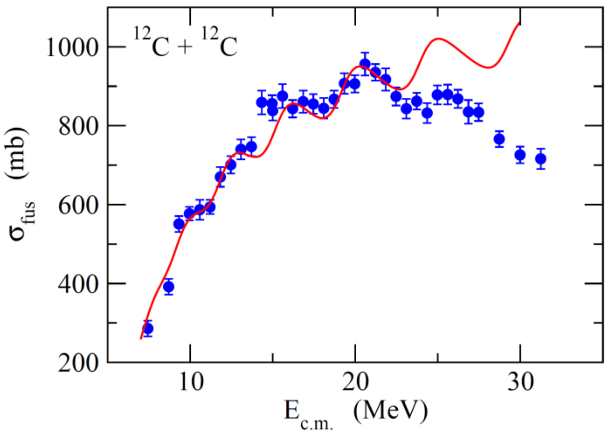
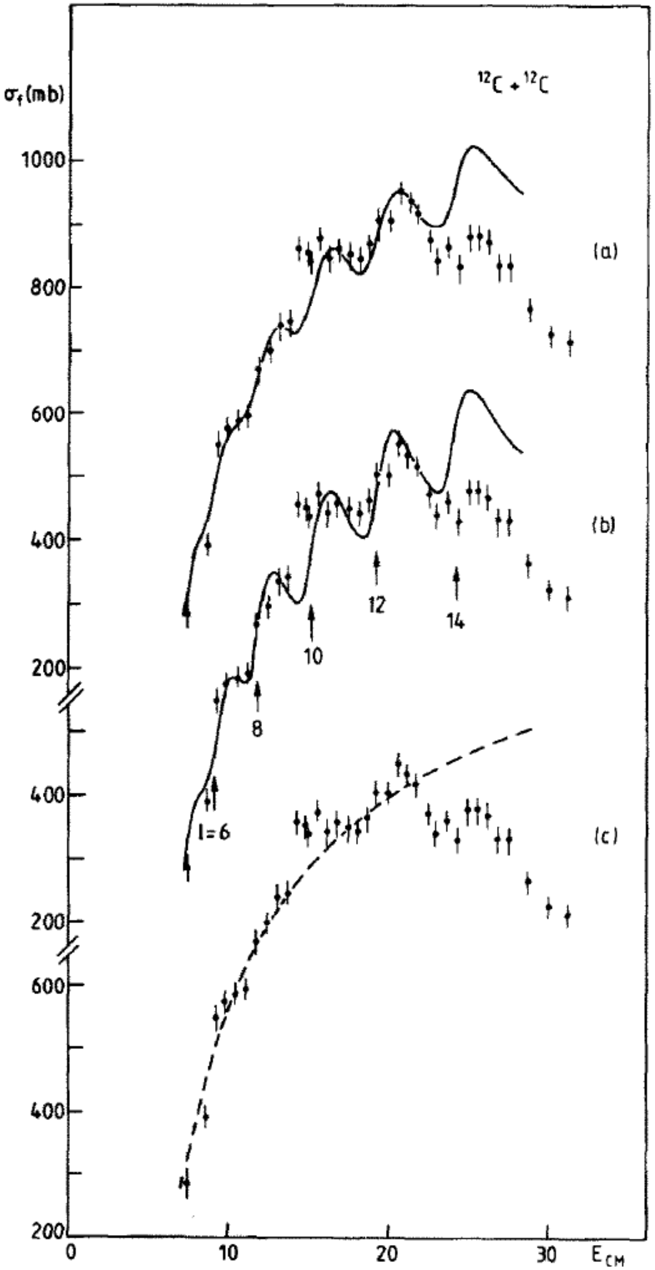
Fusion oscillations in C+C systems
a comparison among $^{12}C+^{12}C$, $^{13}C+^{13}C$ and $^{12}C+^{13}C$



Kouichi Hagino, *Tohoku University*
Neil Rowley, *IPN Orsay*

TOHOKU UNIVERSITY

1. *Introduction: Wong formula*
2. *Fusion oscillations*
   - *$^{12}C + ^{12}C$ (spin zero bosons)*
   - *$^{13}C + ^{13}C$ (spin 1/2 fermions)*
   - *$^{12}C + ^{13}C$ (elastic transfer)*
3. *Summary*



Poffe, Rowley, Lindsay, NPA410('83) 498

two recent publications

PHYSICAL REVIEW C **85**, 064611 (2012)

Structures in high-energy fusion data

H. Esbensen

PHYSICAL REVIEW C **86**, 064603 (2012)

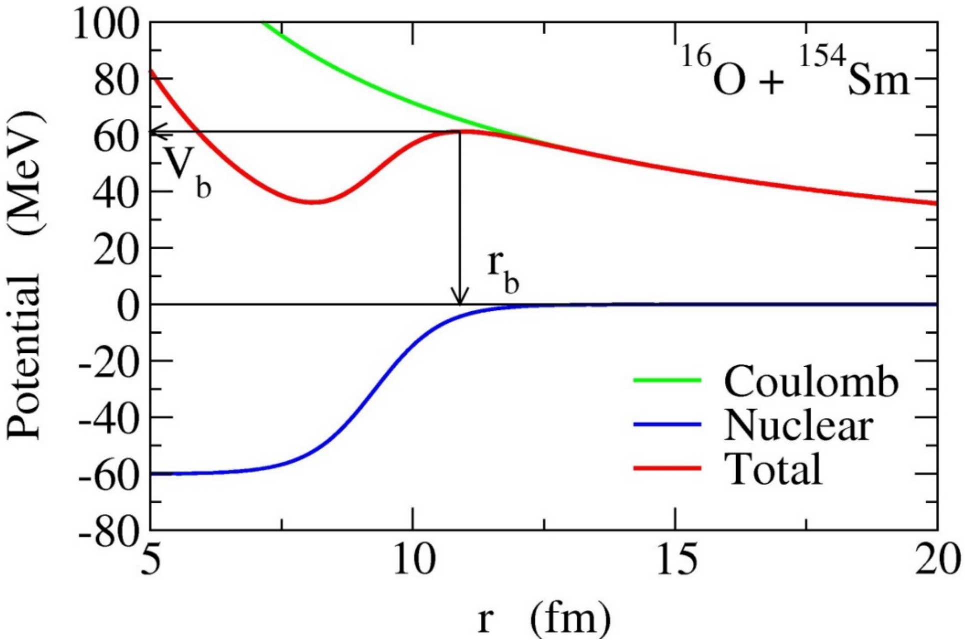
Reaction cross sections in heavy-ion collisions

Cheuk-Yin Wong

also
MUSIC (E. Rehm)

Introduction

The simplest approach to fusion: one-dimensional potential model



$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

In this talk: potential model analyses for C+C fusion (above the barrier)
cf. channel coupling effect → Rowley's talk

Wong's formula

C.Y. Wong, Phys. Rev. Lett. 31 ('73)766

i) Approximate the Coul. barrier by a parabola: $V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$

$$\longrightarrow \quad P_0(E) = 1 \Big/ \left(1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]\right)$$

ii) *l*-independent barrier position and curvature:

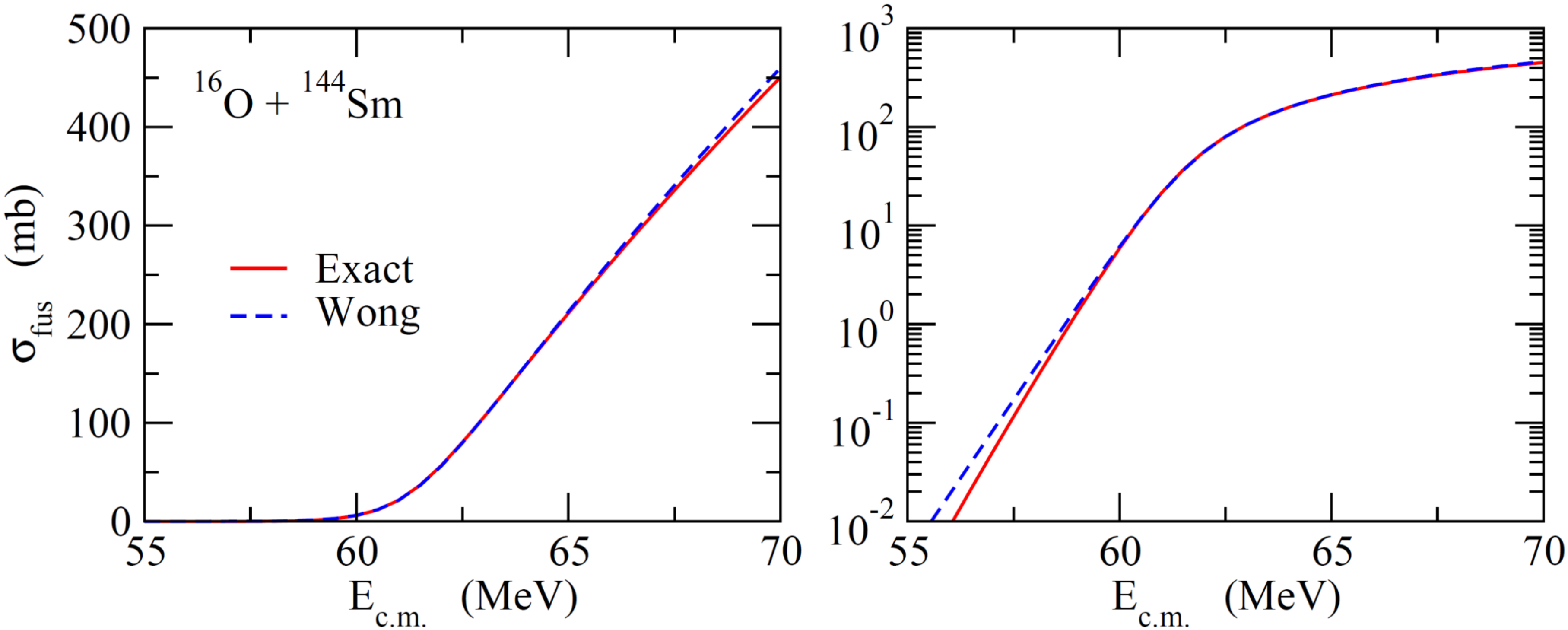
$$\longrightarrow \quad P_l(E) \sim P_0\left(E - \frac{l(l+1)\hbar^2}{2\mu R_b^2}\right)$$

iii) Replace the sum of *l* with an integral

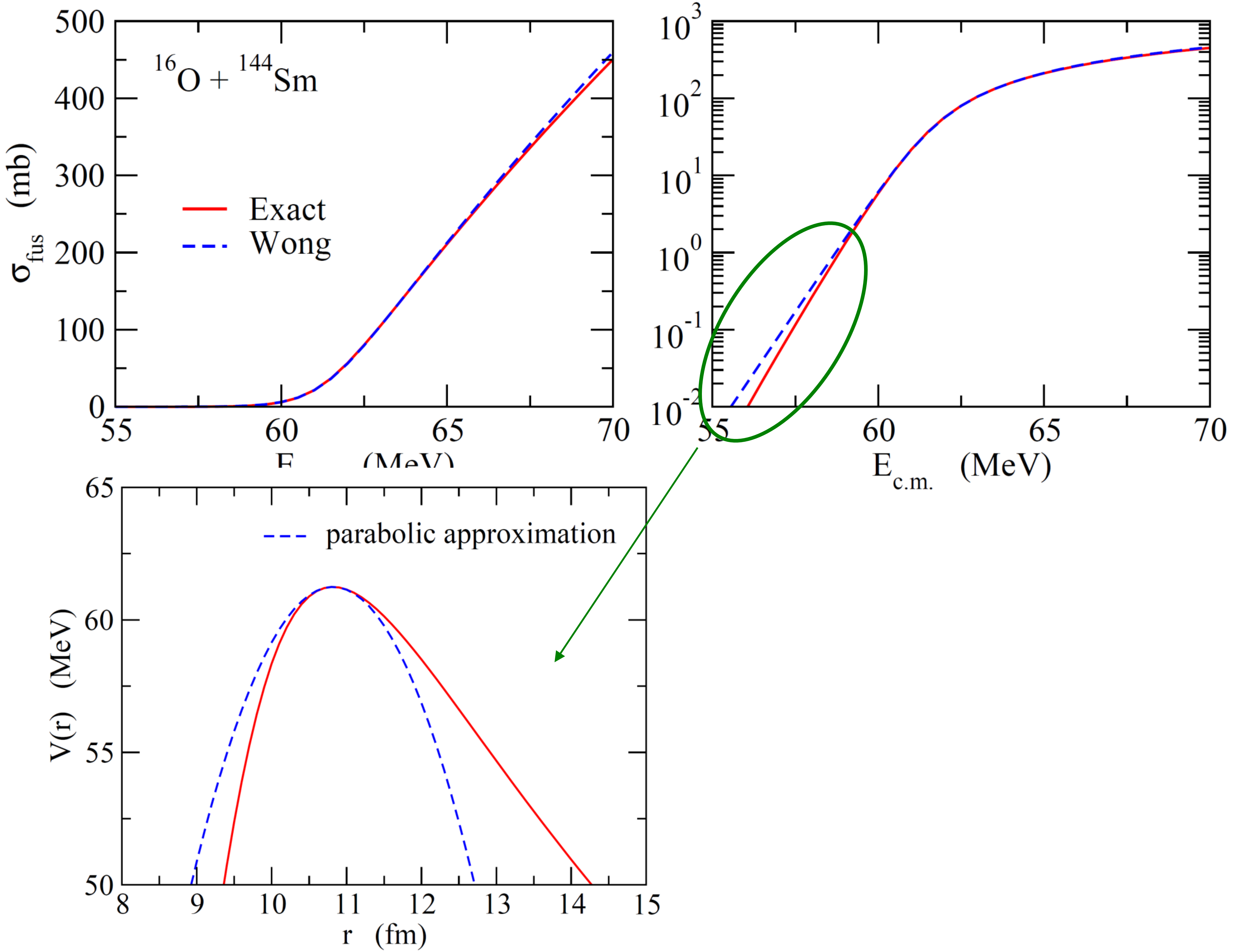
$$\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

