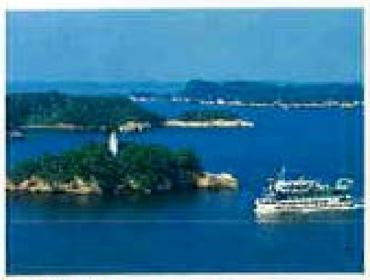
Shape of Lambda hypernuclei studied with self-consitent mean-field methods

Kouichi Hagino 萩野浩一
Tohoku Univ. 東北大学

Myaing Thi Win（東北大学）→
Yusuke Tanimura（東北大学）
谷村雄介

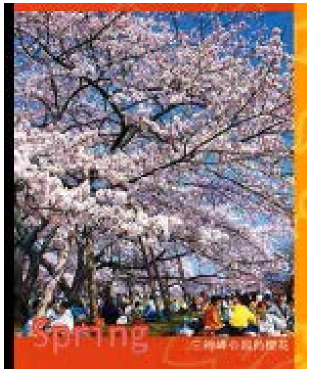
留学生（缅甸）

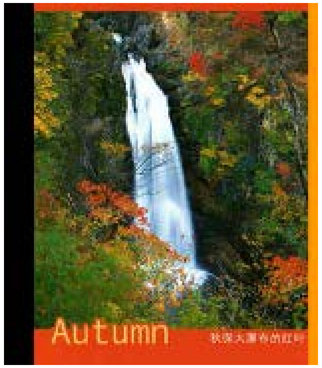
1. Introduction
2. Deformation of Lambda hypernuclei
3. Towards a 3D-mesh RMF calculation
~ inverse Hamiltonian method ~
4. Summary

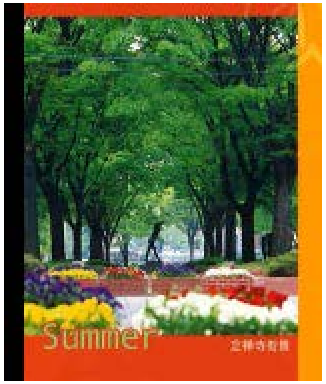


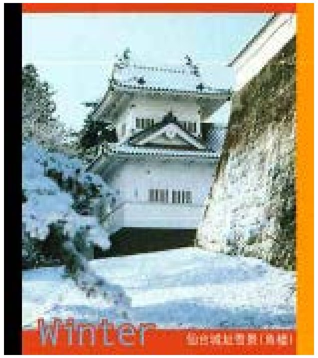
6 松岛【宫城县松岛町】

松岛是日本三景之一，您可乘游船饱览松岛湾内260余座的岛屿，国家级保护文物瑞严寺和五大堂等也是不能错过的历史性建筑。









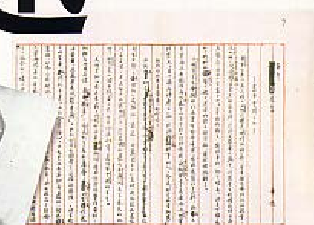
MIYAGI HISTORY

摄于1906年3月,左第一人为鲁迅,即将离开仙台时与同班同学的合影。

历史和鲁迅

史迹,鲁迅生活过的地方

约400年前,作为伊达六十万石的城邑而发展起来,
与中国著名文学家鲁迅有深缘的仙台,
还有受伊达政宗藩主之命支仓常长一行
罗马旅行的出发地石卷。
向您介绍宫城县各地的历史风情。



回忆仙台时代生活的段落。
写于1926年,引自《朝花夕拾》。

鲁迅最初寄宿的
"佐藤屋"旧址,
现在的米袋一丁目。

藤野嚴九郎教授
藤野严九郎教授

藤野教授

· 鲁迅和东北大学

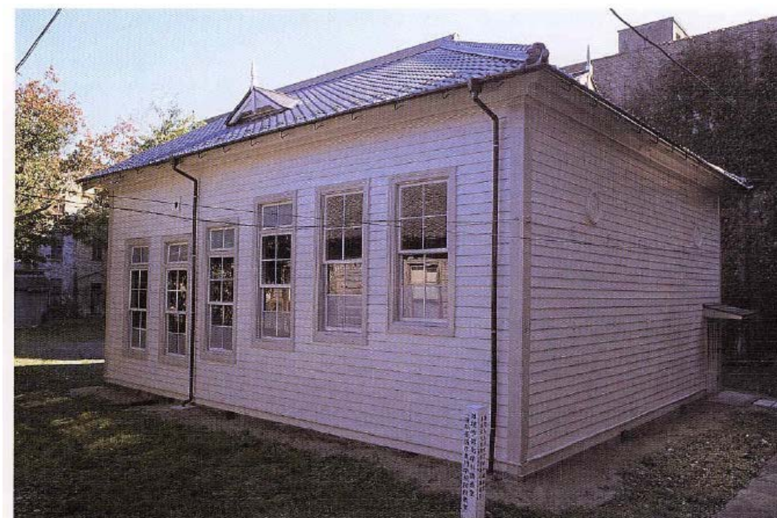
東北大学

鲁迅(原名:周树人)1881年9月25日出生于清朝(现在的中华人民共和国)的长江下游浙江省绍兴县。1902年1月毕业南京的江南陆师学堂附属矿务铁路学堂之后,同年4月作为清朝留学生来我国留学,先就读于东京的弘文学院普通速成科。在此学院鲁迅学习了日语和基础科目。

应鲁迅的要求,1904年5月20日当时的清朝·杨公使向仙台医学专门学校(现在的东北大学医学部)提出了就鲁迅的入学要求进行妥善处理的照会信。

仙台医学专门学校对此以文部省有关入学规则为依据进行探讨之后,决定允许免试入学。并于5月23日给杨公使寄送了入学许可通知书。

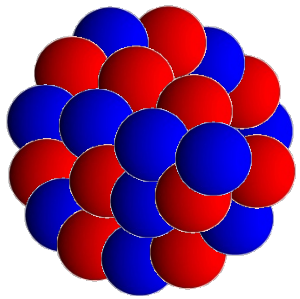
同年9月,鲁迅进入了仙台医学专门学校。



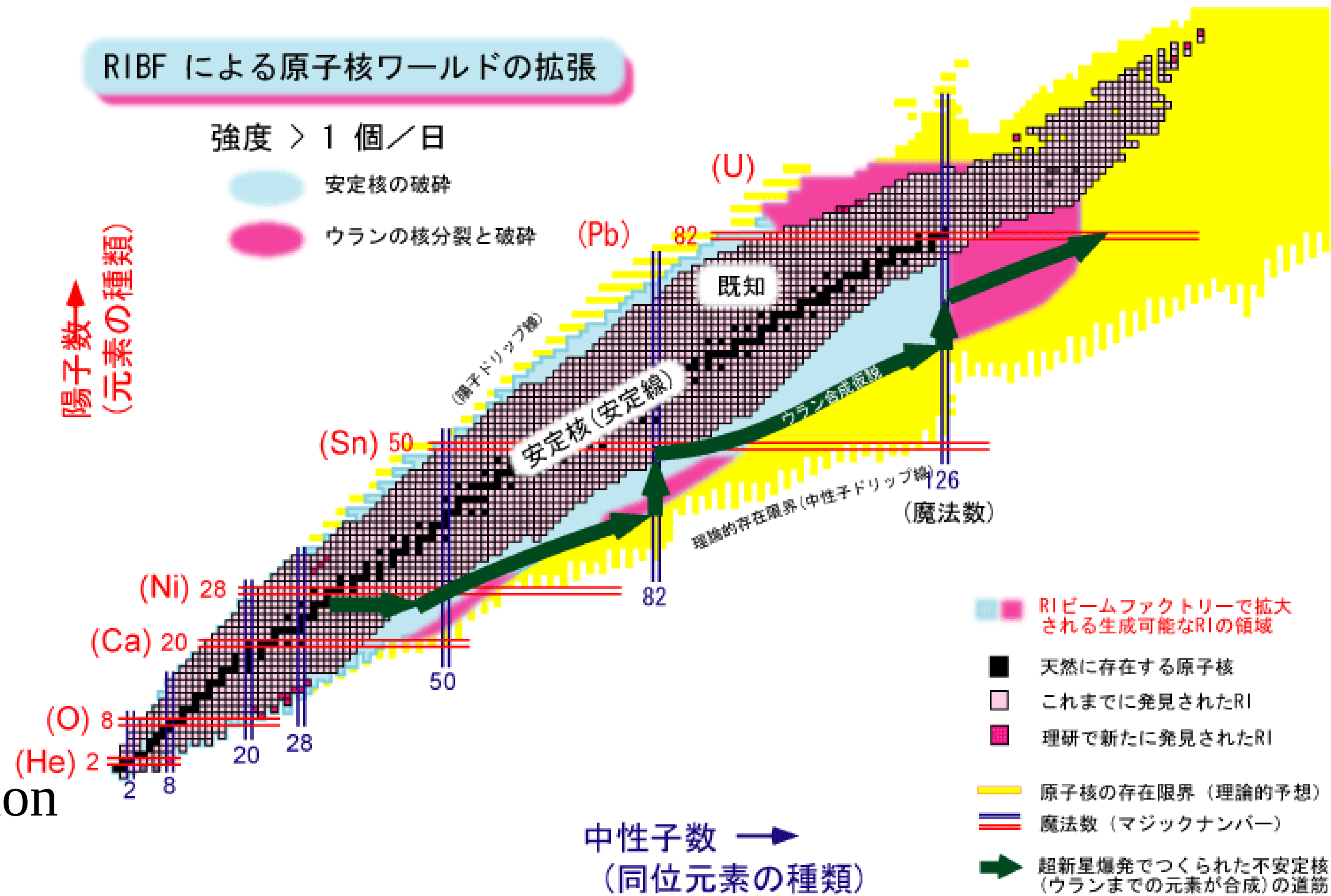
鲁迅が学んだ仙台医学専門学校階段教室外景
(鲁迅曾就读的仙台医学专门学校教学楼外景)

鲁迅阶梯教室

Introduction

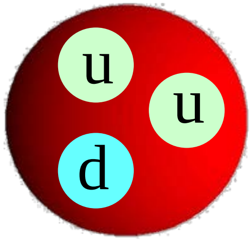


nuclei = many-body systems consisted of neutrons and protons

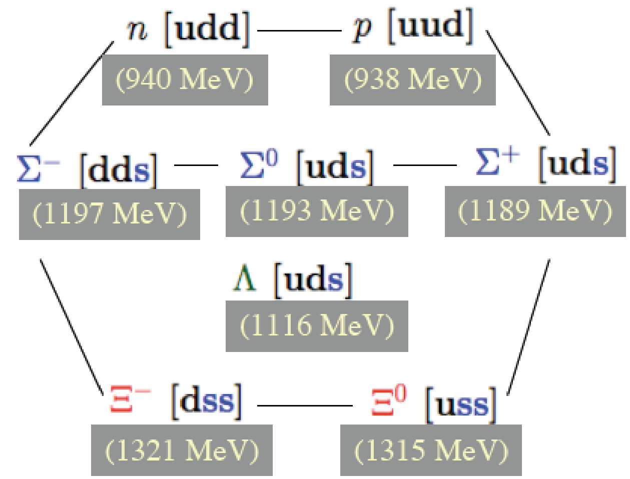


weakly bound exotic nuclei

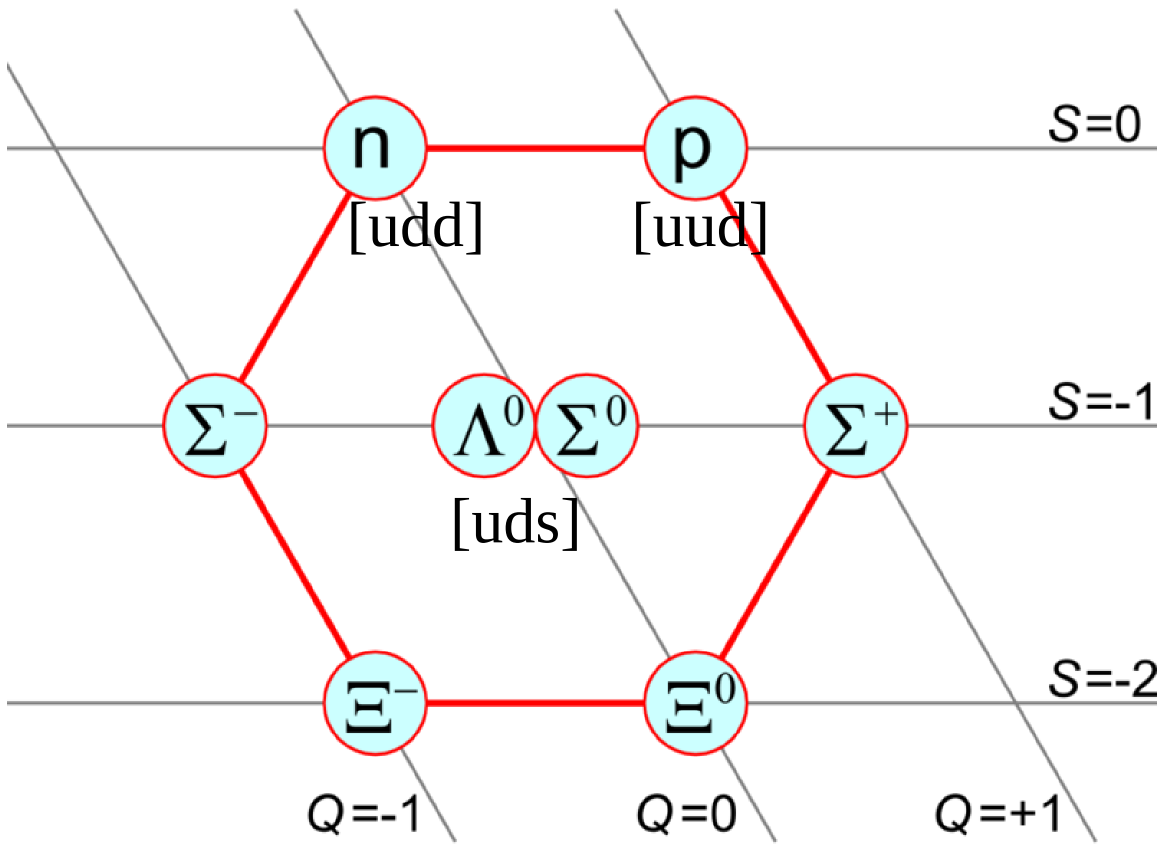
- halo, skin
- large E1
- shell evolution
- ……



p = uud
n = udd

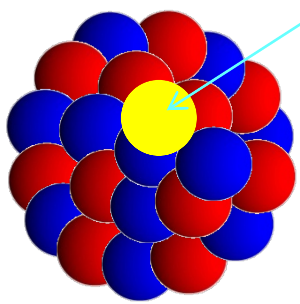


with u,d, and s quarks ⟶ baryon octet



(wikipedia)

Λ hypernuclei

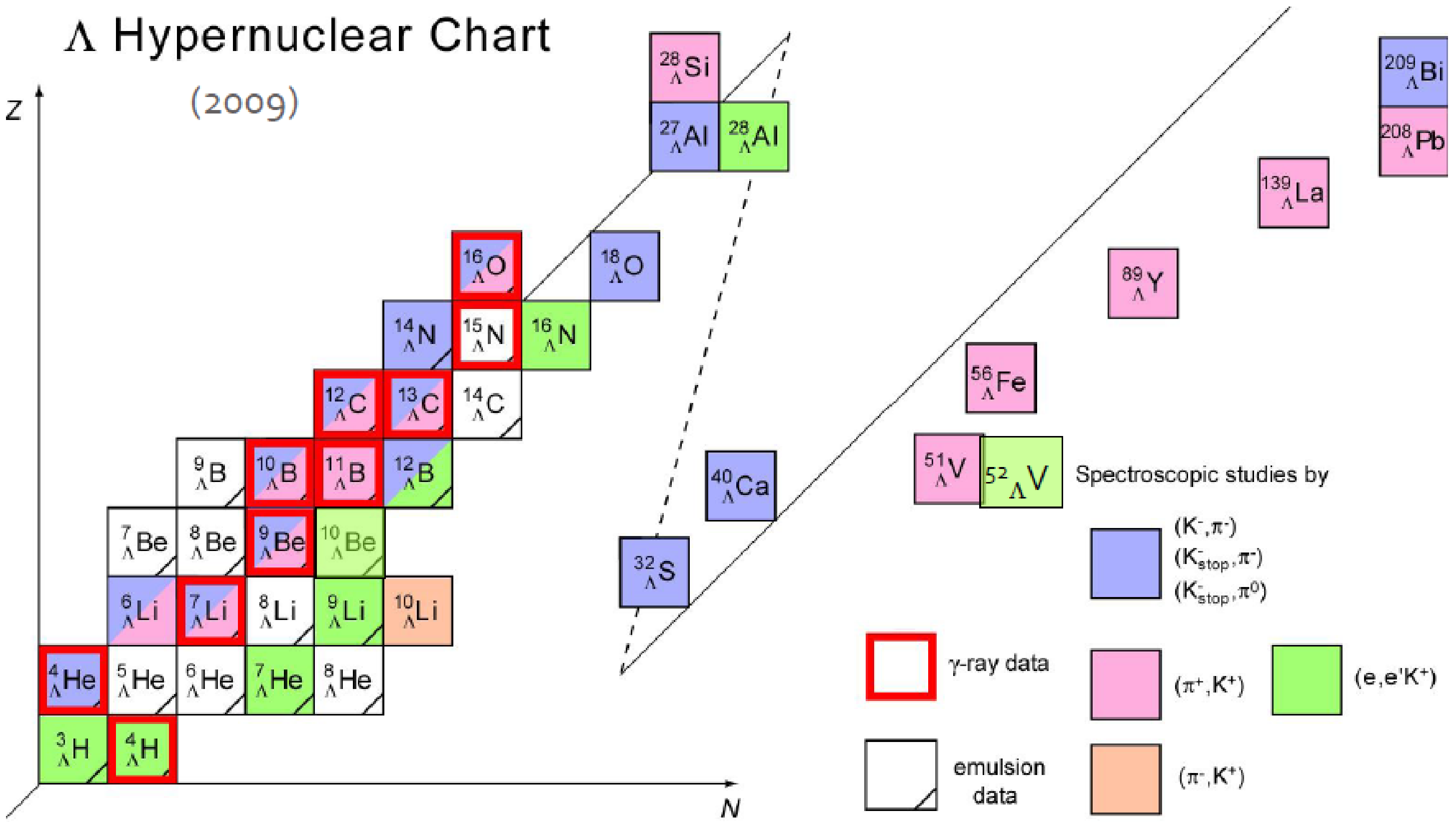


Λparticle: the lightest hyperon
(no charge, no isospin)

