“Recent Progress in Nuclear Structure Physics 2016”, YITP, 2016.12.05-23

Application of beyond-mean-field approach to hypernuclei and heavy-ion fusion reactions

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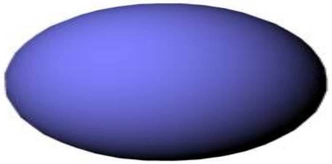
→ 北 Carolina U.)

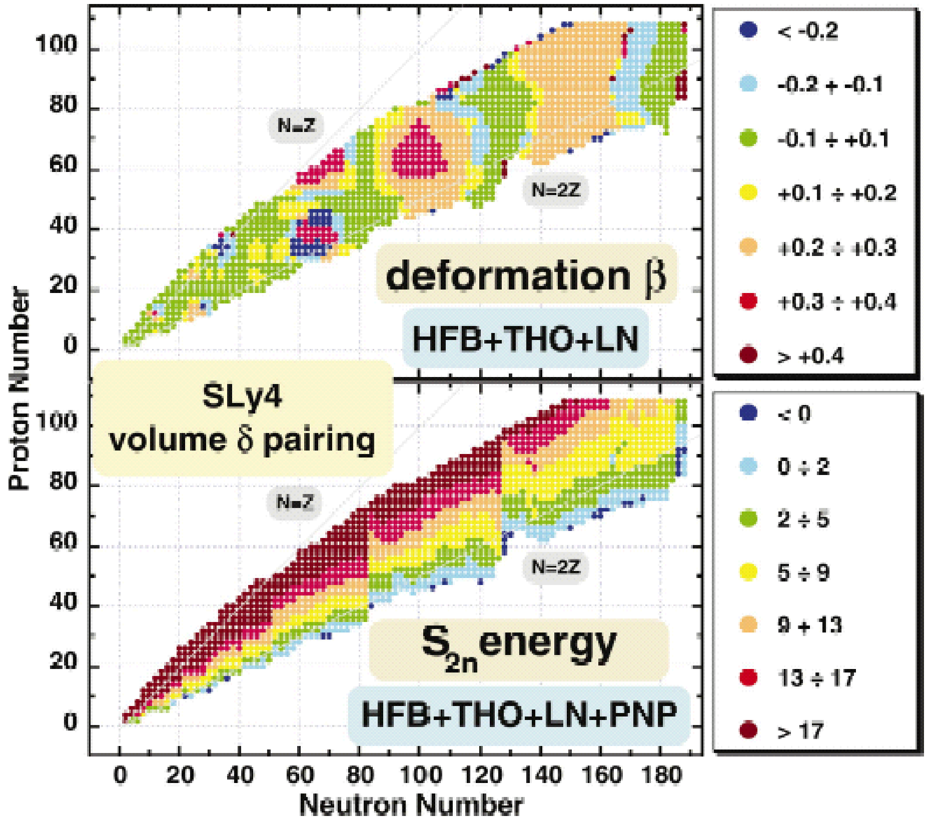
1. Introduction: mean-field approximation and beyond
2. Application to heavy-ion fusion reactions
3. Microscopic particle-rotor model for hypernuclei
4. Summary

Mean-field approximation and beyond



Self-consistent mean-field (Hartree-Fock) method:

- independent particles in a mean-field potential
- global theory for **the whole nuclear chart**
- body-fixed frame → intuitive picture for nuclear deformation
- optimized shape can be automatically determined



M.V. Stoitsov et al., PRC68('03)054312

Mean-field approximation and beyond

✓ body-fixed frame formalism $\longrightarrow$ intuitive picture of nuclear def.

Drawbacks of the mean-field approximation : nuclear spectrum

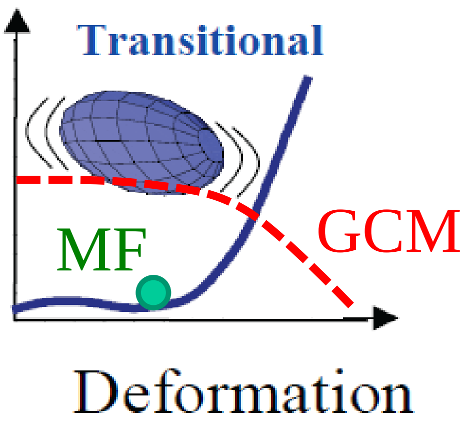
✓ spectrum: lab-frame $\longleftarrow$ transformation from intrinsic to lab. frames

nuclear spectrum: requires to go beyond the mean-field approximation

$$\boxed{|\Psi_{IM}(\beta)\rangle = \hat{P}^{I}_{MK}\hat{P}^{N}\hat{P}^{Z}\,|\Psi_{\mathrm{MF}}(\beta)\rangle}$$

angular momentum + particle number projections

✓ quantum fluctuation

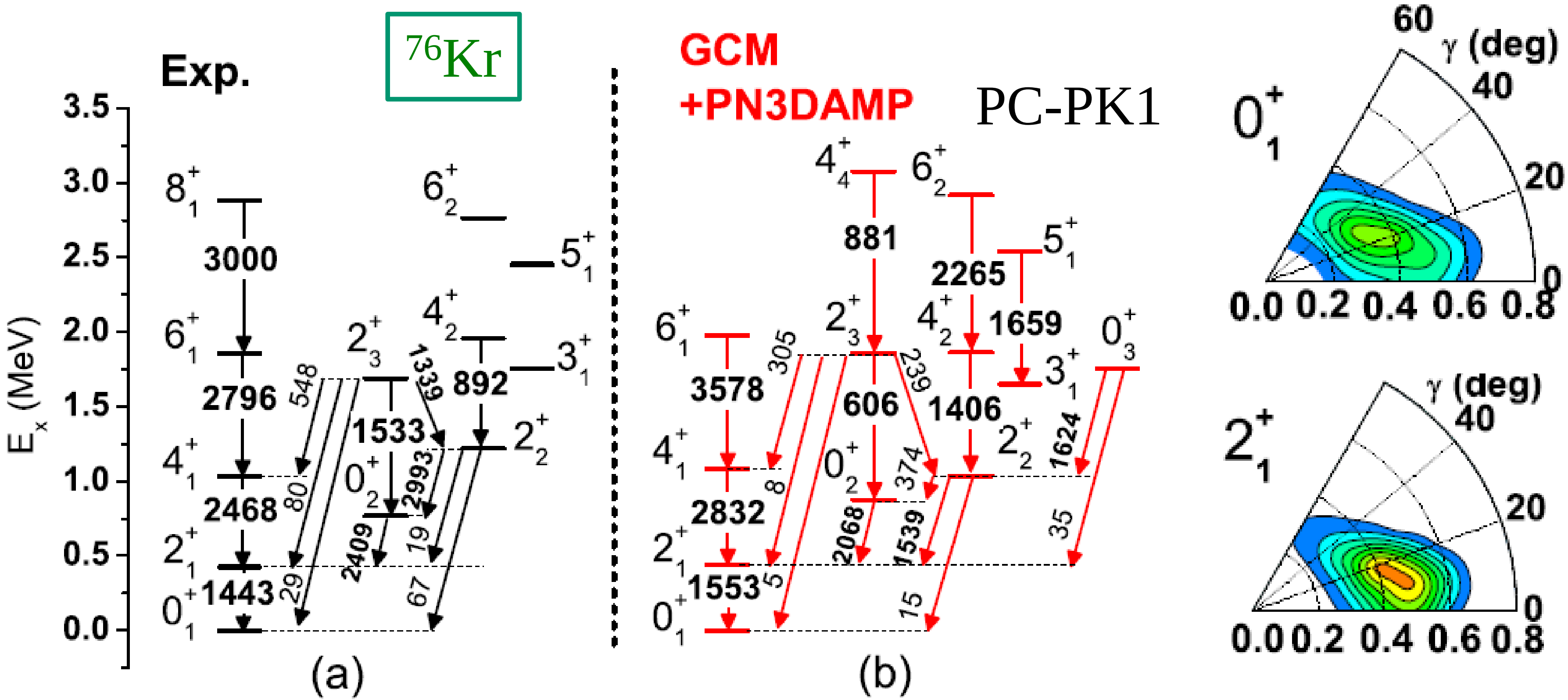


$$\boxed{|\Phi_{IM}\rangle = \int d\beta\, f(\beta)|\Psi_{IM}(\beta)\rangle}$$

generator coordinate method (GCM)

single $\longrightarrow$ multi Slater determinants beyond mean-field approximation

- ✓ angular momentum + particle number projections
- ✓ quantum fluctuation (GCM)



J.M. Yao, K.H., Z.P. Li, J. Meng, and P. Ring, PRC89 (‘14) 054306

cf. collective coordinates: Quadrupole moments (local operators) beyond mean-field approximation

- ✓ angular momentum + particle number projections
- ✓ quantum fluctuation (GCM)

In this talk:

applications to

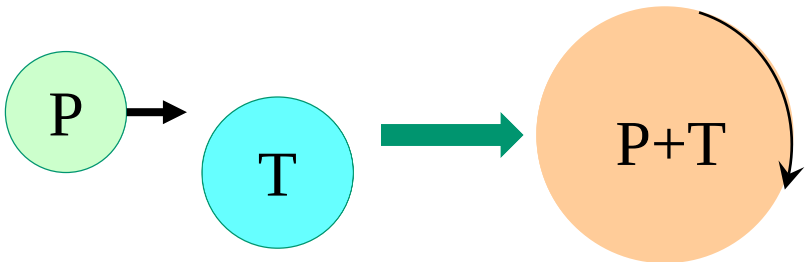
1. Heavy-ion fusion reactions at sub-barrier energies
2. Low-lying spectrum of Λ-hypernuclei

Heavy-ion fusion reactions at sub-barrier energies

K.H. and J.M. Yao, PRC91 (‘15) 064606
J.M. Yao and K.H., PRC94 (‘16) 011303(R)

