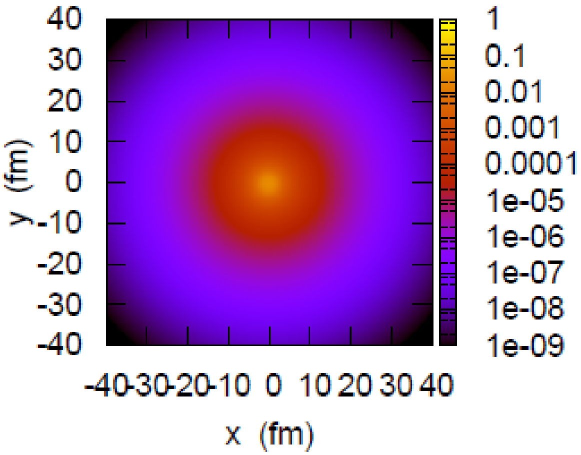
Pairing correlations and odd-even staggering in reaction cross sections of weakly-bound nuclei

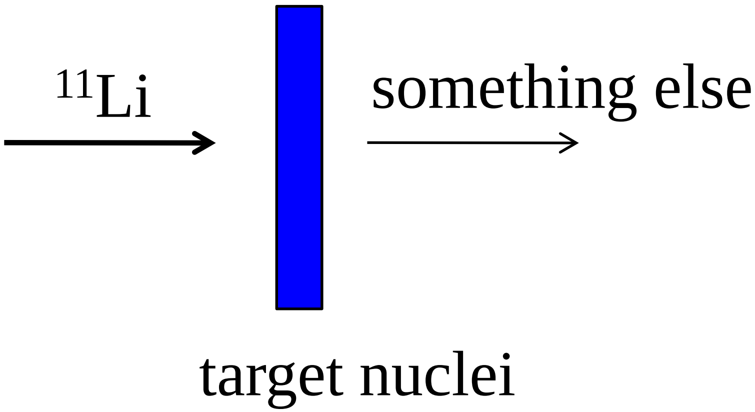


K. Hagino (Tohoku U.)
H. Sagawa (U. of Aizu)

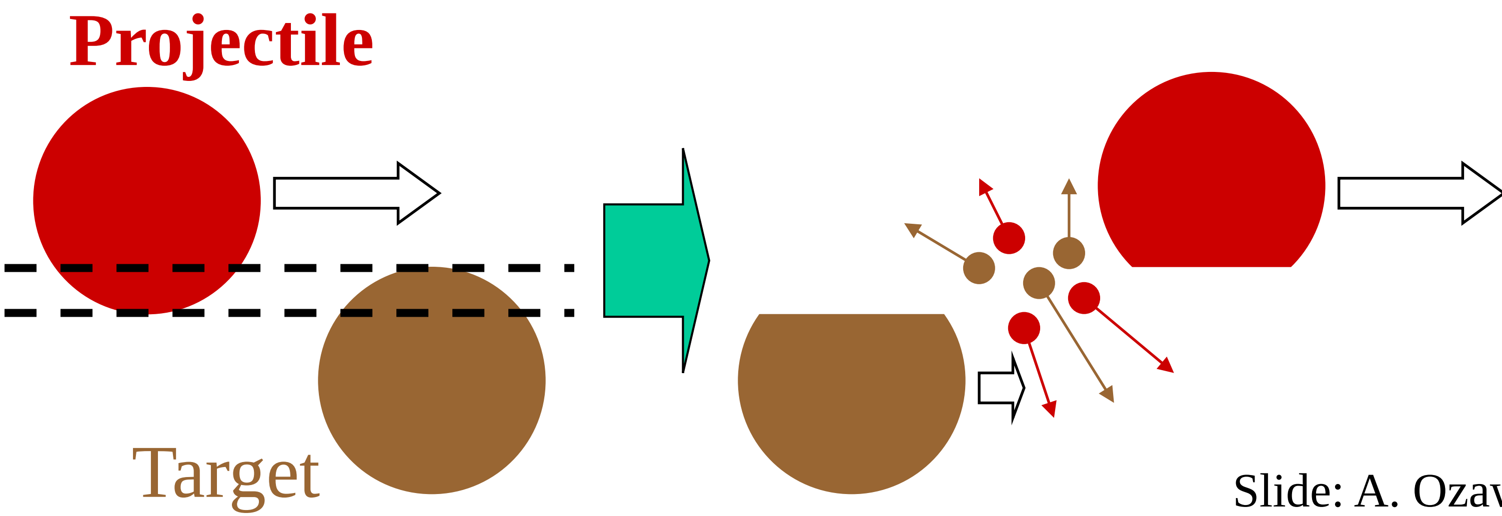
- PRC84('11)011303(R)
- PRC85('12)014303
- arXiv:1202.2725 [nucl-th]

1. Introduction: interaction cross section and nuclear size
2. Odd-even staggering of interaction cross sections (σ_I)
3. Pairing correlation in weakly-bound nuclei and σ_R
4. Staggering parameter
5. Summary

Introduction: interaction cross section



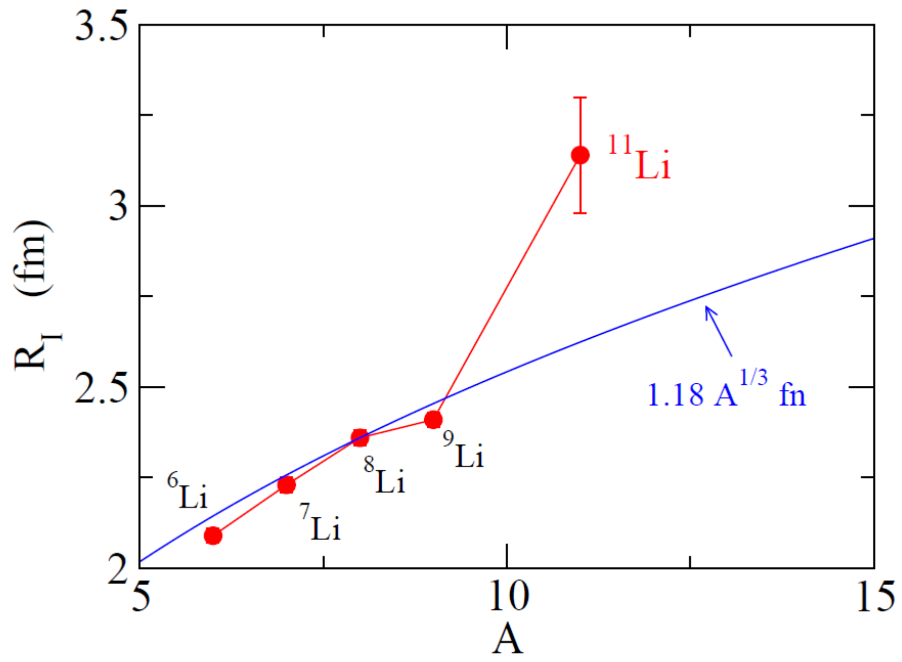
interaction cross section σ_I
= cross section for the change
of Z a/o N in the incident nucleus

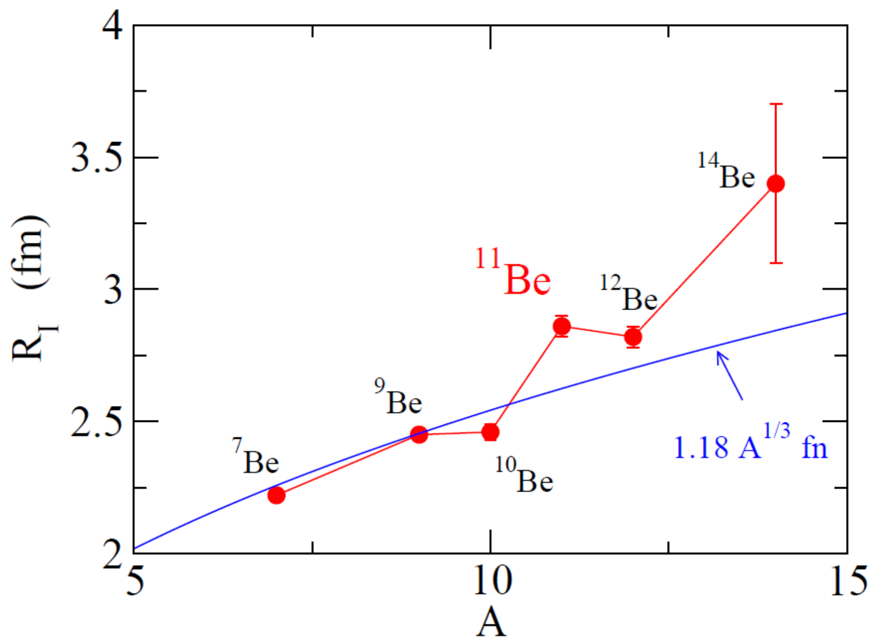


Slide: A. Ozawa

$$\sigma_I \sim \pi[R_I(P) + R_I(T)]^2 \longrightarrow R_I(P)$$

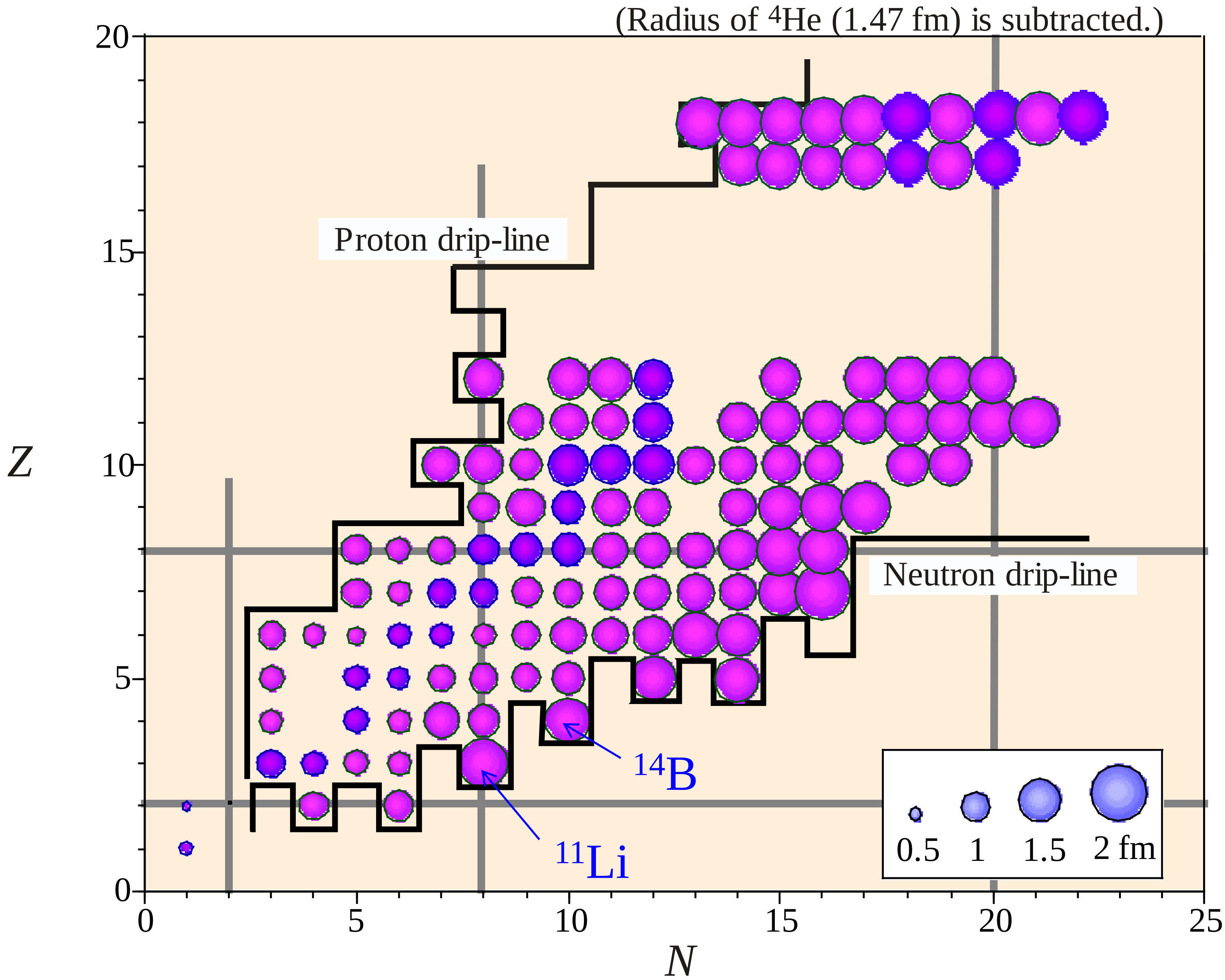
Discovery of **halo nuclei**





I. Tanihata, T. Kobayashi et al.,
PRL55(‘85)2676; PLB206(‘88)592

Nuclear radii determined from σ_R at ~1 *A* GeV

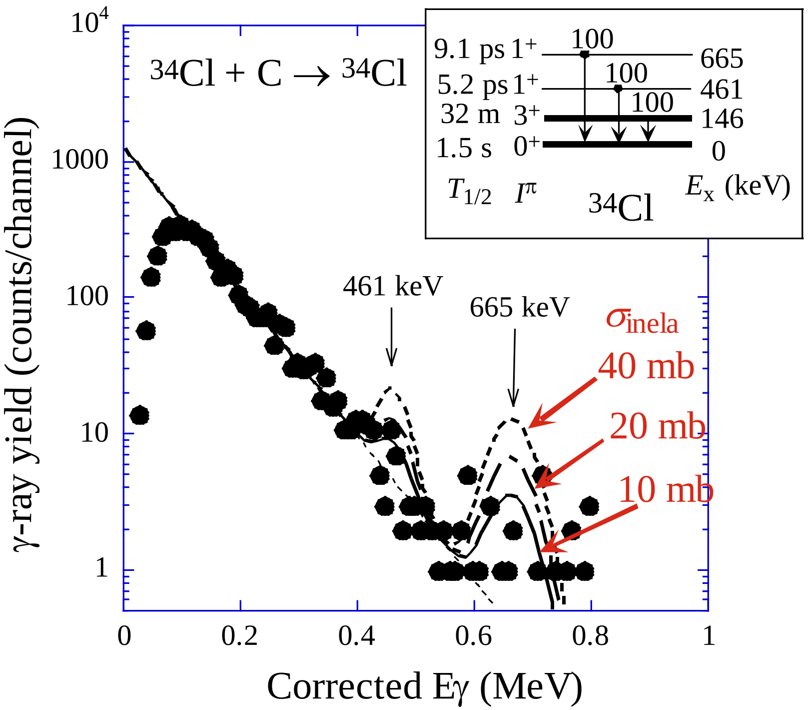


A. Ozawa

Interaction versus reaction cross sections

interaction cross section σ_I : experimentally easier
= cross section for the change of Z a/o N in the incident nucleus

reaction cross section σ_R : theoretically easier
= cross section for processes other than elastic scattering
= $\sigma_I + \sigma_{inel}$



