

Heavy-Ion Fusion Reactions around the Coulomb Barrier

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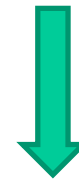
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- ✧ Fusion reactions and quantum tunneling
- ✧ **Basics of the Coupled-channels method**
How to run CCFULL
- ✧ Concept of Fusion barrier distribution

3.11 earthquake

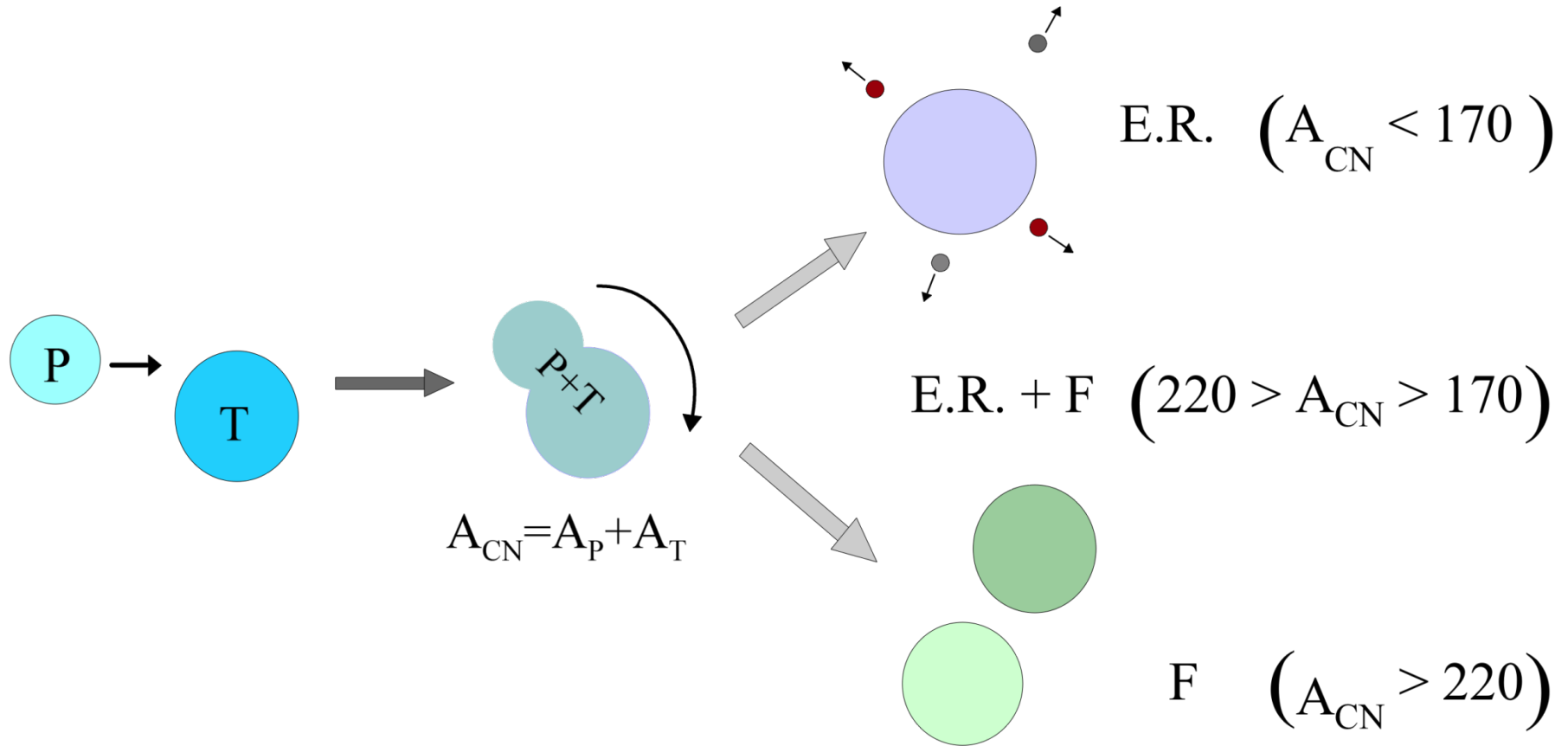


after 1 month



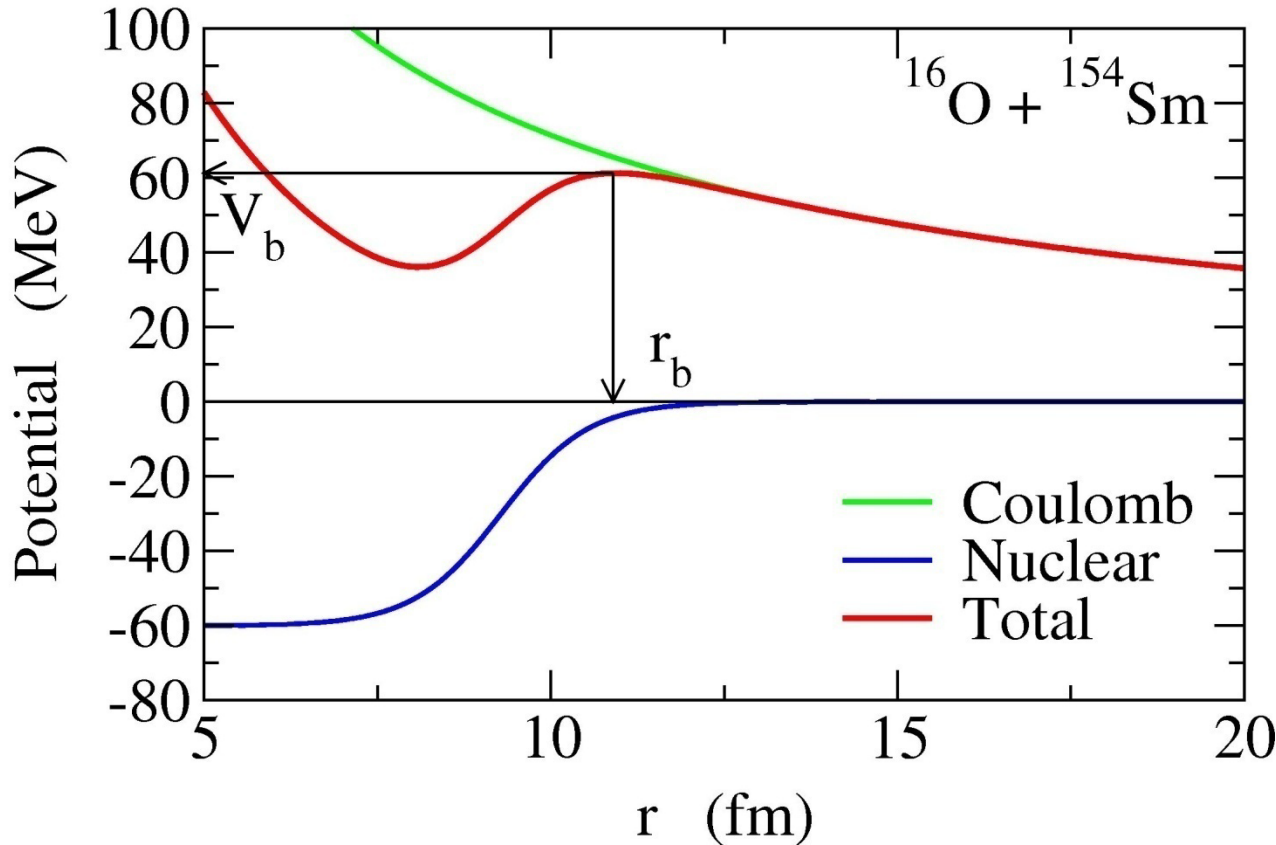
Lecture 1

Fusion: compound nucleus formation



courtesy: Felipe Canto

Inter-nucleus potential



- above barrier
- sub-barrier
- deep subbarrier

Two forces:

1. Coulomb force

Long range,
repulsive

2. Nuclear force

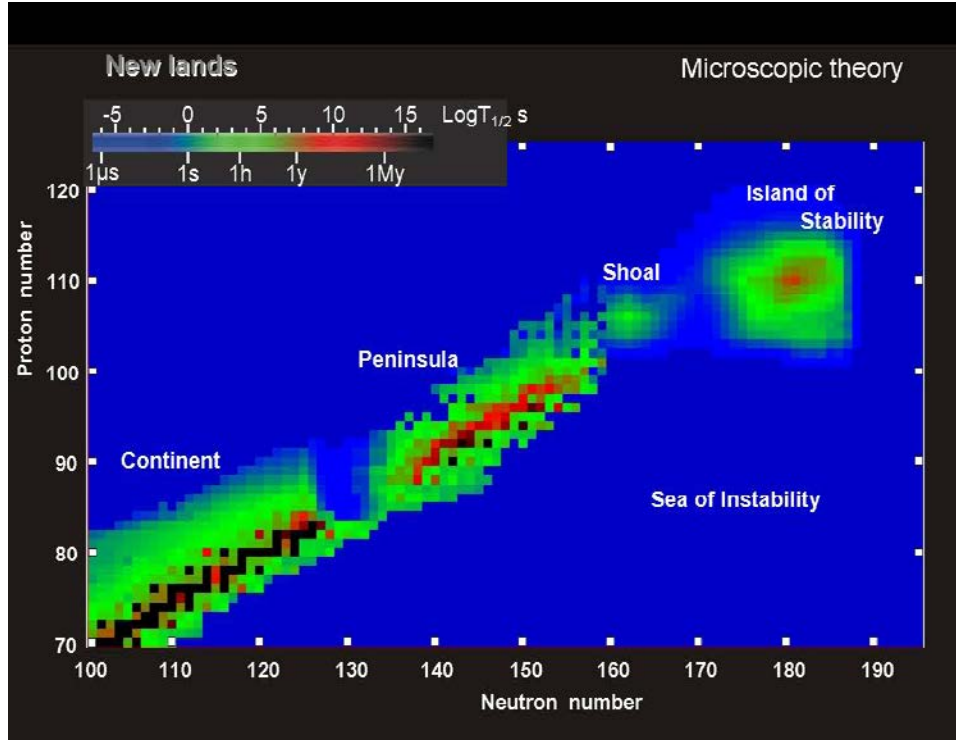
Short range,
attractive



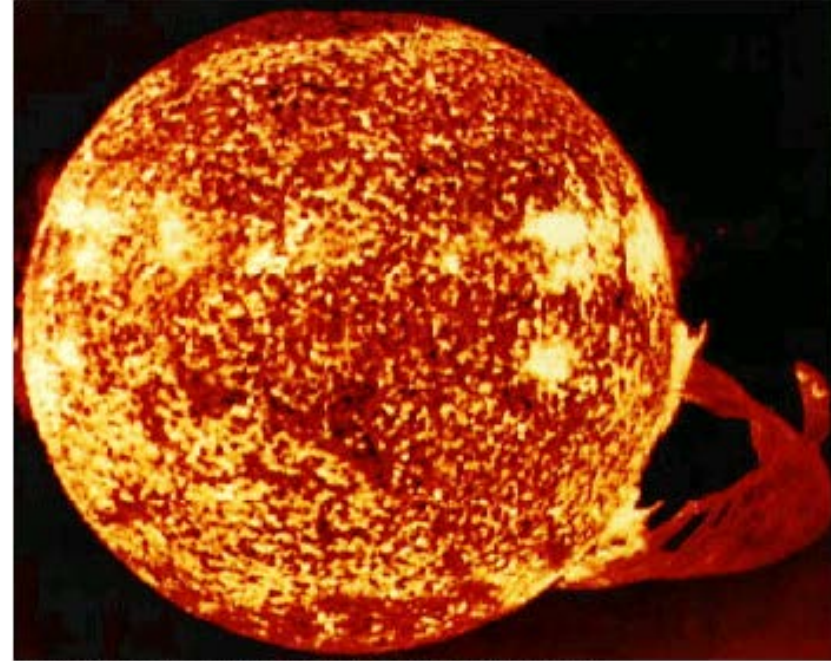
Potential barrier due to the compensation between the two
(Coulomb barrier)

Why subbarrier fusion?

Two obvious reasons:



discovering new elements
(SHE by cold fusion reactions)



NASA, Skylab space station December 19, 1973, solar flare reaching 588 000 km off solar surface

nuclear astrophysics
(fusion in stars)

Why subbarrier fusion?

Two obvious reasons:

- ✓ discovering new elements (SHE)
- ✓ nuclear astrophysics (fusion in stars)

Other reasons:

- ✓ reaction mechanism

strong interplay between reaction and structure

(channel coupling effects)

cf. high E reactions: much simpler reaction mechanism

- ✓ many-particle tunneling

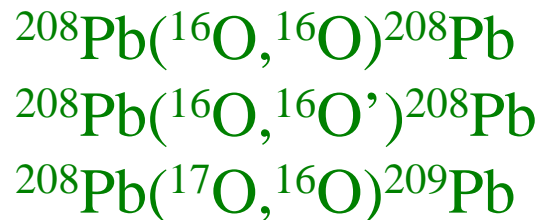
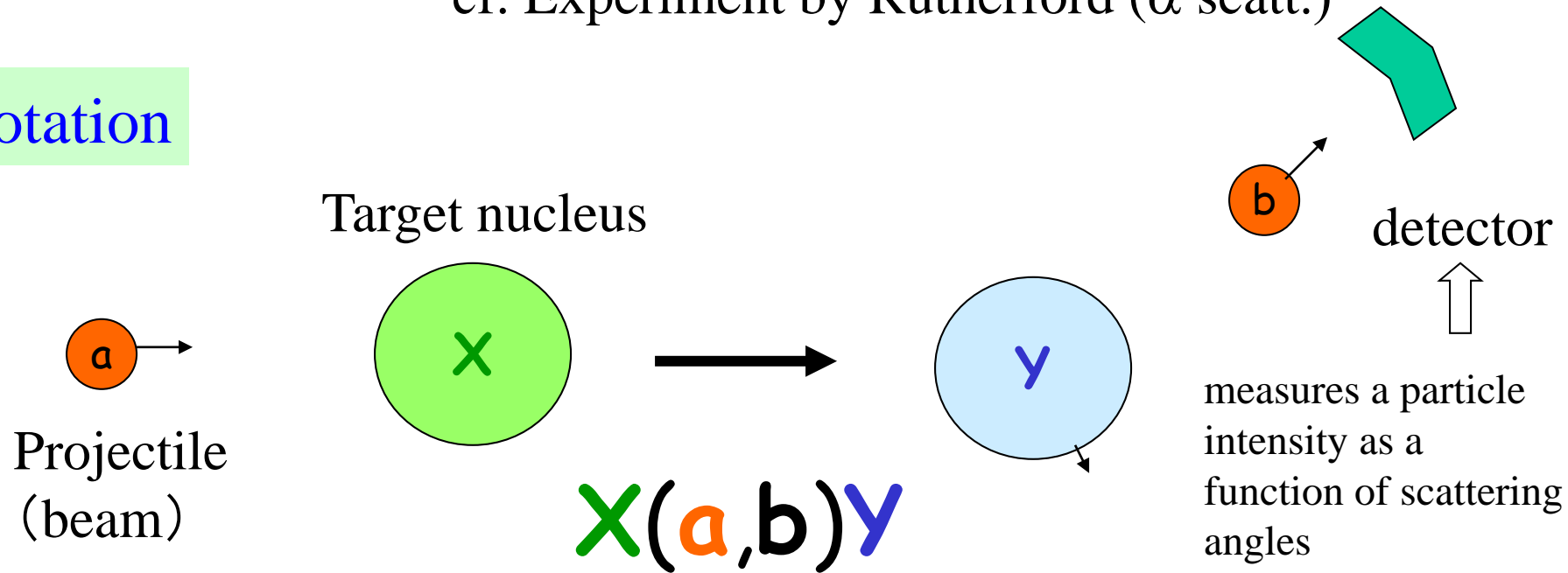
cf. alpha decay: fixed energy

tunneling in atomic collision: less variety of intrinsic motions

Basic of nuclear reactions

Shape, interaction, and excitation structures of nuclei ← scattering expt.
cf. Experiment by Rutherford (α scatt.)

Notation

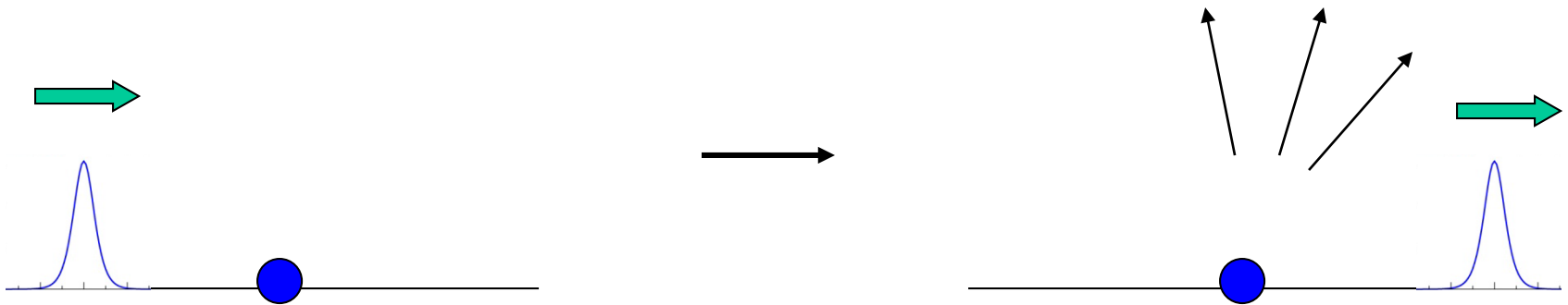


- : $^{16}\text{O} + ^{208}\text{Pb}$ elastic scattering
- : $^{16}\text{O} + ^{208}\text{Pb}$ inelastic scattering
- : 1 neutron transfer reaction

Scattering Amplitude

$$\psi(\mathbf{r}) \rightarrow e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta)\frac{e^{ikr}}{r}$$

= (incident wave) + (scattering wave)



differential cross section:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

Scattering Amplitude

partial wave decomposition

Motion of Free particle: $-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi = \frac{k^2\hbar^2}{2m}\psi$

$$\begin{aligned}\psi(\mathbf{r}) &= e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_{l=0}^{\infty} (2l+1) i^l j_l(kr) P_l(\cos\theta) \\ &\rightarrow \frac{i}{2kr} \sum_{l=0}^{\infty} (2l+1) i^l \left[e^{-i(kr-l\pi/2)} - e^{i(kr-l\pi/2)} \right] P_l(\cos\theta)\end{aligned}$$

In the presence of a potential: $\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) - E \right] \psi = 0$

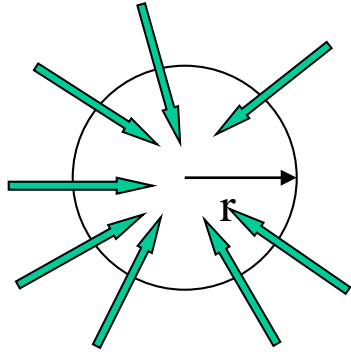
Asymptotic form of wave function

$$\psi(\mathbf{r}) \rightarrow \frac{i}{2kr} \sum_{l=0}^{\infty} (2l+1) i^l \left[e^{-i(kr-l\pi/2)} - \underline{S_l} e^{i(kr-l\pi/2)} \right] P_l(\cos\theta)$$

(note)

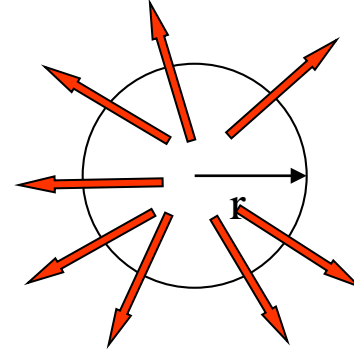
$$\psi(\mathbf{r}) \rightarrow \frac{i}{2k} \sum_l (2l + 1) i^l \frac{1}{r} \left[\underbrace{e^{-i(kr - l\pi/2)}}_{\psi_{\text{in}}} - \underbrace{S_l e^{i(kr - l\pi/2)}}_{\psi_{\text{out}}} \right] P_l(\cos \theta)$$

Total incoming flux



$$j_{\text{in}}^{\text{net}} = \frac{k\hbar}{m} \cdot \frac{\pi}{k^2} \sum_l (2l + 1)$$

Total outgoing flux



$$j_{\text{out}}^{\text{net}} = \frac{k\hbar}{m} \cdot \frac{\pi}{k^2} \sum_l (2l + 1) |S_l|^2$$

➡ If only elastic scattering:

$$|S_l| = 1 \quad (\text{flux conservation})$$

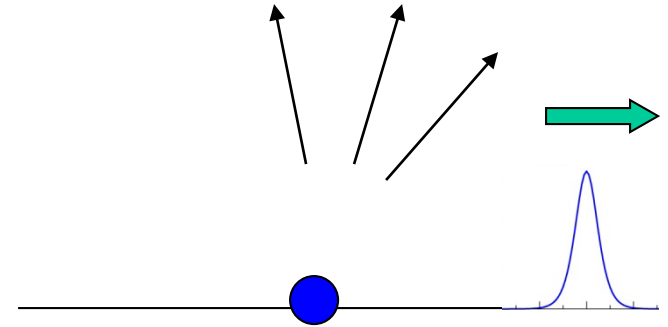
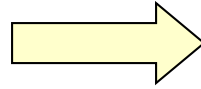
$$S_l = e^{2i\delta_l}$$

δ_l : phase shift

Optical potential and Absorption cross section

Reaction processes

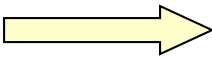
- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- **Compound nucleus formation (fusion)**



Loss of incident flux
(absorption)

Optical potential

$$V_{\text{opt}}(\mathbf{r}) = V(\mathbf{r}) - iW(\mathbf{r}) \quad (W > 0)$$

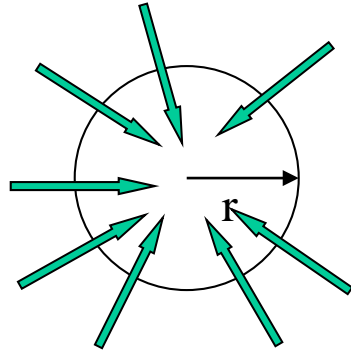

$$\nabla \cdot \mathbf{j} = \dots = -\frac{2}{\hbar}W|\psi|^2$$

(note) Gauss's law

$$\int_S \mathbf{j} \cdot \mathbf{n} dS = \int_V \nabla \cdot \mathbf{j} dV$$

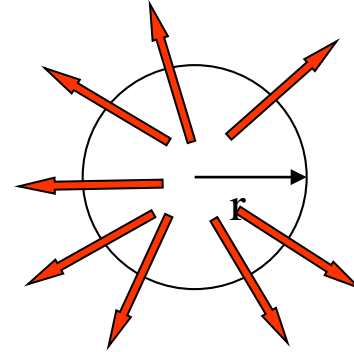
$$\psi(\mathbf{r}) \rightarrow \frac{i}{2k} \sum_l (2l + 1) i^l \frac{1}{r} \left[\underbrace{e^{-i(kr - l\pi/2)}}_{\psi_{\text{in}}} - \underbrace{S_l e^{i(kr - l\pi/2)}}_{\psi_{\text{out}}} \right] P_l(\cos \theta)$$

Total incoming flux



$$j_{\text{in}}^{\text{net}} = \frac{k\hbar}{m} \cdot \frac{\pi}{k^2} \sum_l (2l + 1)$$

Total outgoing flux



$$j_{\text{out}}^{\text{net}} = \frac{k\hbar}{m} \cdot \frac{\pi}{k^2} \sum_l (2l + 1) |S_l|^2$$

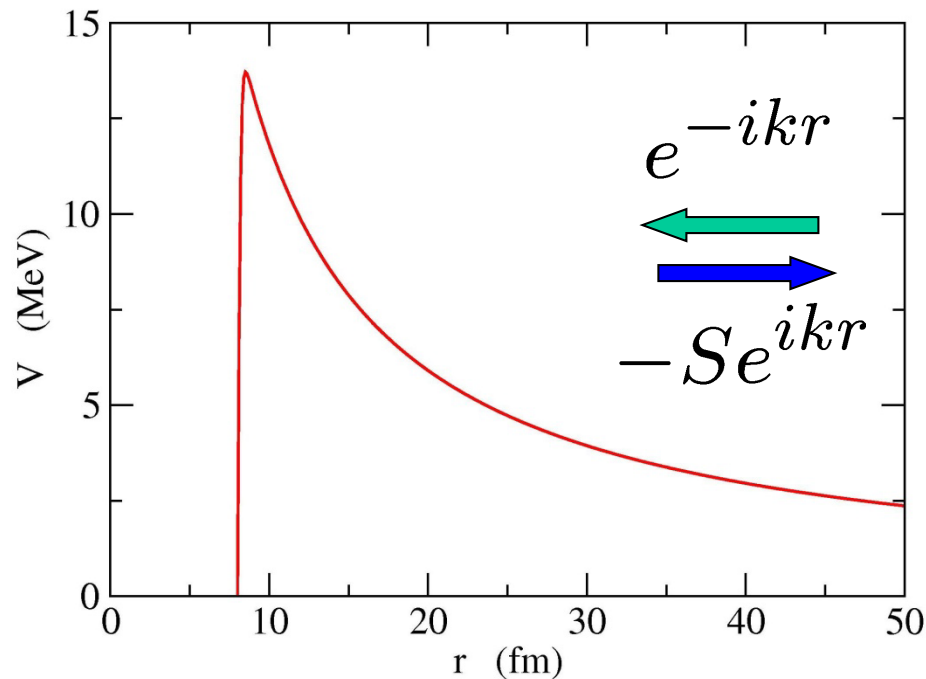
Net flux loss:

$$j_{\text{in}}^{\text{net}} - j_{\text{out}}^{\text{net}} = \frac{k\hbar}{m} \cdot \frac{\pi}{k^2} \sum_l (2l + 1) (1 - |S_l|^2)$$

Absorption cross section:

$$\sigma_{\text{abs}} = \frac{\pi}{k^2} \sum_l (2l + 1) (1 - |S_l|^2)$$

$$\psi(\mathbf{r}) \rightarrow \frac{i}{2k} \sum_l (2l + 1) i^l \frac{1}{r} \left[e^{-i(kr - l\pi/2)} - S_l e^{i(kr - l\pi/2)} \right] P_l(\cos \theta)$$

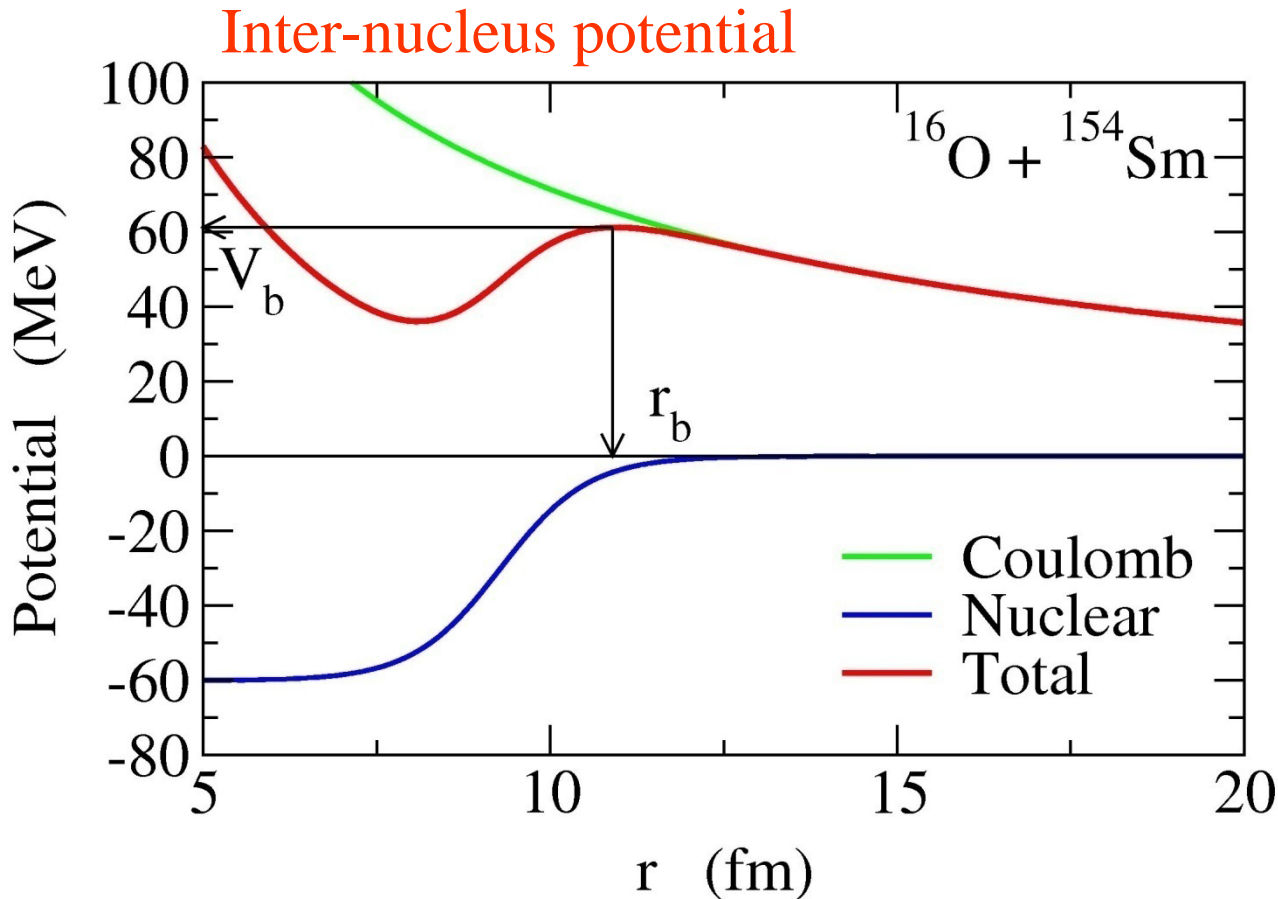


$$-S_l \sim R \text{ (reflection coeff.)} \longrightarrow P = |T|^2 = 1 - |S_l|^2$$

$$\sigma_{\text{abs}} = \frac{\pi}{k^2} \sum_l (2l + 1)(1 - |S_l|^2) = \frac{\pi}{k^2} \sum_l (2l + 1) P_l$$

Overview of heavy-ion reactions

Heavy-ion: Nuclei heavier than ^4He



Two forces:

1. Coulomb force

Long range,
repulsive

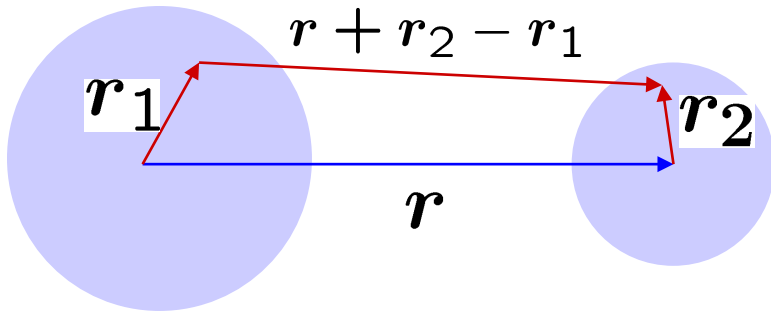
2. Nuclear force

Short range,
attractive



Potential barrier due to the compensation between these two
(Coulomb barrier)

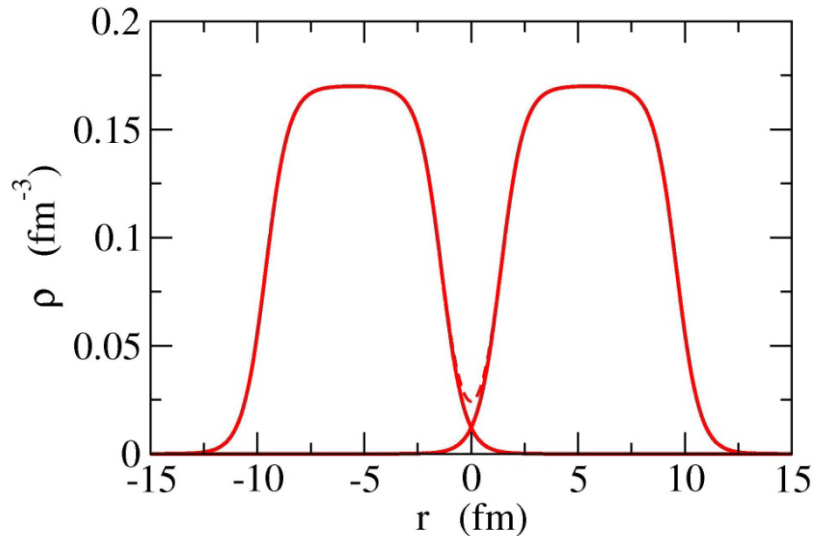
• Double Folding Potential



$$V_{DF}(r) = \int d\mathbf{r}_1 d\mathbf{r}_2 \rho_1(\mathbf{r}_1) \rho_2(\mathbf{r}_2) \times v_{nn}(r + r_2 - r_1)$$

cf. Michigan 3 range Yukawa (M3Y) interaction

$$v_{nn}(r) = 7999 \frac{e^{-4r}}{4r} - 2134 \frac{e^{-2.5r}}{2.5r} - 276 \delta(r) \quad (\text{MeV})$$



$$\rho(r) \sim \frac{\rho_0}{1 + \exp[(r - R_d)/a_d]}$$

• Phenomenological potential

$$V_{WS}(r) = - \frac{V_0}{1 + \exp[(r - R_0)/a]}$$

$$a_d \sim 0.54 \quad (\text{fm})$$

$$a \sim 0.63 \quad (\text{fm})$$

Three important features of heavy-ion reactions

1. Coulomb interaction: important

2. Reduced mass: large

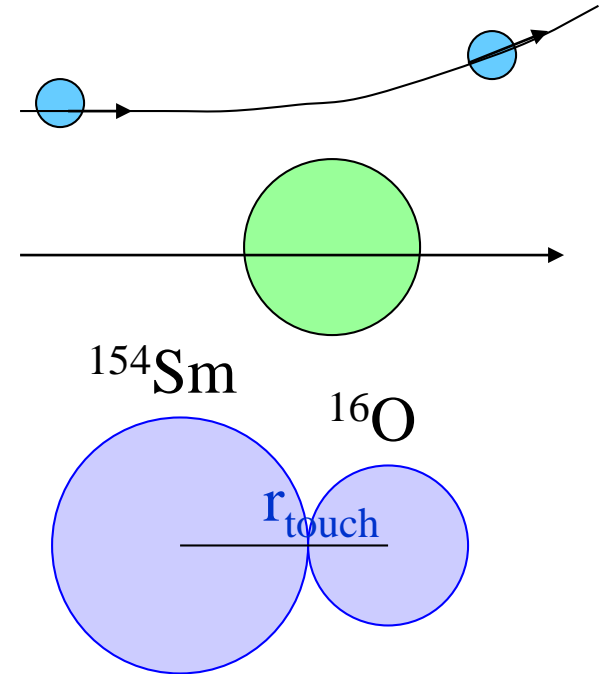


(semi-) classical picture

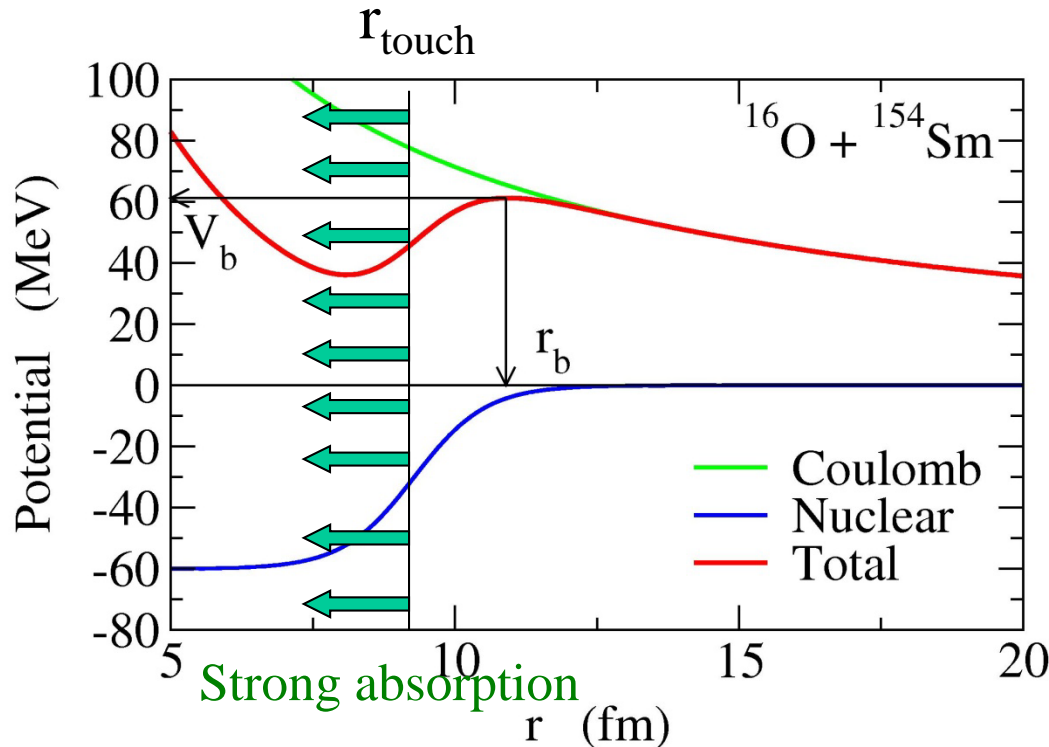
concept of trajectory

$$\mu = m_T m_P / (m_T + m_P)$$

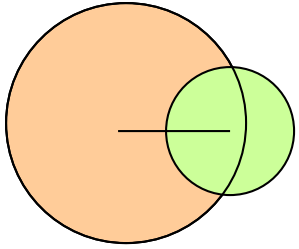
3. Strong absorption inside the Coul. barrier



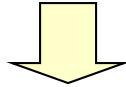
Automatic compound nucleus formation once touched (assumption of strong absorption)



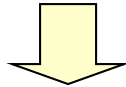
the region of large overlap



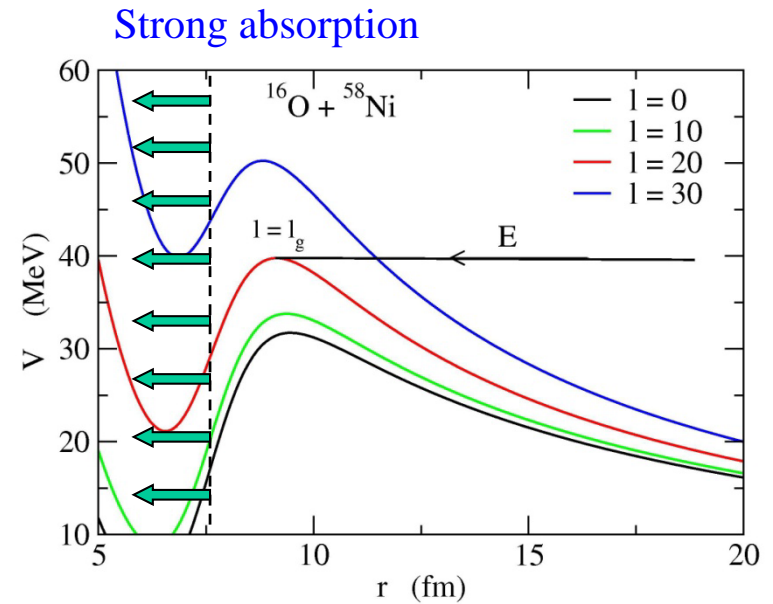
- High level density (CN)
- Huge number of d.o.f.