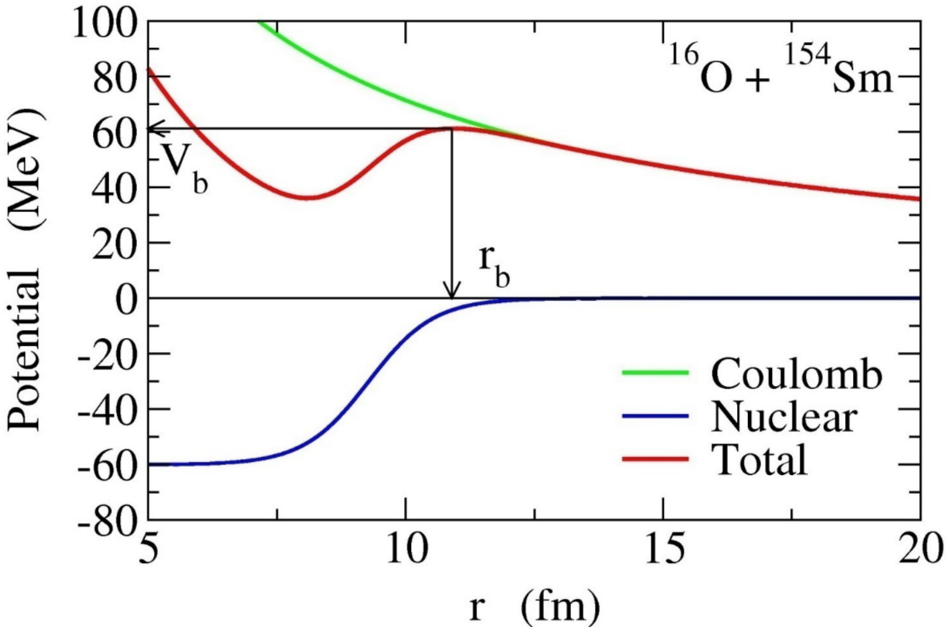
Heavy-ion fusion reactions

Fusion: compound nucleus formation



$A_{CN} = A_P + A_T$

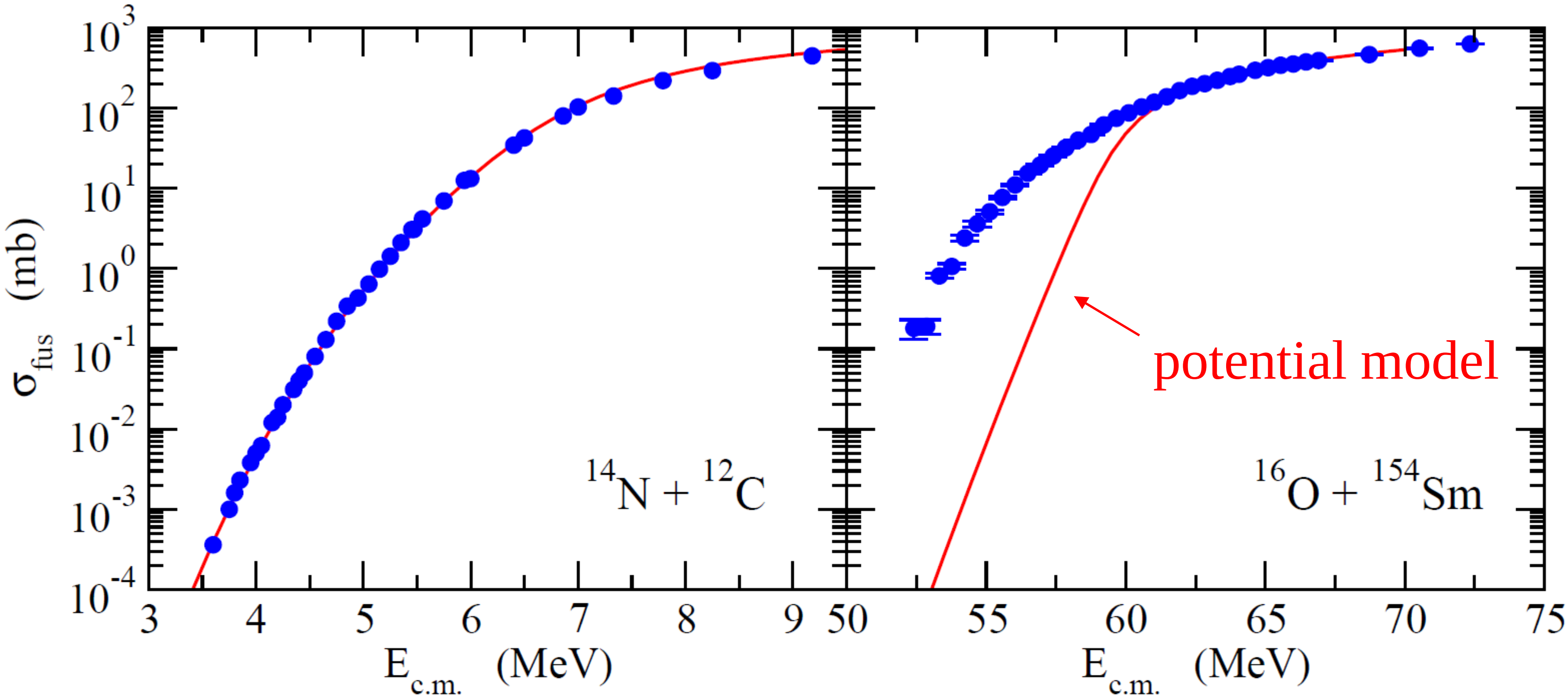
potential model: $V(r)$ + absorption



$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l(E)|^2)$$

Discovery of large sub-barrier enhancement of σ_{fus}

potential model: $V(r)$ + absorption

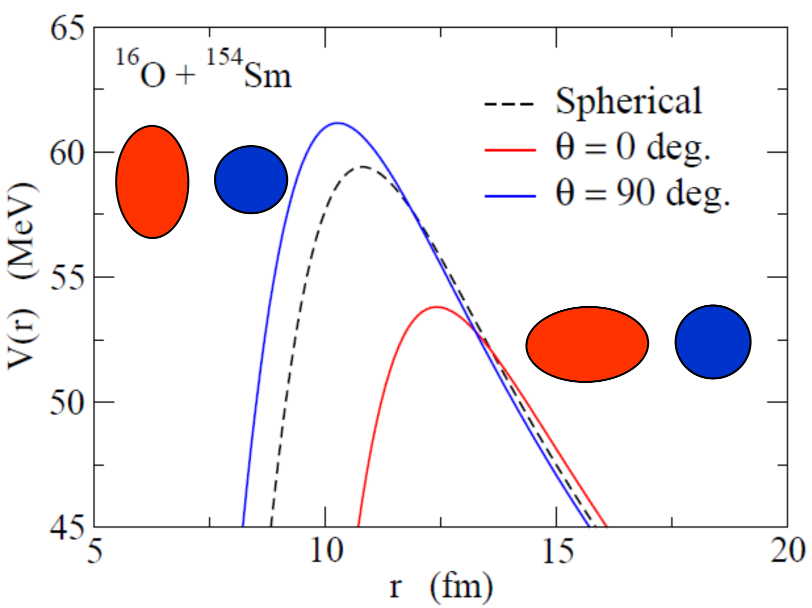


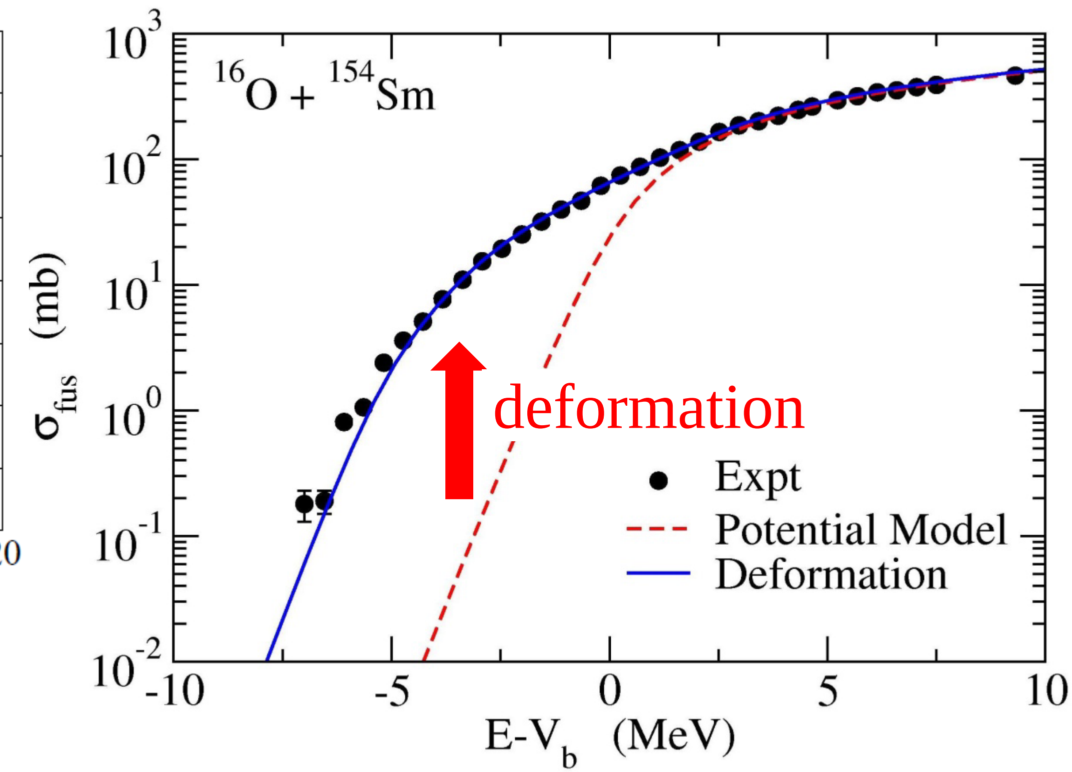
cf. seminal work:

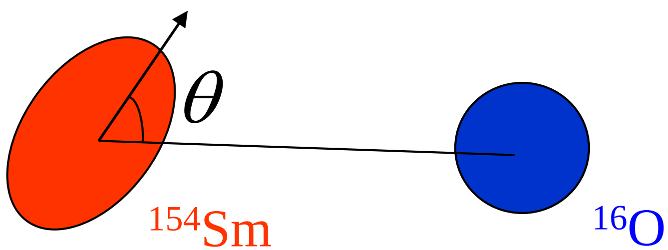
R.G. Stokstad et al., PRL41('78) 465

Effect of nuclear deformation

^{154}Sm : a deformed nucleus with $\beta_2 \sim 0.3$

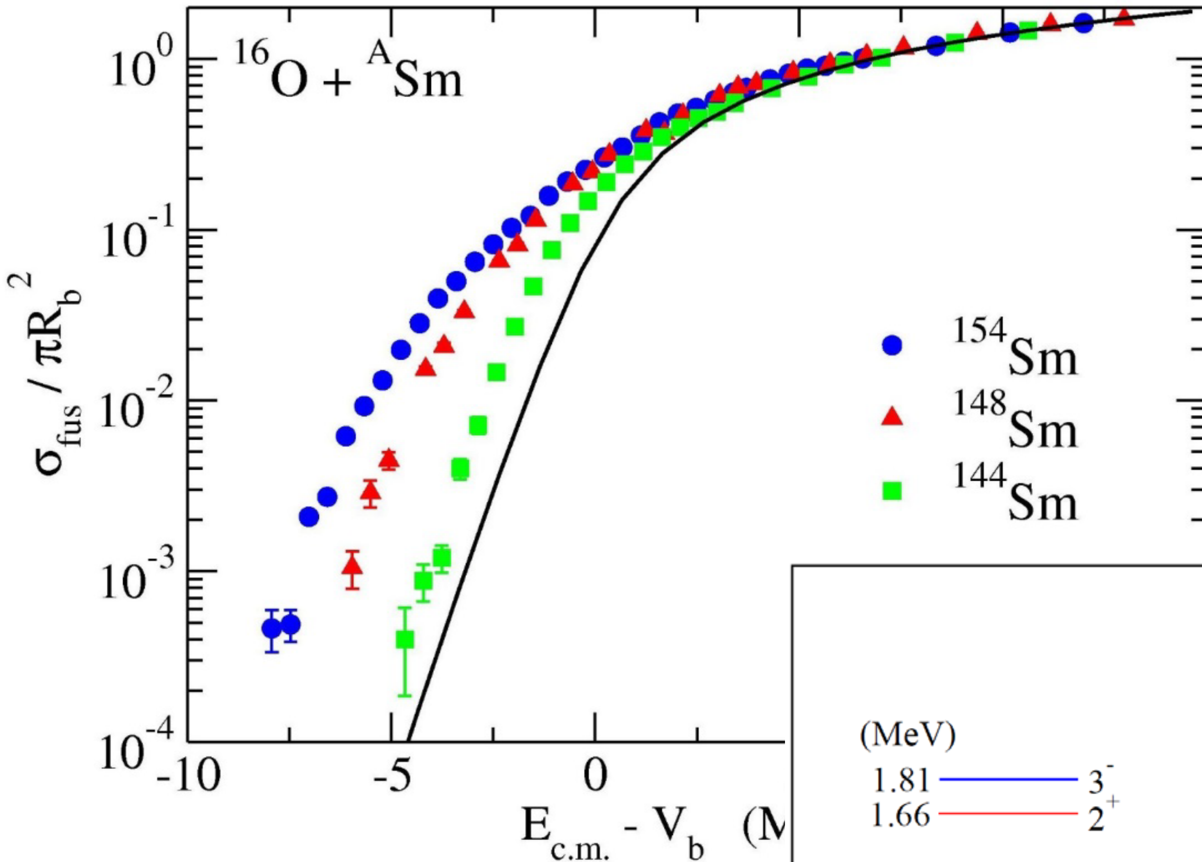


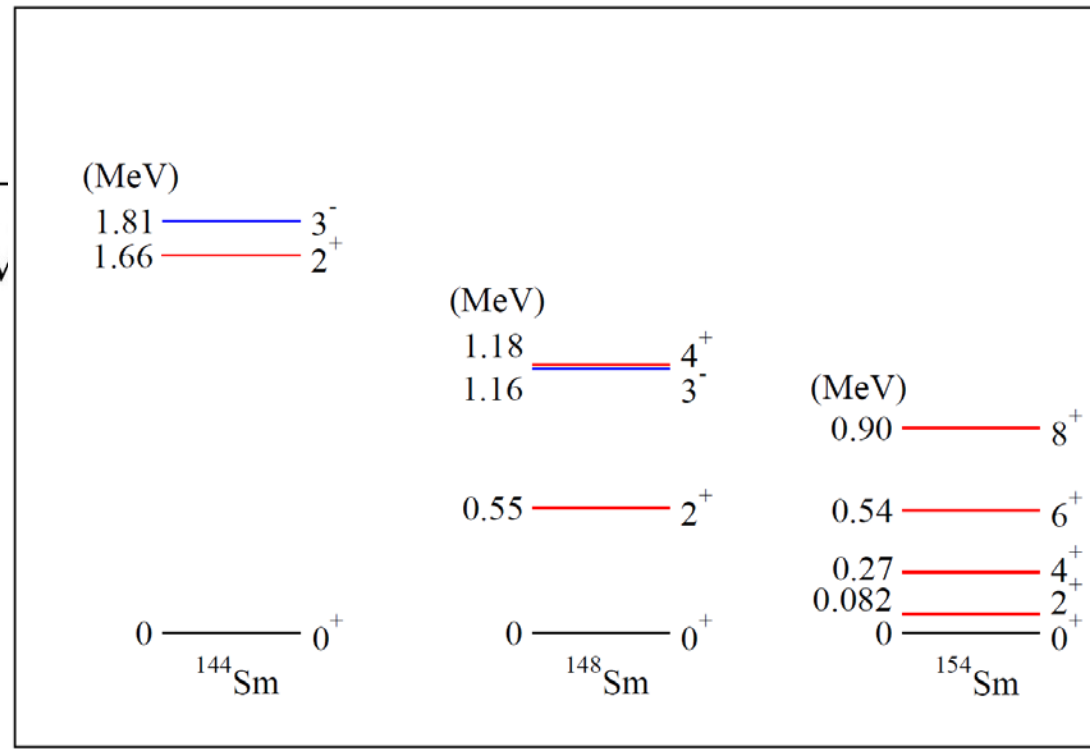




$$\sigma_{\mathsf{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\mathsf{fus}}(E;\theta)$$