(note) For $E \gg V_b$ $\quad 1 \ll \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)$

$$\longrightarrow \quad \sigma_{\text{fus}}(E) \sim \pi R_b^2\left(1 - \frac{V_b}{E}\right) = \sigma_{\text{fus}}^{cl}(E)$$

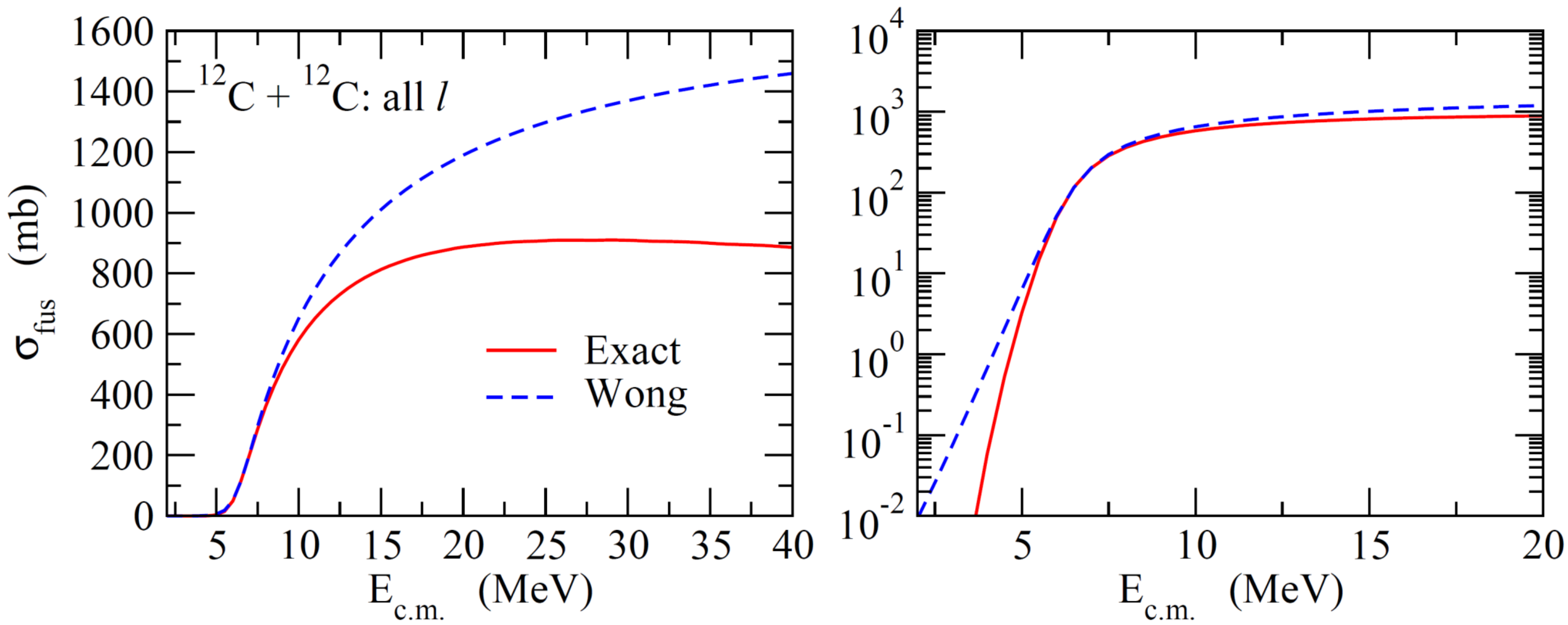


500
400
300
200
100
0
55
60
65
70
^{16}O + ^{144}Sm
Exact
Wong
σ_{fus} (mb)
$E_{c.m.}$ (MeV)
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
55
60
65
70
$E_{c.m.}$ (MeV)



^{16}O + ^{144}Sm
Exact
Wong
σ_{fus} (mb)
500
400
300
200
100
0
55
60
65
70
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
$E_{c.m.}$ (MeV)
parabolic approximation
V(r) (MeV)
65
60
55
50
8
9
10
11
12
13
14
15
r (fm)

Wong formula for light heavy-ion fusion



Wong formula:

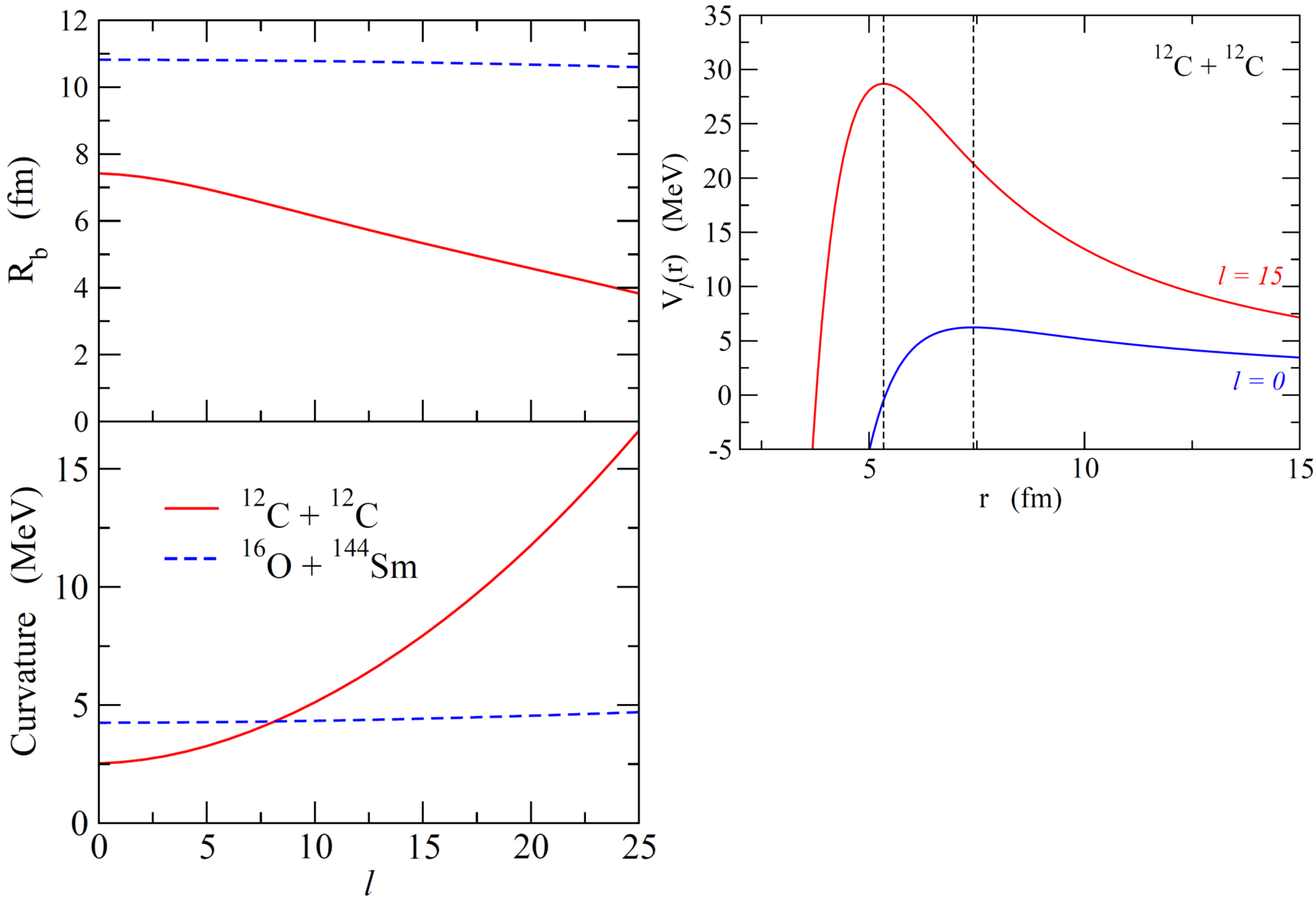
i) Approximate the Coul. barrier by a parabola

ii) l-independent barrier position and curvature ←

iii) Replace the sum of l with an integral

$$V_{\rm cent}(r) = \frac{l(l+1)\hbar^2}{2\mu r^2}$$

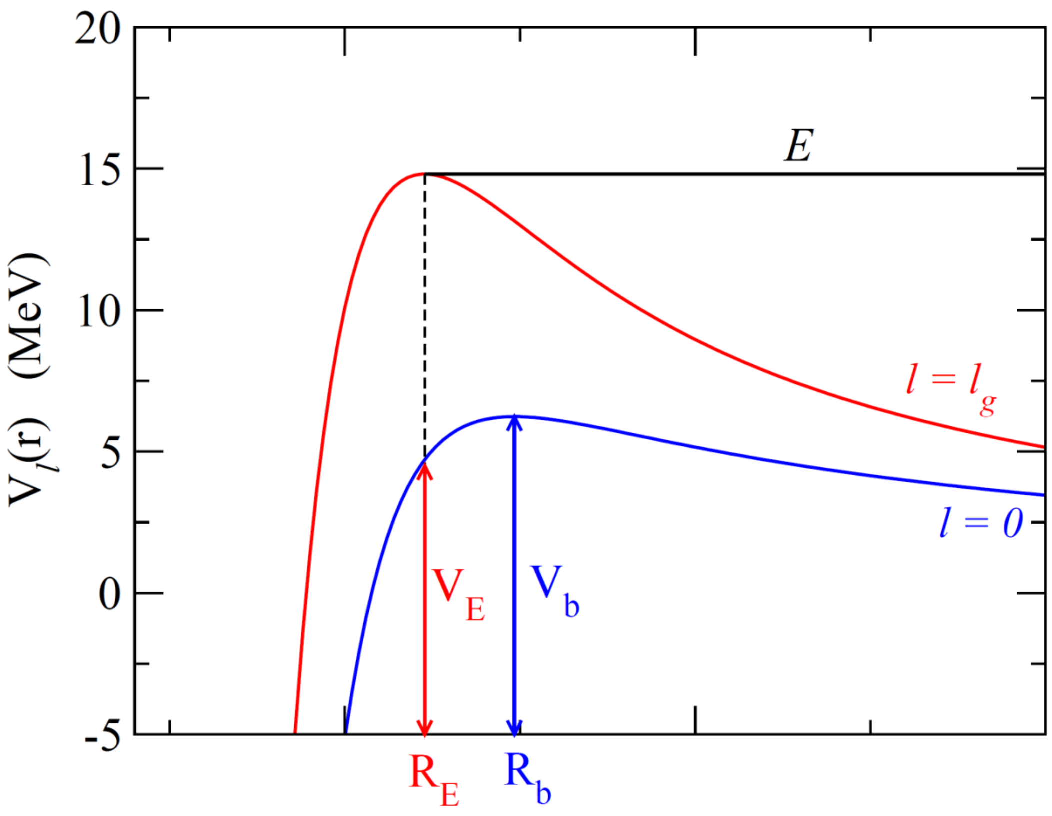
small



12
10
8
6
4
2
0
R_b (fm)
15
10
5
0
Curvature (MeV)
0
5
10
15
20
25
l
12C + 12C
16O + 144Sm
35
30
25
20
15
10
5
0
-5
V_l(r) (MeV)
5
10
15
r (fm)
12C + 12C
l = 15
l = 0

E-dependent Wong formula

N. Rowley, A. Kabir, and R. Lindsay, J. of Phys. G15('89)L269
N. Rowley and K. Hagino, in preparation



use V_b, R_b, and Ω for the grazing angular momentum, l_g

(note)

$$\begin{cases} \sigma_{cl} = \pi b_g^2 \\ E = V_E + \dfrac{(kb_g)^2\hbar^2}{2\mu R_E^2} \end{cases}$$

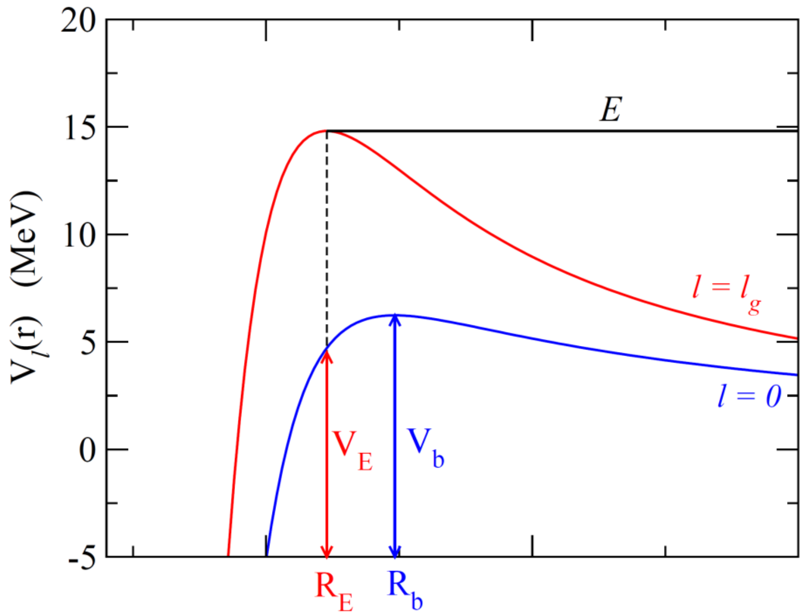
$$\longrightarrow \sigma_{cl} = \pi R_E^2 (1 - V_E/E)$$

$$\sigma_{fus}(E) = \frac{\hbar\Omega}{2E} R_b^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$

$$\Longrightarrow \sigma_{fus}(E) = \frac{\hbar\Omega_E}{2E} R_E^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega_E}(E - V_E)\right)\right]$$

grazing anuglar momentum:

$$\begin{cases} E = V_{l=0}(R_E) + \dfrac{l_g(l_g+1)\hbar^2}{2\mu R_E^2} \\ 0 = V'_{l=0}(R_E) - \dfrac{l_g(l_g+1)\hbar^2}{\mu R_E^3} \end{cases}$$