Relative energy is quickly lost
and converted to internal energy



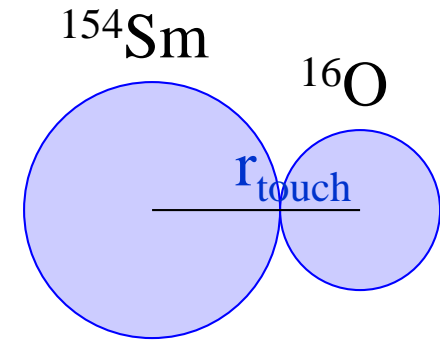
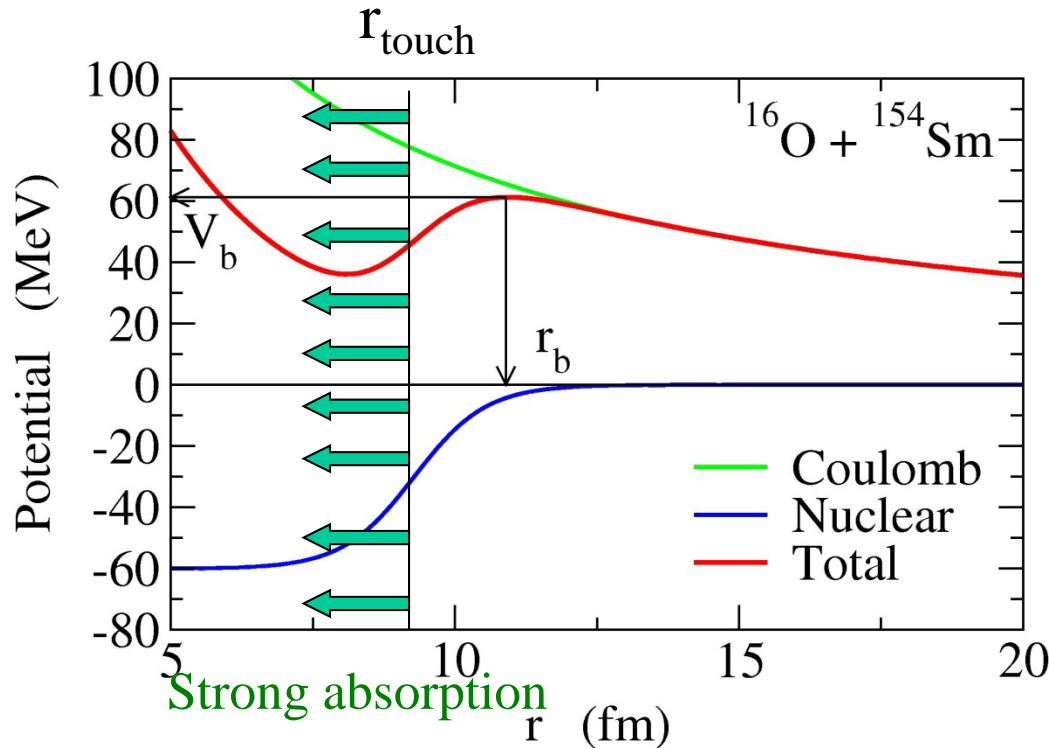
Formation of hot CN (fusion reaction)



$l < l_g$: can access to the strong absorption

$l \geq l_g$: cannot access classically

Fusion reaction and Quantum Tunneling



Automatic CN formation once touched (assumption of strong absorption)



Probability of fusion = prob. to access to r_{touch}

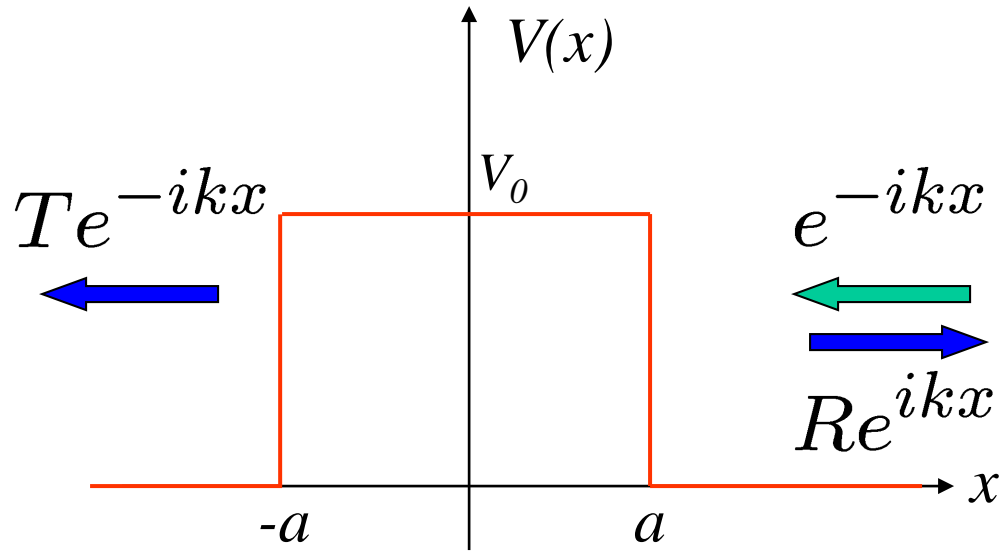


Penetrability of barrier

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) P_l(E)$$

Fusion takes place by quantum tunneling at low energies!

Quantum Tunneling Phenomena

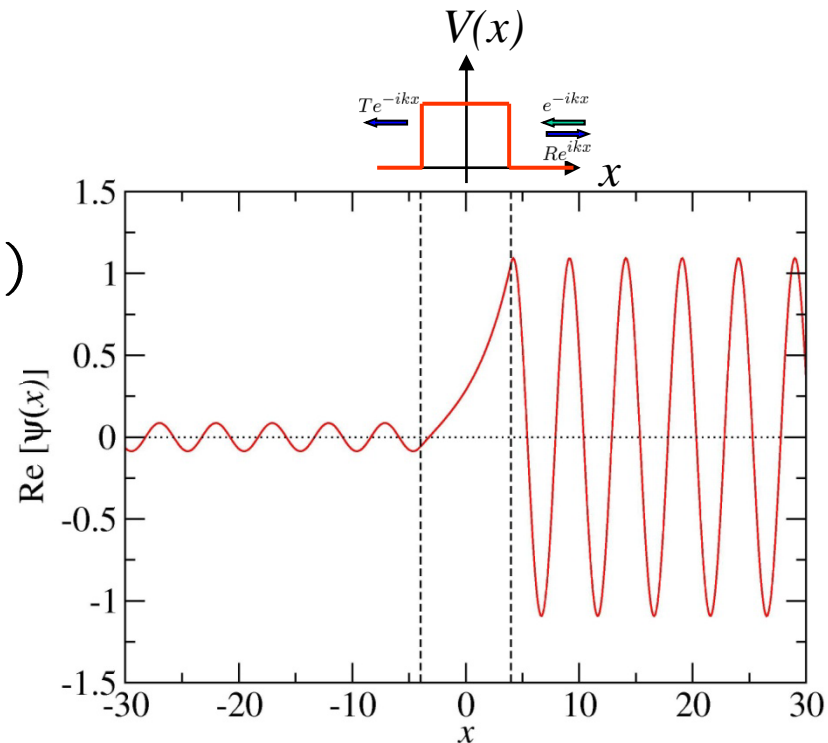


$$\begin{aligned} \psi(x) &= T e^{-ikx} & (x \leq -a) \\ &= A e^{-\kappa x} + B e^{\kappa x} & (-a < x < a) \\ &= e^{-ikx} + R e^{ikx} & (x \geq a) \end{aligned}$$

$$k = \sqrt{2mE/\hbar^2}$$

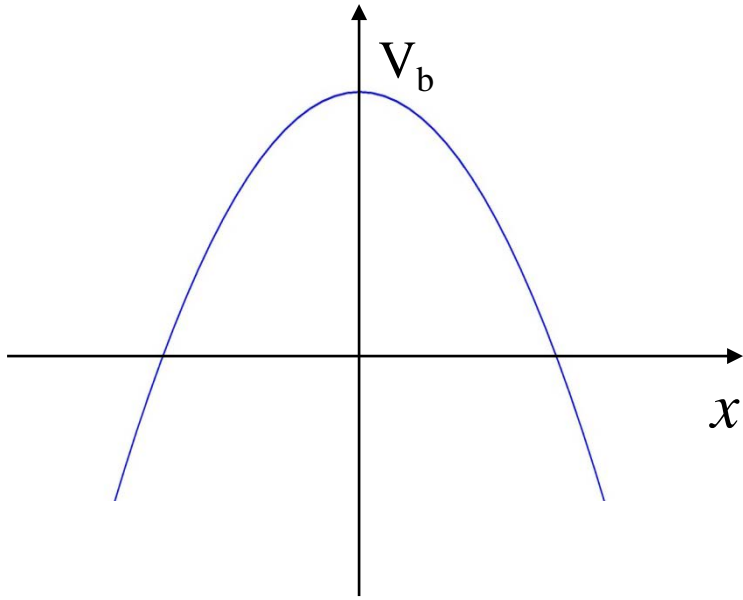
$$\kappa = \sqrt{2m(V_0 - E)/\hbar^2}$$

Tunnel probability: $P(E) = |T|^2$

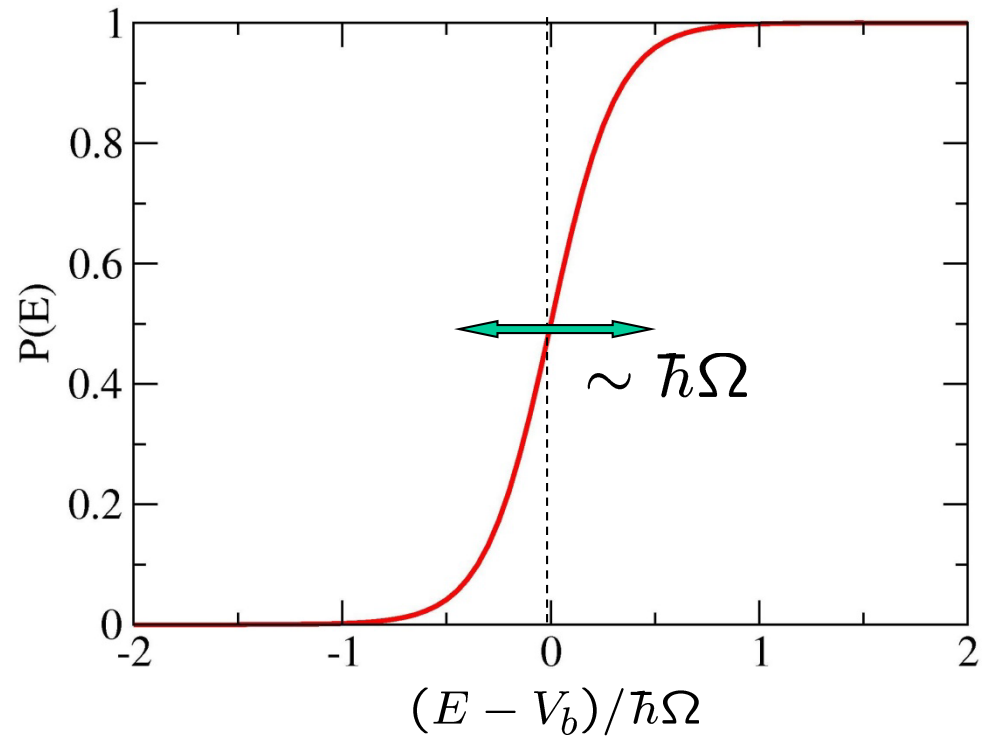


For a parabolic barrier.....

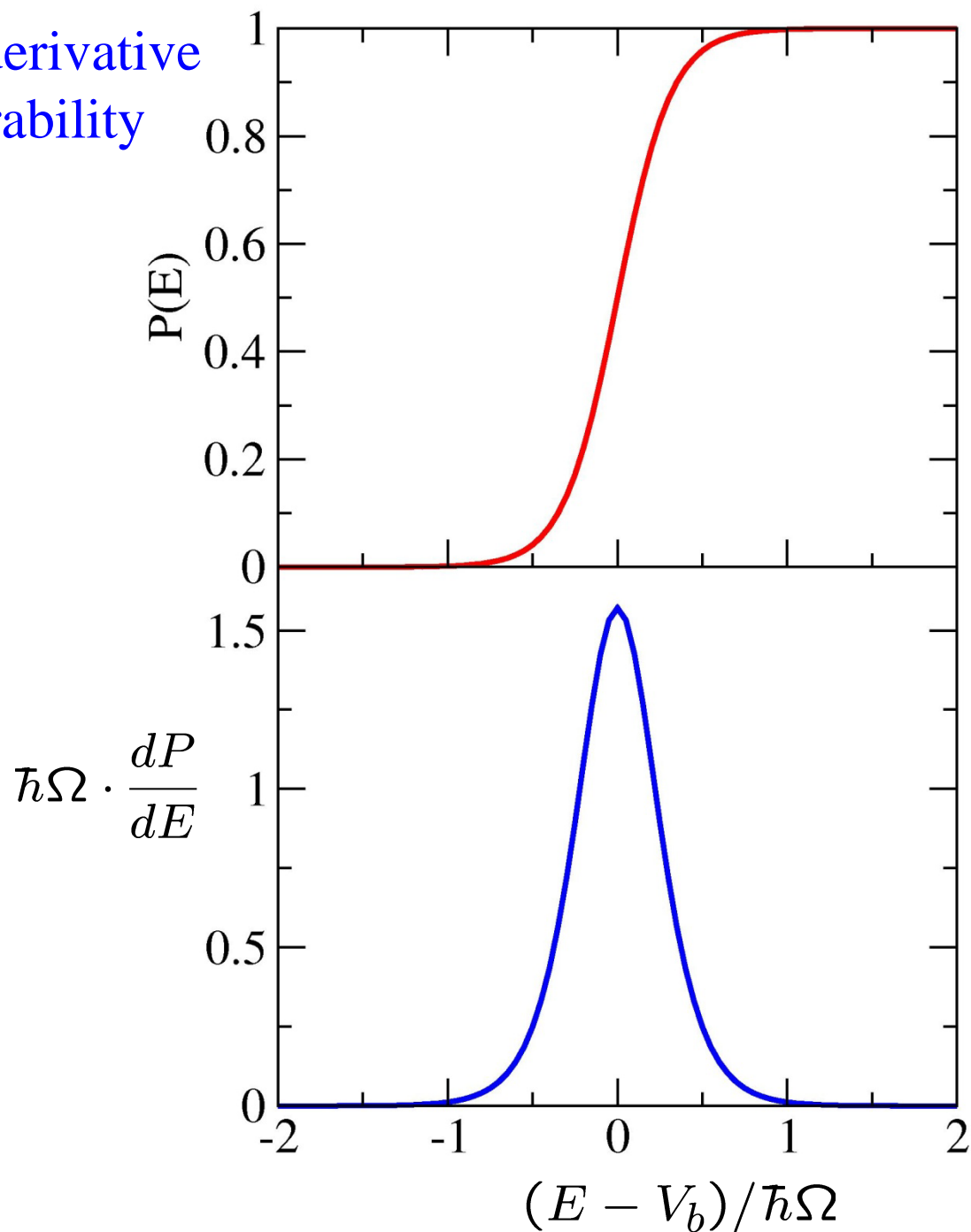
$$V(x) = V_b - \frac{1}{2}m\Omega^2 x^2$$



$$P(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}$$



Energy derivative
of penetrability



(note) Classical limit

$$P(E) = \theta(E - V_b)$$

$$dP/dE = \delta(E - V_b)$$

Potential Model: its success and failure

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + V(r) + \frac{l(l+1)\hbar^2}{2\mu r^2} - E \right] u_l(r) = 0$$

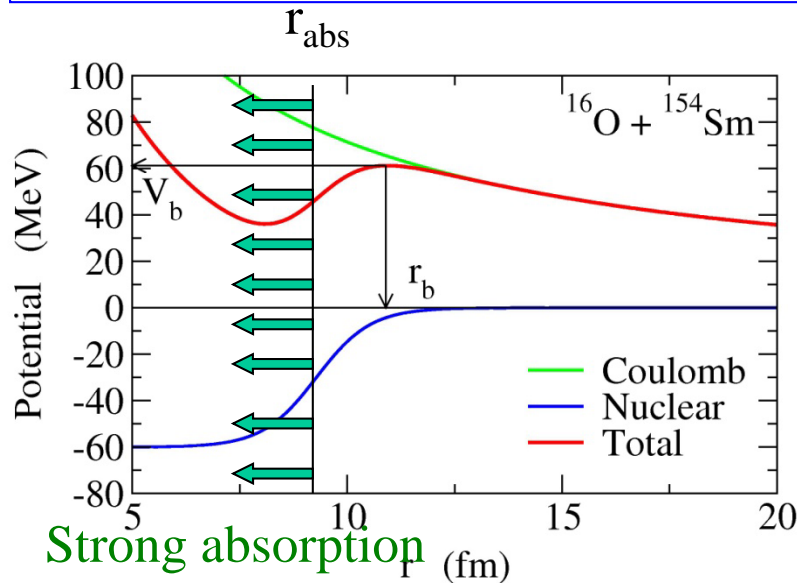
Asymptotic boundary condition: $u_l(r) \rightarrow H_l^{(-)}(kr) - S_l H_l^{(+)}(kr)$

Fusion cross section:

$$\sigma_{\text{fus}} = \frac{\pi}{k^2} \sum_l (2l+1) P_l$$

Mean angular mom. of CN:

$$\langle l \rangle = \frac{\sum_l l(2l+1) P_l}{\sum_l (2l+1) P_l}$$



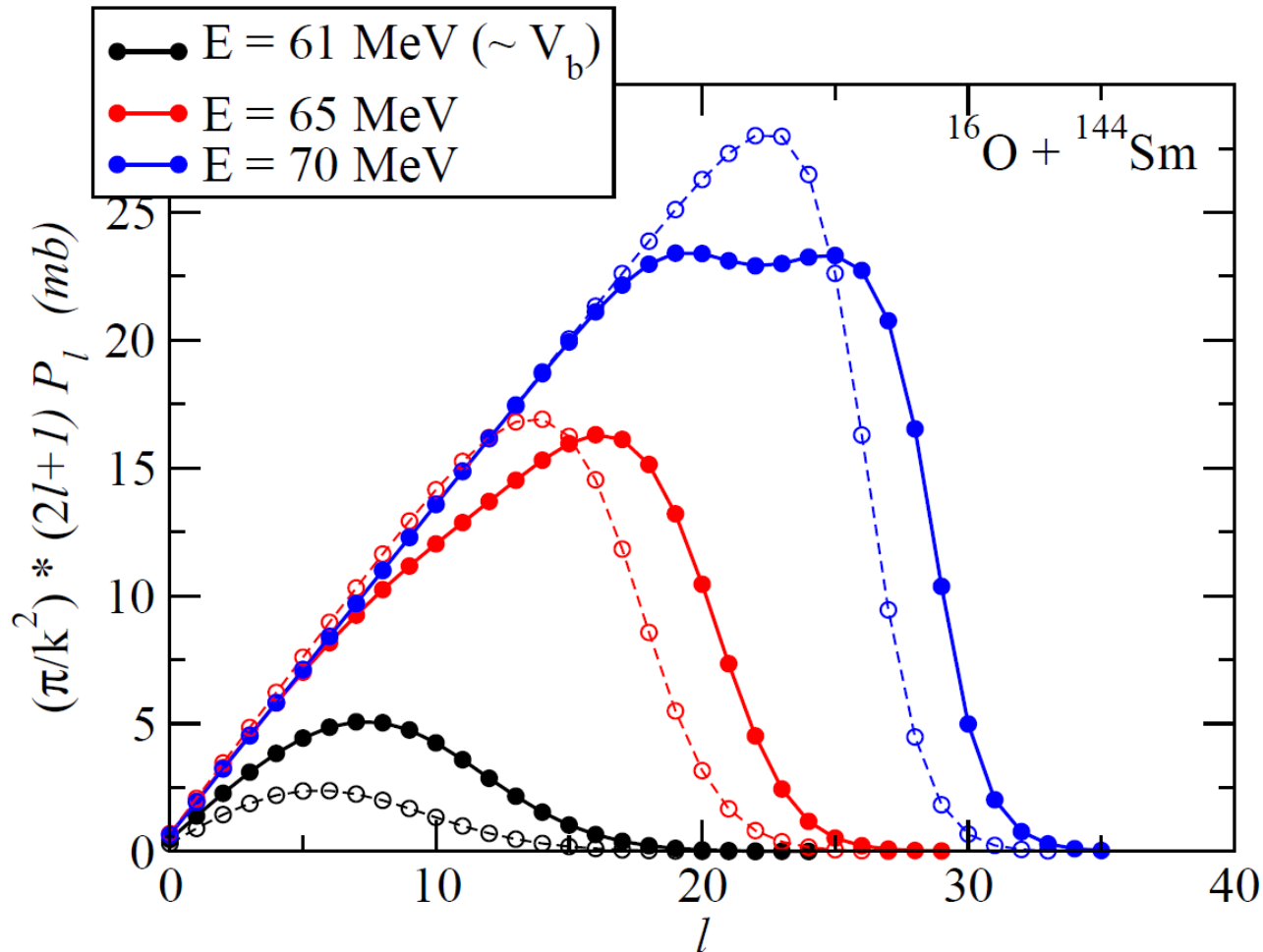
$$P_l = 1 - |S_l|^2$$

Fusion cross section:

$$\sigma_{\text{fus}} = \frac{\pi}{k^2} \sum_l (2l + 1) P_l$$

Mean angular mom. of CN:

$$\langle l \rangle = \frac{\sum_l l(2l + 1) P_l}{\sum_l (2l + 1) P_l}$$



Wong's formula

C.Y. Wong, Phys. Rev. Lett. 31 ('73)766

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) P_l(E)$$

i) Approximate the Coul. barrier by a parabola: $V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$

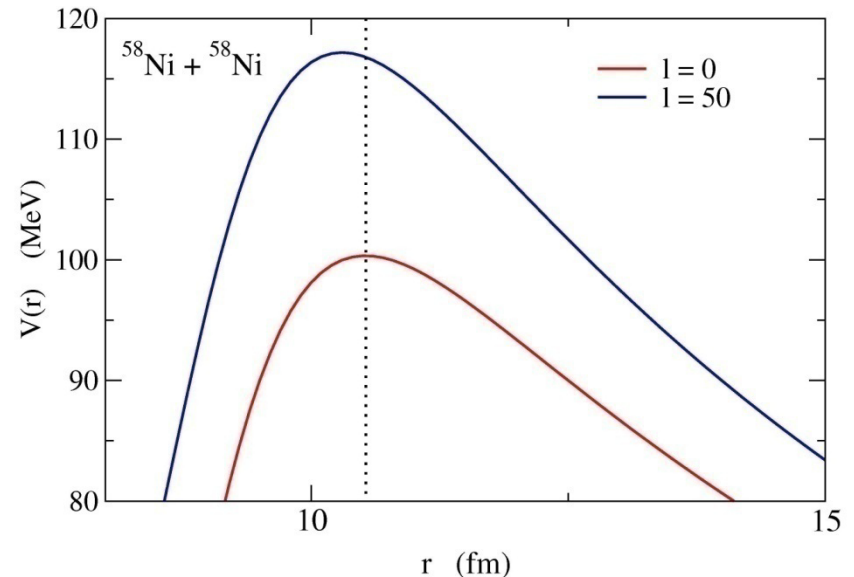
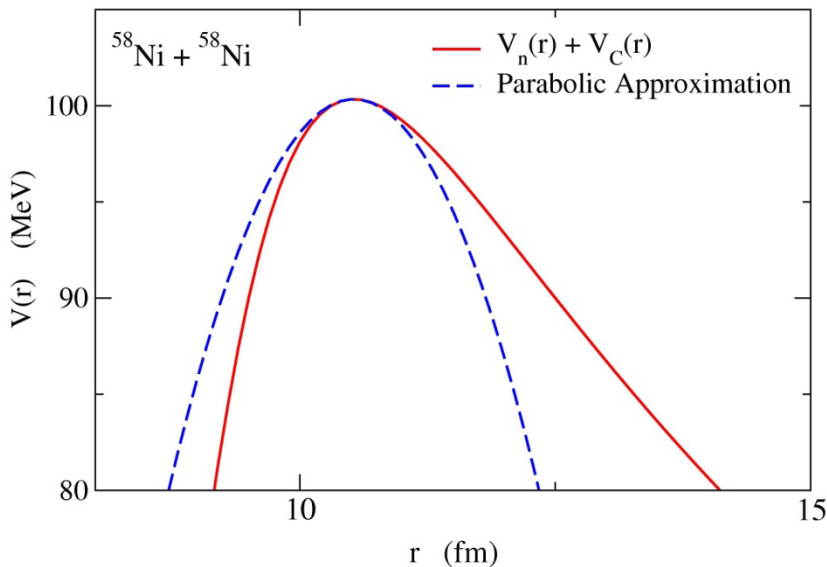
→ $P_0(E) = 1 / \left(1 + \exp \left[\frac{2\pi}{\hbar\Omega} (V_b - E) \right] \right)$

ii) Approximate P_l by P_0 :

$$P_l(E) \sim P_0 \left(E - \frac{l(l+1)\hbar^2}{2\mu R_b^2} \right)$$

(assume l -independent Rb and curvature)

iii) Replace the sum of l with an integral





$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \log \left[1 + \exp \left(\frac{2\pi}{\hbar\Omega} (E - V_b) \right) \right]$$

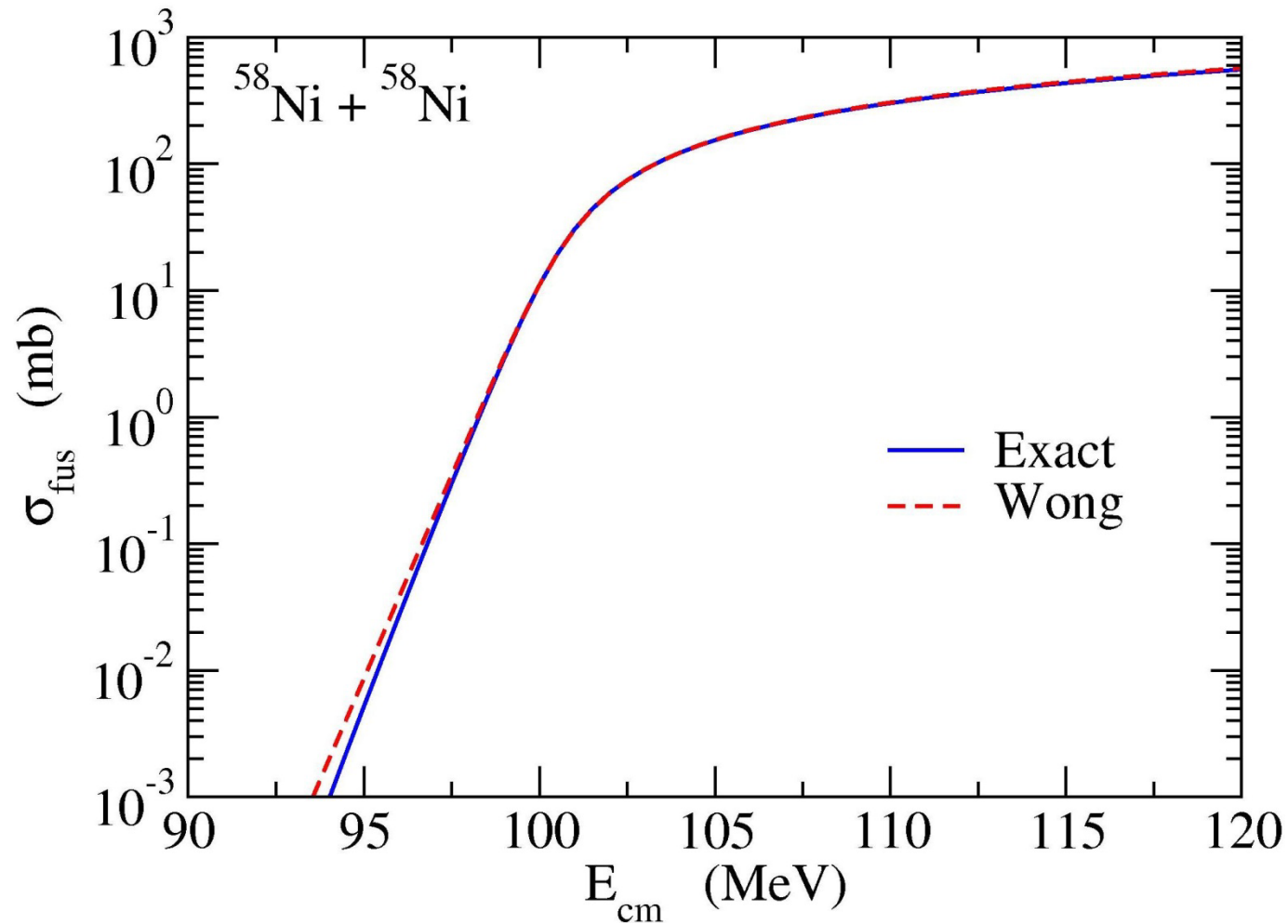
(note) For $E \gg V_b$ $1 \ll \exp \left(\frac{2\pi}{\hbar\Omega} (E - V_b) \right)$

$$\implies \sigma_{\text{fus}}(E) \sim \pi R_b^2 \left(1 - \frac{V_b}{E} \right) = \sigma_{\text{fus}}^{\text{cl}}(E)$$

(note)

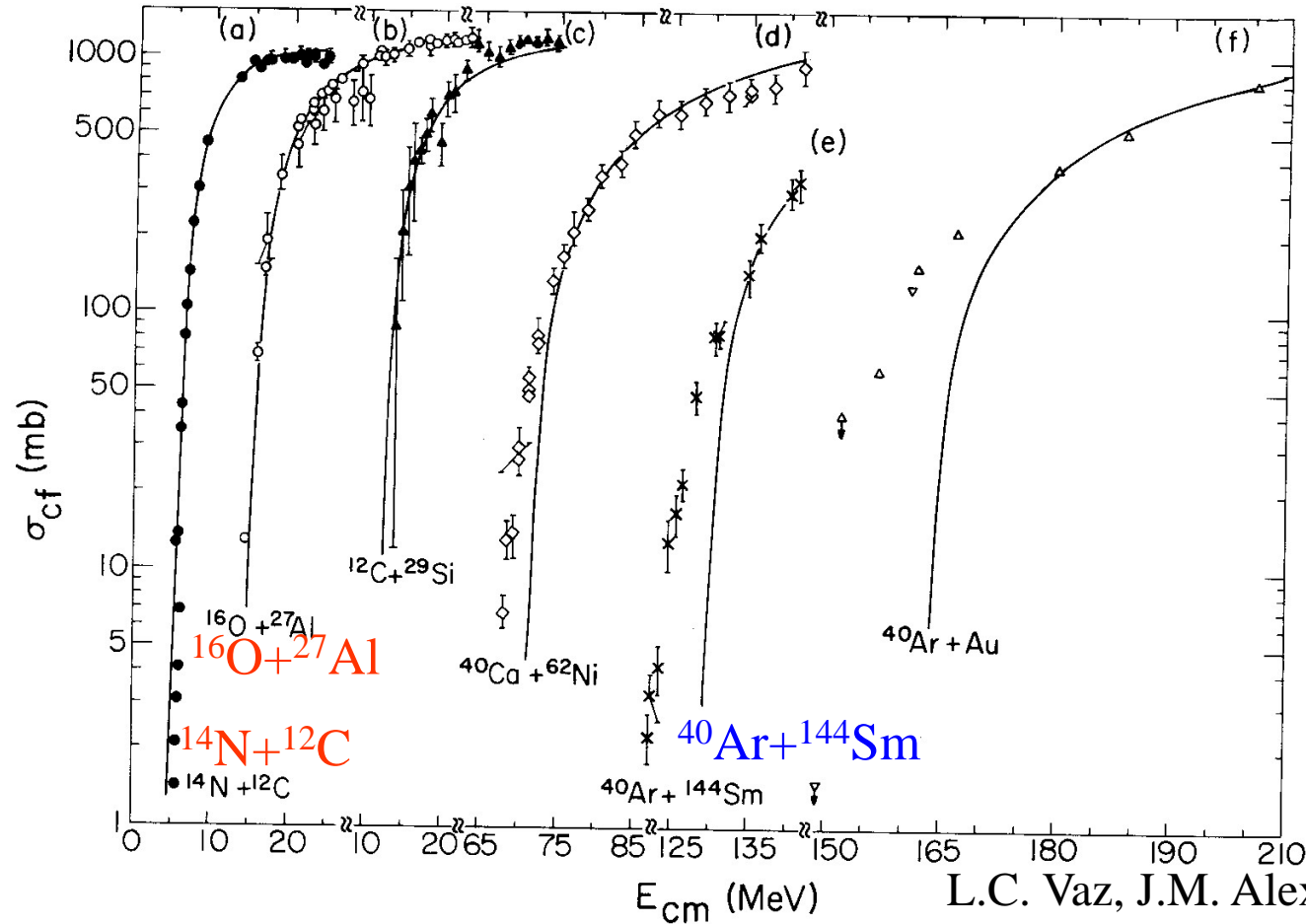
$$\frac{d(E\sigma_{\text{fus}})}{dE} = \frac{\pi R_b^2}{1 + \exp \left[\frac{2\pi}{\hbar\Omega} (V_b - E) \right]} = \pi R_b^2 \cdot P_{l=0}(E)$$

$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \log \left[1 + \exp \left(\frac{2\pi}{\hbar\Omega} (E - V_b) \right) \right]$$



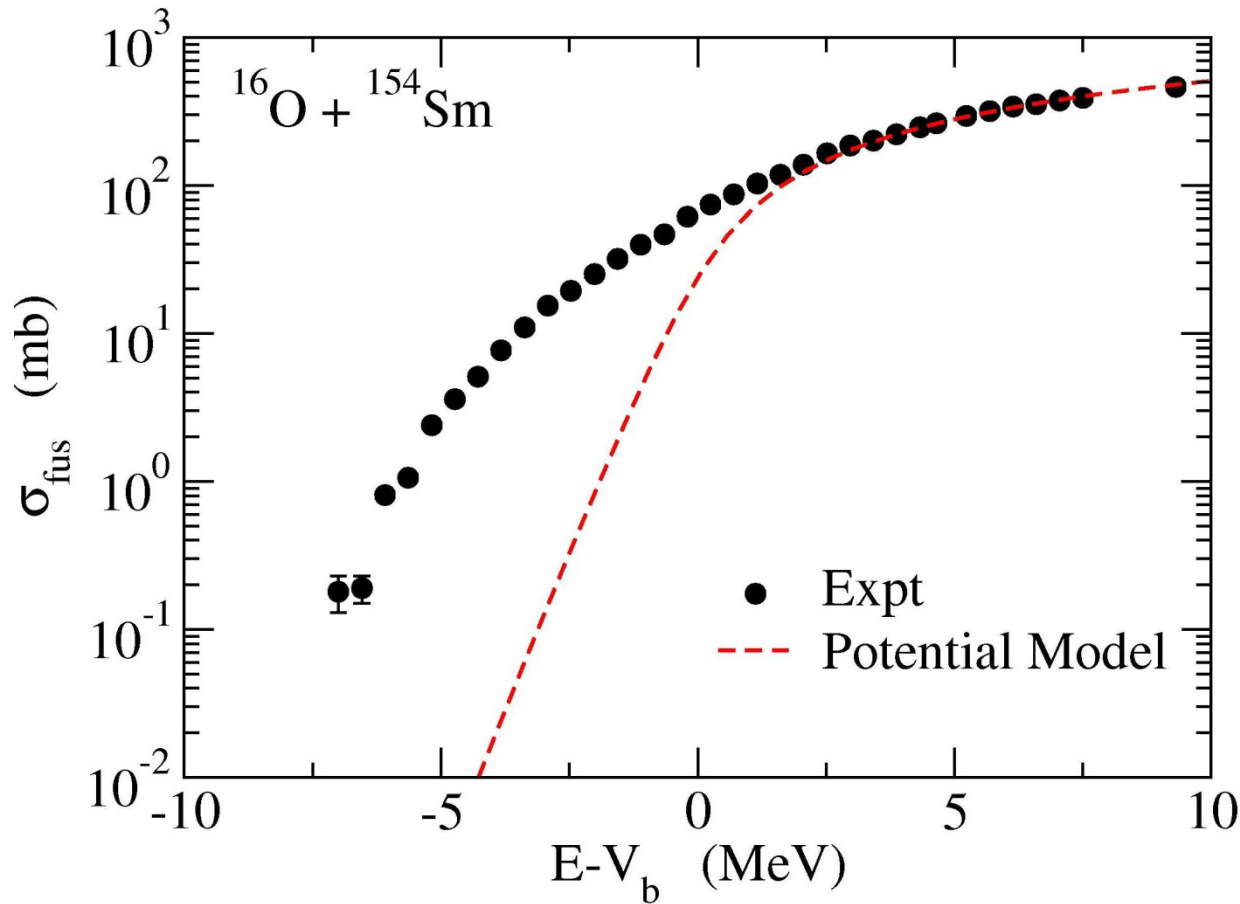
Comparison between prediction of pot. model with expt. data

Fusion cross sections calculated with a static energy independent potential



L.C. Vaz, J.M. Alexander, and
G.R. Satchler, Phys. Rep. 69('81)373

- Works well for relatively light systems
- Underpredicts σ_{fus} for heavy systems at low energies

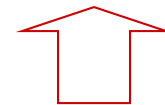


Potential model:
Reproduces the data reasonably well for

$$E > V_b$$

Underpredicts σ_{fus} for

$$E < V_b$$



What is the origin?

