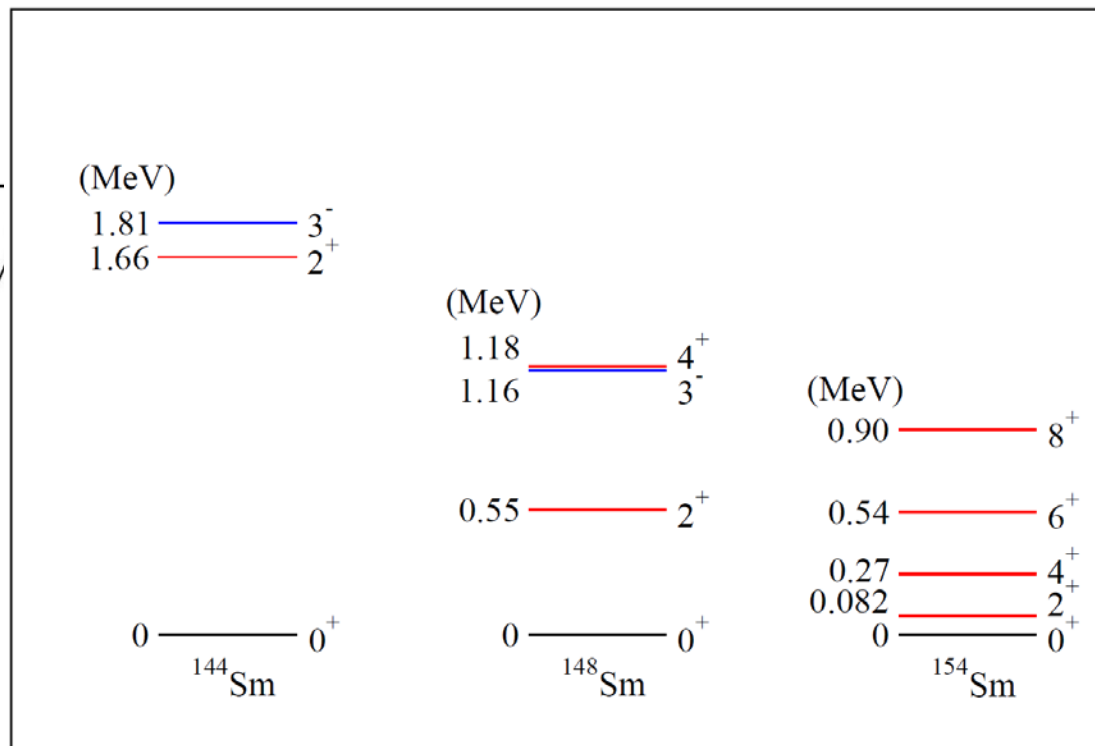


sub-barrier enhancement  
in general

→ couplings to low-lying  
collective motions



**coupled-channels method**



# C.C. method: a standard tool for H.I. sub-barrier fusion reactions

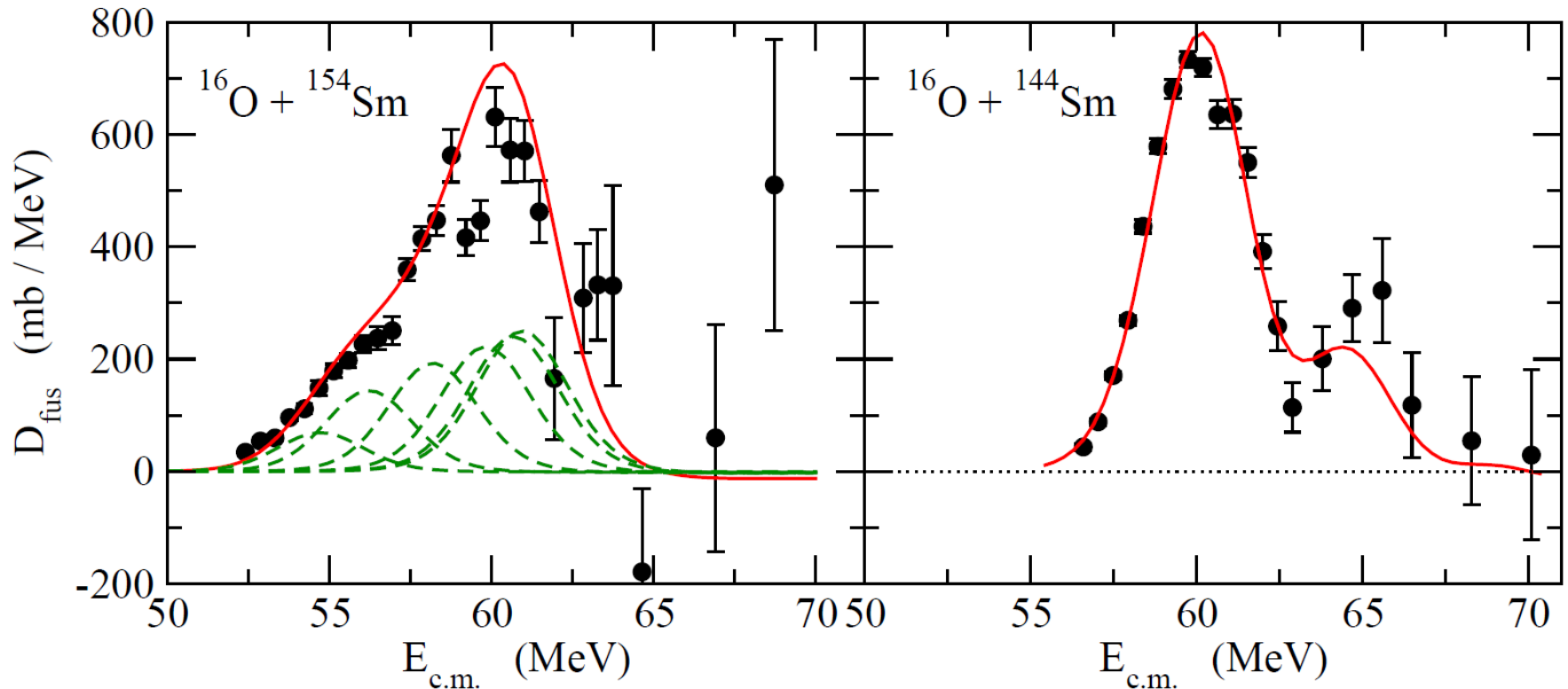
CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 ('99) 143)

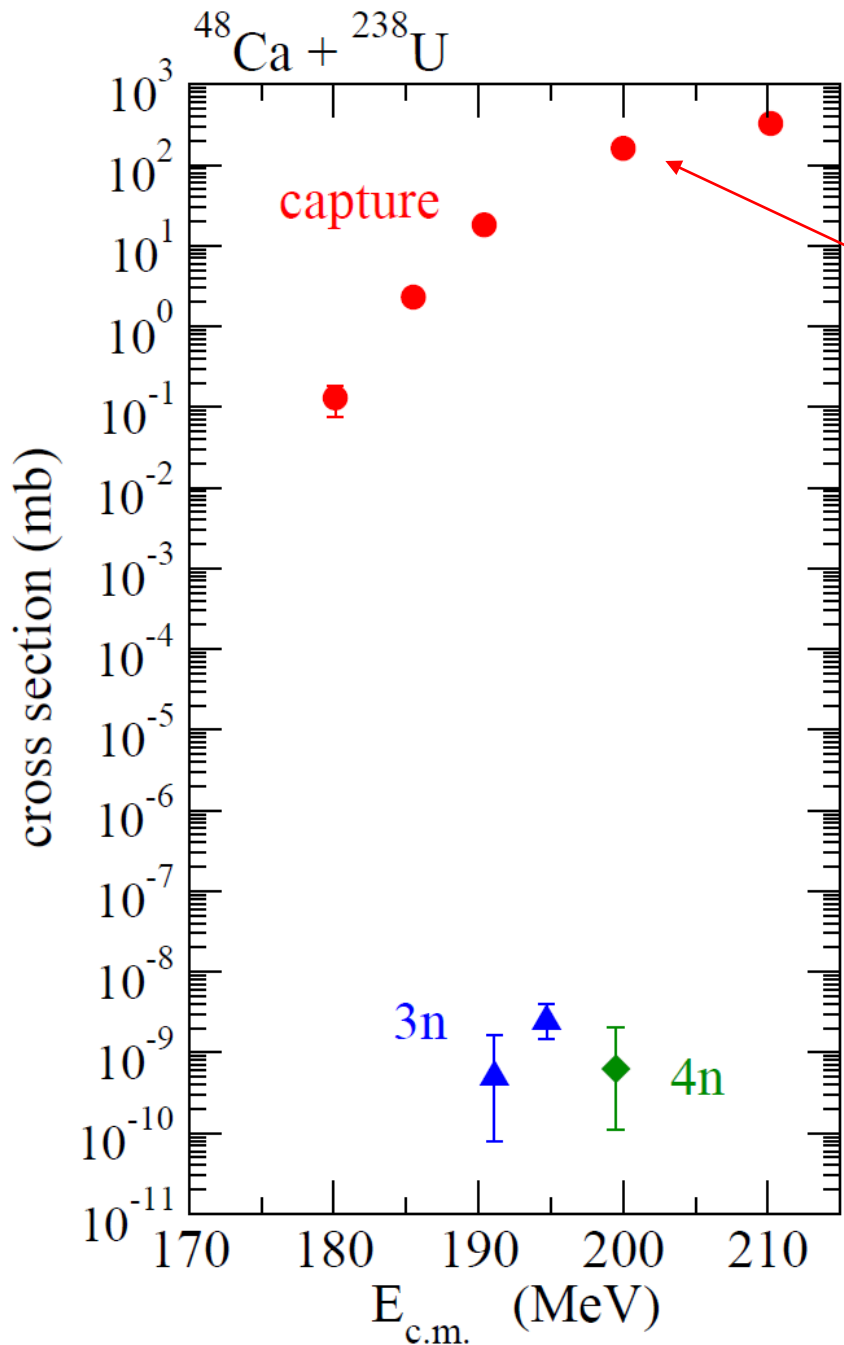
cf. Coulomb excitations: important for SHE

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91))