Fusion: strong interplay between nuclear structure and nuclear reaction

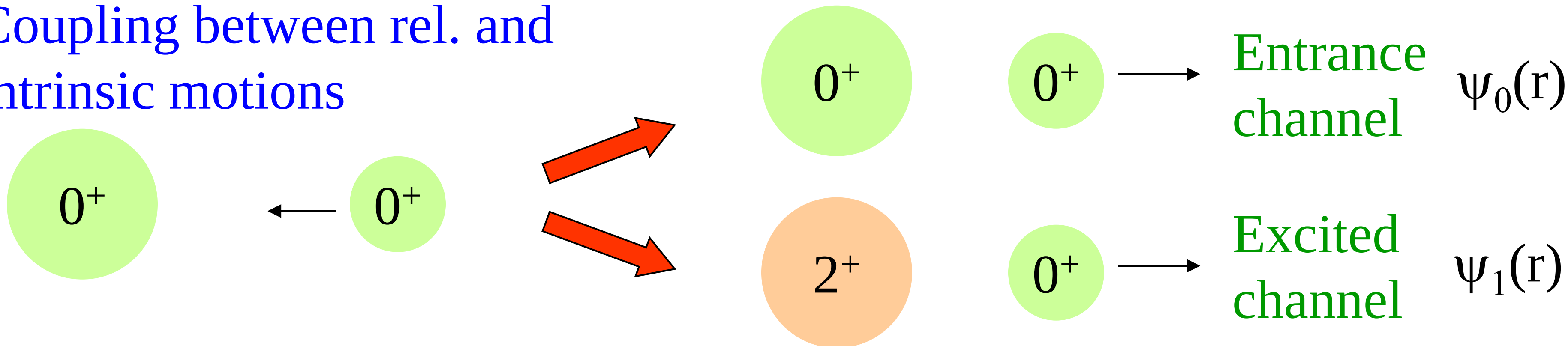


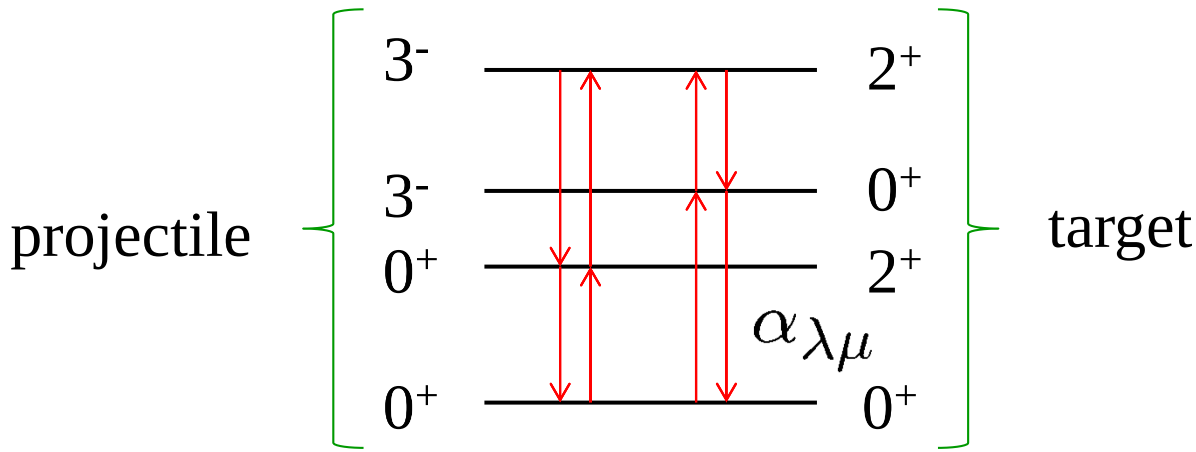


Strong target dependence
at $E < V_b$

Coupled-Channels method

Coupling between rel. and intrinsic motions





$$\boxed{\Psi(\boldsymbol{r},\xi)=\sum_k \psi_k(\boldsymbol{r})\phi_k(\xi)}$$

coupled Schroedinger equations for $\psi_k(r)$

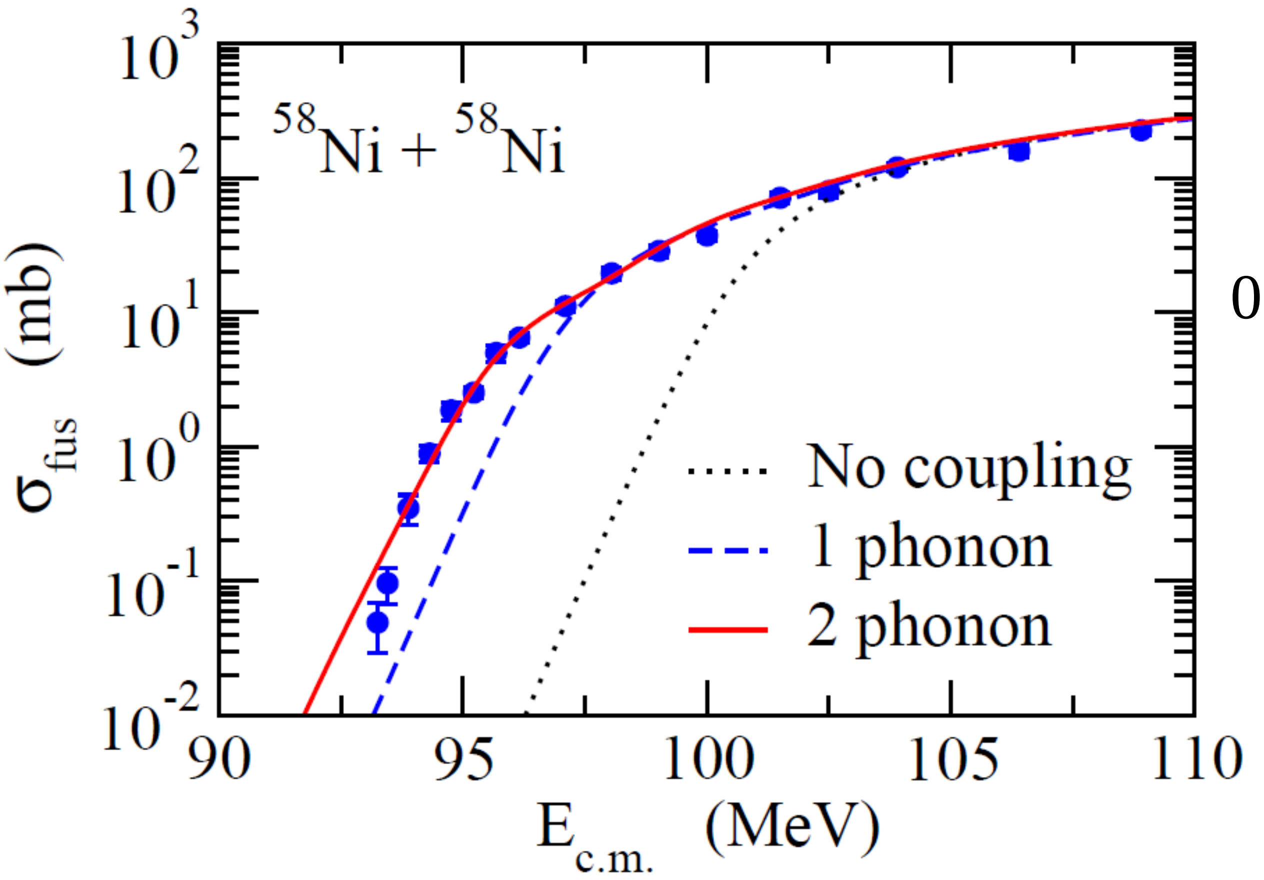
$$V_{\rm coup}(r,\xi)\sim -R_T\frac{dV_N(r)}{dr}\alpha_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}})$$

excitation operator

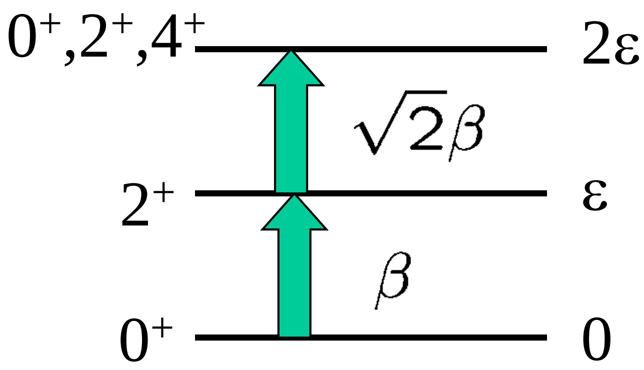
Semi-microscopic modeling of sub-barrier fusion

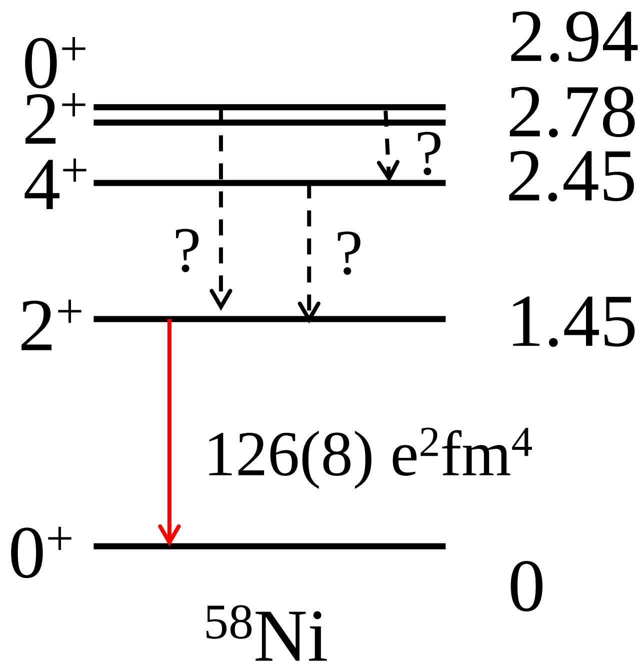
K.H. and J.M. Yao, PRC91('15) 064606

multi-phonon excitations



simple harmonic oscillator





$Q(2_1^+) = -10 \pm 6\ e\mathrm{fm}^2$

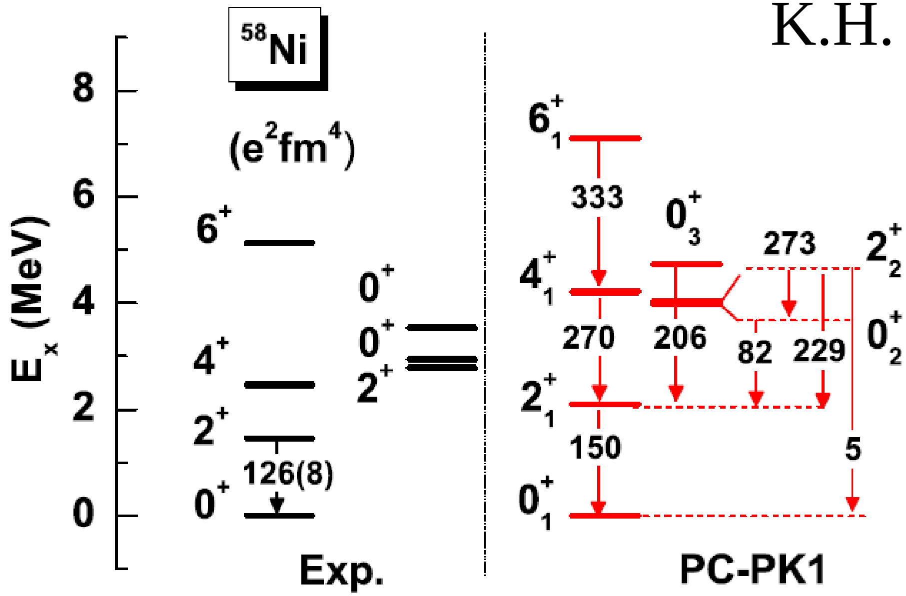
Simple harmonic oscillator
: justifiable?

beyond mean-field approach
to account for anharmonicity

cf. GCM: large amplitude
collective motions

Semi-microscopic coupled-channels model for sub-barrier fusion

K.H. and J.M. Yao, PRC91 (‘15) 064606



microscopic
multi-pole operator

✓ $$V_{\rm coup} \sim -R_T\frac{dV_N}{dr}\alpha_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}}) \to -R_T\frac{dV_N}{dr}Q_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}})$$

✓ *M*(E2) from MR-DFT calculation ← among higher members of phonon states

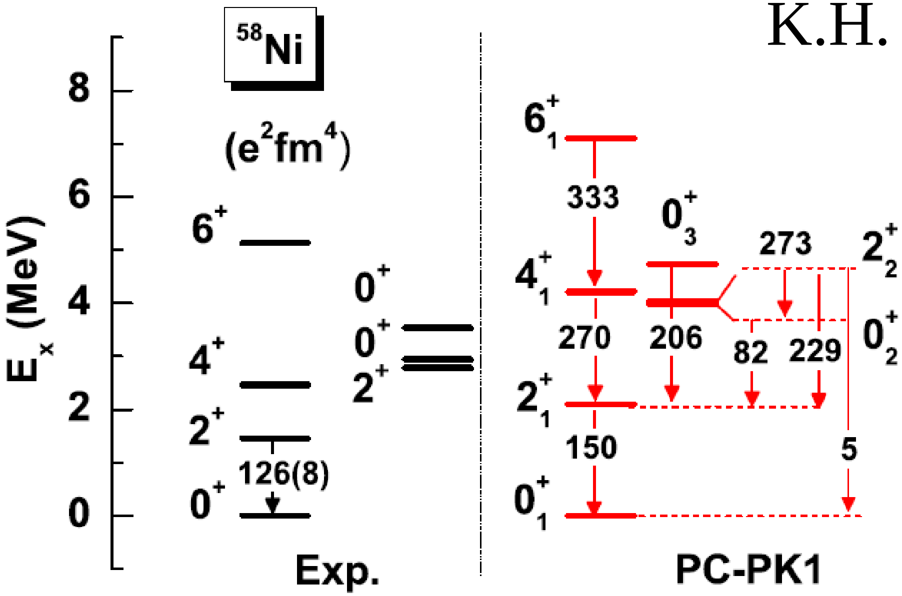
✓ scale to the empirical B(E2; $2_1^+ \to 0_1^+$)

$$B(E2;I_i\to I_f)\to f\times B(E2;I_i\to I_f)$$

$$f=\frac{B(E2;2^+\to 0^+)_{\rm exp}}{B(E2;2^+\to 0^+)_{\rm calc.}}$$

Semi-microscopic coupled-channels model for sub-barrier fusion

K.H. and J.M. Yao, PRC91 ('15) 064606



microscopic
multi-pole operator

✓ $$V_{\rm coup} \sim -R_T\frac{dV_N}{dr}\alpha_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}}) \to -R_T\frac{dV_N}{dr}Q_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}})$$

✓ *M*(E2) from MR-DFT calculation ← among higher members of phonon states

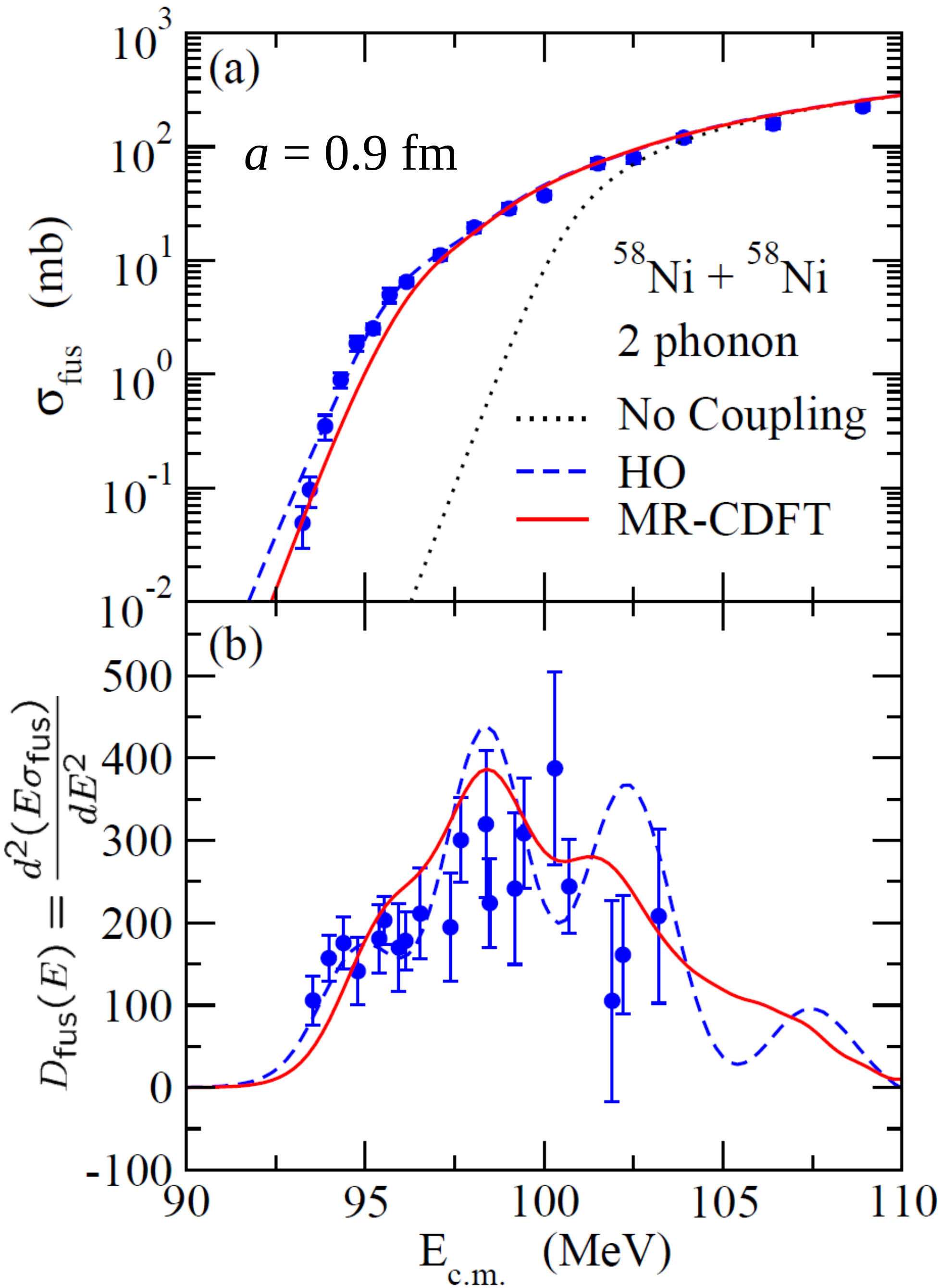
✓ scale to the empirical B(E2; $2_1^+ \to 0_1^+$)

✓ still use a phenomenological potential

✓ use the experimental values for E_x

✓ β_N and β_C from M_n/M_p for each transition

✓ axial symmetry (no 3^+ state)