Data: J.R. Leigh et al., PRC52('95)3151

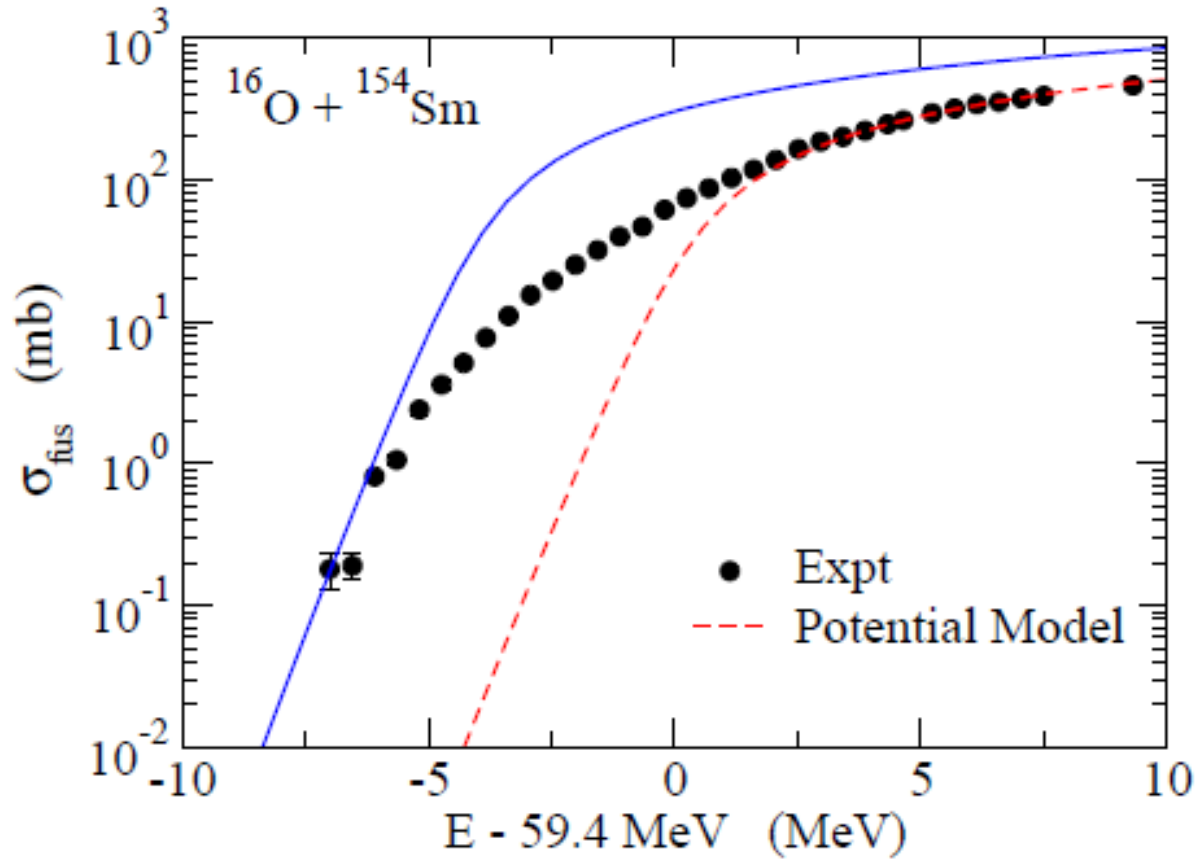
cf. seminal work:

R.G. Stokstad et al., PRL41('78)465

PRC21('80)2427

Inter-nuclear Potential is poorly parametrized?
Other origins?

With a deeper nuclear potential (but still within a potential model).....



Potential Inversion

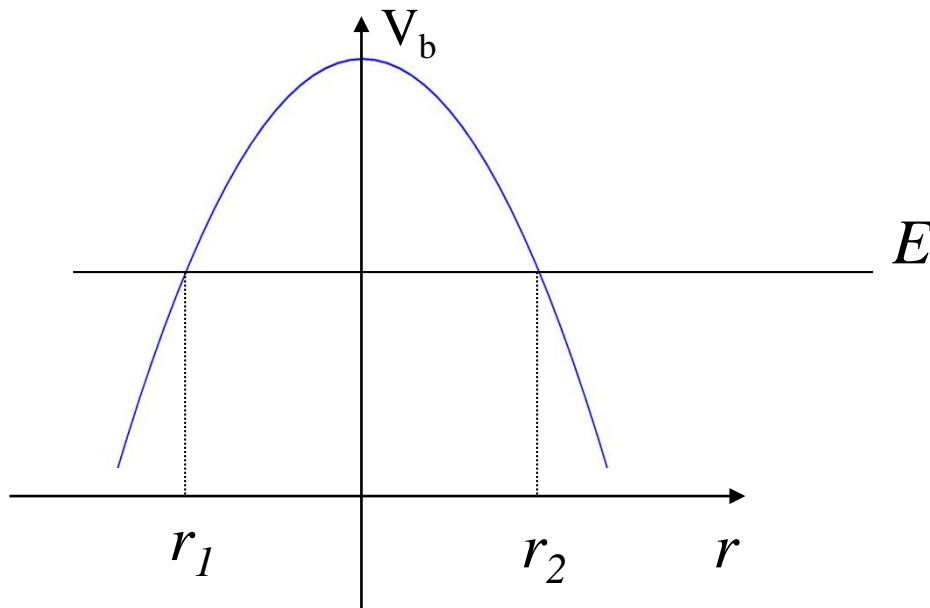
$$P_0(E) = \frac{1}{\pi R_b^2} \frac{d(E\sigma_{\text{fus}})}{dE}$$

(note)

$$P_0(E) = 1/[1 + S_0(E)], \quad S_0(E) = \int_{r_1}^{r_2} dr \sqrt{\frac{2\mu}{\hbar^2}(V(r) - E)}$$



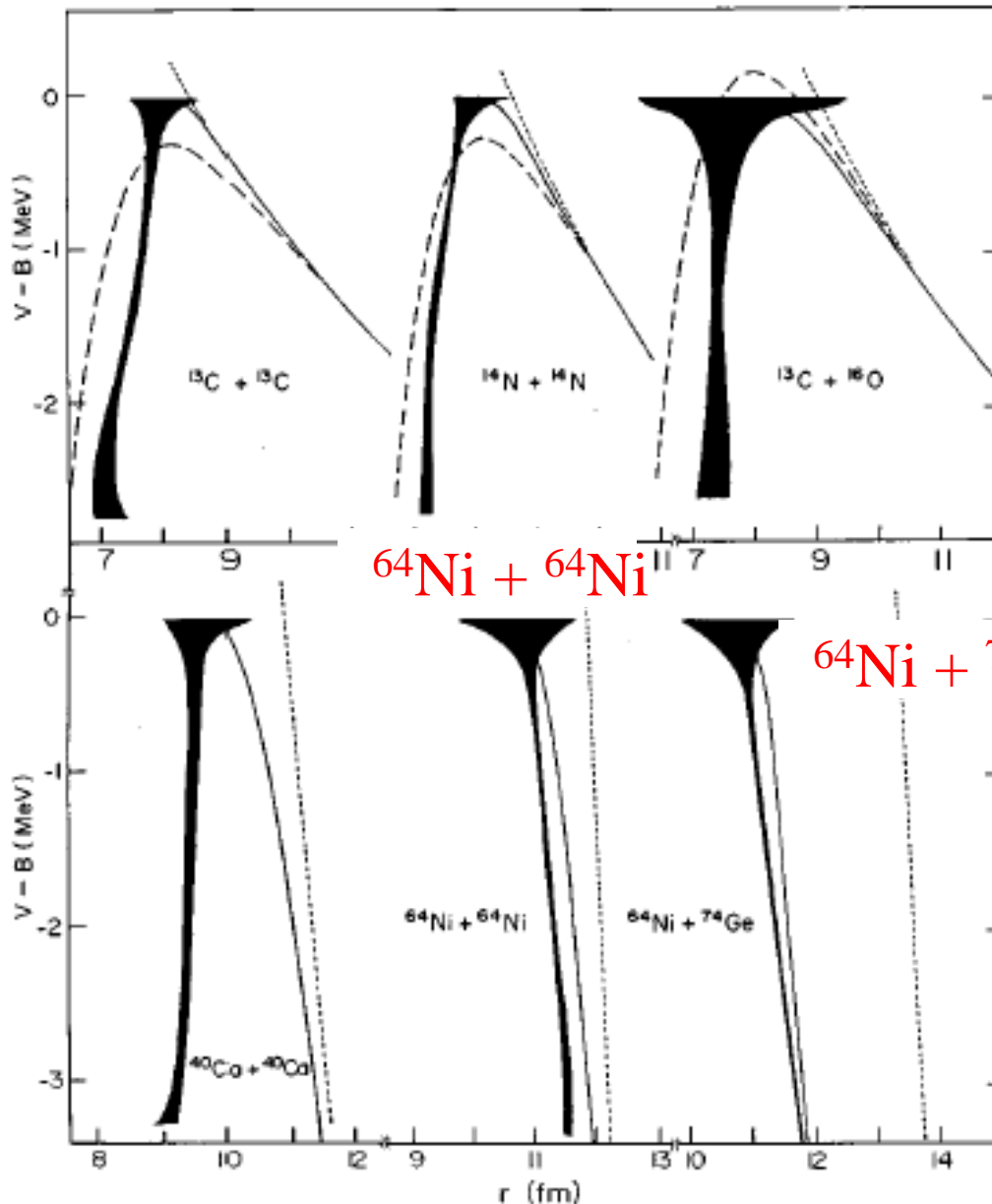
$$t(E) \equiv r_2 - r_1 = -\frac{2}{\pi} \sqrt{\frac{\hbar^2}{2\mu}} \int_E^{V_b} \frac{\frac{dS_0(E')}{dE'}}{\sqrt{E' - E}} dE'$$



$\sigma_{\text{fus}} \xrightarrow{\quad} V(r)$
↑
Semi-classical app.

- Energy independent
- local
- single-ch.

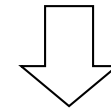
● Potential inversion



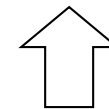
$$\sigma_{\text{fus}} \xrightarrow{\quad} V(r)$$

Semi-classical app.

- Energy independent
- local
- single-ch.



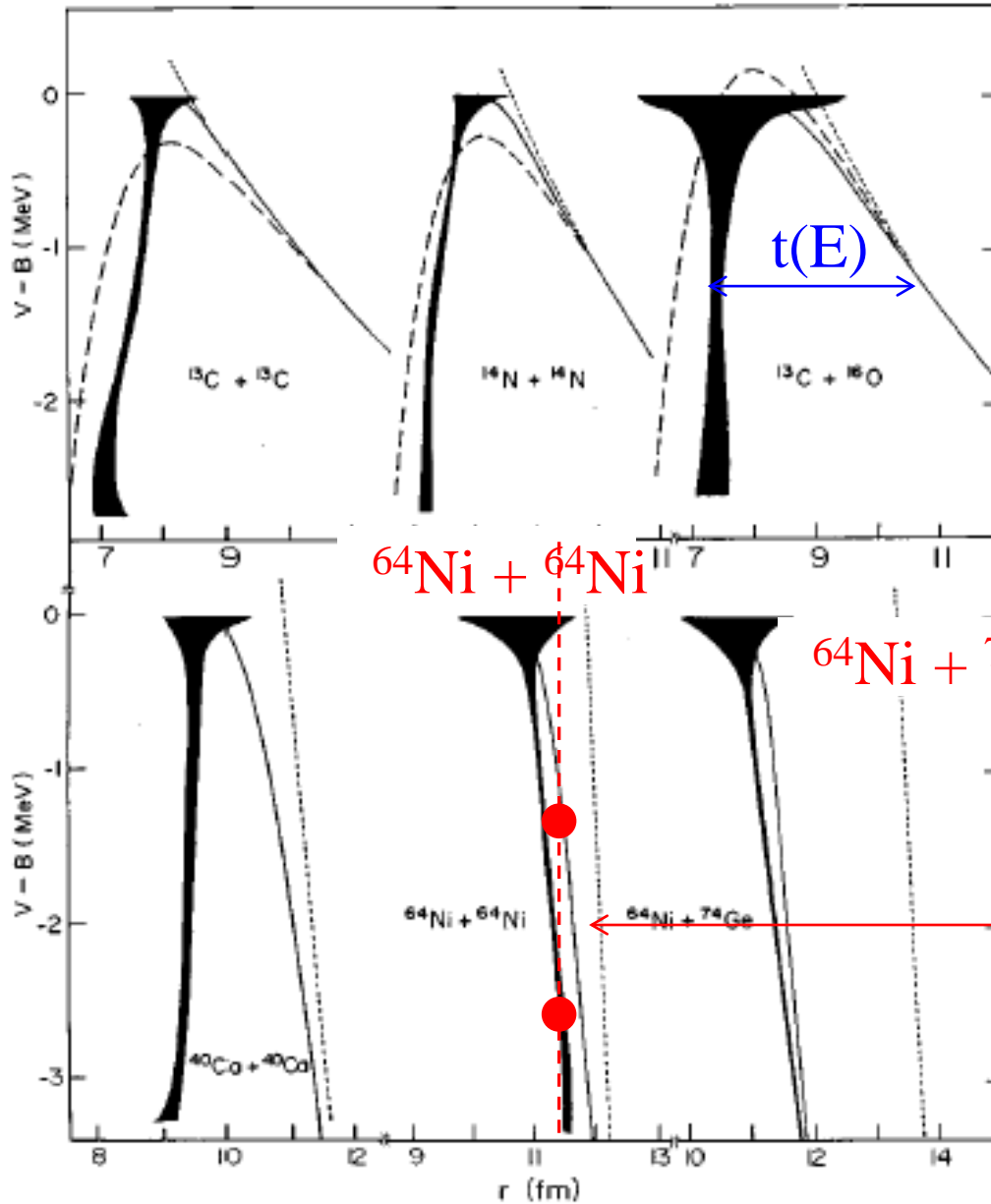
Unphysical potentials



Beautiful demonstration
of C.C. effects

A.B. Balantekin, S.E. Koonin, and
J.W. Negele, PRC28('83)1565

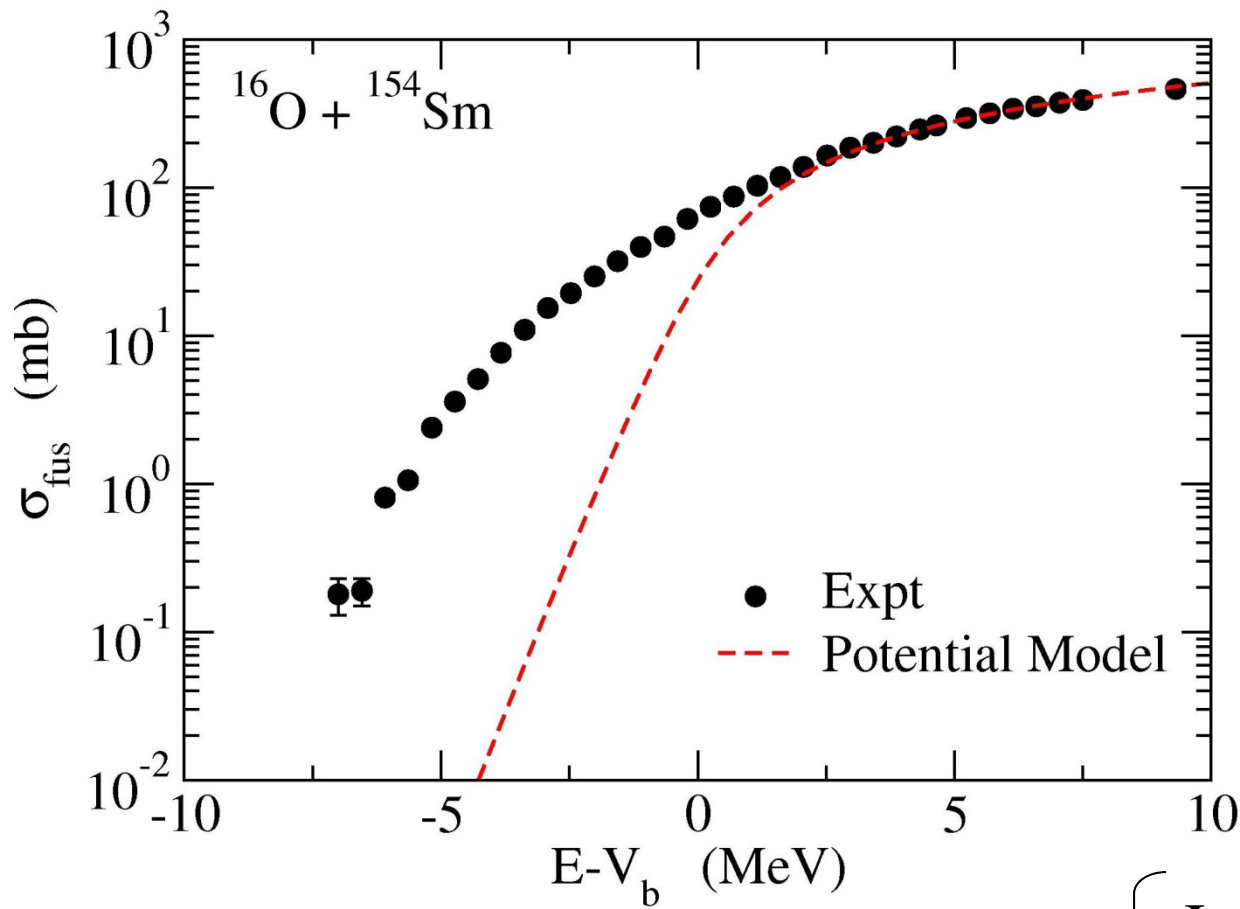
• Potential inversion



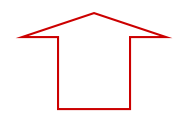
e.g.,
 $t(E) = 3 \pm 0.2$ fm

double valued potential
(unphysical !!)

Fusion cross sections calculated with a static energy independent potential



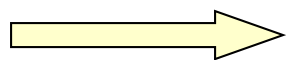
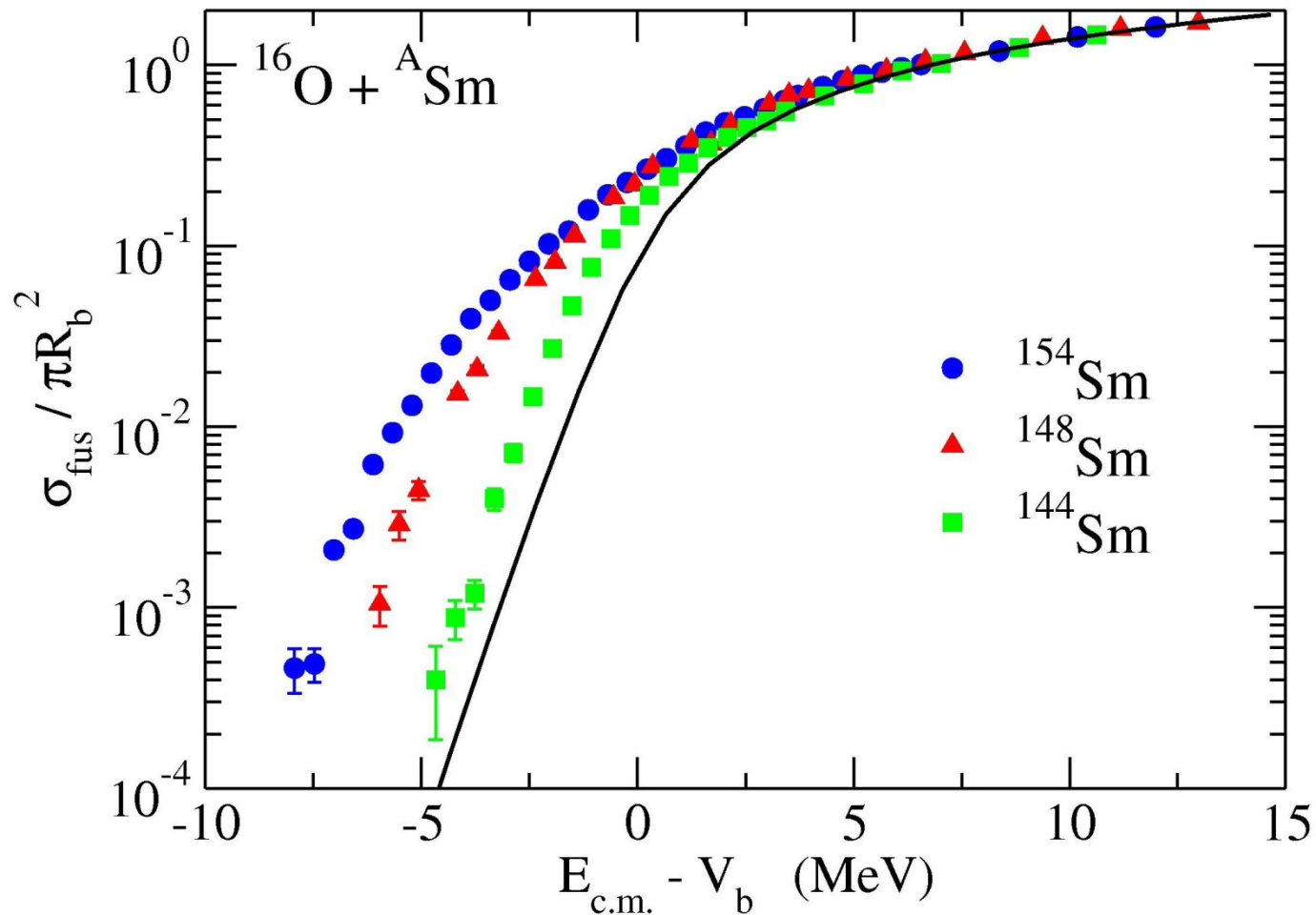
Potential model:
Reproduces the data reasonably well for $E > V_b$
Underpredicts σ_{fus} for $E < V_b$



What is the origin?

~~Inter-nuclear Potential is poorly parametrized?~~
Other origins?

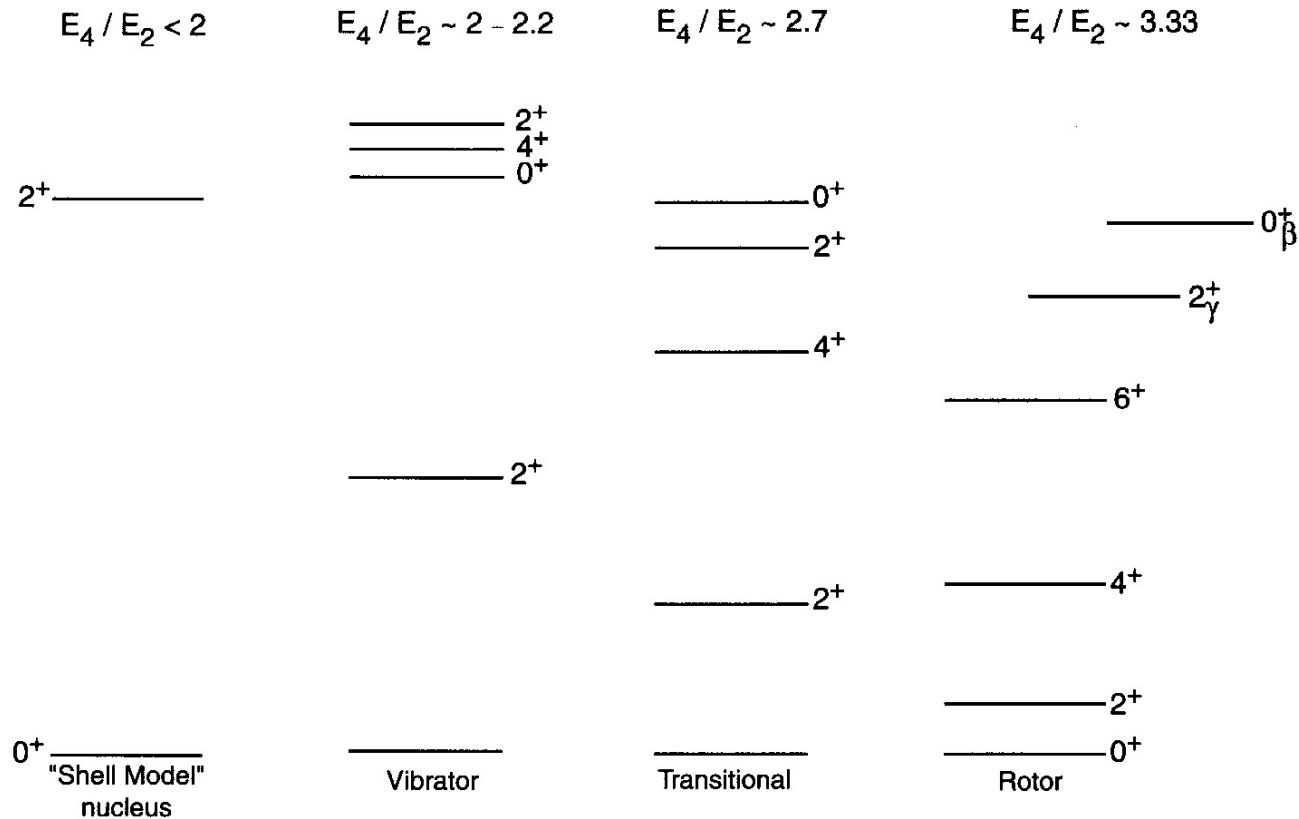
Target dependence of fusion cross section



Strong target dependence at $E < V_b$

Low-lying collective excitations in atomic nuclei

Low-lying excited states in even-even nuclei are collective excitations, and strongly reflect the pairing correlation and shell structure



SCHMATIC EVOLUTION OF STRUCTURE
NEAR CLOSED - SHELL \rightarrow MID SHELL

Taken from R.F. Casten,
"Nuclear Structure from a
Simple Perspective"

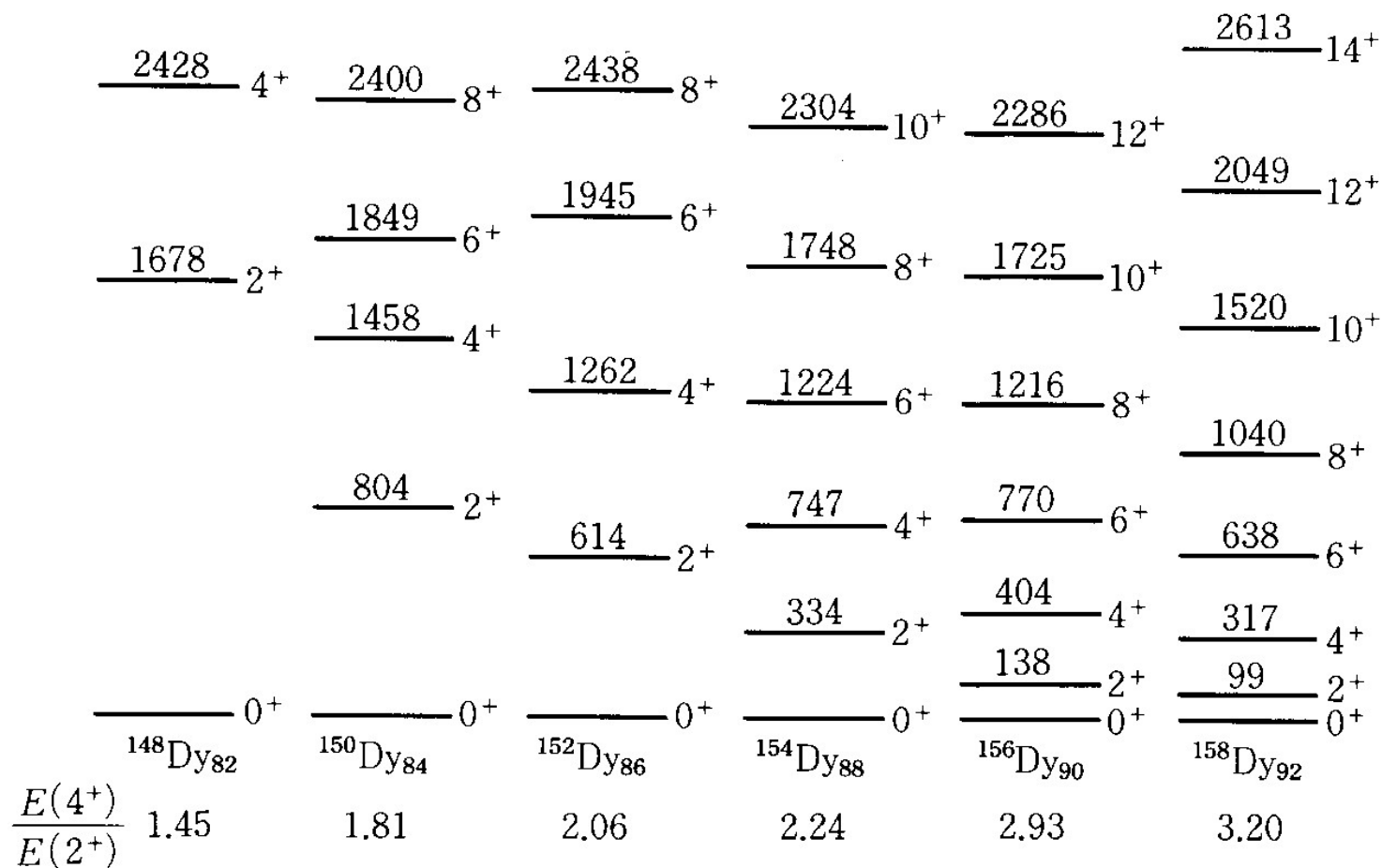
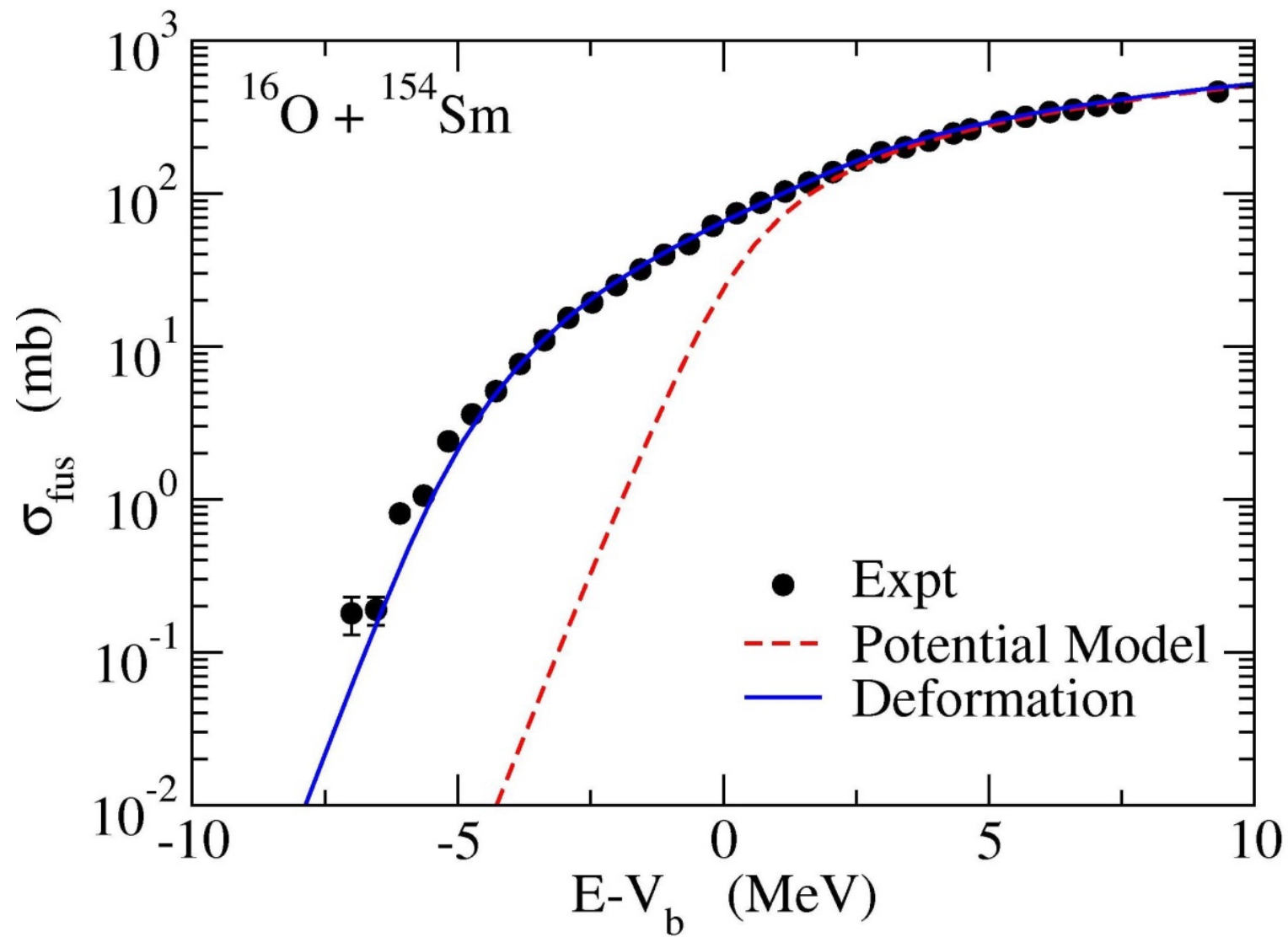


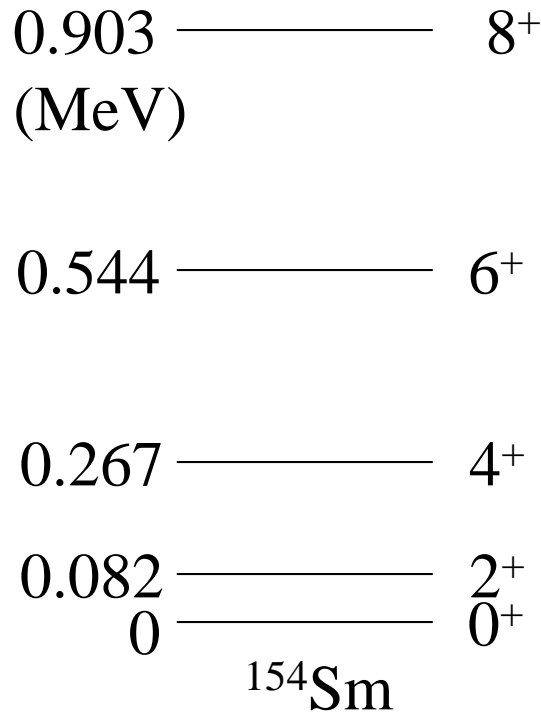
図 3-4 Dy アイソトープの低励起スペクトル. 励起エネルギーの単位は keV.