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

— c.c. calculations





Experimental data exist for  $\sigma_{\text{cap}}$

$$\sigma_{\text{cap}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) T_l(E)$$

inputs for C.C. calculations

i) inter-nucleus potential

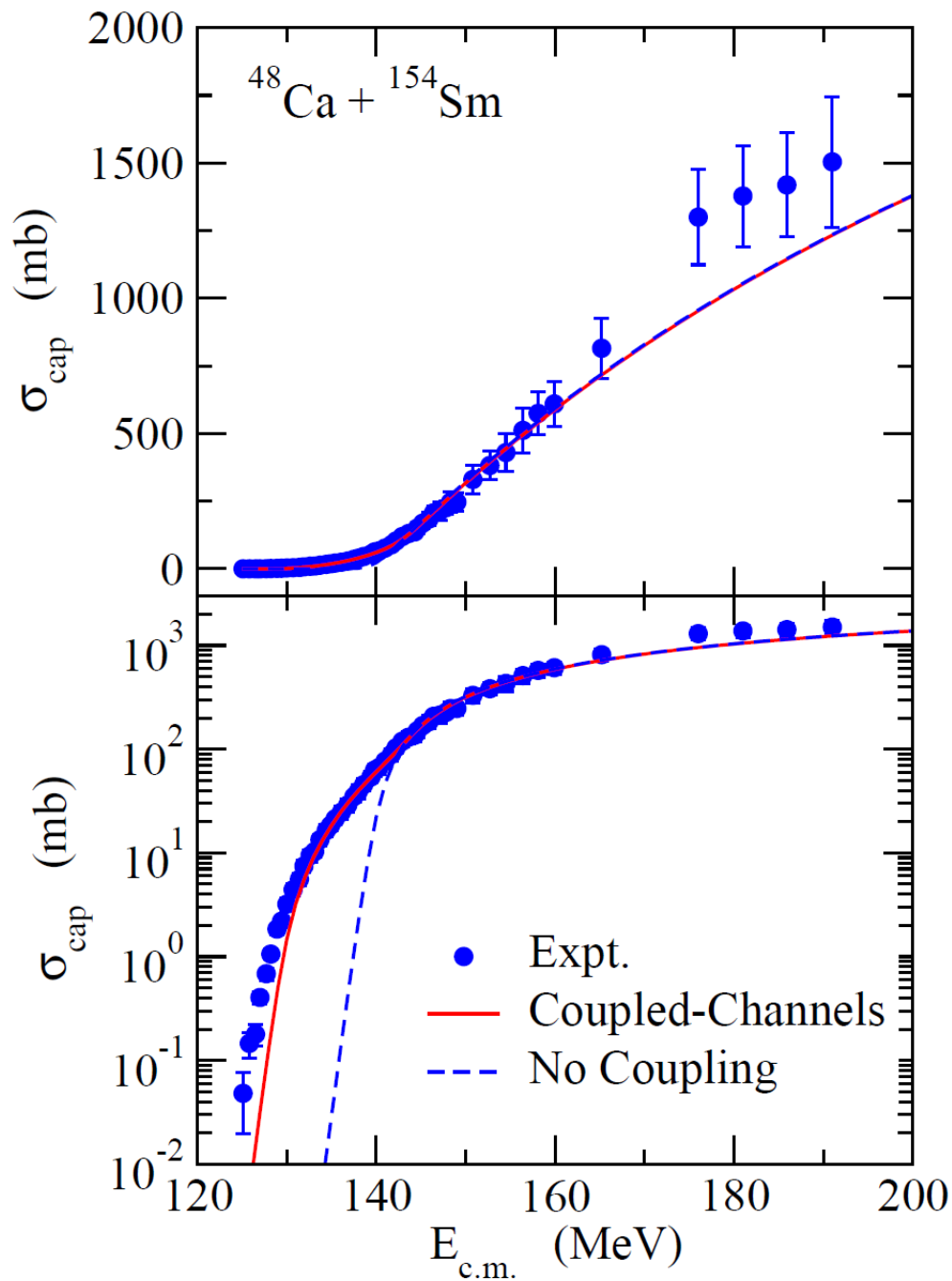
fit to exp. data for capture

ii) intrinsic motions

collective model

(rigid rotor / harmonic vibrators)





calculations with CCFULL  
 $^{48}\text{Ca} + ^{154}\text{Sm}$  (rot: up to  $16^+$ )

$$\sigma_{\text{exp}} \longrightarrow \sigma_{\text{cc}} \longrightarrow T_l(E)$$

\* enhancement at very low energies  $\rightarrow$  transfer process

# Recent development: CCFULL + nuclear structure

inputs for C.C. calculations

i) inter-nucleus potential

fit to exp. data for capture

ii) intrinsic motions

collective model

(rigid rotor / harmonic vibrators)

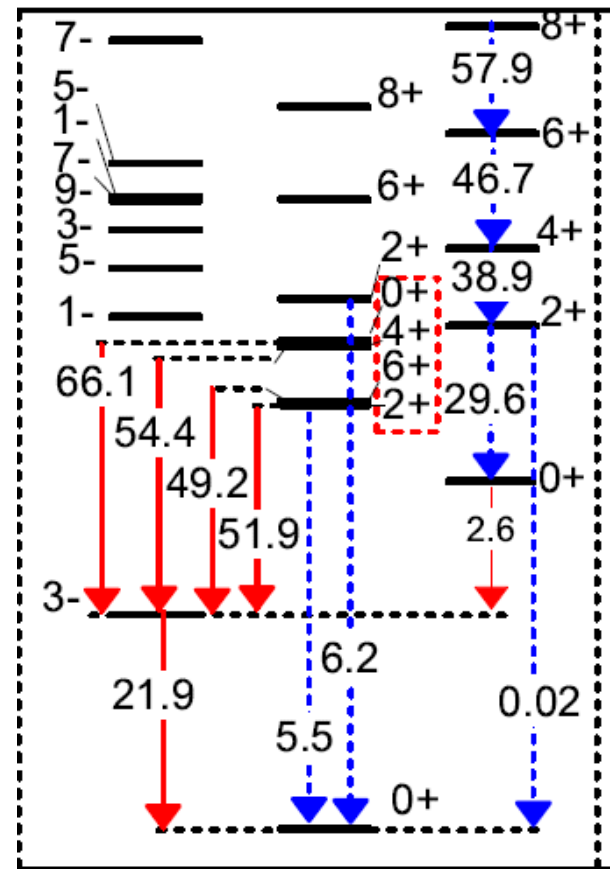


CCFULL

+ nuclear structure calculations

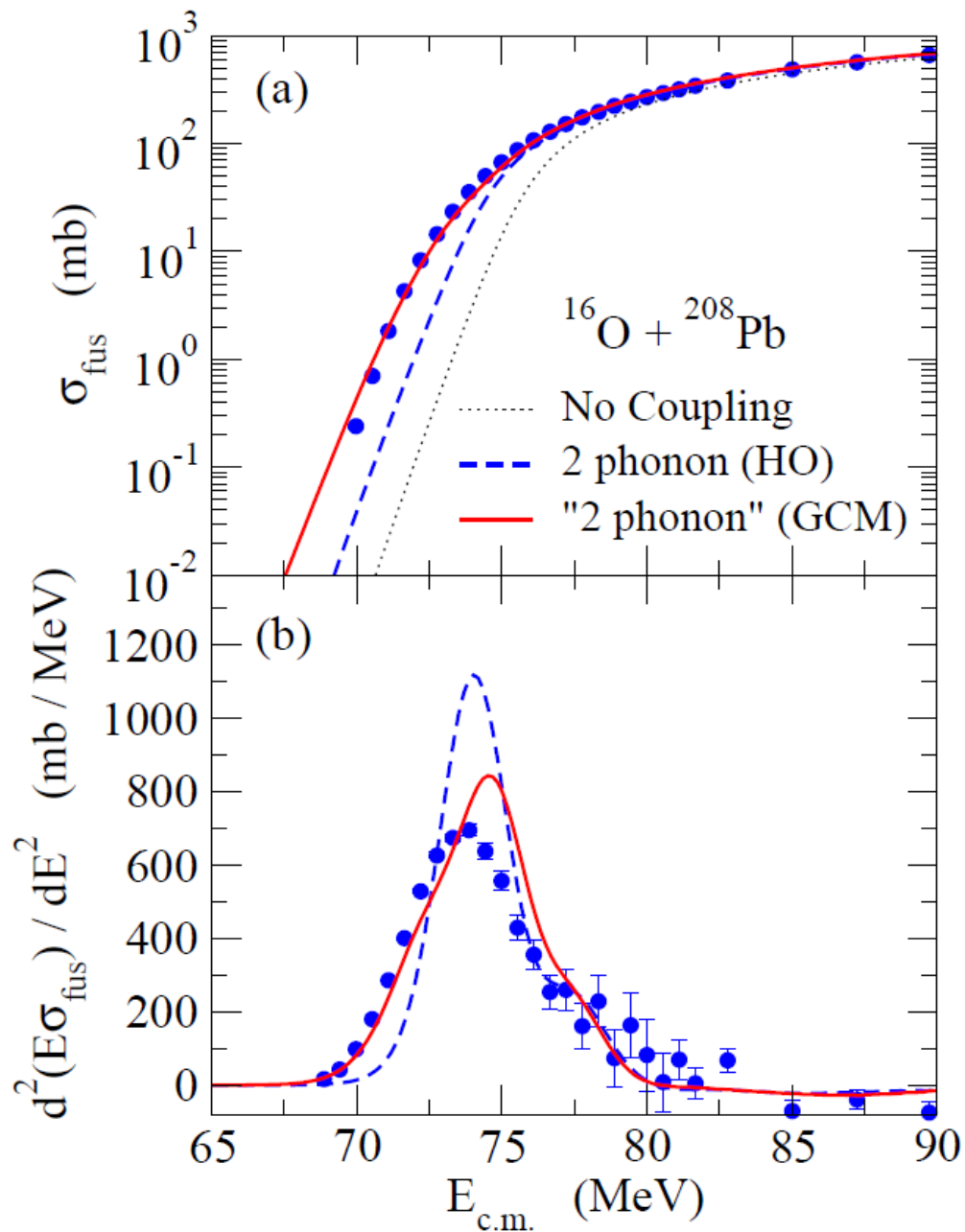
(GCM, Shell Model, IBM.....)

(b) GCM ( $\beta_2$ - $\beta_3$ )  $^{208}\text{Pb}$

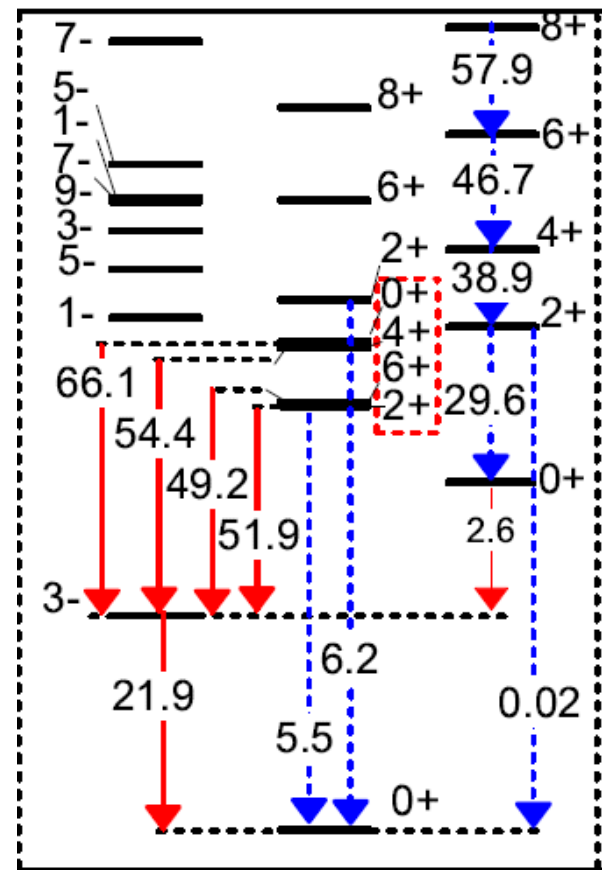


J.M. Yao and K.H.,  
PRC94 ('16) 11303(R)