for an exponential potential, $V_N(r) = -V_0\, e^{-r/a}$

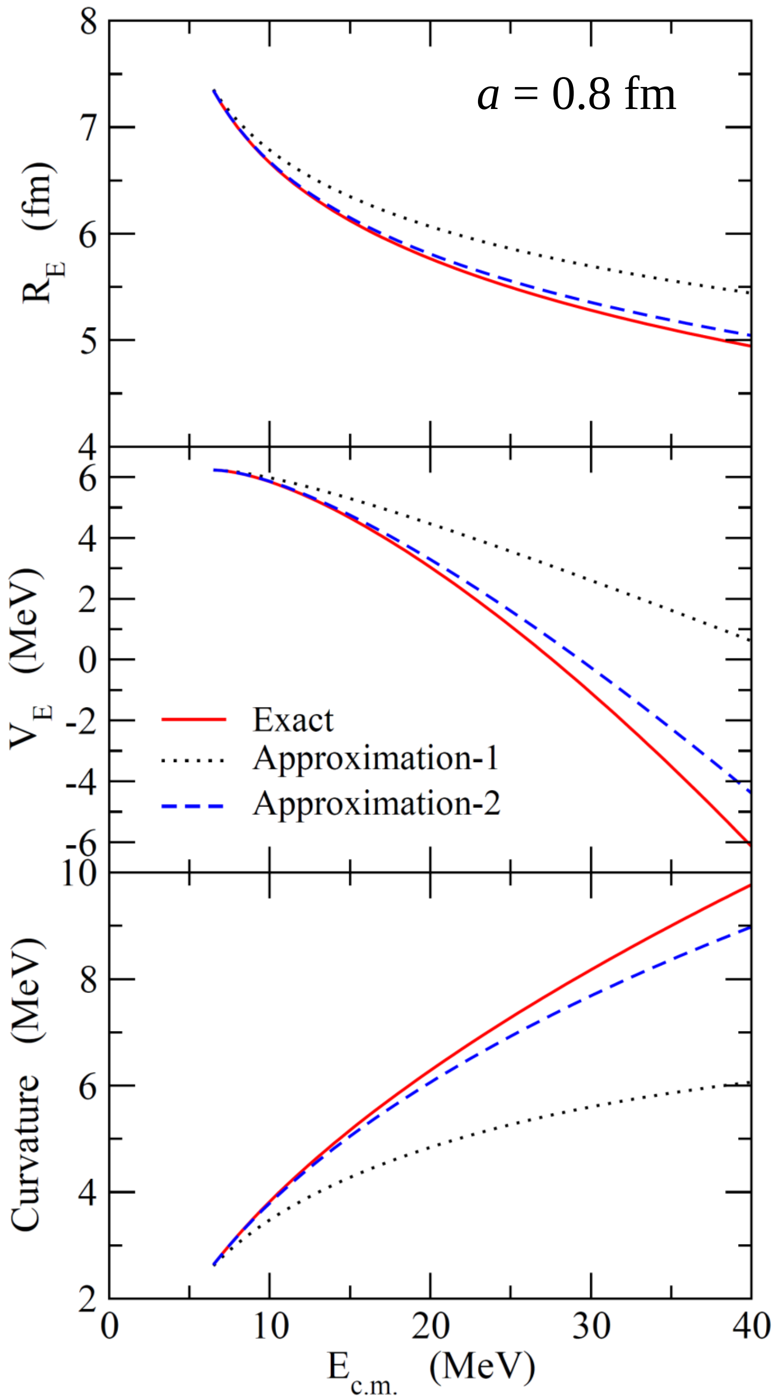
$$R_E \sim R_b - a\ln(2x+1), \quad x = (E - V_b)/V_b$$

or a better approximation:

$$R_E \sim R_b - a\ln(g)$$

$$g = \frac{2(1-\beta)}{(1-f_0)(1-f_0-2\beta)}\left((1-f_0)(x+1) - \frac{1}{2}\frac{1}{1-\beta}\right)$$

$$\beta = a/R_b, \quad f_0 = \beta\ln(2x+1)$$



Exact:

$$e^{f/\beta} = \frac{2(1-\beta)}{(1-f)(1-f-2\beta)} \times \left((1-f)(x+1) - \frac{1}{2}\frac{1}{1-\beta}\right)$$

Approximation-1:

$$e^{f/\beta} \sim 2x+1$$

Approximation-2:

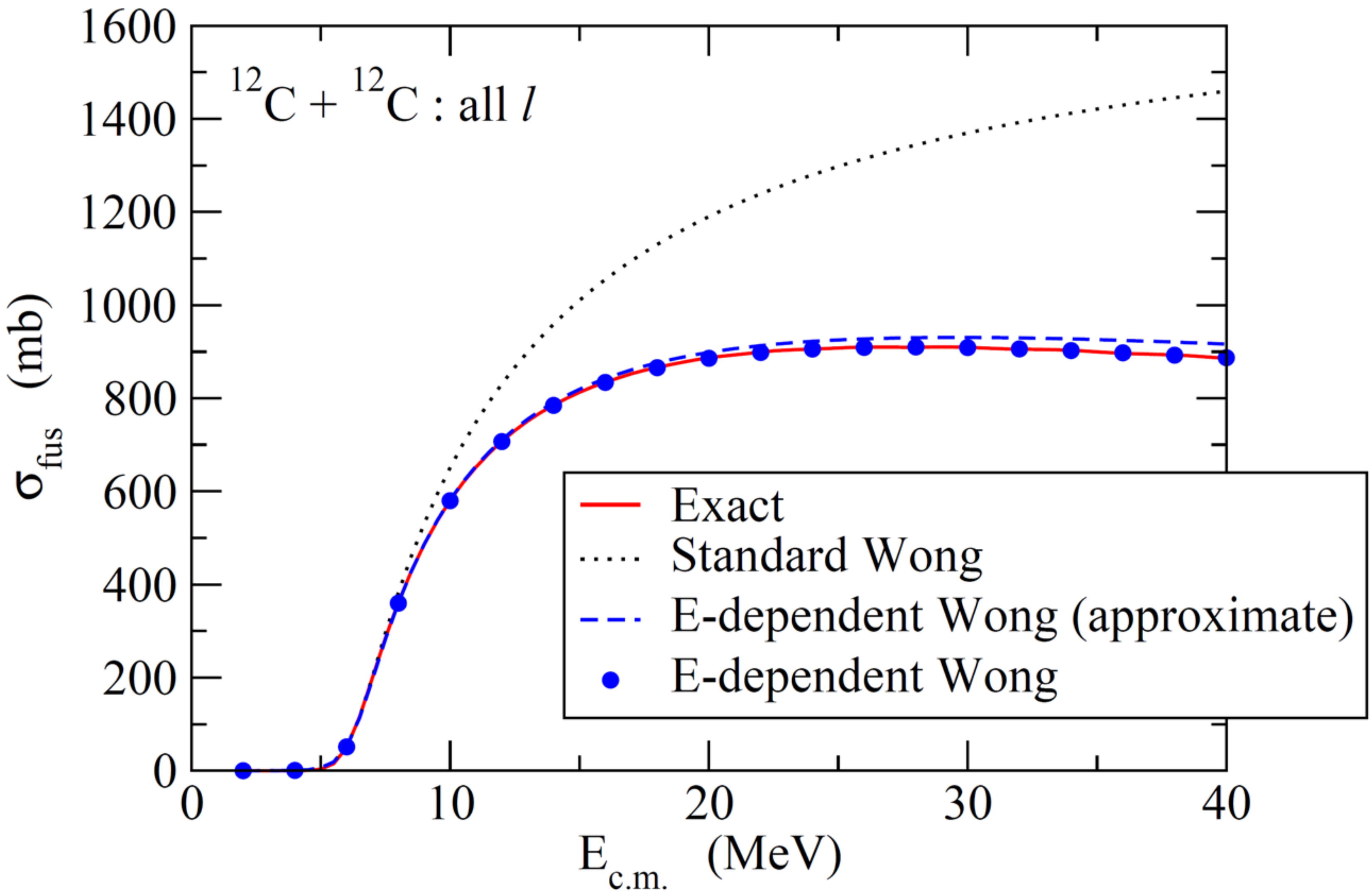
$$e^{f/\beta} \sim \frac{2(1-\beta)}{(1-f_0)(1-f_0-2\beta)} \times \left((1-f_0)(x+1) - \frac{1}{2}\frac{1}{1-\beta}\right)$$

$$f_0 = \beta \ln(2x+1)$$

$$\beta = a/R_b, \quad x = (E-V_b)/V_b,$$

$$f = (R_b - R_E)/R_b$$

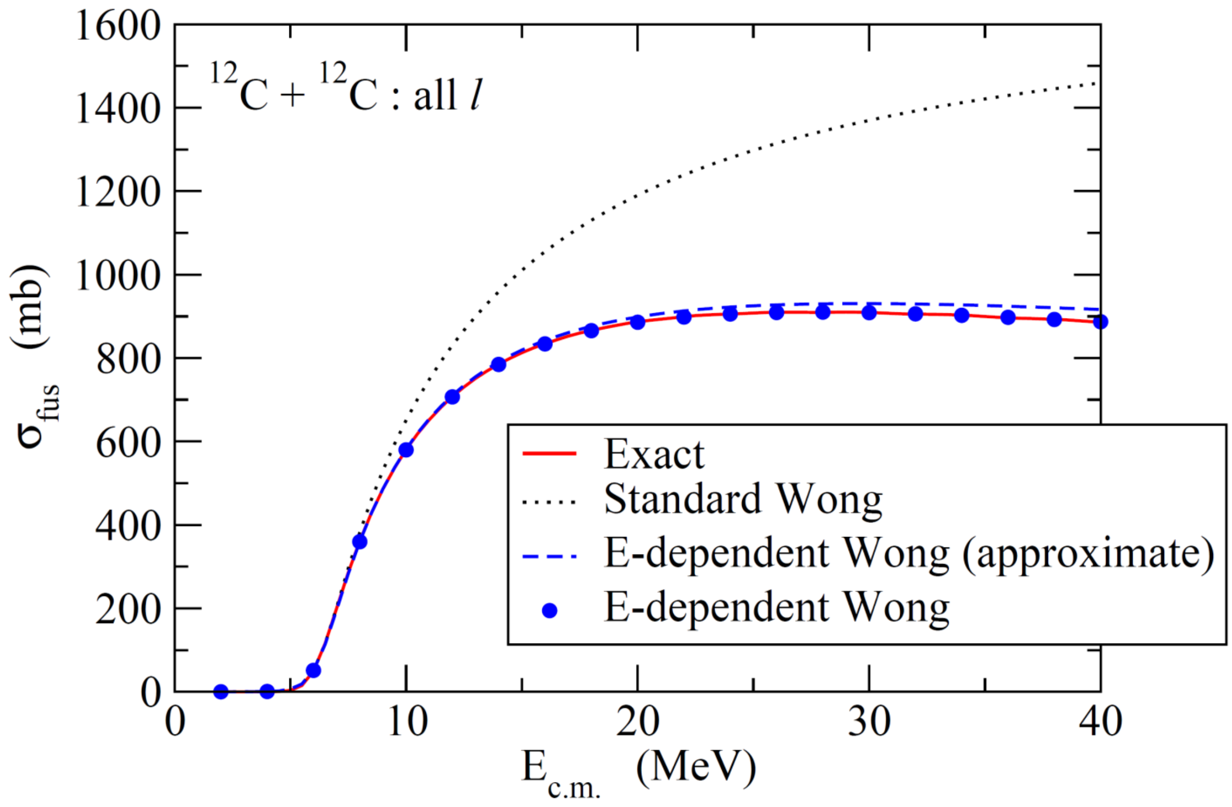
$$\sigma_{\rm fus}(E) = \frac{\hbar\Omega_E}{2E} R_E^2 \ln\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega_E}(E - V_E)\right)\right]$$



Continuum approximation

Wong formula:

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \rightarrow \frac{\pi}{k^2}\int dl\,(2l+1)P(l,E)$$

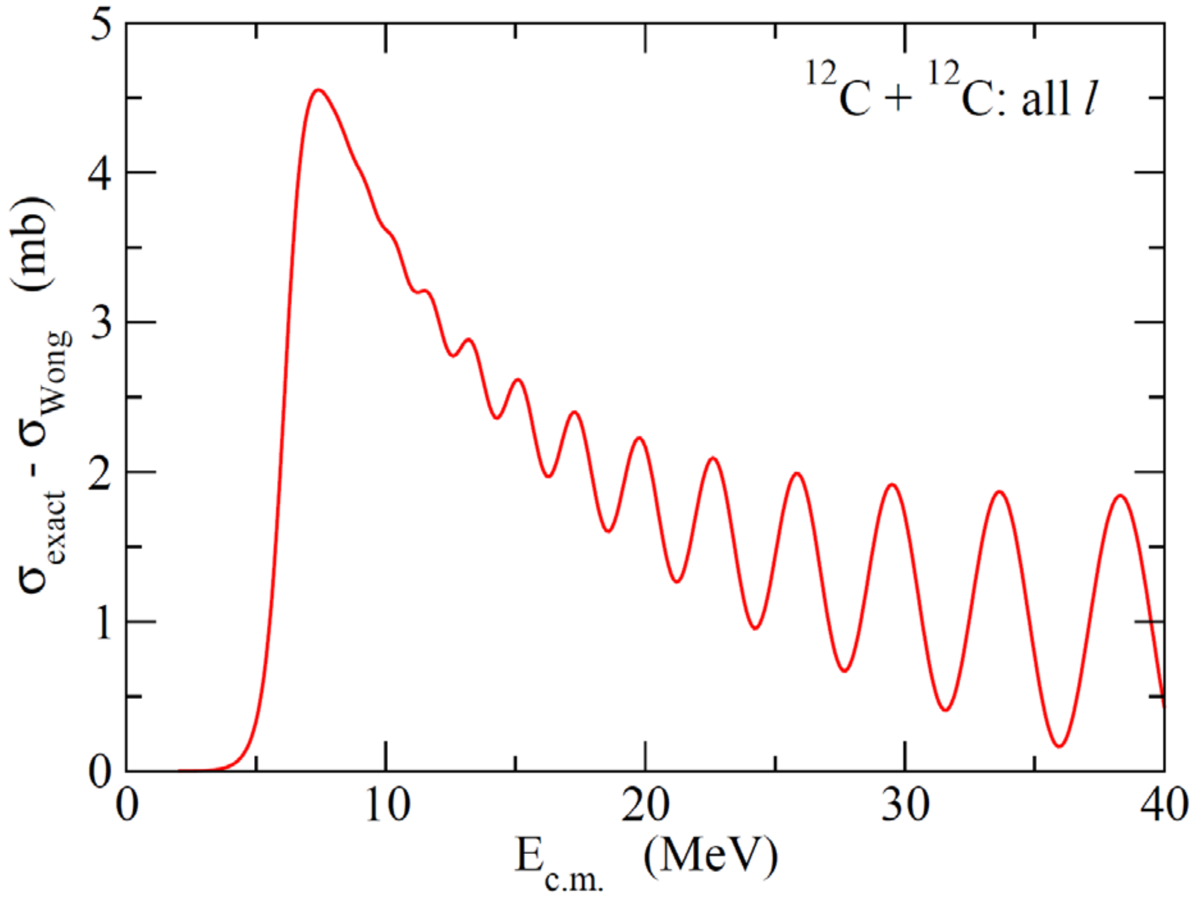


the continuum approximation: appears very good
but if you take a closer look…..