E = 950 MeV/A

σ_{inel} ~ 10 mb

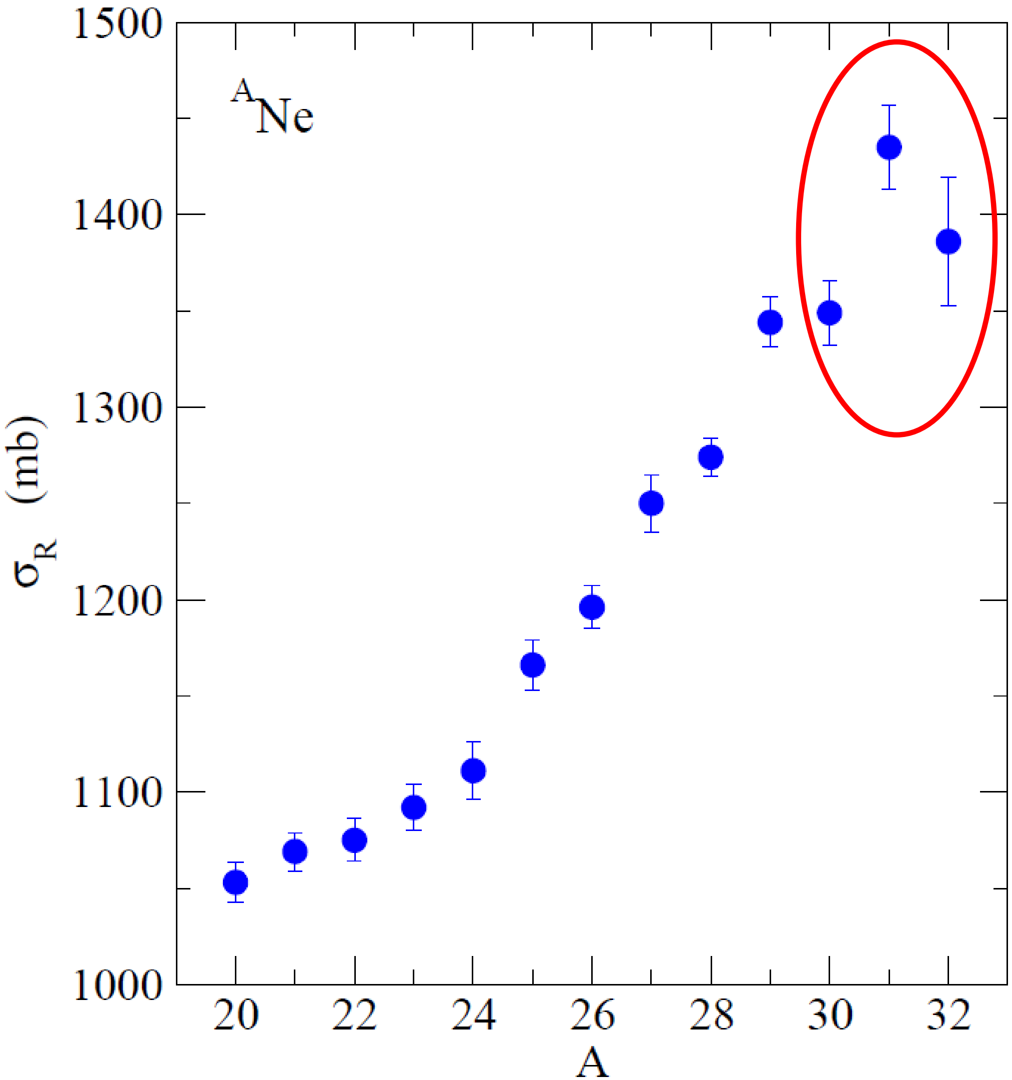
cf. σ_I = 1334 +/- 28 mb

$\sigma_I \sim \sigma_R$ for unstable nuclei

A. Ozawa et al., NPA709 ('02)60

Odd-even staggering of interaction cross sections

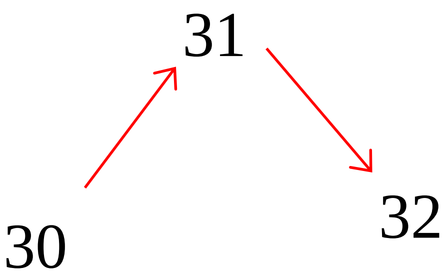
σ_I of unstable nuclei: often show a large odd-even staggering



Typical example:

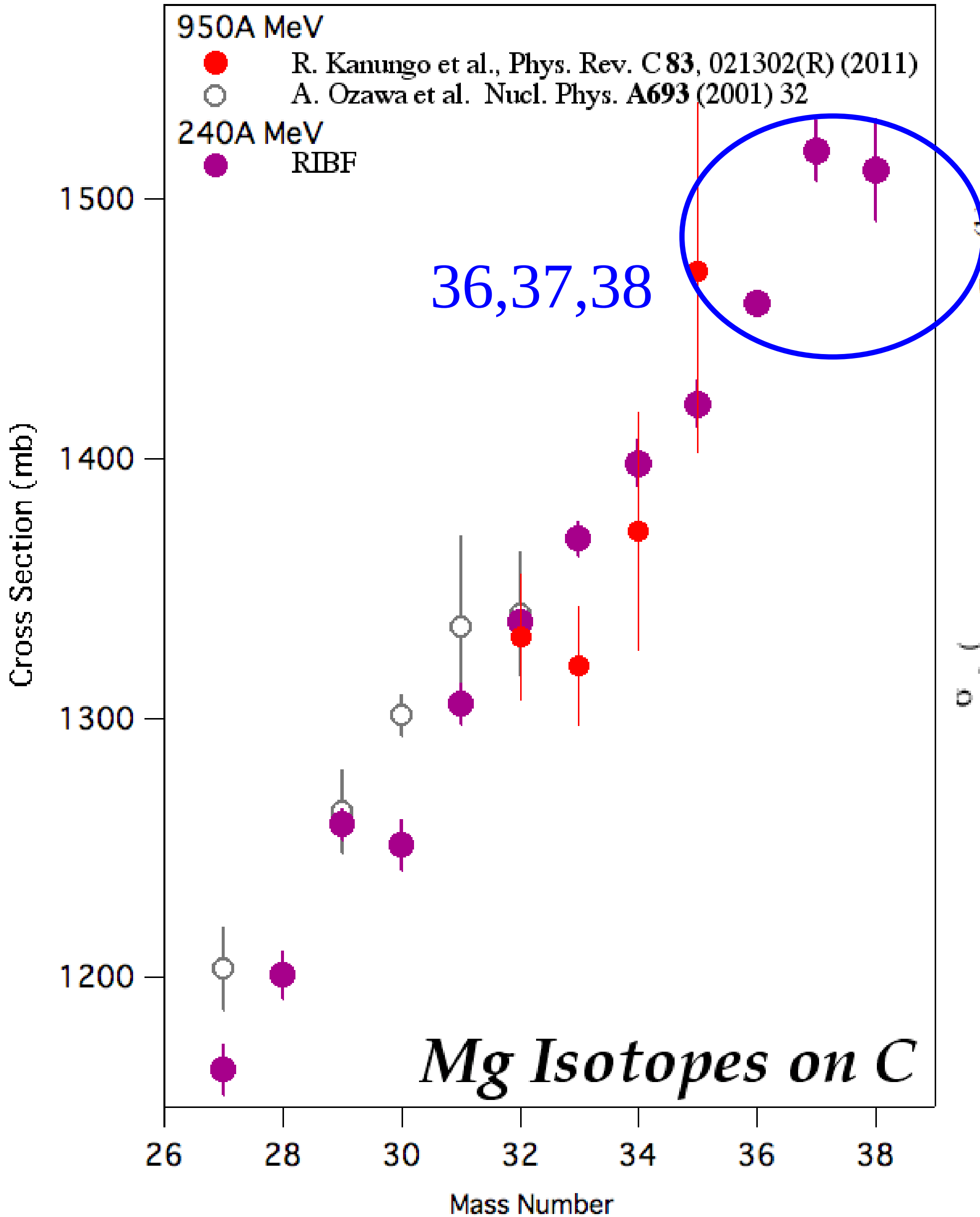
Recent experimental data on Ne isotopes

M. Takechi et al., Phys. Lett. B707 ('12) 357

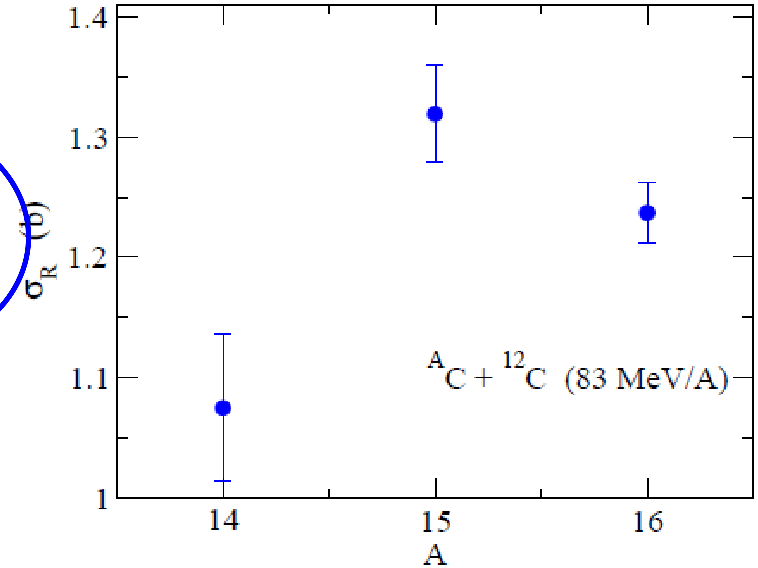


clear odd-even effect

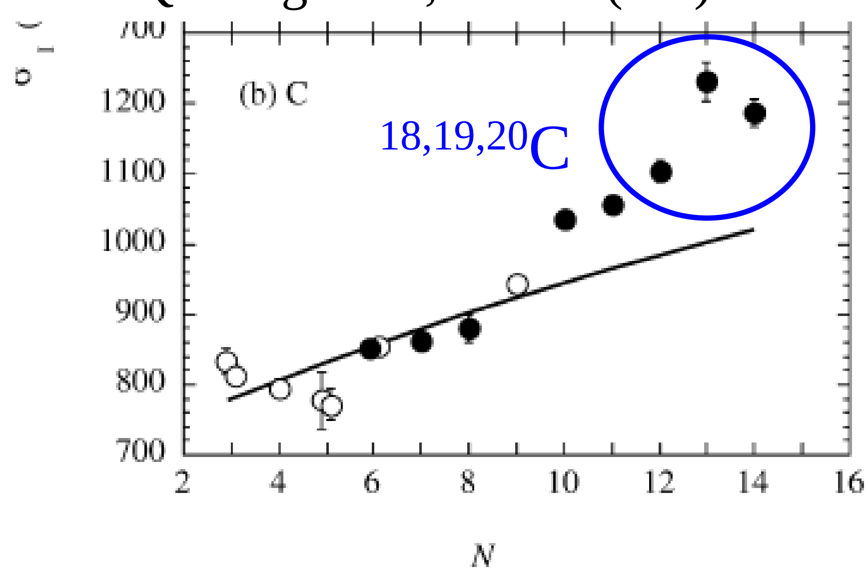
Other systems



M. Takechi, private communications



D.Q. Fang et al., PRC69('04)034613



A. Ozawa et al., NPA691('01)599

cf. Other examples of odd-even staggering in nuclear physics

➢isotope shifts: smaller charge radius for odd-A nuclei

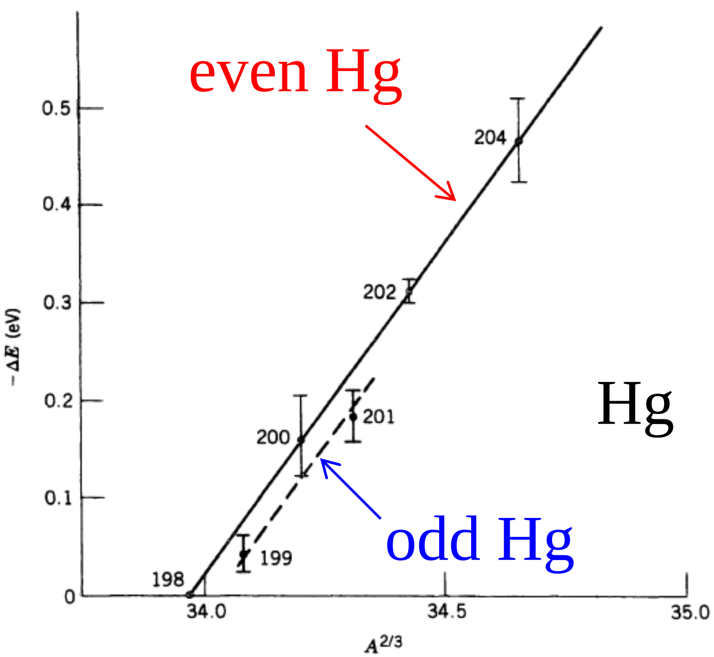


Figure 3.6 K X-ray isotope shifts in Hg. The energy of the K X ray in Hg is about 100 keV, so the relative isotope shift is of the order of 10^{-6}. The data show the