# GCM + CCFULL



# (b) GCM ( $\beta_2$ - $\beta_3$ ) $^{208}\text{Pb}$

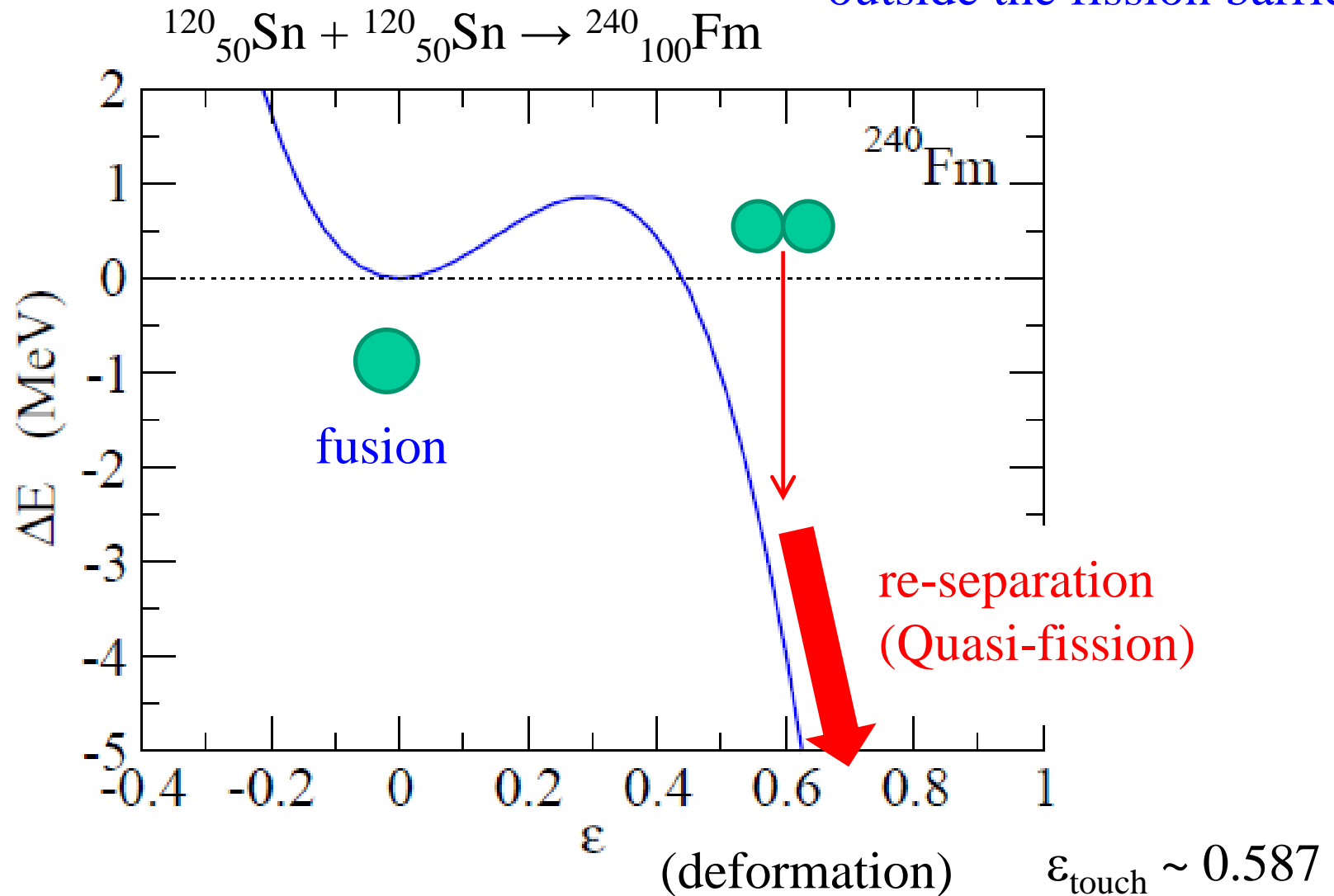


J.M. Yao and K.H.,  
PRC94 ('16) 11303(R)

# Formation phase: Langevin method

Fission barrier (Liquid Drop Model)

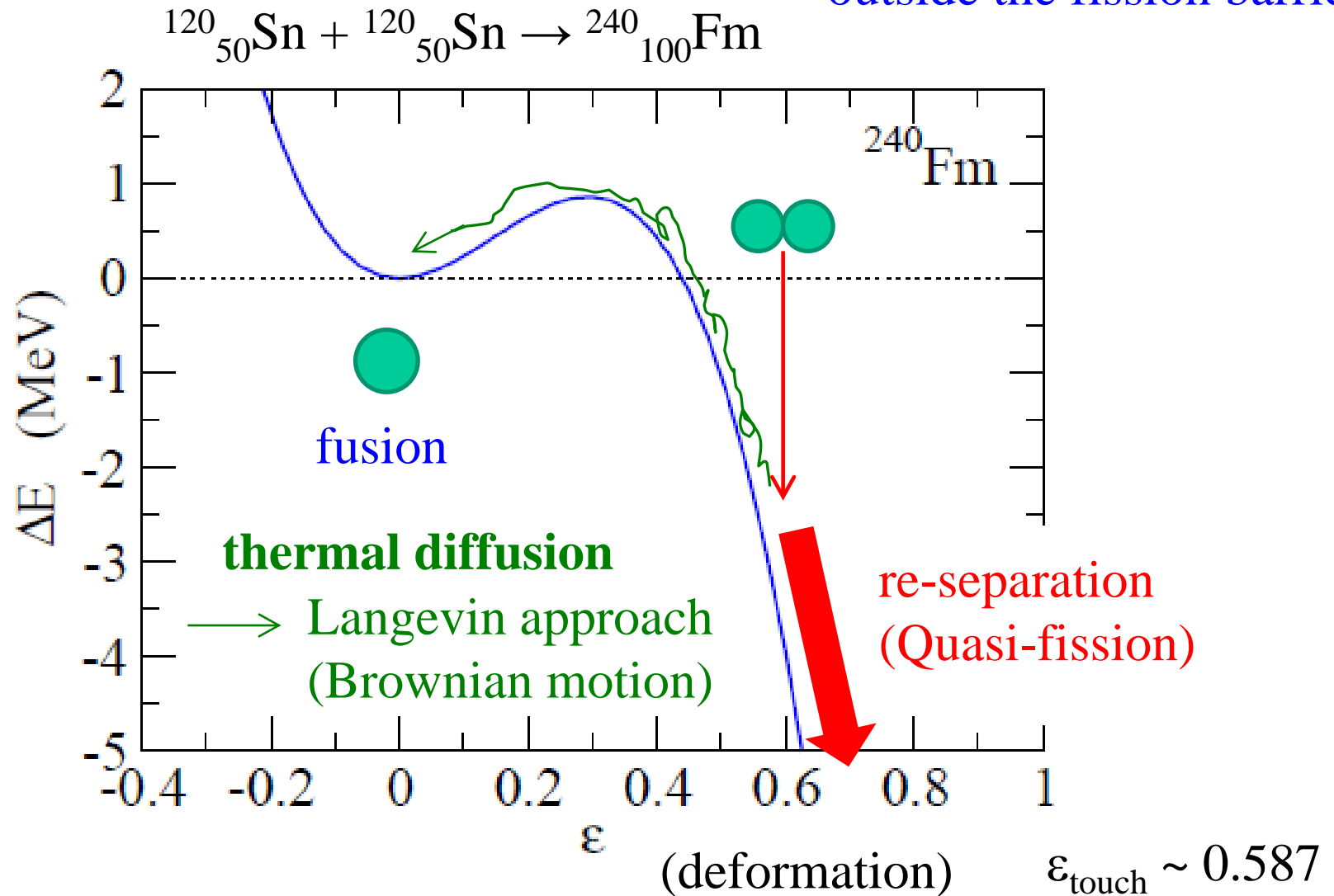
touching point:  
outside the fission barrier



# Formation phase: Langevin method

Fission barrier (Liquid Drop Model)

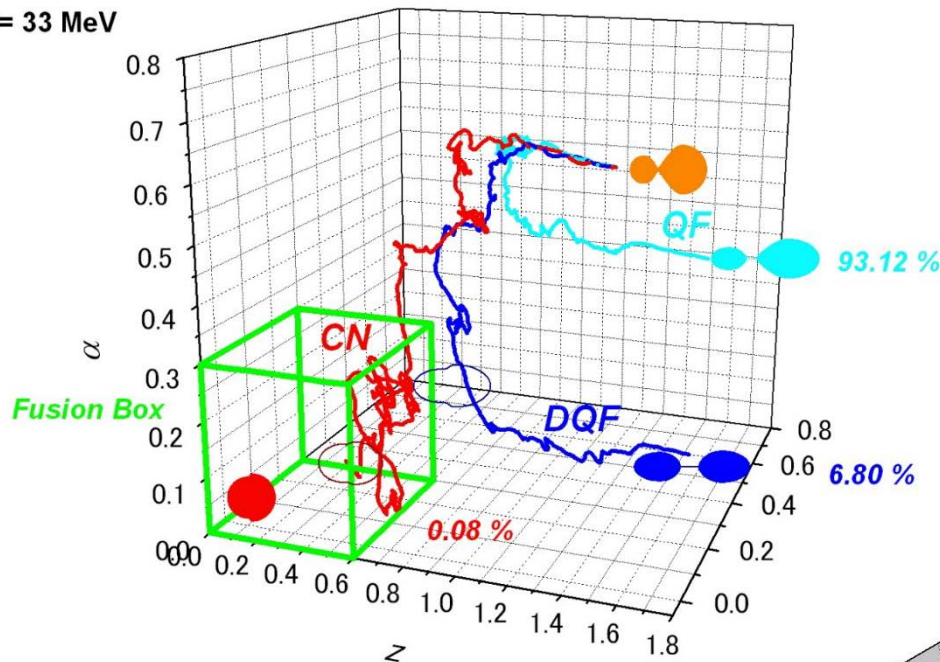
touching point:  
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# Langevin approach



$E^* = 33 \text{ MeV}$



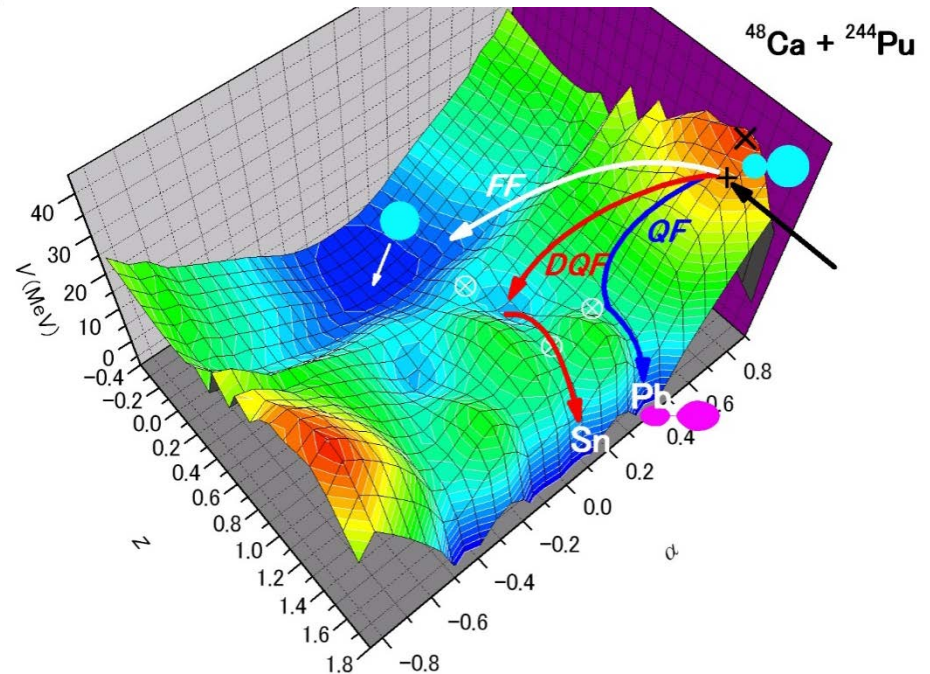
$q$ : ▪ internuclear separation ( $z$ )  
 ▪ deformation ( $\delta$ )  
 ▪ mass asymmetry ( $\alpha$ )  
 of the two fragments