^{58}Ni+^{58}Ni

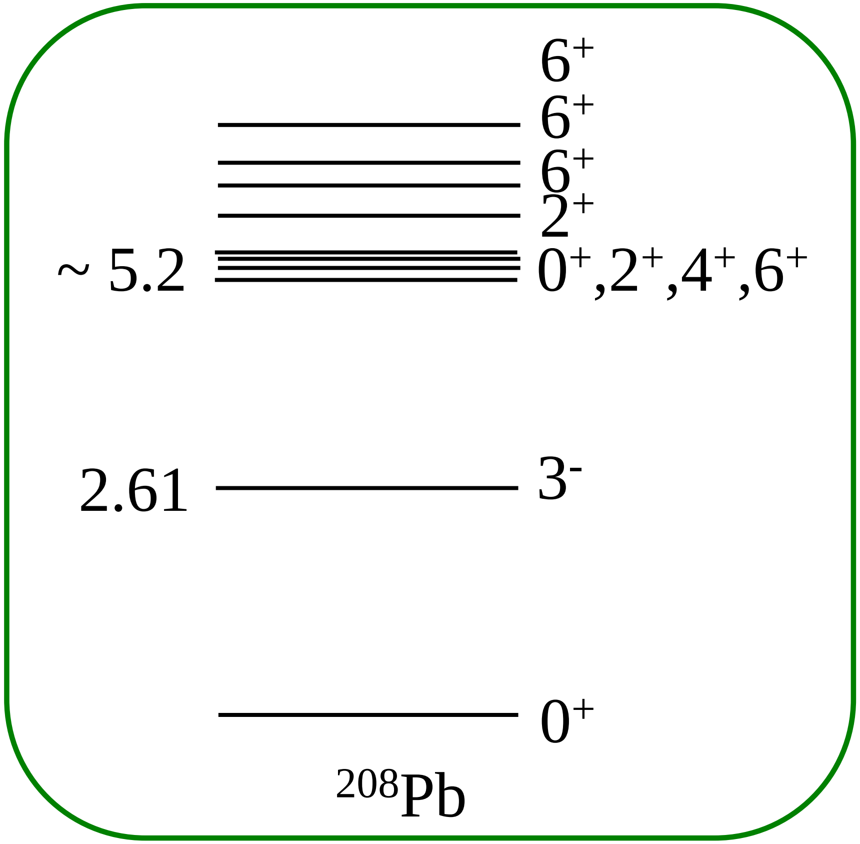
anharmonicity of 2^+ phonon
$\rightarrow$ only a minor improvement

Next, more non-trivial case with 2^+ - 3^- coupling:
anharmonicity of oct. vib. in ^{208}Pb

K.H. and J.M. Yao, PRC91 ('15) 064606

Application to ^{16}O + ^{208}Pb fusion reaction

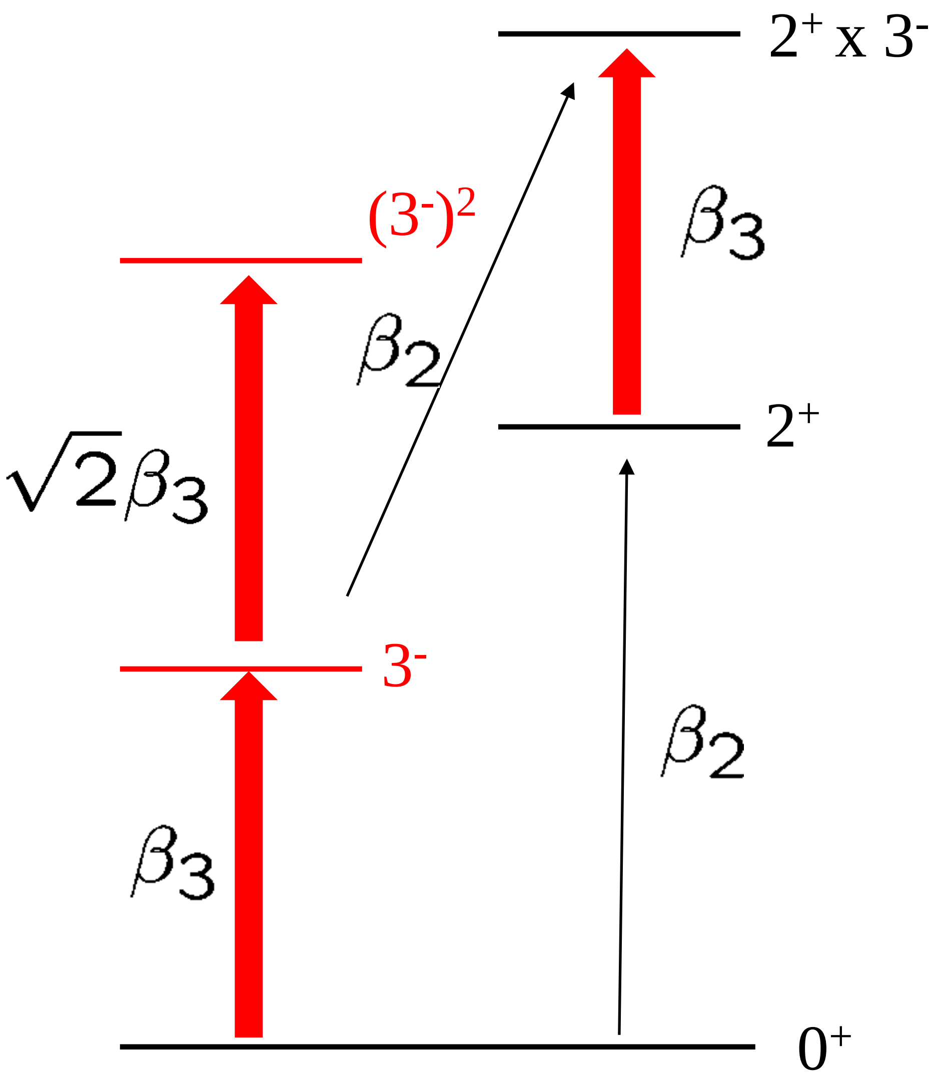
double-octupole phonon states in ^{208}Pb

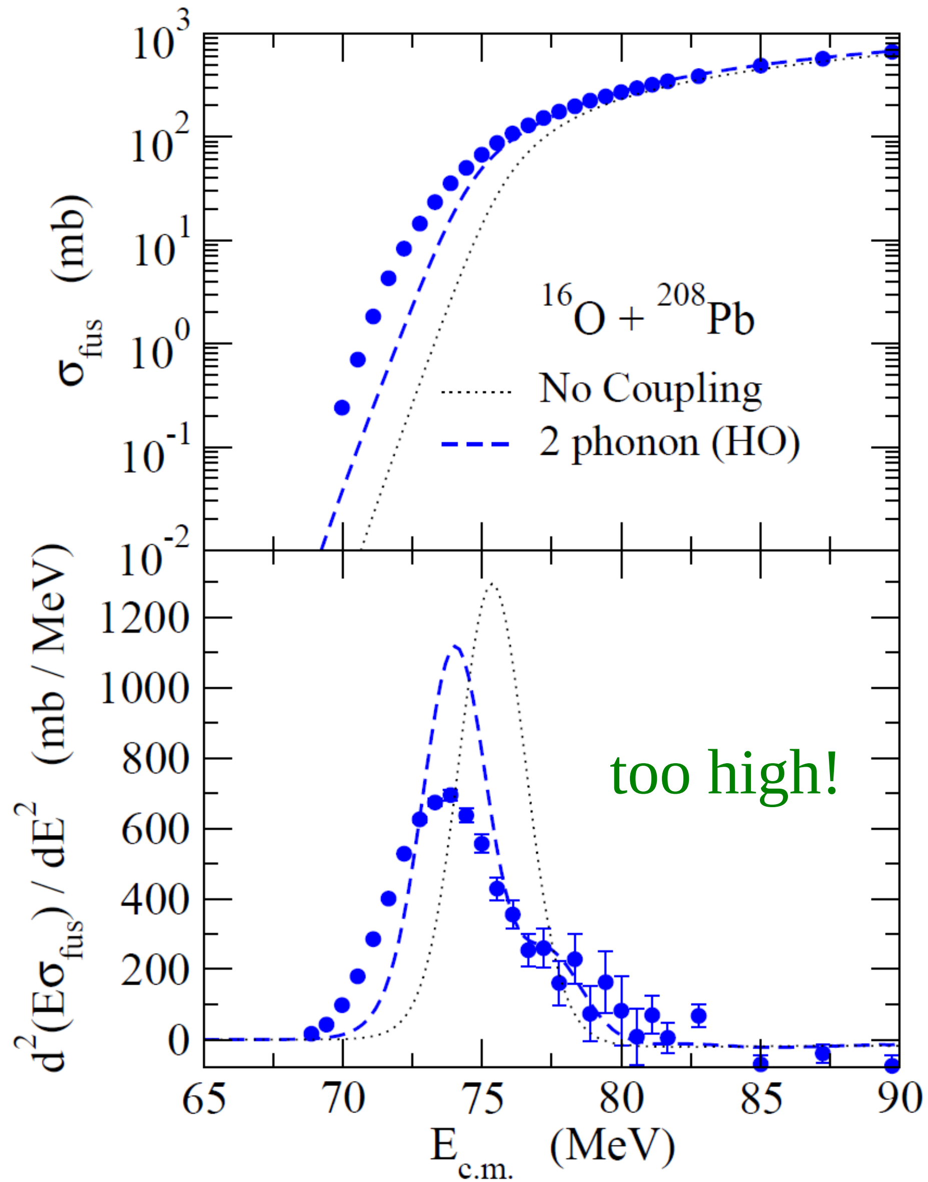


M. Yeh, M. Kadi, P.E. Garrett et al., PRC57 (‘98) R2085
K. Vetter, A.O. Macchiavelli et al., PRC58 (‘98) R2631
V. Yu. Pnomarev and P. von Neumann-Cosel, PRL82 (‘99) 501
B.A. Brown, PRL85 (‘00) 5300