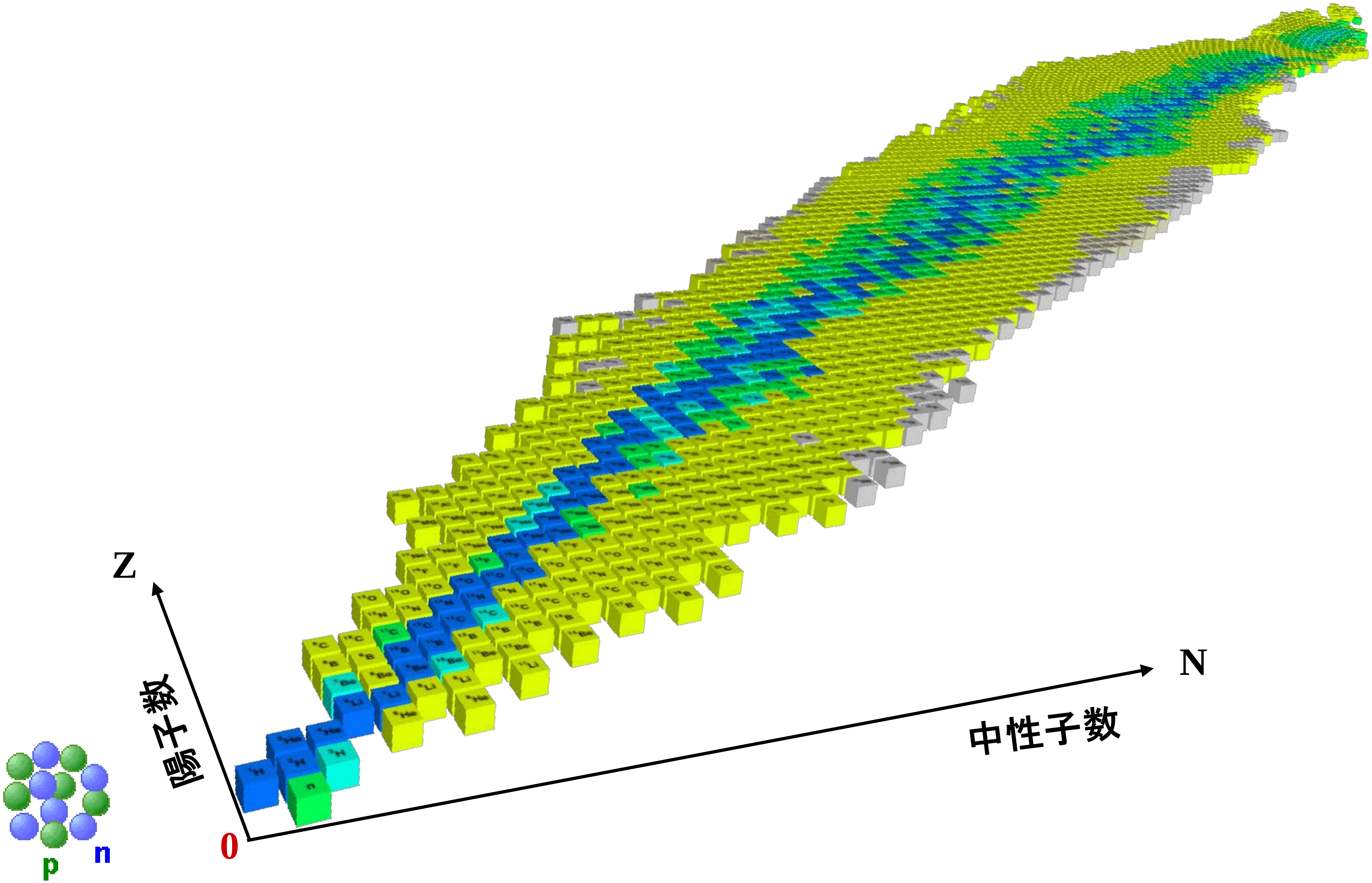
*no Pauli principle between nucleons and a Λ particle

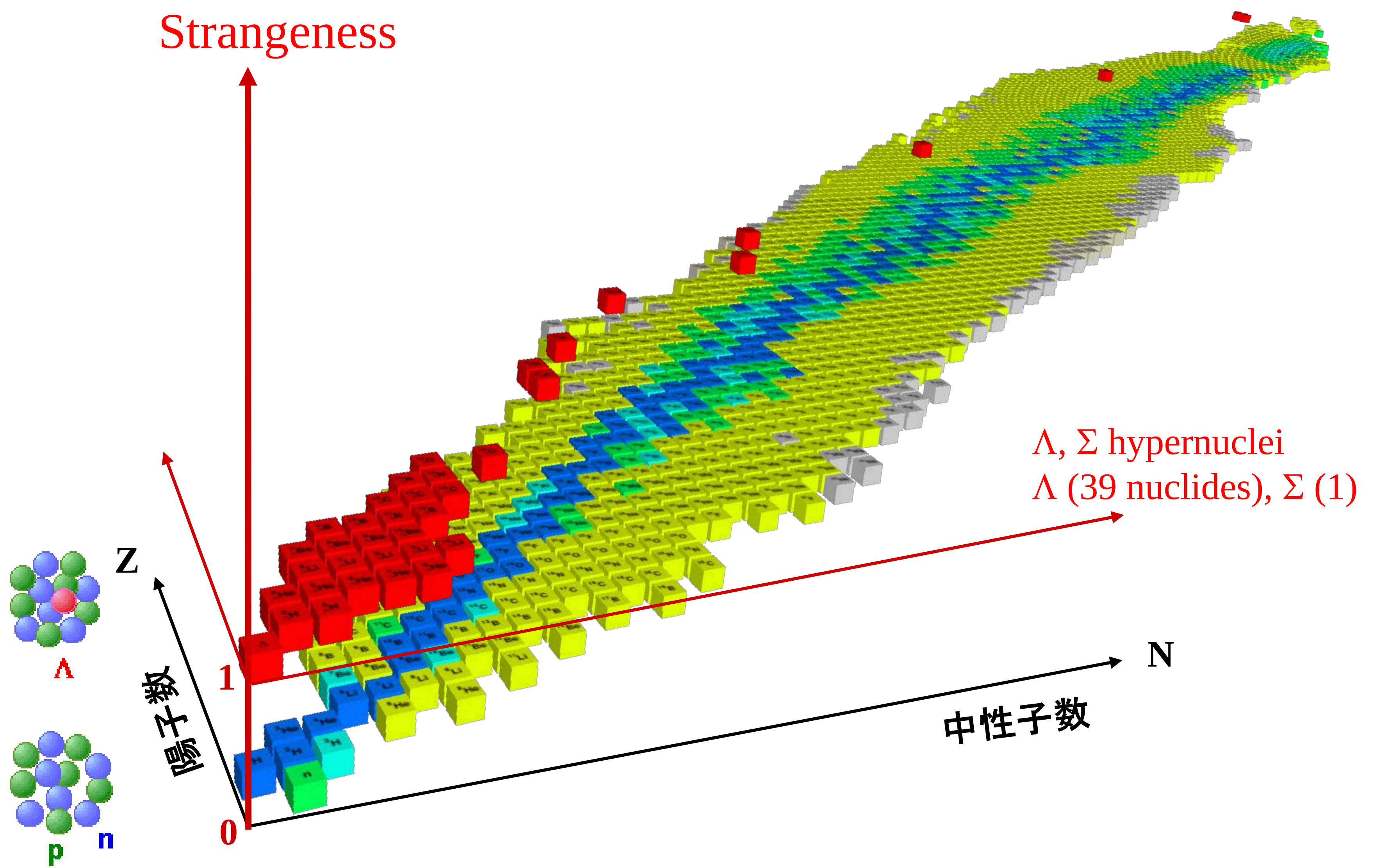
proton

neutron

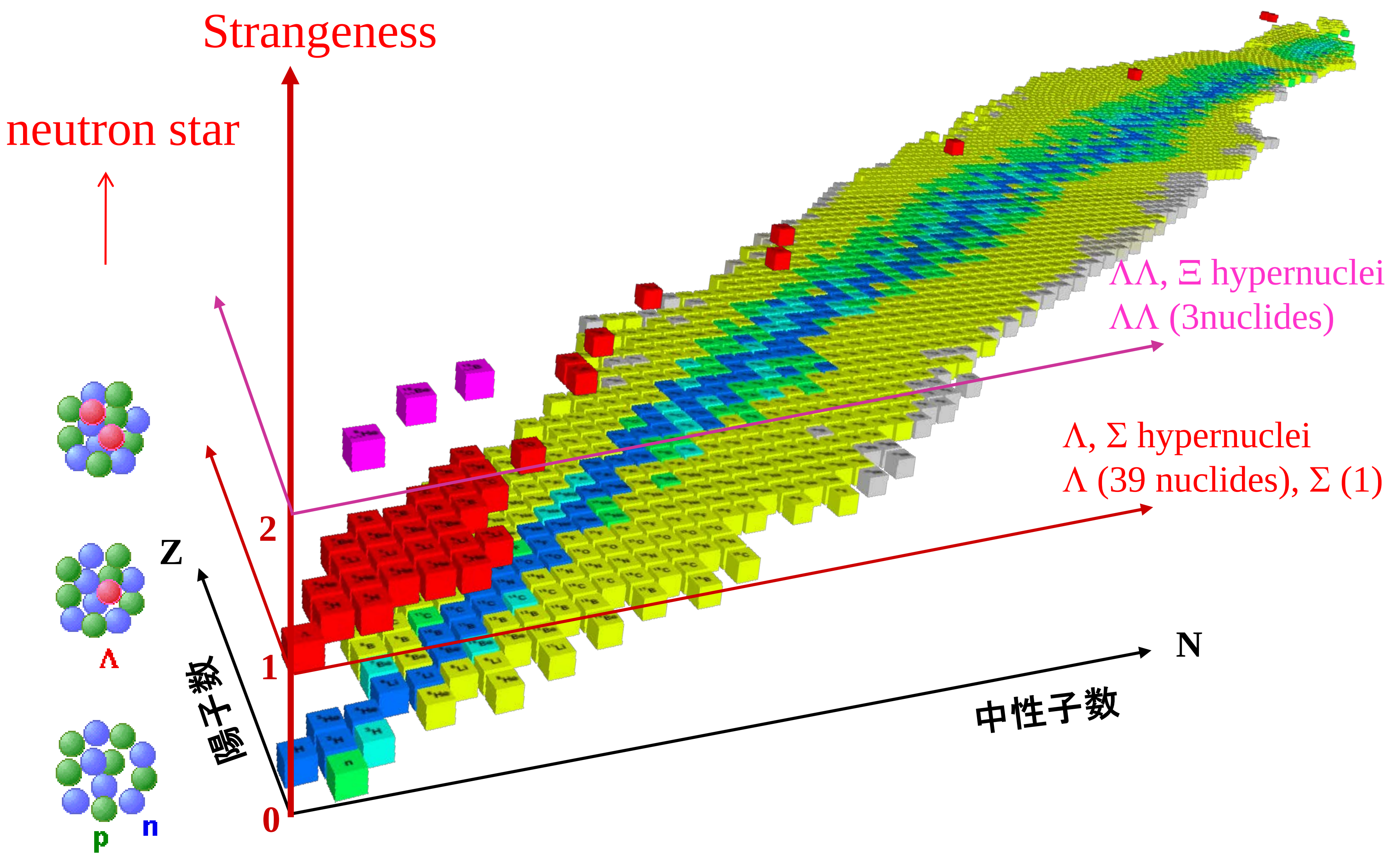


O. Hashimoto and H. Tamura, Prog. Part. Nucl. Phys. 57('06)564


Z
陽子数
N
中性子数
0
p
n



Strangeness
Λ, Σ hypernuclei
Λ (39 nuclides), Σ (1)
Z
陽子数
N
中性子数
1
0
Λ
p
n


Strangeness
neutron star
ΛΛ, Ξ hypernuclei
ΛΛ (3nuclides)
Λ, Σ hypernuclei
Λ (39 nuclides), Σ (1)
Z
陽子数
N
中性子数
Λ
p
n
0
1
2

sd-shell nuclei and deformation

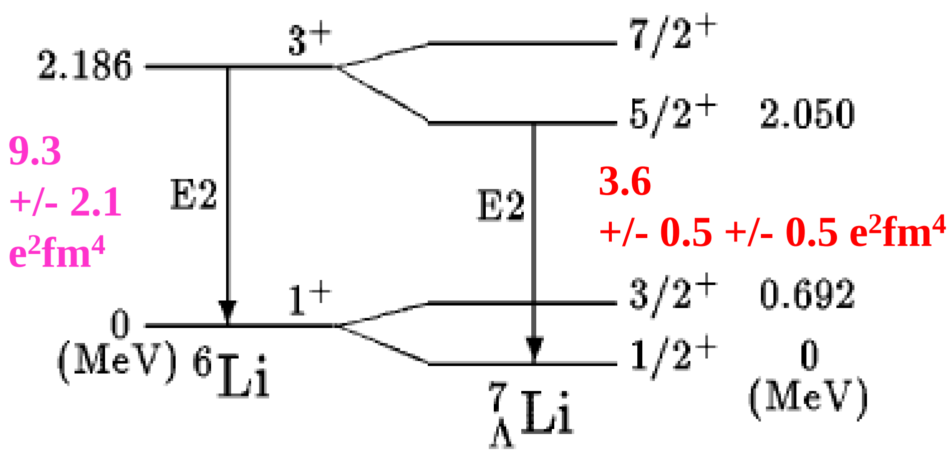
Impurity effects: one of the main interests of hypernuclear physics

how does Λ affect several properties of atomic nuclei?