- ✓ Y. Aritomo et al.
- ✓ Zagrebaev-Greiner

multi-dimensional extension of:

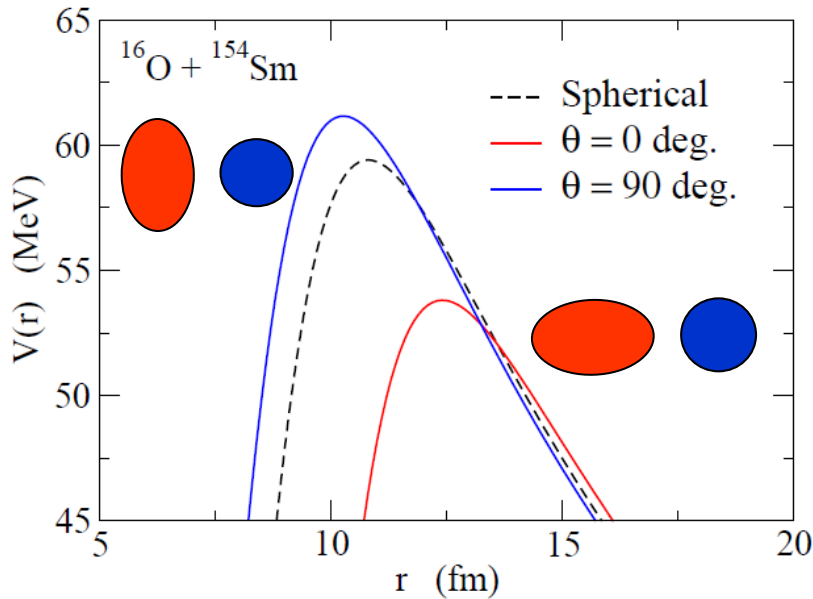
$$m \frac{d^2 q}{dt^2} = - \frac{dV(q)}{dq} - \gamma \frac{dq}{dt} + R(t)$$

$\gamma$ : friction coefficient  
 $R(t)$ : random force



## extension to deformed systems

hot fusion: actinide targets

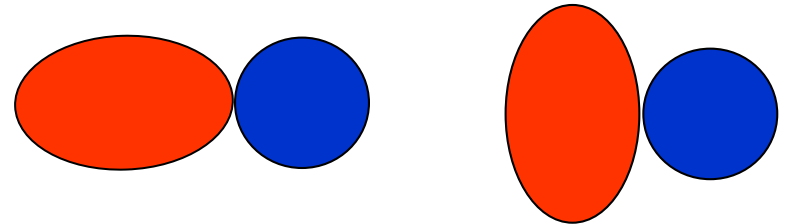


$$\sigma_{\text{cap}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{cap}}(E; \theta)$$

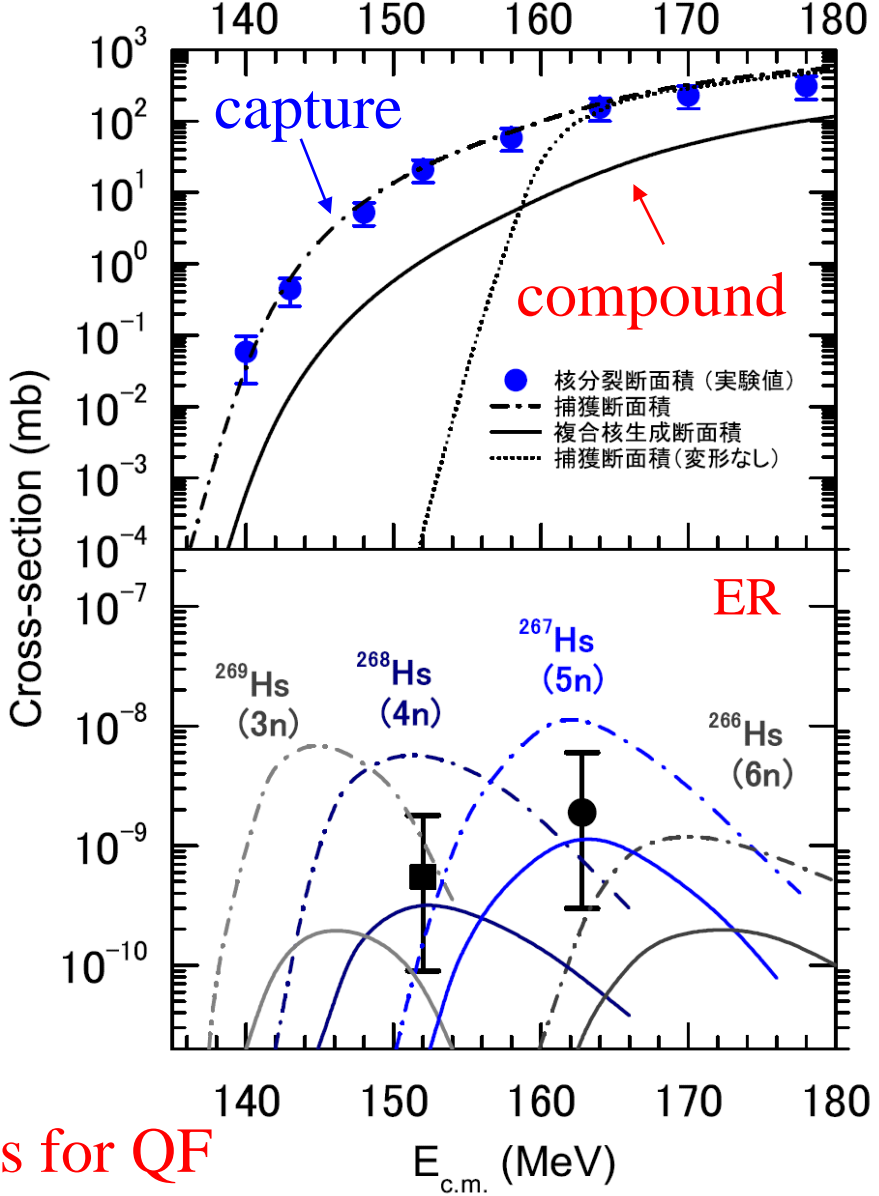
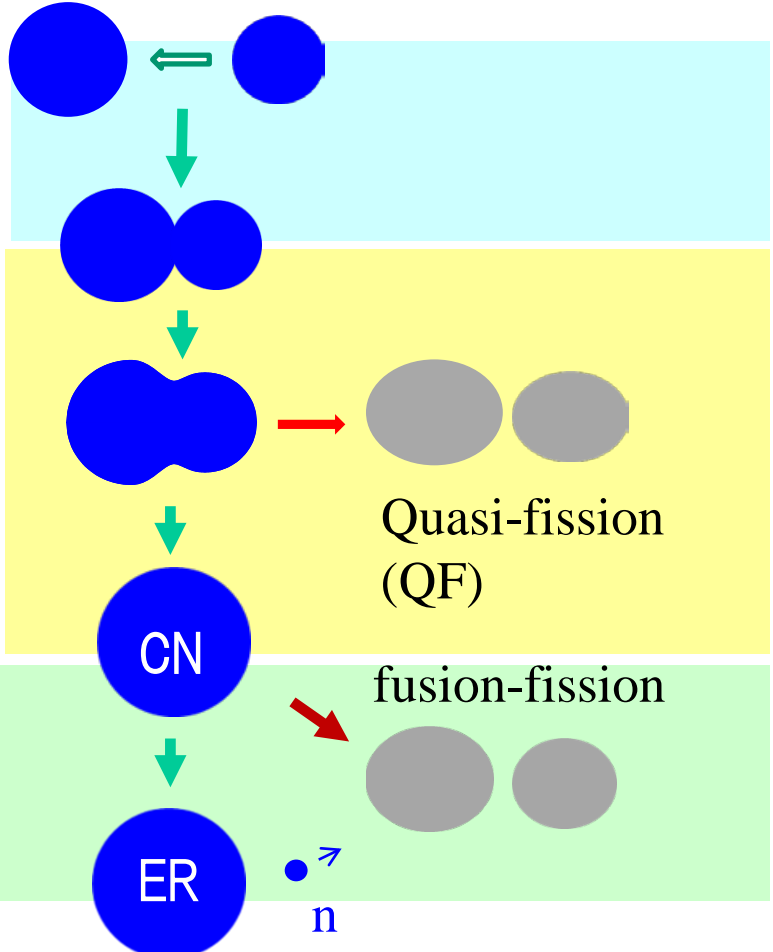
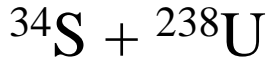


$$\sigma_{\text{ER}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{ER}}(E; \theta)$$