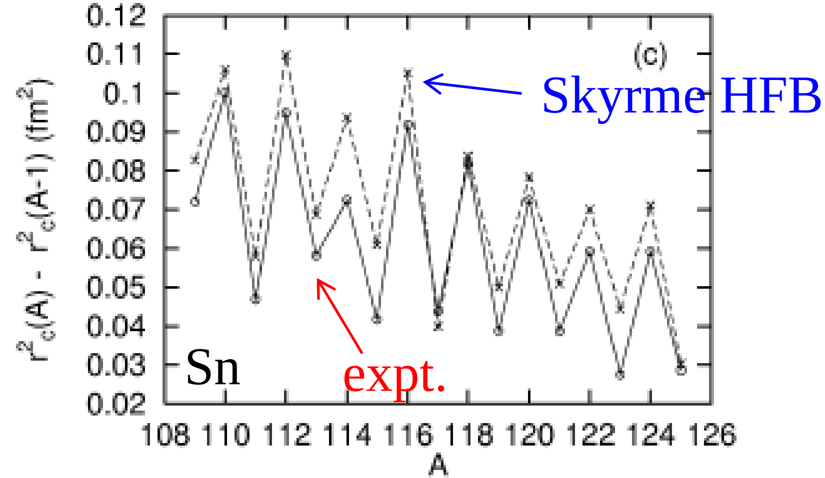
K.S. Krane, "Introductory Nuclear Physics"

$$\Delta E \sim -\frac{2}{5}\frac{Z^4e^2}{a_0^3}\left(\langle r^2\rangle_A - \langle r^2\rangle_{A'}\right)$$

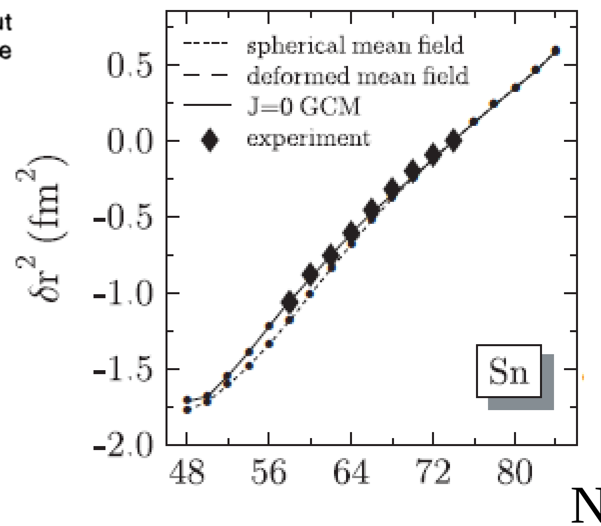
cf. Bohr-Mottelson, eq. (2.85)

$$\gamma \equiv \frac{\langle r^2\rangle_{A+1} - \langle r^2\rangle_A}{\langle r^2\rangle_{A+2} - \langle r^2\rangle_A}$$

- deformation effect? -pairing effect?



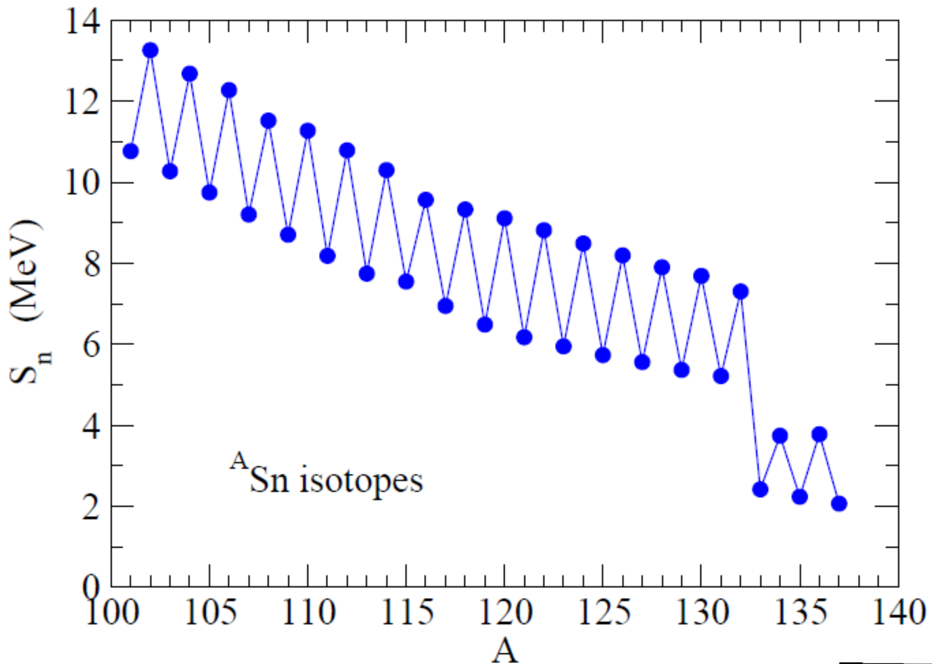
S. Sakakihara and Y. Tanaka, NPA691('01)649



M.Bender, G.F. Bertsch, P.-H. Heenen, PRC73('06)034322 (even-even only)

cf. Other examples of odd-even staggering in nuclear physics

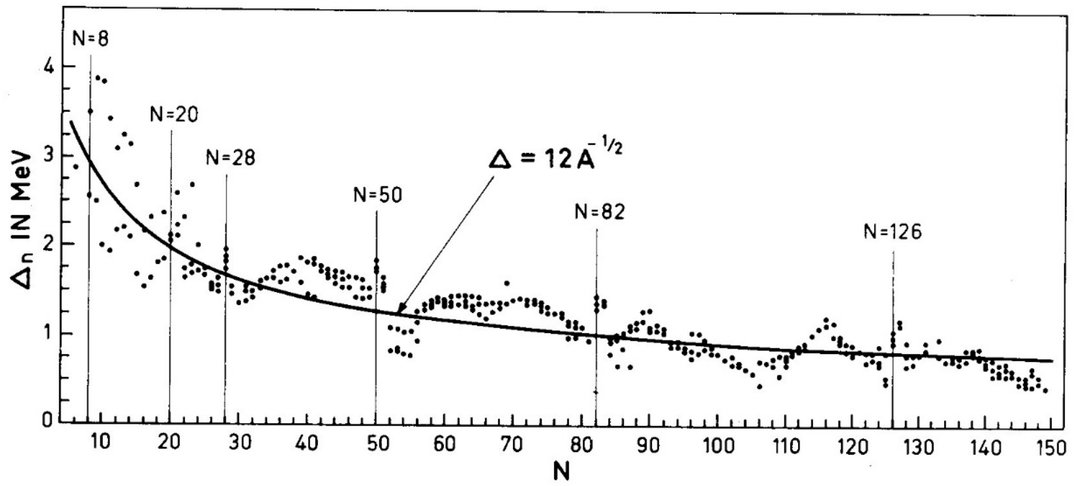
- binding energy



pairing gap parameter

$$\Delta(N) = \frac{(-)^N}{2}(B(N-1) - 2B(N) + B(N+1)) = \frac{(-)^N}{2}(S_n(N-1) - S_n(N))$$

$$S_n(N) = B(N) - B(N-1)$$



➢pairing anti-halo effect

K. Bennaceur, J. Dobaczewski, and M. Ploszajczak, PLB496(‘00)154

pairing

⇩

asymptotic behavior of s.p. wave functions

⇩

suppression of density distribution

➢pairing anti-halo effect

K. Bennaceur, J. Dobaczewski, and M. Ploszajczak, PLB496('00)154

pairing

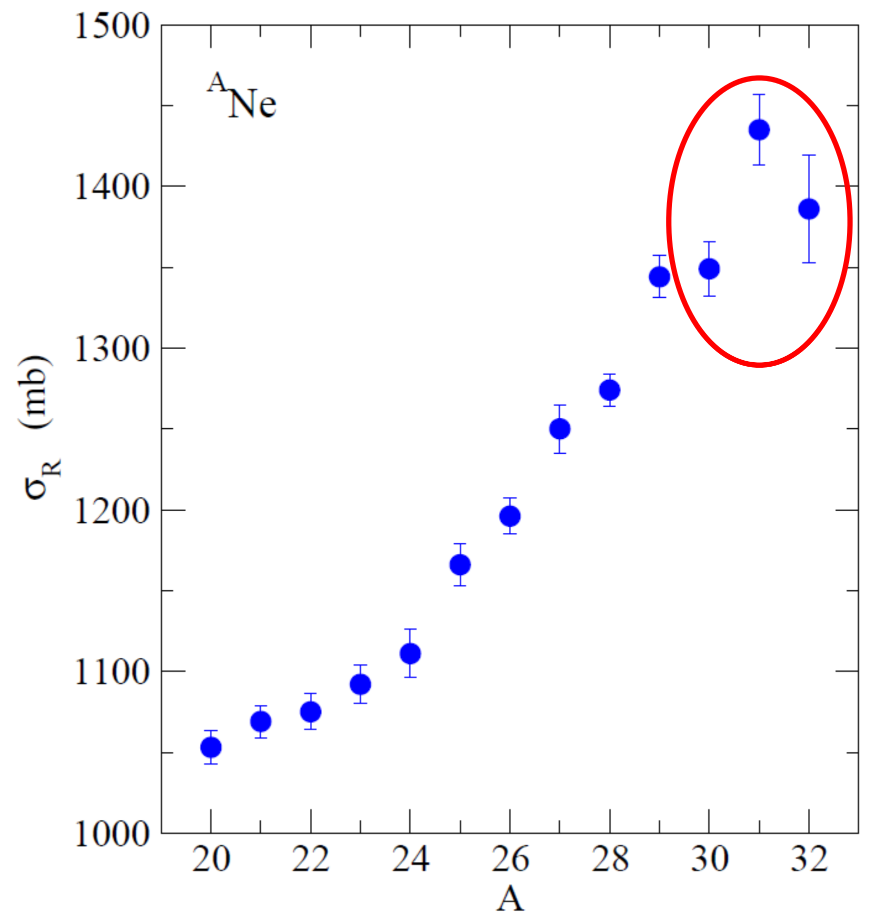
⬇

asymptotic behavior of s.p. wave functions

⬇

suppression of density distribution

➢odd-even staggering of σ_R



Our motivation:

Relation between the odd-mass staggering (OES) of σ_R and pairing (anti-halo) effect?

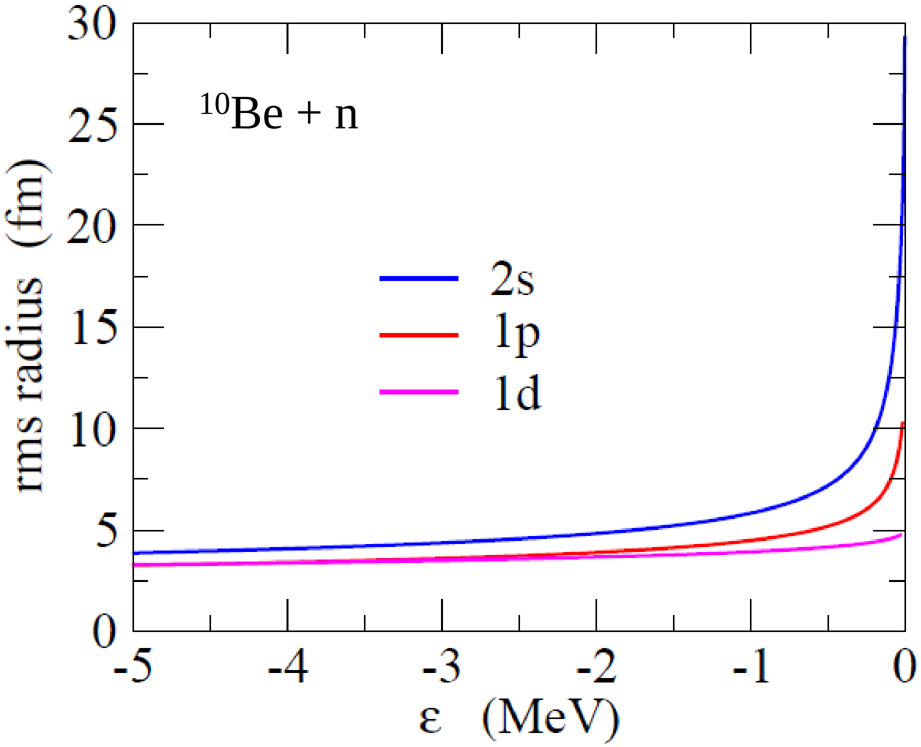
First experimental evidence for the anti-halo effect?

Effect of pairing on radius of a weakly-bound orbit

asymptotic behavior of a s.p. wave function for s-wave:

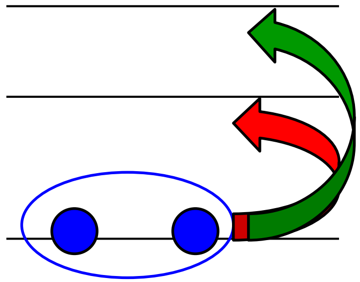
$$\psi(r) \sim \exp(-\kappa r) \qquad \kappa = \sqrt{\frac{2m|\epsilon|}{\hbar^2}}$$

$$\langle r^2 \rangle_{\rm HF} = \frac{\int r^2 |\psi(r)|^2 dr}{\int |\psi(r)|^2 dr} \propto \frac{1}{\kappa^2} = \frac{\hbar^2}{2m|\epsilon|} \to \infty$$



$$\langle r^2 \rangle \propto \begin{cases} \dfrac{1}{|\epsilon|} & (l=0) \\ \dfrac{1}{\sqrt{|\epsilon|}} & (l=1) \\ \text{const.} & (l=2) \end{cases}$$

For even-mass system:



Cooper pair

Hartree-Fock-Bogoliubov (HFB) equations:

$$\begin{pmatrix} \hat{h}-\lambda & \Delta(r) \\ \Delta(r) & -\hat{h}+\lambda \end{pmatrix} \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix} = E_k \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix}$$

Δ(r): pair potential
λ: chemical potential

density: $$\rho(\boldsymbol{r}) = \sum_k |V_k(\boldsymbol{r})|^2$$

Asymptotic form of $V_k(r)$:

$$V_k(r) \sim \exp(-\beta_k r)$$

$$\beta_k = \sqrt{\frac{2m(E_k-\lambda)}{\hbar^2}} \sim \sqrt{\frac{2m\Delta}{\hbar^2}}$$

↑

$$E_k \sim \sqrt{(\epsilon-\lambda)^2+\Delta^2} \sim \Delta$$

$$(\epsilon, \lambda \to 0)$$

$$\langle r^2 \rangle_{\rm HFB} \propto \frac{\hbar^2}{2m\Delta}$$

"pairing anti-halo effect"