- size, shape, density distribution, single-particle energy, shell effect, fission barrier……

the most prominent example:

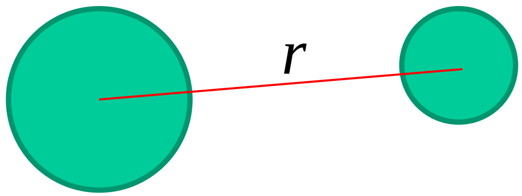
the reduction of B(E2) in $^7_\Lambda$Li

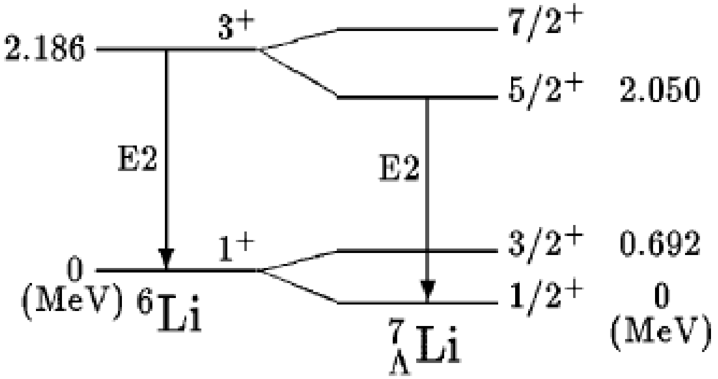


about 19% reduction of nuclear size (shrinkage effect)

K. Tanida et al., PRL86('01)1982

For a two-body system ….
(if no excitation of each fragment)





K. Tanida et al., PRL86(‘01)1982

E2 operator :

$$\hat{Q}_\mu = e_{\mathrm{E2}}\, r^2 Y_{2\mu}(\hat{\boldsymbol{r}})$$

E2 effective charge :

$$e_{\mathrm{E2}} = \frac{A_2^2 Z_1 + A_1^2 Z_2}{(A_1 + A_2)^2}\, e$$

e_{E2} = 0.667 e for α + d
0.673 e for $^5{}_\Lambda$He + d

reduction of B(E2)

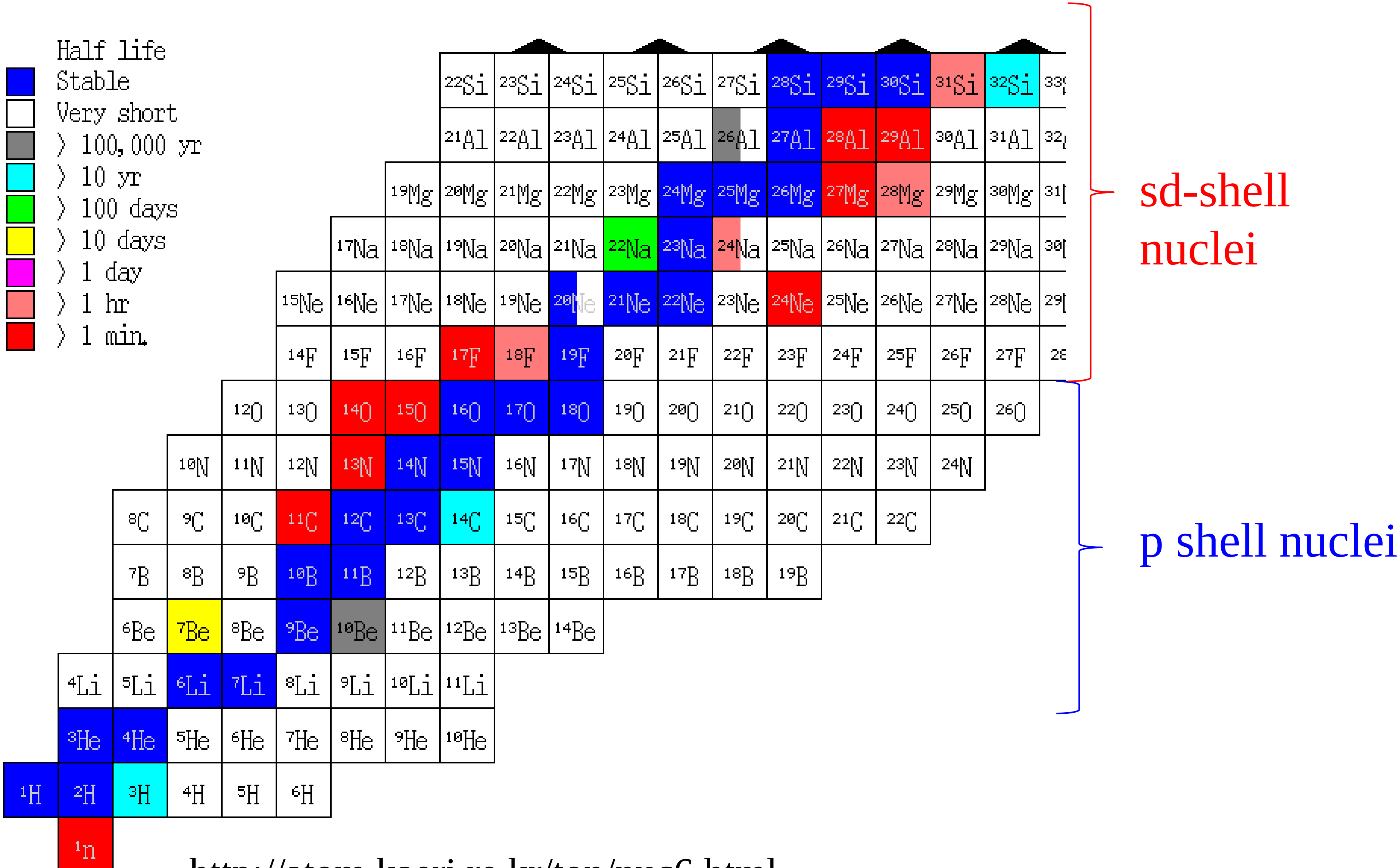
⇕

reduction of $\left[\langle r^2 \rangle_{i\to f}\right]^2$

T. Motoba, H. Bando, K. Ikeda, PTP70(‘83)189
E. Hiyama, M. Kamimura, K. Miyazaki, T. Motoba, PRC59(‘99)2351

$$\begin{aligned} B(E2:5/2^+ \to 1/2^+) &= 1/6 \cdot \left| \left\langle [3^+ \otimes 1/2^+]^{(5/2)} || Q_2 || [1^+ \otimes 1/2^+]^{(1/2)} \right\rangle \right|^2 \\ &= 1/9 \cdot |\langle 3^+ || Q_2 || 1^+ \rangle|^2 \end{aligned}$$

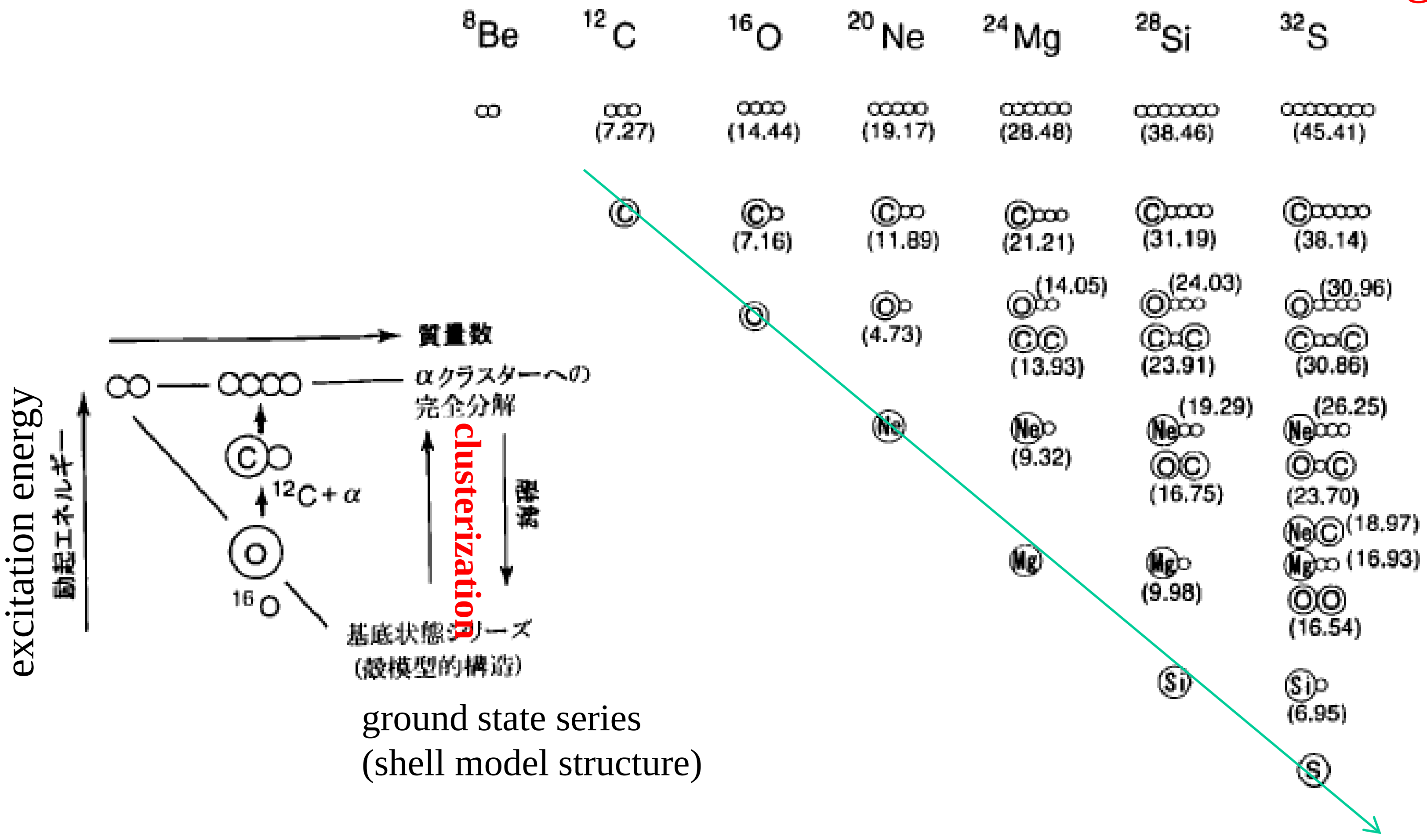
how about heavier nuclei?



http://atom.kaeri.re.kr/ton/nuc6.html

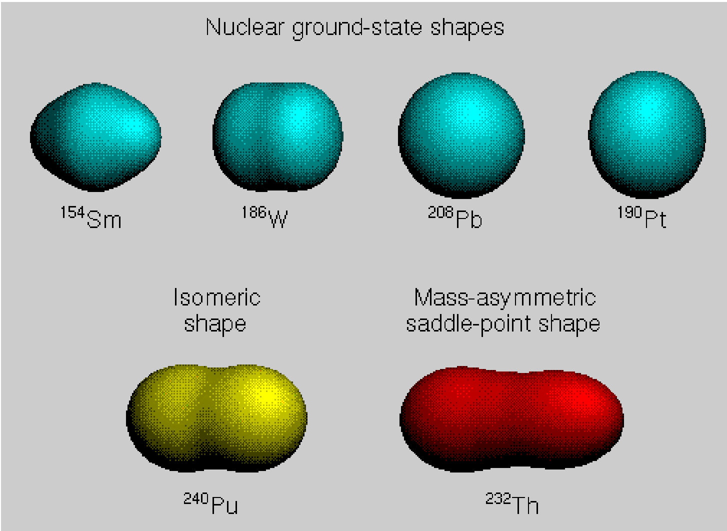
how about heavier nuclei?

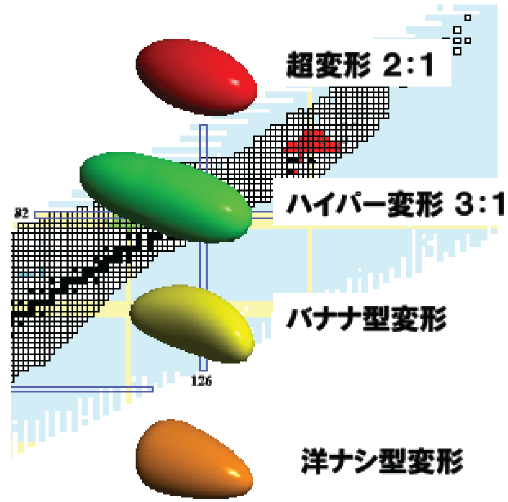
Ikeda diagram



the g.s. has a shell model-like structure for nuclei heavier than Be
(cluster-like structure appears in the excited states：threshold rule)

Shell model (mean-field) structure and nuclear deformation





http://t2.lanl.gov/tour/sch001.html

- many open-shell nuclei are deformed in the ground state
  - ✓characterstic feature of finite many-body systems
  - ✓spontaneous symmetry breaking of (rotational) symmetry

- B(E2) for deformed nuclei

$$B(E2:2^+\to 0^+) = \frac{1}{16\pi}\cdot Q_0^2$$

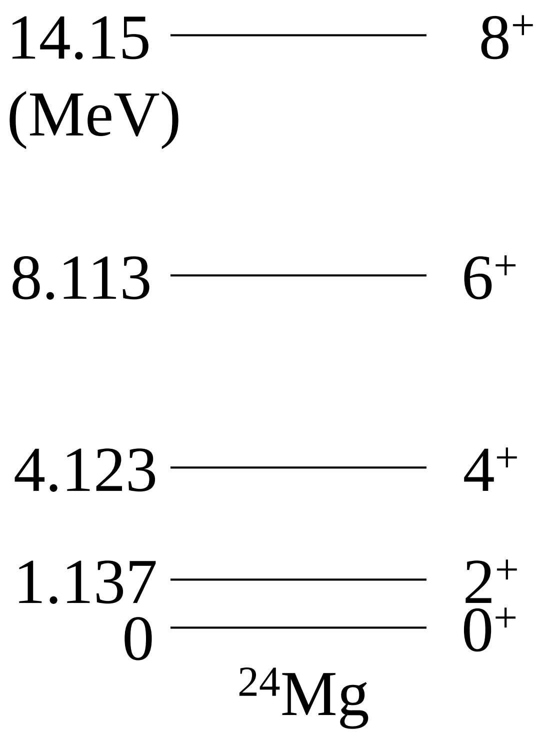
$$Q_0 \sim \sqrt{\frac{16\pi}{5}}\frac{3}{4\pi} Ze\, R_0^2\, \beta$$

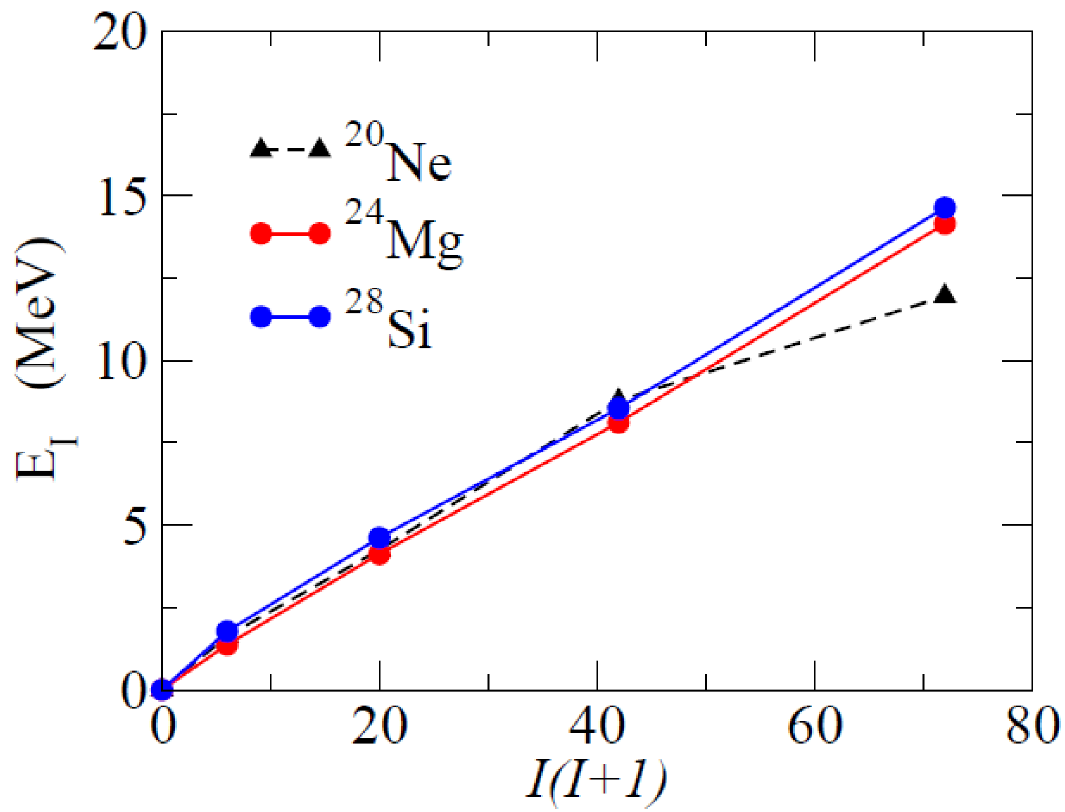
⇨ A change in B(E2) can be interpreted as a change in β

sd-shell nuclei : prominent nuclear deformation

an evidence for deformation

rotational spectrum

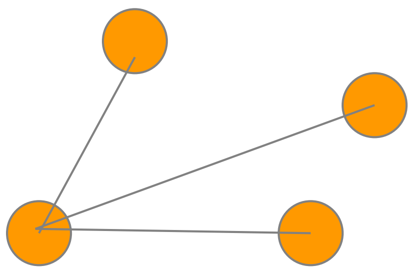


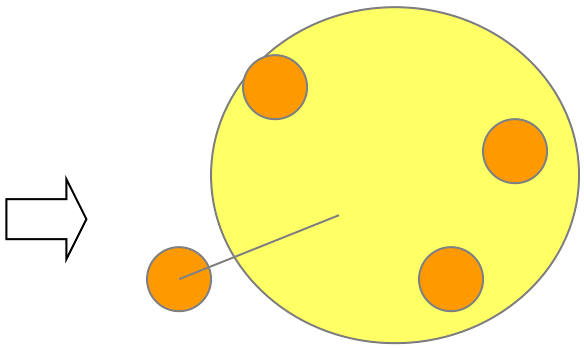


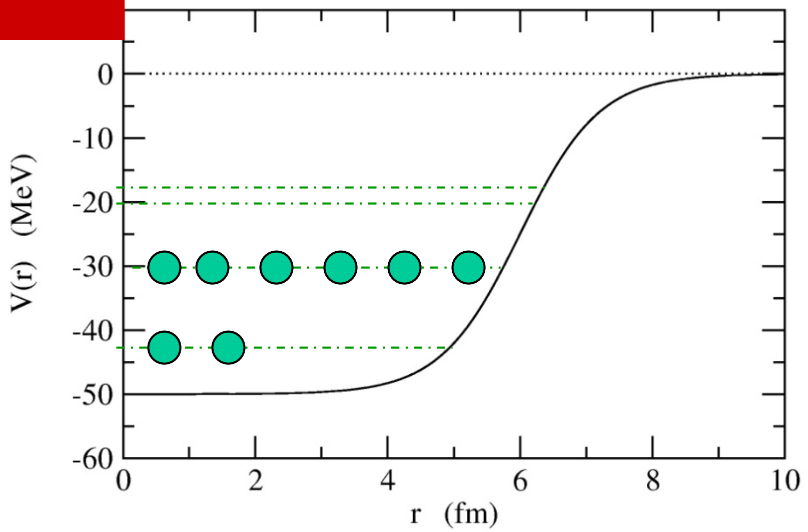
$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

How is the deformation altered due to an addition of Λ particle?