Lecture 2

Effect of collective excitation on σ_{fus} : rotational case

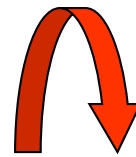
Excitation spectra of ^{154}Sm



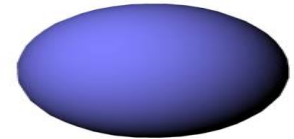
cf. Rotational energy of a rigid body
(Classical mechanics)

$$E = \frac{1}{2} \mathcal{J} \omega^2 = \frac{I^2}{2\mathcal{J}}$$

$$(I = \mathcal{J}\omega, \omega = \dot{\theta})$$



^{154}Sm is deformed



$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

Effect of collective excitation on σ_{fus} : rotational case

Comparison of energy scales

$$V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$$

- Tunneling motion: $E_{\text{tun}} \sim \hbar\Omega \sim 3.5 \text{ MeV}$ (barrier curvature)
- Rotational motion: $E_{\text{rot}} \sim E_{2^+} \sim 0.08 \text{ MeV}$

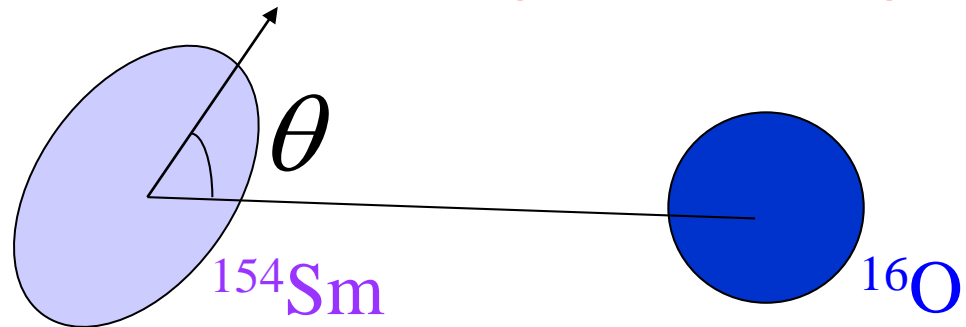
$\curvearrowright E_{\text{tun}} \gg E_{\text{rot}} = I(I + 1)\hbar^2/2\mathcal{J} \rightarrow 0$

$\longleftrightarrow \mathcal{J} \rightarrow \infty$

\curvearrowright The orientation angle of ^{154}Sm does not change much during fusion

(note)

Ground state (0^+ state) when reaction starts

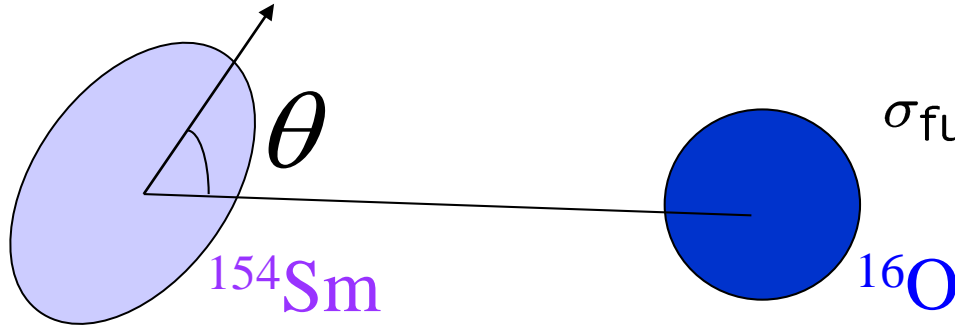


Mixing of all orientations with an equal weight

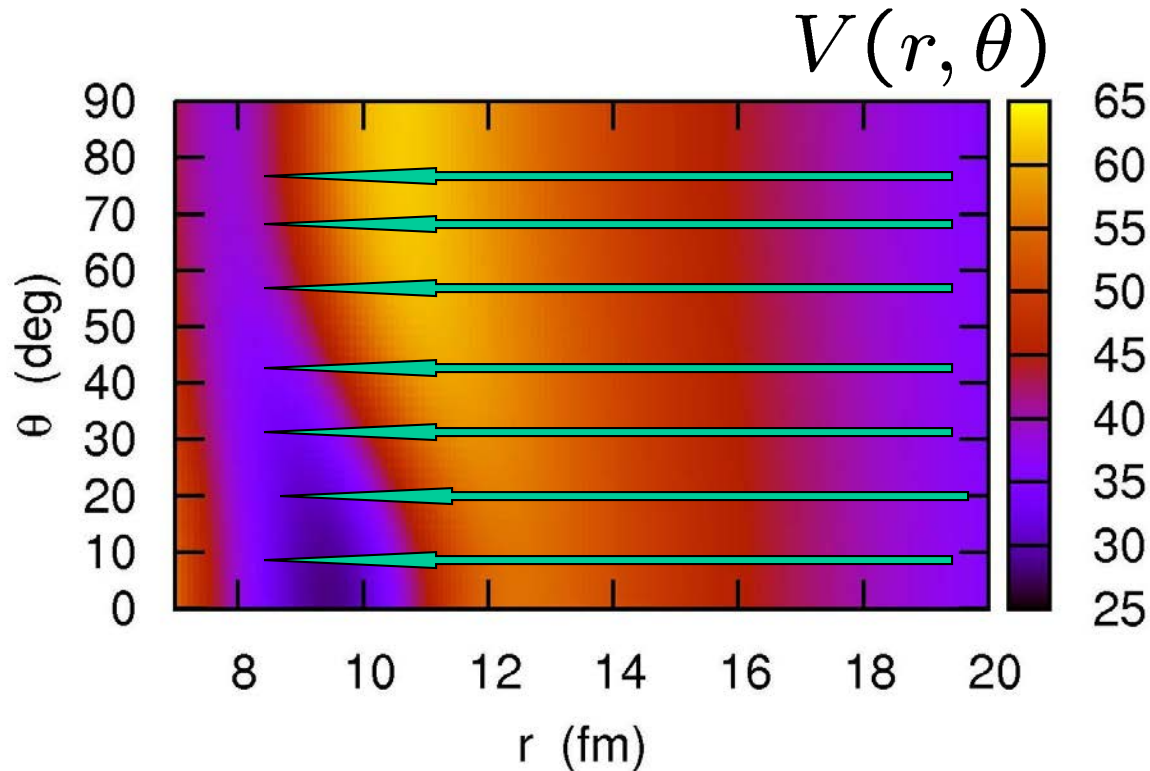
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$

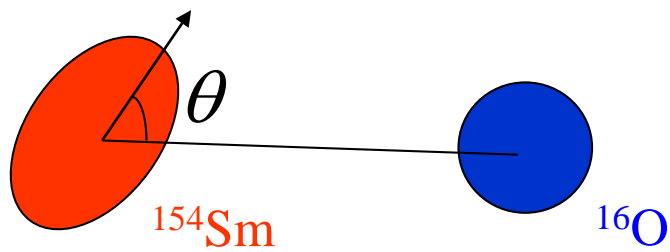
Effect of collective excitation on σ_{fus} : rotational case

↪ The orientation angle of ^{154}Sm does not change much during fusion

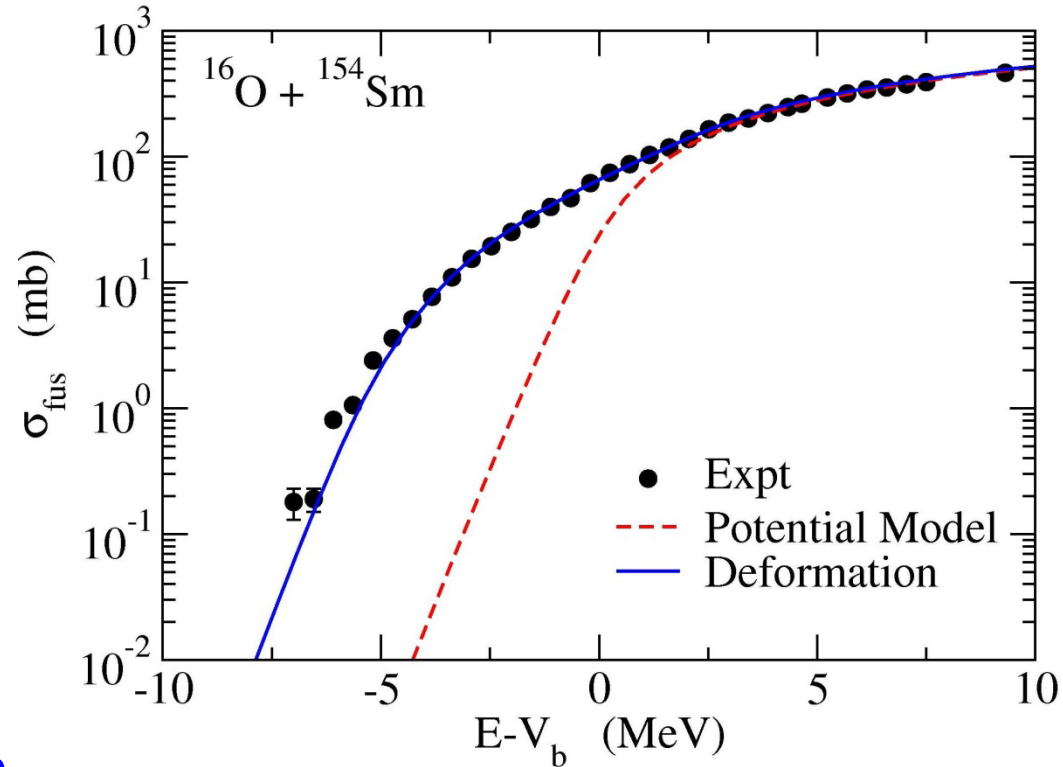
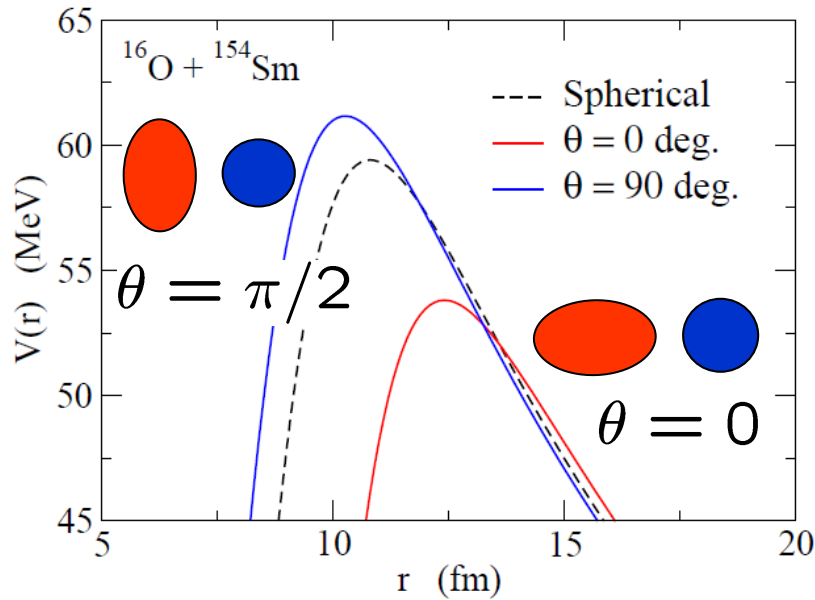


$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$





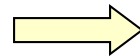
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$



The barrier is lowered for $\theta=0$ because an attraction works from large distances.

The barrier increases for $\theta=\pi/2$. because the rel. distance has to get small for the attraction to work

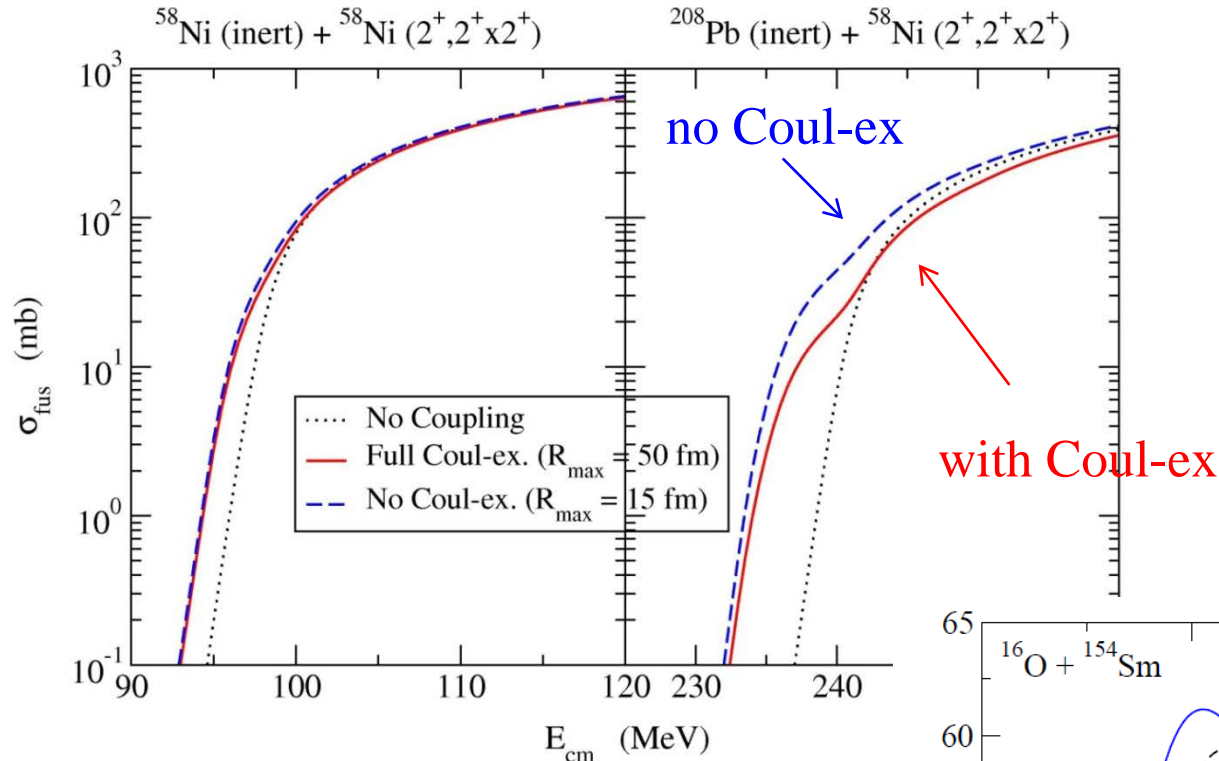
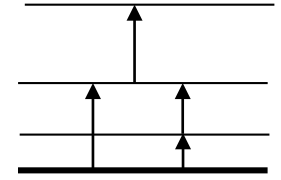
Def. Effect: enhances σ_{fus} by a factor of 10 ~ 100



Fusion: interesting probe for nuclear structure

Two effects of channel couplings

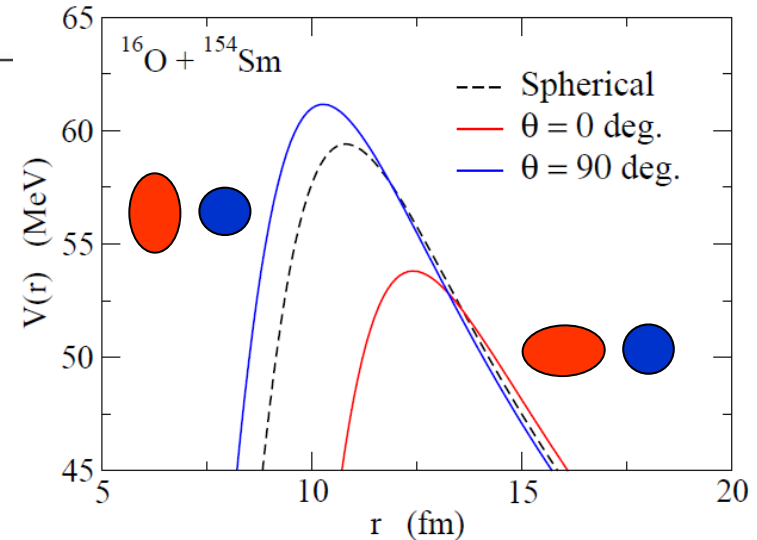
✓ energy loss due to inelastic excitations



✓ dynamical modification of the Coulomb barrier



large enhancement of fusion cross sections

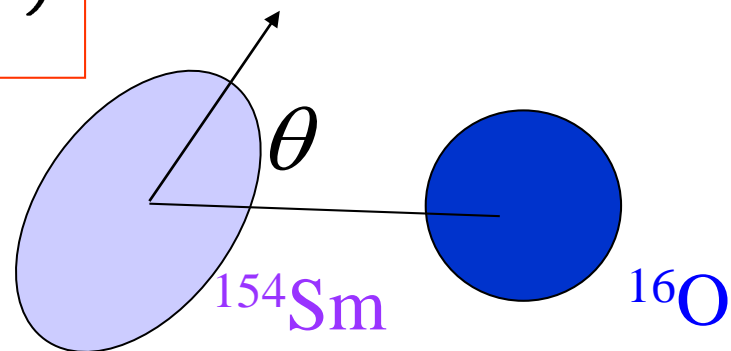


cf. 2-level model: Dasso, Landowne, and Winther, NPA405('83)381

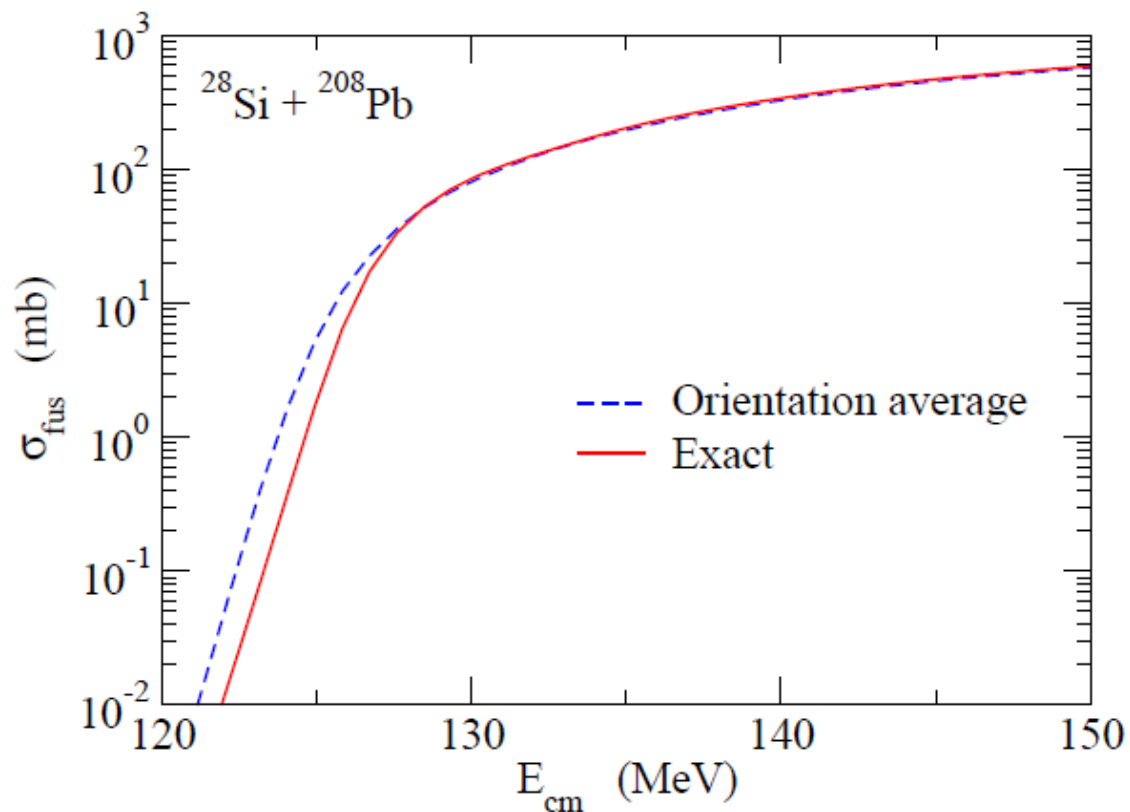
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$

One warning:

Don't use this formula for light deformed nuclei, e.g., ^{28}Si

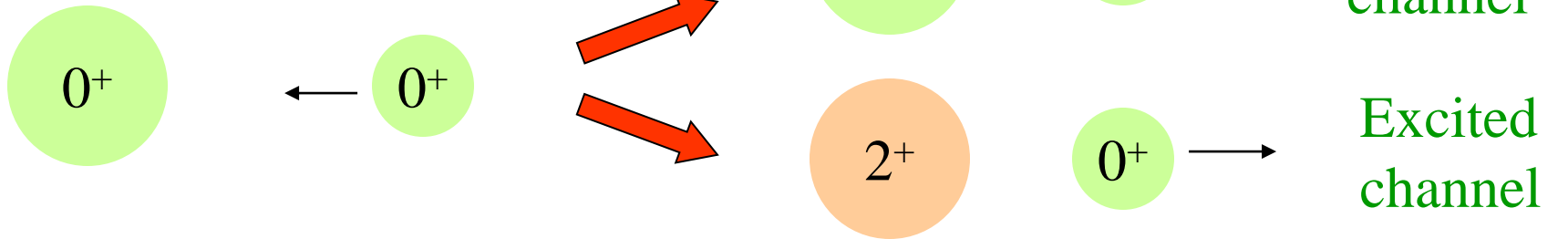


4 ⁺	—————	4.618
		(MeV)
2 ⁺	—————	1.779
		(MeV)
0 ⁺	—————	0
	^{28}Si	

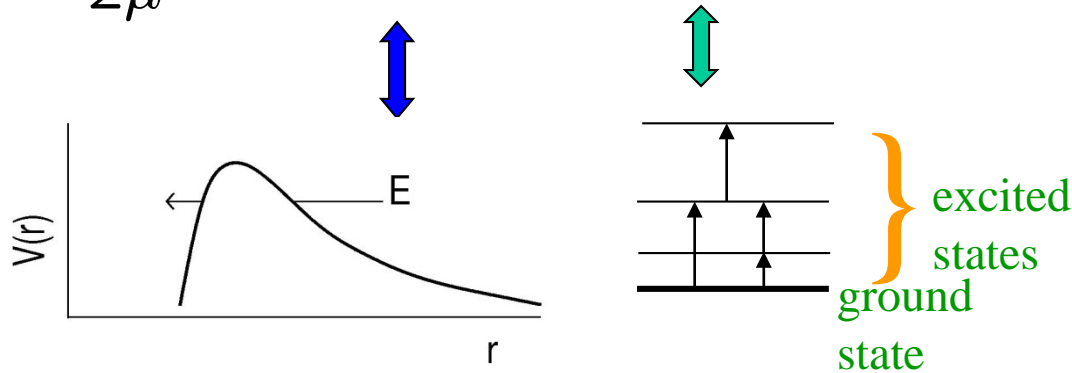


More quantal treatment: Coupled-Channels method

Coupling between rel. and intrinsic motions



$$H = -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + H_0(\xi) + V_{\text{coup}}(r, \xi)$$



$$H_0(\xi) \phi_k(\xi) = \epsilon_k \phi_k(\xi)$$

$$\Psi(r, \xi) = \sum_k \psi_k(r) \phi_k(\xi)$$

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\text{coup}}(\mathbf{r}, \xi)$$

$$\Psi(\mathbf{r}, \xi) = \sum_k \psi_k(\mathbf{r})\phi_k(\xi) \quad H_0(\xi)\phi_k(\xi) = \epsilon_k \phi_k(\xi)$$

Schroedinger equation: $(H - E)\Psi(\mathbf{r}, \xi) = 0$

$$\langle \phi_k | \rightarrow$$

$$\langle \phi_k | H - E | \Psi \rangle = 0$$

or

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \epsilon_k - E \right] \psi_k(\mathbf{r}) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(\mathbf{r}) = 0$$

Coupled-channels equations

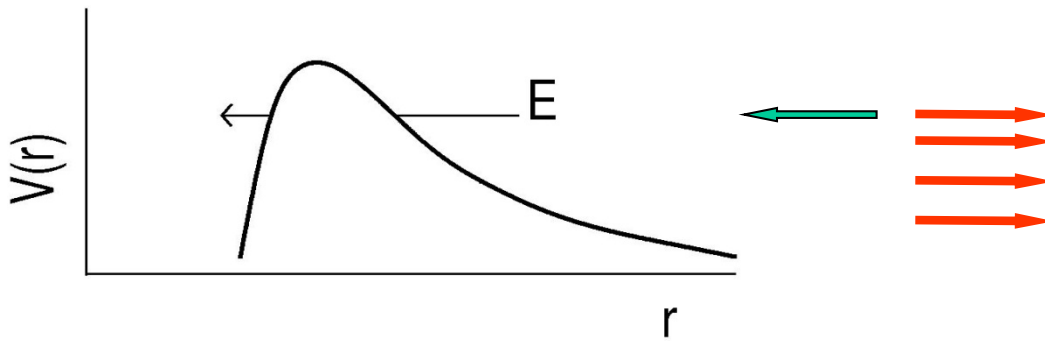
Coupled-channels equations

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \epsilon_k - E \right] \psi_k(\mathbf{r}) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(\mathbf{r}) = 0$$

equation for ψ_k

transition from $\phi_{k'}$ to ϕ_k

boundary condition:



$$\psi_n(r) \rightarrow \begin{cases} e^{-ik_0 r} - S_0 e^{ik_0 r} & (n = 0) \\ -S_n e^{ik_n r} & (n \neq 0) \end{cases}$$

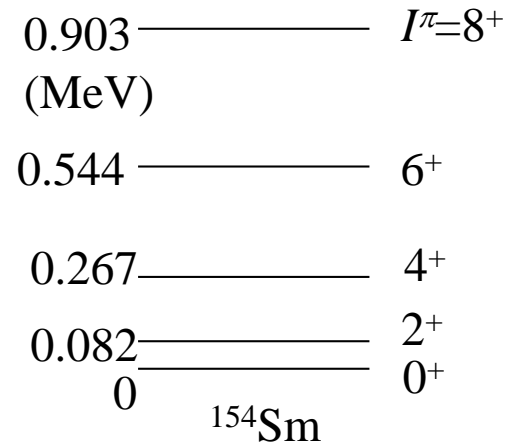
$$P(E) = 1 - \sum_n |S_n|^2$$

$$k_n = \sqrt{2\mu(E - \epsilon_n)/\hbar^2}$$

Angular momentum coupling

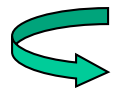
$$H_0(\xi)\phi_{nIm_I}(\xi) = \epsilon_{nI}\phi_{nIm_I}(\xi)$$

Total ang. mom.: $I + l = J$



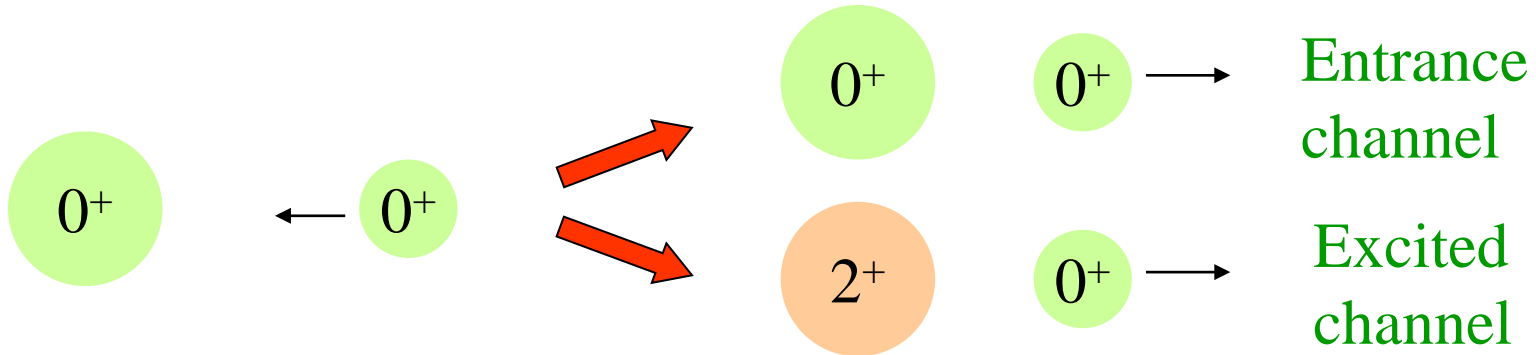
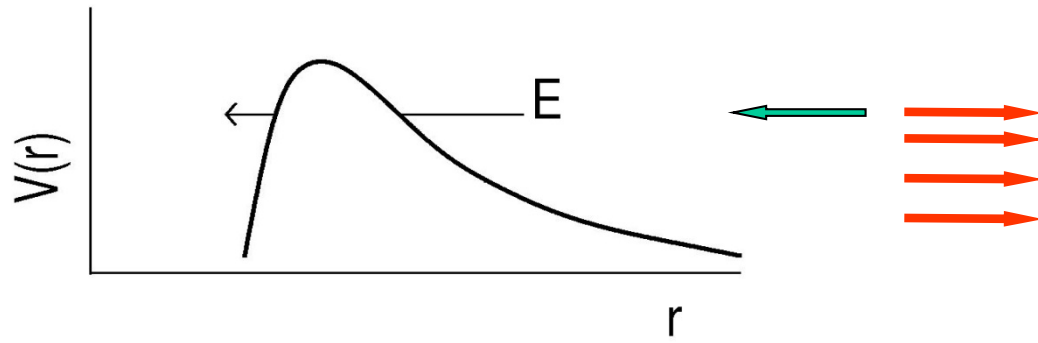
$$\Psi(\mathbf{r}, \xi) = \sum_k \psi_k(\mathbf{r})\phi_k(\xi) = \sum_{n,l,I} \frac{u_{nlI}(r)}{r} [Y_l(\hat{\mathbf{r}})\phi_{nI}(\xi)]^{(JM)}$$

$$\langle [Y_l\phi_{nI}]^{(JM)} | H - E | \Psi \rangle = 0$$



$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V_0(r) - E + \epsilon_{nI} \right] u_{nlI}(r) + \sum_{n'l'I'} \langle [Y_l\phi_{nI}]^{(JM)} | V_{\text{coup}}(r) | [Y_{l'}\phi_{n'I'}]^{(JM)} \rangle u_{n'l'I'}(r) = 0$$

Boundary condition
(with ang. mom. coupling)



$$\Psi(r, \xi) = \sum_{n,l,I} \frac{u_{nlI}(r)}{r} [Y_l(\hat{r}) \phi_{nI}(\xi)]^{(JM)}$$

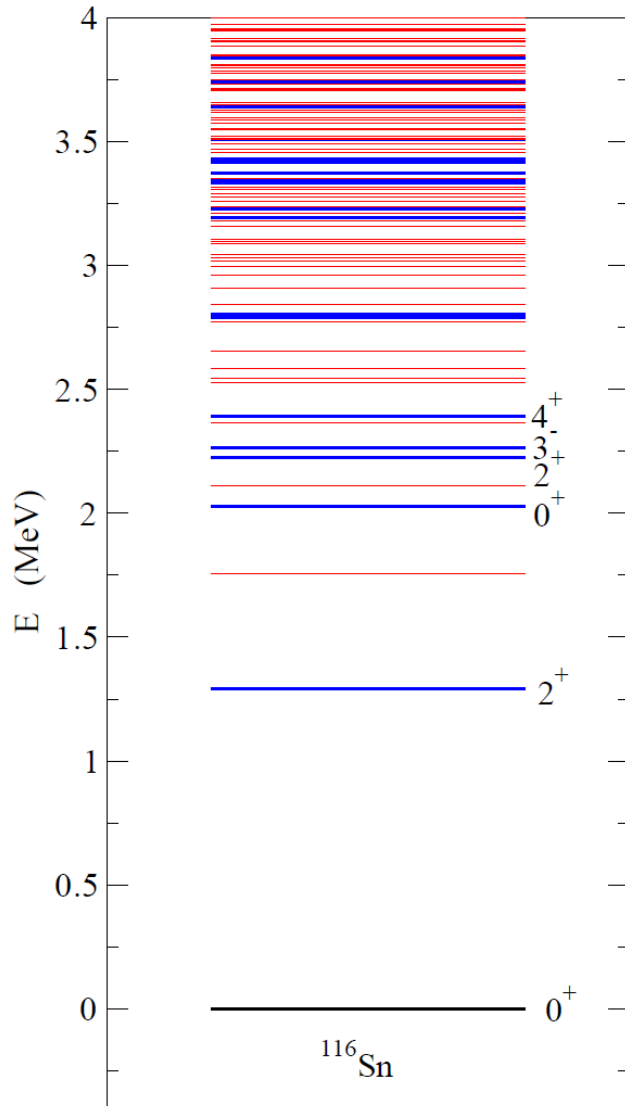
$$u_{nlI}(r) \rightarrow H_l^{(-)}(k_{nI}r) \delta_{n,n_i} \delta_{l,l_i} \delta_{I,I_i} - \sqrt{\frac{k_0}{k_{nI}}} S_{nlI} H_l^{(+)}(k_{nI}r)$$

$$P_l(E) = 1 - \sum_{nI} |S_{nlI}|^2$$

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) P_l(E)$$

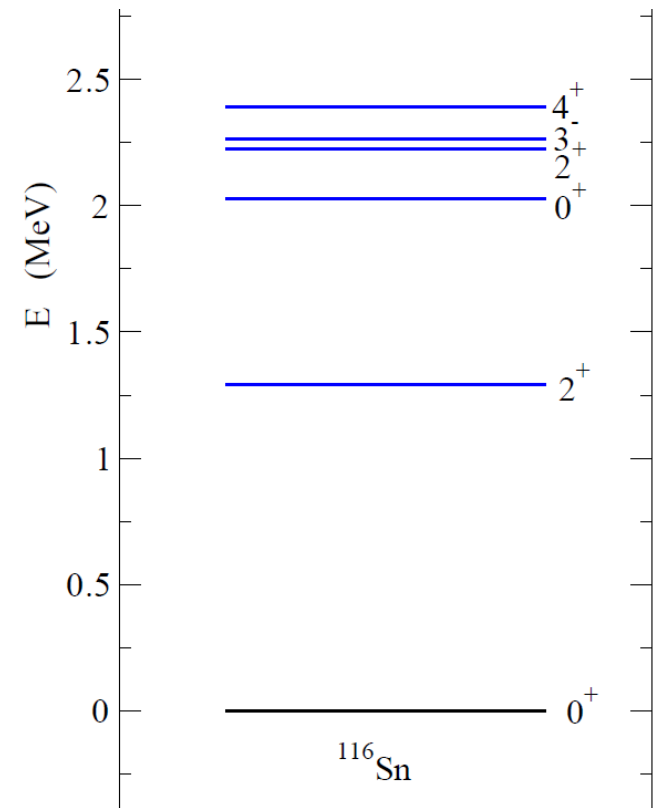
How to perform coupled-channels calculations?

1. Modeling: selection of excited states to be included



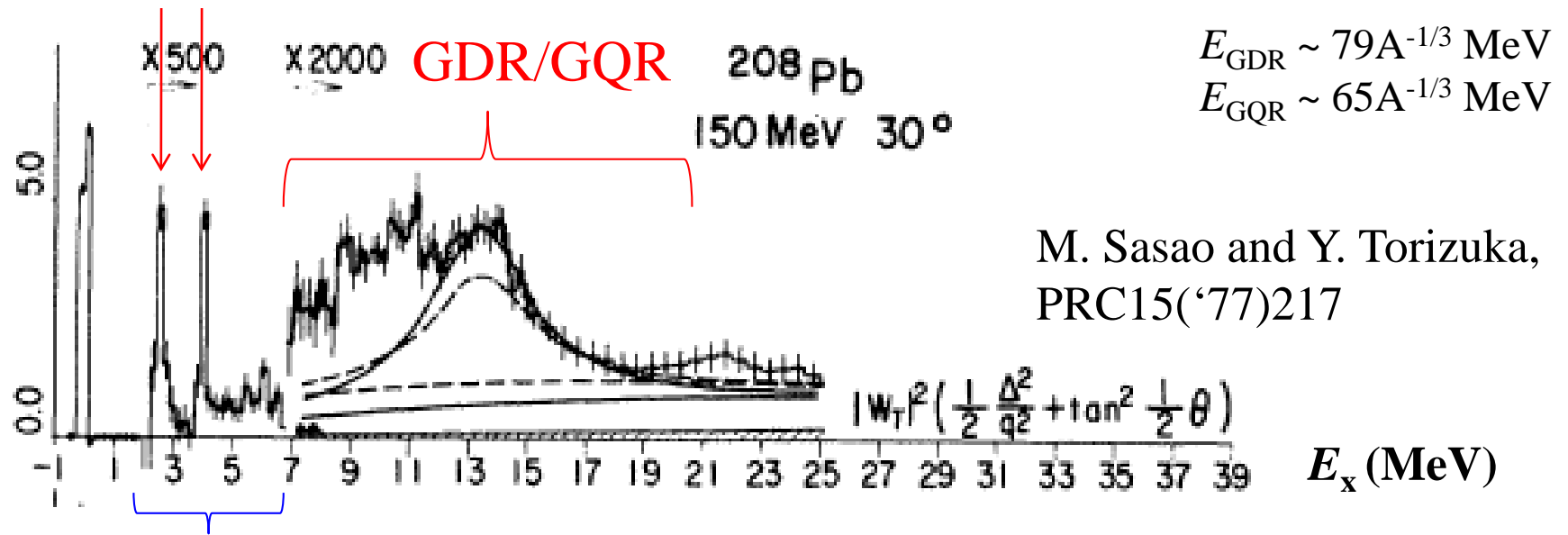
low-lying
collective
states only

S. Raman et al.,
PRC43('91)521



typical excitation spectrum: electron scattering data

low-lying collective excitations

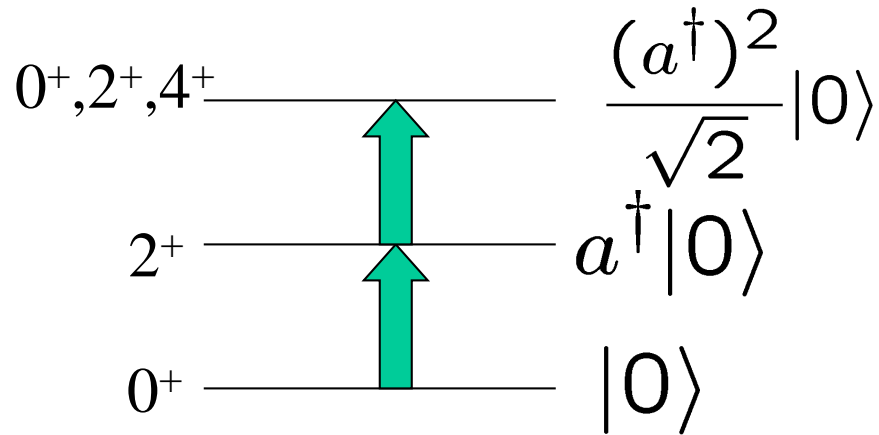


low-lying non-collective excitations

- Giant Resonances: high E_x , smooth mass number dependence
→ adiabatic potential renormalization
- Low-lying collective excitations: barrier distributions,
strong isotope dependence
- Non-collective excitations: either neglected completely or
implicitly treated through an absorptive potential

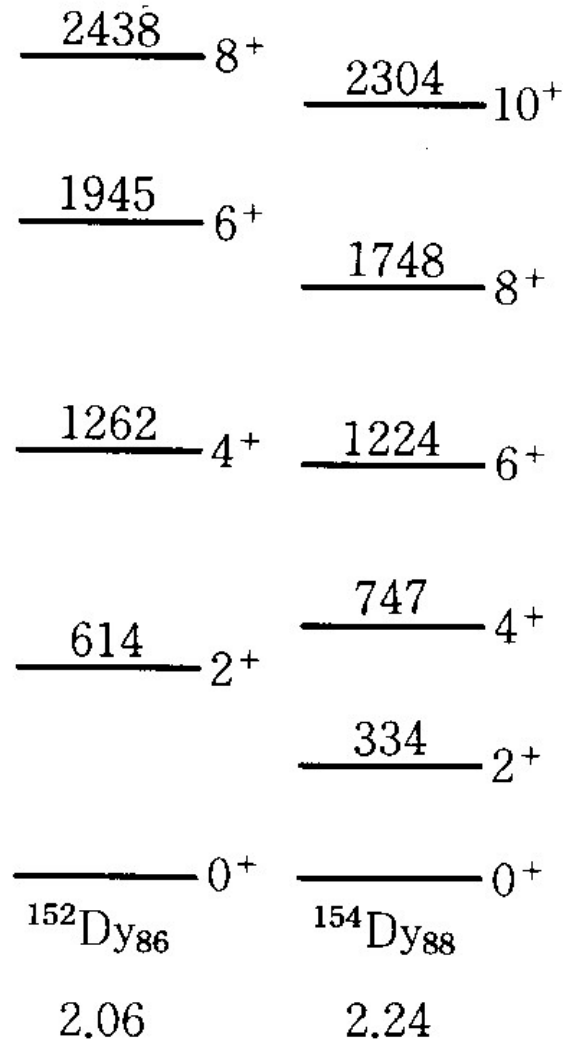
2. Nature of collective states: vibration? or rotation?

a) Vibrational coupling



$$|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$$

$$\epsilon_n = (n + 1/2)\hbar\omega$$

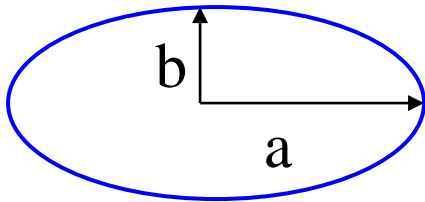


Vibrational excitations

Bethe-Weizacker formula: Mass formula based on Liquid-Drop Model

(A, Z) $B(N, Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N - Z)^2}{A}$

→ For a deformed shape,



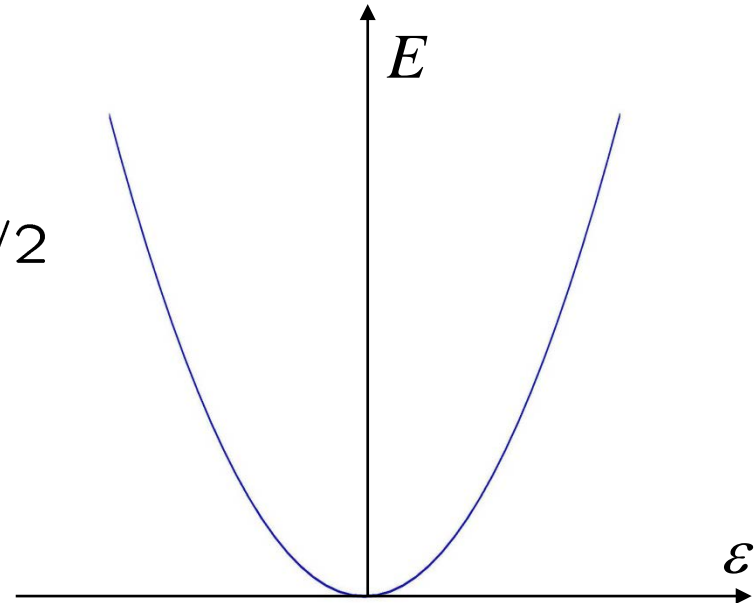
$$a = R \cdot (1 + \epsilon)$$

$$b = R \cdot (1 + \epsilon)^{-1/2}$$

↪

$$E_{\text{surf}} = E_{\text{surf}}^{(0)} (1 + 2\epsilon^2/5 + \dots)$$

$$E_C = E_C^{(0)} (1 - \epsilon^2/5 + \dots)$$

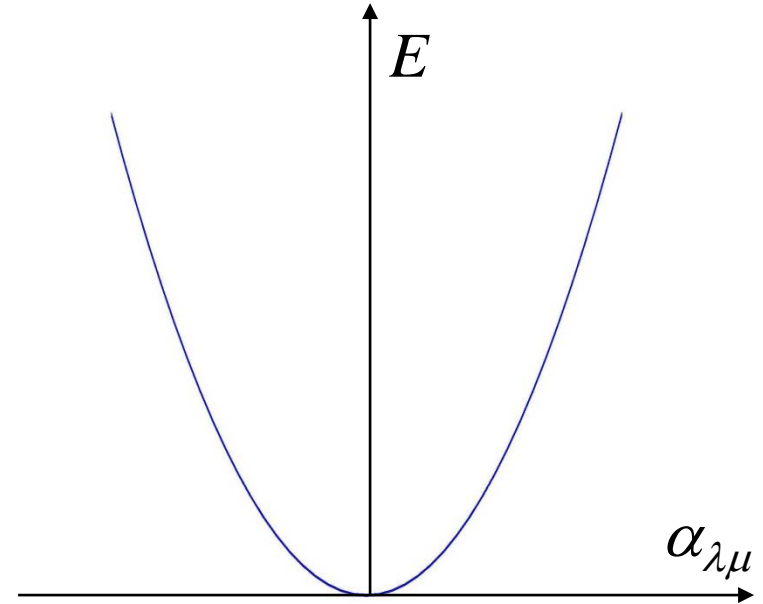


In general

$$R(\theta, \phi) = R_0 \left(1 + \sum_{\lambda, \mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^* \right)$$