large fragmentations, especially 6^+ state

Application to $^{16}O + ^{208}Pb$ fusion reaction

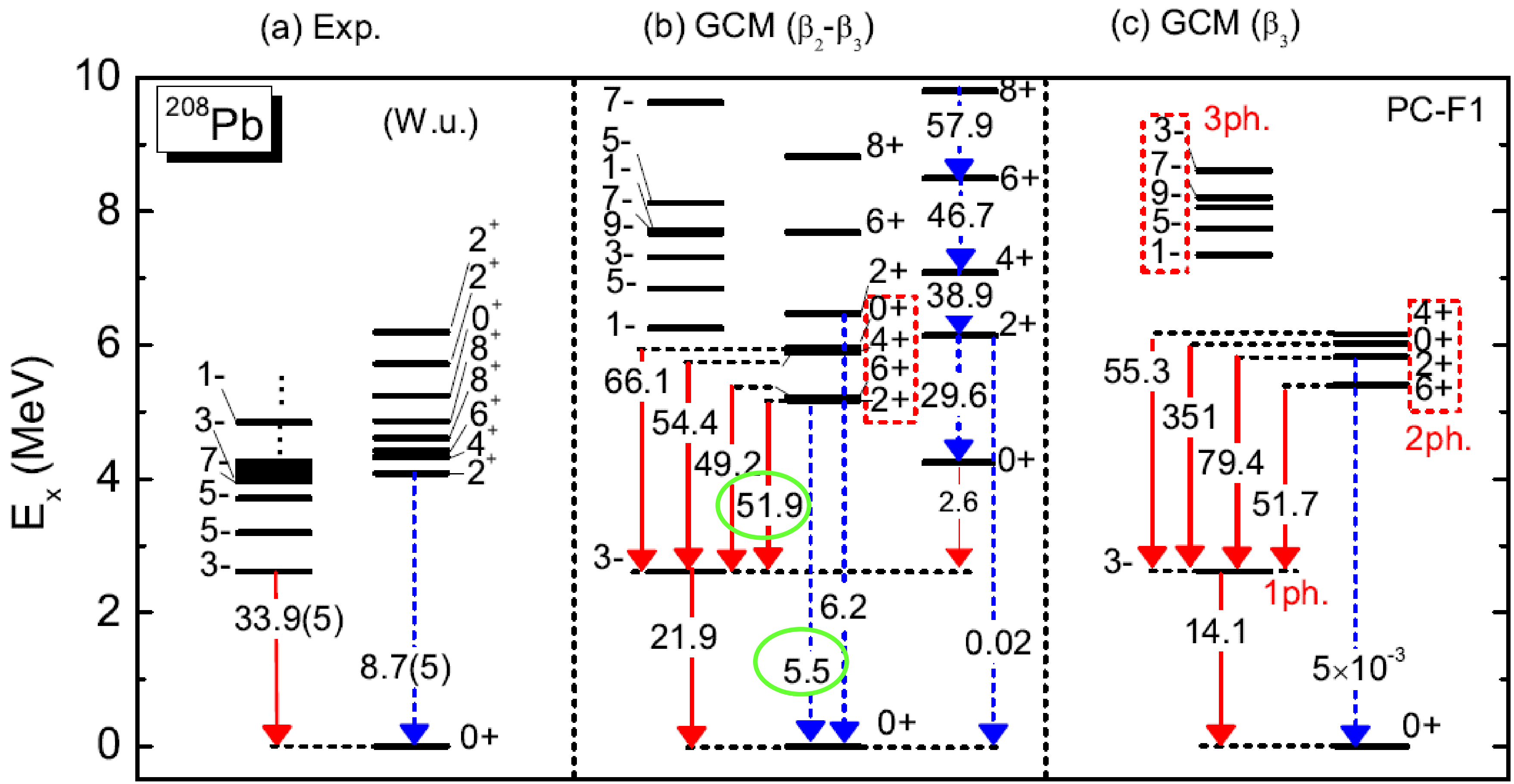




cf. C.R. Morton et al., PRC60('99) 044608 expt. data

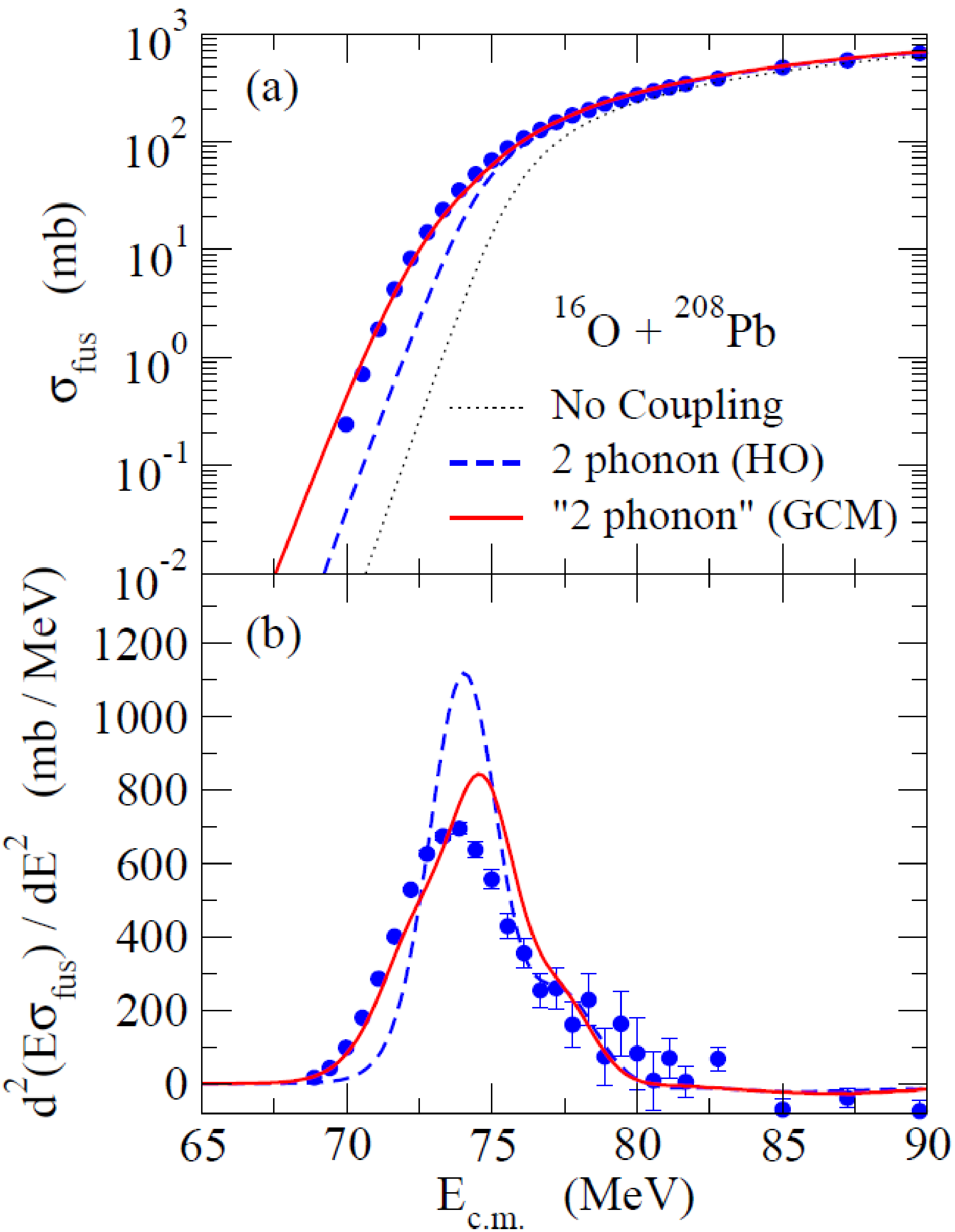
fluctuation both in β_3 and β_2

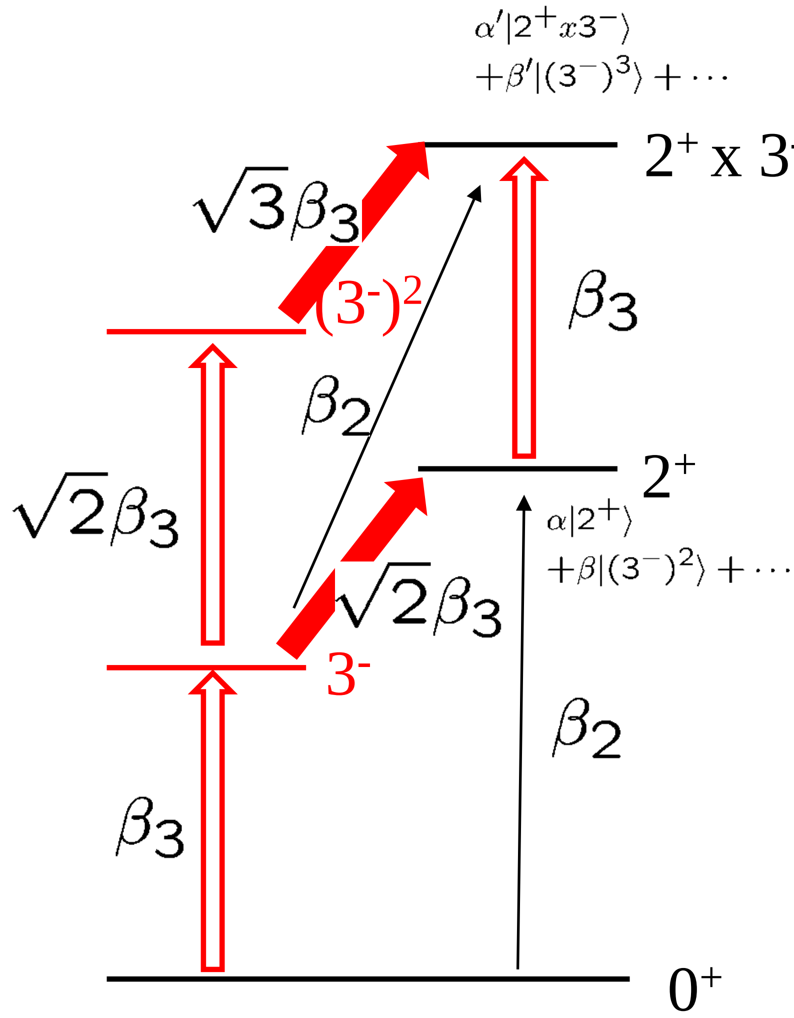
fluctuation in β_3 frozen at $\beta_2=0$



2_1^+ state: strong coupling both to g.s. and 3_1^-

$$\longrightarrow \quad |2_1^+\rangle = \alpha|2^+\rangle_{\rm HO} + \beta|[3^- \otimes 3^-]^{(I=2)}\rangle_{\rm HO} + \cdots$$





J.M. Yao and K.H.,
PRC94 ('16) 11303(R)

Low-lying spectra of Λ-hypernuclei

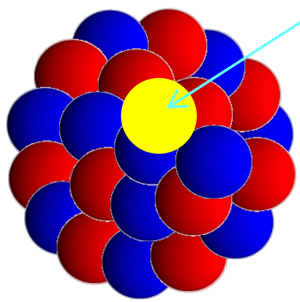
H. Mei, K.H., J.M. Yao, T. Motoba, PRC90 ('14) 064302
PRC91 ('15) 064305
PRC93 ('16) 044307
two more papers
in preparation
H. Mei, K.H., J.M. Yao, PRC93 ('16) 011301(R)

Introduction

Λ hypernucleus

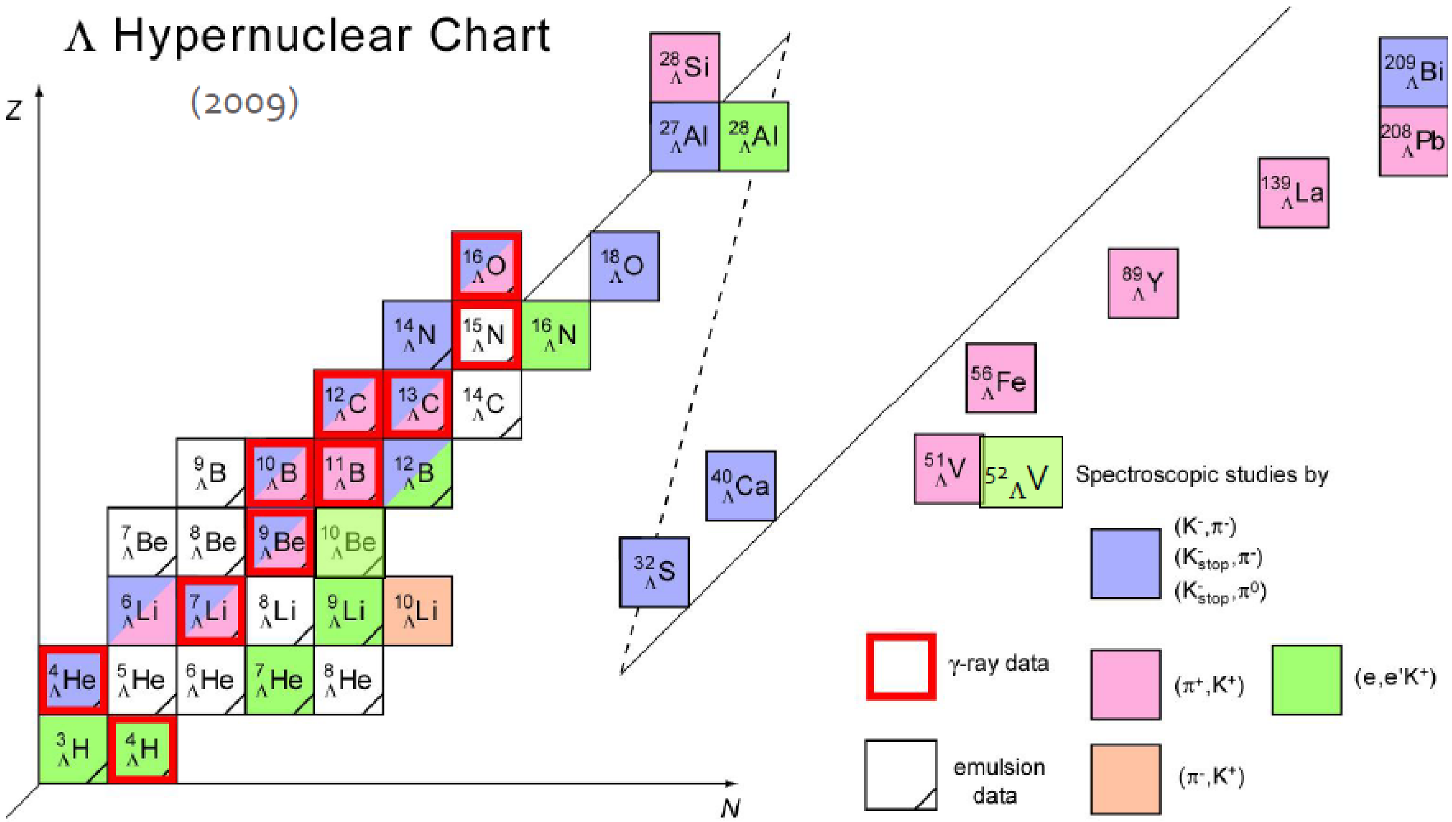
Λ particle: the lightest hyperon
(no charge, no isospin)

*no Pauli principle between nucleons and a Λ particle



proton

neutron



O. Hashimoto and H. Tamura, Prog. Part. Nucl. Phys. 57('06)564

Introduction

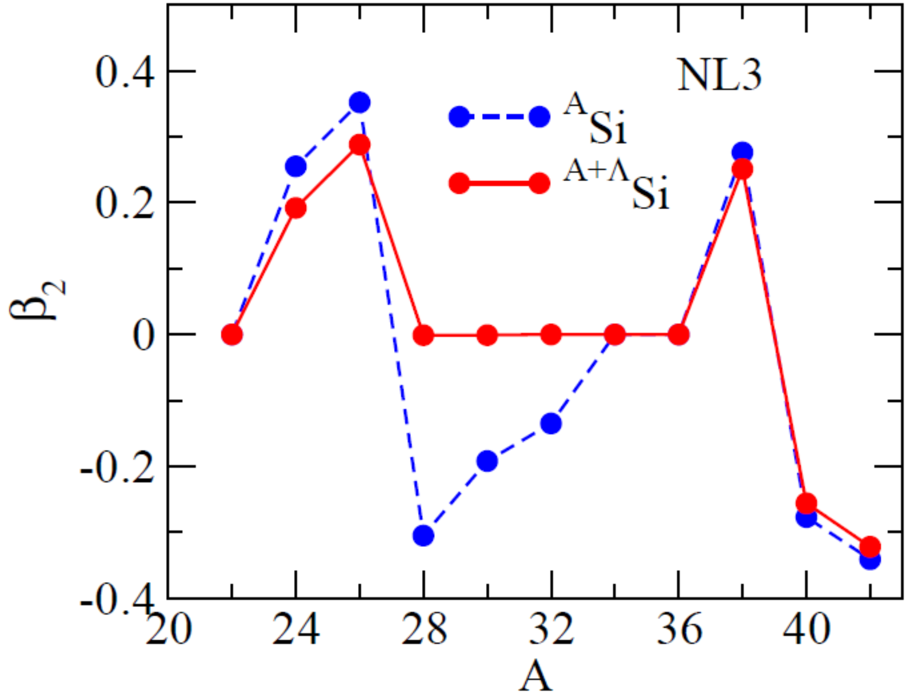
Impurity effects: one of the main interests of hypernuclear physics

how does Λ affect several properties of atomic nuclei?

- size, shape, density distribution, single-particle energy, shell structure, fission barrier……

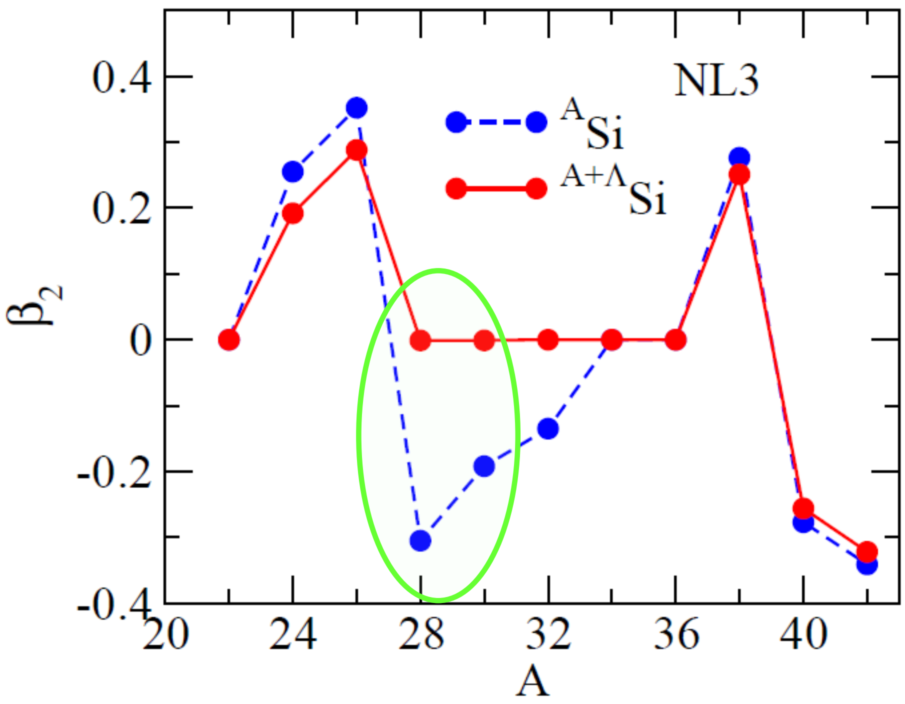
Theoretical approaches:

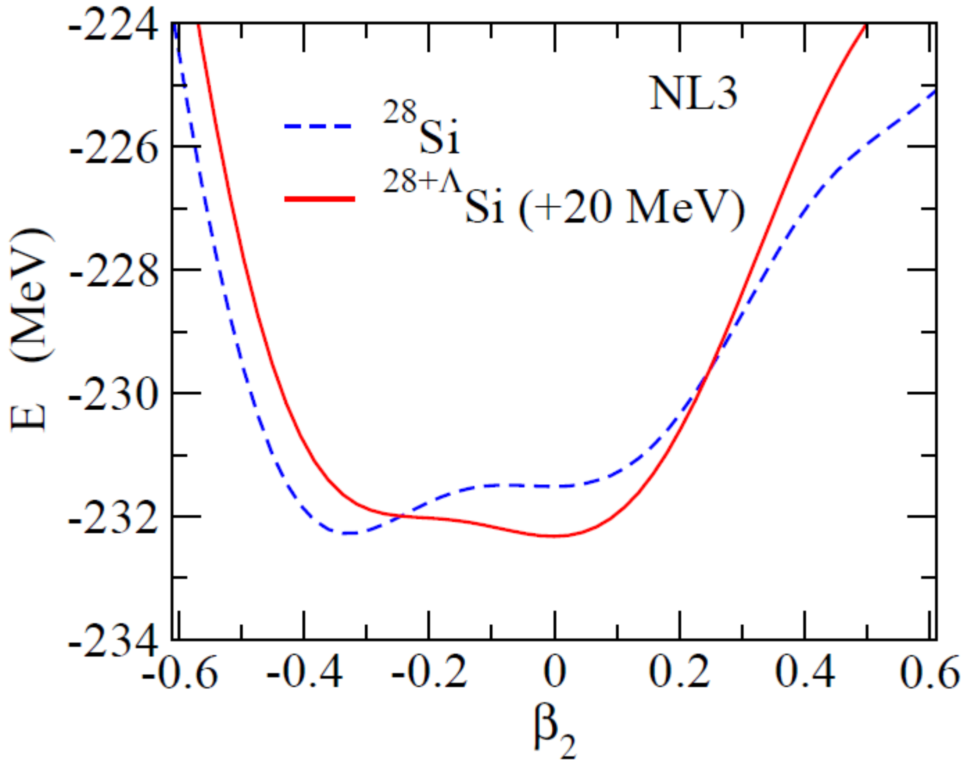
- ✓ cluster model
- ✓ shell model
- ✓ AMD
- ✓ self-consistent mean-field models