Self-consistent mean-field theory and nuclear deformation







treat the interaction with the other nucleons only on average

⇒ one-body problem with an effective potential

← the potential is determined so as to minimize the total energy (variational principle)

independent particle motion in a potential well

put nucleons from the bottom of the well according to Pauli principle (Slater determinant)

Hartree-Fock method and symmetry

Ψ_{HF} = an approximate solution of H (i.e., never eigen state)
= does not necessarily possess the symmetries that H has.

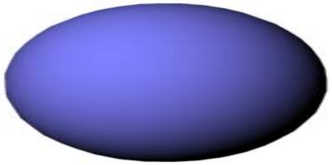
(Spontaneous) symmetry breaking

Advantage: a large part of many-body correlation can be taken into account without losing the independent particle picture

Disadvantage: a need to restore the symmetry (in principle) to compute experimental observables

- Translational symmetry: always broken in nuclear systems
- Rotational symmetry

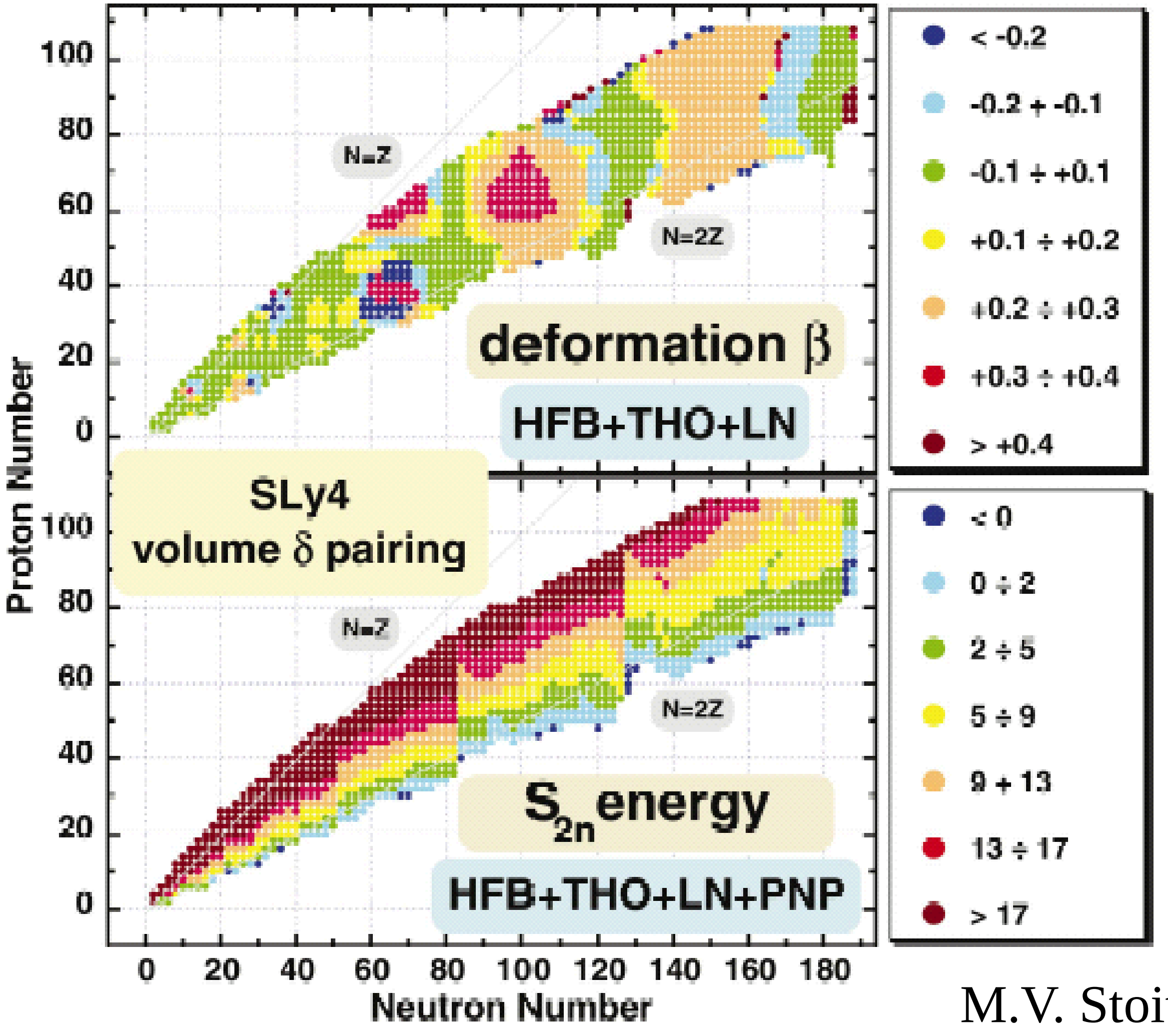
Deformed solution



optimized shape can be automatically determined
= suitable for discussion of shape of hypernuclei

well employed effective nucleon-nucleon interactions

- ✓Skyrme interaction (non-rel., density-dependent delta function)
- ✓Gogny interaction (non-rel., finite range)
- ✓Relativistic mean-field model (relativistic, “meson exchange”)

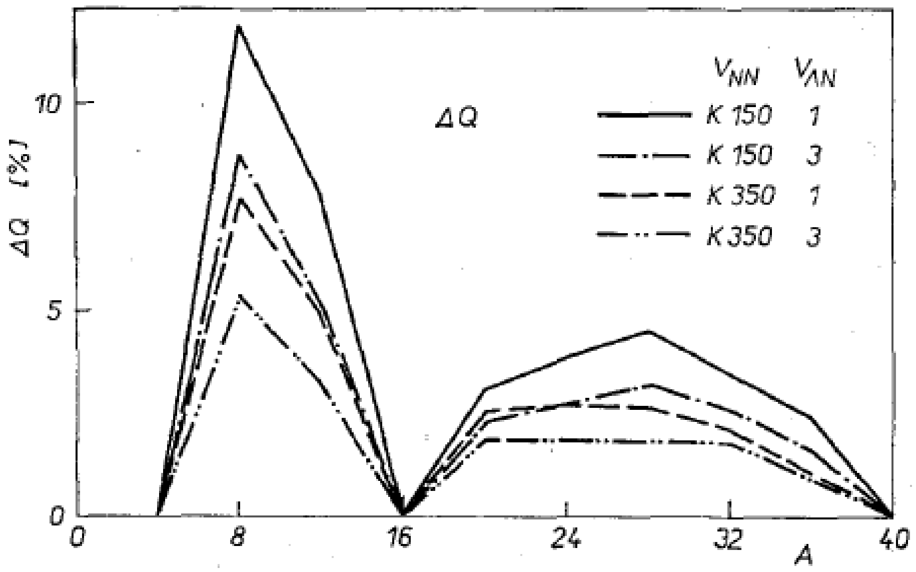


M.V. Stoitsov et al., PRC68(’03)054312

Shape of hypernuclei

J. Zofka, Czech. J. Phys. B30('80)95

Hartree-Fock calculations with V_{NN}: 3 range Gauss
$V_{\Lambda N}$: 2 range Gauss

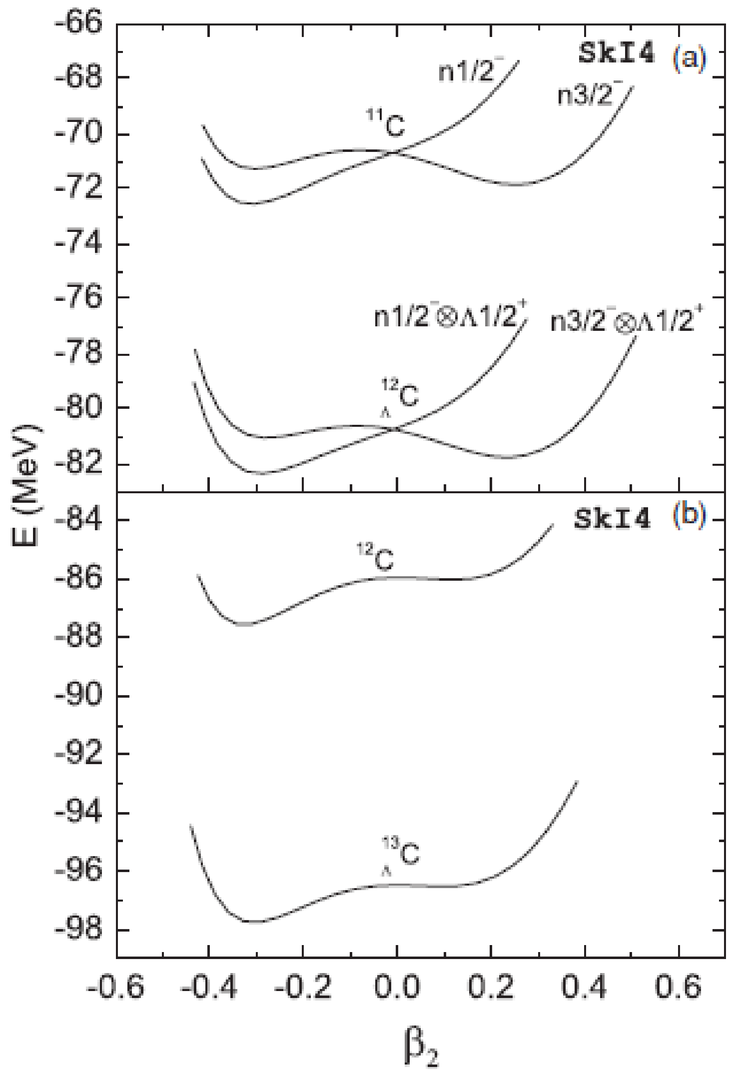


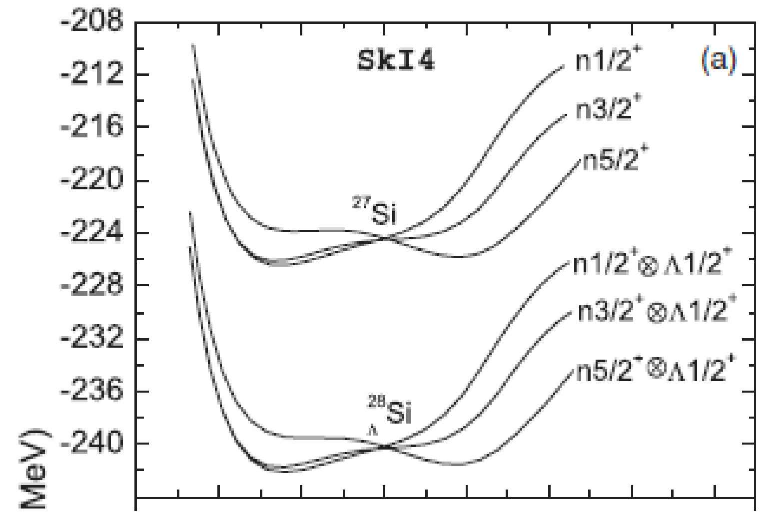
Λ changes the Q-moment (deformation) at most by 5%
e.g., $\beta = 0.5 \longrightarrow \beta = 0.475$

Shape of hypernuclei

Recent Skyrme-Hartree-Fock +BCS calculation by Zhou *et al.*

(with assumption of axial symmetry for simplicity)





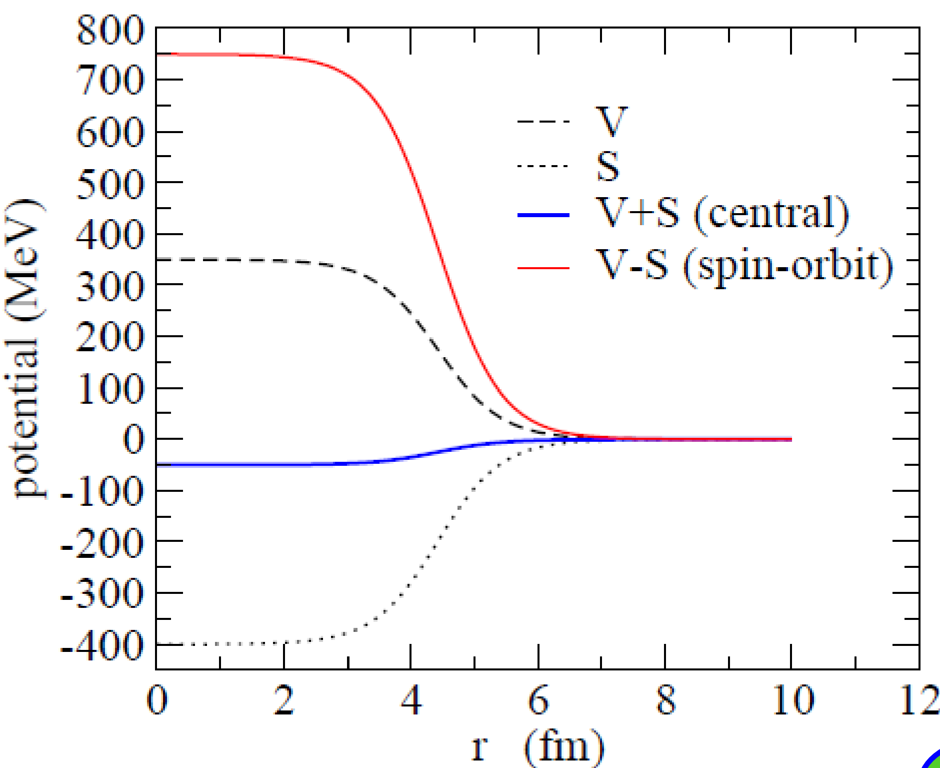
- ➢similar deformation between the hypernuclei and the core nuclei
- ➢hypernuclei: slightly smaller deformation than the core

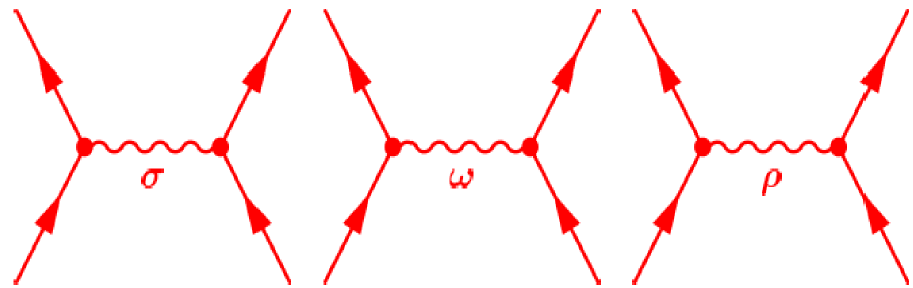
X.-R. Zhou *et al.*, PRC76(‘07) 034312

Deformation of Λ hypernuclei

Recent Skyrme-Hartree-Fock calculations by Zhou *et al.*

How about Relativistic Mean-Field (RMF) approach?





non-relativistic reduction

$$V_{\mathrm{cent}} = V + S$$

(strong cancellation between *V* and *S*)

$$V_{\mathrm{ls}} = \frac{m}{m - (V-S)/2}(V - S)$$

changes in *V* and *S* due to a Λ particle are emphasized
(only in RMF)

cf. D. Vretenar et al.,
PRC57('98)R1060

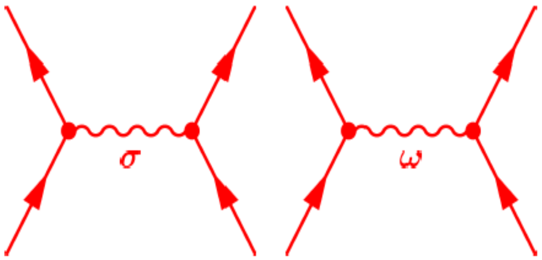
RMF for deformed hypernuclei

$$\mathcal{L} = \mathcal{L}_N + \bar{\psi}_\Lambda \left[\gamma_\mu (i\partial^\mu - g_{\omega\Lambda}\omega^\mu) - m_\Lambda - g_{\sigma\Lambda}\sigma\right]\psi_\Lambda$$

$$g_{\omega\Lambda} = \frac{2}{3} g_{\omega N} \longleftarrow \text{quark model}$$

$$g_{\sigma\Lambda} = 0.621 g_{\sigma N} \longleftarrow {}^{17}_{\Lambda}\text{O}$$

cf. D. Vretenar et al., PRC57('98)R1060



Λσ and Λω couplings

variational principle

$$\left[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta\left(m_\Lambda + g_{\sigma\Lambda}\sigma(\boldsymbol{r})\right) + g_{\omega\Lambda}\omega^0(\boldsymbol{r})\right]\psi_\Lambda = \epsilon_\Lambda\psi_\Lambda$$

$$[-\boldsymbol{\nabla}^2 + m_\omega^2]\omega^0(\boldsymbol{r}) = g_\omega\rho_v(\boldsymbol{r}) + g_{\omega\Lambda}\psi_\Lambda^\dagger(\boldsymbol{r})\psi_\Lambda(\boldsymbol{r})$$

etc.