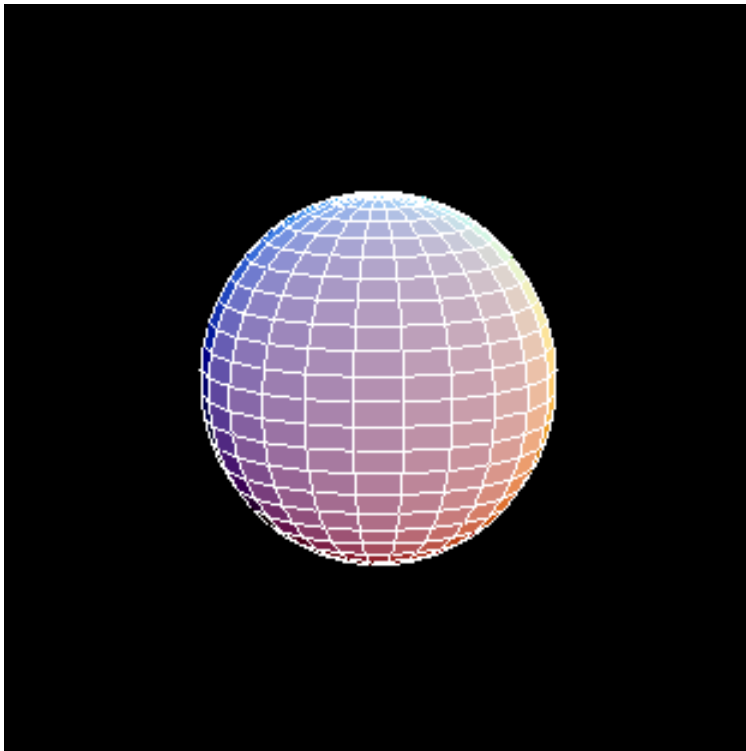
$$V = \frac{1}{2} \sum_{\lambda, \mu} C_{\lambda} |\alpha_{\lambda\mu}|^2$$

Harmonic oscillation



$\lambda=2$: Quadrupole vibration

Movie: Dr. K. Arita (Nagoya Tech. U.)
<http://www.phys.nitech.ac.jp/~arita/>

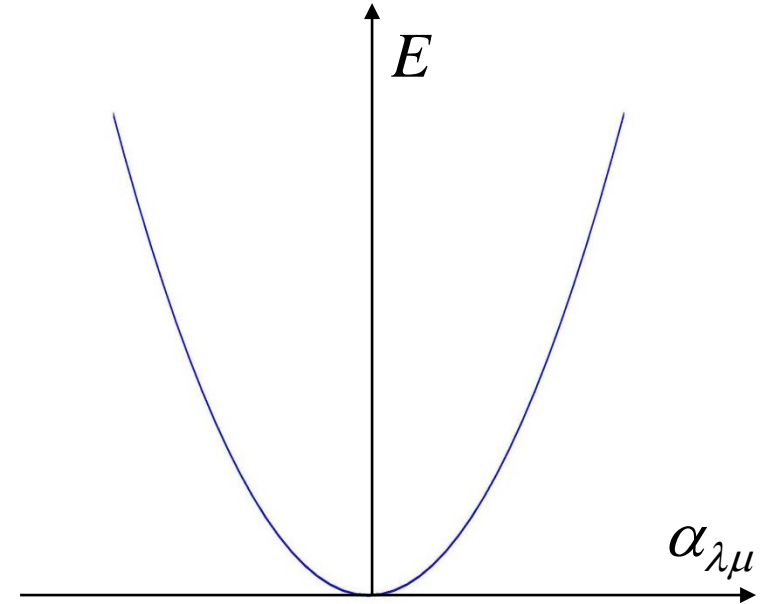


In general

$$R(\theta, \phi) = R_0 \left(1 + \sum_{\lambda, \mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^* \right)$$

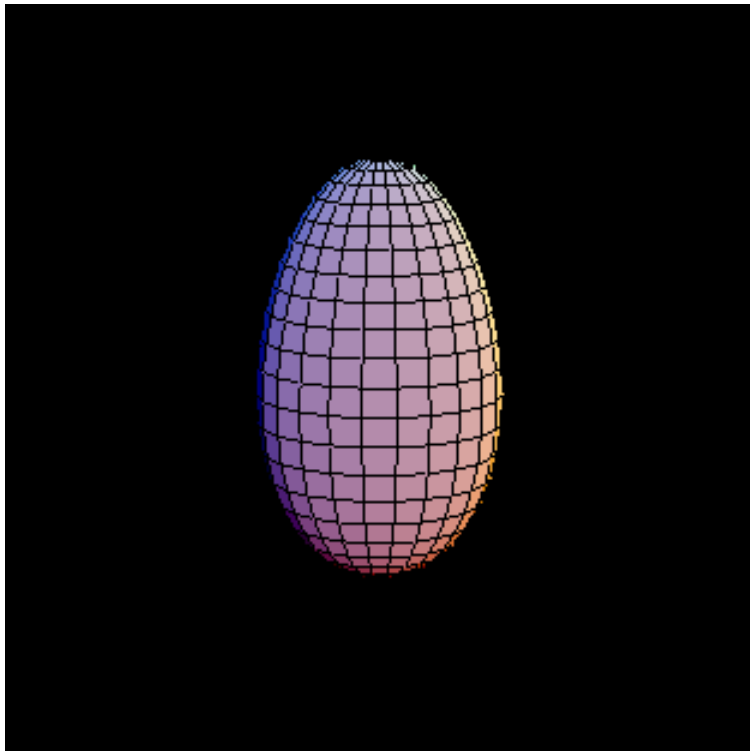
$$V = \frac{1}{2} \sum_{\lambda, \mu} C_{\lambda} |\alpha_{\lambda\mu}|^2$$

Harmonic oscillation



$\lambda=3$: Octupole vibration

Movie: Dr. K. Arita (Nagoya Tech. U.)
<http://www.phys.nitech.ac.jp/~arita/>



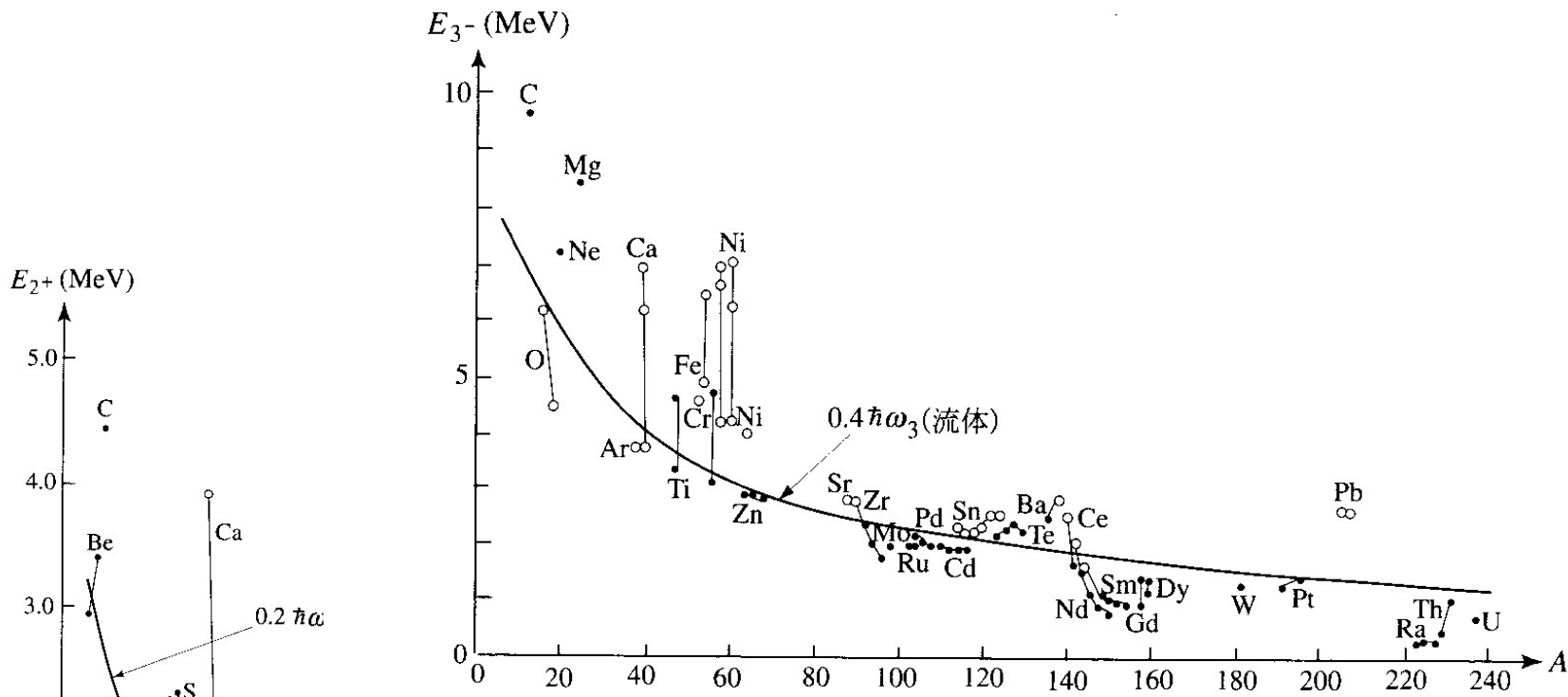


図 3.3 偶々核の第 1 励起 3^- 状態の励起エネルギー

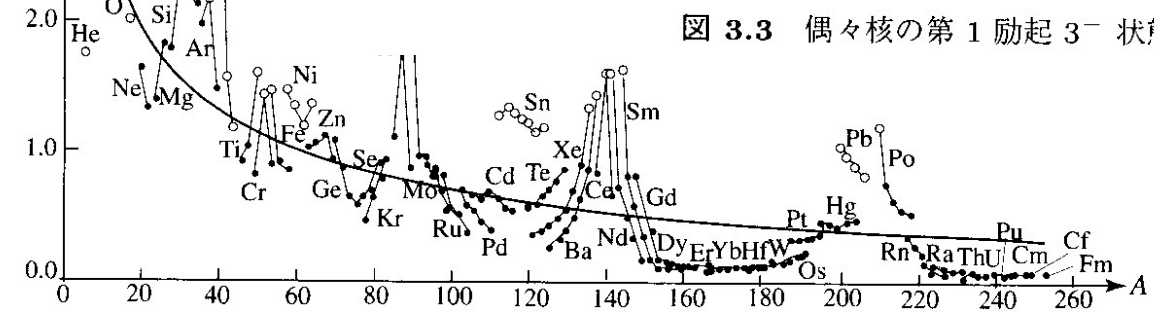


図 3.2 偶々核の第 1 励起 2^+ 状態の励起エネルギー

Double phonon states

4^+ ————— 1.282 MeV
 2^+ ————— 1.208 MeV
 0^+ ————— 1.133 MeV

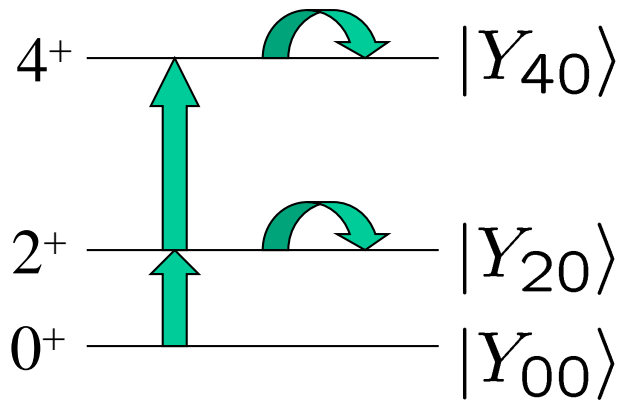
2^+ ————— 0.558 MeV

0^+ —————

^{114}Cd

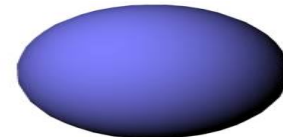
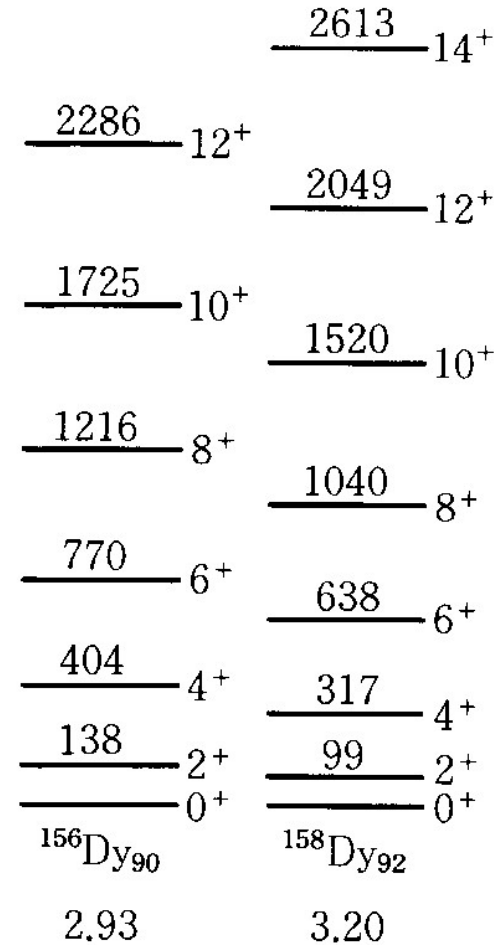
2. Nature of collective states: vibration? or rotation?

b) Rotational coupling



$$|I\rangle = |Y_{I0}\rangle$$

$$\epsilon_I = \frac{I(I+1)}{6} \cdot E_{I=2}$$




3. Coupling constants and coupling potentials

Deformed Woods-Saxon model:

$$\begin{aligned} V_{WS}(r) &= -\frac{V_0}{1 + \exp[(r - R_0)/a]} \\ &= -\frac{V_0}{1 + \exp[(r - R_P - R_T)/a]} \end{aligned}$$

$$R_T \rightarrow R_T \left(1 + \sum_{\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi) \right)$$

excitation operator


$$V_{WS}(r) = -\frac{V_0}{1 + \exp[(r - R_0 - R_T \alpha_{\lambda} \cdot Y_{\lambda}(\hat{r}))]/a]}$$

Coupling Potential: Collective Model

$$R(\theta, \phi) = R_T \left(1 + \sum_{\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi) \right)$$

➤ Vibrational case

$$\alpha_{\lambda\mu} = \frac{\beta_{\lambda}}{\sqrt{2\lambda + 1}} (a_{\lambda\mu}^{\dagger} + (-)^{\mu} a_{\lambda\mu})$$

➤ Rotational case

Coordinate transformation to the body-fixed frame

$$\alpha_{\lambda\mu} = \sqrt{\frac{4\pi}{2\lambda + 1}} \beta_{\lambda} Y_{\lambda\mu}(\theta_d, \phi_d) \quad (\text{for axial symmetry})$$

In both cases
$$\beta_{\lambda} = \frac{4\pi}{3Z_T R_T^{\lambda}} \sqrt{\frac{B(E\lambda) \uparrow}{e^2}}$$

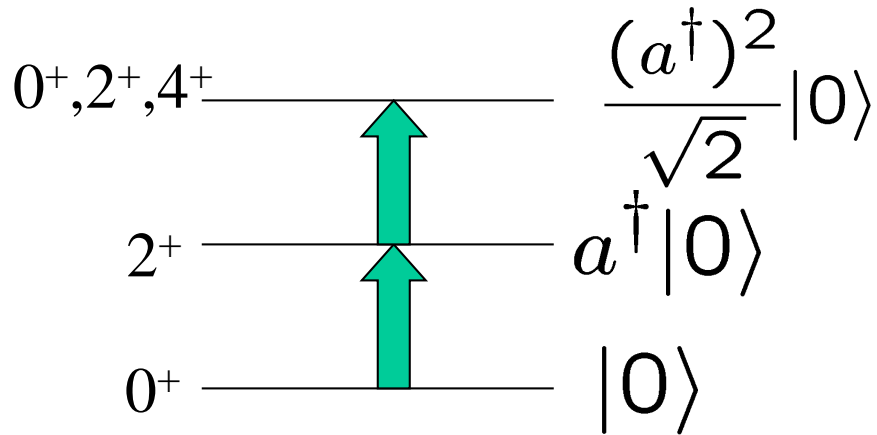
(note) coordinate transformation to the rotating frame ($\hat{r} = 0$)

$$\sum_{\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi) \rightarrow \sqrt{\frac{2\lambda + 1}{4\pi}} \alpha_{\lambda 0}$$

Coupling schemes

a) Vibrational coupling

excitation operator: $\hat{O} = \frac{\beta}{\sqrt{4\pi}}(a + a^\dagger)$



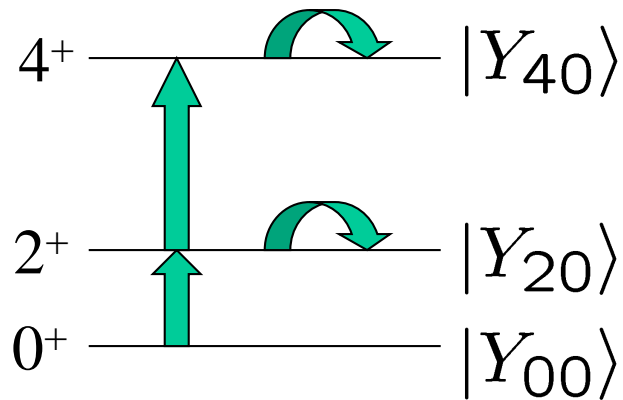
$$|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$$
$$\epsilon_n = (n + 1/2)\hbar\omega$$

$$\langle n|O|n'\rangle = \frac{\beta}{\sqrt{4\pi}} \left(\sqrt{n'} \delta_{n,n'-1} + \sqrt{n'+1} \delta_{n,n'+1} \right)$$
$$= \begin{pmatrix} 0 & F & 0 \\ F & \epsilon & \sqrt{2}F \\ 0 & \sqrt{2}F & 2\epsilon \end{pmatrix}$$

Coupling schemes

b) Rotational coupling

excitation operator: $\hat{O} = \beta Y_{20}(\theta) (+\beta_4 Y_{40}(\theta) + \dots)$

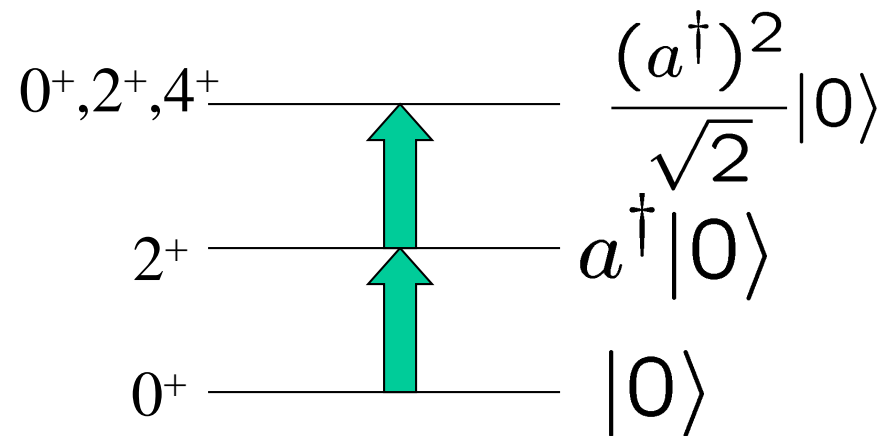


$$|I\rangle = |Y_{I0}\rangle$$
$$\epsilon_I = \frac{I(I+1)}{6} \cdot E_{I=2}$$

$$\langle I|O|I'\rangle = \sqrt{\frac{5 \cdot (2I+1)(2I'+1)}{4\pi}} \begin{pmatrix} I & 2 & I' \\ 0 & 0 & 0 \end{pmatrix}^2$$
$$= \begin{pmatrix} 0 & F & 0 \\ F & \epsilon + \frac{2\sqrt{5}}{7}F & \frac{6}{7}F \\ 0 & \frac{6}{7}F & \frac{10\epsilon}{3} + \frac{20\sqrt{5}}{77}F \end{pmatrix}$$

Vibrational coupling

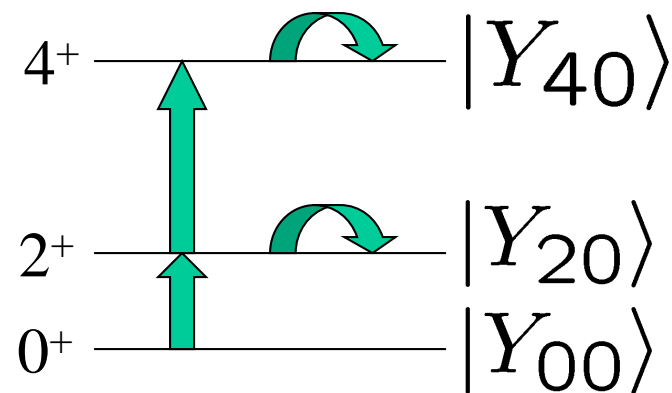
$$\hat{O} = \frac{\beta}{\sqrt{4\pi}}(a + a^\dagger)$$



$$\begin{pmatrix} 0 & F & 0 \\ F & \epsilon & \sqrt{2}F \\ 0 & \sqrt{2}F & 2\epsilon \end{pmatrix}$$

Rotational coupling

$$\hat{O} = \beta Y_{20}(\theta)$$



$$\begin{pmatrix} 0 & F & 0 \\ F & \epsilon + \frac{2\sqrt{5}}{7}F & \frac{6}{7}F \\ 0 & \frac{6}{7}F & \frac{10\epsilon}{3} + \frac{20\sqrt{5}}{77}F \end{pmatrix}$$

$$F = \frac{\beta}{\sqrt{4\pi}}$$

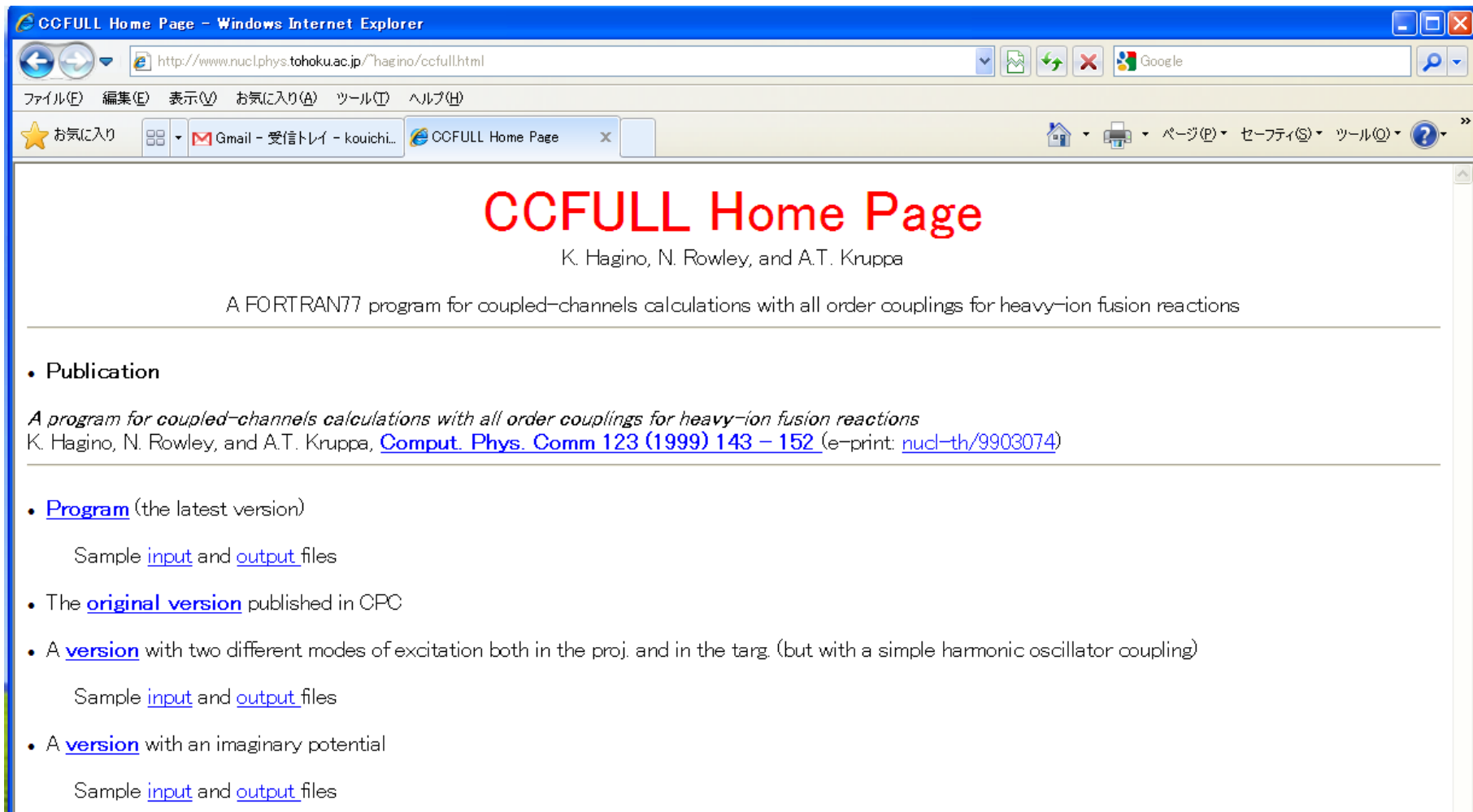
cf. reorientation term

Lecture 3

Deformed Woods-Saxon model (collective model)

CCFULL

K.H., N. Rowley, and A.T. Kruppa,
Comp. Phys. Comm. 123('99)143



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http://www.nucl.phys.tohoku.ac.jp/~hagino/ccfull.html

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K. Hagino, N. Rowley, and A.T. Kruppa

A FORTRAN77 program for coupled-channels calculations with all order couplings for heavy-ion fusion reactions

- **Publication**
A program for coupled-channels calculations with all order couplings for heavy-ion fusion reactions
K. Hagino, N. Rowley, and A.T. Kruppa, [Comput. Phys. Comm 123 \(1999\) 143 - 152](#) (e-print: [nucl-th/9903074](#))

- **Program** (the latest version)
 - Sample [input](#) and [output](#) files
- The **original version** published in CPC
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i) all order couplings

$$V_{\text{coup}}(r, \hat{O}) = V_{\text{coup}}^{(N)}(r, \hat{O}) + V_{\text{coup}}^{(C)}(r, \hat{O})$$

Nuclear coupling:

$$V_{\text{coup}}^{(N)}(r, \hat{O}) = -\frac{V_0}{1 + \exp[(r - R_0 - R_T \hat{O})/a]}$$

Coulomb coupling:

$$V_{\text{coup}}^{(C)}(r, \hat{O}) = \frac{3}{2\lambda + 1} Z_P Z_T e^2 \frac{R_T^\lambda}{r^{\lambda+1}} \hat{O}$$

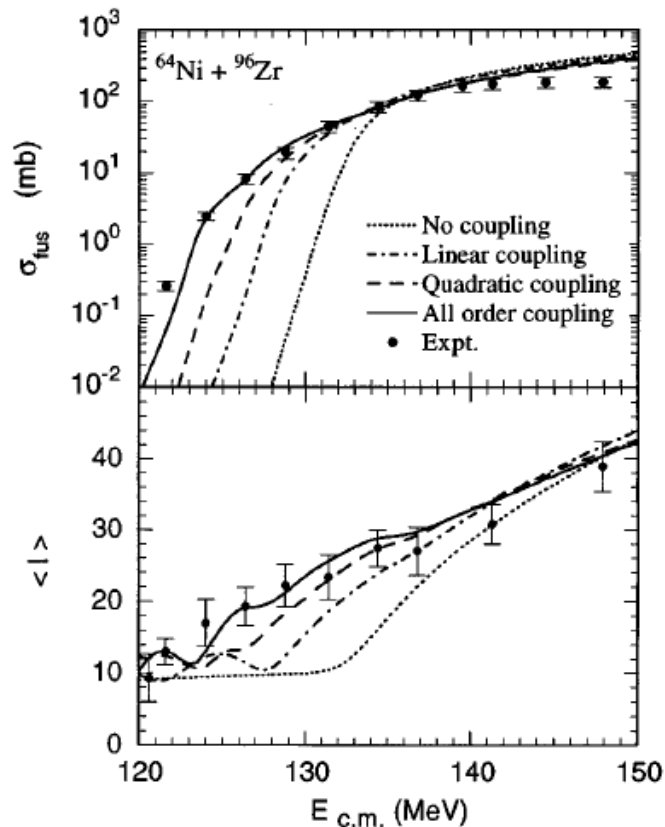
i) all order couplings

$$V_{\text{coup}}^{(N)}(r, \hat{O}) = -\frac{V_0}{1 + \exp[(r - R_0 - R_T \hat{O})/a]}$$
$$\sim V_N(r) - R_T \hat{O} \frac{dV_N(r)}{dr}$$

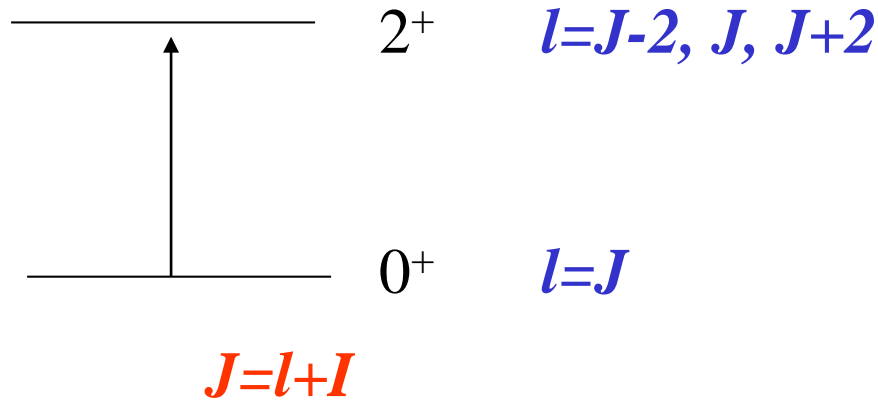
i) all order couplings

$$V_{\text{coup}}^{(N)}(r, \hat{O}) = \frac{V_0}{1 + \exp[(r - R_0 - R_T \hat{O})/a]}$$

$$\sim \cancel{V_N(r) - R_T \hat{O} \frac{dV_N(r)}{dr}}$$



K.H., N. Takigawa, M. Dasgupta,
D.J. Hinde, and J.R. Leigh, PRC55('97)276

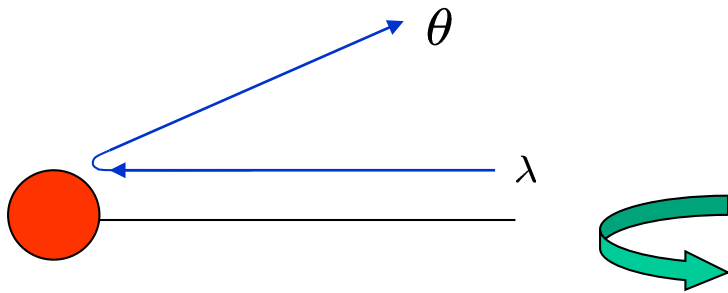
ii) isocentrifugal approximation

Truncation	Dimension
2 ⁺	4 → 2
4 ⁺	9 → 3
6 ⁺	16 → 4
8 ⁺	25 → 5

Iso-centrifugal approximation:

λ : independent of excitations

$$\frac{l(l+1)\hbar^2}{2\mu r^2} \rightarrow \frac{J(J+1)\hbar^2}{2\mu r^2}$$



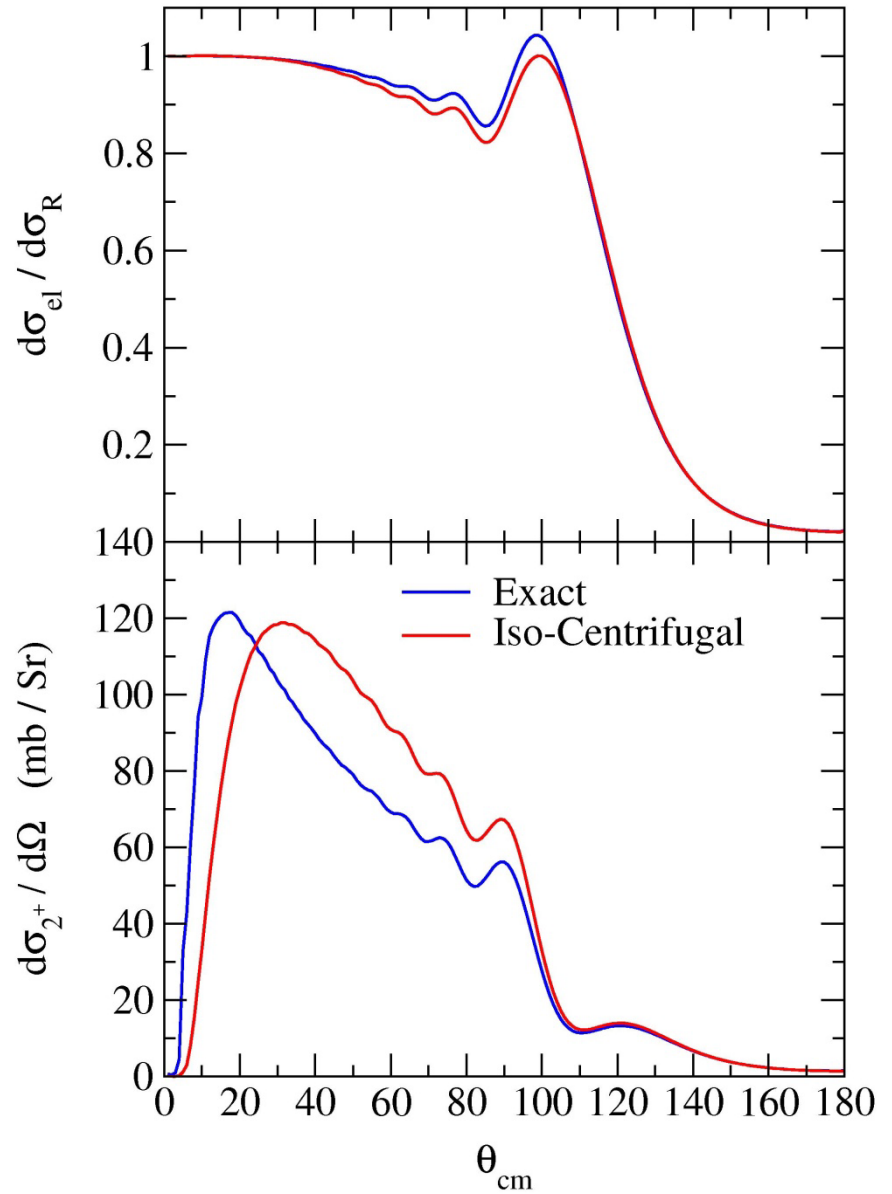
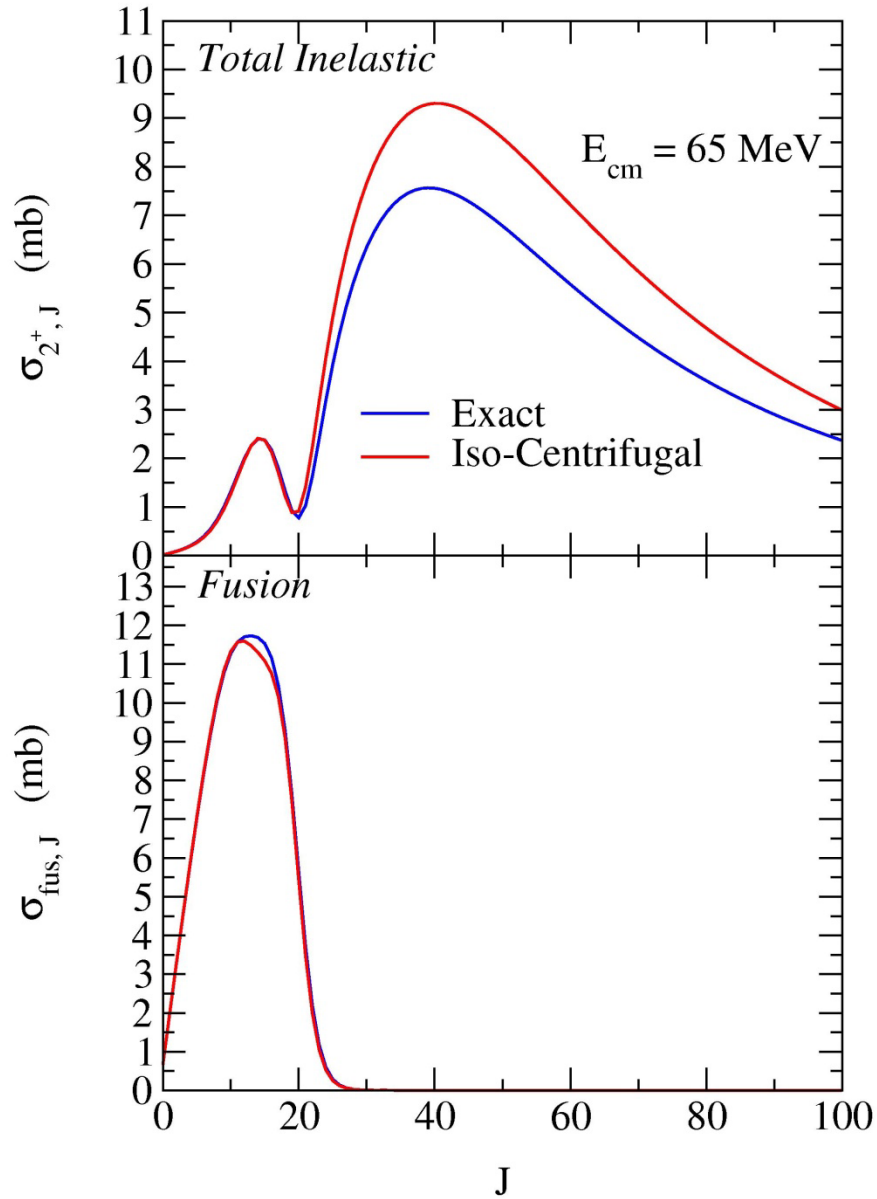
$$V_{\text{coup}}(\mathbf{r}, \xi) = f(r) Y_{\lambda}(\hat{\mathbf{r}}) \cdot T_{\lambda}(\xi)$$

transform to
the rotating frame

$$\rightarrow \sqrt{\frac{2\lambda+1}{4\pi}} f(r) T_{\lambda 0}(\xi)$$

“Spin-less system”

$^{16}\text{O} + ^{144}\text{Sm} (2^+)$



iii) incoming wave boundary condition (IWBC)

$$\sigma_{\text{fus}} = \frac{\pi}{k^2} \sum_l (2l + 1) P_l \quad (P_l = 1 - |S_l|^2)$$

(1) Complex potential

$$V(r) = V_R(r) - iW(r)$$

(2) IWBC

limit of large W (strong absorption)

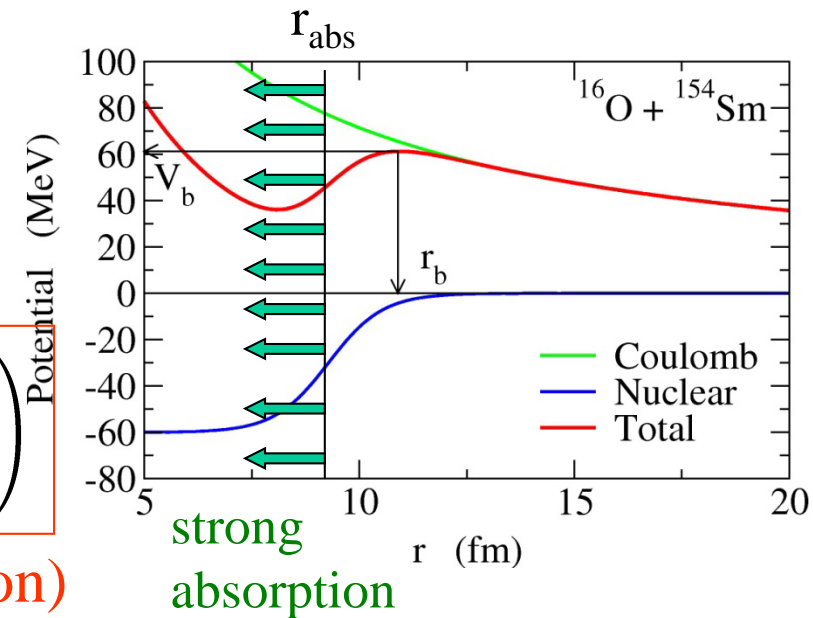
$$u_l(r) = T_l \exp \left(-i \int_{r_{\text{abs}}}^r k_l(r') dr' \right)$$

(Incoming Wave Boundary Condition)

$$k_l(r) = \sqrt{2\mu/\hbar^2 [E - V_R(r) - l(l+1)\hbar^2/2\mu r^2]}$$

- Only Real part of Potential
 - More efficient at low energies
- $$P_l = |T_l|^2$$

cf. $|S_l| \sim 1$ at low E



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16.,8.,144.,62.
1.2,-1,1.06,0
1.81,0.205,3,1
1.66,0.11,2,0
6.13,0.733,3,1
0,0.1,0.3
105.1,1.1,0.75
55.,70.,1.
30,0.05

← reaction system

($A_p=16$, $Z_p=8$, $A_t=144$, $Z_t=62$)

←

r_p , $Ivibrot_p$, r_t , $Ivibrot_t$
(inert projectile, and vib. for targ.)