Continuum approximation

Wong formula:

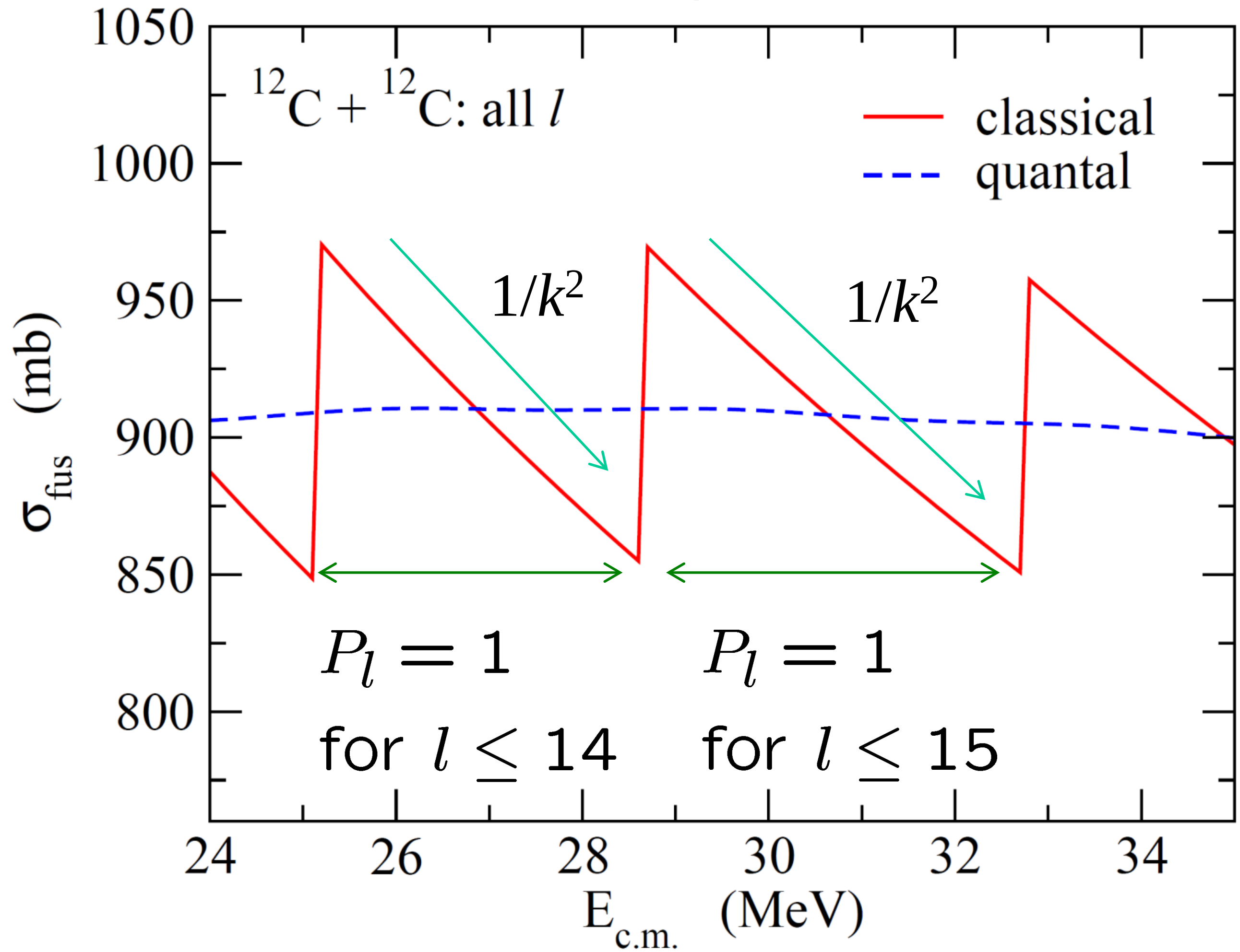
$$\sigma_{\mathsf{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \to \frac{\pi}{k^2}\int dl\,(2l+1)P(l,E)$$



* Exact: a parabolic barrier with R_E, V_E, Ω_E obtained with an exp. pot.

the origin of the oscillations

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

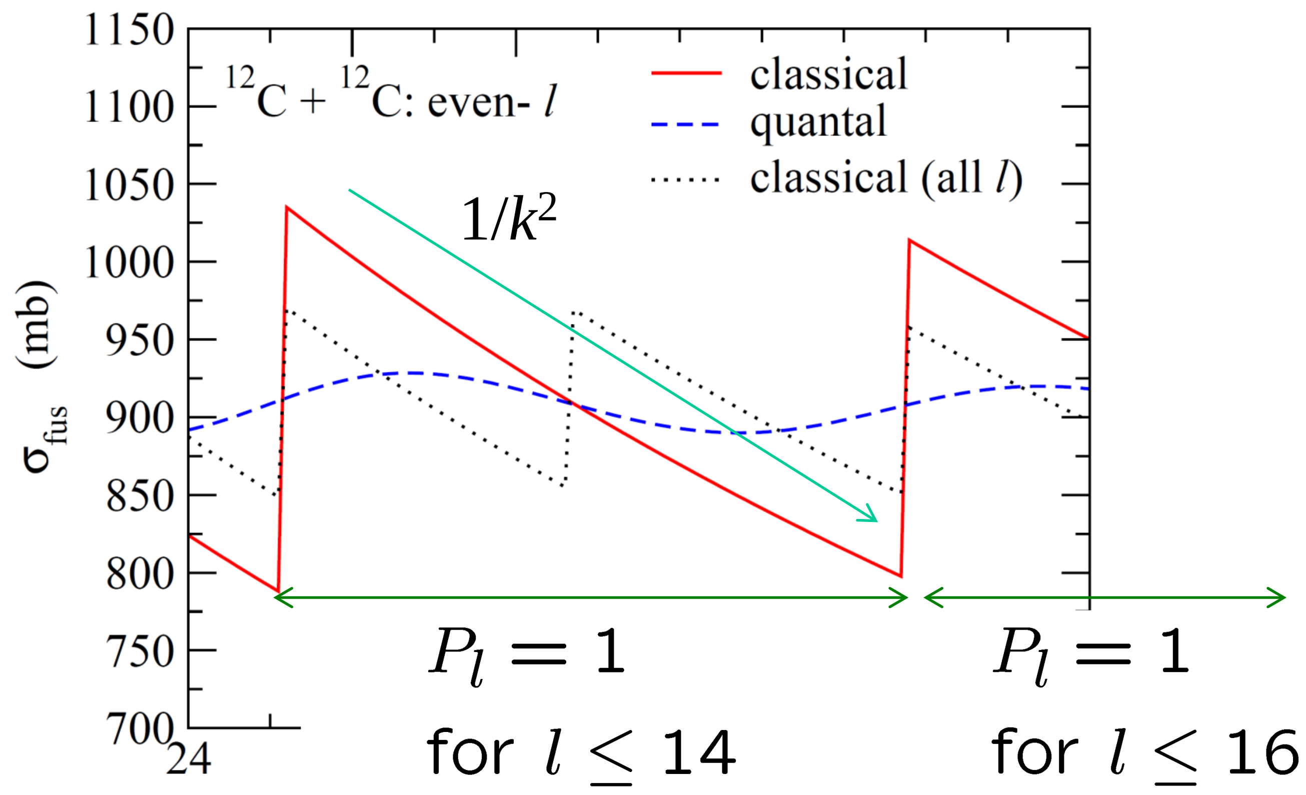


* practically, the oscillations are invisible ($\Delta\sigma$ ~ 1 mb in quantal calc.)

effect of symmetrization: fusion oscillations in light symmetric systems

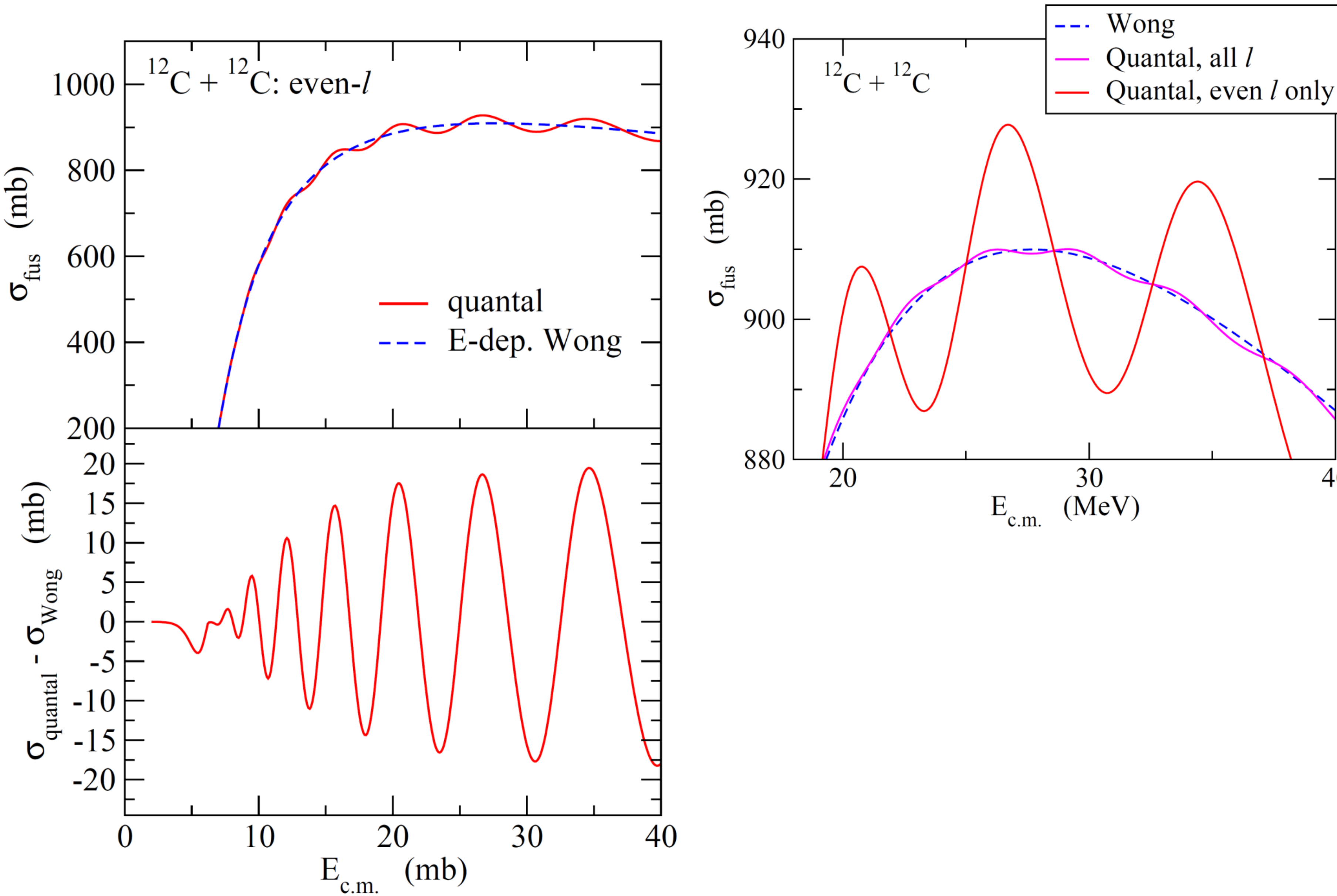
fusion of identical spin-zero bosons: wf has to be symmetric

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E) \to \frac{\pi}{k^2}\sum_l (1+(-)^l)(2l+1)P_l(E)$$