K. Bennaceur, J. Dobaczewski, and M. Ploszajczak, PLB496('00)154

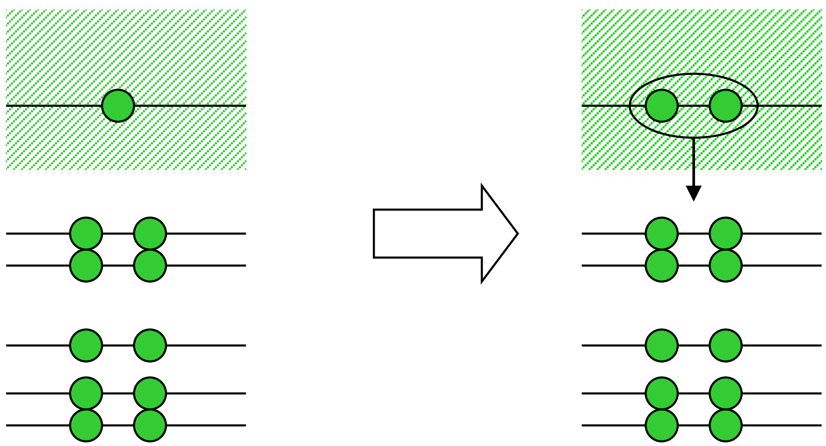
Pairing correlation in weakly-bound nuclei

$$\langle r^2 \rangle_{\mathrm{HFB}} \propto \frac{\hbar^2}{2m\Delta}$$

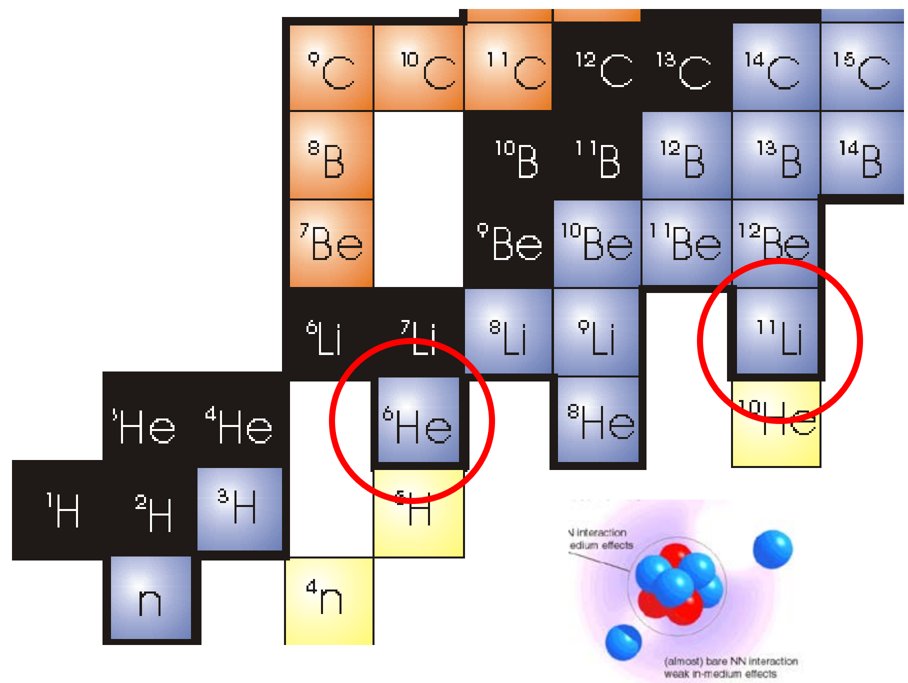
"pairing anti-halo effect"

$$\boxed{\Delta \neq 0 \quad \text{as } \epsilon, \lambda \to 0?}$$

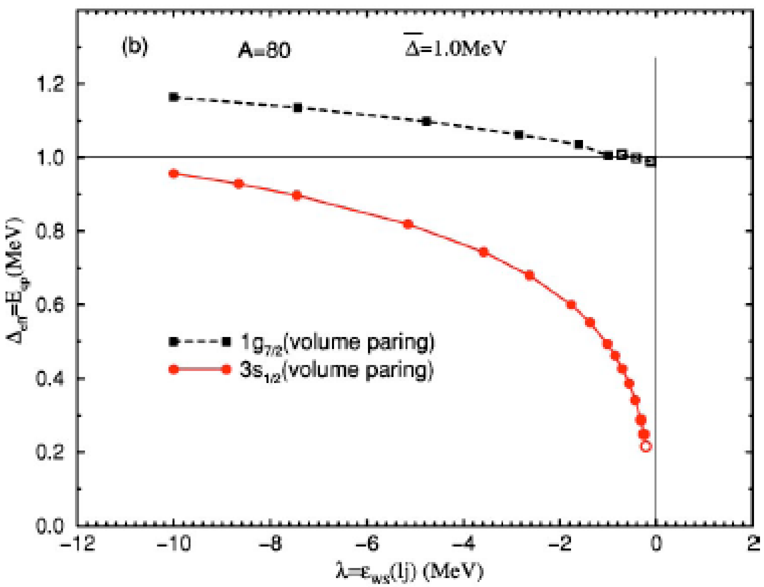
cf. for light neutron-rich nuclei (Borromean nuclei)



unbound   bound



For heavier nuclei: controvertial arguments based on HFB



I. Hamamoto and H. Sagawa, PRC70(‘04)034317

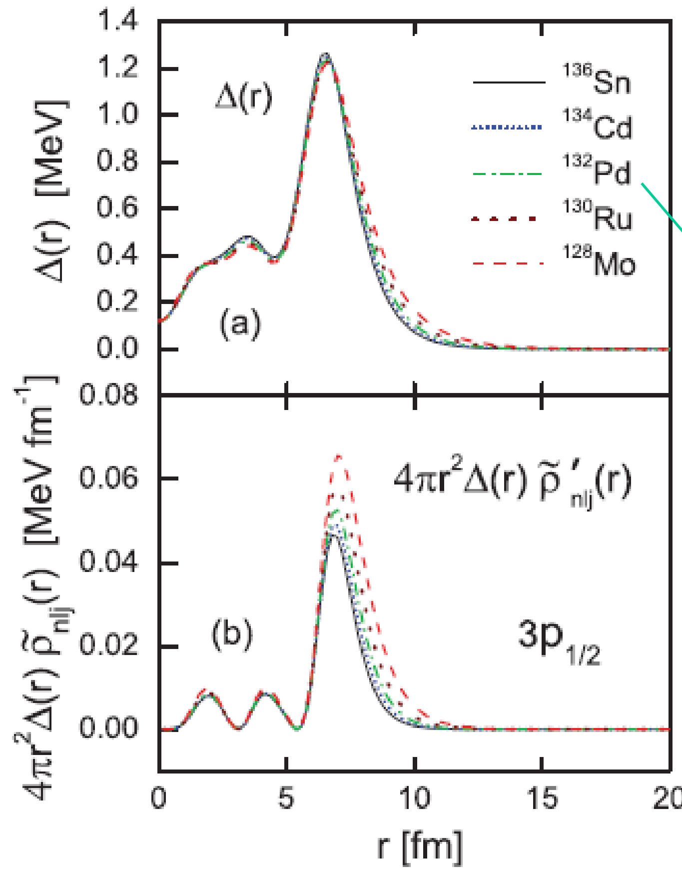
■simplified HFB model

✓Δ(r): prefixed

✓set $\lambda = \varepsilon_{HF}$

✓define Δ_{eff} = lowest E_{qp}

$\Rightarrow$ $\Delta_{eff} \to 0 \quad (\epsilon \to 0)$



e.g.

ε=-0.01 (MeV)

Δ = 0.57 (MeV)

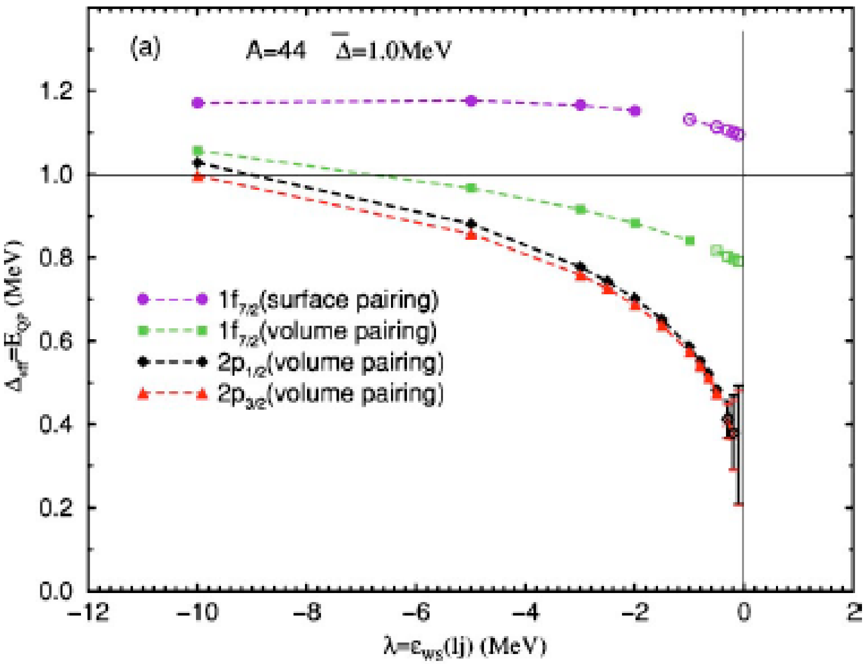
($3p_{3/2}$)

Y. Zhang, M. Matsuo, J. Meng, PRC83(‘11)054301

■self-consistent HFB

$\Delta_{eff} \neq 0 \quad (\epsilon \to 0)$

see also M. Yamagami, PRC72(‘05)064308



I. Hamamoto and H. Sagawa,
PRC70('04)034317

Pairing gap at neutron drip line

- different models
  - ✓simplified HFB (Hamamoto)
  - ✓self-consistent HFB (Zhang-Matsuo-Meng)

- different definitions for effective pairing gap
  - ✓lowest quasi-particle energy (Hamamoto)

$$E_k \sim \sqrt{(\epsilon_k - \lambda)^2 + \Delta_k^2} \sim \Delta_k$$ (BCS approximation)

  - ✓HFB wave functions (Zhang-Matsuo-Meng)

$$\Delta_k \sim \frac{\int d\boldsymbol{r} \Delta(r) U_k^*(\boldsymbol{r}) V_k(\boldsymbol{r})}{\int d\boldsymbol{r} U_k^*(\boldsymbol{r}) V_k(\boldsymbol{r})}$$

$$\begin{pmatrix} \hat{h} - \lambda & \Delta(r) \\ \Delta(r) & -\hat{h} + \lambda \end{pmatrix} \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix} = E_k \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix}$$

  - ✓canonical basis

$$\Delta_k \sim \int d\boldsymbol{r} \Delta(r) |\phi_k^{\rm can}(\boldsymbol{r})|^2$$

canonical basis (natural orbit)

$$\int d\boldsymbol{r}'\rho(\boldsymbol{r},\boldsymbol{r}')\phi_k^{(\mathrm{can})}(\boldsymbol{r}') = \left(v_k^{(can)}\right)^2 \phi_k^{(\mathrm{can})}(\boldsymbol{r})$$

$$\begin{pmatrix} \hat{h}-\lambda & \Delta(r) \\ \Delta(r) & -\hat{h}+\lambda \end{pmatrix} \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix} = E_k \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix}$$

$$\rho(\boldsymbol{r},\boldsymbol{r}') = \sum_k V_k(\boldsymbol{r})V_k^*(\boldsymbol{r})$$

With canonical basis, HFB $\longrightarrow$ an intuitive BCS-like form

cf. BCS approximation: Δ(r) = const.

$$U_k(\boldsymbol{r}) = u_k\phi_k^{(\mathrm{HF})}(\boldsymbol{r}), \quad V_k(\boldsymbol{r}) = v_k\phi_k^{(\mathrm{HF})}(\boldsymbol{r})$$

$$\longrightarrow \quad \phi_k^{(\mathrm{can})}(\boldsymbol{r}) = \phi_k^{(\mathrm{HF})}(\boldsymbol{r})$$

canonical basis: effective s.p. orbit including pairing

Pairing gap at neutron drip line

- different models
  - ✓simplified HFB (Hamamoto)
  - ✓self-consistent HFB (Zhang-Matsuo-Meng)
- different definitions for effective pairing gap
  - ✓lowest quasi-particle energy (Hamamoto)
  - ✓HFB wave functions (Zhang-Matsuo-Meng)
  - ✓canonical basis