16.,8.,144.,62.

← reaction system

(A_p=16, Z_p=8, A_t=144, Z_t=62)

1.2,-1,1.06,0

←

r_p, Ivibrot_p, r_t, Ivibrot_t

(inert projectile, and vib. for targ.)

1.81,0.205,3,1

1.66,0.11,2,0

$$V_{\text{coup}}^{(N)}(r, \hat{O})$$

$$= - \frac{V_0}{1 + \exp[(r - R_0 - R_T \hat{O})/a]}$$

6.13,0.733,3,1

$$R_T = r_t A_t^{1/3} \quad (\text{fm})$$

0,0.1,0.3

If Ivibrot_t = 0: O = O_{vib}Ivibrot_t = 1: O = O_{rot}Ivibrot_t = -1: O = 0 (inert)

55.,70.,1.

30,0.05

similar for the projectile

16.,8.,144.,62.

 $(A_p=16, Z_p=8, A_t=144, Z_t=62)$

1.2,-1,1.06,0

(inert projectile, and vib. for targ.)

1.81,0.205,3,1

properties of the targ. excitation

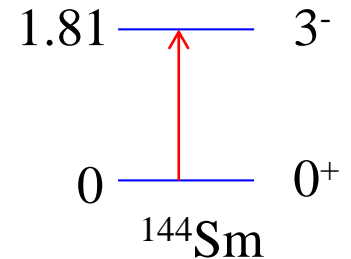
1.66,0.11,2,0

$$E_{1st} = 1.81 \text{ MeV}$$

$$\beta = 0.205$$

$$\lambda = 3$$

$$N_{\text{phonon}} = 1$$



6.13,0.733,3,1

0,0.,0.3

coupling to 3^- vibrational state in
the target with def. parameter $\beta = 0.205$

105.1,1.1,0.75

55.,70.,1.

$$\alpha_{\lambda\mu} = \frac{\beta_\lambda}{\sqrt{2\lambda + 1}} (a_{\lambda\mu}^\dagger + (-)^\mu a_{\lambda\mu})$$

30,0.05

$$\beta_\lambda = \frac{4\pi}{3Z_T R_T^\lambda} \sqrt{\frac{B(E\lambda) \uparrow}{e^2}}$$

16.,8.,144.,62.
 1.2,-1,1.06,0
 1.81,0.205,3,1
 1.66,0.11,2,0
 6.13,0.733,3,1
 0,0.1,0.3
 105.1,1.1,0.75
 55.,70.,1.
 30,0.05

$(A_p=16, Z_p=8, A_t=144, Z_t=62)$

$(\text{inert projectile, and vib. for targ.})$

properties of the targ. excitation

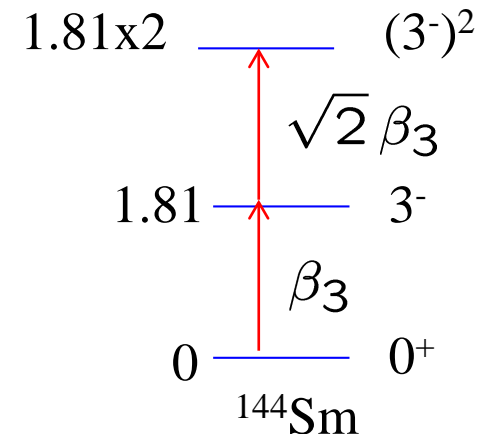
(note) if $N_{\text{phonon}} = 2$: double phonon excitation


$$E_{1\text{st}} = 1.81 \text{ MeV}$$

$$\beta = 0.205$$

$$\lambda = 3$$

$$N_{\text{phonon}} = 2$$



16.,8.,144.,62.	← (A _p =16, Z _p =8, A _t =144, Z _t =62)
1.2,-1,1.06,0	← (inert projectile, and vib. for targ.)
1.81,0.205,3,1	← properties of the targ. excitation
1.66,0.11,2,0	(note) if Ivibrott = 1 (rot. coup.)
6.13,0.733,3,1	the input line would look like:
0,0.1,0.3	0.08,0.306,0.05,3 instead of 1.81,0.205,3,1
105.1,1.1,0.75	
55.,70.,1.	E_{2+} β_2 β_4 N_{rot}
30,0.05	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> 3 excited states ($N_{rot}=3$) + g.s. </div> <div style="font-size: 3em; margin-right: 10px;">}</div> <div style="display: flex; flex-direction: column; align-items: center;"> <div style="margin-bottom: 5px;">$6 \times 7 \times 0.08 / 6$ ——— 6^+</div> <div style="margin-bottom: 5px;">$4 \times 5 \times 0.08 / 6$ ——— 4^+</div> <div style="margin-bottom: 5px;">0.08 ——— 2^+</div> <div style="margin-bottom: 5px;">0 ——— 0^+</div> </div> </div>

16.,8.,144.,62.	← (A _p =16, Z _p =8, A _t =144, Z _t =62)
1.2,-1,1.06,0	← (inert projectile, and vib. for targ.)
1.81,0.205,3,1	← properties of the targ. excitation
1.66,0.11,2,0	← same as the previous line, but the second mode of excitation in the target nucleus (vibrational coupling only)
6.13,0.733,3,1	
0,0.1,0.3	
	N _{phonon} = 0 → no second mode
105.1,1.1,0.75	
55.,70.,1.	
30,0.05	

16.,8.,144.,62.	←	$(A_p=16, Z_p=8, A_t=144, Z_t=62)$
1.2,-1,1.06,0	←	(inert projectile, and vib. for targ.)
1.81,0.205,3,1	←	properties of the targ. excitation
1.66,0.11,2,1	←	second mode in the targ.
6.13,0.733,3,1		(note) if $N_{\text{phonon}} = 1$: the code will ask you while you run it whether your coupling scheme is (a) or (b)
0,0.1,0.3		
105.1,1.1,0.75		
55.,70.,1.		
30,0.05		

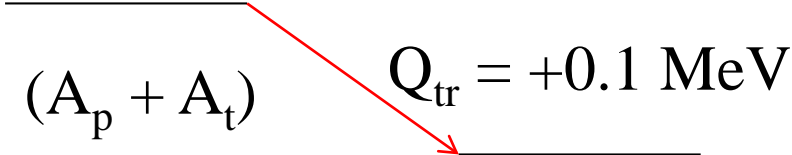
(a)

^{144}Sm

(b)

^{144}Sm

16.,8.,144.,62.	←	$(A_p=16, Z_p=8, A_t=144, Z_t=62)$
1.2,-1,1.06,0	←	(inert projectile, and vib. for targ.)
1.81,0.205,3,1	←	properties of the targ. excitation
1.66,0.11,2,1	←	second mode in the targ.
6.13,0.733,3,1	←	properties of the proj. excitation (similar as the third line)
0,0.1,0.3		(will be skipped for an inert projectile)
105.1,1.1,0.75		
55.,70.,1.		
30,0.05		

16.,8.,144.,62.	←	$(A_p=16, Z_p=8, A_t=144, Z_t=62)$
1.2,-1,1.06,0	←	(inert projectile, and vib. for targ.)
1.81,0.205,3,1	←	properties of the targ. excitation
1.66,0.11,2,1	←	second mode in the targ.
6.13,0.733,3,1	←	properties of the proj. excitation (similar as the third line)
0,0.1,0.3	←	transfer coupling (g.s. to g.s.)
105.1,1.1,0.75		
55.,70.,1.		$F_{tr}(r) = F \frac{dV_N}{dr}$
30,0.05		* no transfer coup. for $F = 0$

16.,8.,144.,62.	←	$(A_p=16, Z_p=8, A_t=144, Z_t=62)$
1.2,-1,1.06,0	←	(inert projectile, and vib. for targ.)
1.81,0.205,3,1	←	properties of the targ. excitation
1.66,0.11,2,1	←	second mode in the targ.
6.13,0.733,3,1	←	properties of the proj. excitation (similar as the third line)
0,0.1,0.3	←	transfer coupling (g.s. to g.s.)
105.1,1.1,0.75	←	potential parameters
55.,70.,1.		$V_N(r) = -\frac{V_0}{1 + \exp[(r - R_0)/a]}$
30,0.05		$V_0 = 105.1 \text{ MeV}, a = 0.75 \text{ fm}$ $R_0 = 1.1 * (A_p^{1/3} + A_t^{1/3}) \text{ fm}$

16.,8.,144.,62.	←	$(A_p=16, Z_p=8, A_t=144, Z_t=62)$
1.2,-1,1.06,0	←	(inert projectile, and vib. for targ.)
1.81,0.205,3,1	←	properties of the targ. excitation
1.66,0.11,2,1	←	second mode in the targ.
6.13,0.733,3,1	←	properties of the proj. excitation (similar as the third line)
0,0.1,0.3	←	transfer coupling (g.s. to g.s.)
105.1,1.1,0.75	←	potential parameters
55.,70.,1.	←	$E_{\min}, E_{\max}, \Delta E$ (c.m. energies)
30,0.05	←	$R_{\max}, \Delta r$

ccfull.inp

OUTPUT

```

16.,8.,144.,62.
1.2,-1,1.06,0
1.81,0.205,3,1
1.66,0.11,2,1
6.13,0.733,3,1
0,0.1,0.3
105.1,1.1,0.75
55.,70.,1.
30,0.05

```

16O + 144Sm Fusion reaction

Phonon Excitation in the targ.: beta_N= 0.205, beta_C= 0.205,
r0= 1.06(fm), omega= 1.81(MeV), Lambda= 3, Nph= 1

Potential parameters: V0= 105.10(MeV), r0= 1.10(fm),
a= 0.75(fm),power= 1.00

Uncoupled barrier: Rb=10.82(fm), Vb= 61.25(MeV),
Curv=4.25(MeV)

Ecm (MeV)	sigma (mb)	<l>
55.00000	0.97449E-02	5.87031
56.00000	0.05489	5.94333
57.00000	0.28583	6.05134
58.00000	1.36500	6.19272
59.00000	5.84375	6.40451
.....		
69.00000	427.60179	17.16365
70.00000	472.46037	18.08247

In addition, "cross.dat" : fusion cross sections only

Coupled-channels equations and barrier distribution

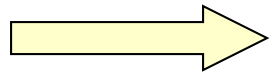
$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_0(r) - E + \epsilon_n \right] u_n(r) + \sum_{n'} \langle \phi_n | V_{\text{coup}}(r, \xi) | \phi_{n'} \rangle u_{n'}(r) = 0$$

$$u_n(r) \rightarrow H_J^{(-)}(k_n r) \delta_{n, n_i} - \sqrt{\frac{k_0}{k_n}} S_n H_J^{(+)}(k_n r)$$

$$P_J(E) = 1 - \sum_n |S_n|^2$$

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2} \sum_J (2J+1) P_J(E)$$

Calculate σ_{fus} by numerically solving the coupled-channels equations

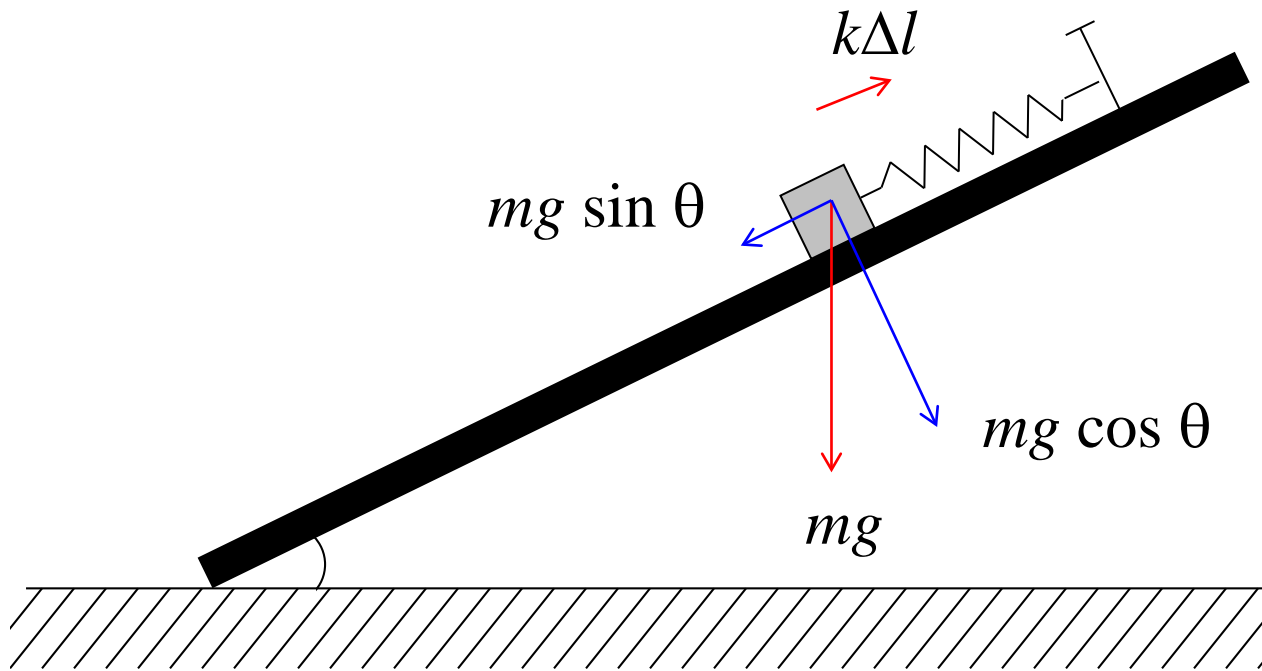


Let us consider a limiting case in order to understand (interpret) the numerical results

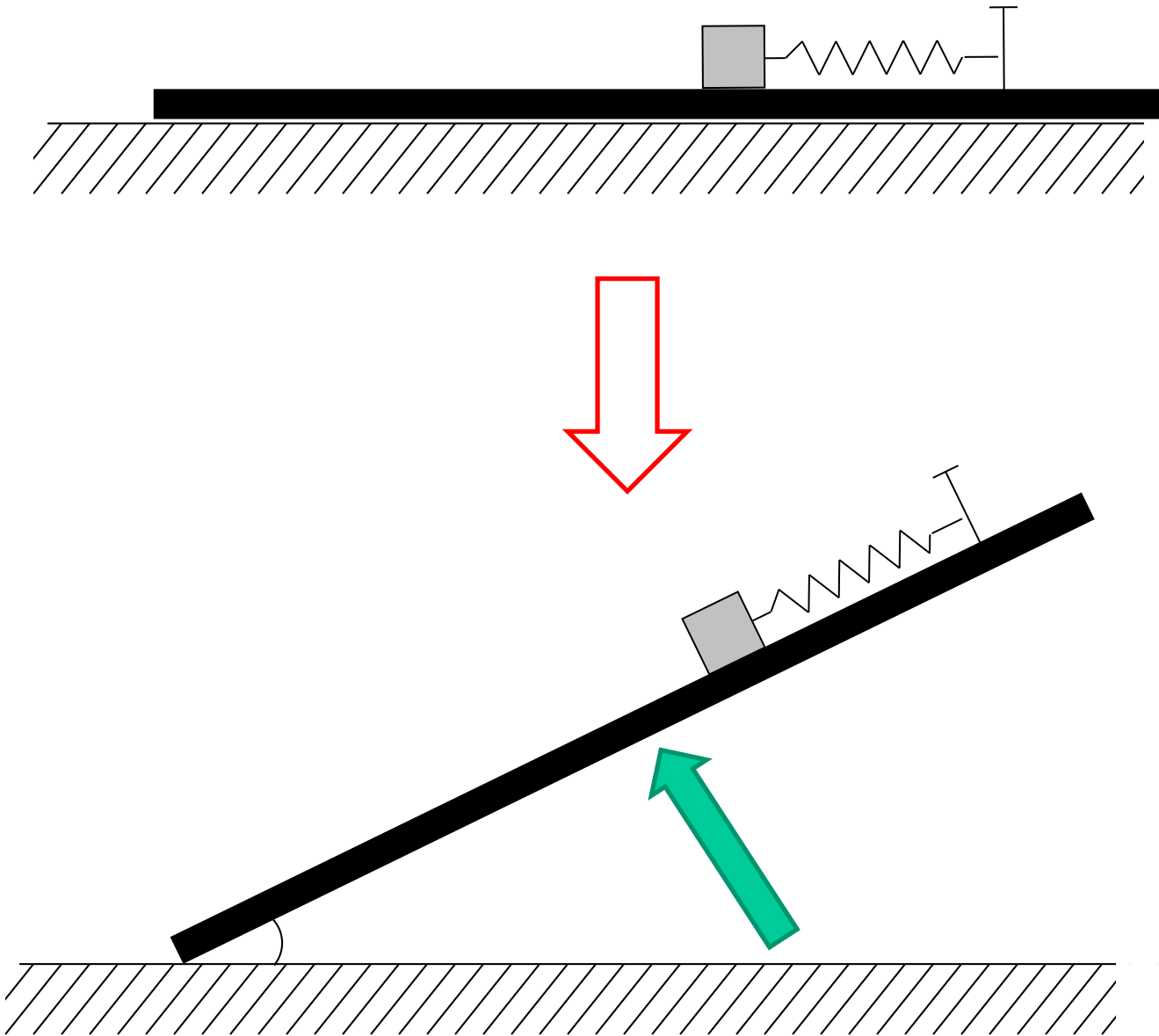
- ϵ_{nI} : very large *Adiabatic limit*
- $\epsilon_{nI} = 0$ *Sudden limit*

Comparison of two time scales

a spring on a board



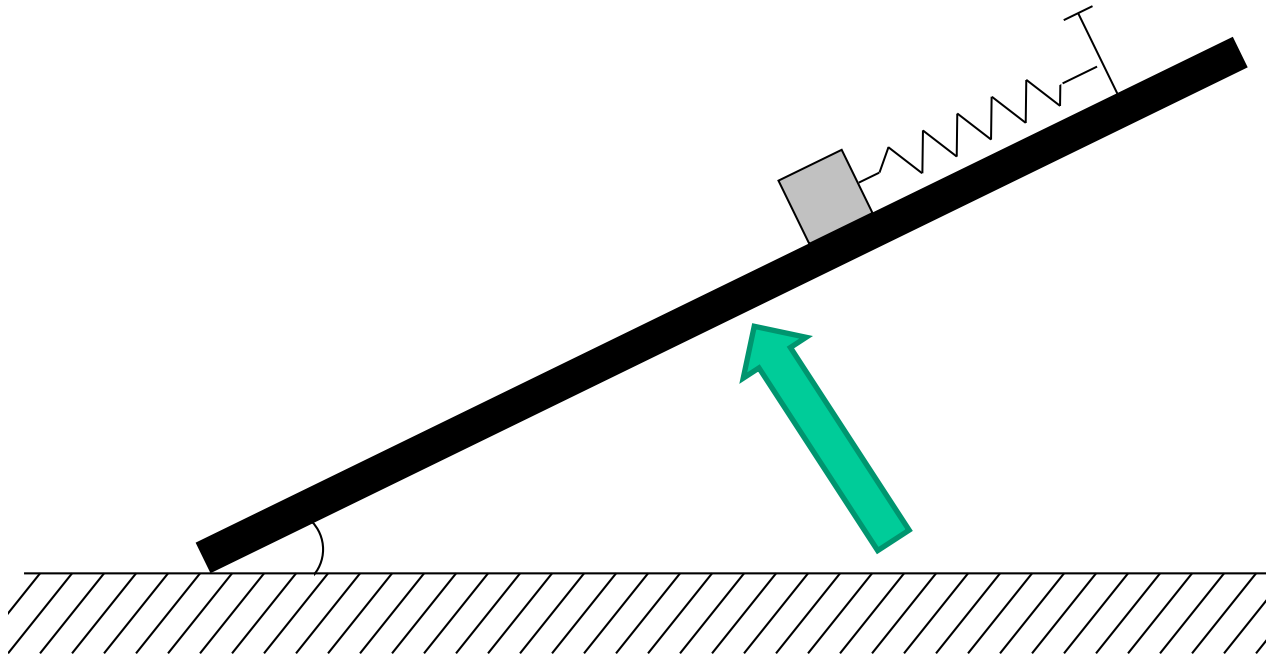
static case: $mg \sin \theta = k\Delta l \quad \rightarrow \quad \Delta l = mg \sin \theta / k$



move very **slowly?** or move **instantaneously?**

Comparison of two time scales

similar related example: spring on a moving board



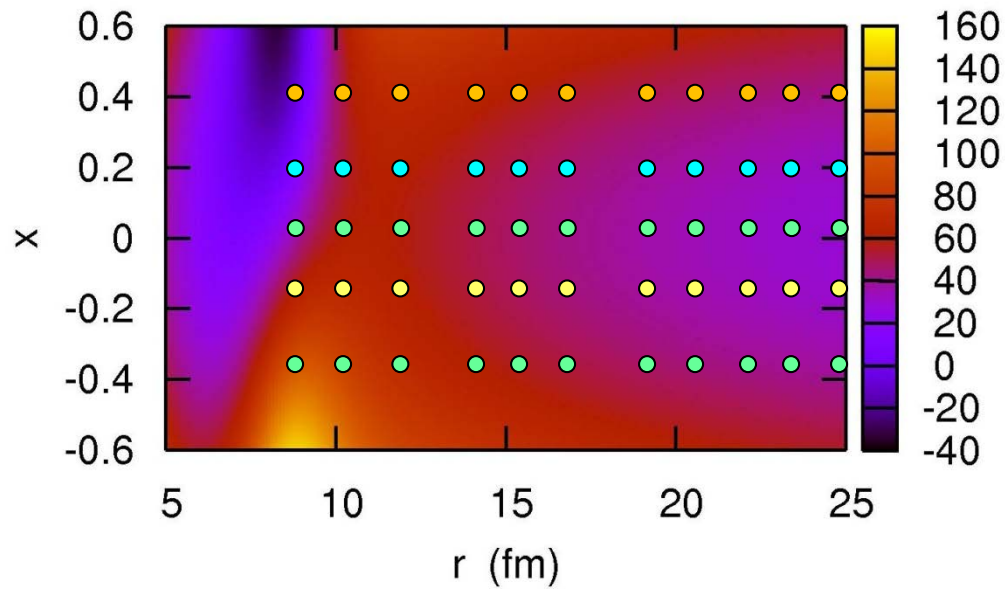
move very **slowly?** or move **instantaneously?**



keep the original length ($\Delta l = 0$) “sudden limit”

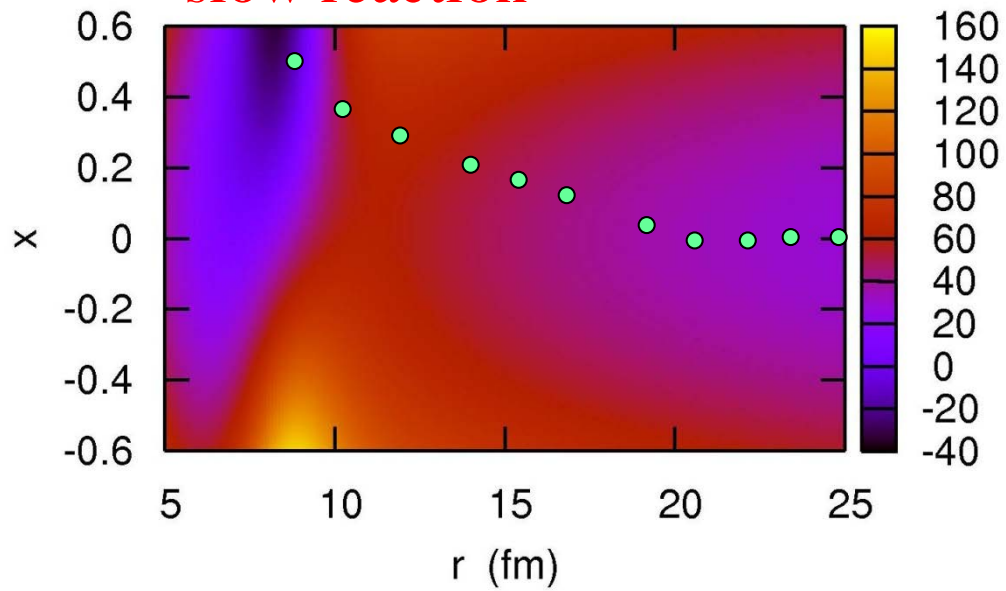
always at the equilibrium length ($\Delta l = mg \sin \theta / k$) “adiabatic limit”

fast reaction



large fluctuation

slow reaction



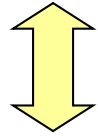
+ small fluctuation
around the adiabatic path

relative distance

Two limiting cases: (i) adiabatic limit

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\text{coup}}(\mathbf{r}, \xi)$$

much slower rel. motion than the intrinsic motion



much larger energy scale for intrinsic motion than the typical energy scale for the rel. motion

$$\hbar\Omega \ll \epsilon$$

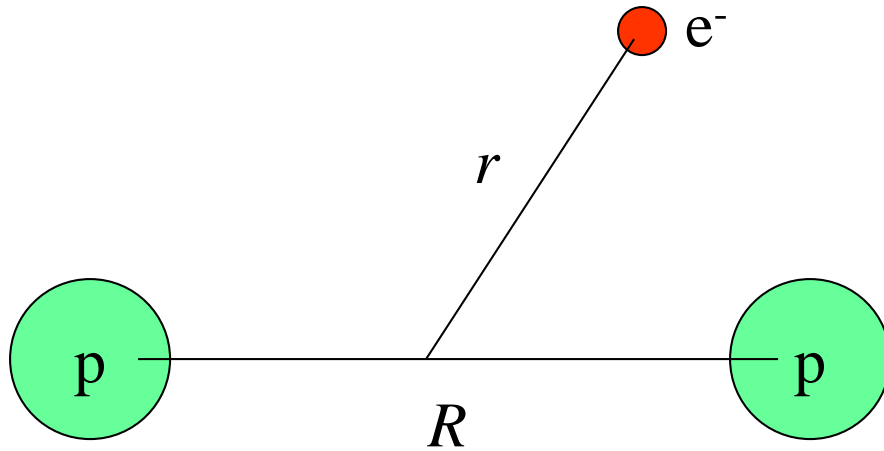
(Barrier curvature v.s. Intrinsic excitation energy)


$$[H_0(\xi) + V_{\text{coup}}(\mathbf{r}, \xi)]\varphi_0(\xi; \mathbf{r}) = \epsilon_0(r) \varphi_0(\xi; \mathbf{r})$$



$$H_0(\xi) + V_{\text{coup}}(\mathbf{r}, \xi) \rightarrow \epsilon_0(r)$$

c.f. Born-Oppenheimer approximation for hydrogen molecule



$$[T_R + T_r + V(r, R)]\Psi(r, R) = E\Psi(r, R)$$

1. Consider first the electron motion for a fixed R

$$[T_r + V(r, R)]u_n(r; R) = \epsilon_n(R)u_n(r; R)$$

2. Minimize $\epsilon_n(R)$ with respect to R

Or 2'. Consider the proton motion in a potential $\epsilon_n(R)$

$$[T_R + \epsilon_n(R)]\phi_n(R) = E\phi_n(R)$$

Adiabatic Potential Renormalization

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\text{coup}}(\mathbf{r}, \xi)$$

When ε is large,

$$H_0(\xi) + V_{\text{coup}}(\mathbf{r}, \xi) \rightarrow \epsilon_0(r)$$

where

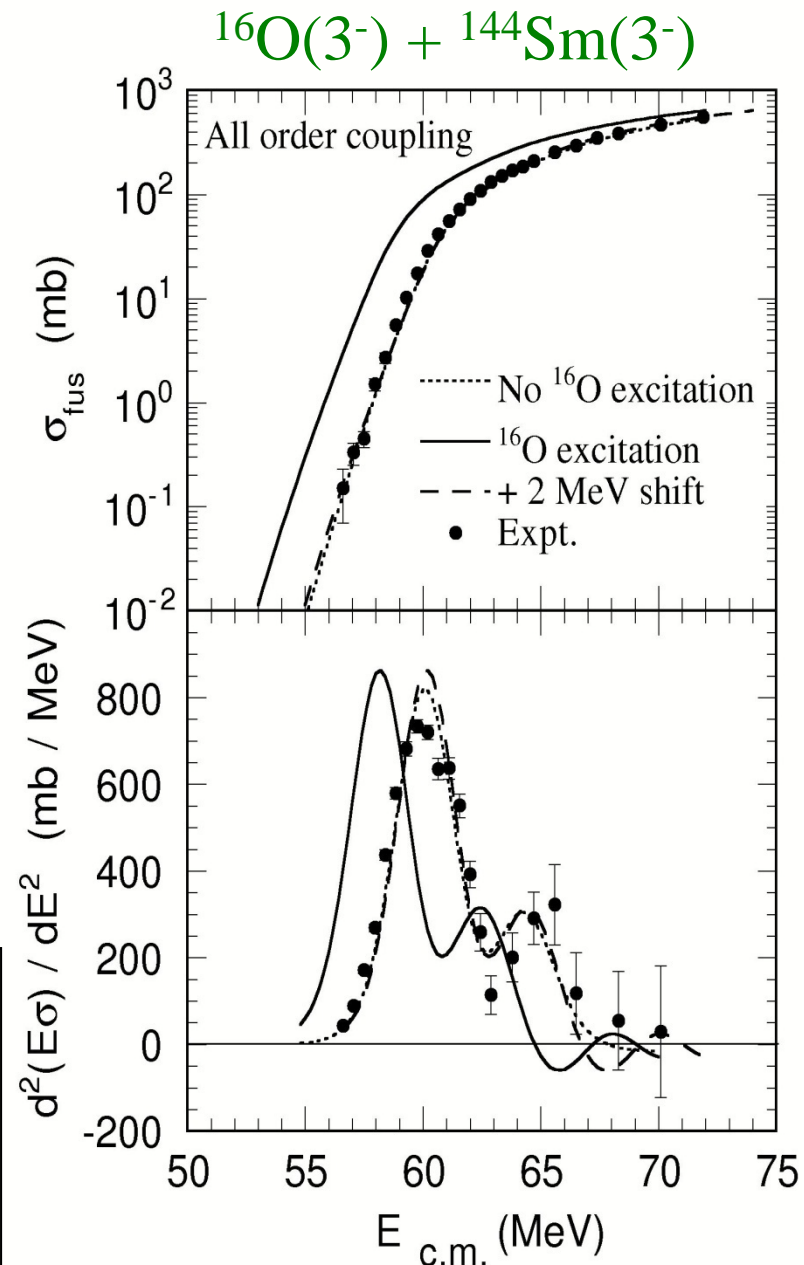
$$\begin{aligned} [H_0(\xi) + V_{\text{coup}}(\mathbf{r}, \xi)]\varphi_0(\xi; \mathbf{r}) \\ = \epsilon_0(r) \varphi_0(\xi; \mathbf{r}) \end{aligned}$$

Fast intrinsic motion

→ Adiabatic potential renormalization

$$V_{\text{ad}}(r) = V_0(r) + \epsilon_0(r)$$

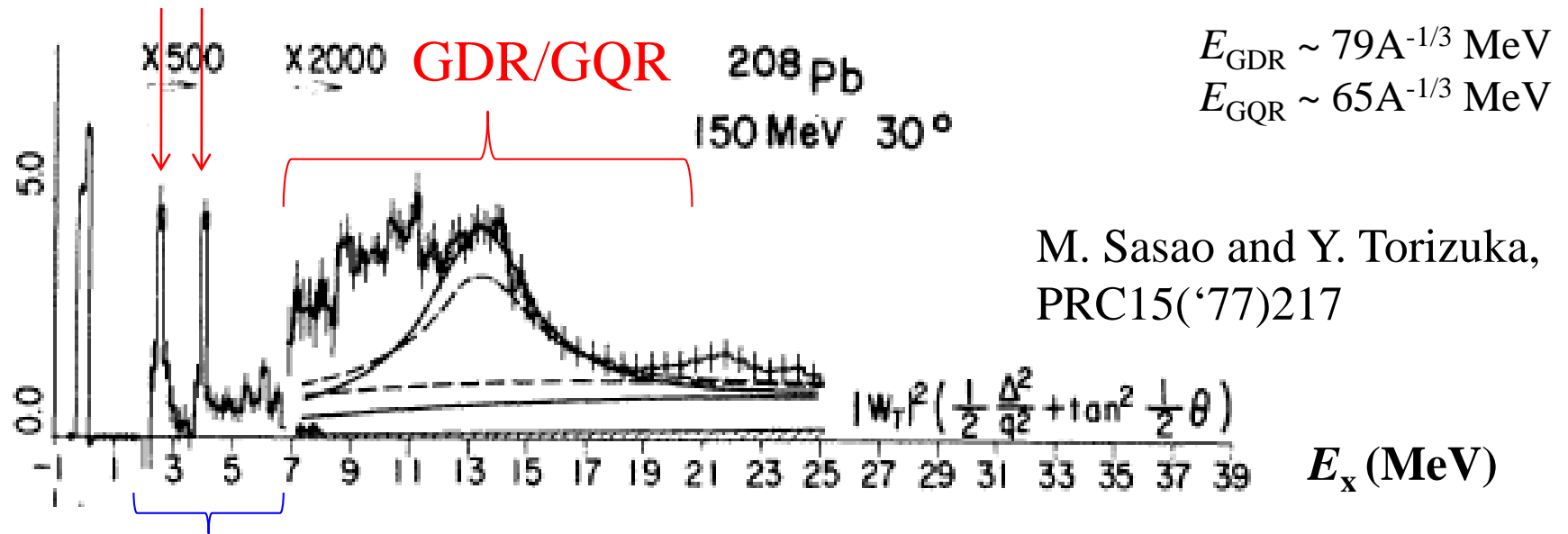
Giant Resonances, $^{16}\text{O}(3^-)$ [6.31 MeV]



K.H., N. Takigawa, M. Dasgupta,
D.J. Hinde, J.R. Leigh, PRL79('99)2014

typical excitation spectrum: electron scattering data

low-lying collective excitations



low-lying non-collective excitations

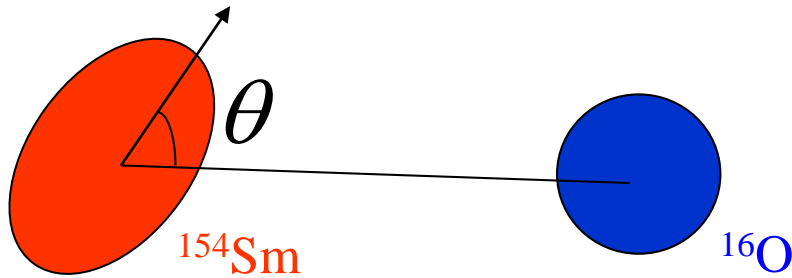
- Giant Resonances: high E_x , smooth mass number dependence
→ adiabatic potential renormalization
- Low-lying collective excitations: barrier distributions,
strong isotope dependence
- Non-collective excitations: either neglected completely or
implicitly treated through an absorptive potential