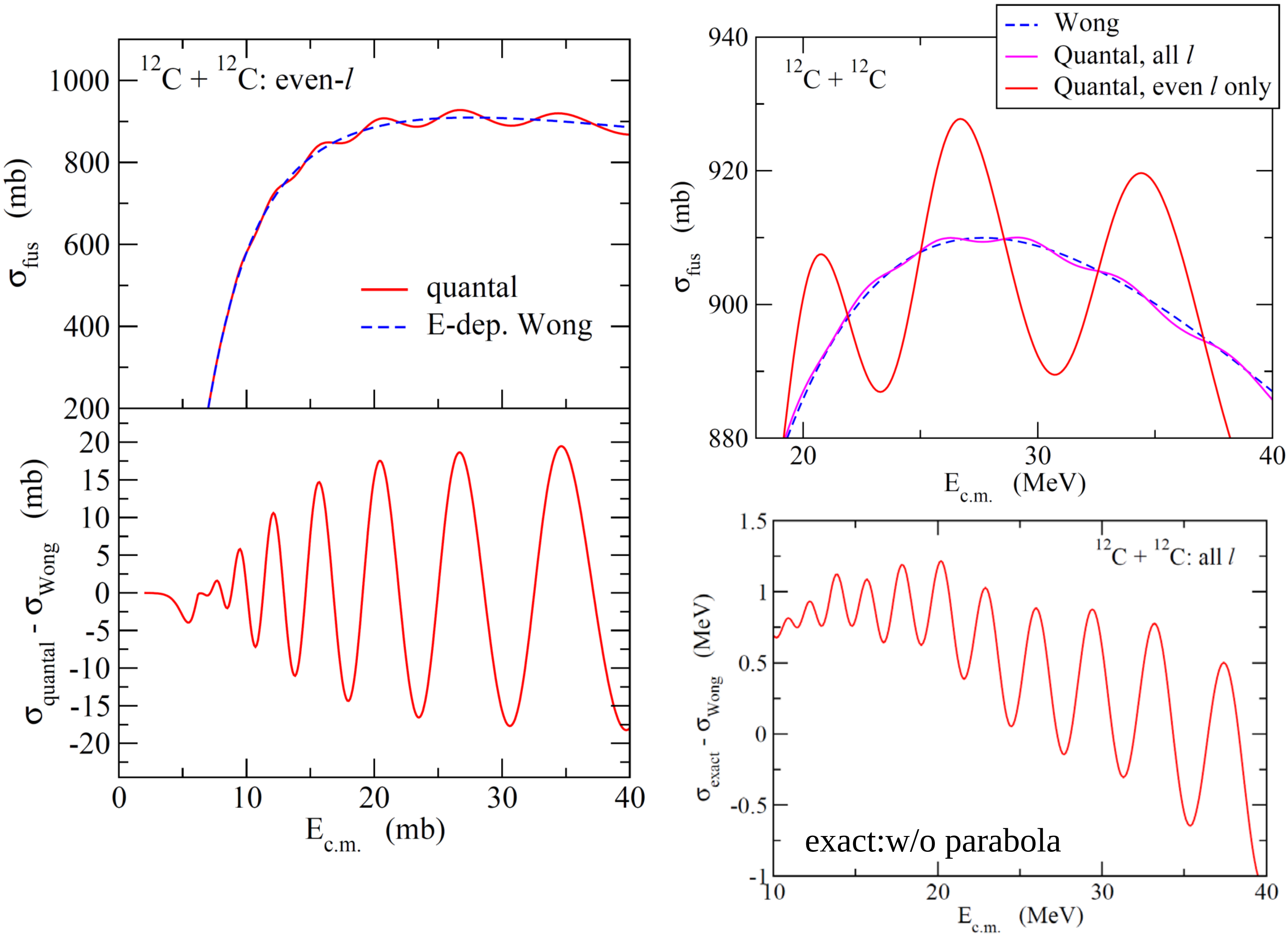


✓the angular mom. is quantized in units of 2-hbar

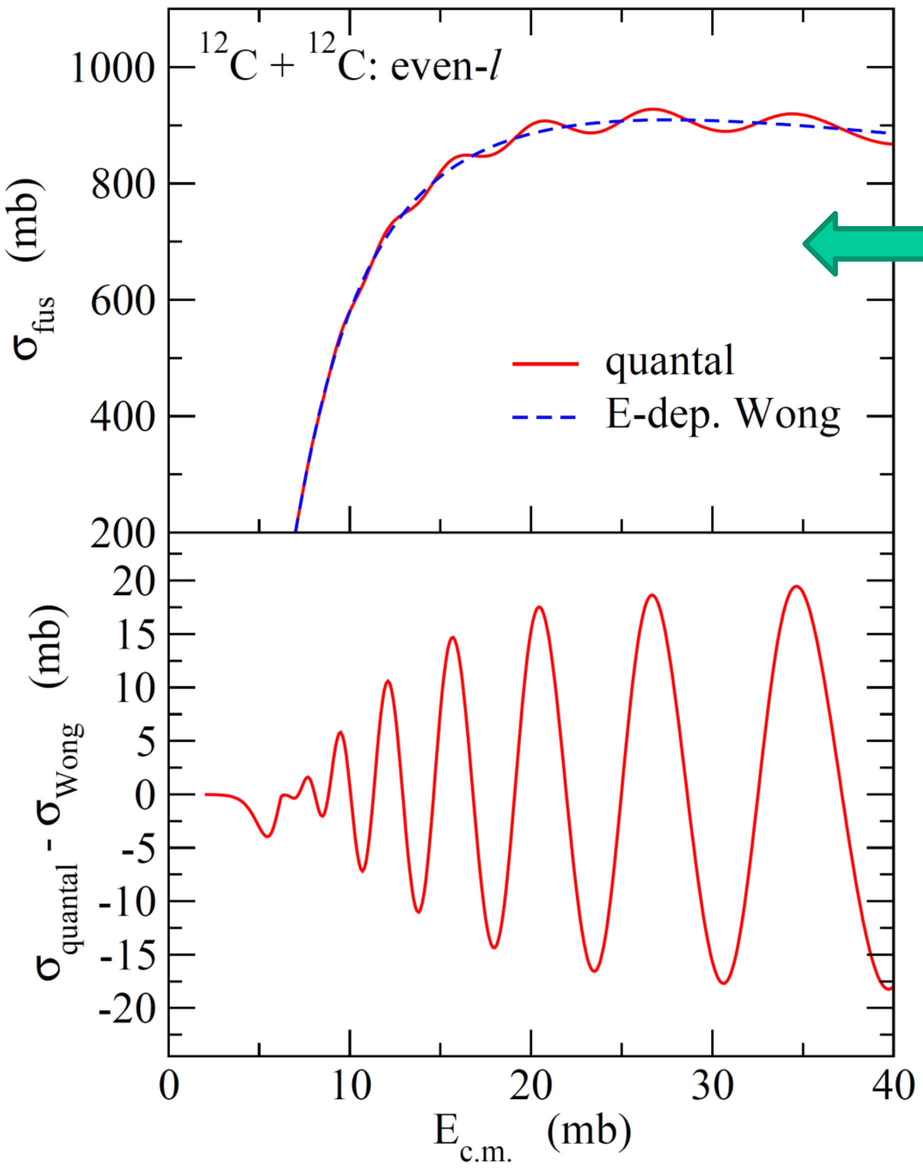
✓a larger amplitude of fusion oscillations

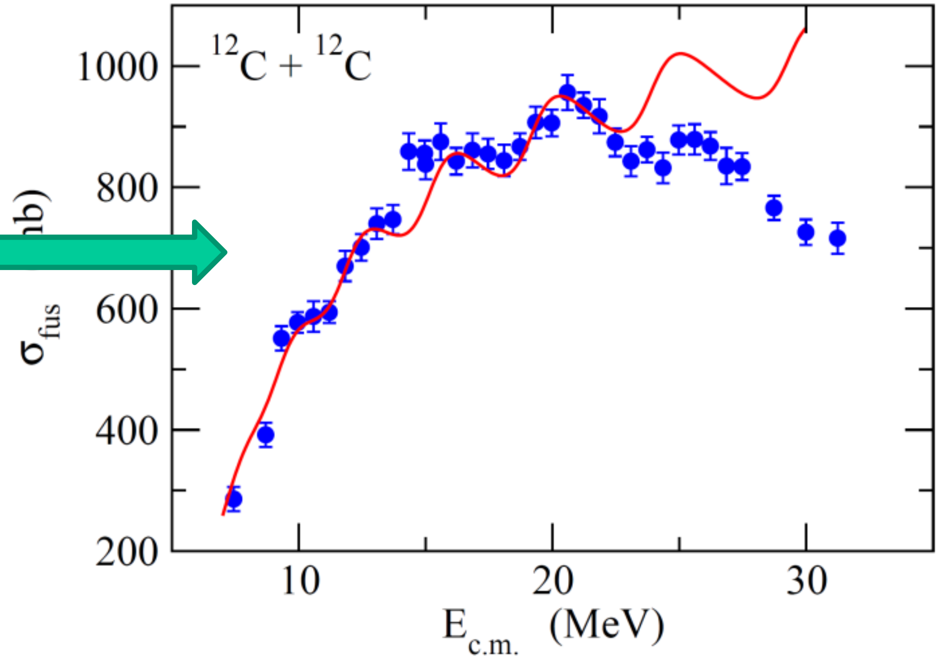


^{12}C + ^{12}C: even-l
quantal
E-dep. Wong
σ_{fus} (mb)
$\sigma_{quantal} - \sigma_{Wong}$ (mb)
$E_{c.m.}$ (mb)
Wong
Quantal, all l
Quantal, even l only
^{12}C + ^{12}C
σ_{fus} (mb)
$E_{c.m.}$ (MeV)



^{12}C + ^{12}C: even-l
quantal
E-dep. Wong
σ_{fus} (mb)
$\sigma_{quantal}$ - σ_{Wong} (mb)
$E_{c.m.}$ (mb)
Wong
Quantal, all l
Quantal, even l only
^{12}C + ^{12}C
σ_{fus} (mb)
$E_{c.m.}$ (MeV)
^{12}C + ^{12}C: all l
σ_{exact} - σ_{Wong} (MeV)
exact:w/o parabola
$E_{c.m.}$ (MeV)





N. Poffe, N. Rowley, R. Lindsay, NPA410('83) 498

Analytic formula for fusion oscillations

N. Poffe, N. Rowley, and R. Lindsay, Nucl. Phys. A410 ('83) 498
N. Rowley and K. Hagino, in preparation

Poisson sum rule

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (1\pm(-)^l)(2l+1)P_l(E)$$

$$= \frac{2\pi}{k^2}\sum_{m=-\infty}^{\infty}\int_0^\infty (1\mp ie^{i\pi\lambda})\lambda P(E;\lambda)e^{2\pi m i\lambda}d\lambda \qquad \text{(still exact)}$$

$$P(E;\lambda=l+1/2)=P_l(E)$$

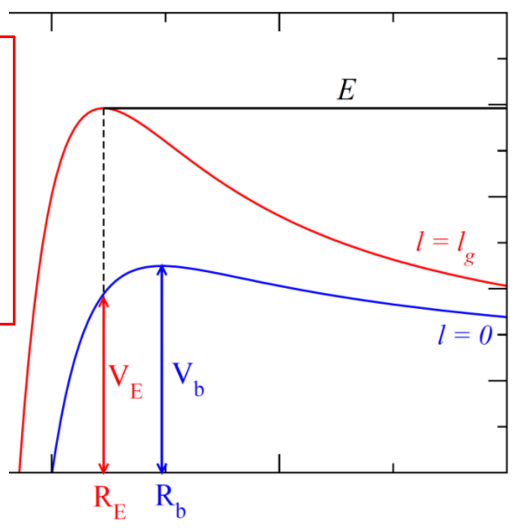
Approximation:

$$\sigma_{\rm fus}(E) \sim \sigma_{m=0}+\sigma_{m=1}+\sigma_{m=-1}$$

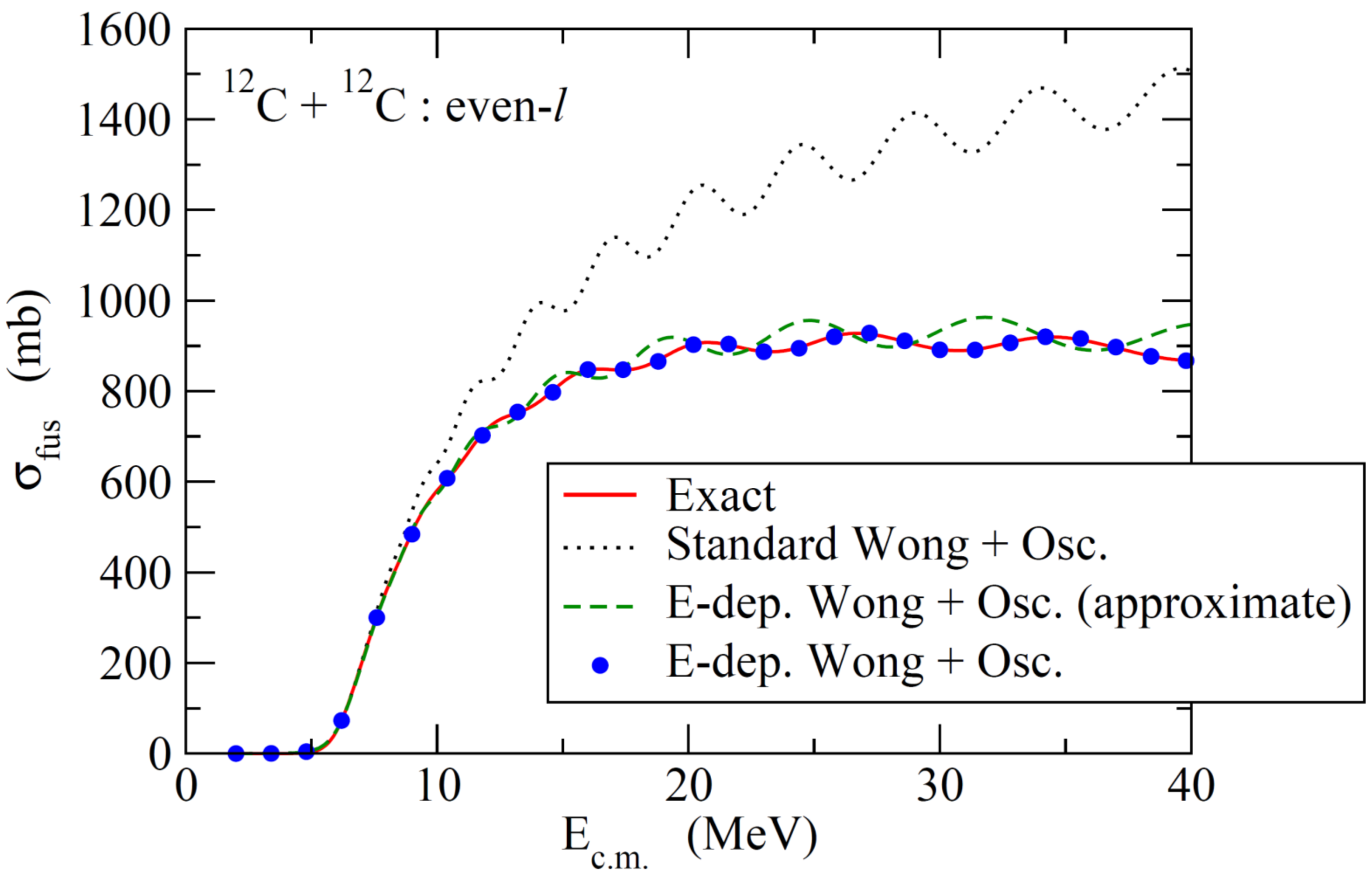
$$\sigma_{m=0}=\sigma_{\rm Wong}$$

$$\sigma_{m=1}+\sigma_{m=-1} = \pm 2\pi R_b^2\frac{\hbar\Omega}{E}e^{-\xi}\sin(\pi l_g)$$

$$\xi = \pi\cdot\frac{\hbar\Omega}{2l_g+1}\cdot\frac{\mu R_b^2}{\hbar^2}$$



$$\sigma_{\rm osc}(E) = 2\pi R_b^2 \frac{\hbar\Omega}{E} e^{-\xi} \sin(\pi l_g), \qquad \xi = \pi \cdot \frac{\hbar\Omega}{2l_g+1} \cdot \frac{\mu R_b^2}{\hbar^2}$$