Myaing Thi Win and K.H., PRC78('08)054311

Si isotopes





Myaing Thi Win and K.Hagino, PRC78('08)054311

soft energy surface → role of beyond-mean-field effect?

beyond mean-field approximation

◆ Projection+GCM for the whole (A_c+1) system

H. Mei, K.H. and J.M. Yao, PRC93 (‘16) 011301(R)

$$\boxed{|\Psi_{IM}\rangle = \int d\beta f(\beta)\, \hat{P}^I_{MK} \hat{P}^N \hat{P}^Z \left[|\Psi_{A_c}(\beta)\rangle |\Psi_\Lambda(\beta)\rangle\right]}$$

cf. Hill-Wheeler eq. for each *I*

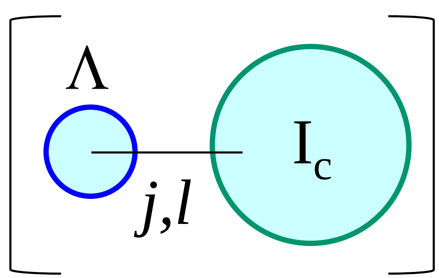
◆ Microscopic particle-rotor model for single-Λ hypernuclei

H. Mei, K.H., J.M. Yao, and T. Motoba, PRC90(‘14)064302, PRC91(‘15) 064305
PRC93 (‘16) 044307

i) beyond mean-field calculations for e-e core: $|\Phi_{0+}\rangle, |\Phi_{2+}\rangle, |\Phi_{4+}\rangle, \cdots$

ii) coupling of Λ to the core states

$$|\Phi_{IM}\rangle = \sum_{j,l,I_c}$$



(IM)

Λ+core model with core excitations

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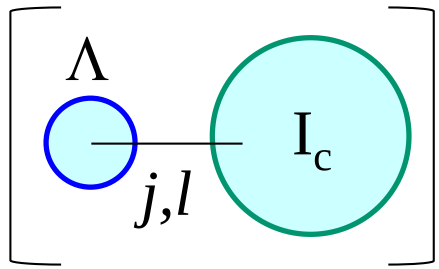


(IM)

Λ+core model with core excitations

$$\Psi_{JM}(\boldsymbol{r}_\Lambda, \{\boldsymbol{r}_i\}) = \sum_{n,j,\ell,I} R_{j\ell nI}(r_\Lambda)[\mathcal{Y}_{j\ell}(\hat{\boldsymbol{r}}_\Lambda) \otimes \Phi_{nI}(\{\boldsymbol{r}_i\})]^{(JM)}$$

$$H = H_c + T_\Lambda + \sum_{i=1}^{A_c} v_{N\Lambda}(\boldsymbol{r}_\Lambda, \boldsymbol{r}_i)$$

⇒ coupled equations for R_{jlnI}

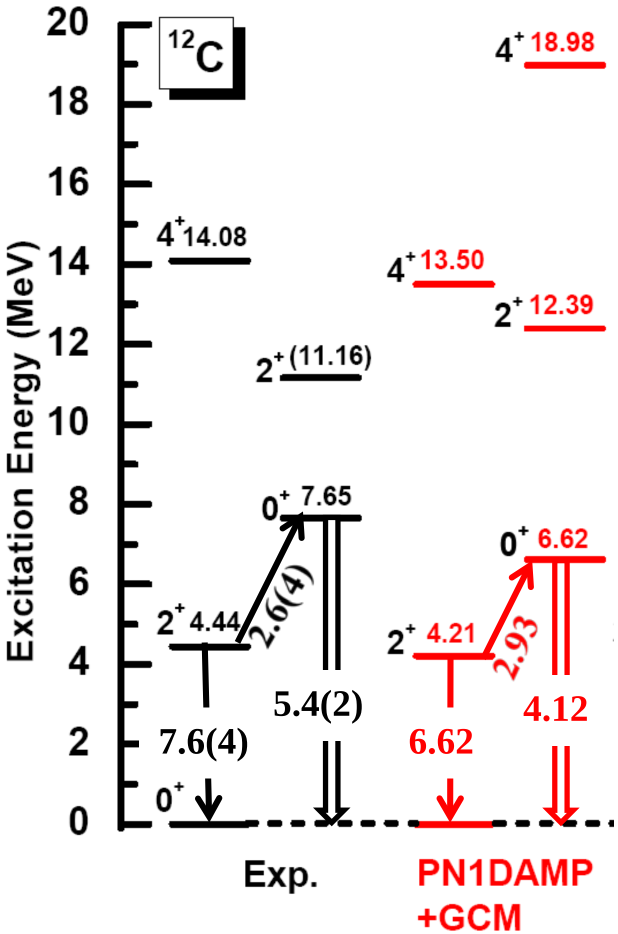
$v_{N\Lambda}$: point coupling relativistic NΛ interaction

Microscopic Particle-Rotor Model for Λ hypernuclei

Example: $^{13}{}_{\Lambda}C$

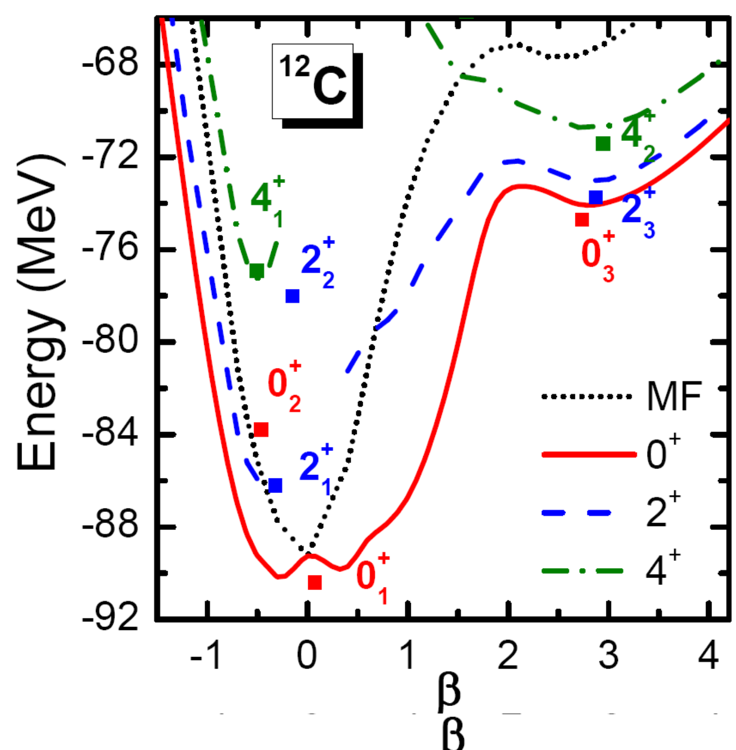
i) beyond mean-field calculations for e-e core (^{12}C) : GCM + projections

$$|\Phi_{I_cM_c}\rangle = \int d\beta\, f(\beta)|\Psi_{I_cM_c}(\beta)\rangle = \int d\beta\, f(\beta)\hat{P}^{I_c}_{M_cK_c}\hat{P}^{N}\hat{P}^{Z}|\Psi_{\mathrm{MF}}(\beta)\rangle$$



✓ axial symmetry

✓ relativistic PC-F1



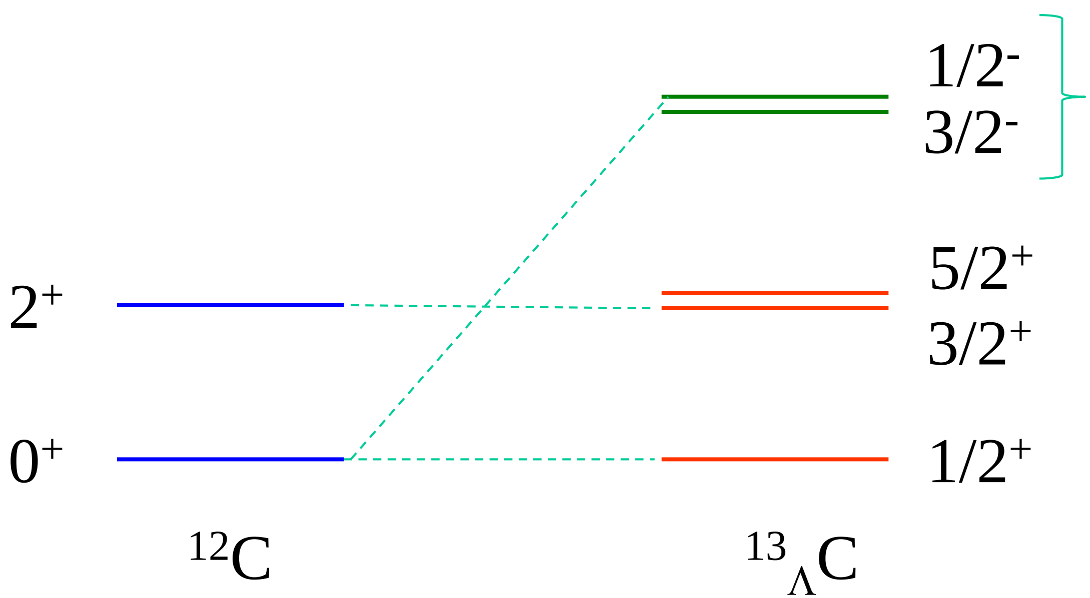
Microscopic Particle-Rotor Model for Λ hypernuclei

Example: $^{13}_{\Lambda}C$

(i) beyond mean-field calculations for e-e core (^{12}C)

(ii) coupling of Λ to the core states

$$|\Phi_{IM}\rangle = \sum_{j,l,I_c} \Big[\underbrace{\psi_{jl}(\boldsymbol{r}_\Lambda)}_{\Lambda} \otimes \underbrace{|\Phi_{I_c}\rangle}_{\text{core}}\Big]^{(IM)}$$

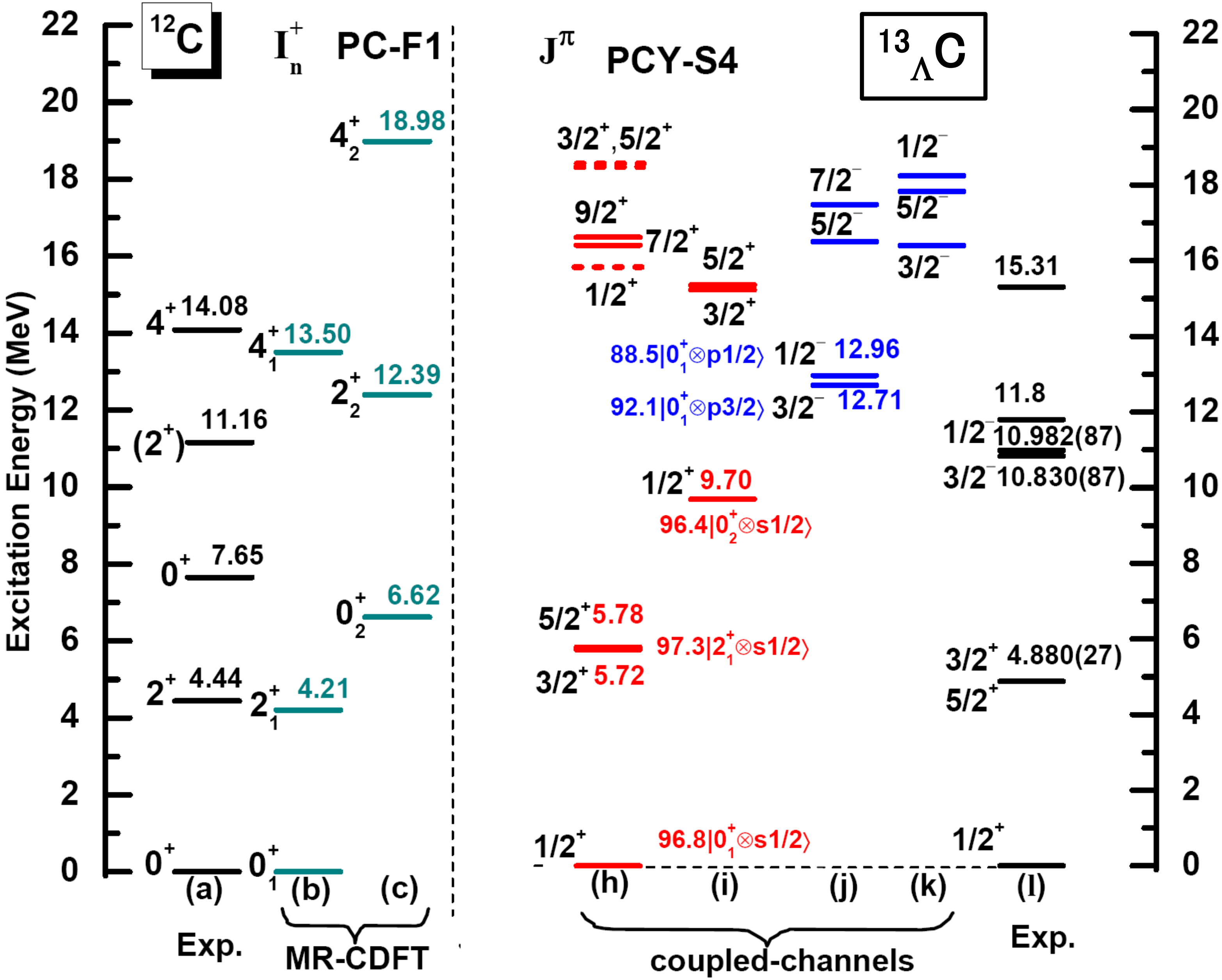


$1/2^-$: $\sim 88.5\% |p_{1/2} \otimes 0^+\rangle + 11\% |p_{3/2} \otimes 2^+\rangle$

$3/2^-$: $\sim 92.1\% |p_{3/2} \otimes 0^+\rangle + 5\% |p_{1/2} \otimes 2^+\rangle$