→ self-consistent solution (iteration)

RMF for deformed hypernuclei

self-consistent solution (iteration)

→ (intrinsic) Quadrupole moment

$$Q = \sqrt{\frac{16\pi}{5}} \int d\boldsymbol{r} \, [\rho_v(\boldsymbol{r}) + \psi^\dagger_\Lambda(\boldsymbol{r})\psi_\Lambda(\boldsymbol{r})] \, r^2 Y_{20}(\hat{\boldsymbol{r}})$$

Application to hypernuclei

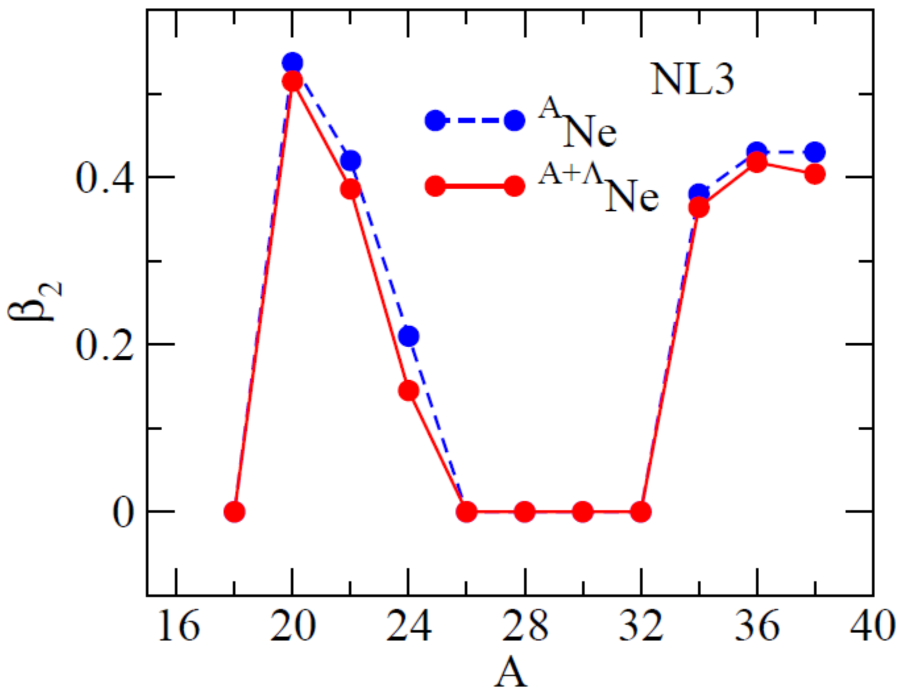
- parameter sets: NL3 and NLSH
- Axial symmetry
- pairing among nucleons: Const. gap approach

  $\Delta_n = 4.8/N^{1/3} \quad \Delta_p = 4.8/Z^{1/3}$ (MeV)
- Λ particle: the lowest s.p. level ($K^\pi = 1/2^+$)
- Basis expansion with deformed H.O. wf
- Deformation parameter:

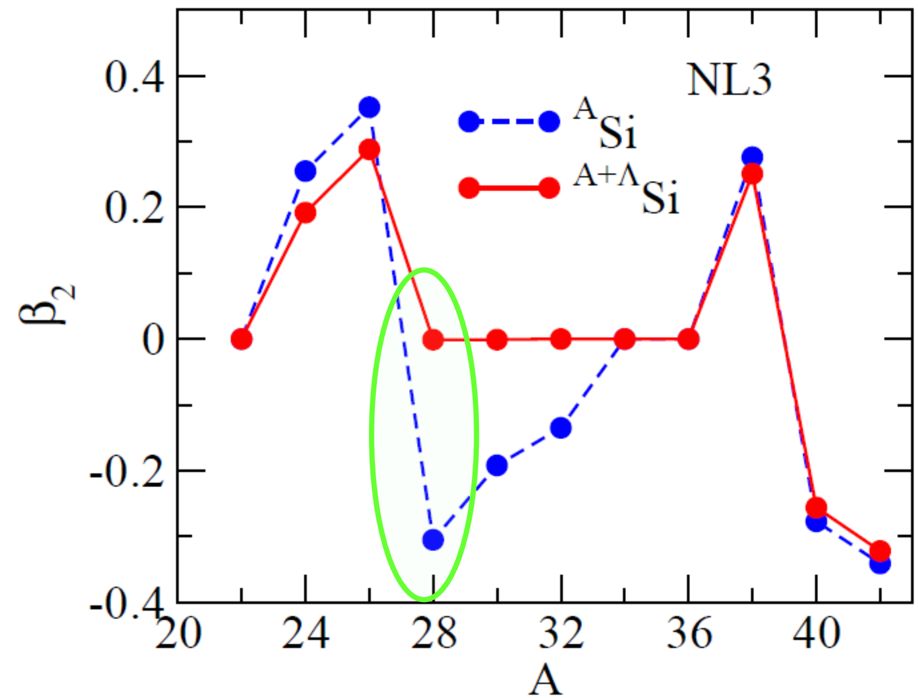
$$Q = \sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} (A_c + 1) R_0^2 \, \beta$$

$$R_0 = 1.2 \, A_c^{1/3} \text{ (fm)}$$

Ne isotopes



Si isotopes



- in most cases, similar deformation between the core and the hypernuclei
- hypernuclei: slightly smaller deformation than the core

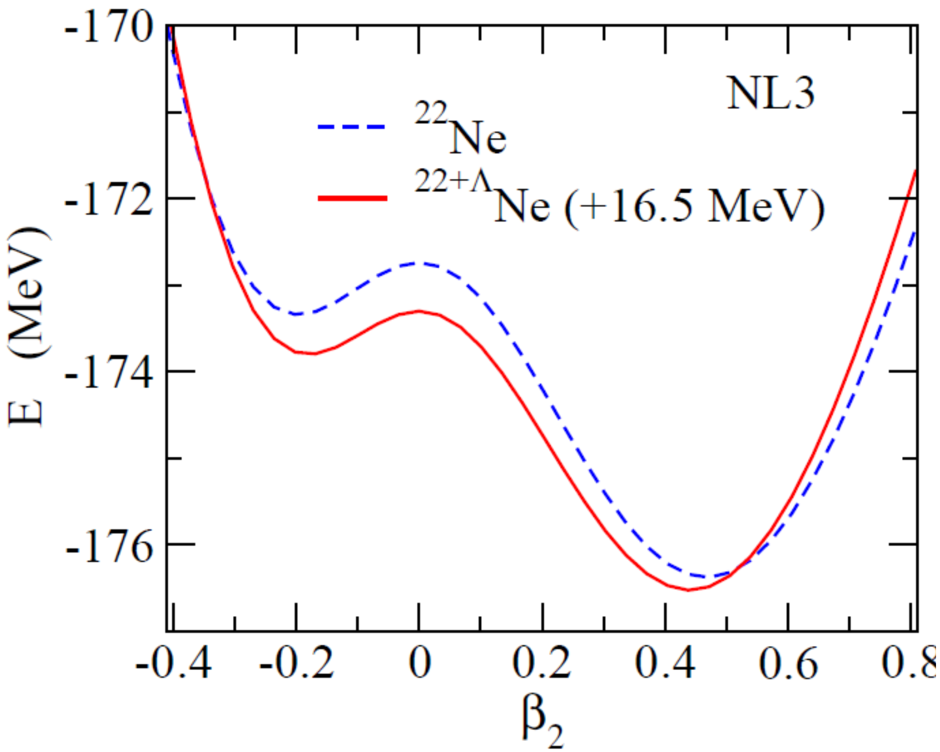
⟶ conclusions similar to Skyrme-Hartree-Fock (Zhou *et al.*)

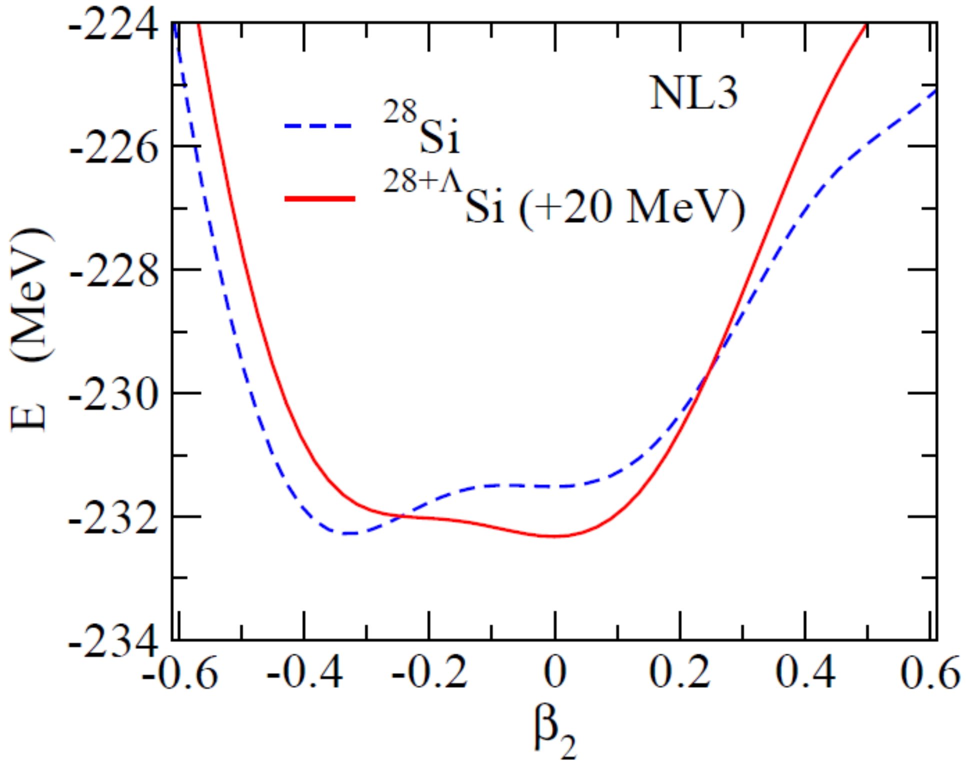
Exception: $^{29}_{\Lambda}$Si

oblate (^{28}Si) $\xrightarrow{\Lambda}$ spherical ($^{29}_{\Lambda}$Si)

Myaing Thi Win and K.H., PRC78('08)054311

Potential energy surface (constraint Hartree-Fock)





If the energy curve is relatively flat, a large change in nuclear deformation can occur due to an addition of Λ particle

the same conclusion also with NLSH and/or with another treatment of pairing correlation (constant G approach)

Myaing Thi Win and K.H., PRC78('08)054311

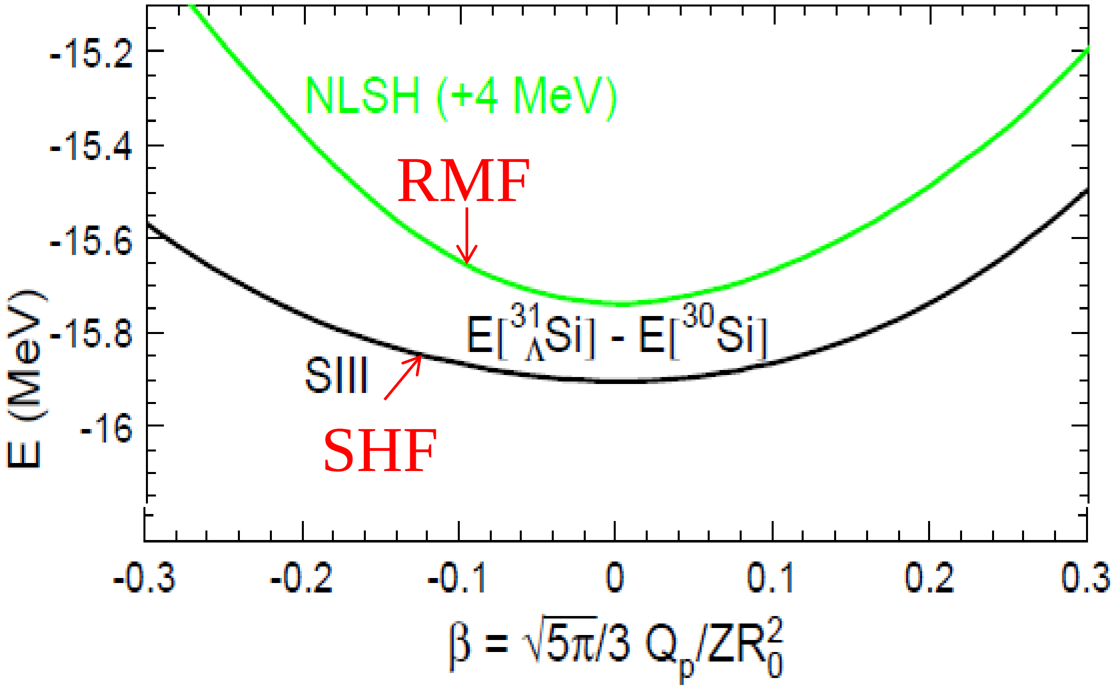
Comparison between RMF and SHF

- Gain of binding energy= $E_{^{30+\Lambda}\mathrm{Si}} - E_{^{30}\mathrm{Si}}$
  - ➢ in spherical configuration

$$\Delta E = E(\beta = -0.2) - E(\beta = 0) \approx 0.14\mathrm{MeV} \quad \mathbf{(SHF)}$$

$$\Delta E = E(\beta = -0.2) - E(\beta = 0) \approx 0.3\mathrm{MeV} \quad \mathbf{(RMF)}$$

- Larger effect of NΛ force in RMF



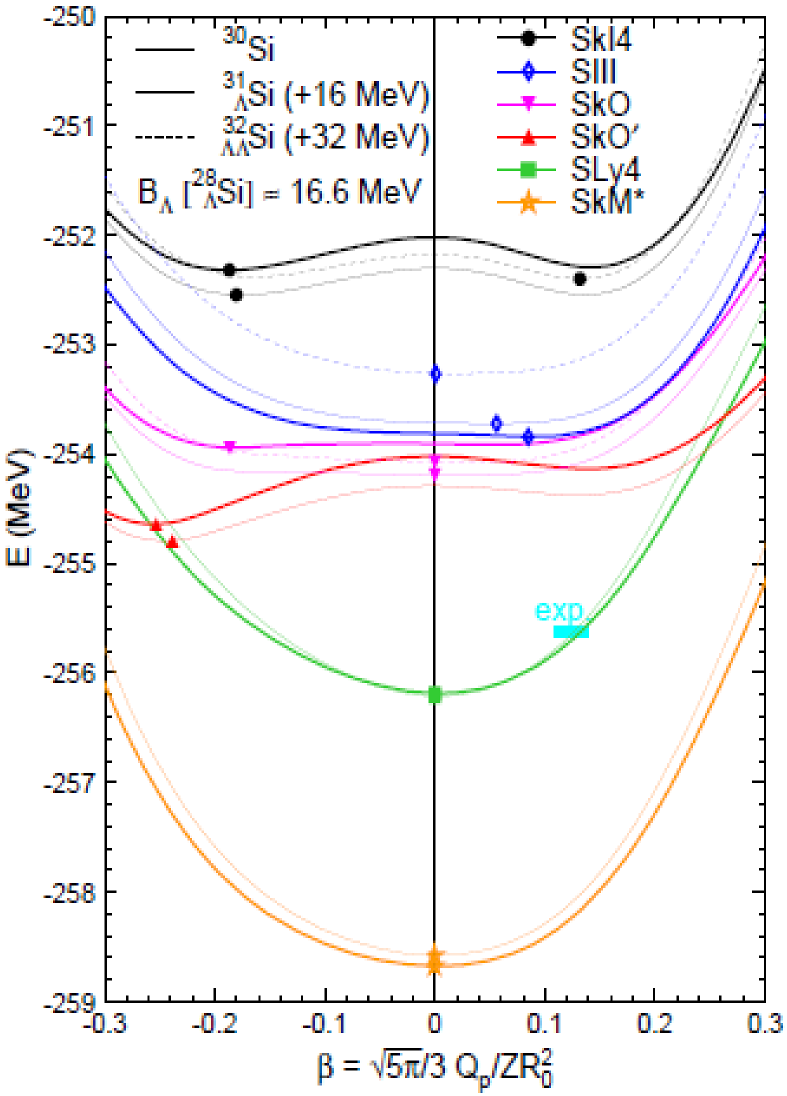
H.-J.Schulze, M.Thi Win, K.Hagino,H.Sagawa Prog.Theo.Phys123('10) 569

Systematic comparison with Skyrme-Hartree-Fock method：

- ➢Stronger influence of Λ in RMF than in SHF
- ➢Disappearance of deformation can happen also with SHF if the energy curve is very flat