$$R_{\text{touch}}(\theta) = R_p + R_T(1 + \beta_2 Y_{20}(\theta))$$



Y. Aritomo, K.H., K. Nishio, S. Chiba,  
PRC85('12)044614

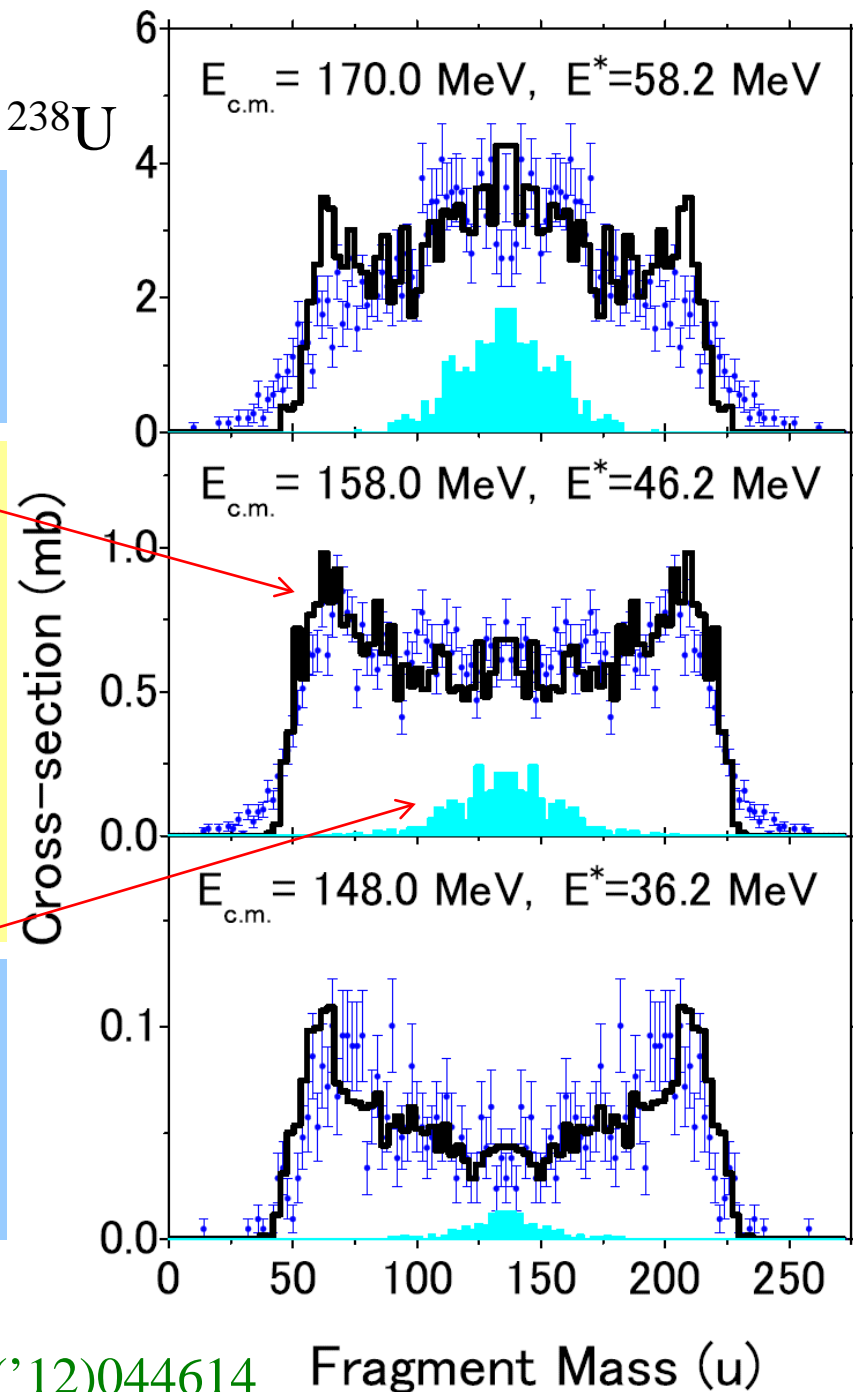
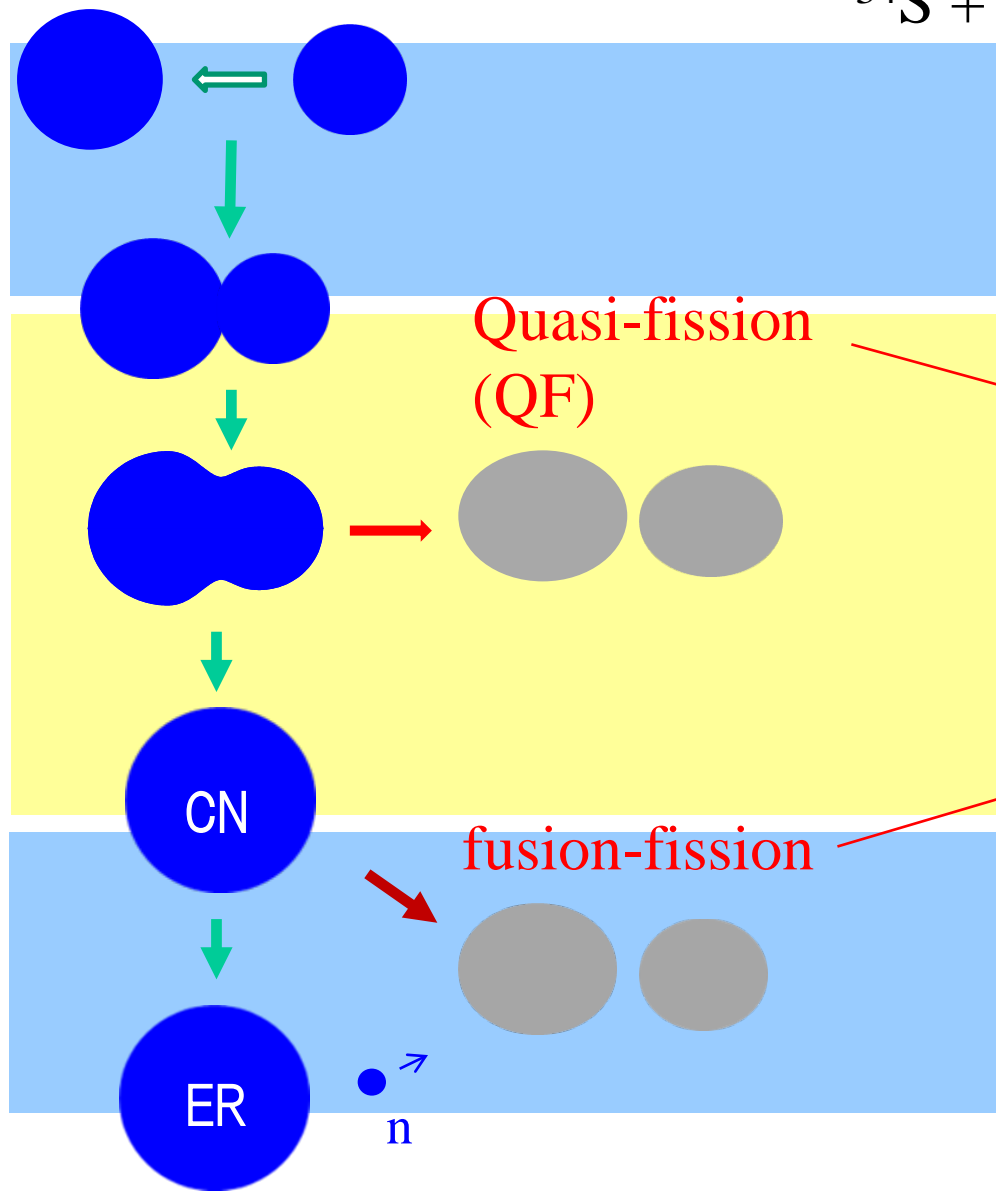


No data for  $\sigma_{\text{CN}}$   
 → important to check whether the model reproduces other quantities for QF

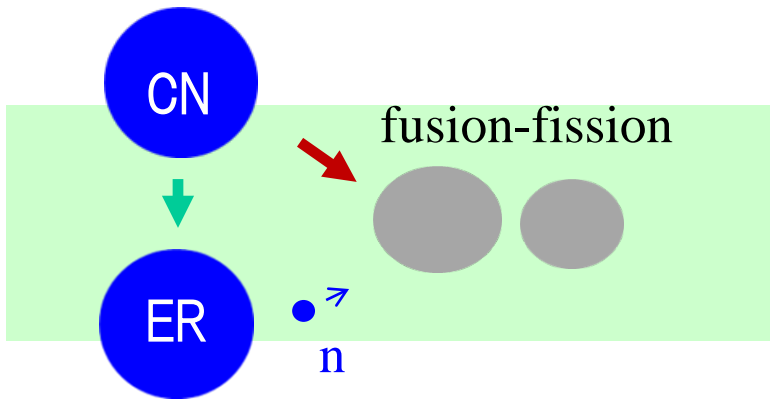


# fragment mass distribution for QF

$^{34}\text{S} + ^{238}\text{U}$



# Survival phase: statistical model

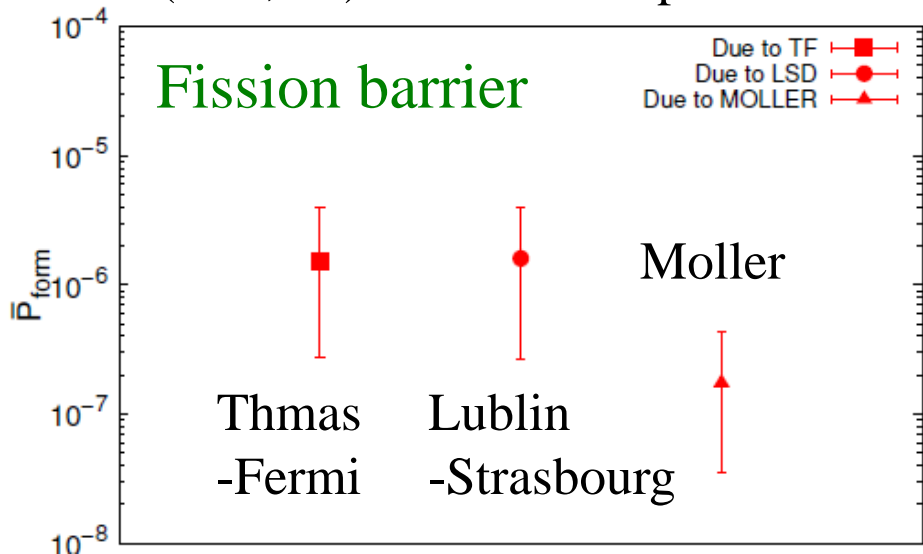


statistical model

$$\frac{\Gamma_n}{\Gamma_f} \sim \frac{\int_0^{E^* - S_n} d\epsilon \rho_{A-1}(E^* - S_n - \epsilon)}{\int_0^{E^* - B_f} d\epsilon \rho_f(E^* - B_f - \epsilon)}$$

method well established, but too many parameters/model dependence  
 $B_f$ , level densities, friction.....

$^{208}\text{Pb}$  ( $^{58}\text{Fe}$ , 1n)  $^{265}\text{Hs}$  at the optimum energy



$$\langle P_{\text{form}} \rangle = \frac{\sigma_{\text{ER}}^{(\text{exp})}}{\frac{\pi}{k^2} \sum_l (2l + 1) W(E^*)}$$

KEWPIE2

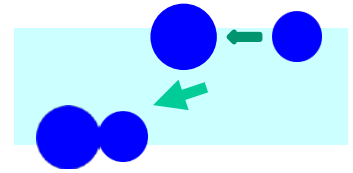
H. Lu, D. Boilley, Y. Abe,  
 and C. Shen, preprint.