Two limiting cases: (ii) sudden limit

$$\epsilon \rightarrow 0$$

$$\epsilon_I = I(I + 1)\hbar^2/2\mathcal{J}$$

$$\mathcal{J} \rightarrow \infty$$




$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$

Coupled-channels:

$$\begin{pmatrix} 0 & f(r) & 0 \\ f(r) & \frac{2\sqrt{5}}{7}f(r) & \frac{6}{7}f(r) \\ 0 & \frac{6}{7}f(r) & \frac{20\sqrt{5}}{77}f(r) \end{pmatrix} \xrightarrow{\text{diagonalize}} \begin{pmatrix} \lambda_1(r) & 0 & 0 \\ 0 & \lambda_2(r) & 0 \\ 0 & 0 & \lambda_3(r) \end{pmatrix}$$

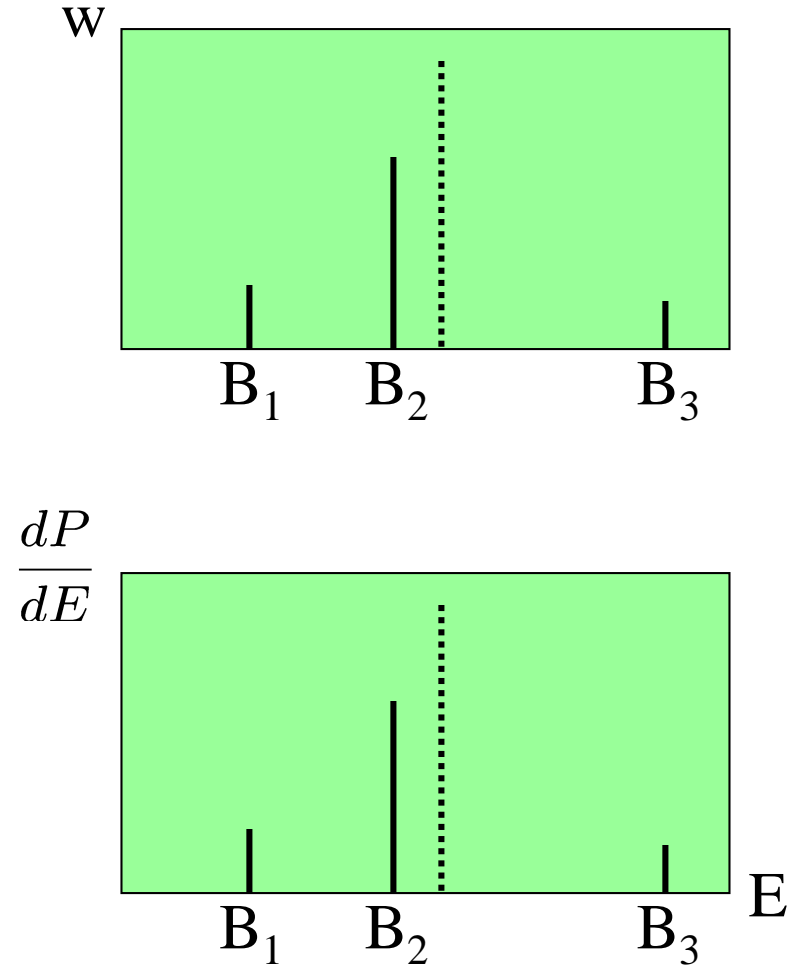
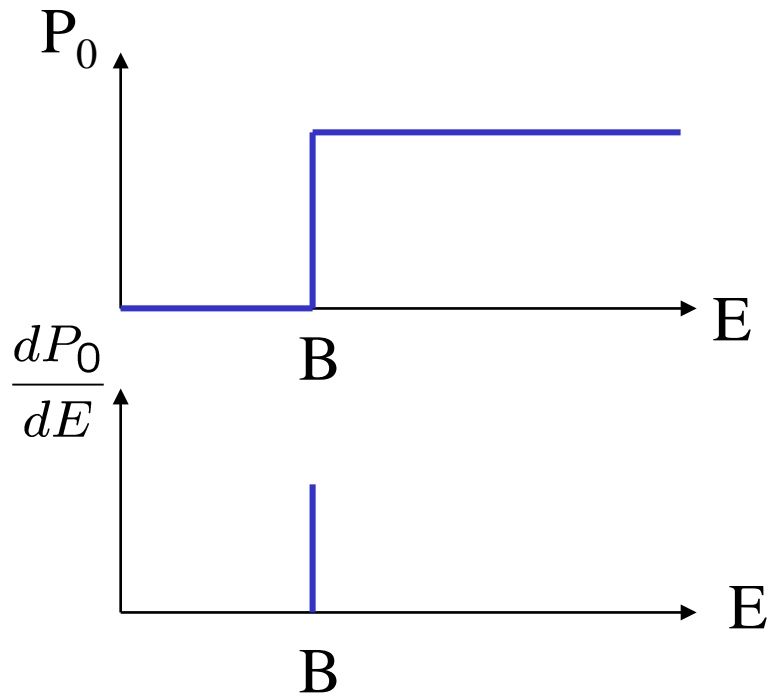
$$\Rightarrow P(E) = \sum_i w_i P(E; V_0(r) + \lambda_i(r))$$

Slow intrinsic motion

 Barrier Distribution

Barrier distribution

$$P(E) = \sum_i w_i P(E; V_0(r) + \lambda_i(r))$$

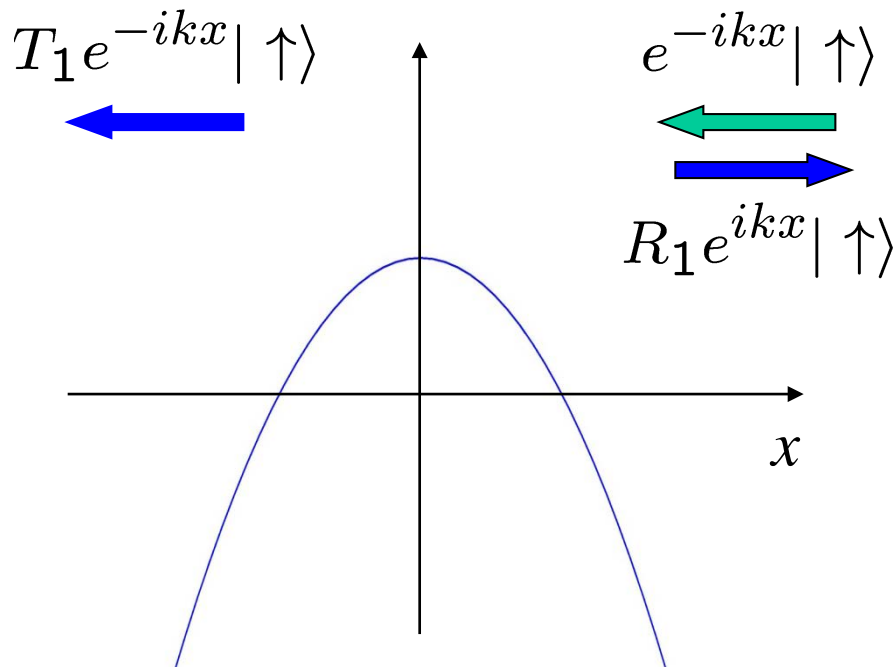


Barrier distribution: understand the concept using a spin Hamiltonian

Hamiltonian (example 1):
$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V_0(x) + \hat{\sigma}_z \cdot V_s(x)$$

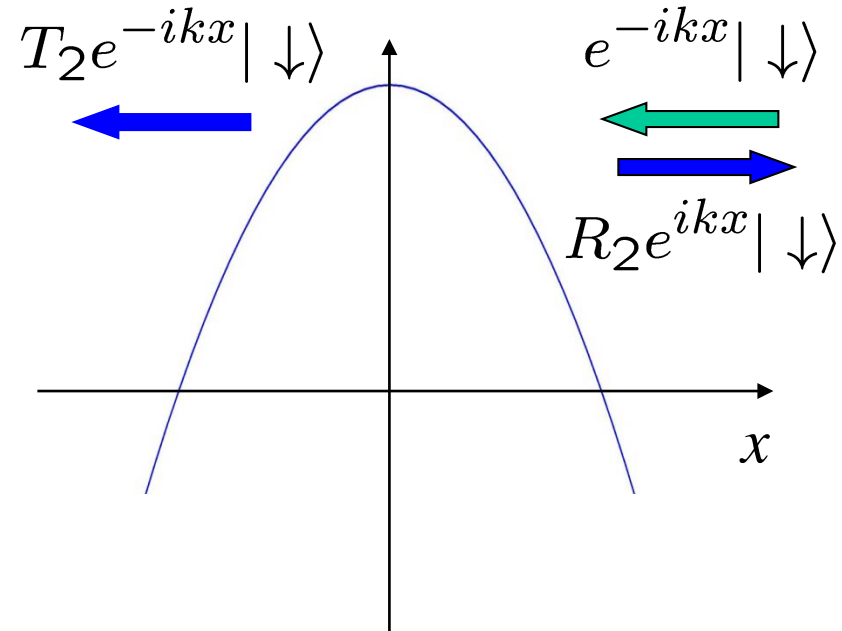
$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For Spin-up



$$V_1(x) = V_0(x) + V_s(x)$$

For Spin-down



$$V_2(x) = V_0(x) - V_s(x)$$

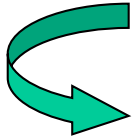
$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V_0(x) + \hat{\sigma}_z \cdot V_1(x)$$



Wave function
(general form)

$$\begin{aligned} \Psi(x) &= \psi_1(x) |\uparrow\rangle + \psi_2(x) |\downarrow\rangle \\ &= \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} \end{aligned}$$

The spin direction does not change during tunneling:



$$P(E) = w_{\uparrow} P_1(E) + w_{\downarrow} P_2(E)$$

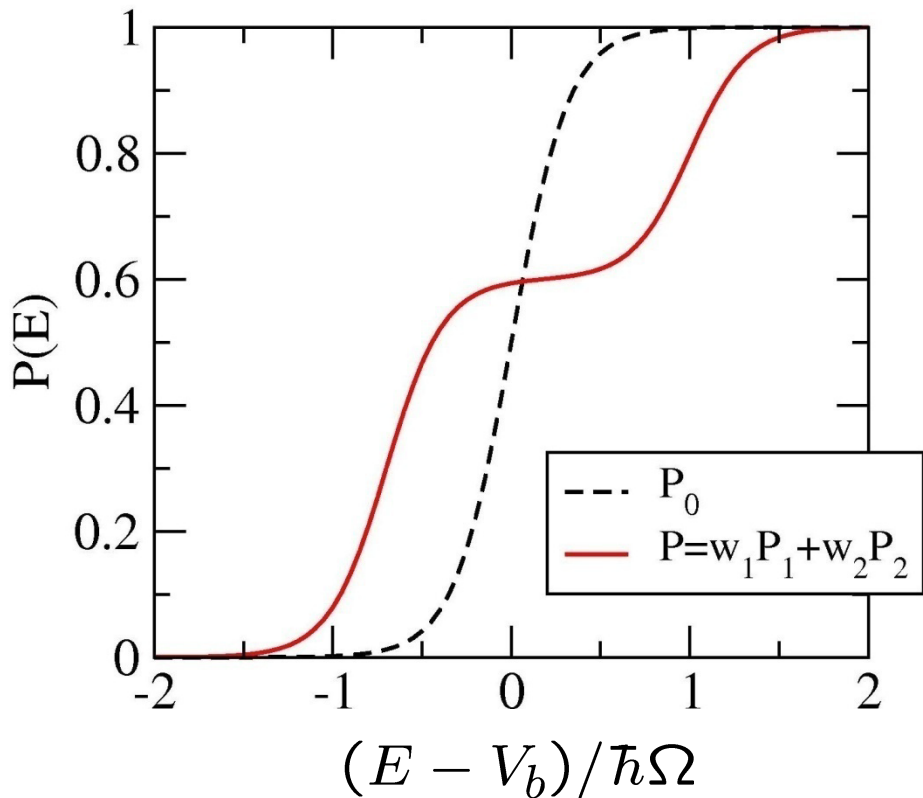
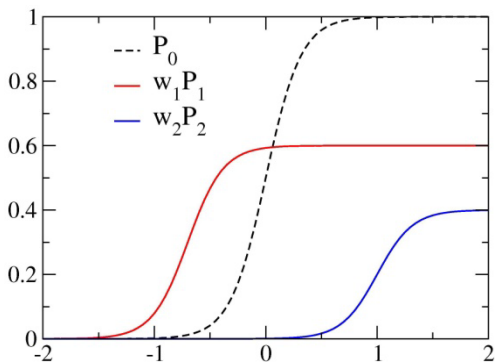
$$w_{\uparrow} + w_{\downarrow} = 1$$

$$P(E) = w_{\uparrow}P_1(E) + w_{\downarrow}P_2(E)$$

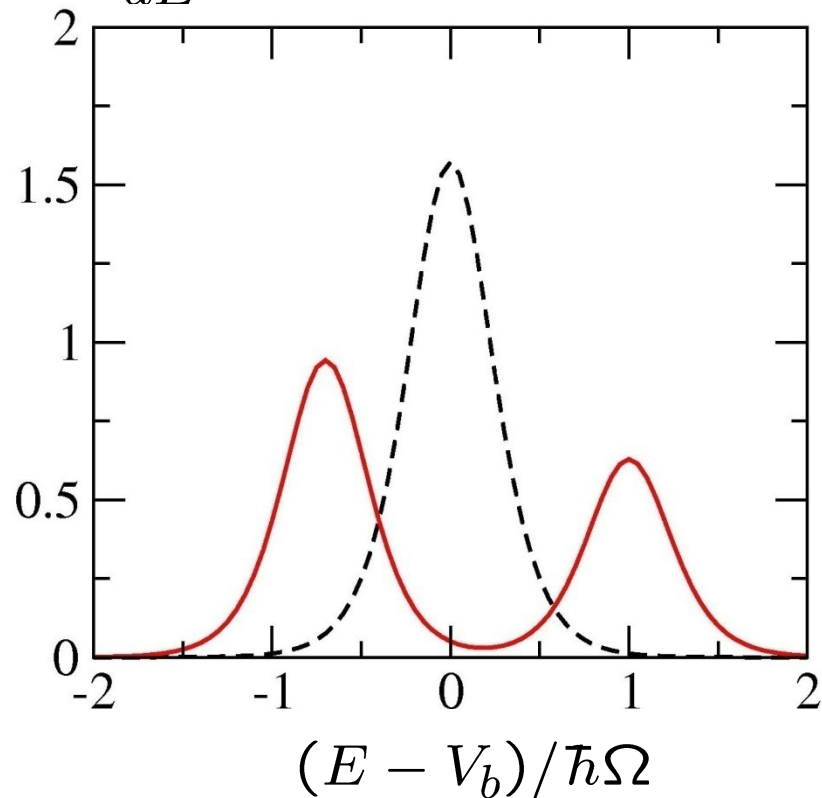


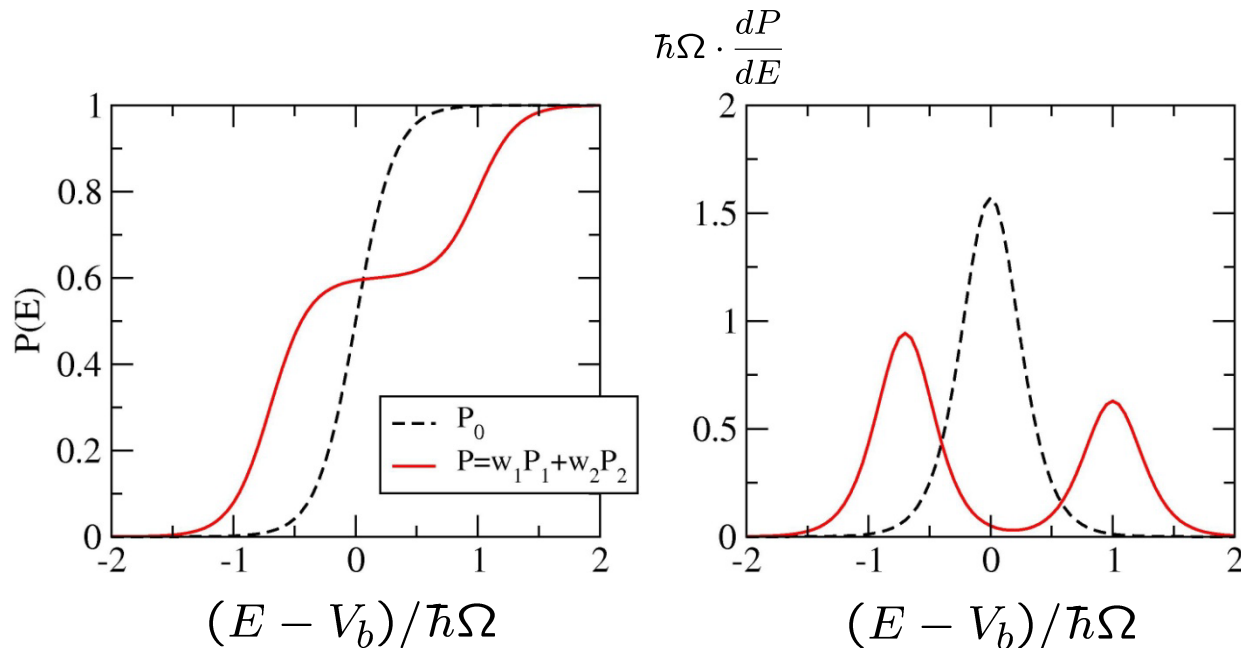
Tunneling prob. is a weighted sum of tunnel prob. for two barriers

$$\left\{ \begin{array}{l} V_1(x) = V_0(x) + V_s(x) \quad \leftarrow \text{ } | \uparrow \rangle \\ V_2(x) = V_0(x) - V_s(x) \quad \leftarrow \text{ } | \downarrow \rangle \end{array} \right.$$



$$\hbar\Omega \cdot \frac{dP}{dE}$$





- Tunnel prob. is enhanced at $E < V_b$ and hindered $E > V_b$
- dP/dE splits to two peaks \longrightarrow “barrier distribution”
- The peak positions of dP/dE correspond to each barrier height
- The height of each peak is proportional to the weight factor

$$P(E) = w_{\uparrow} P_1(E) + w_{\downarrow} P_2(E)$$


$$\frac{dP}{dE} = w_{\uparrow} \frac{dP_1}{dE} + w_{\downarrow} \frac{dP_2}{dE}$$

simple 2-level model (Dasso, Landowne, and Winther, NPA405('83)381)

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + V_l(r) + \begin{pmatrix} 0 & F \\ F & \epsilon \end{pmatrix} - E \right] \begin{pmatrix} u_0(r) \\ u_1(r) \end{pmatrix} = 0$$

entrance channel

excited channel



$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + V_l(r) + \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} - E \right] \begin{pmatrix} \phi_0(r) \\ \phi_1(r) \end{pmatrix} = 0$$

$$\begin{cases} \phi_0(r) = \alpha \cdot u_0(r) + \beta \cdot u_1(r) \\ \phi_1(r) = \beta \cdot u_0(r) - \alpha \cdot u_1(r) \end{cases}$$

simple 2-level model (Dasso, Landowne, and Winther, NPA405('83)381)

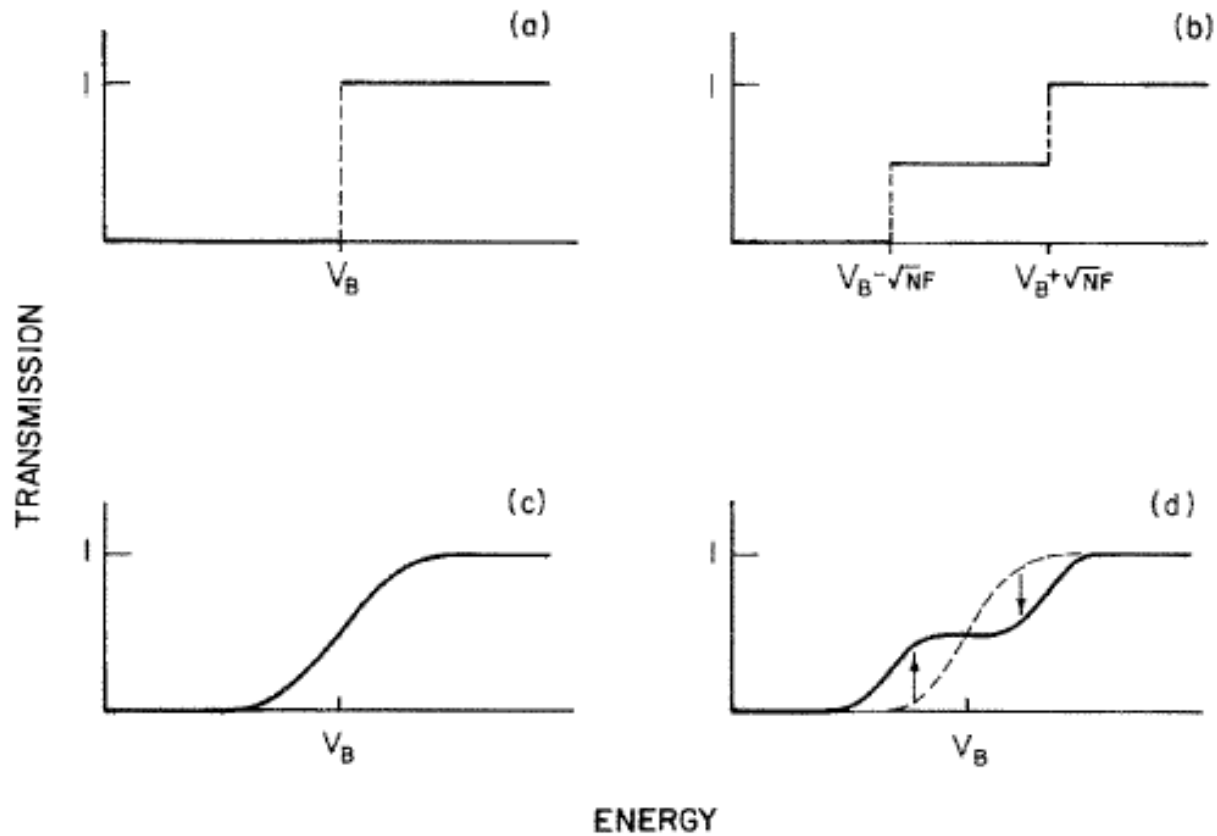


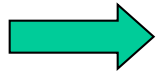
Fig. 1. Illustration of how channel coupling increases transmission at energies below the barrier and decreases it above. Parts (a) and (b) indicate the classical limits for no coupling and coupling, respectively, while parts (c) and (d) indicate how quantum mechanical effects modify the corresponding curves.

Sub-barrier Fusion and Barrier distribution method

$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \log \left[1 + \exp \left(\frac{2\pi}{\hbar\Omega} (E - V_b) \right) \right]$$



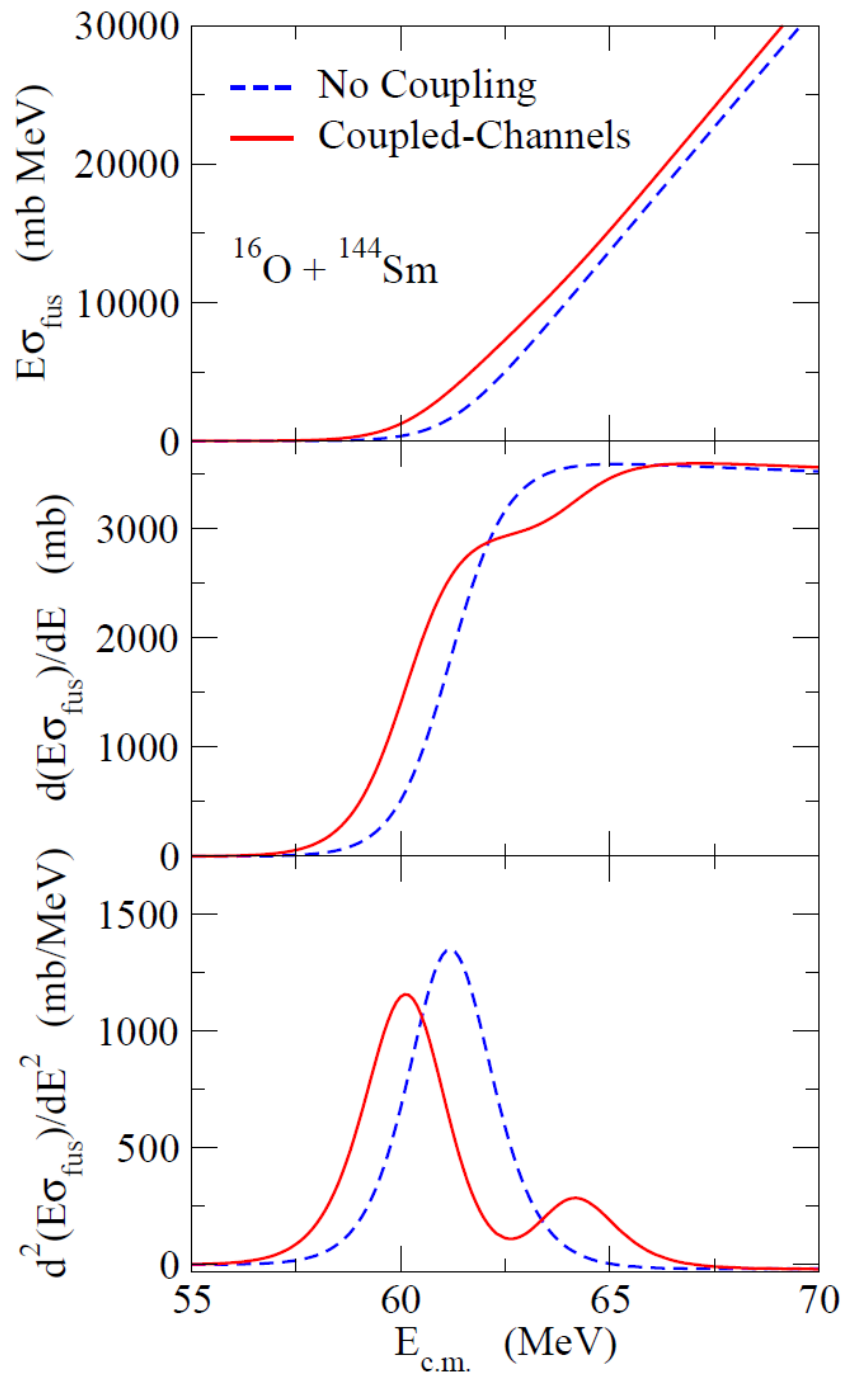
$$\frac{d(E\sigma_{\text{fus}})}{dE} = \frac{\pi R_b^2}{1 + \exp \left[\frac{2\pi}{\hbar\Omega} (V_b - E) \right]} = \pi R_b^2 \cdot P_{l=0}(E)$$



$$D_{\text{fus}}(E) \equiv \frac{d^2(E\sigma_{\text{fus}})}{dE^2} \simeq \pi R_b^2 \frac{dP_{l=0}}{dE}$$

(Fusion barrier distribution)

N. Rowley, G.R. Satchler,
P.H. Stelson, PLB254('91)25



N. Rowley, G.R. Satchler,
 P.H. Stelson, PLB254('91)25

$$\frac{d}{dE}[E\sigma_{\text{fus}}(E)] \propto P(E)$$

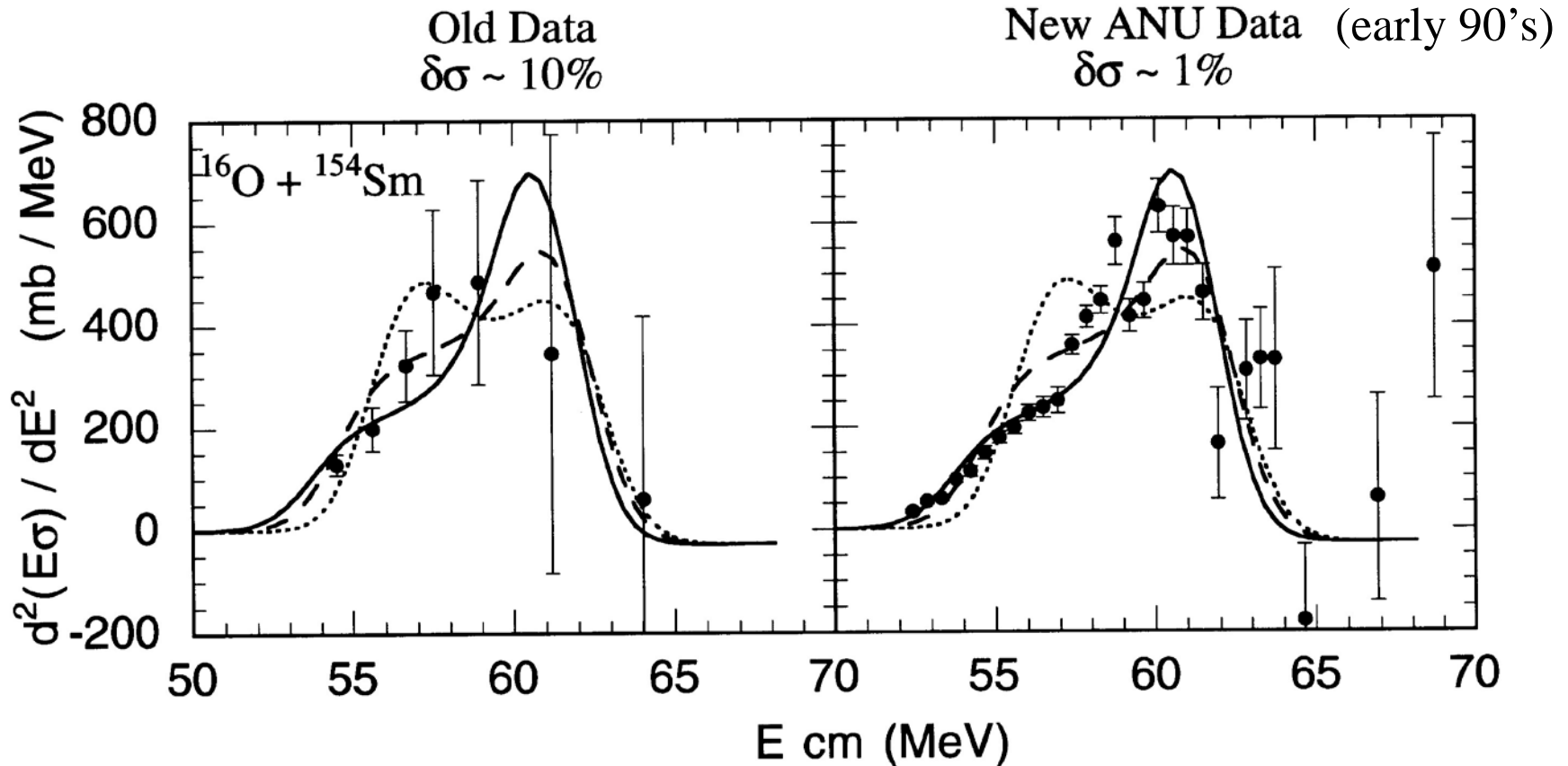
$$\frac{d^2}{dE^2}[E\sigma_{\text{fus}}(E)] \propto \frac{dP}{dE}$$

centered on $E = V_b$

Barrier distribution measurements

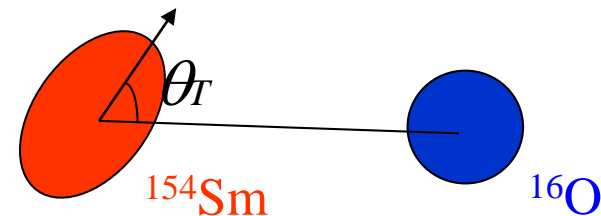
Fusion barrier distribution $D_{\text{fus}}(E) = \frac{d^2(E\sigma)}{dE^2}$

Needs high precision data in order for the 2nd derivative to be meaningful

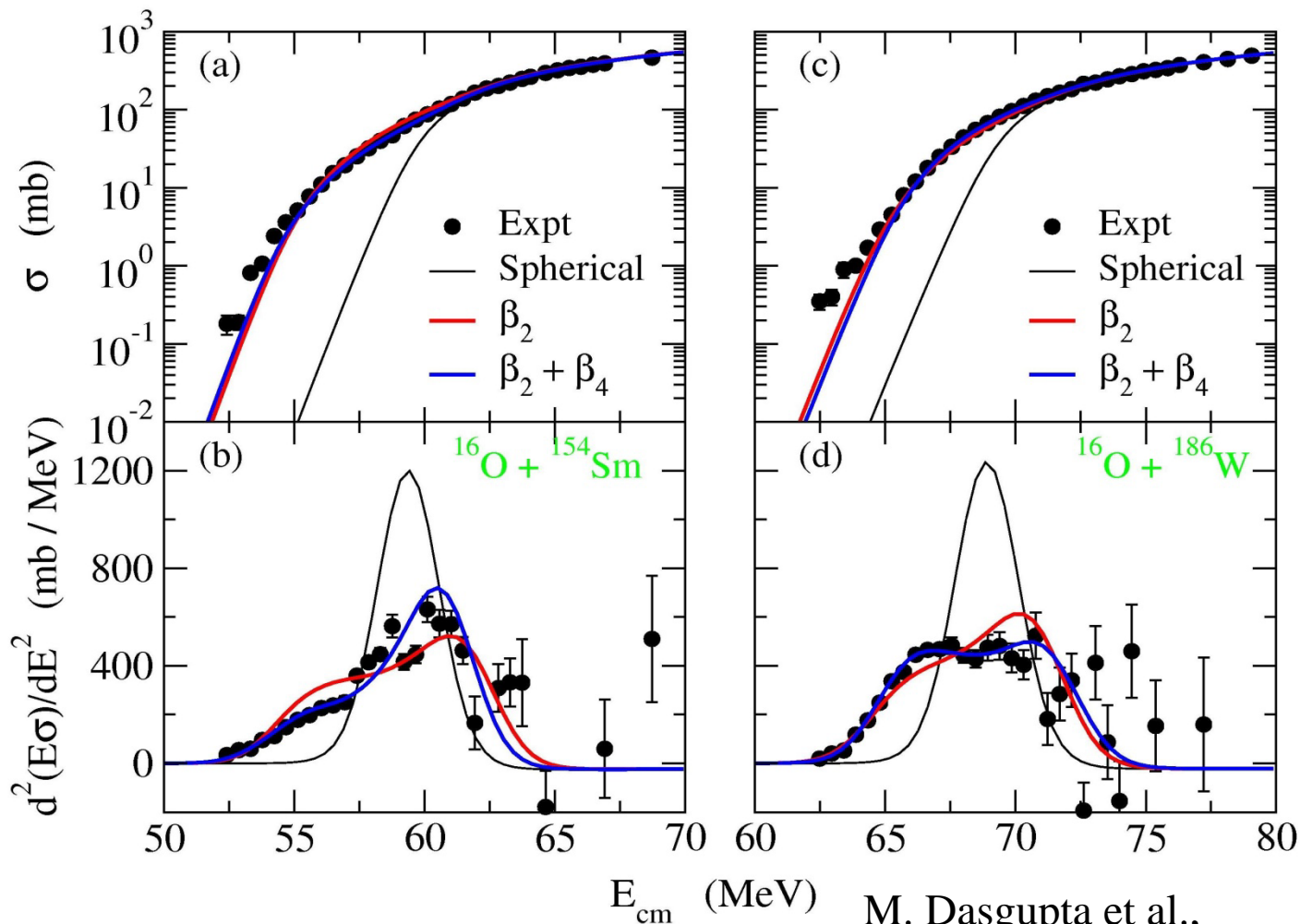


Experimental Barrier Distribution

Requires high precision data

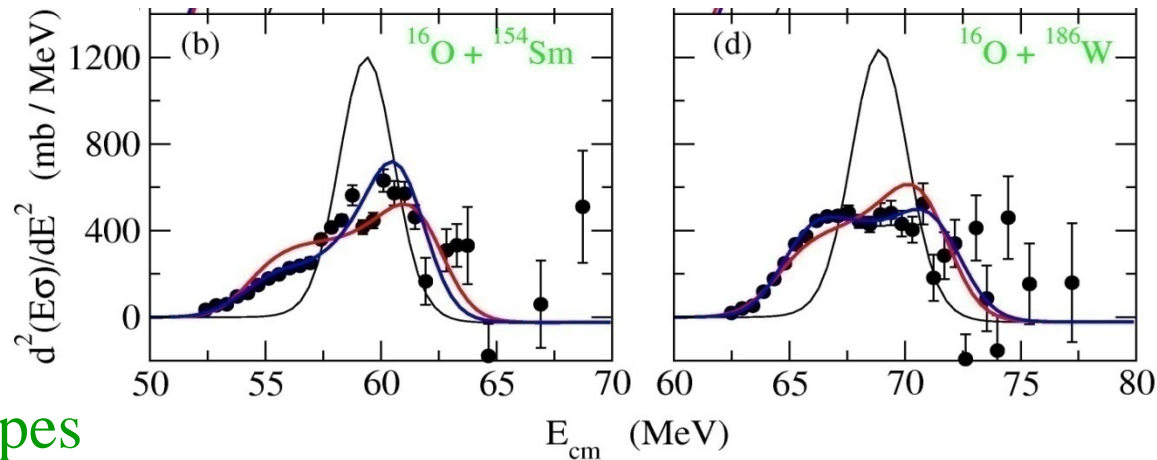


$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta_T) \sigma_{\text{fus}}(E; \theta_T)$$



M. Dasgupta et al.,
Annu. Rev. Nucl. Part. Sci. 48('98)401

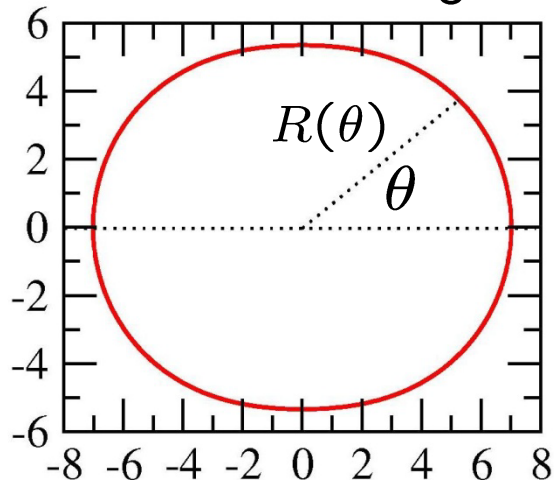
Investigate nuclear shape through barrier distribution