H.-J. Schulze, Myaing Thi Win, K.H., H. Sagawa, PTP123(‘10)569

A key point is a flatness of potential energy curve

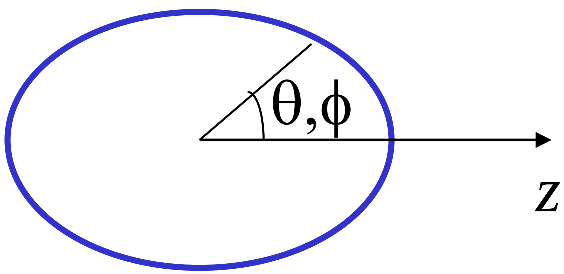


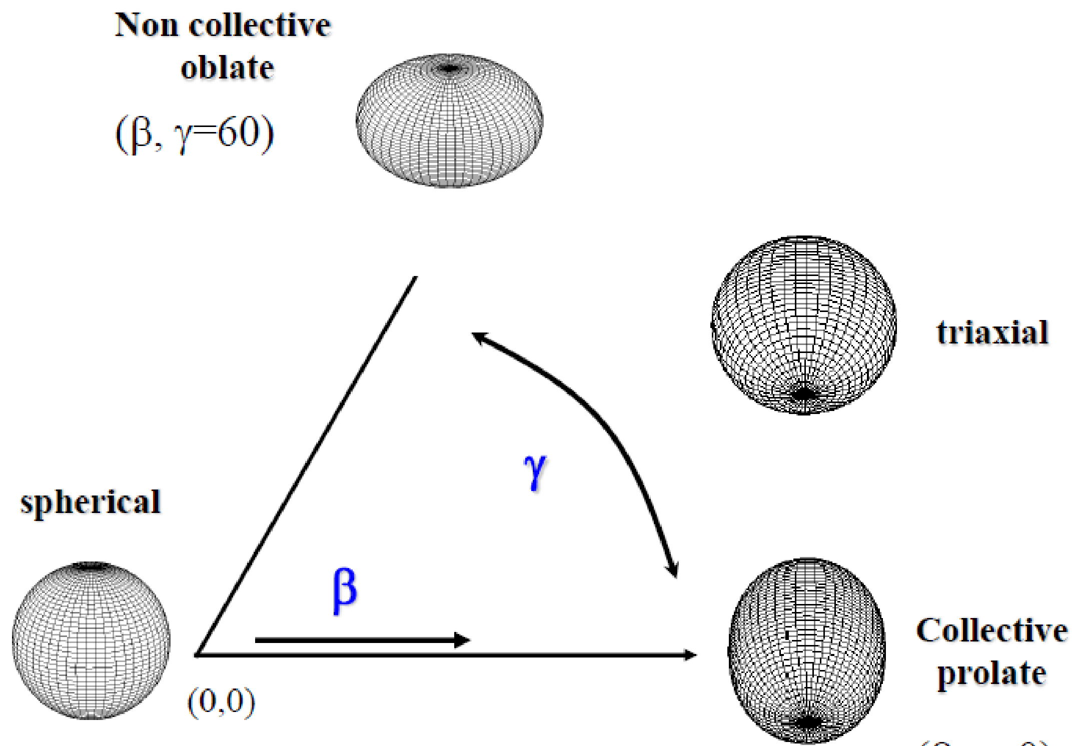
3D Hartree-Fock calculation for hypernuclei

So far, axial symmetric shape has been assumed for simplicity

→ Effect of Λ particle on triaxial deformation?

$$R(\theta,\phi) = R_0\left[1+\beta\cos\gamma Y_{20}(\theta)+\frac{1}{\sqrt{2}}\beta\sin\gamma(Y_{22}(\theta,\phi)+Y_{2-2}(\theta,\phi))\right]$$



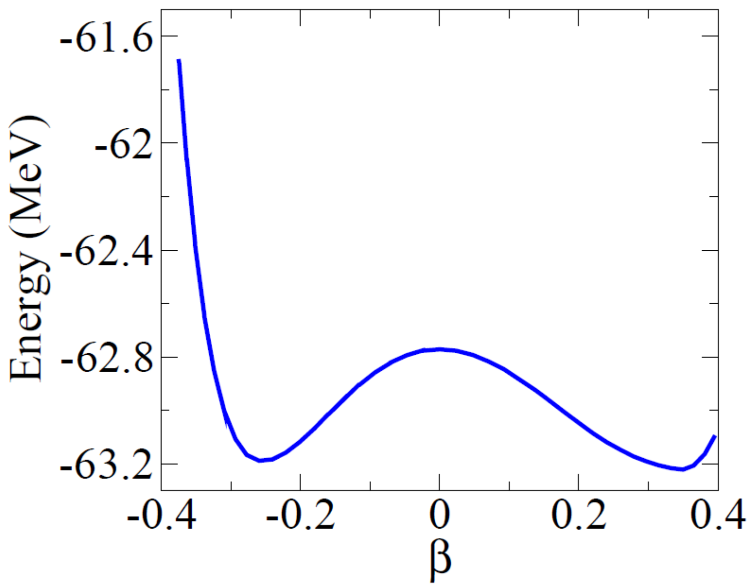


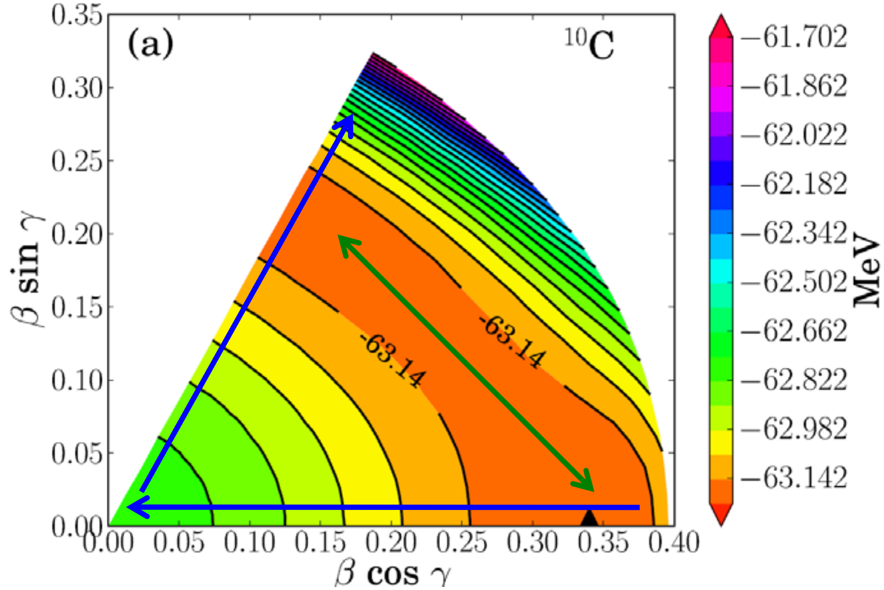
Courtesy: Takeshi Koike

3D Hartree-Fock calculation for hypernuclei

It is often said:

even if the barrier is heigh along the axial deformation,
the potential surface may be flat along triaxiality (shape coexistence)





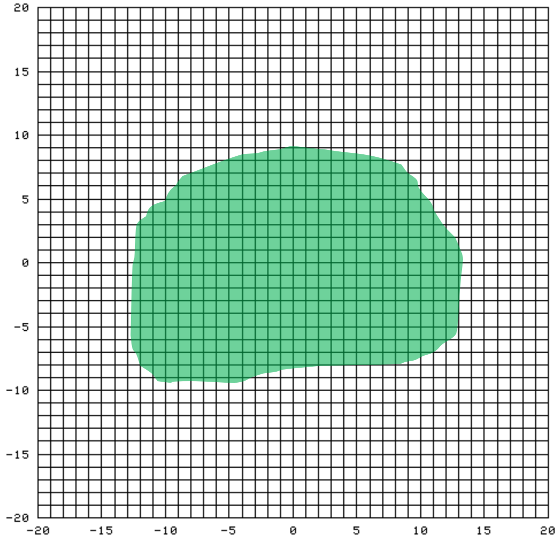
Important to discuss the energy surface in 3D (β,γ) deformtion plane?

Skyrme-Hartree-Fock calculations for hypernuclei

3D calcaulations with non-relativistic Skyrme-Hartree-Fock:
the most convenient and the easiest way

- 3D mesh calculation ("lattice Hartree-Fock")
- Imaginary time evolution of single-particle wave functions
- computer code "ev8" available

  P. Bonche, H. Flocard, and P.-H. Heenen,
  NPA467('87)115, CPC171('05)49



$$\phi_k(x,y,z) \sim \phi_k(n_x\Delta x, n_y\Delta y, n_z\Delta z)$$
$$\phi_k(x,y,z) = \lim_{\tau\to\infty} e^{-\hat{h}\tau}\phi_k^{(0)}(x,y,z)$$

(note)
$$\begin{aligned} e^{-\hat{h}\tau}\phi^{(0)} &= e^{-\hat{h}\tau}\sum_k C_k\phi_k \\ &= \sum_k e^{-e_k\tau}C_k\phi_k \\ &\to e^{-e_0\tau}C_0\phi_0 \qquad (\tau\to\infty) \end{aligned}$$

Skyrme-Hartree-Fock calculations for hypernuclei

3D calcaulations with non-relativistic Skyrme-Hartree-Fock:
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→ extension to hypernuclei

$$
\begin{aligned}
v_{\Lambda N}(\boldsymbol{r}_\Lambda, \boldsymbol{r}_N) &= t_0(1 + x_0 P_\sigma)\delta(\boldsymbol{r}_\Lambda - \boldsymbol{r}_N) + \cdots \\
v_{\Lambda NN}(\boldsymbol{r}_\Lambda, \boldsymbol{r}_1, \boldsymbol{r}_2) &= t_3\delta(\boldsymbol{r}_\Lambda - \boldsymbol{r}_1)\delta(\boldsymbol{r}_\Lambda - \boldsymbol{r}_2)
\end{aligned}
$$

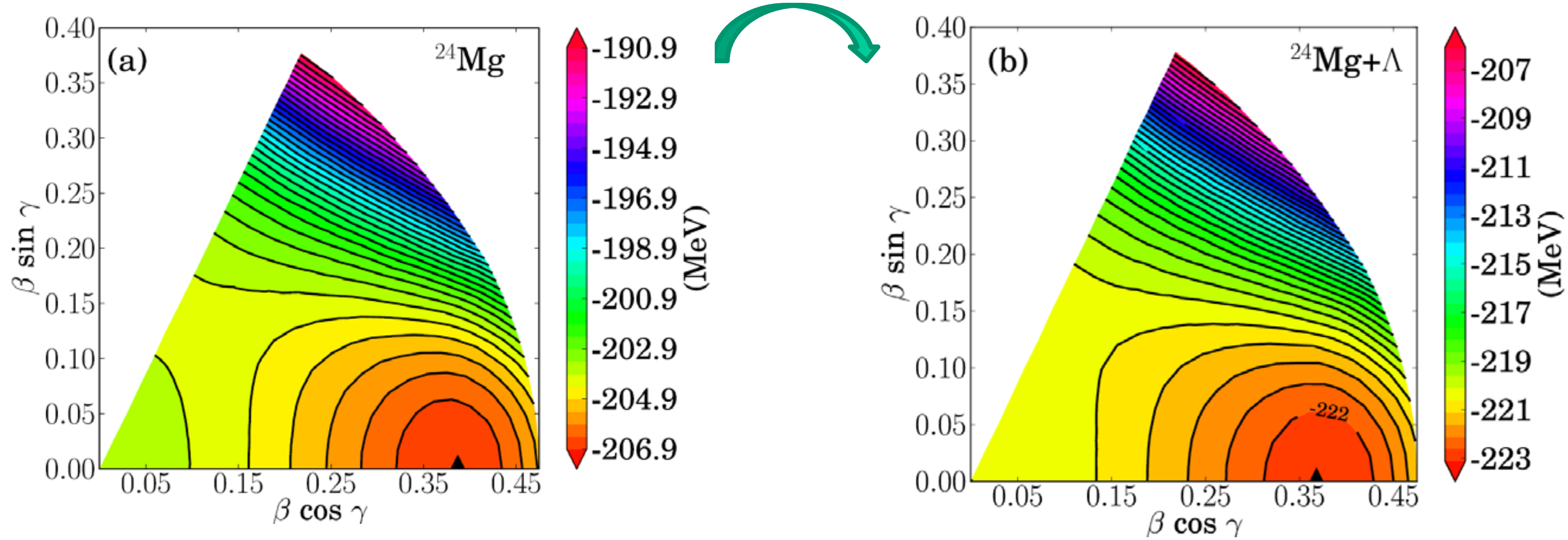
M. Rayet, NPA367('81)381

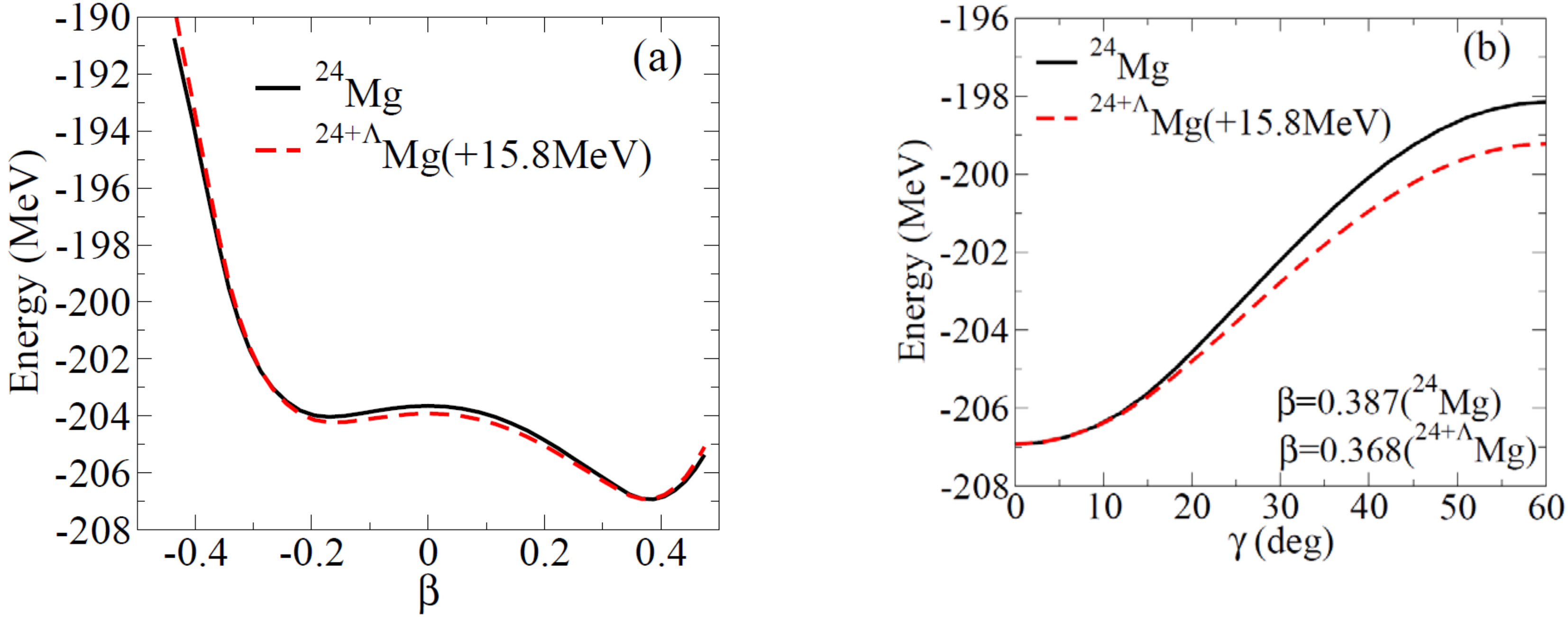
∗ Interaction No.1 of Yamamoto *et al.* + SGII (NN)
(Y. Yamamoto, H. Bando, and J. Zofka, PTP80('88)757)

