Microscopic particle-rotor model for low-lying spectrum of $\Lambda$ hypernuclei

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Impurity effects: one of the main interests of hypernuclear physics

**how does \( \Lambda \) affect several properties of atomic nuclei?**

- size, shape, density distribution, single-particle energy, shell structure, fission barrier……

✓ the most prominent example:
the reduction of B(E2) in \( ^7\Lambda Li \)

about 19% reduction of nuclear size (shrinkage effect)

K. Tanida et al., PRL86 (‘01) 1982
Mean-field approximation and beyond

Self-consistent mean-field (Hartree-Fock) method:

- independent particles in a mean-field potential
- global theory for the whole nuclear chart
- intuitive picture for nuclear deformation
- optimized shape can be automatically determined
  = suitable for a discussion on shape of hypernuclei

Myaing Thi Win and K.H., PRC78(‘08)054311
Mean-field approximation and beyond

Problems of mean-field approximation

- body-fixed frame formalism $\rightarrow$ intuitive picture of nuclear deformation
- spectrum: lab-frame $\leftrightarrow$ transformation from intrinsic to lab. frames
  \[
  |\psi_{IcM_c}(\beta)\rangle = \hat{P}_{McK_c}^I \hat{P}^N \hat{P}^Z |\psi_{\text{MF}}(\beta)\rangle
  \]
  angular momentum + particle number projections
- quantum fluctuation

“beyond mean-field approximation”
beyond mean-field approximation

Difficulties for odd-mass nuclei (single-$\Lambda$ hypernuclei)

- half-integer spins
- broken time-reversal symmetry

Our aim

Construct an alternative way to describe low-energy spectrum of single-$\Lambda$ hypernuclei based on “beyond mean-field” approach

J.M. Yao, K.H. et al., PRC89 (‘14) 054306
Microscopic Particle-Rotor Model for $\Lambda$ hypernuclei


$\Lambda +$ even-even core nucleus  e.g., $^{9}_\Lambda$Be = $^8$Be + $\Lambda$

- beyond mean-field calculations for e-e core

\[
|\Phi_{IcM_c}\rangle = \int d\beta f(\beta) |\Psi_{IcMc}(\beta)\rangle \\
|\Psi_{IcMc}(\beta)\rangle = \hat{P}_{McKc}^{Ic} \hat{P}^N \hat{P}^Z |\Psi_{MF}(\beta)\rangle
\]

$^8$Be

Energy (MeV)

\(\beta\) values:
- $\beta = 1.2$
- $\beta = 2.0$
- $\beta = 3.0$
- $\beta = 4.0$

relativistic point coupling model with PCF-1
Microscopic Particle-Rotor Model for $\Lambda$ hypernuclei


$\Lambda$ + even-even core nucleus  e.g., $^9\Lambda$Be = $^8$Be + $\Lambda$

- beyond mean-field calculations for e-e core
- coupling of $\Lambda$ to the core states

$$|\Phi_{IM}\rangle = \left[\psi_{jl}(r_{\Lambda}) \otimes |\Phi_{0+}\rangle\right]^{(IM)} + \left[\psi_{j'l'}(r_{\Lambda}) \otimes |\Phi_{2+}\rangle\right]^{(IM)} + \cdots$$

particle-core model with core excitations

cf. conventional particle-rotor model:
  core states $\rightarrow$ macroscopic rotor (Wigner’s D-functions)
  with a fixed deformation

our approach: a microscopic version of particle-rotor model

cf. no Pauli principle for $\Lambda$ particle
Results for $^9_{\Lambda}\text{Be}$

$\mathcal{L}_{\Lambda N} = -\alpha_{V_{\Lambda}}^{N\Lambda} \delta(r_{\Lambda} - r_N) - \alpha_{S_{\Lambda}}^{N\Lambda} \gamma_{\Lambda}^0 \delta(r_{\Lambda} - r_N) \gamma_N^0$

- coupling to $0_1^+$, $2_1^+$, and $4_1^+$ of $^8\text{Be}$

$B_\Lambda$ of $^9_{\Lambda}\text{Be}$

cf. $\alpha$ cluster model

T. Motoba et al., PTP70 (‘83) 189

$$|IM\rangle = |L^\pi\rangle \otimes |\chi_\Lambda\rangle$$
Results for $^9\Lambda\text{Be}$

wave function components

microscopic particle-rotor

- almost pure $\Lambda_s$ states for the g.s. rotational band
- large admixture of $0^+$ and $2^+$ states for the second band

$92.8(s_{1/2} \otimes 0^+) + 51.6(p_{1/2} \otimes 0^+) + \ldots$

$54.8(01)_{1/2} + 41.6(21)_{1/2} + \ldots$

$94.5(00)_{1/2} + 5.3(22)_{1/2} + \ldots$

$\alpha$-cluster model (Motoba et al., 1983)
Results for $^9_{\Lambda}\text{Be}$


$\text{B(E2) transition rates } (e^2fm^4)$

microscopic particle-rotor

$\alpha$-cluster model (Motoba et al., 1983)

$^8\text{Be}$ \quad $^9_{\Lambda}\text{Be}$

$0^+$ \rightarrow $2^+$ \rightarrow $4^+$

$47.3$ \quad $25.0$ \quad $41.6$ \quad $22.6$ \quad $39.3$ \quad $22.4$ \quad $13.5$ \quad $11.3$

$1/2^+, 3/2^+$ \quad $3/2^+, 5/2^+$ \quad $4^+$ \quad $5/2^+, 7/2^+$

$\triangleright$ much smaller reduction in $\text{B(E2)}$ : role of higher states?
Summary

Microscopic particle-rotor model for $\Lambda$-hypernuclei

- $\Lambda + \text{GCM states for core}$
- microscopic version of particle-rotor model
- first calculation for low-lying spectrum based on mean-field type calculations
- application to $^9\Lambda\text{Be}$: nice agreement with $\alpha$ cluster model (except for EM transitions)

Future perspectives

- applications to many $\Lambda$-hypernuclei (both rotational and vibrational core)
- extension to include triaxiality (cf. $^{25}\Lambda\text{Mg}$)

Challenging problem

- application to formation reactions of hypernuclei
  - description of ordinary odd-mass nuclei: Pauli principle?