⇒ comparison of the different definitions within the same model

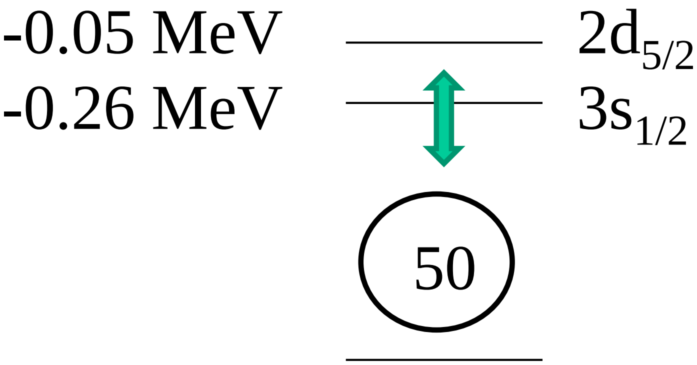
Model HFB with a Woods-Saxon mean-field potential

$$\begin{pmatrix} \hat{h}-\lambda & \Delta(r) \\ \Delta(r) & -\hat{h}+\lambda \end{pmatrix} \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix} = E_k \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix}$$

$$\hat{h} = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{WS}}(r)$$

↑

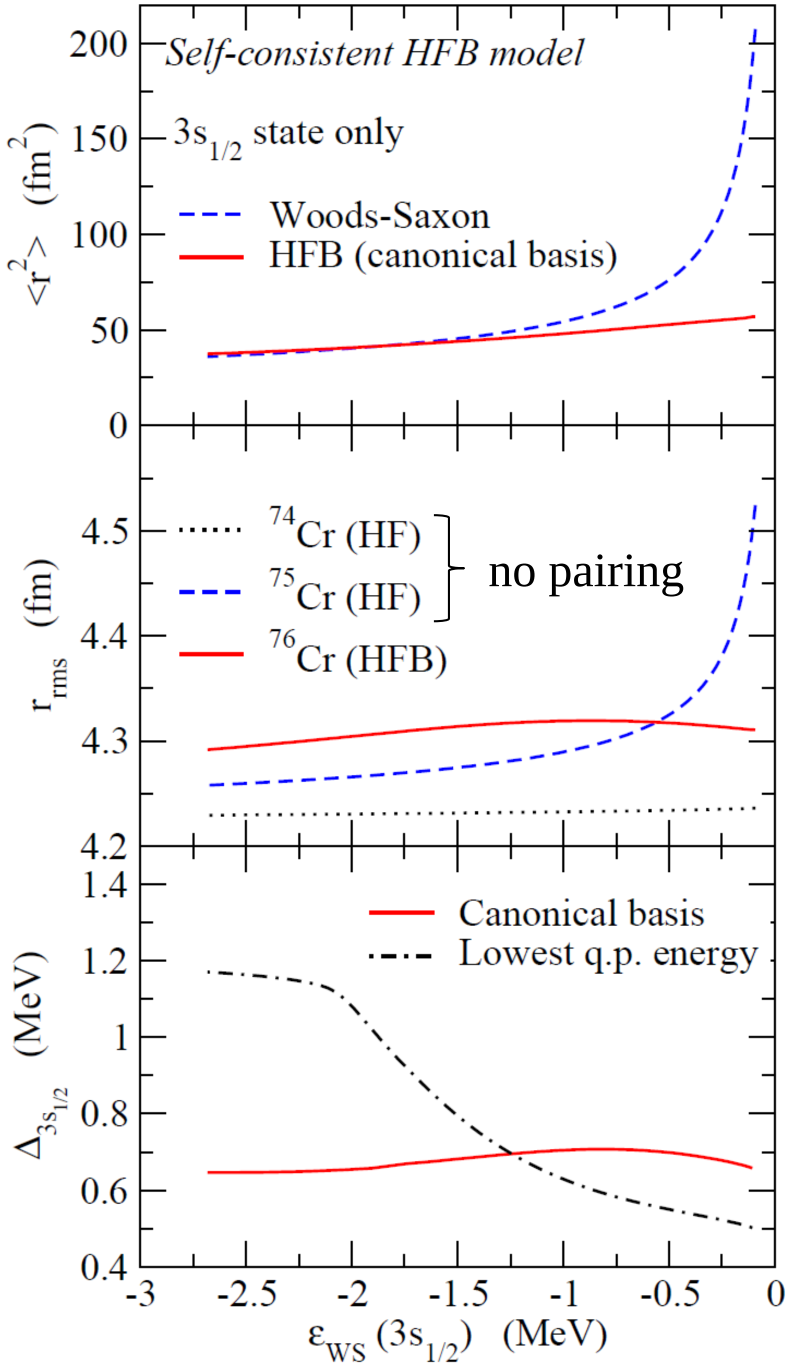
$^{76}_{24}\mathrm{Cr}_{52}$



$$\Delta(r) = \frac{V_{\mathrm{pair}}}{2}\left(1-\frac{\rho(r)}{\rho_0}\right)\tilde{\rho}_n(r) \qquad V_{\mathrm{pair}} \leftarrow \bar{\Delta} = 1.0\ \mathrm{MeV}$$

$$\tilde{\rho}_n(r) = -\sum_{k=n} U_k^*(\boldsymbol{r})V_k(\boldsymbol{r})$$

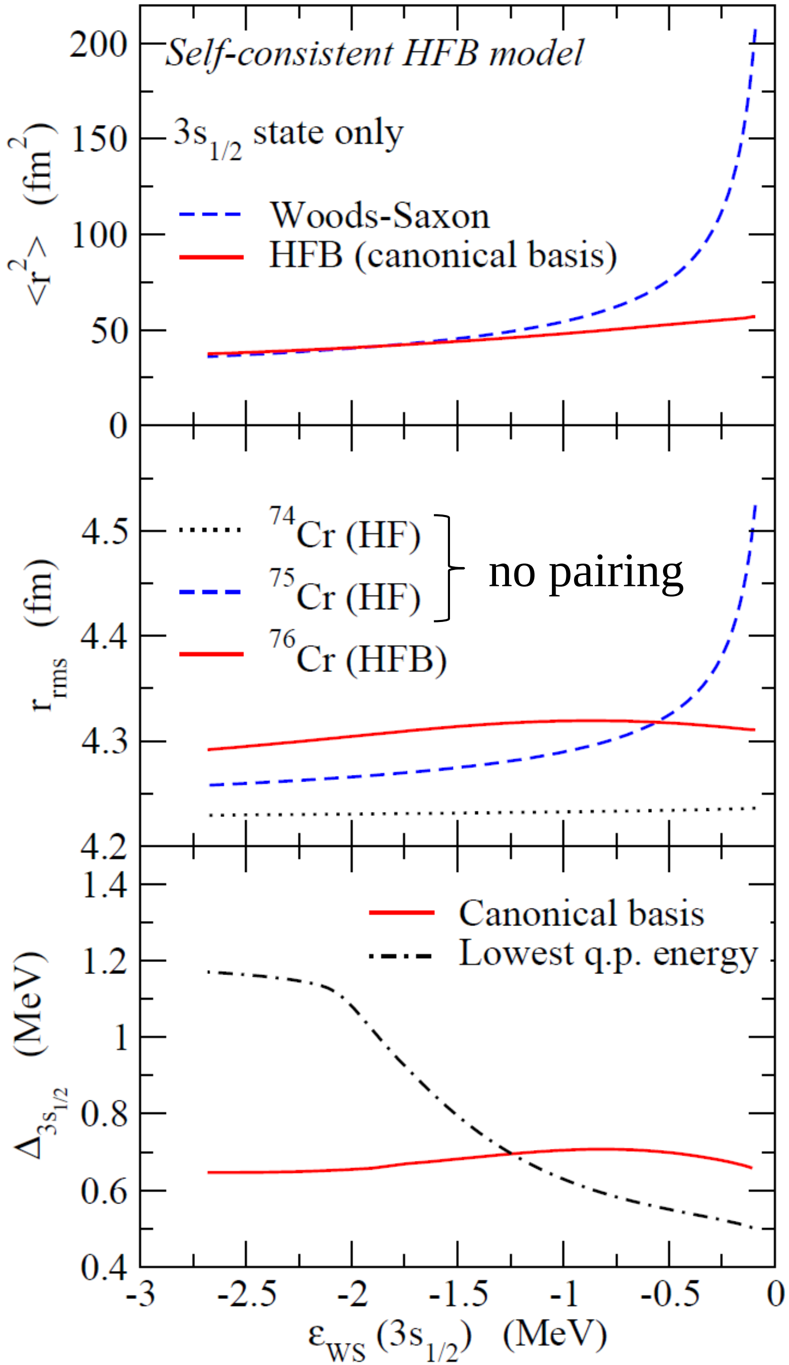
- ✓ λ: self-consistently determined so that N=52
- ✓ E_{cut} = 50 MeV above λ
- ✓ R_{box}=60 fm

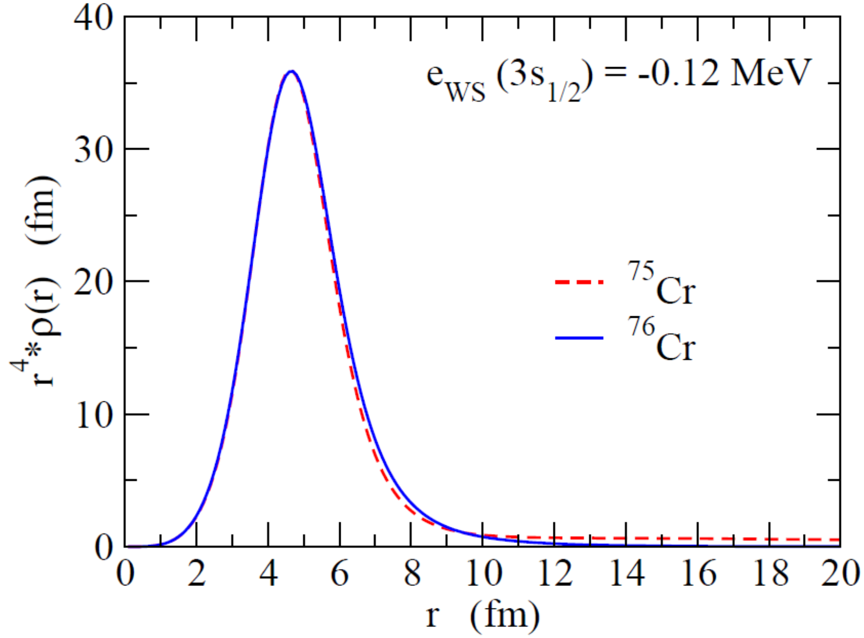


suppression of the radius

the effective pairing gap persists for both the definitions (agreement with Zhang-Matsuo-Meng)

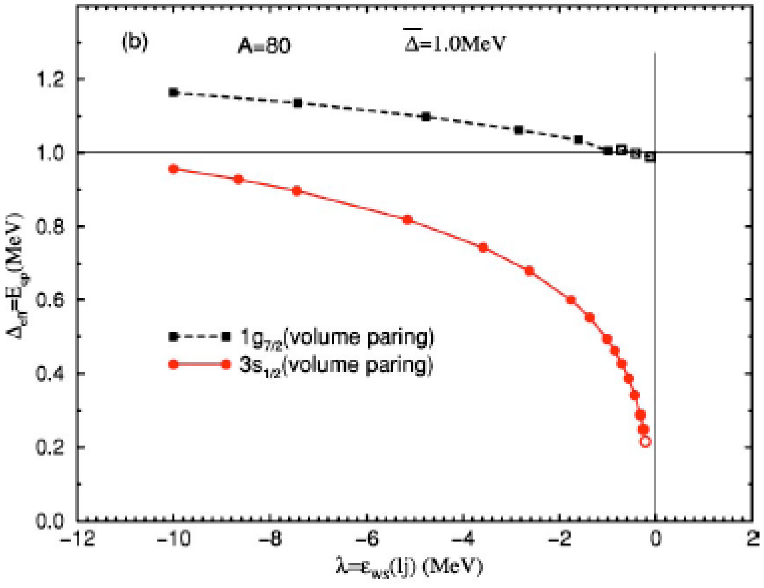
K. H. and H. Sagawa, PRC85('12)014303





K. H. and H. Sagawa, PRC85('12)014303

Relation to the Hamamoto-san's result



I. Hamamoto and H. Sagawa, PRC70('04)034317

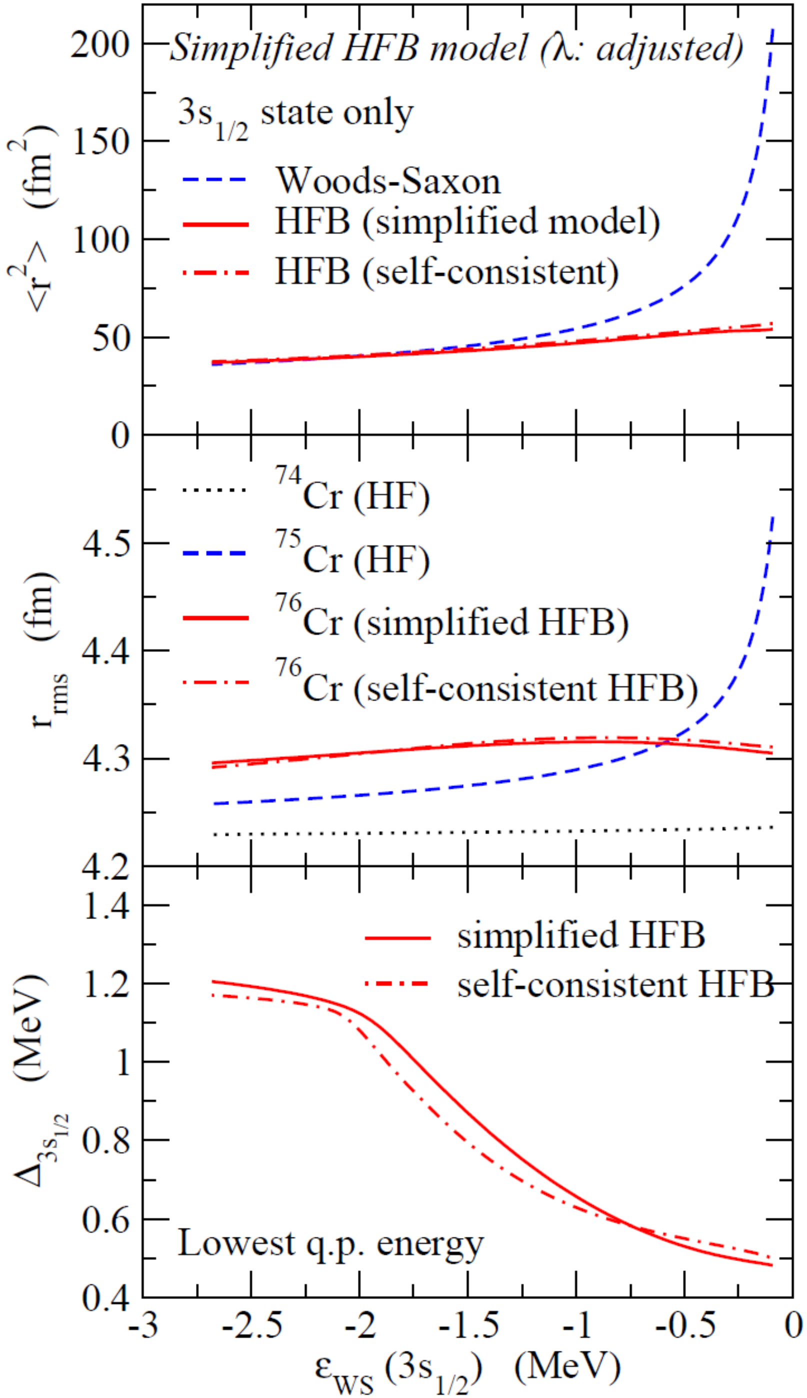
- ■simplified HFB model
  - ✓$\Delta(r)$: prefixed
  - ✓set $\lambda = \varepsilon_{HF}$
  - ✓define Δ_{eff} = lowest E_{qp}

$$\Delta(r) = \Delta_0 \cdot r \frac{d}{dr}\left(\frac{1}{1+\exp((r-R)/a)}\right)$$

Role of self-consistency in:

i) $\Delta(r)$?

ii) λ ?