# Theoretical challenges towards $Z=119-120$ and island of stability

## 1. Approaching phase

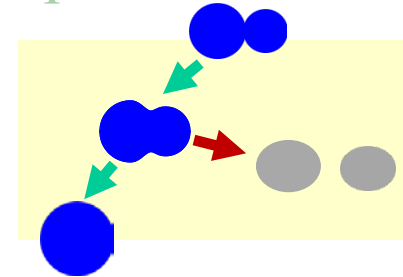
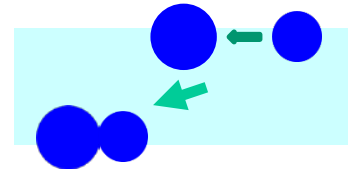
- ✓ coupled-channels for stable nuclei: well established
- ✓ still challenging: breakup and multi-nucleon transfer processes
- ✓ barrier distribution  $\rightarrow$  Bayesian statistics



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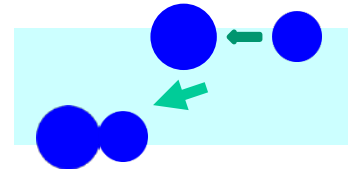
## 2. Formation phase

- ✓ Langevin approach
- ✓ challenge: remove the restriction of  $\delta_1 = \delta_2$  (4-dim. Langevin)  
→ important for Ti, V, Cr induced fusion
- ✓ neutron emissions during the shape evolution
- ✓ a change in the potential surface due to the neutron emissions

# Theoretical challenges towards Z=119-120 and island of stability

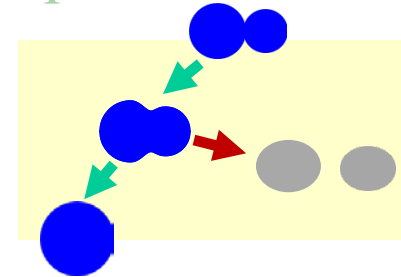
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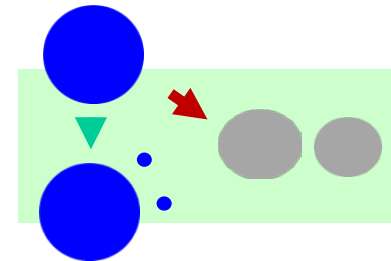
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## 3. Survival phase

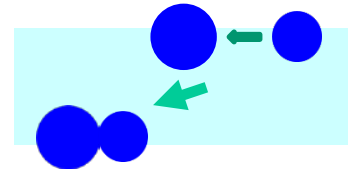
- ✓ statistical model
- ✓ many parameters ( $B_f$ , level density,....)
- ✓ difficult for an absolute value, but how about relative quantities?



# Theoretical challenges towards Z=119-120 and island of stability

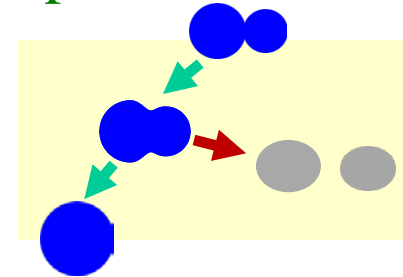
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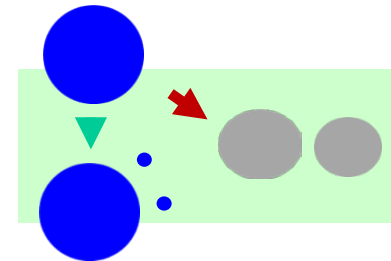
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**need continuous efforts both from theory and experiment**





# Bayesian approach to sub-barrier fusion

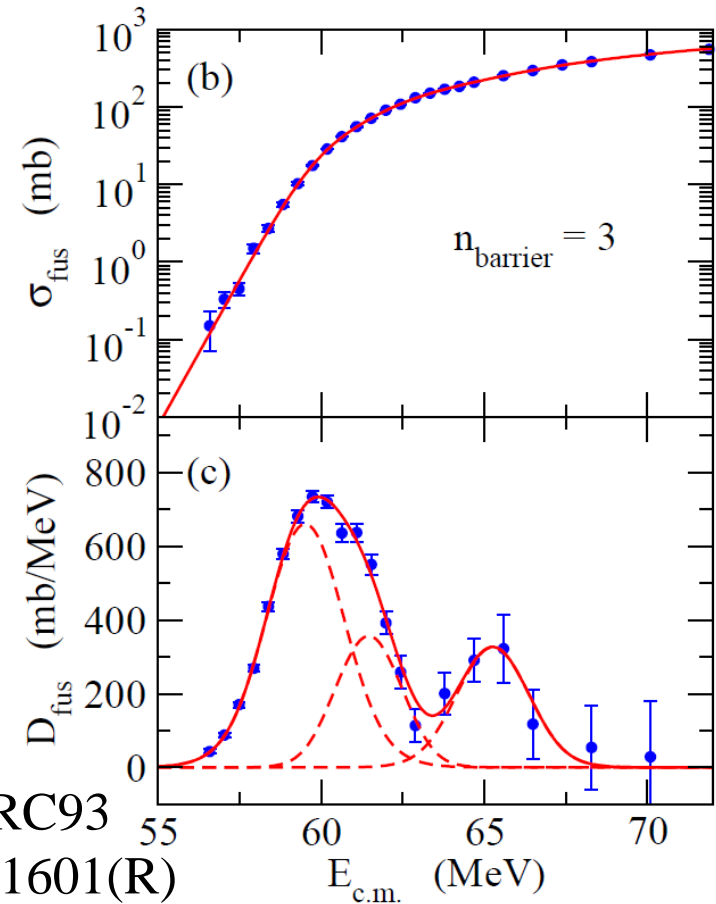
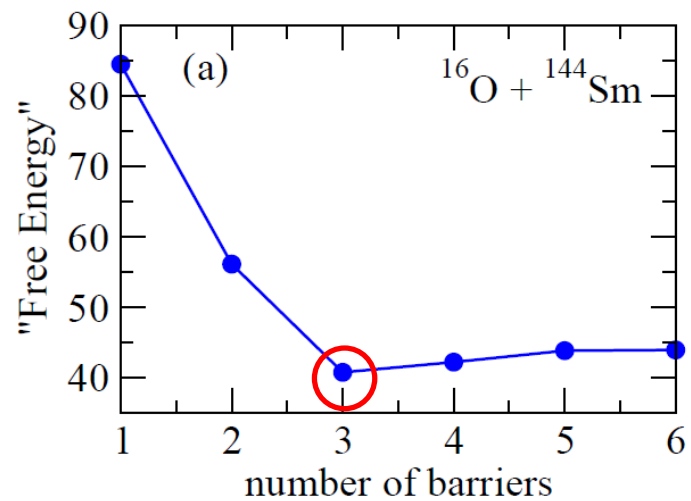
$$D_{\text{exp}}(E) = \sum_{i=1}^K w_k D_0(E; B_k, s_k)$$

optimize  $K, w_k, B_k, s_k$   
in order to reproduce data

problem: how to fix  $K$  (# of barrier)?  
(over-fitting)

Guideline for  $K$  using the Bayesian  
spectrum deconvolution

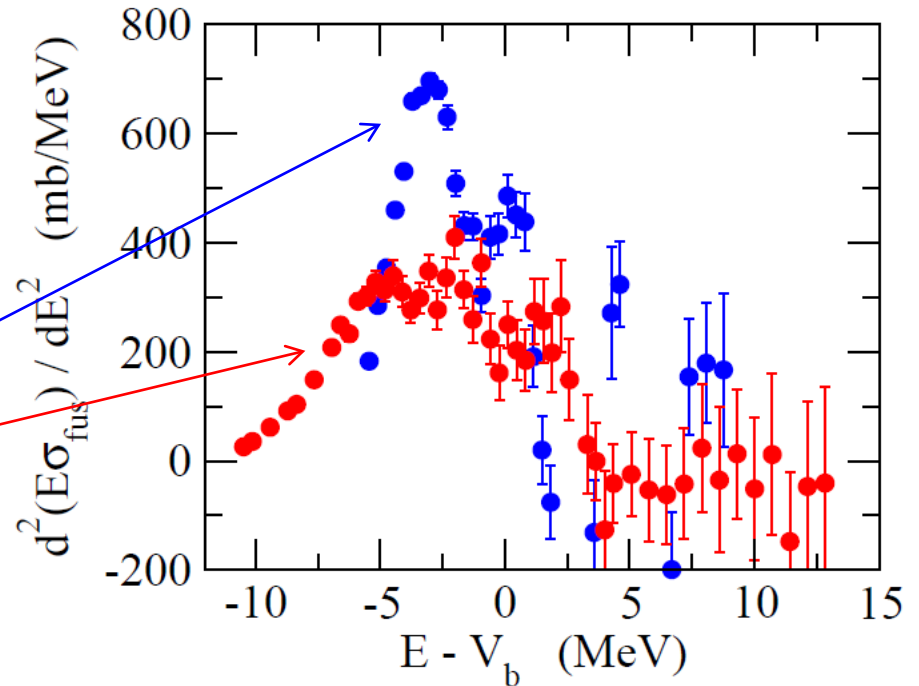
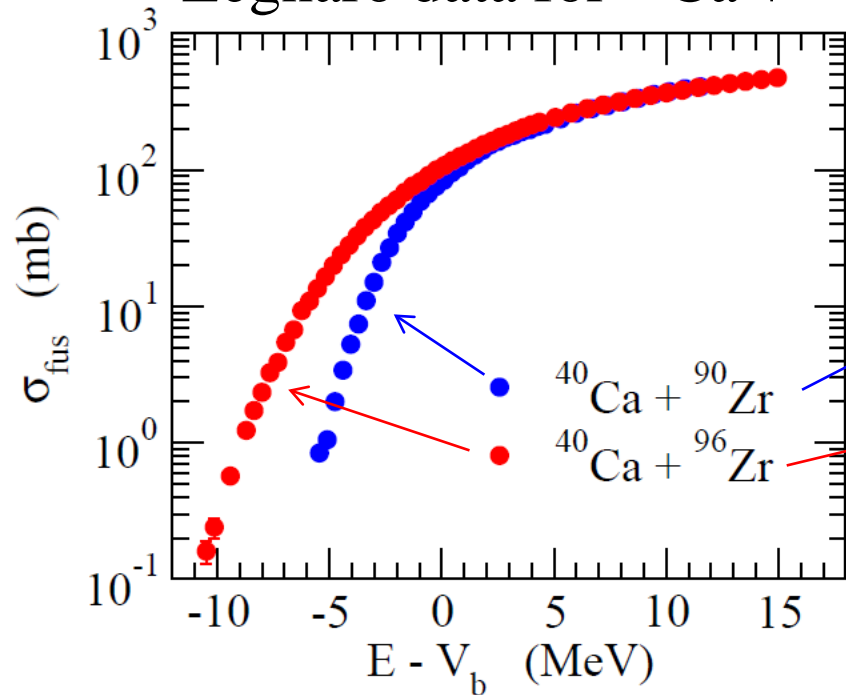
→ extraction of  $T_l$   
from barrier distribution



K.H., PRC93  
(‘16) 061601(R)