(note)

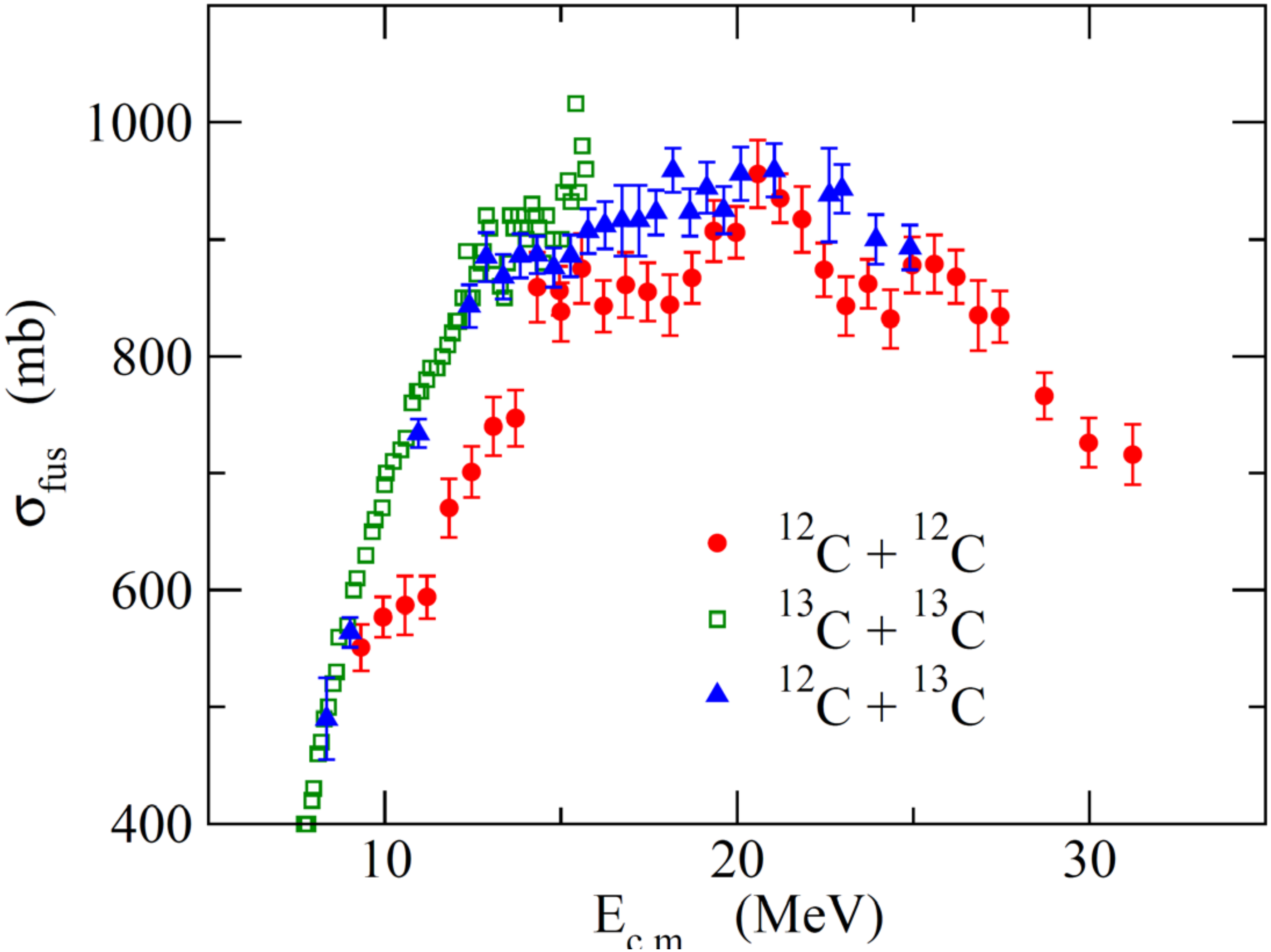
$$\frac{|\sigma_{\rm osc}|}{\sigma_{\rm Wong}} \sim \frac{2\hbar\Omega}{E - V_b} \cdot e^{-\xi}$$

$$2l_g + 1 \gg \pi\hbar\Omega \cdot \frac{\mu R_b^2}{\hbar^2}$$

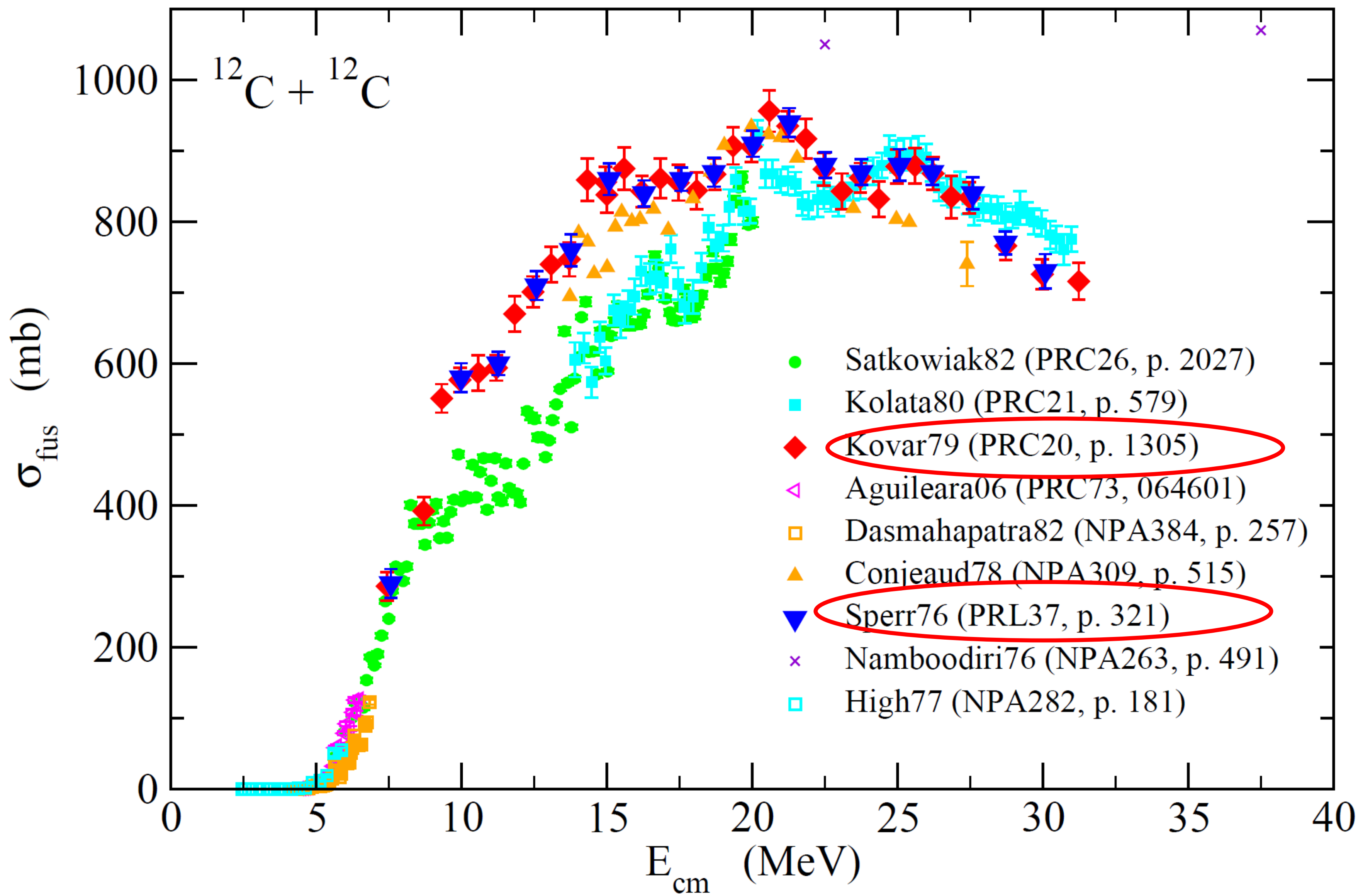
in order for the osc. to be visible

$\longrightarrow$ light symmetric systems

Comparison with experimental data



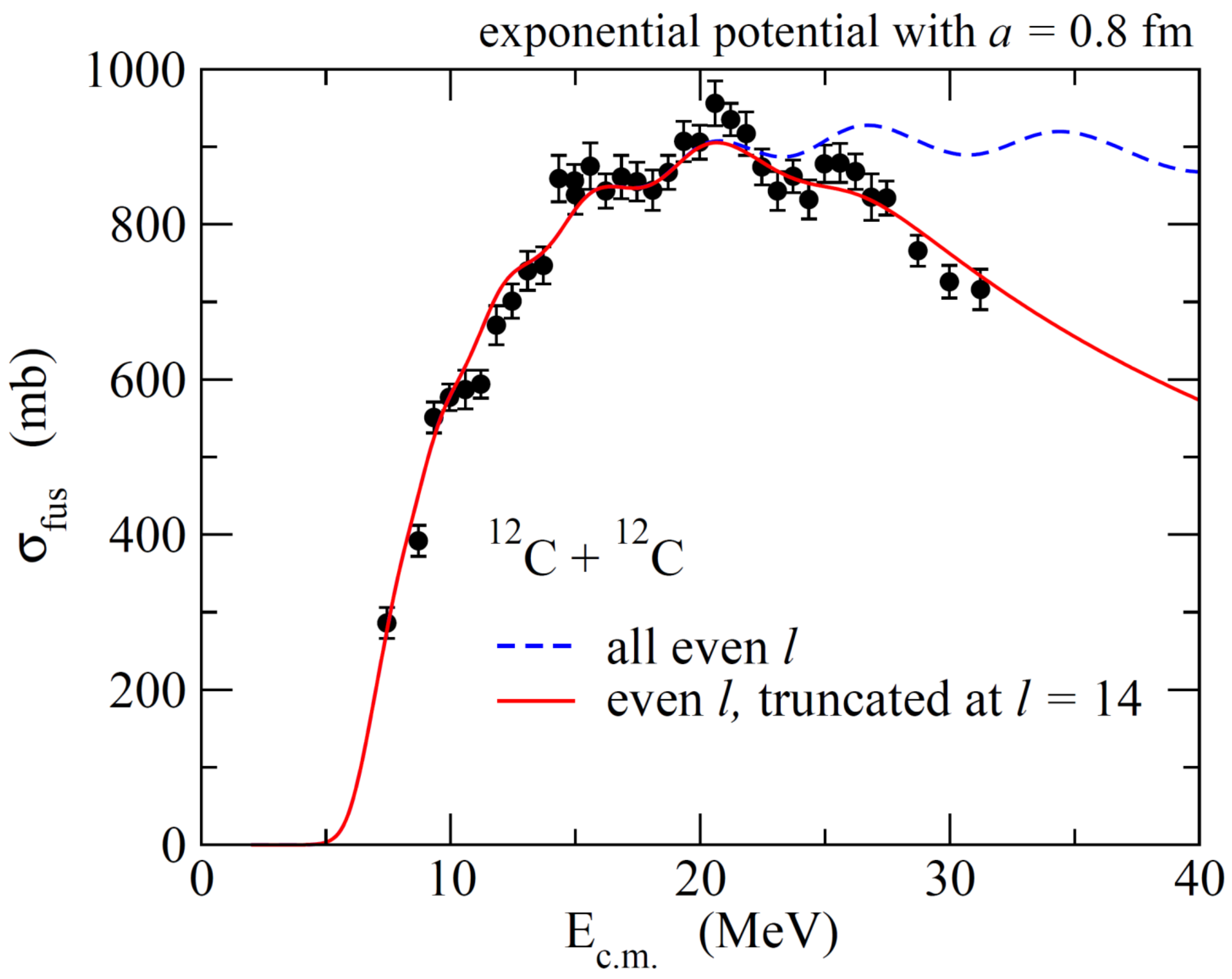
i) $^{12}C + ^{12}C$



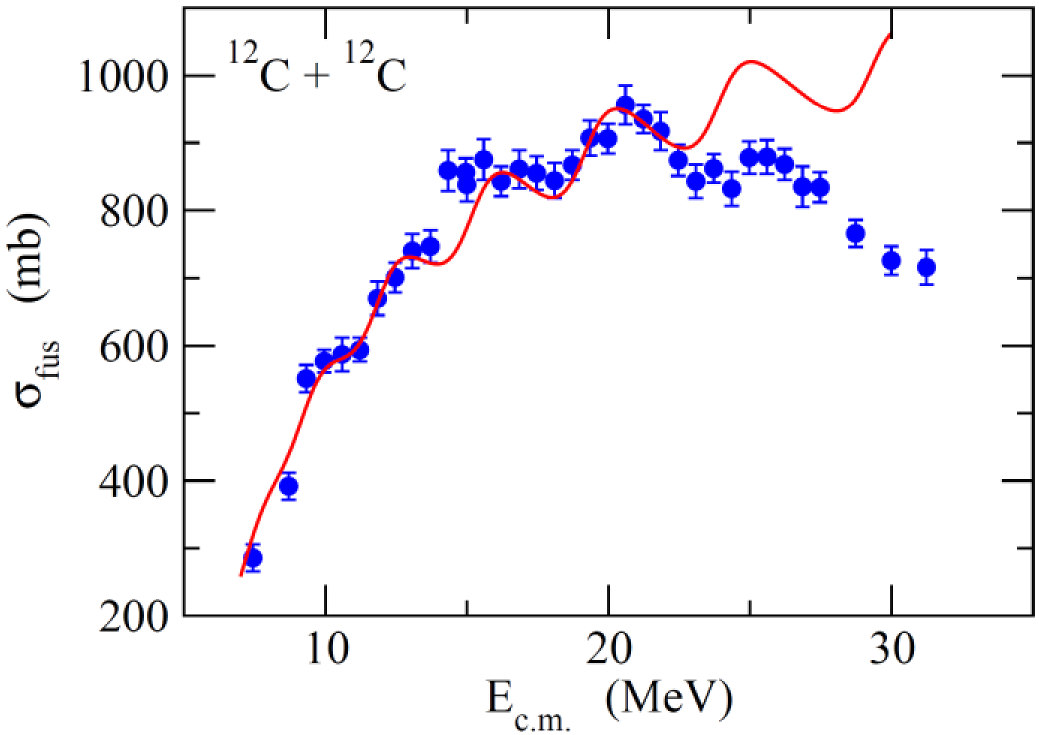
12C + 12C
1000
800
600
400
200
0
0
5
10
15
20
25
30
35
40
σfus (mb)
Ecm (MeV)
Satkowiak82 (PRC26, p. 2027)
Kolata80 (PRC21, p. 579)
Kovar79 (PRC20, p. 1305)
Aguileara06 (PRC73, 064601)
Dasmahapatra82 (NPA384, p. 257)
Conjeaud78 (NPA309, p. 515)
Sperr76 (PRL37, p. 321)
Namboodiri76 (NPA263, p. 491)
High77 (NPA282, p. 181)

i) $^{12}C + {}^{12}C$

$^{12}C_{g.s.} : 0^+ \longrightarrow$ the relative w.f. has to be spatially symmetric