"self-consistent HFB"

$$\Delta(r) = \frac{V_{\mathrm{pair}}}{2}\left(1 - \frac{\rho(r)}{\rho_0}\right)\tilde{\rho}_n(r)$$

$$\tilde{\rho}_n(r) = -\sum_{k=n} U_k^*(\boldsymbol{r})V_k(\boldsymbol{r})$$

"simplified HFB"

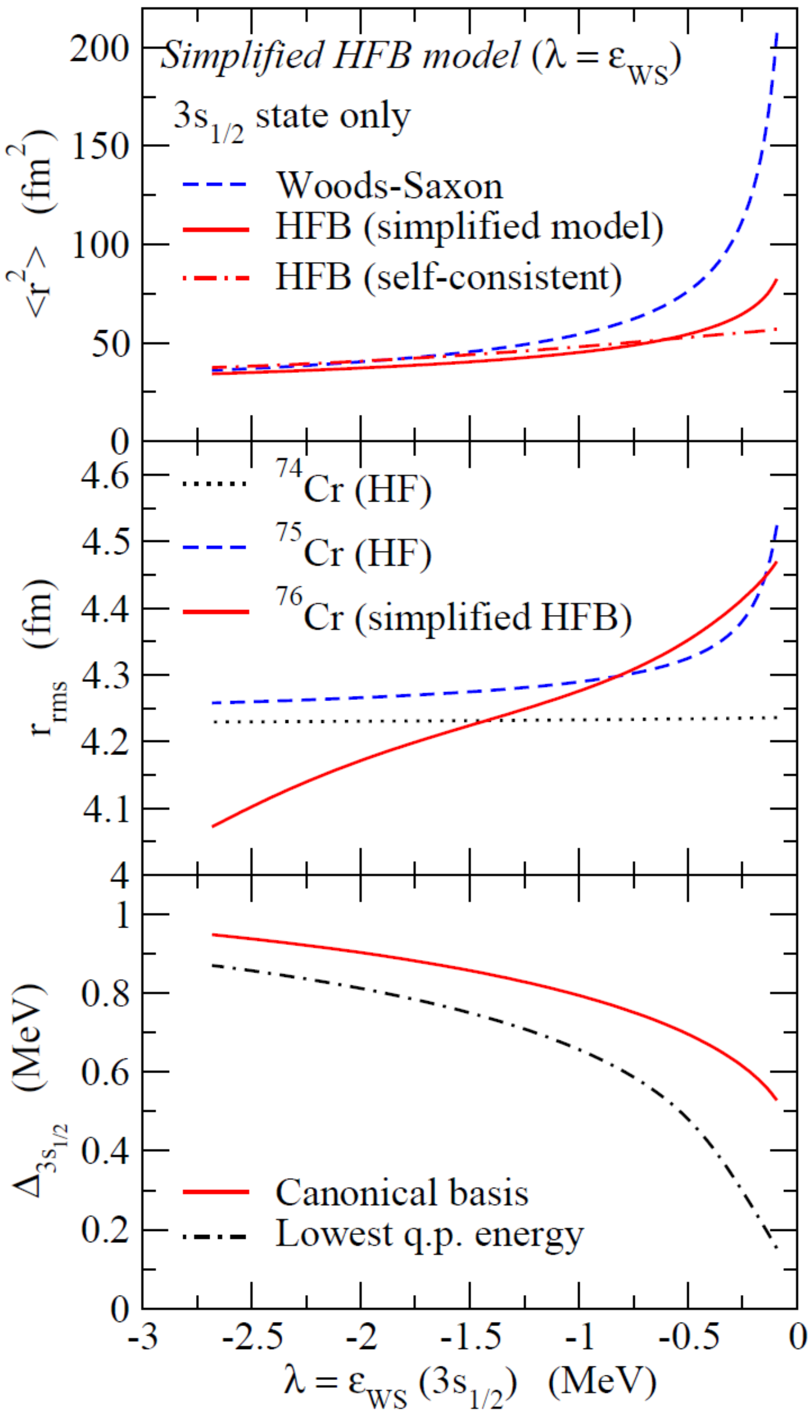
$$\Delta(r) = \Delta_0 \cdot r\,\frac{d}{dr}\left(\frac{1}{1+\exp((r-R)/a)}\right)$$

$$\Delta_0 = -1.107 \text{ MeV}$$

In both the models, λ is determined self-consistently so that $N = 52$.

- Very small difference

→ self-consistency in Δ(r) is not that important



"self-consistent HFB"

$$\Delta(r) = \frac{V_{\rm pair}}{2}\left(1 - \frac{\rho(r)}{\rho_0}\right)\tilde{\rho}_n(r)$$

$$\tilde{\rho}_n(r) = -\sum_{k=n} U_k^*(\boldsymbol{r})V_k(\boldsymbol{r})$$

λ : determined so that $N = 52$.

"simplified HFB"

$$\Delta(r) = \Delta_0 \cdot r\frac{d}{dr}\left(\frac{1}{1+\exp((r-R)/a)}\right)$$

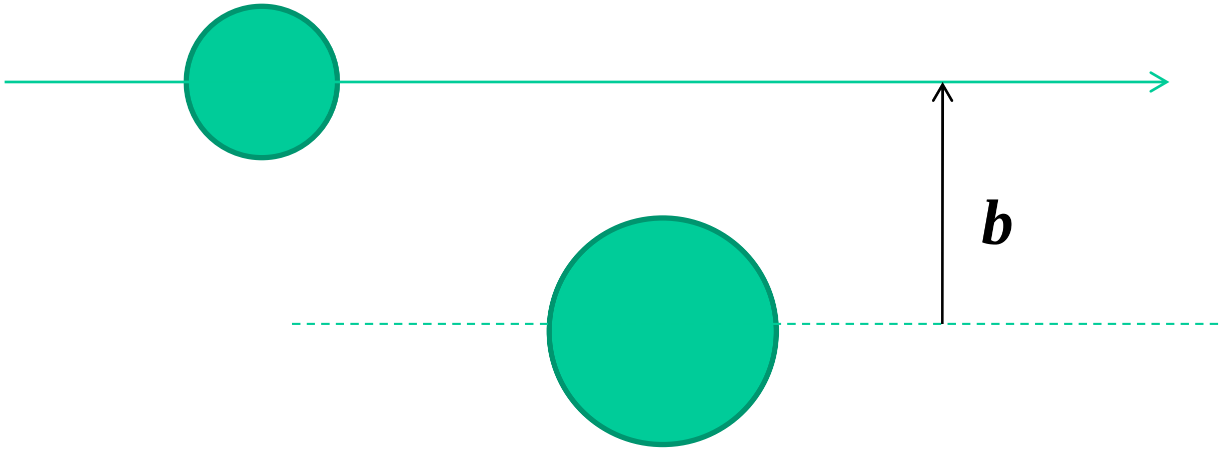
$\Delta_0 = -1.107$ MeV

λ : set to be ε_{WS}

- Similar result to Hamamoto-san

→ vanishing Δ : artifact of $\lambda = \varepsilon_{WS}$

Reaction cross sections



Glauber theory (optical limit approximation)

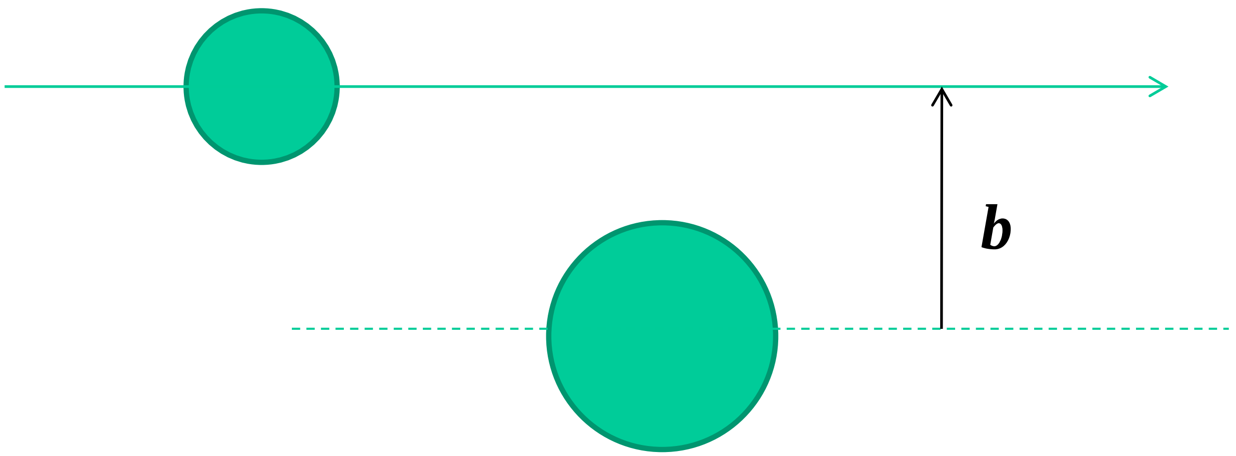
$$\sigma_{\rm R} = \int d^2 b \left(1 - |e^{i\chi(b)}|^2\right)$$

$$e^{i\chi(b)} = \exp\left[-\int d\boldsymbol{r}_P \boldsymbol{r}_T \rho_P(\boldsymbol{r}_P)\rho_T(\boldsymbol{r}_T)\Gamma(\boldsymbol{b}+\boldsymbol{s}_P-\boldsymbol{s}_T)\right]$$

$$\Gamma(\boldsymbol{b}) = \frac{1-i\alpha}{4\pi\beta}\sigma_{NN}^{\rm tot}\exp\left(-\frac{b^2}{2\beta}\right)$$

- straight-line trajectory
- adiabatic approximation
- simplified treatment for multiple scattering: $(1-x)^N \to e^{-Nx}$

Reaction cross sections



Glauber theory (optical limit approximation:OLA)

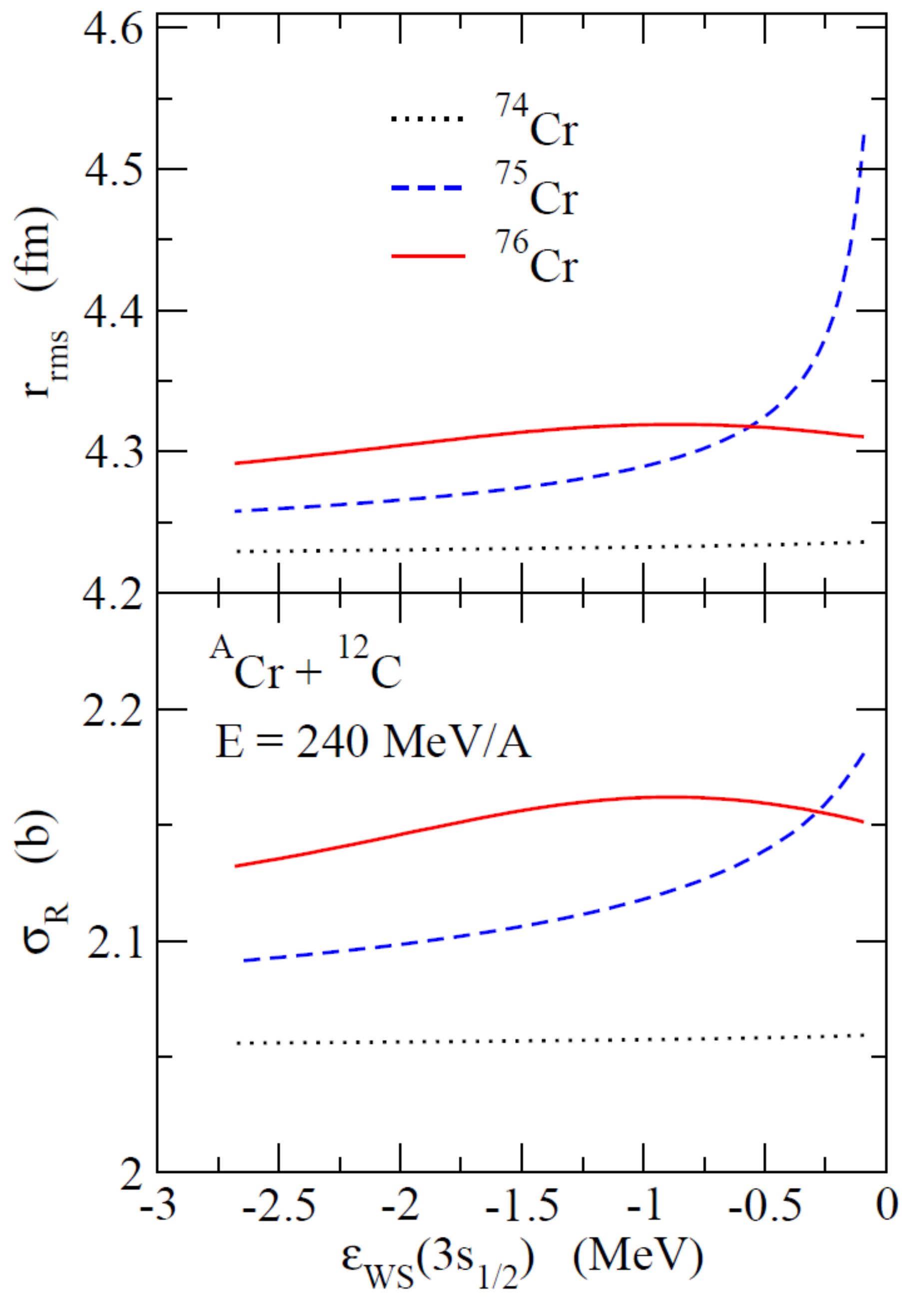
$$\sigma_{\rm R} = \int d^2b \left(1 - |e^{i\chi(b)}|^2\right)$$

$$e^{i\chi(b)} = \exp\left[-\int d\boldsymbol{r}_P \boldsymbol{r}_T \rho_P(\boldsymbol{r}_P)\rho_T(\boldsymbol{r}_T)\Gamma(\boldsymbol{b}+\boldsymbol{s}_P-\boldsymbol{s}_T)\right]$$

- Correction to the OLA

B. Abu-Ibrahim and Y. Suzuki, PRC61('00)051601(R)

$$i\chi(b) \to -\int d\boldsymbol{r}_P\, \rho_P(\boldsymbol{r}_P)\left[1 - e^{-\int \boldsymbol{r}_T \rho_T(\boldsymbol{r}_T)\Gamma(\boldsymbol{b}+\boldsymbol{s}_P-\boldsymbol{s}_T)}\right]$$