∗ Pairing among nucleons: BCS approximation with d.d. contact force

∗ Λ particle: the lowest energy state

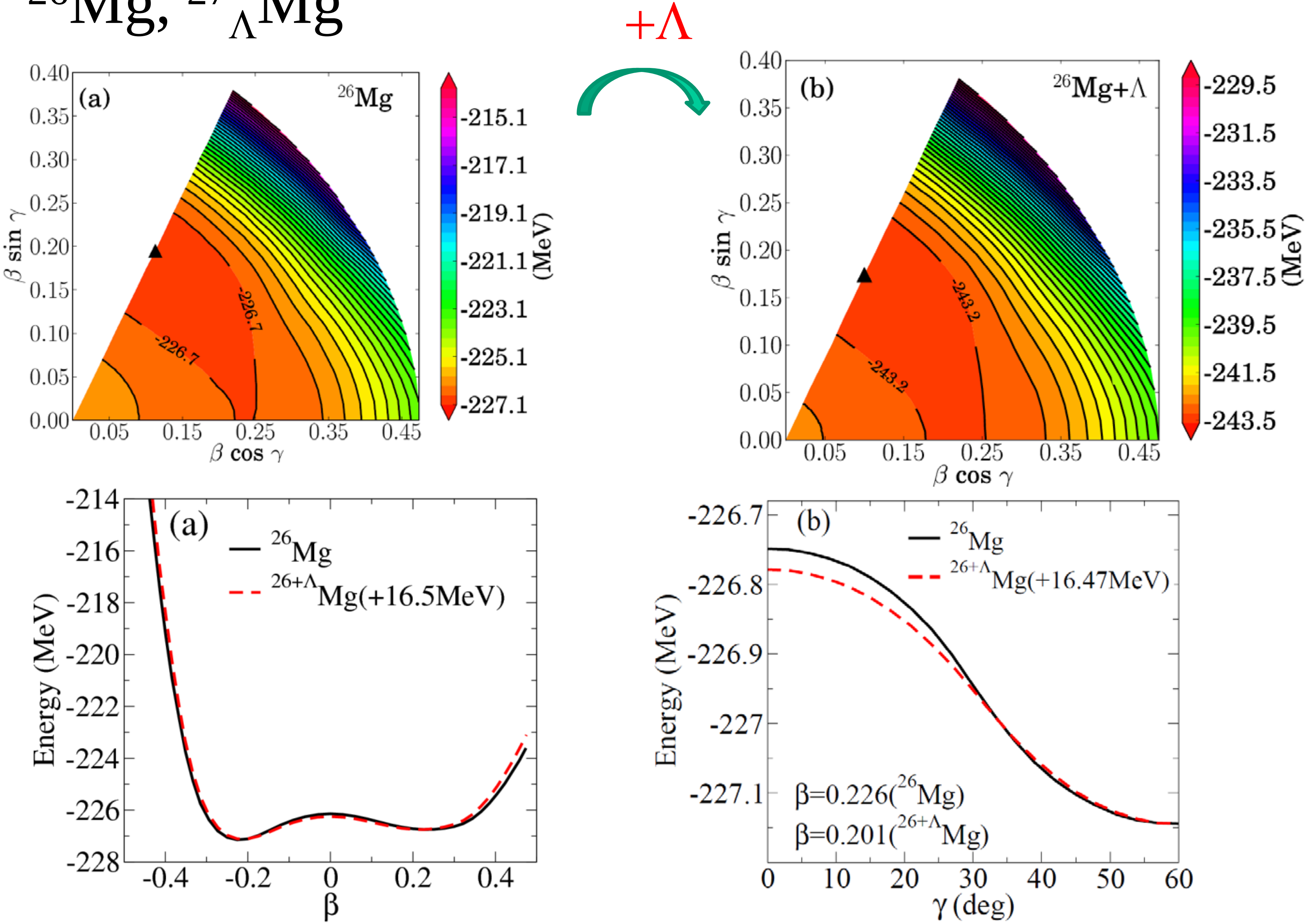
^{24}Mg, $^{25}{}_{\Lambda}$Mg





Myaing Thi Win, K.H., T. Koike, Phys. Rev. C83('11)014301

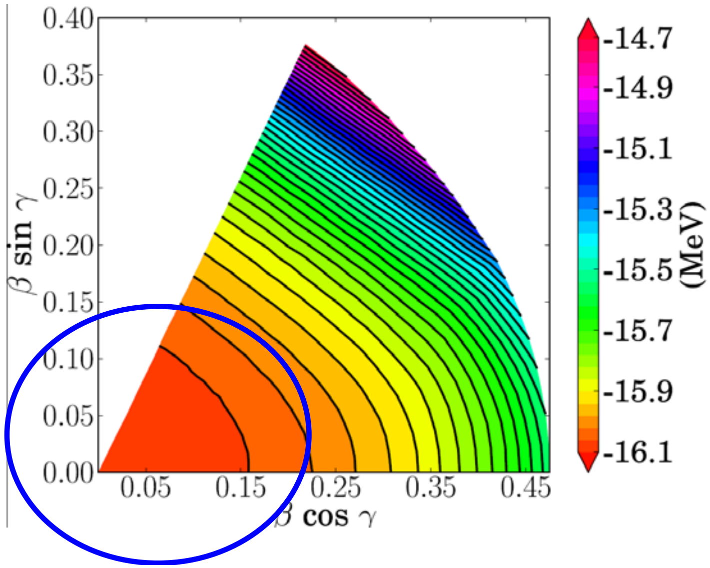
^{26}Mg, $^{27}{}_{\Lambda}$Mg



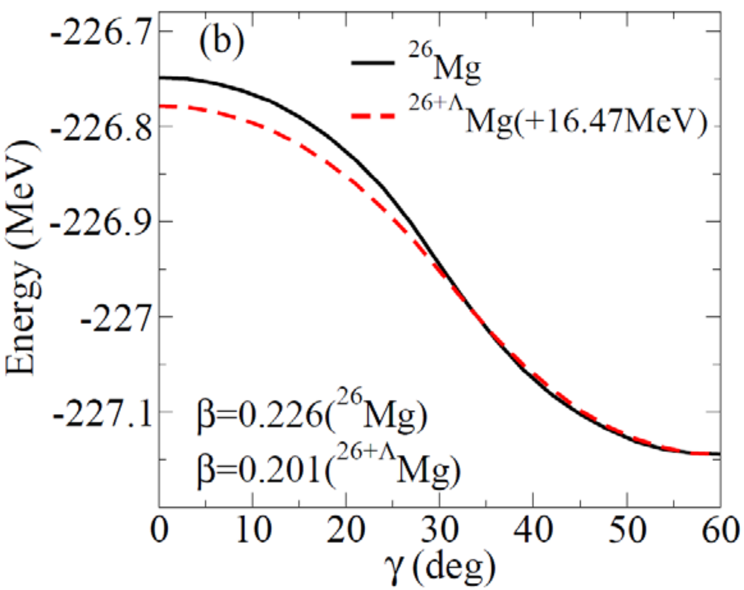
Myaing Thi Win, K.H., T. Koike, Phys. Rev. C83('11)014301

Discussions

$$E_{{}^{25}_{\Lambda}Mg}(\beta,\gamma) - E_{{}^{24}Mg}(\beta,\gamma)$$



- ➢Deformation is driven to speherical when Λ is in the lowest state (⟶how about Λ in an excited state?)
- ➢Prolate configuration is prefered for the same value of β



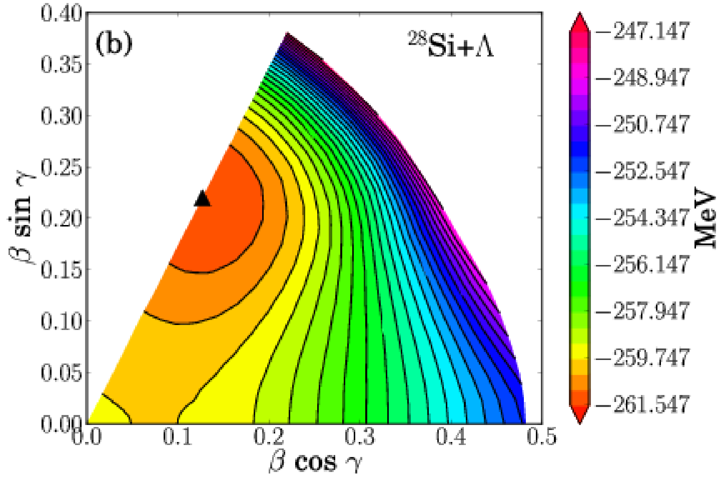
All of ^{24}Mg, ^{26}Mg, ^{26}Si, ^{28}Si show that Λ makes the curvature along the γ direction somewhat smaller

Experiment? (the energy of 2_2^+ state)

quantitative estimat：RPA or GCM

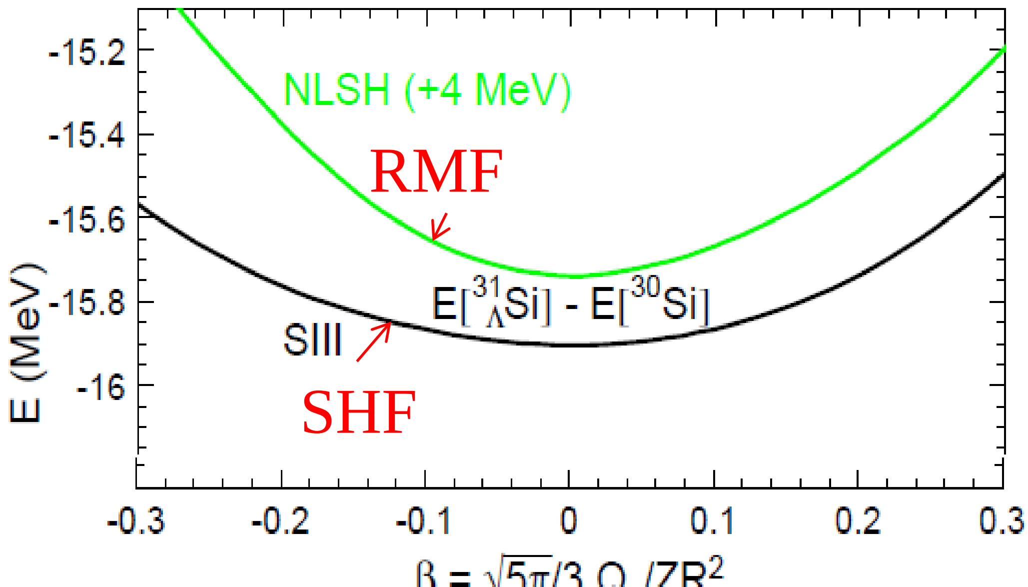
or Bohr Hamiltonian

Towards a 3D-mesh RMF calculation

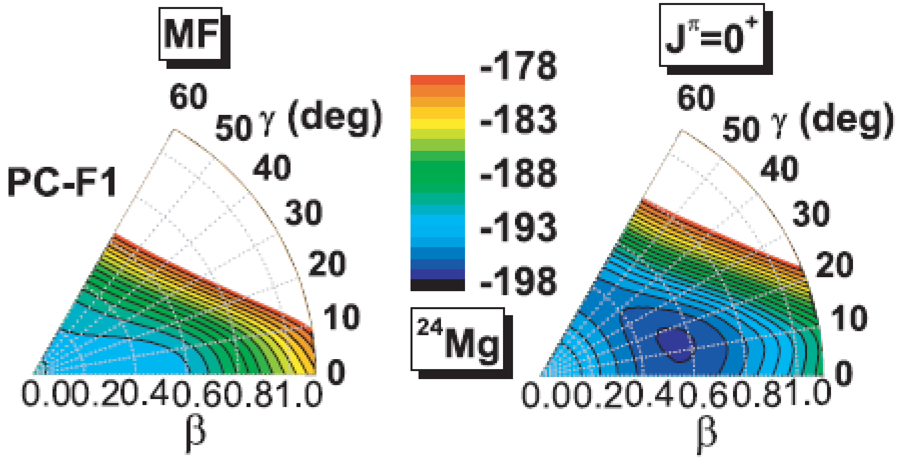


So far, we have done 3D calculations for hypernuclei only with SHF.

It will be interesting to perform similar 3D studies with RMF (stronger Λ effects expected).



there have not been many 3D RMF calculations…..



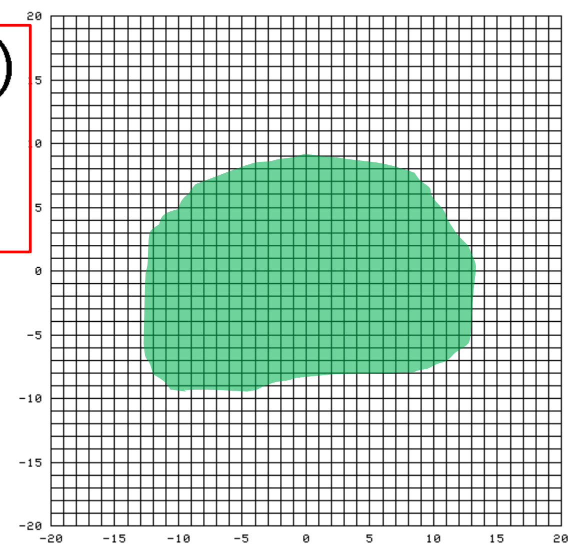
W. Koepf and P. Ring, PLB212('88)397
J.M. Yao *et al.*, PRC81('10)044311
PRC83('11)014308

expansion with 3D HO basis

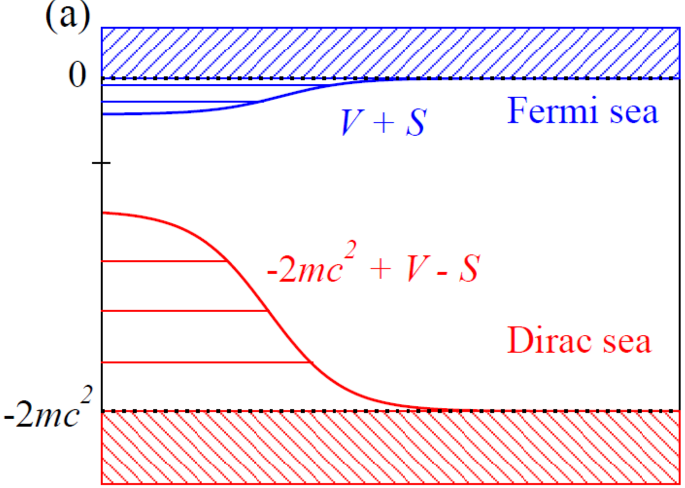
No 3D mesh calculations with RMF!!

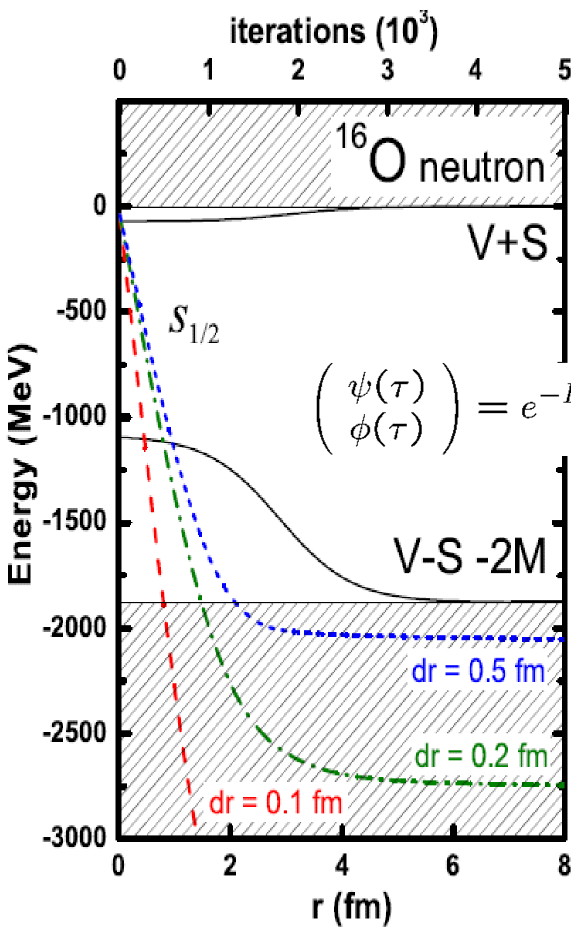
$$\phi_k(x,y,z) \sim \phi_k(n_x\Delta x, n_y\Delta y, n_z\Delta z)$$

$$\phi_k(x,y,z) = \lim_{\tau\to\infty} e^{-\hat{h}\tau}\phi_k^{(0)}(x,y,z)$$



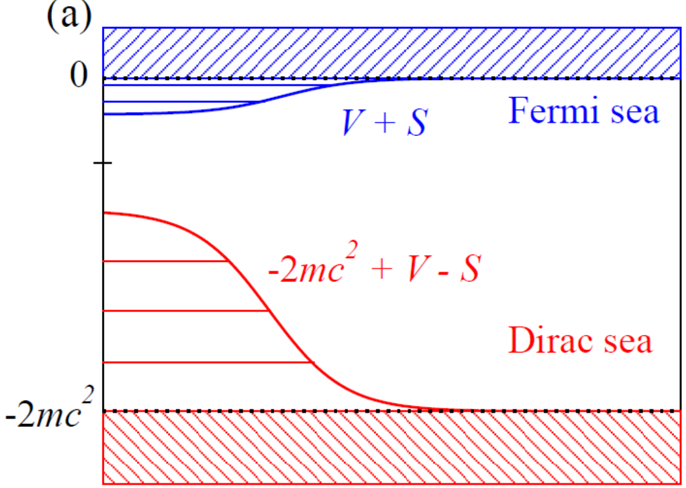
⟶ difficulty with imaginary time evolution
(variational collapse)