$5/2^+$, $3/2^+$: $\sim 97.3\% |s_{1/2} \otimes 2^+\rangle$

$1/2^+$: $\sim 96.8\% |s_{1/2} \otimes 0^+\rangle$



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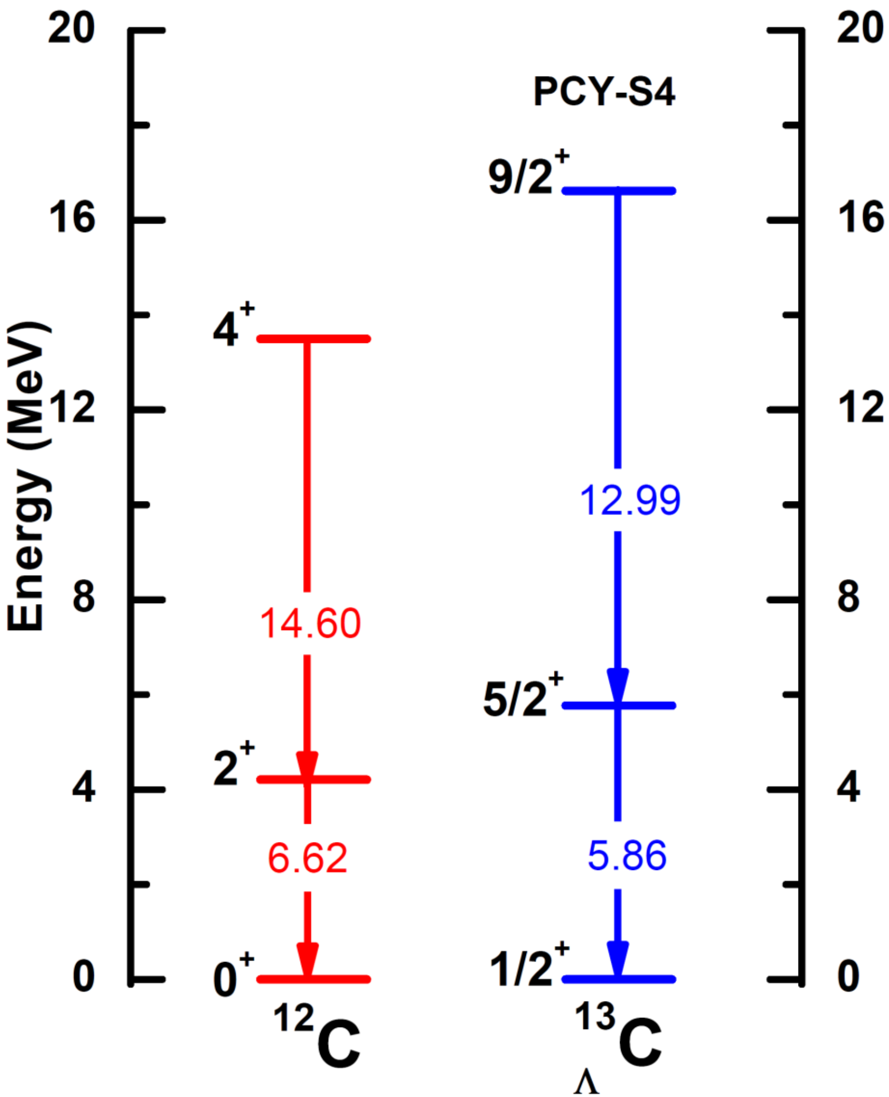
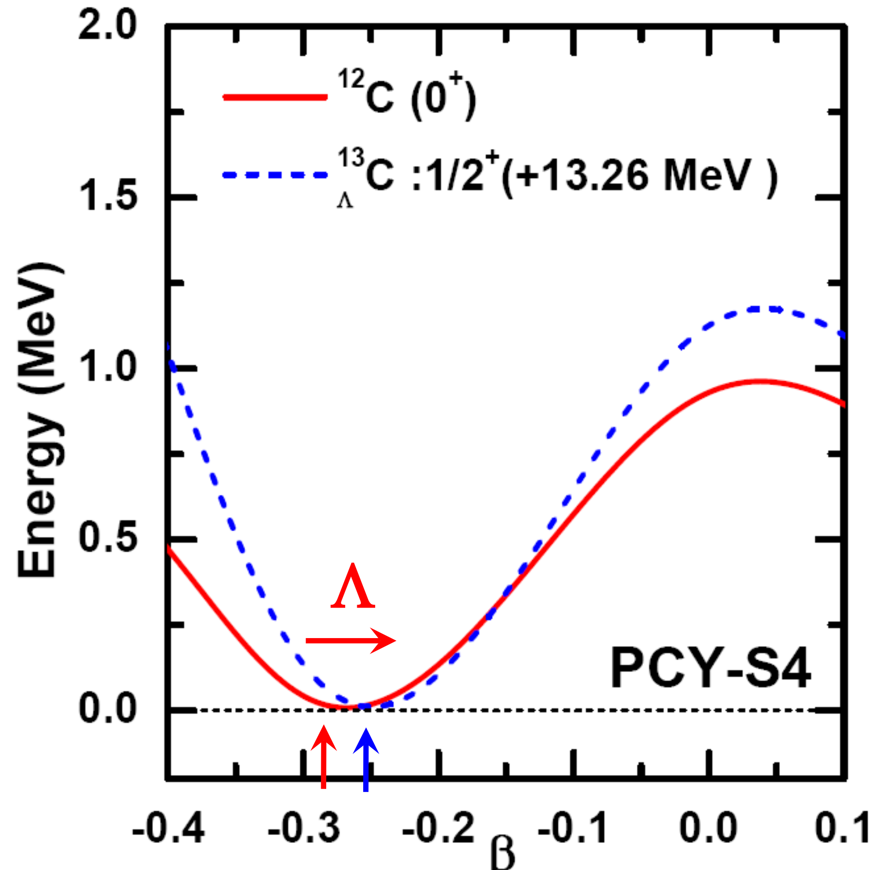
B(E2) transition rates (e^{2}fm^4)



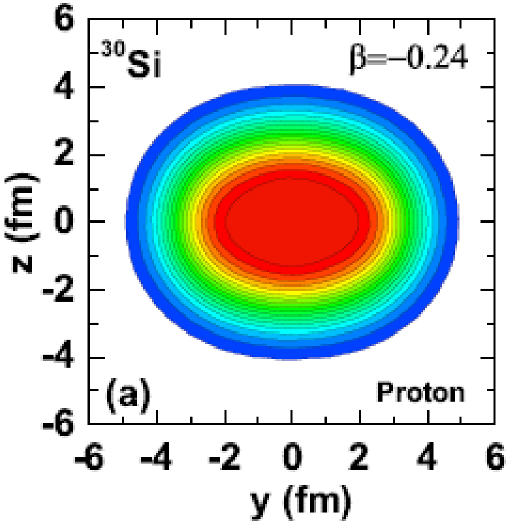


	^{12}C	$^{13}_{\Lambda}$C
β	-0.27	-0.25
r_p (fm)	2.44	2.39

- B(E2) : ~ 11% reduction

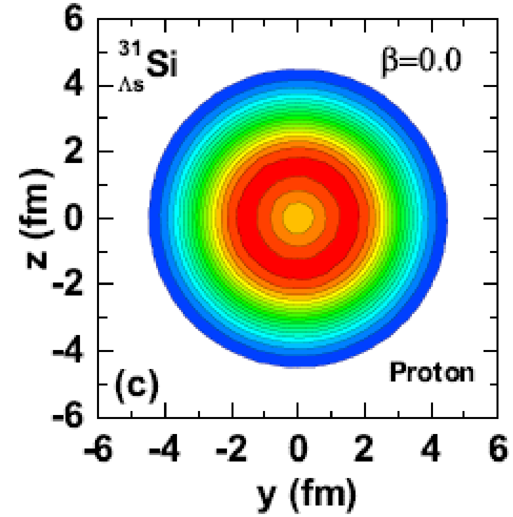
Beyond MF effect on deformation change in $^{31}_{\Lambda}$Si

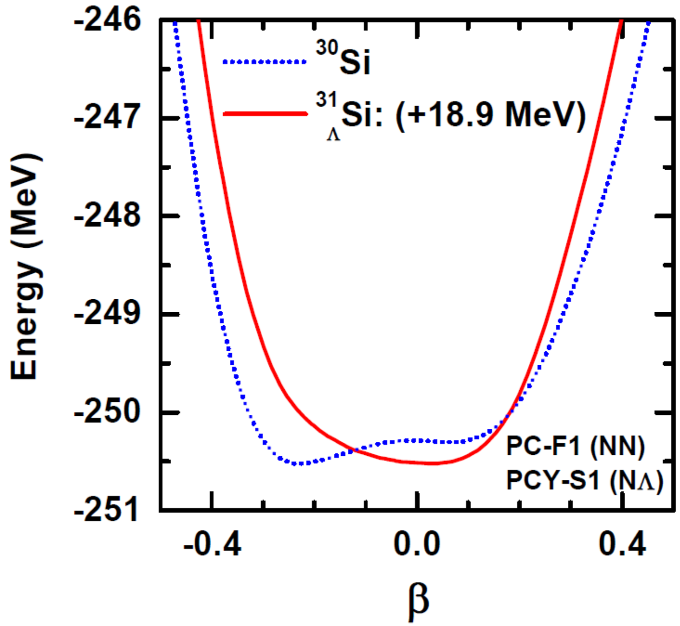
Mean-field calculations:



$\xrightarrow{\Lambda}$

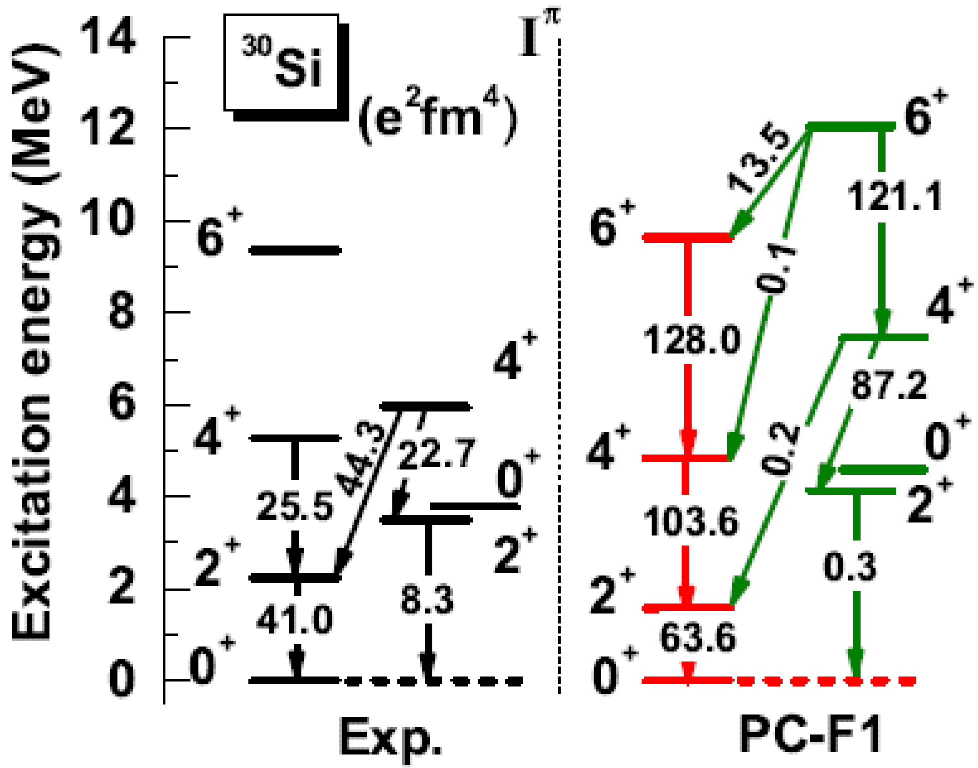
oblate → spherical

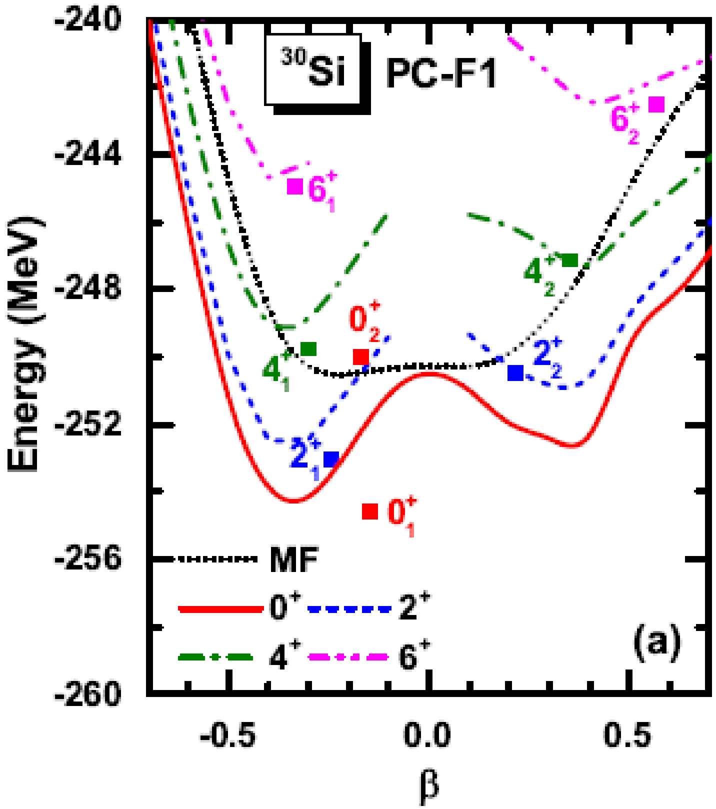




Soft potential energy surface
→ beyond-mean-field effects?