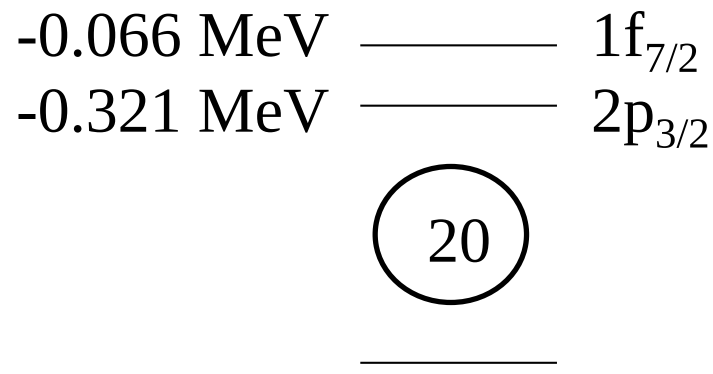


^{74,75,76}Cr + ^{12}C reactions at E=240 MeV/A

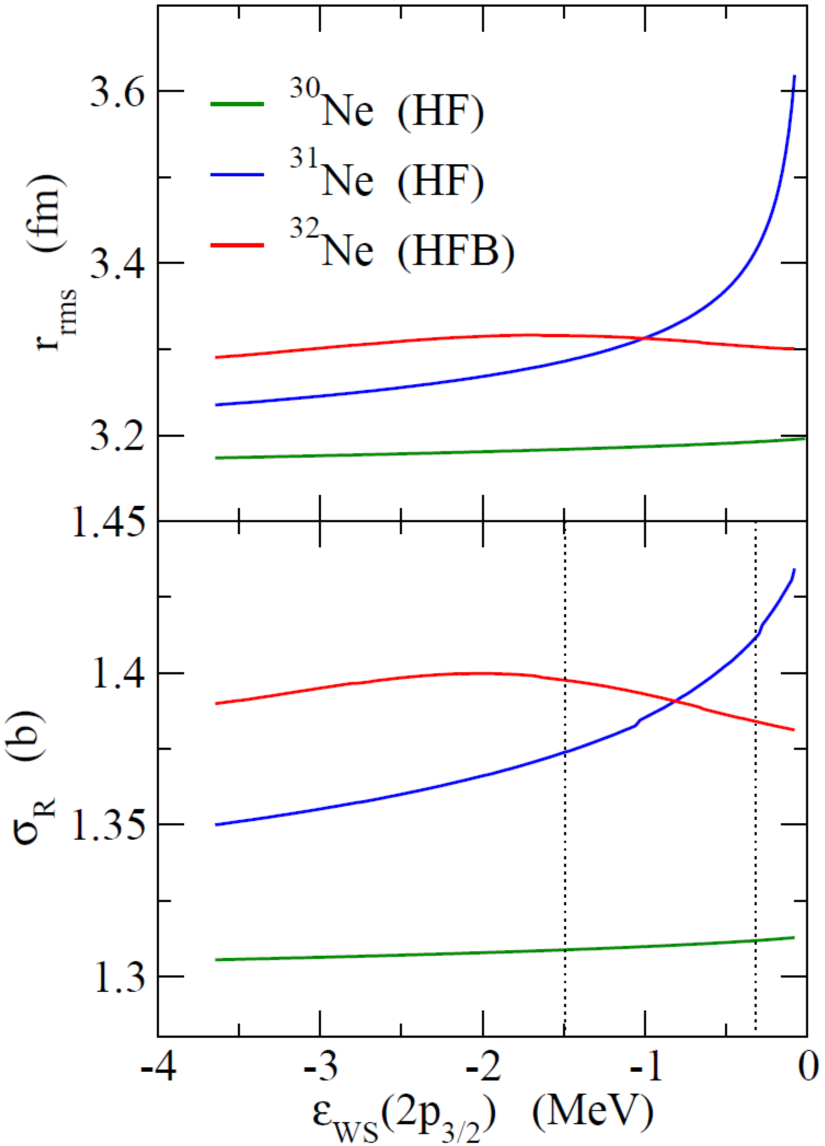
density of ^{74,75,76}Cr : HFB
density of ^{12}C : Gaussian

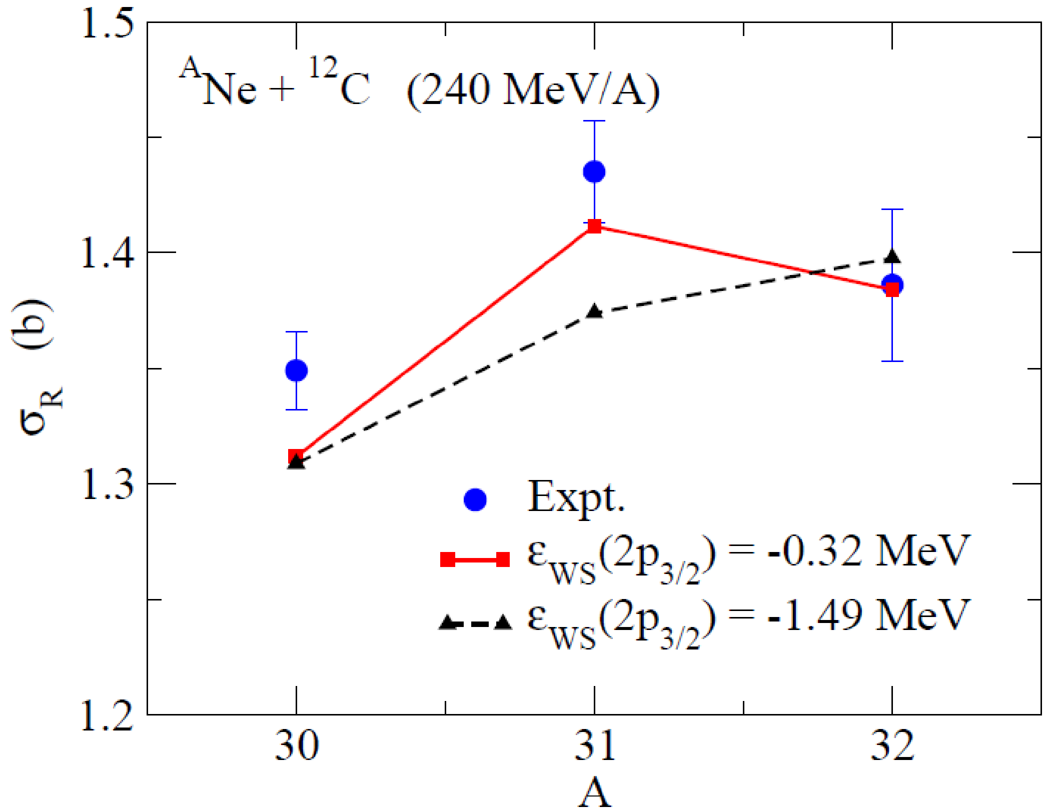
Other system: ^{30,31,32}Ne

HFB with a spherical Woods-Saxon



^{31}Ne (a =0.75 fm)

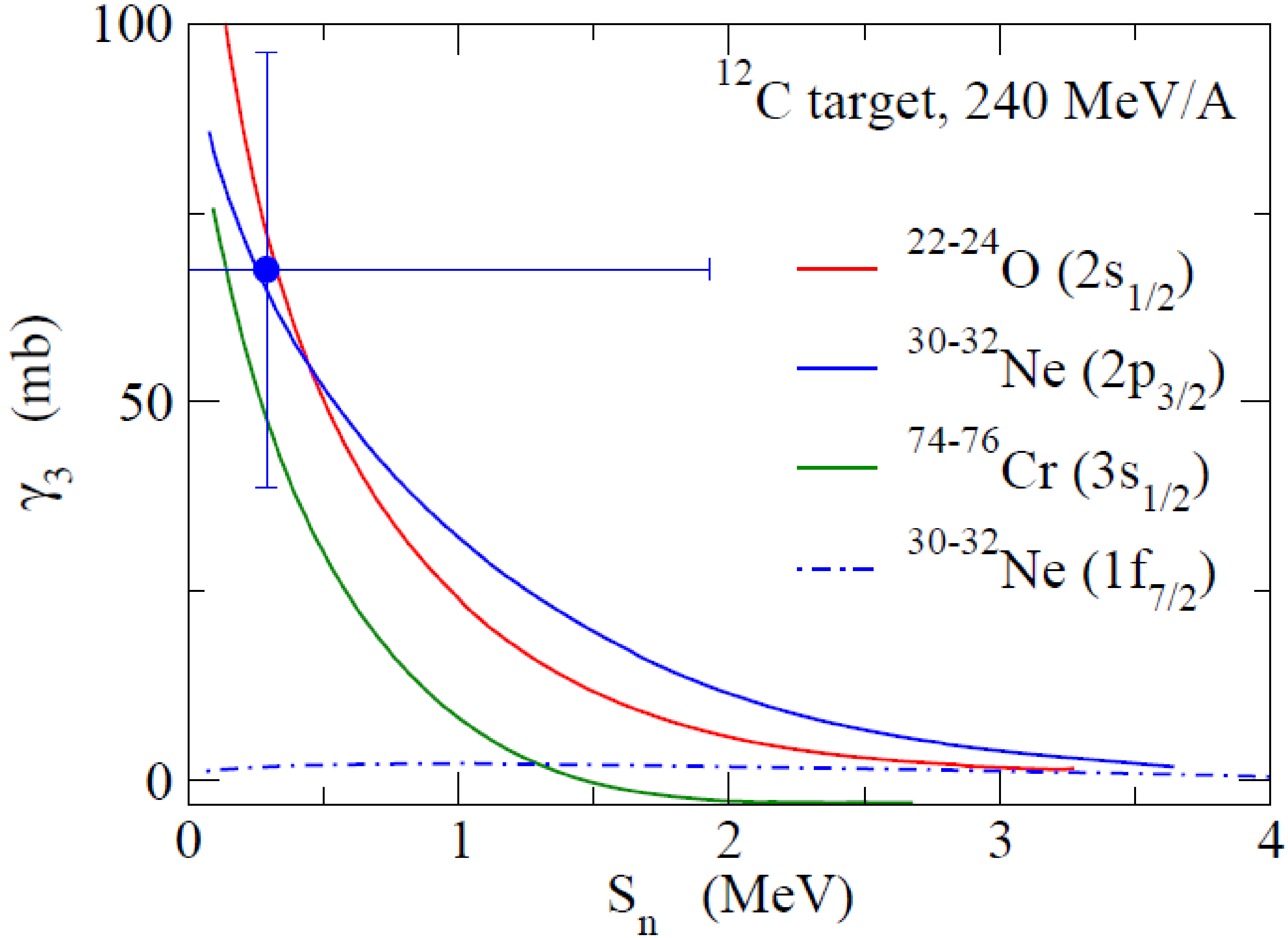




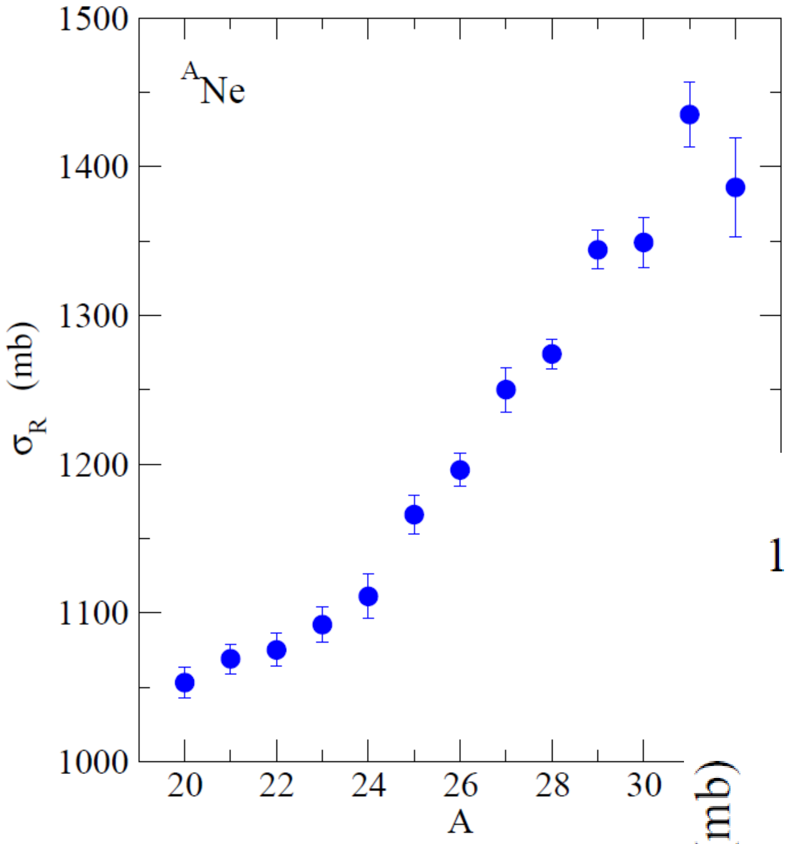
K. H. and H. Sagawa, PRC84('11)011303(R)