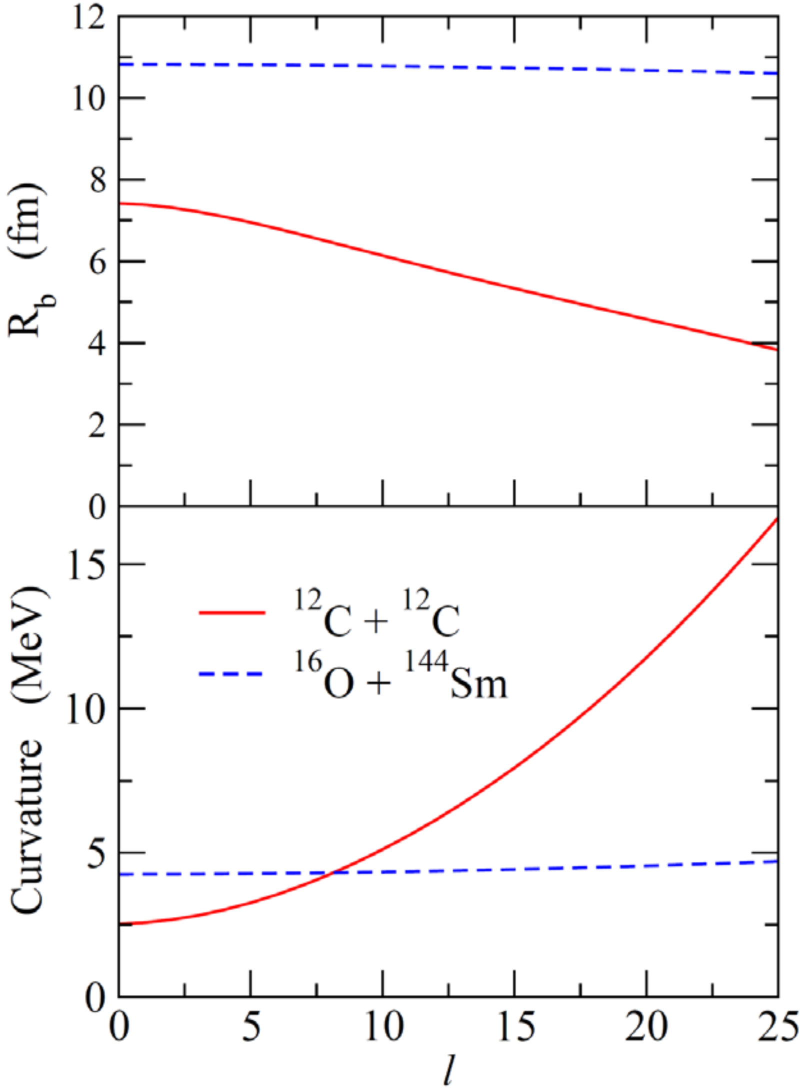


Fit with fixed V_b, R_b, Ω



$V_b = 5.6$ MeV
$R_b = 6.3$ fm
$\Omega = 3.0$ MeV

However, remember this:



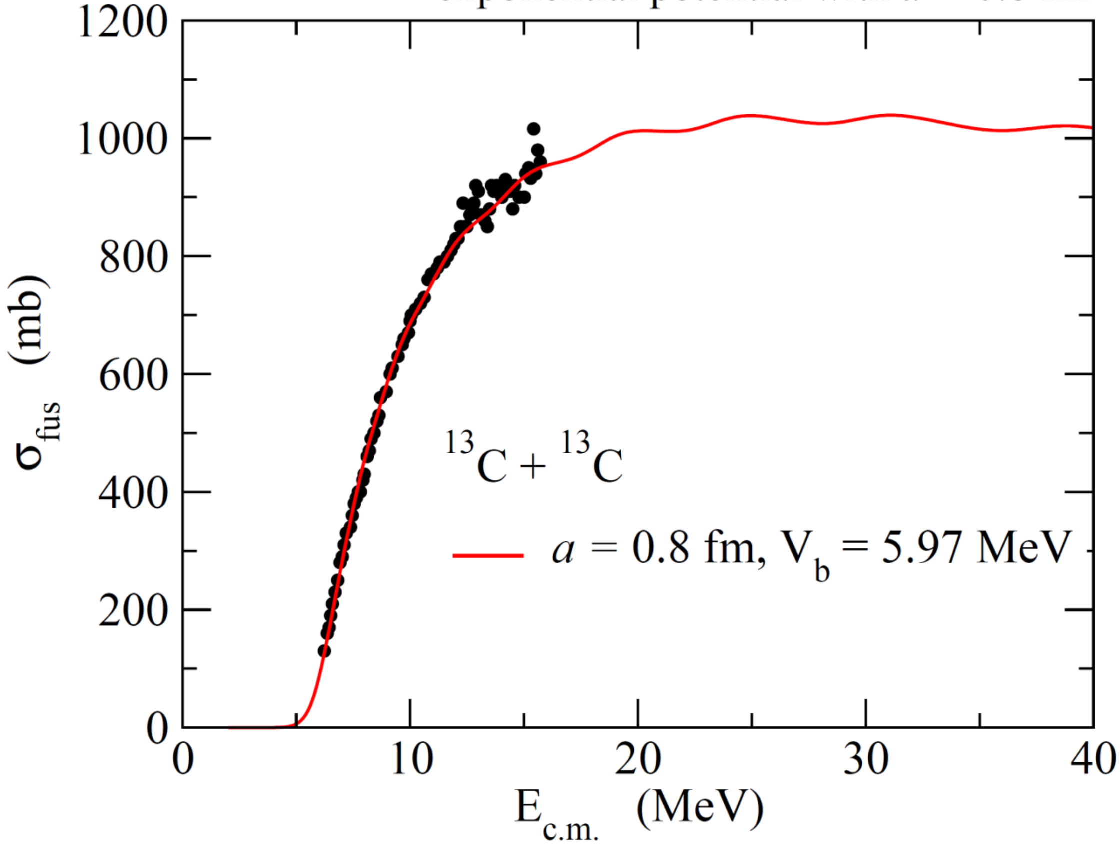
ii) $^{13}C + ^{13}C$

$^{13}C_{g.s.}$: $1/2^-$ $\longrightarrow$ the relative w.f. has to be spatially symmetric for S = 0
spatially anti-symmetric for S = 1

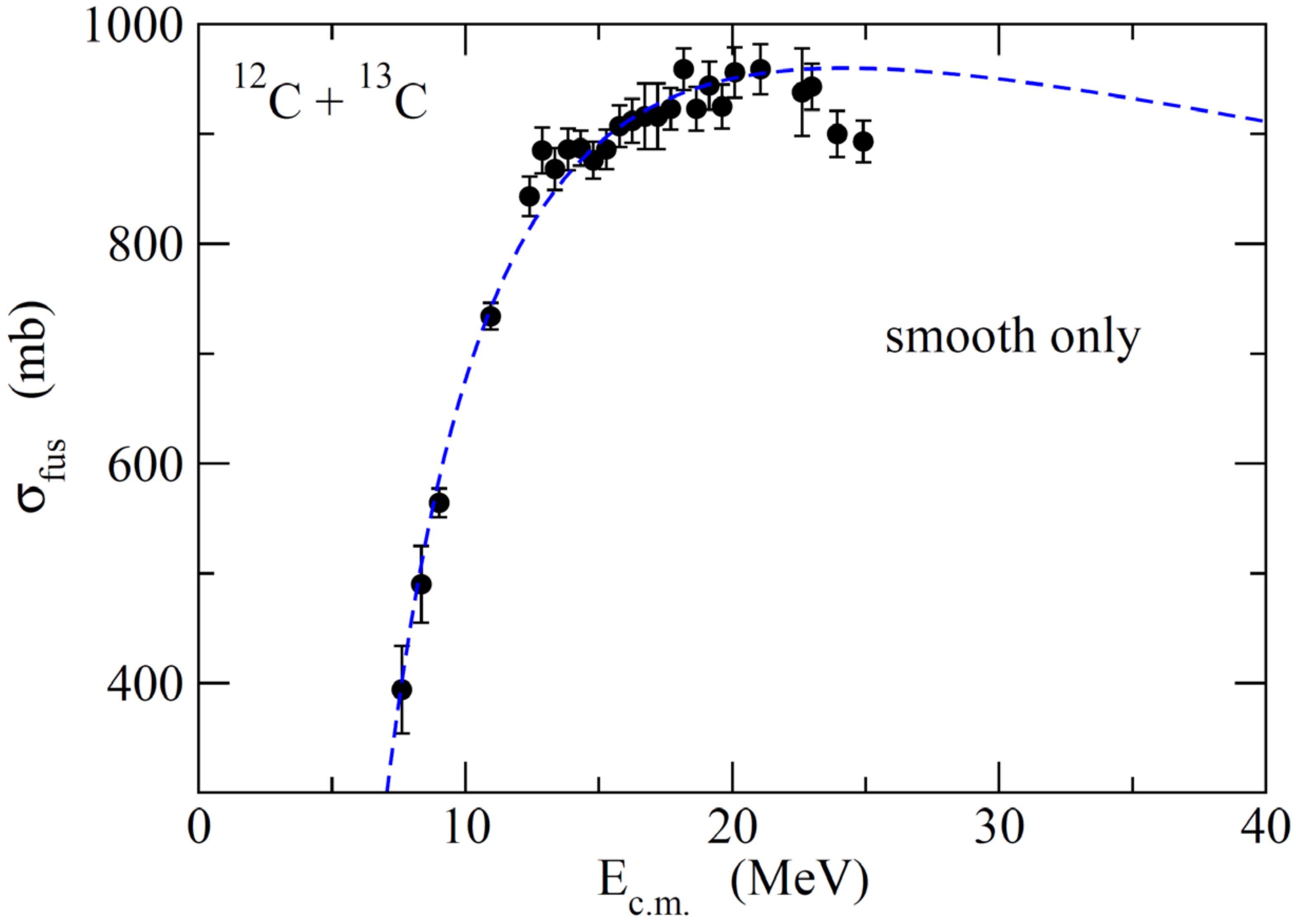
$$\sum_l \to \frac{1}{4}\sum_l \left(1+(-1)^l\right) + \frac{3}{4}\sum_l \left(1-(-1)^l\right)$$

$$\sigma_{\rm osc} = \frac{1}{2}\sigma_{\rm osc}({\rm odd}-l)$$

exponential potential with a = 0.8 fm

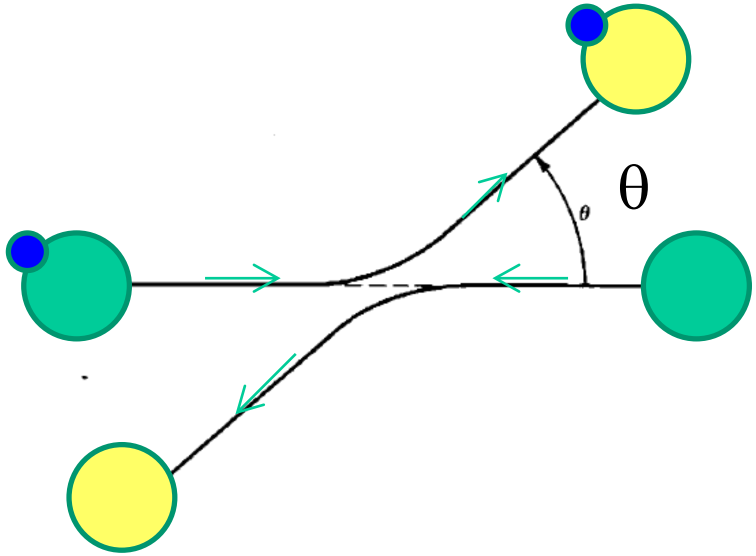


iii) $^{12}C + ^{13}C$



1000
800
600
400
0
10
20
30
40
$^{12}C + ^{13}C$
smooth only
σ_{fus} (mb)
$E_{c.m.}$ (MeV)

role of elastic transfer



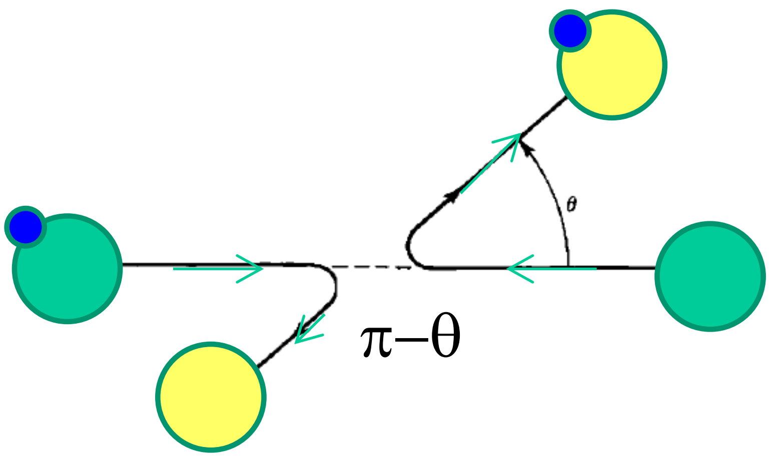
elastic scattering

$f_{\mathrm{el}}(\theta)$

indistinguishable

transfer

$f_{\mathrm{trans}}(\pi - \theta)$



$$f(\theta) \to f_{\mathrm{el}}(\theta) + f_{\mathrm{trans}}(\pi - \theta)$$

role of elastic transfer

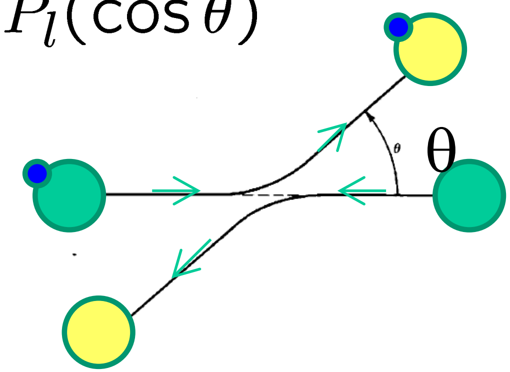
$$f(\theta) \to f_{\text{el}}(\theta) + f_{\text{trans}}(\pi - \theta)$$

$$f_{\text{el}}(\theta) = \sum_l (2l+1)\frac{S_l^{\text{el}} - 1}{2ik} P_l(\cos\theta)$$

$$f_{\text{trans}}(\pi - \theta) = \sum_l (2l+1)\frac{S_l^{\text{trans}}}{2ik} P_l(\cos(\pi - \theta))$$