spectrum of ^{30}Si



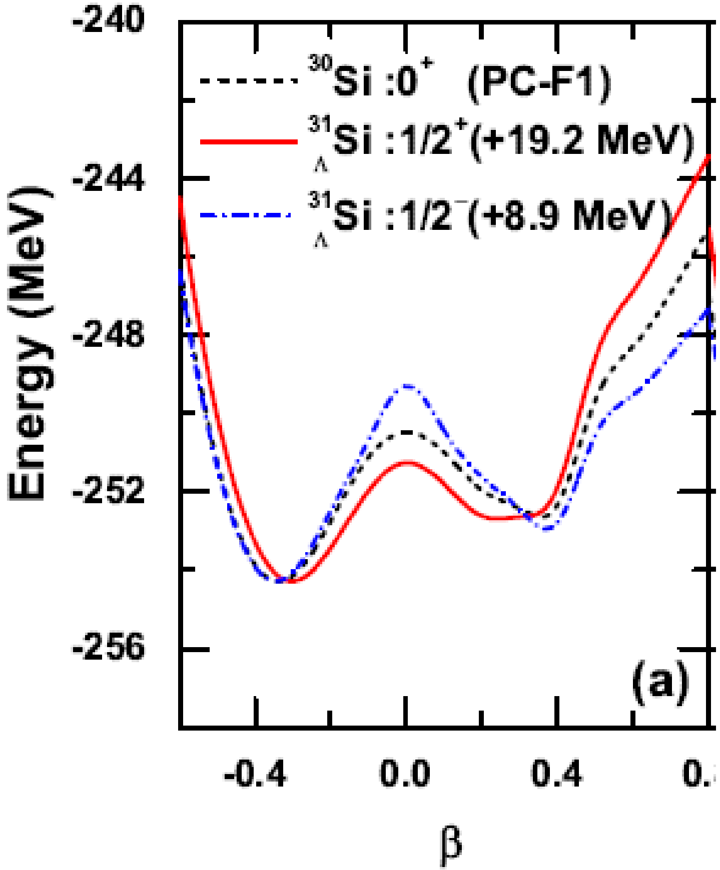


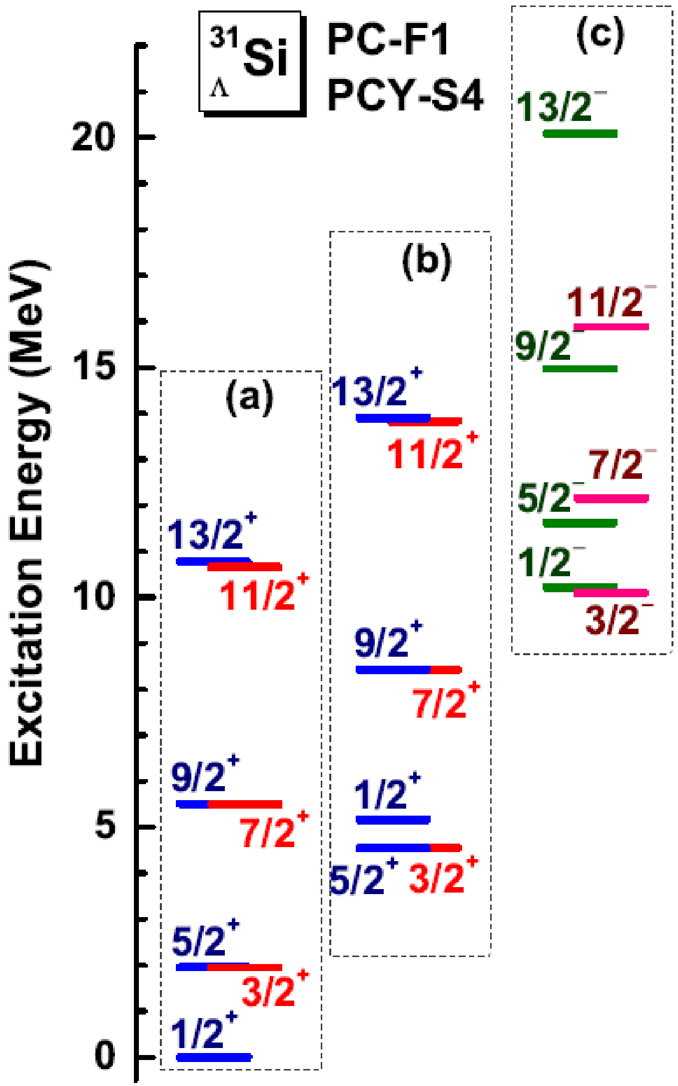
* axial symmetry

$$R_{4/2} = E(4^+) / E(2^+) = 3.083$$

cf. Expt: $R_{4/2} = 2.66$

spectrum of $^{31}{}_{\Lambda}Si$





still rotational spectrum

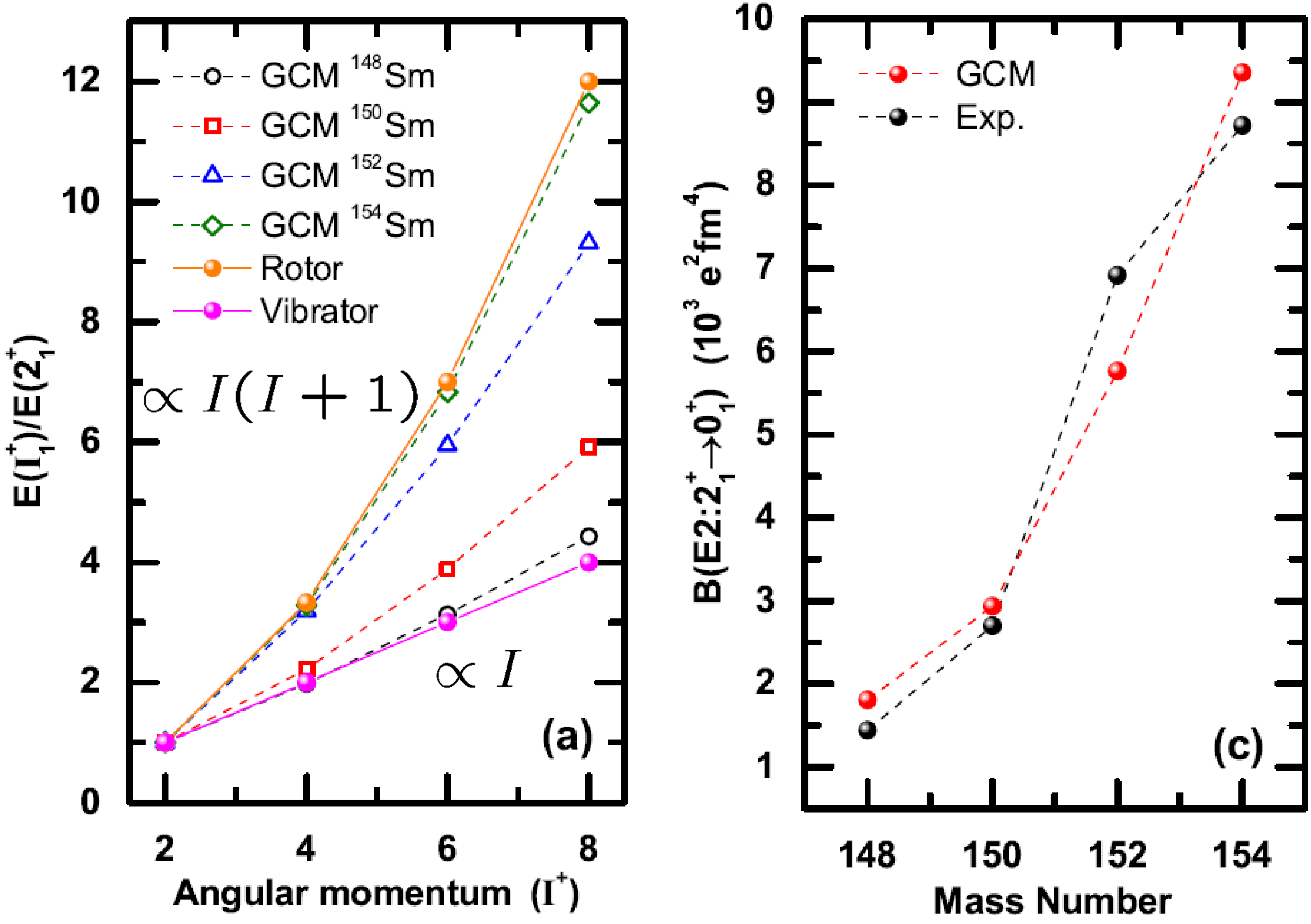
[$R_{4/2}$ = 3.08 → 2.83]

changes towards spherical, but not quite

→ large BMF effects

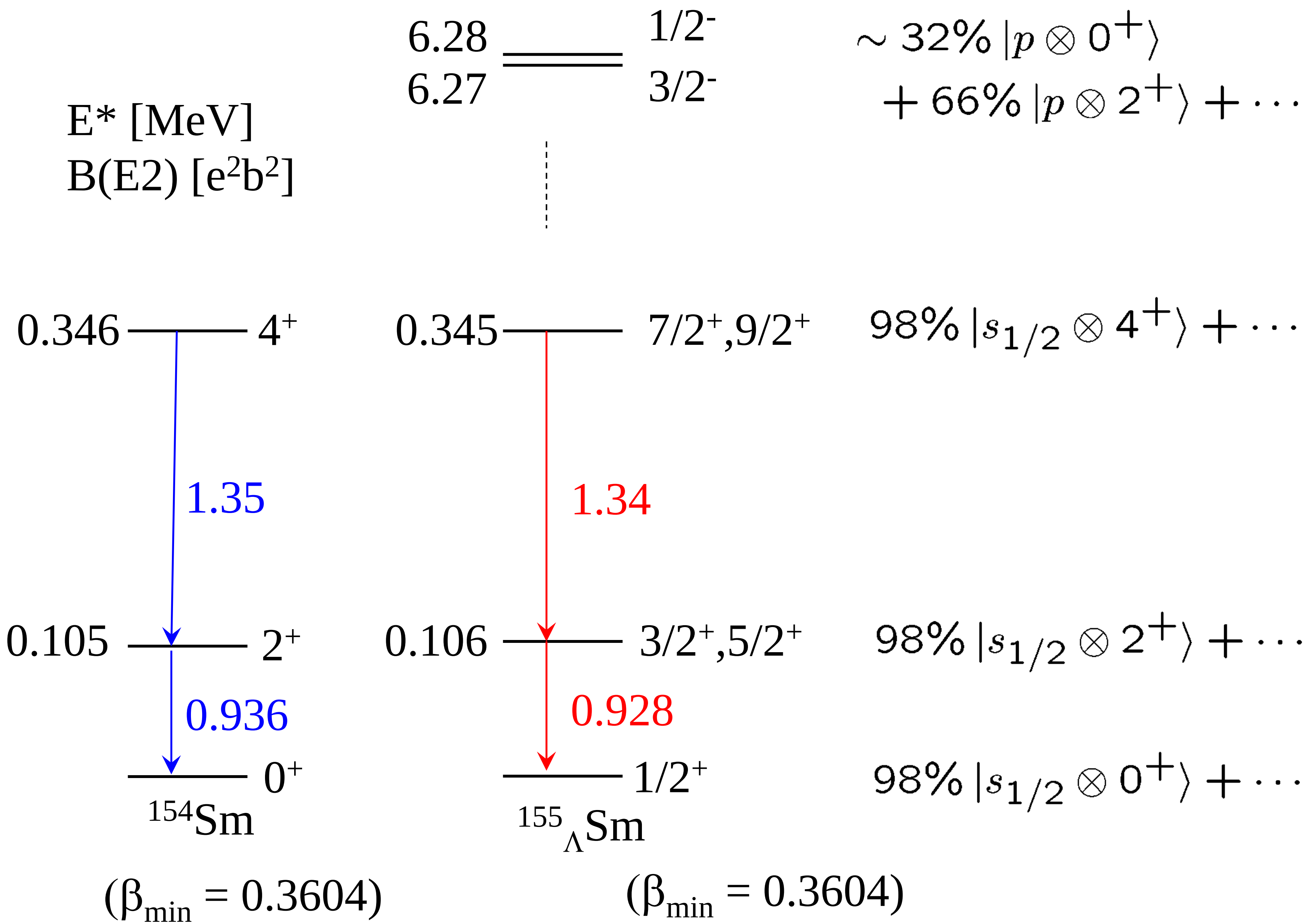
H. Mei, K.H., J.M. Yao, and T. Motoba, in preparation

Application to Sm isotopes

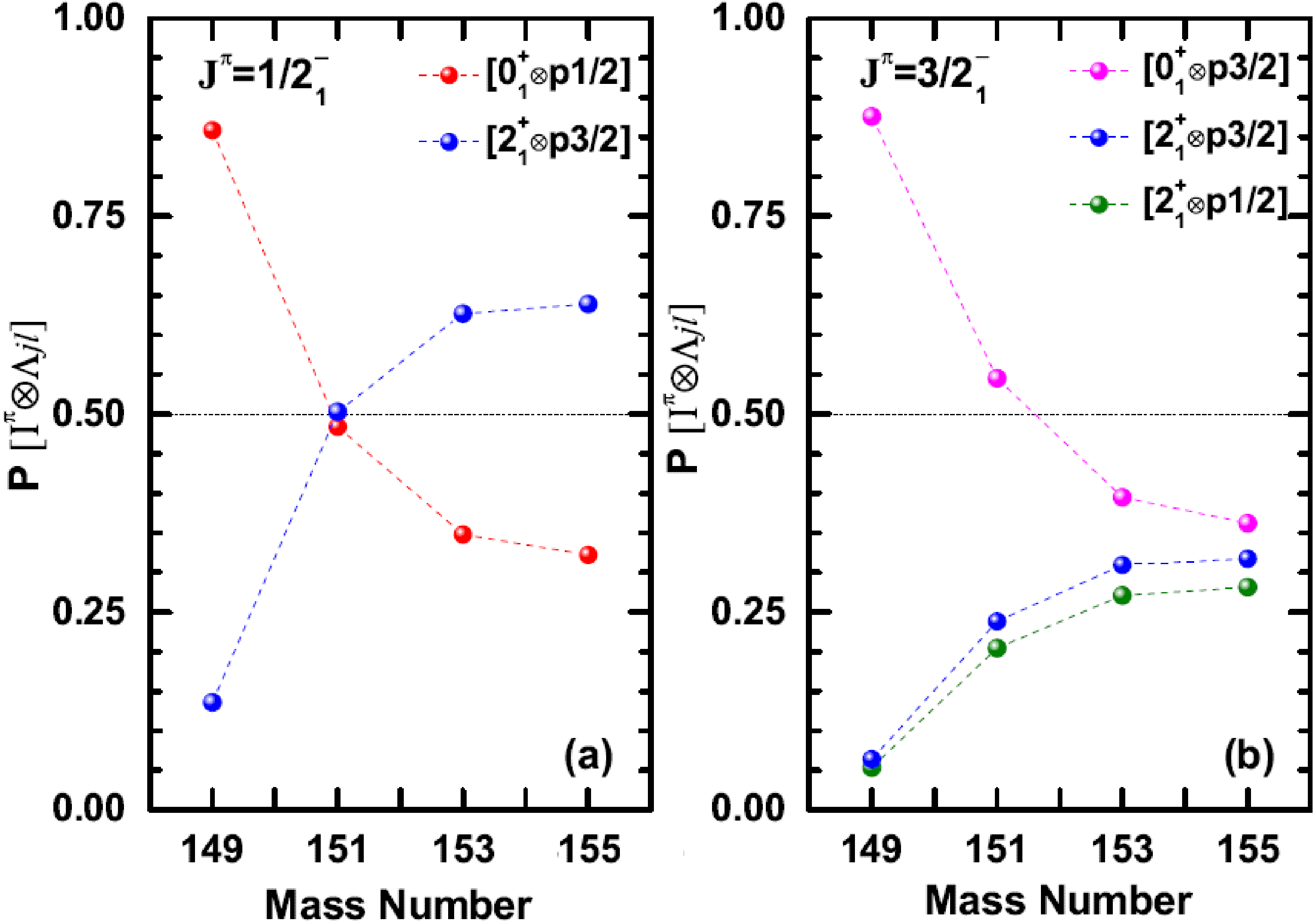


Results for $^{155}_{\Lambda}$Sm

H. Mei, K.H., J.M. Yao, T. Motoba, PRC91('15) 064305



weak coupling → strong coupling



H. Mei, K.H., J.M. Yao, and T. Motoba, in preparation

Summary

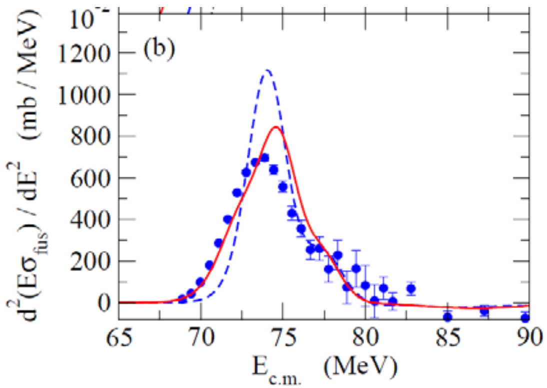
Applications of Beyond-Mean-Field method

- Heavy-ion subbarrier fusion reactions

✓strong interplay between reaction and structure
cf. fusion barrier distributions

C.C. calculations with beyond-MF method

✓ anharmonicity
✓ octupole vibrations: $^{16}O + ^{208}Pb$



- Low-lying spectra of Λ hypernuclei

Microscopic particle-rotor model

✓Λ + GCM states for core
✓first calculation for low-lying spectrum based on mean-field type calculations
✓ re-examination of “disappearance of deformation” in MF calc.