# Role of multi-neutron transfer process in subbarrier fusion

Legnaro data for  $^{40}\text{Ca} + ^{90,96}\text{Zr}$



H. Timmers et al., NPA633('98)421

$^{40}\text{Ca} + ^{96}\text{Zr}$

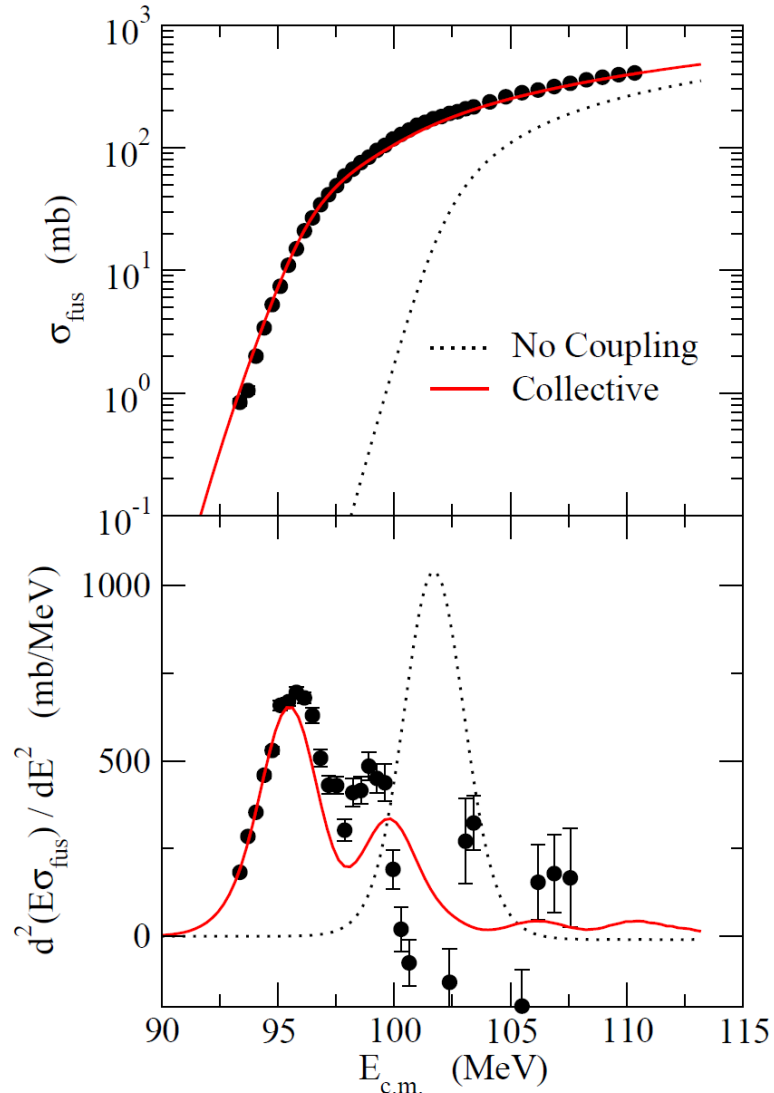
- more enhancement of fusion cross sections
- flatter barrier distribution
  - ✓ stronger octupole collectivity
  - ✓ multi-neutron transfer process

# stronger octupole collectivity in $^{96}\text{Zr}$

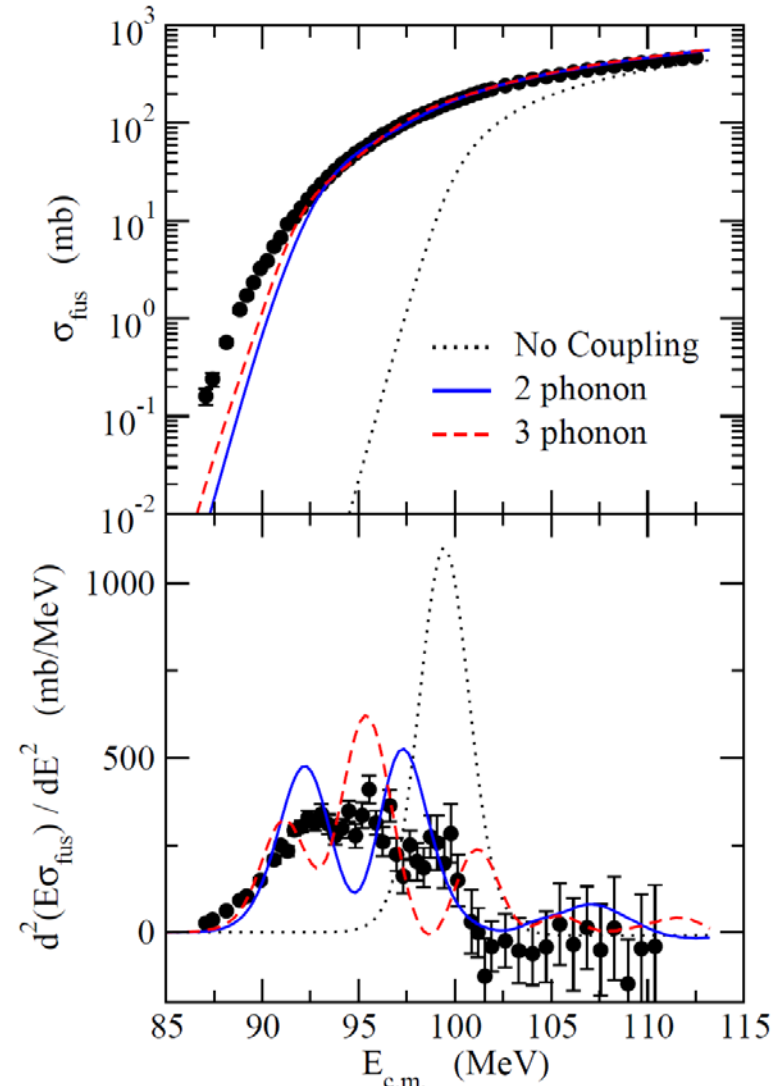
$$^{90}\text{Zr}: B(E3: 3^-_1 \rightarrow 0^+) = 29.1 \text{ W.u.}, E_{3^-} = 2.748 \text{ MeV}$$

$$^{96}\text{Zr}: B(E3: 3^-_1 \rightarrow 0^+) = 52.7 \text{ W.u.}, E_{3^-} = 1.897 \text{ MeV}$$

$^{40}\text{Ca} (3^-) + ^{90}\text{Zr} (2^+ : 2 \text{ phonon}, 3^- : 2 \text{ phonon})$



$^{40}\text{Ca} (3^-) + ^{96}\text{Zr} (2^+ : 2 \text{ phonon}, 3^- : 2 \text{ phonon})$



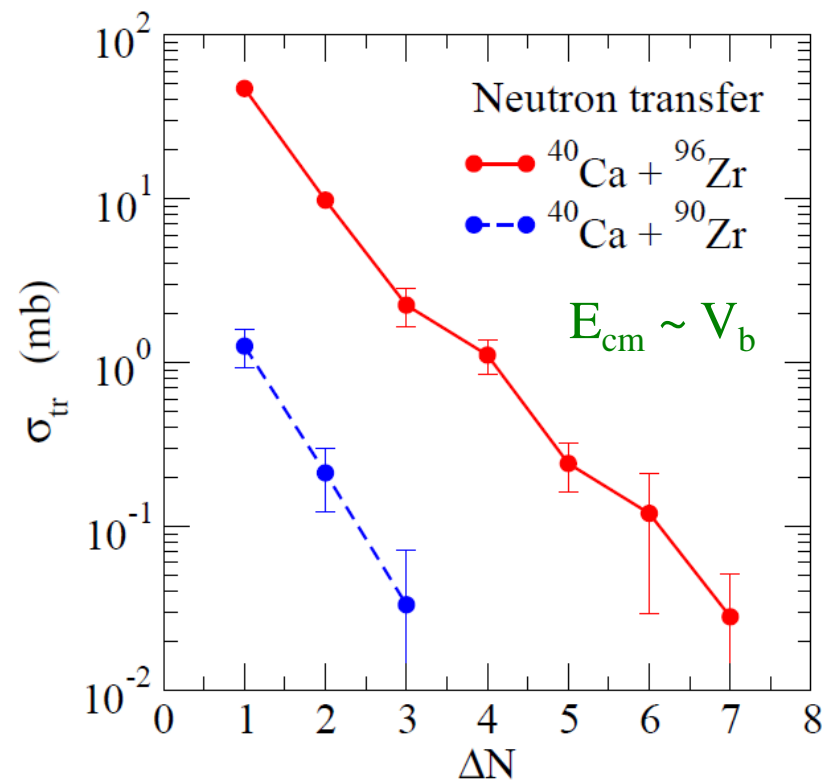
# Q-values for multi-neutron transfer channels

$Q_{gg}$  (MeV)

	$^{40}\text{Ca} + ^{90}\text{Zr}$	$^{40}\text{Ca} + ^{96}\text{Zr}$
+1n	-3.61	+0.51
+2n	-1.44	+5.53
+3n	-5.86	+5.24
+4n	-4.17	+9.64
+5n	-9.65	+8.42
+6n	-9.05	+11.62

cf.  $Q_{gg}(-1n) = -8.45$  MeV for  $^{40}\text{Ca} + ^{90}\text{Zr}$

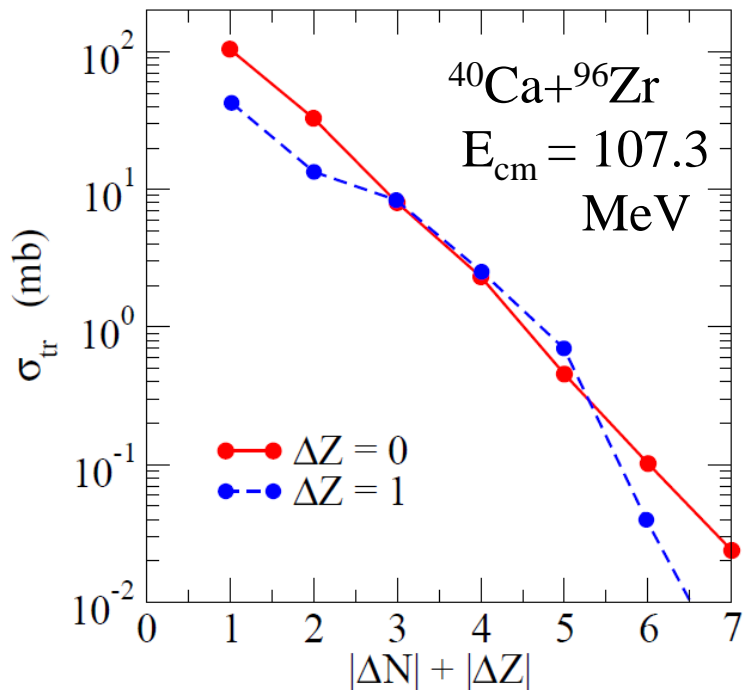
Experimental data for  
total transfer cross sections