Systematic study: OES parameter

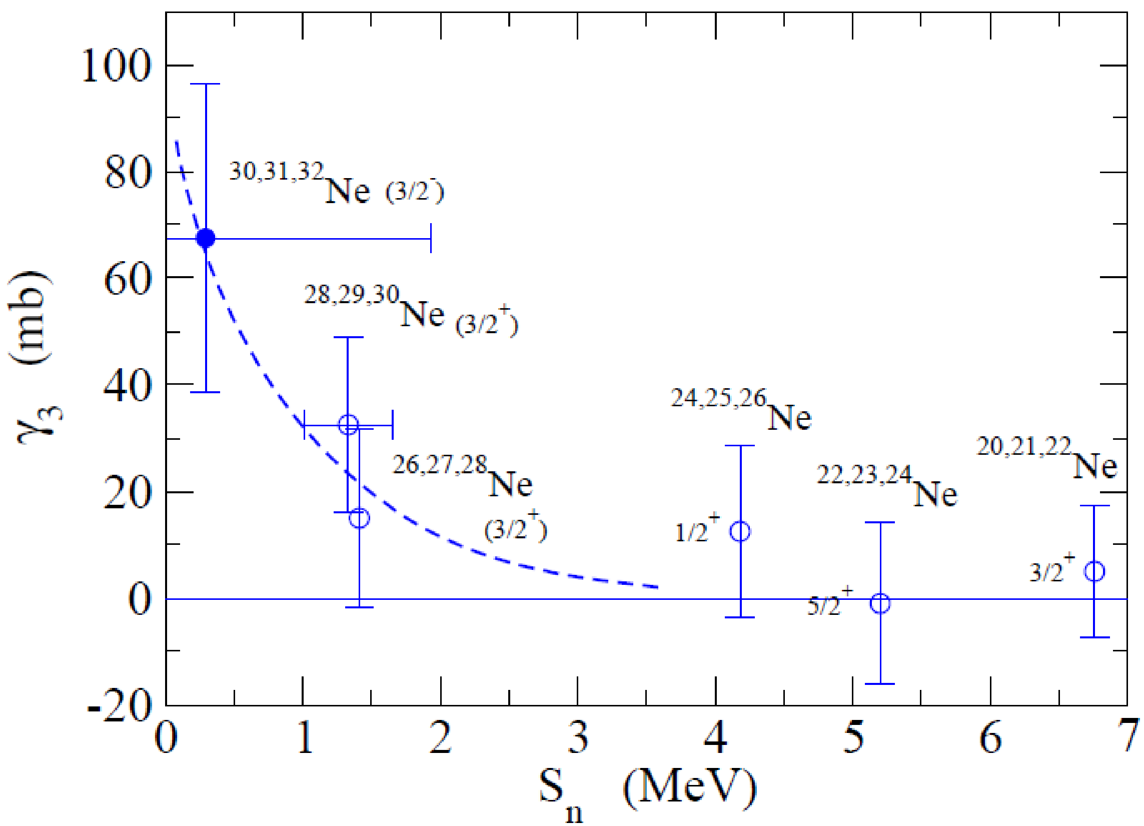
$$\gamma_3 \equiv -\frac{1}{2}[\sigma_R(A+2) - 2\sigma_R(A+1) + \sigma_R(A)]$$



K. H. and H. Sagawa, PRC85(‘12)014303

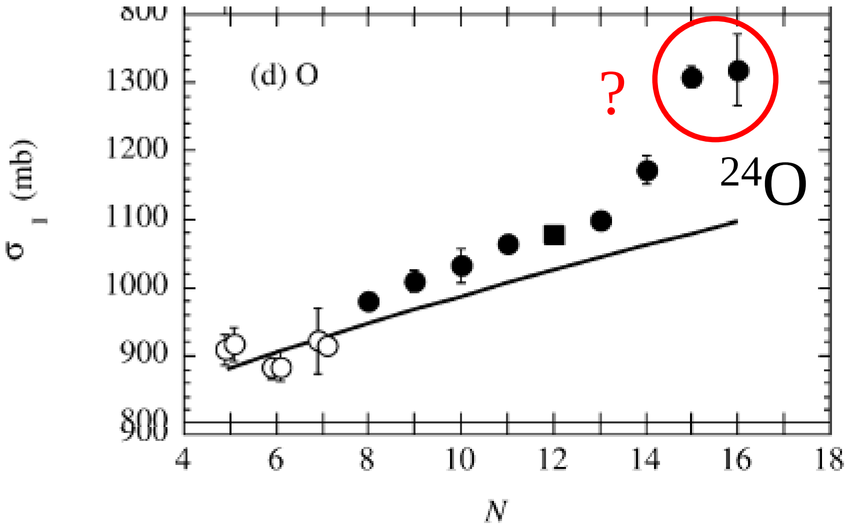


systematics

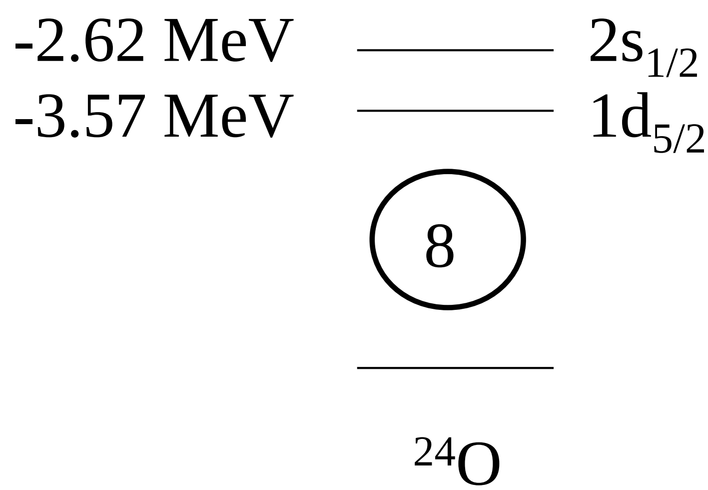


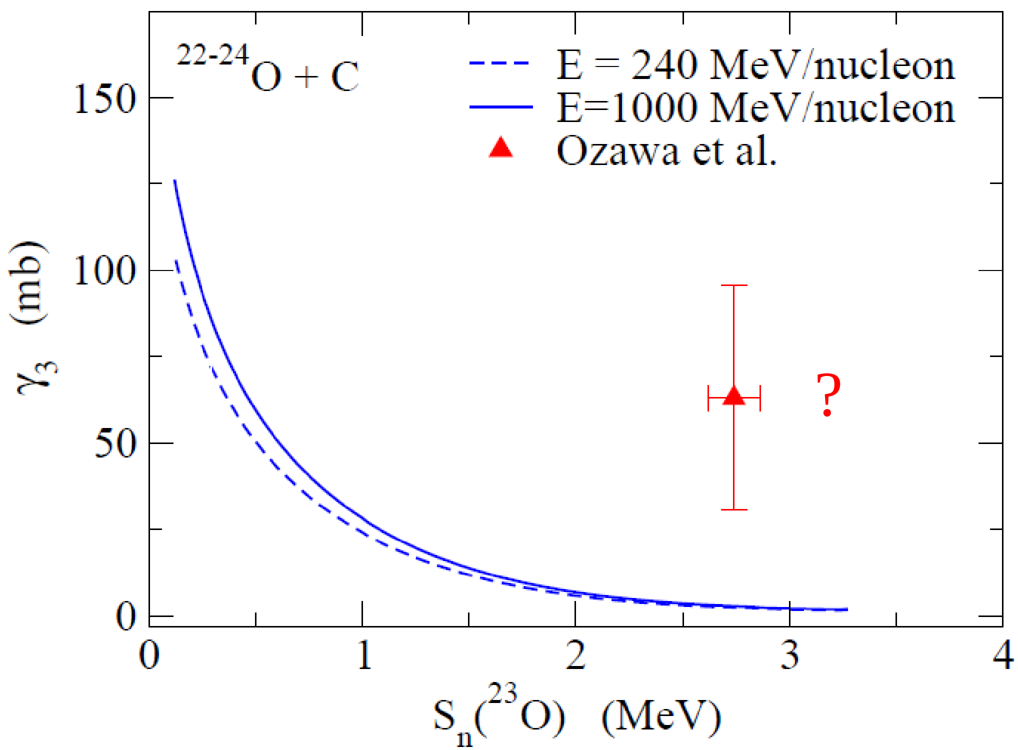
K. H. and H. Sagawa, PRC85('12)014303

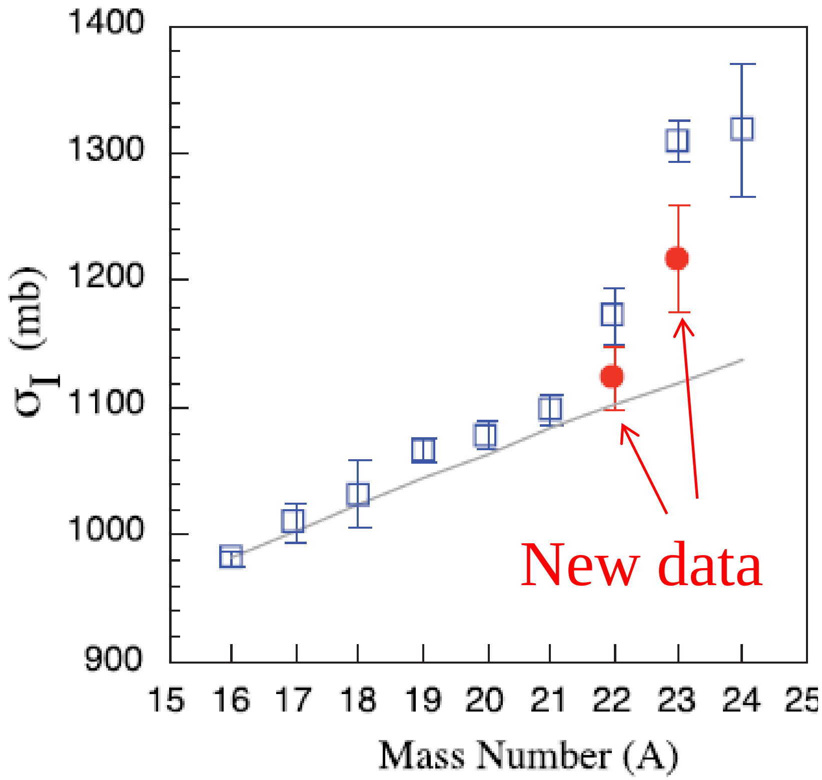
$^{22,23,24}O + ^{12}C$ @ 950 MeV/A



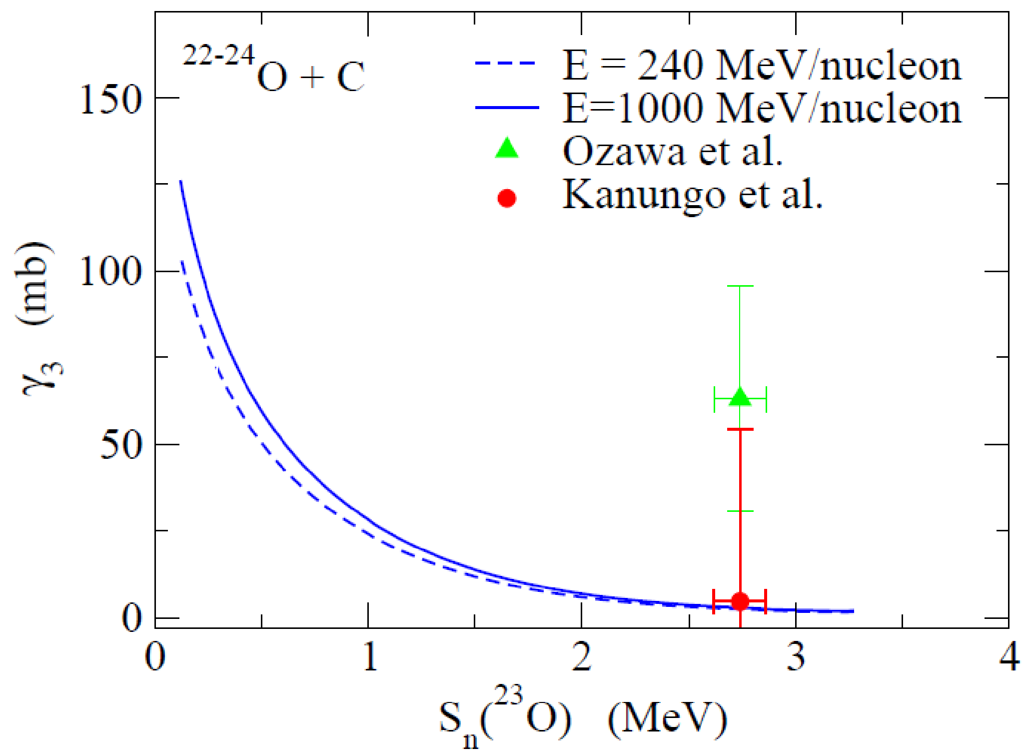
A. Ozawa et al.,NPA691('01)599







R. Kanungo et al.,
PRC84('11)061304(R)



K.H. and H. Sagawa,
arXiv:1202.2725 [nucl-th]

Summary

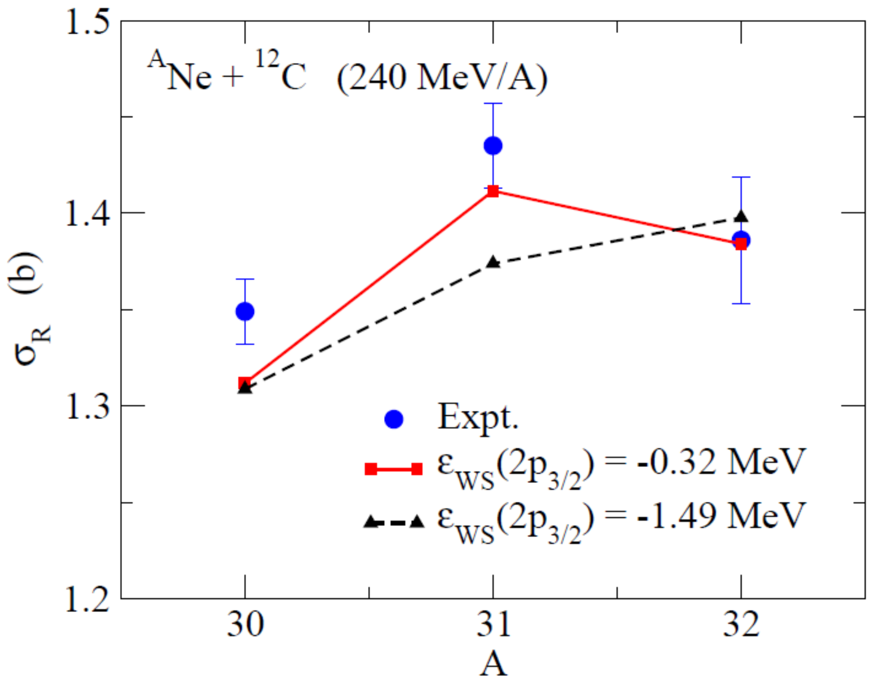
- Analyses of σ_R with HFB + Glauber

weakly-bound even-even nuclei:
✓the pairing correlation persists even at the drip
✓suppression of the radius due to *the pairing correlation* (l = 0, 1)

→ reduction of σ_R

↓

Odd-even staggering of σ_R



- Odd-even staggering parameter

a good tool to investigate the pairing correlation in weakly bound nuclei

- Work in progress: deformation effects