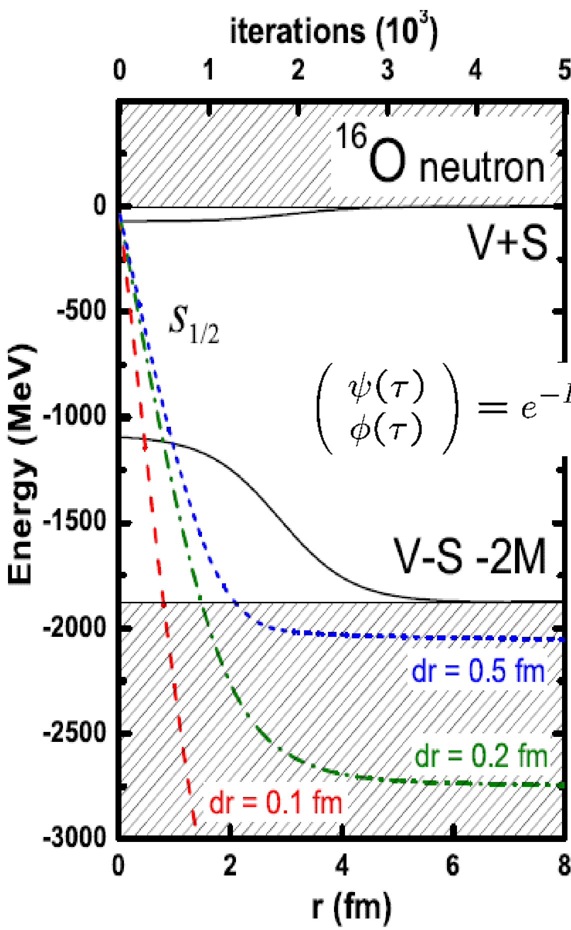




$$\begin{pmatrix} \psi(\tau) \\ \phi(\tau) \end{pmatrix} = e^{-H_{\mathrm{Dirac}}\tau} \begin{pmatrix} \psi(0) \\ \phi(0) \end{pmatrix}$$

Ying Zhang et al.,
Int. J. Mod. Phys. E19('10)55





$$\begin{pmatrix} \psi(\tau) \\ \phi(\tau) \end{pmatrix} = e^{-H_{\text{Dirac}}\tau} \begin{pmatrix} \psi(0) \\ \phi(0) \end{pmatrix}$$

Ying Zhang et al., :
application of im. time method to the Schrodinger-equivalent form of Dirac eq.

$$H_{\text{Dirac}} \begin{pmatrix} \psi \\ \phi \end{pmatrix} = \epsilon \begin{pmatrix} \psi \\ \phi \end{pmatrix}$$

$$\Downarrow$$

$$H_{\text{eff}}(\epsilon)\psi = \epsilon\psi$$

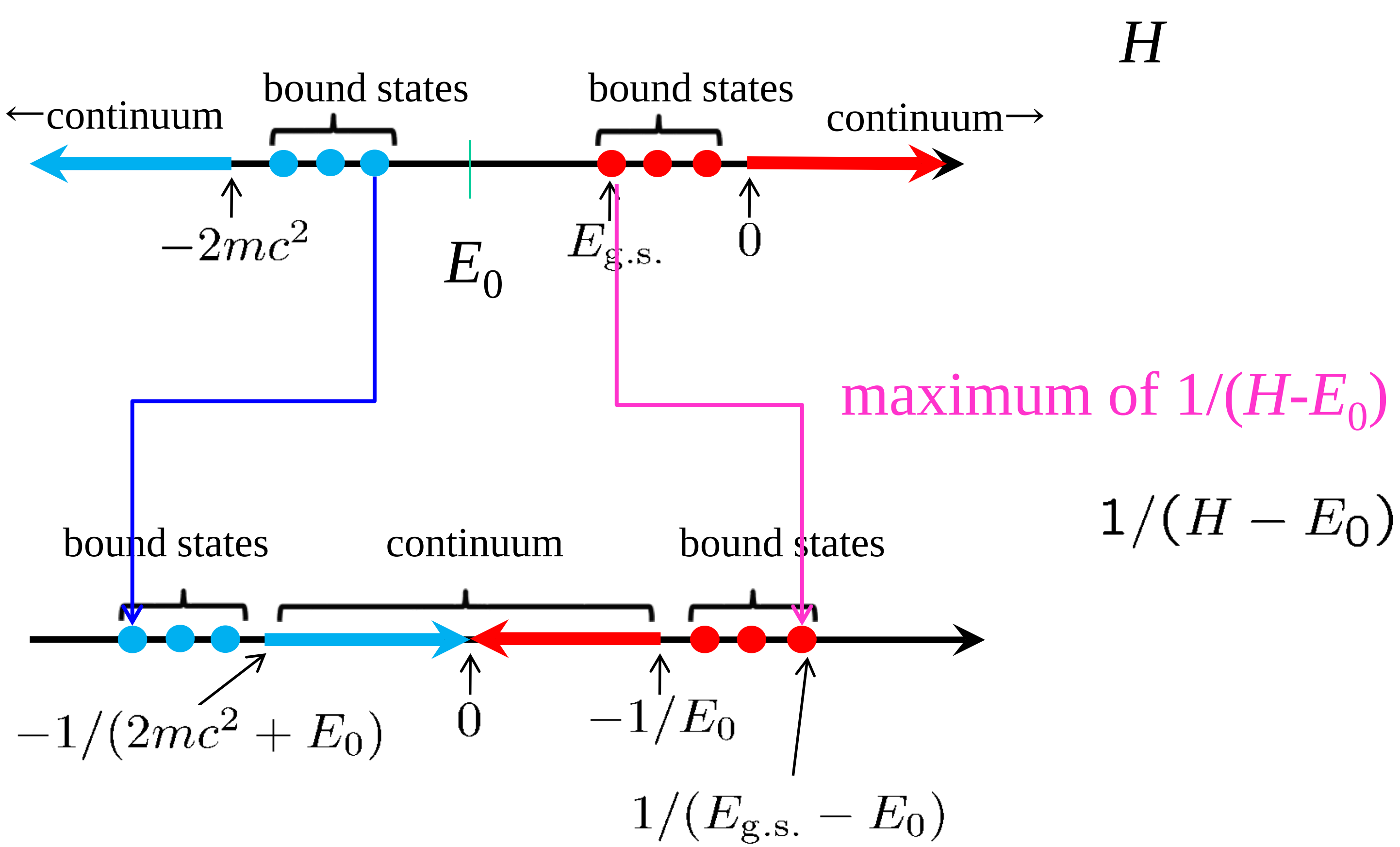
$$\Downarrow$$

$$\psi(\tau) = e^{-H_{\text{eff}}(\epsilon)\tau}\psi(0)$$

Ying Zhang et al.,
Int. J. Mod. Phys. E19(‘10)55

Alternative method: inverse Hamiltonian method

K.H. and Y. Tanimura, PRC82('10)057301



R.N. Hill and C. Krauthauser, PRL72('94)2151

How to maximize $1/(H\text{-}E_0)$?

as T goes to infinity, only the lowest energy state above E_0 survives:

$$\lim_{T\to\infty} \exp\left(\frac{T}{H-E_0}\right)|\Psi\rangle = \lim_{T\to\infty} \sum_n e^{T/(\varepsilon_n - E_0)}|\phi_n\rangle\langle\phi_n|\Psi\rangle \propto |\phi_{\rm g.s.}\rangle$$

In practice

$$|\Psi^{n+1}\rangle \simeq \left(1 + \frac{\Delta T}{H-E_0}\right)|\Psi^n\rangle$$

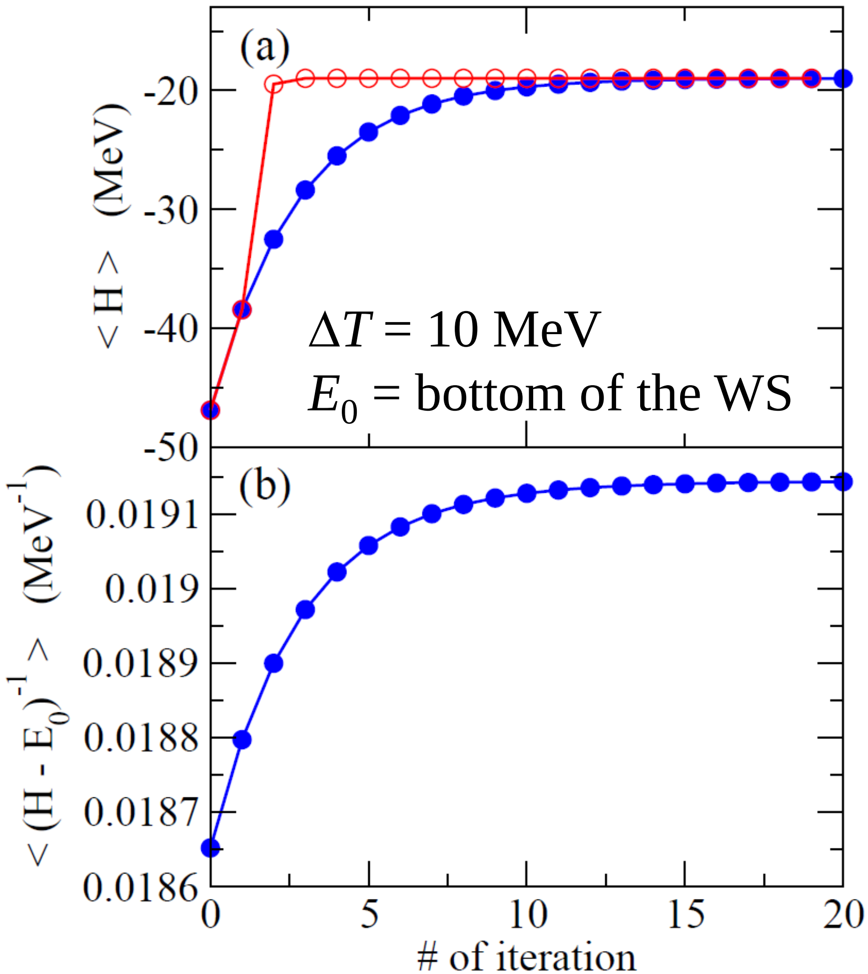
* for a spherical potential, it is easy to take an inverse of H

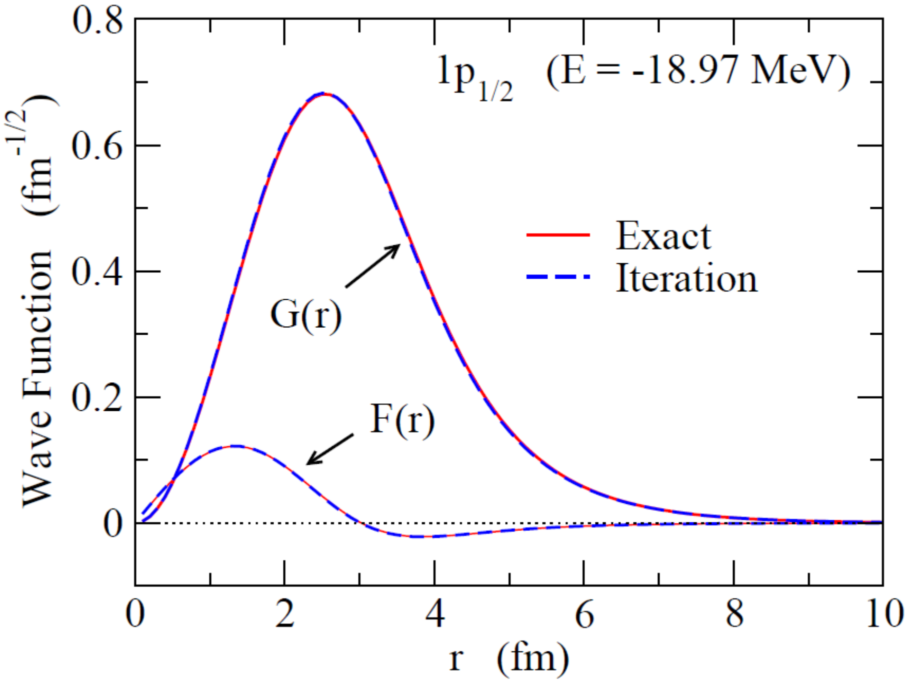
cf. Skyrme TDHF: implicit method for time-evolution

$$|\phi^{n+1}\rangle = \frac{1-iH\frac{\Delta t}{2\hbar}}{1+iH\frac{\Delta t}{2\hbar}}|\phi^n\rangle = \left(\frac{2}{1+iH\frac{\Delta t}{2\hbar}} - 1\right)|\phi^n\rangle$$

S.E. Koonin et al., PRC15 ('77) 1359

$1p_{1/2}$ state of ^{16}O (Woods-Saxon potential)

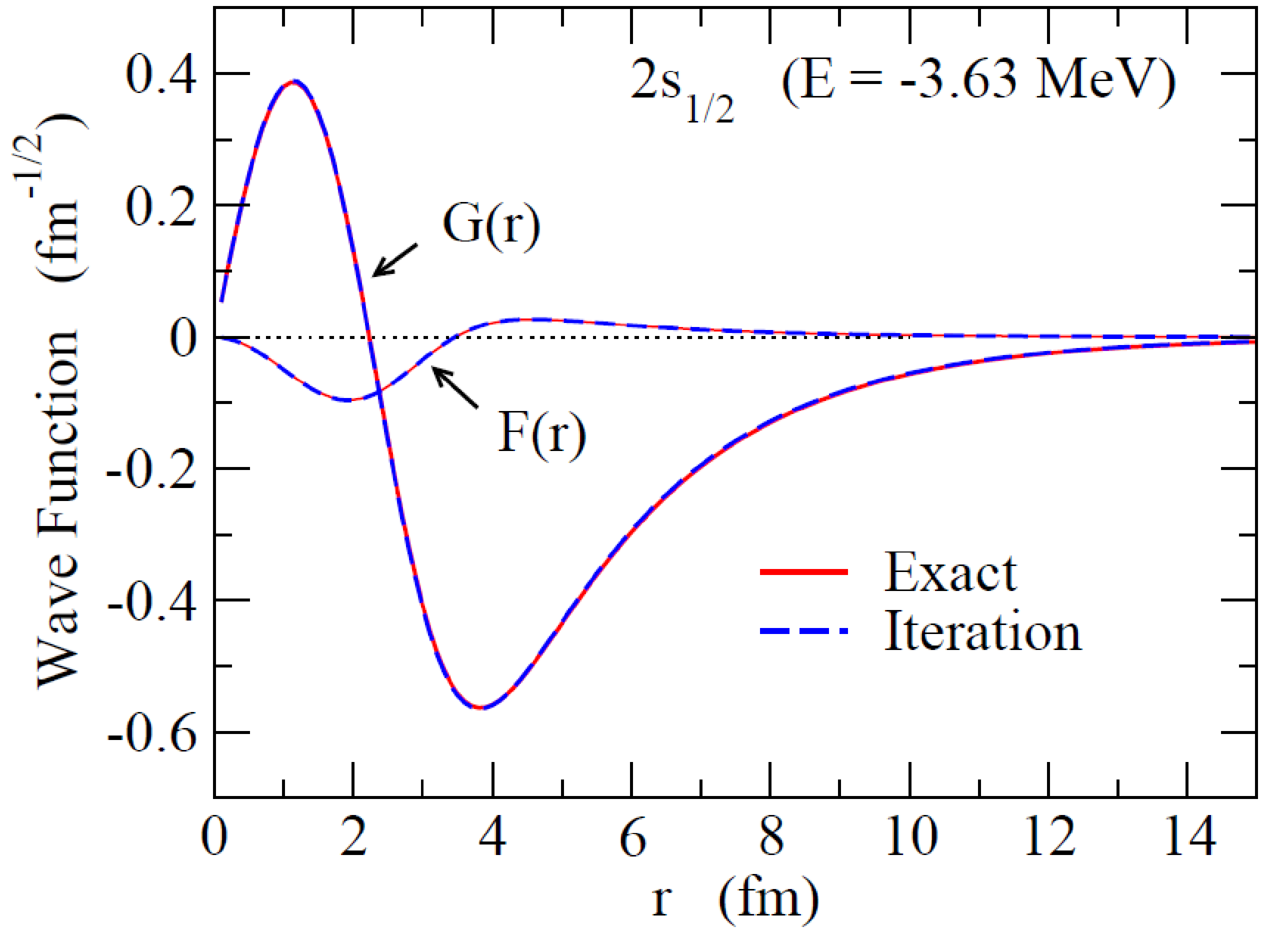




E_{exact} = -18.976 MeV
$E_{iterative}$ = -18.974 MeV

K.H. and Y. Tanimura, PRC82('10)057301

$2s_{1/2}$ state of ^{16}O (Woods-Saxon potential)



K.H. and Y. Tanimura, PRC82('10)057301

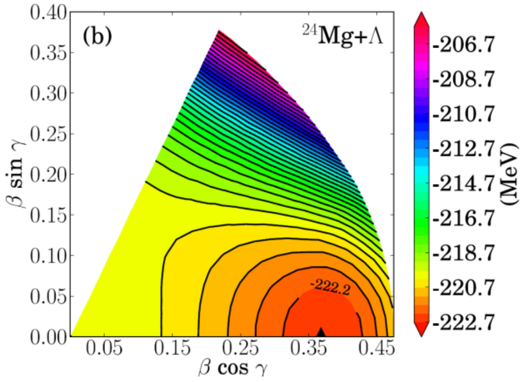
Summary

Shape of Λ hypernuclei: from the view point of mean-field theory

- ➢deformation: in important key work in the sd-shell region
- ➢RMF: stronger influence of Λ particle

  ⟶ Shape of ^{28}Si : drastically changed due to Λ
- ➢SHF: weaker influence of Λ, but large def. change if PES is very flat

  - •3D calcaulations
  - •softening of γ-vibration?



next step :

- ➢estimate the spectrum with beyond-MF methods
  - •Ang. Mom. Proj. (rotational spectrum)
  - •GCM or RPA (vibrational spectrum)
  - •5D Bohr Hamiltonian
- ➢3D-mesh RMF calculations? ⟵ inverse H method