G. Montagnoli et al.,  
J. of Phys. G23('97)1431

# New simple model

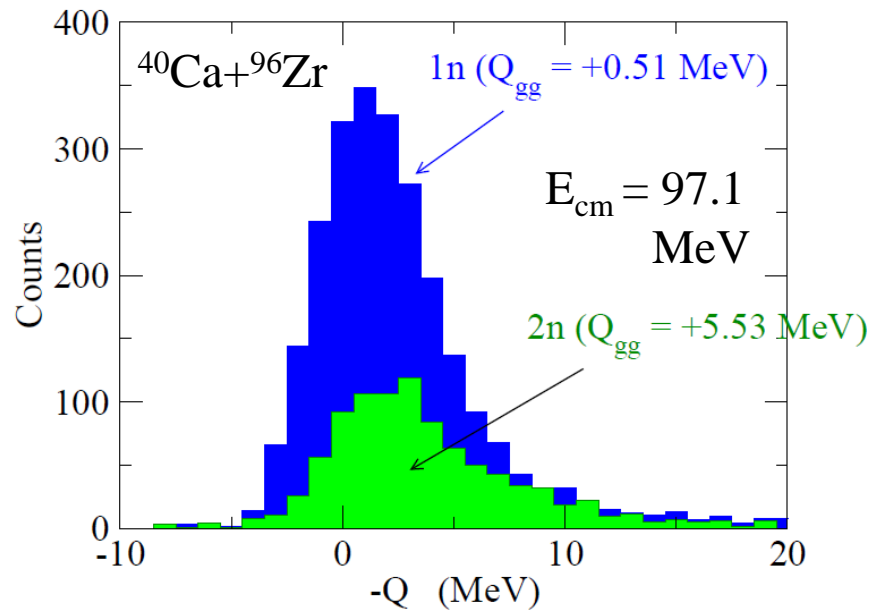
## 1. Neutron transfer chain only



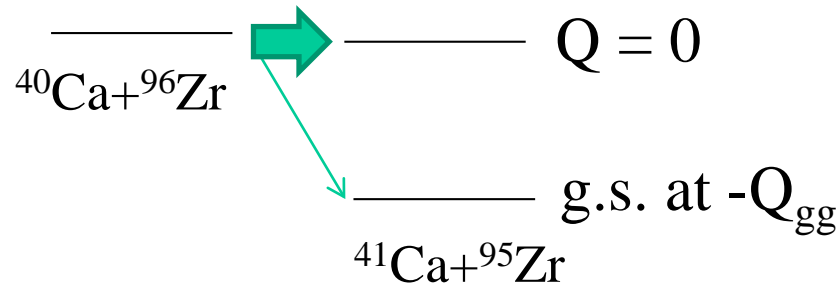
S. Szilner et al., PRC76('07)024604

proton transfer: less strongly coupled to the entrance channel

## 2. Approximate Q-value distribution



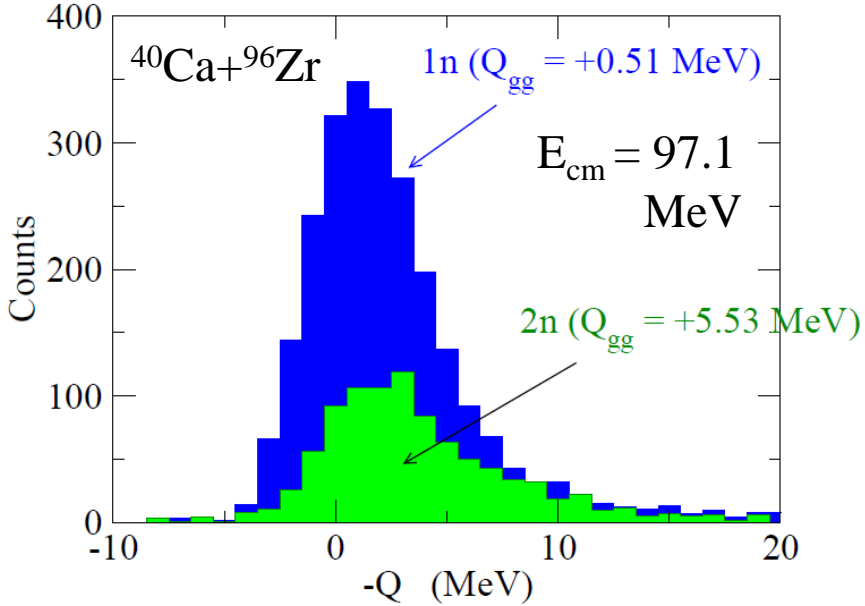
L. Corradi et al., PRC84('11)034603  
 (Recent data with PRISMA)



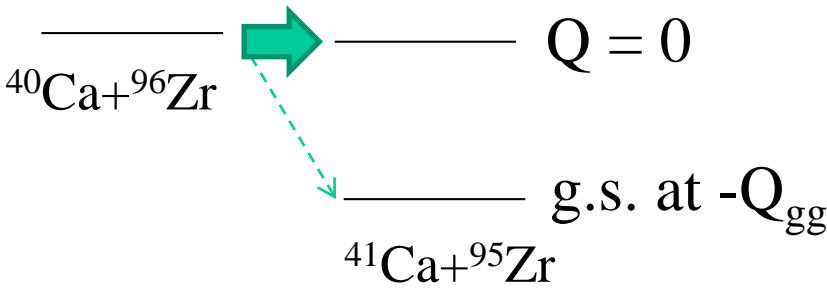
Q-value matching

$\rightarrow$  put all the strength to a single state at  $Q=0$

## 2. Approximate Q-value distribution



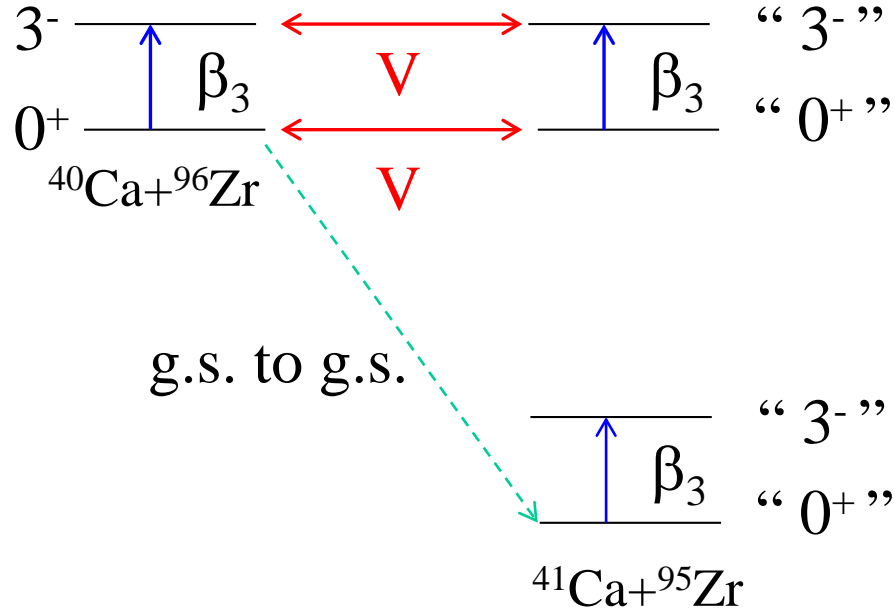
L. Corradi et al., PRC84('11)034603  
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Q-value matching

$\longrightarrow$  put all the strength to a single state at  $Q=0$

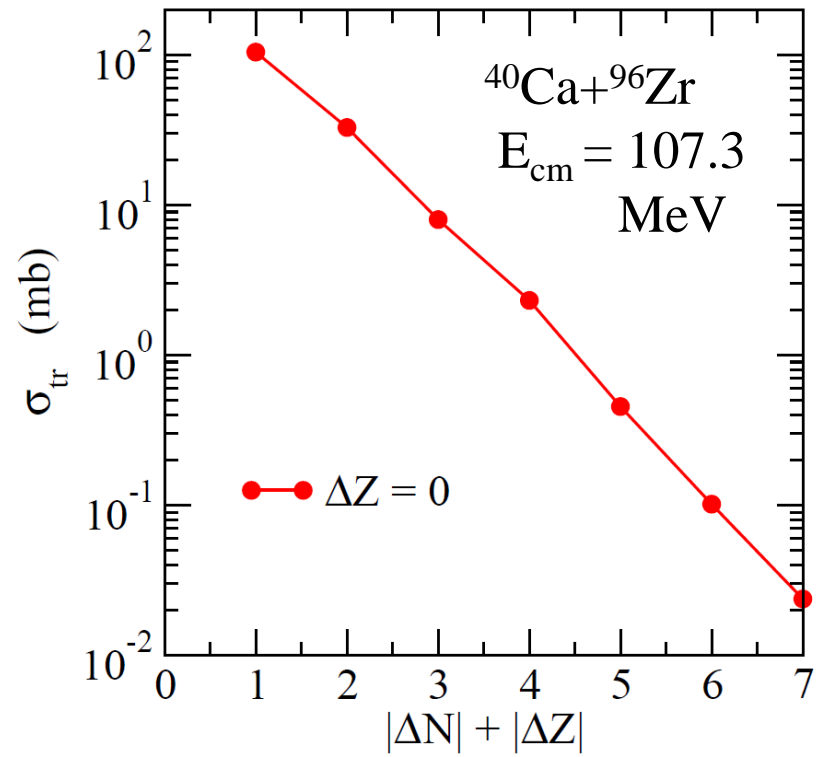
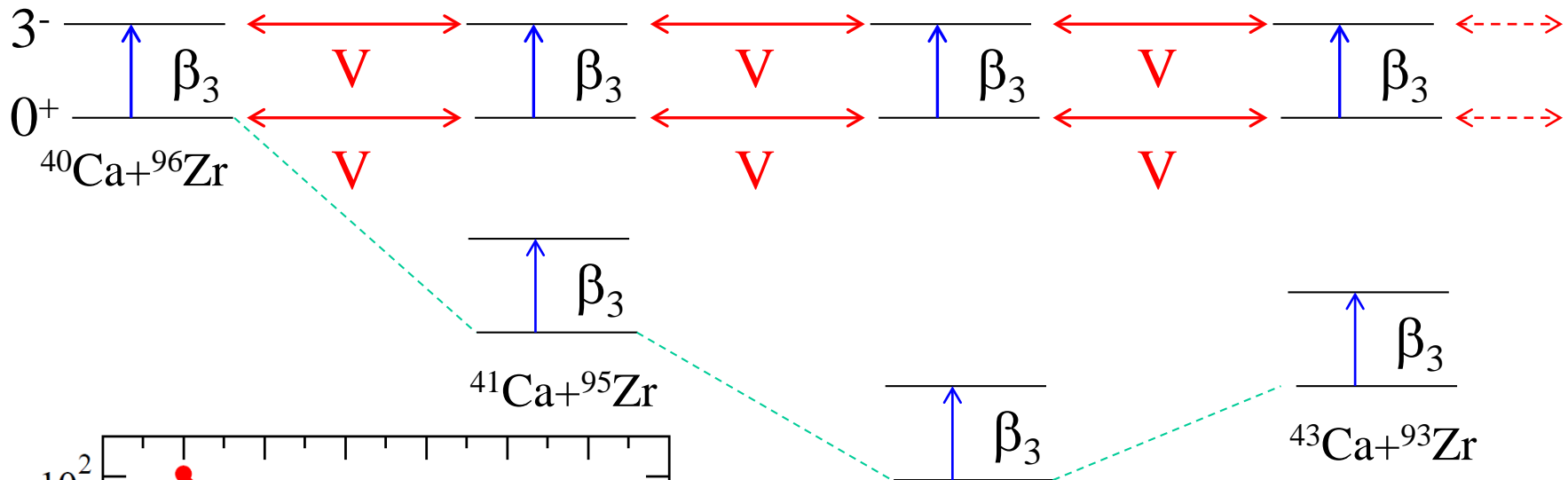
## 3. Same coupling scheme for inelastic excitations



Brink-Axel hypothesis

constant coupling approximation for transfer

# 4. Sequential coupling to each transfer partition with the same strength



Brink-Axel hypothesis

experimental data:

$$\frac{\sigma_{\Delta N}}{\sigma_{\Delta N-1}} \sim \text{const.}$$