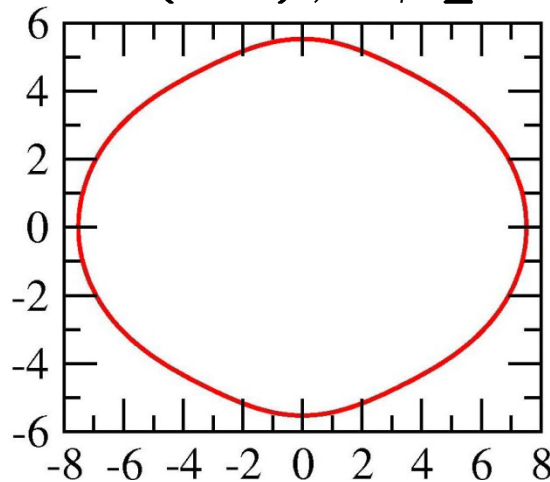
Nuclear shapes

$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \dots)$$

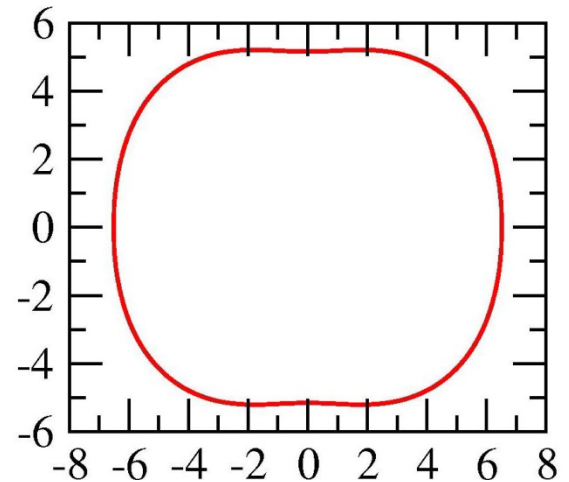
$$R_0 = 5.9 \text{ (fm)}, \quad \beta_2 = 0.3$$



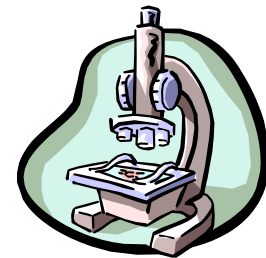
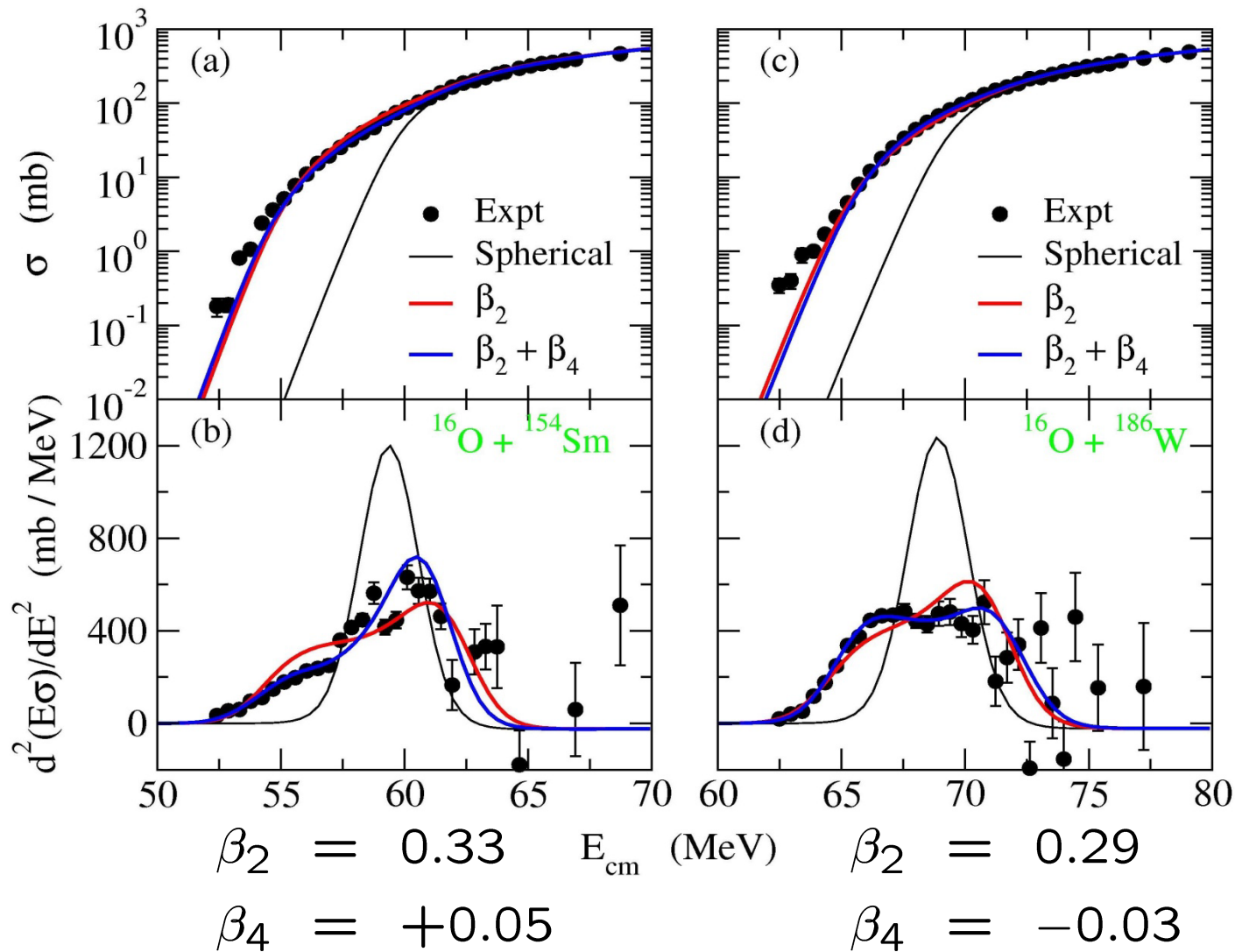
$$\beta_4 = 0$$



$$\beta_4 = 0.1$$



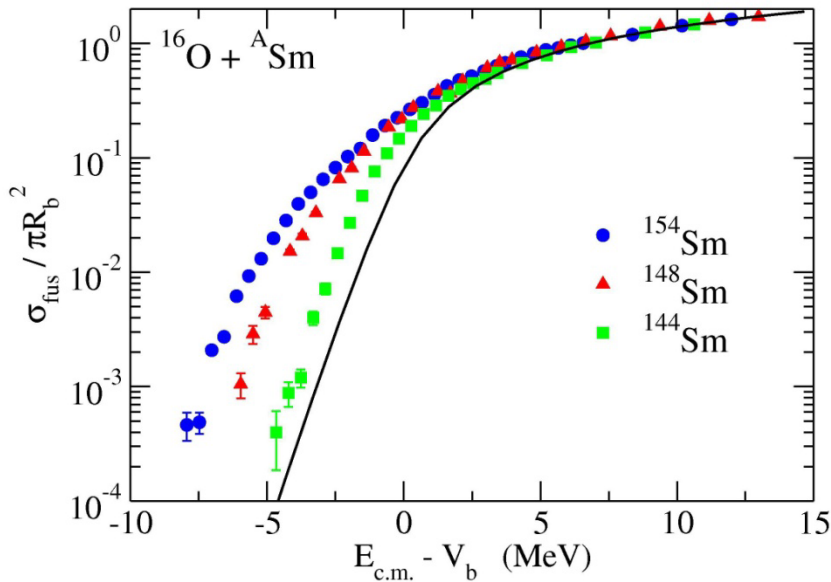
$$\beta_4 = -0.1$$



By taking the barrier distribution, one can very clearly see the difference due to β_4 !

➔ Fusion as a quantum tunneling microscope for nuclei

Advantage of fusion barrier distribution



Fusion Cross sections



Very strong exponential energy dependence



Difficult to see differences due to details of nuclear structure

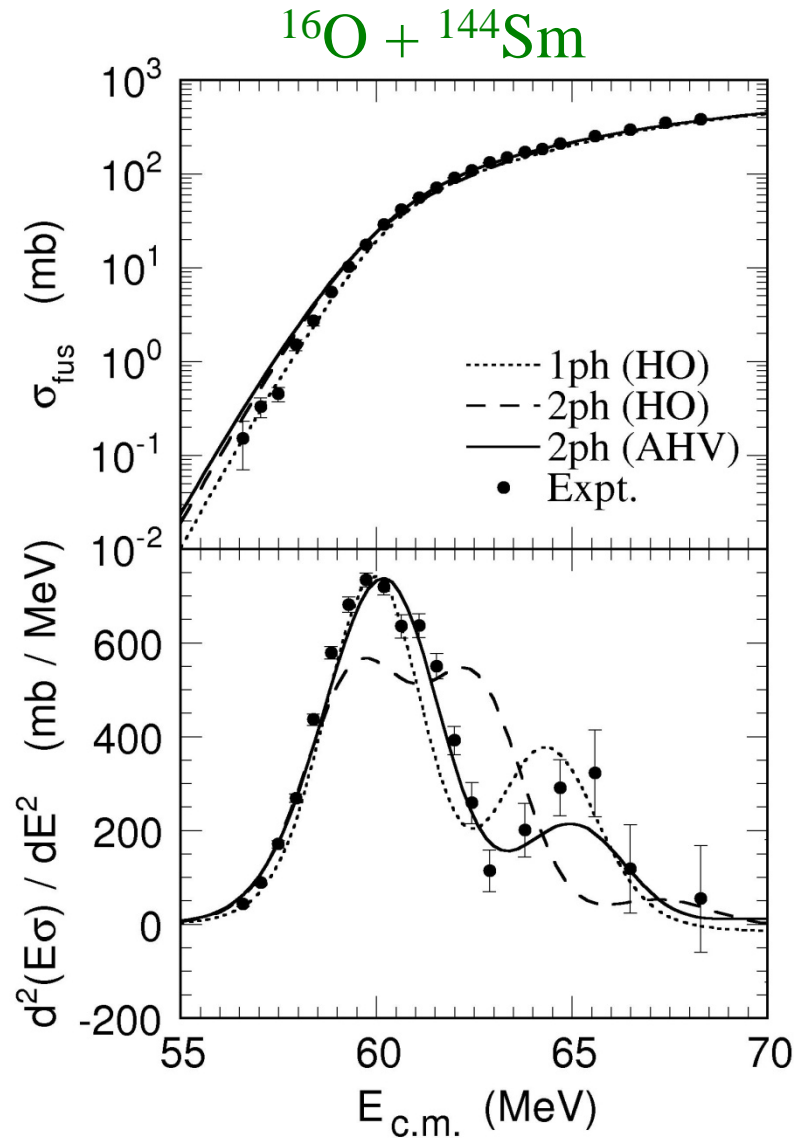
Plot cross sections in a different way: Fusion barrier distribution

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma)}{dE^2}$$

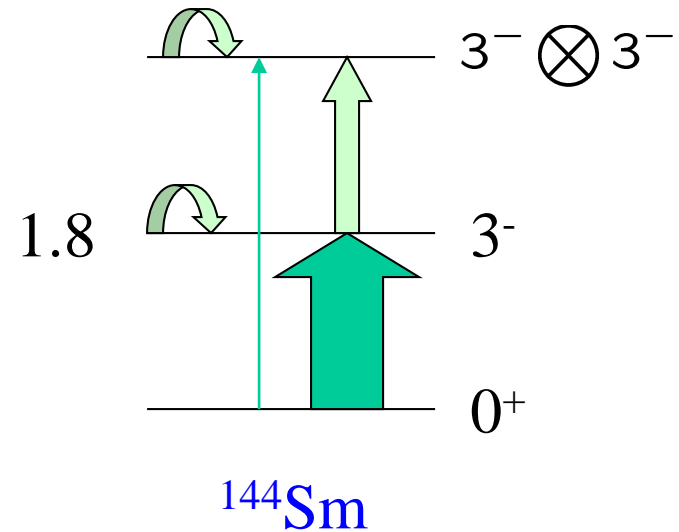
N. Rowley, G.R. Satchler,
P.H. Stelson, PLB254('91)25

→ Function which is sensitive to details of nuclear structure

Example for spherical vibrational system



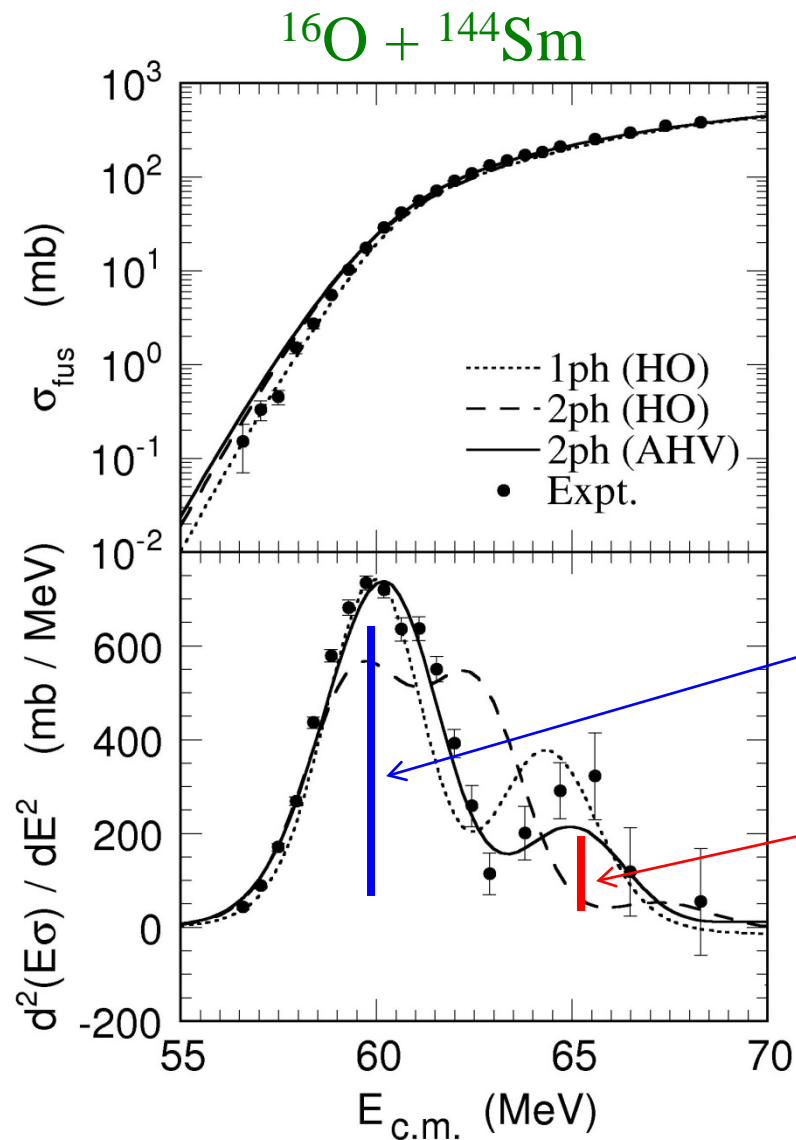
Anharmonicity of octupole vibration



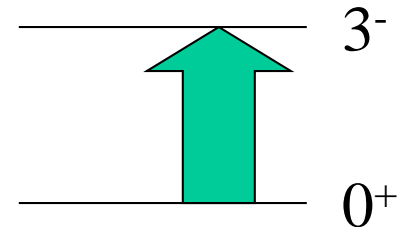
Quadrupole moment:
 $Q(3^-) = -0.70 \pm 0.02\text{b}$

K.Hagino, N. Takigawa, and S. Kuyucak,
 PRL79('97)2943

Barrier distribution



1.8 MeV



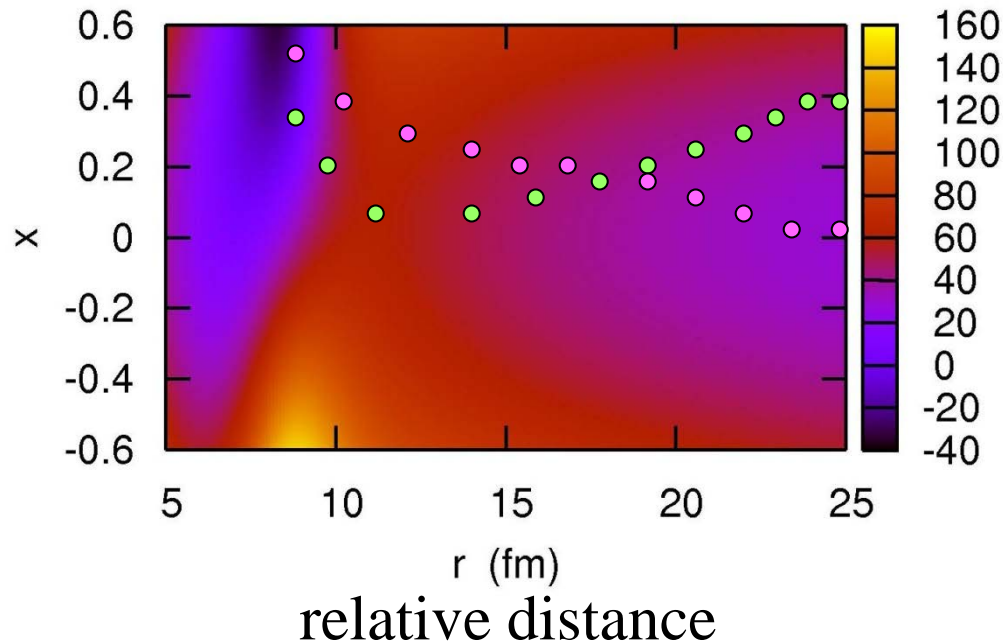
^{144}Sm

$$\alpha|0^+\rangle + \beta|3^-\rangle$$

$$\beta|0^+\rangle - \alpha|3^-\rangle$$

Coupling to excited states \longrightarrow distribution of potential barrier

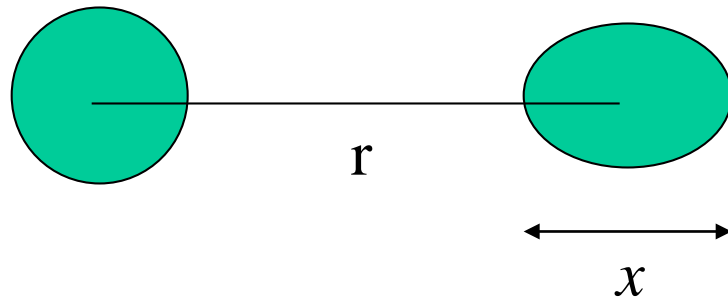
multi-dimensional potential surface



single barrier

\longrightarrow a collection of many barriers

$$P(E) = P[E, V(r)]$$
$$\longrightarrow P(E) = \sum_{\alpha} w_{\alpha} P[E, V_{\alpha}(r)]$$



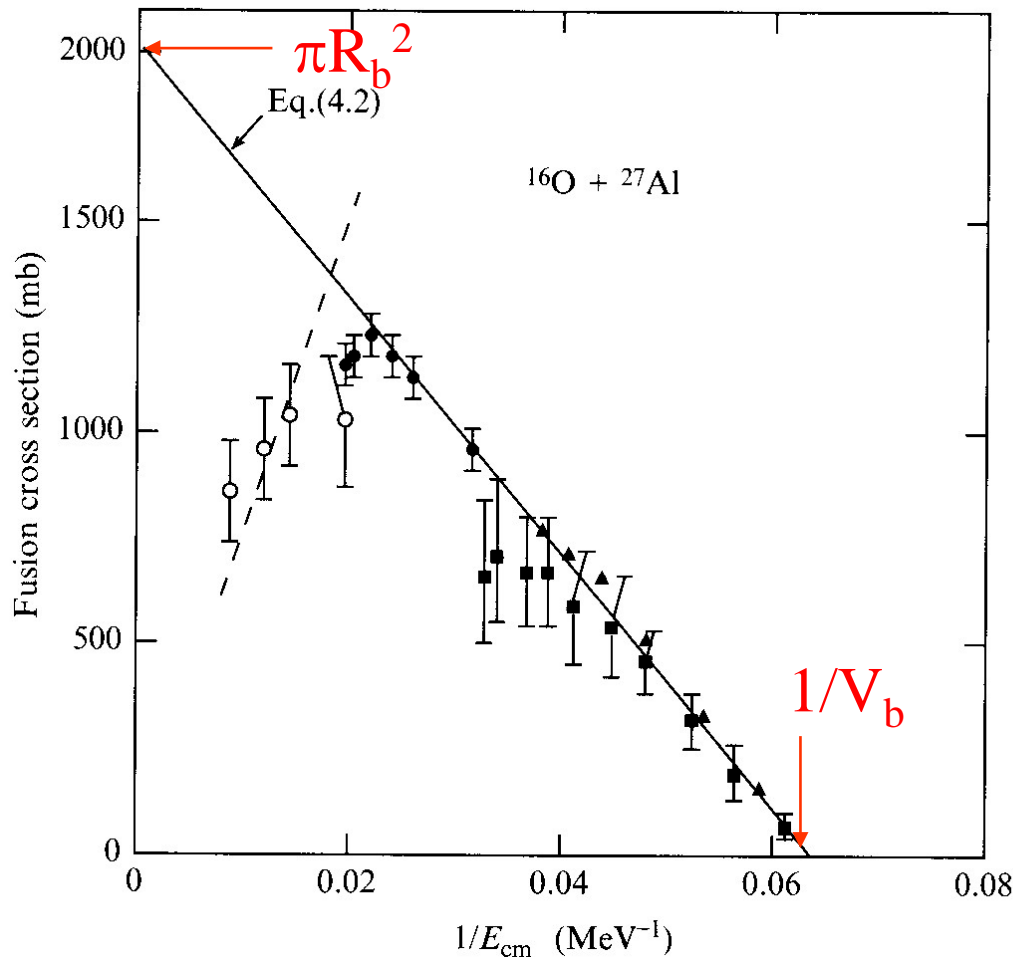
(intrinsic coordinate)

Representations of fusion cross sections

i) σ_{fus} vs $1/E$ (~70's)

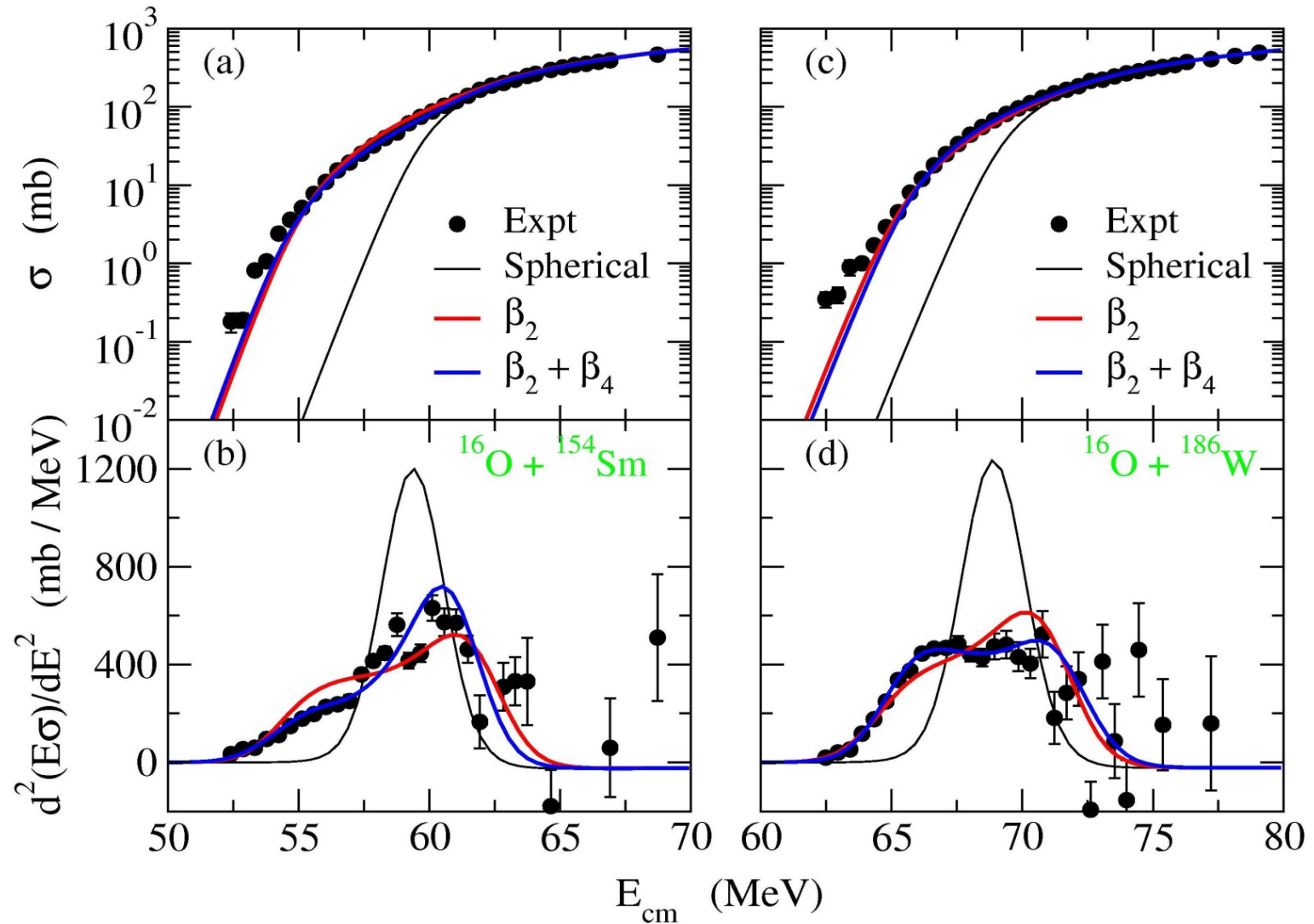
Classical fusion cross section is proportional to $1/E$:

$$\sigma_{\text{fus}}^{\text{cl}}(E) = \pi R_b^2 \left(1 - \frac{V_b}{E}\right)$$



Taken from J.S. Lilley,
“Nuclear Physics”


ii) barrier distribution (~ 90 's)



iii) logarithmic derivative (~ 00 's)

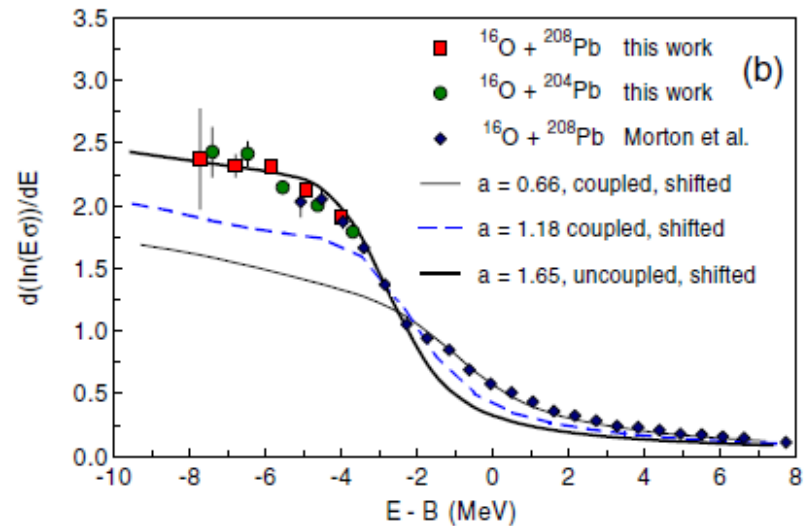
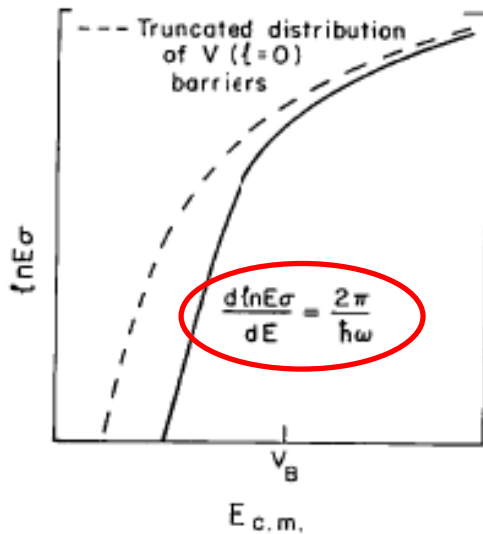
$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \log \left[1 + \exp \left(\frac{2\pi}{\hbar\Omega} (E - V_b) \right) \right]$$

$$\sim \frac{\hbar\Omega}{2E} R_b^2 \exp \left(\frac{2\pi}{\hbar\Omega} (E - V_b) \right) \quad (E \ll V_b)$$

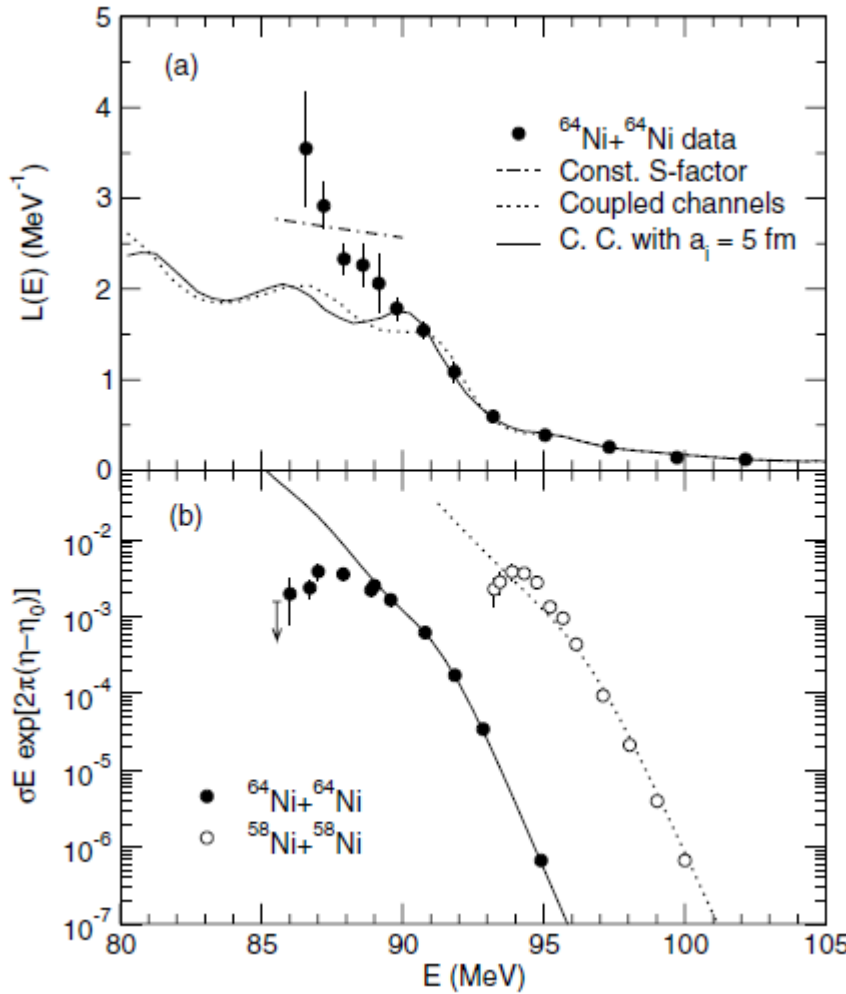
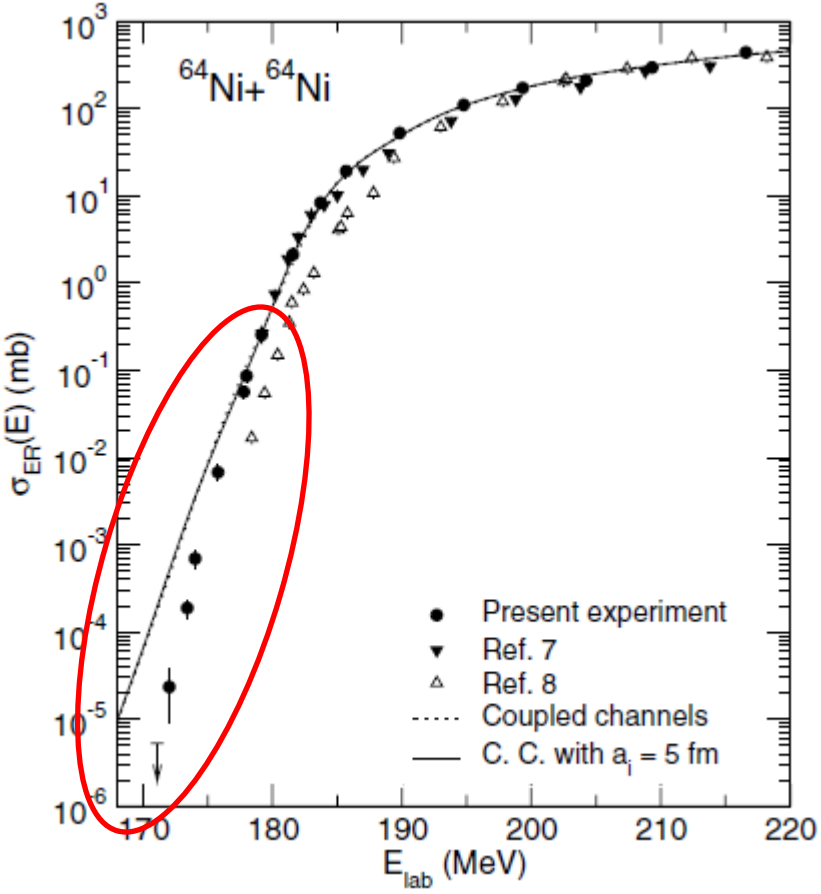


$$\frac{d}{dE} \log(E\sigma) = \frac{(E\sigma)'}{E\sigma} = \frac{2\pi}{\hbar\Omega}$$

cf. $D_{\text{fus}} = (E\sigma)''$

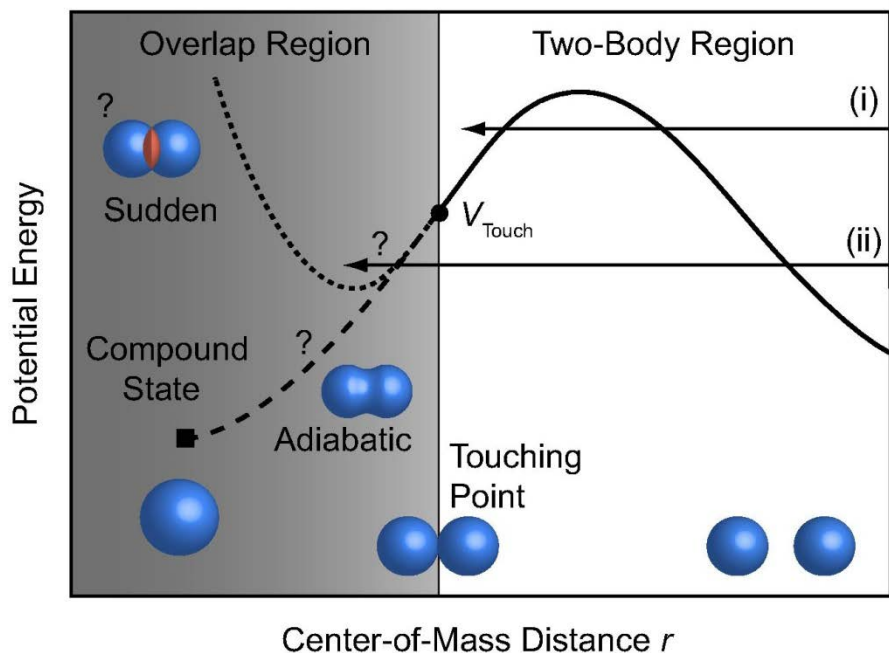


deep subbarrier hindrance of fusion cross sections



C.L. Jiang et al., PRL89('02)052701; PRL93('04)012701

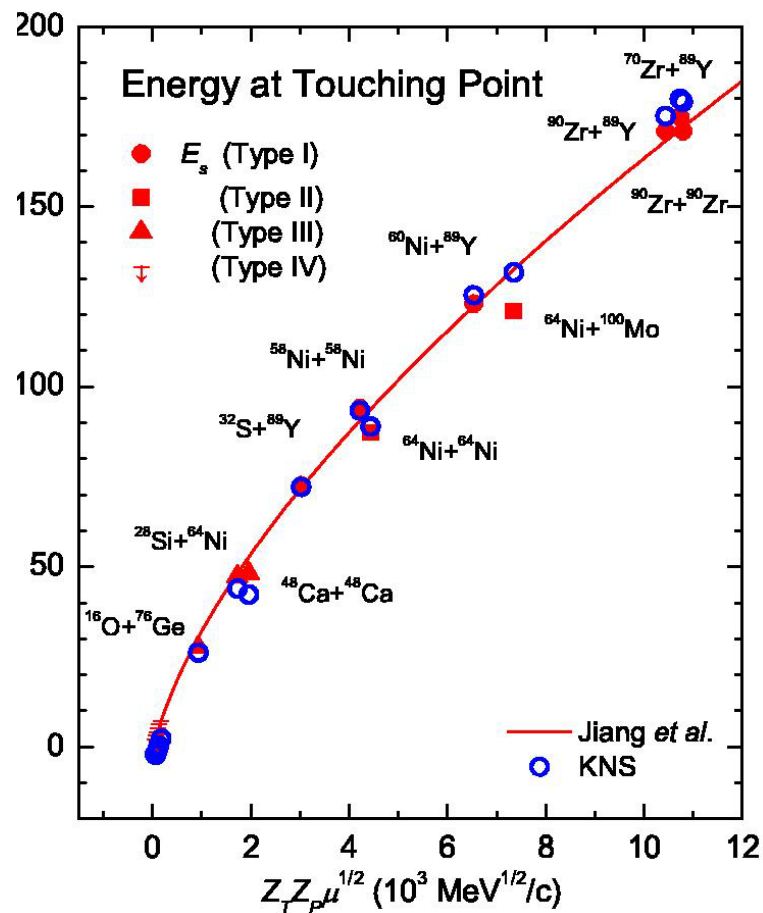
Systematics of the touching point energy and deep subbarrier hindrance



mechanism of deep subbarrier hindrance:
not yet been fully understood



how to model the dynamics after touching?



T. Ichikawa, K.H., A. Iwamoto,
PRC75('07) 064612 & 057603

Summary

Heavy-Ion Fusion Reactions around the Coulomb Barrier

✧ Fusion and quantum tunneling

Fusion takes place by tunneling

✧ Basics of the Coupled-channels method

Collective excitations during fusion

✧ Concept of Fusion barrier distribution

Sensitive to nuclear structure

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

✧ Quasi-elastic scattering and quantum reflection

Complementary to fusion

Computer program: CCFULL

<http://www.nucl.phys.tohoku.ac.jp/~hagino/ccfull.html>

References

Nuclear Reaction in general

- G.R. Satchler, “*Direct Nuclear Reactions*”
- G.R. Satchler, “*Introduction to Nuclear Reaction*”
- R.A. Broglia and A. Winther, “*Heavy-Ion Reactions*”
- “*Treatise on Heavy-Ion Science*”, vol. 1-7
- D.M. Brink, “*Semi-classical method in nucleus-nucleus collisions*”
- P. Frobrich and R. Lipperheide, “*Theory of Nuclear Reactions*”

Heavy-ion Fusion Reactions

- M. Dasgupta et al., Annu. Rev. Nucl. Part. Sci. 48('98) 401
- A.B. Balantekin and N. Takigawa, Rev. Mod. Phys. 70('98) 77
- Proc. of Fusion03, Prog. Theo. Phys. Suppl. 154('04)
- Proc. of Fusion97, J. of Phys. G 23 ('97)
- Proc. of Fusion06, AIP, in press.