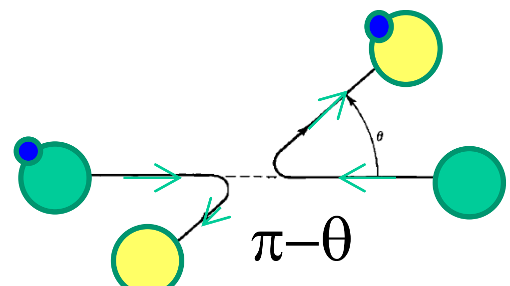
$$= (-)^l P_l(\cos\theta)$$

$$S_l^{\text{eff}} = S_l^{\text{el}} + (-1)^l\, S_l^{\text{trans}}$$



$$\text{if} \quad S_l^{\text{trans}} \sim \alpha \frac{\partial S_l^{\text{el}}}{\partial l}$$

$$S_l^{\text{eff}} = S^{\text{el}}(l + (-1)^l \alpha)$$



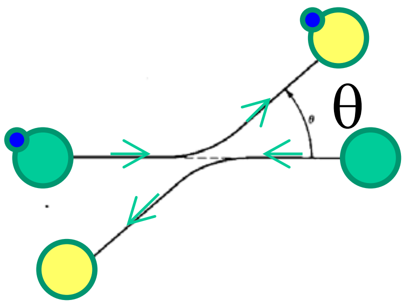
role of elastic transfer

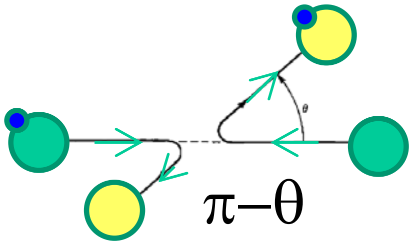
$$S_l^{\text{eff}} = S^{\text{el}}(l + (-1)^l \alpha)$$

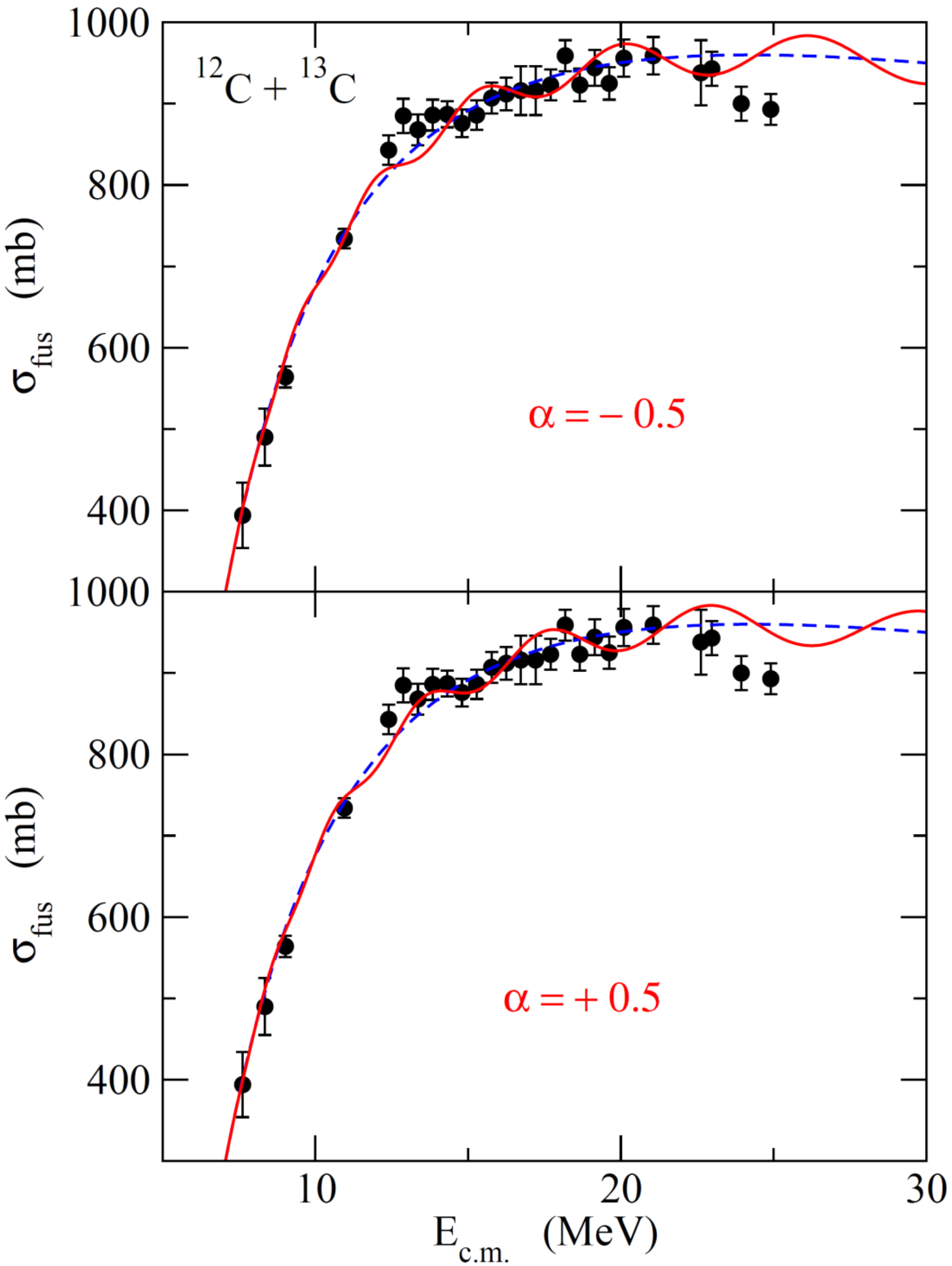
$$\sigma_{\text{osc}}(E) = \pm 2\pi R_b^2 \frac{\hbar\Omega}{E} e^{-\xi} \sin(\pi l_g),$$

$$\begin{aligned} \sin(\pi l_g) &\rightarrow [\sin(\pi(l_g + \alpha)) - \sin(\pi(l_g - \alpha))]/2 \\ &= \cos(\pi l_g)\sin(\pi\alpha) \end{aligned}$$

$$\sigma_{\text{osc}}(E) = 2\pi R_b^2 \frac{\hbar\Omega}{E} e^{-\xi} \cos(\pi l_g)\sin(\pi\alpha)$$







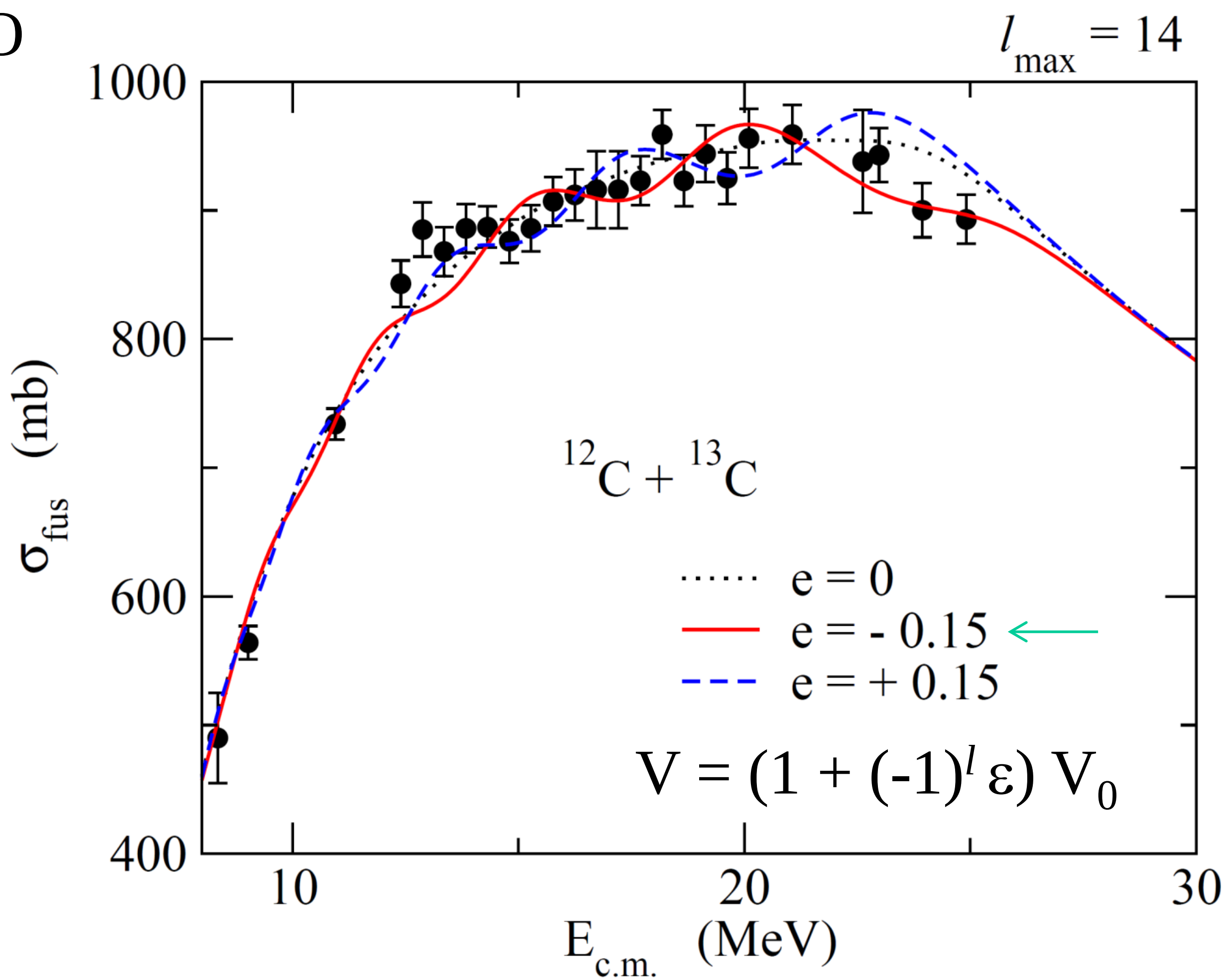
$$\sigma_{\mathsf{osc}}(E) \;=\; 2\pi R_b^2 \frac{\hbar\Omega}{E}\, e^{-\xi} \times \cos(\pi l_g)\sin(\pi\alpha)$$

exponential potential with *a* = 0.9 fm

parity-dependent potential

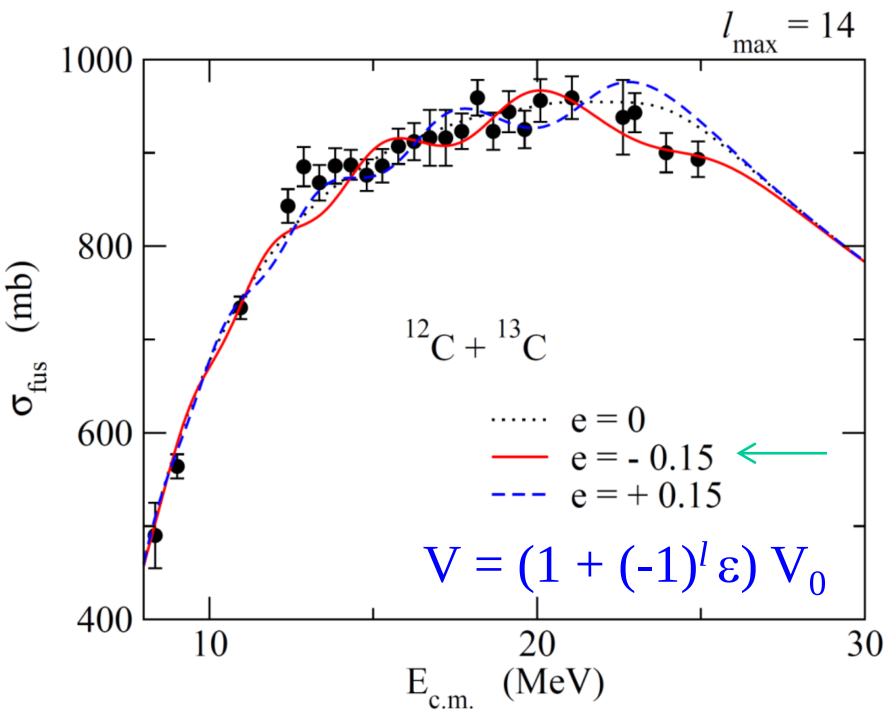
- ✓W. von Oertzen and H.G. Bohlen, Phys. Rep. 19C(‘75) 1
- ✓A. Vitturi and C.H. Dasso, Nucl. Phys. A458 (‘86) 157
- ✓A. Kabir, M.W. Kermode and N. Rowley, Nucl. Phys. A481(‘88) 94

cf. $^{12}C + ^{16}O$



exponential potential with a = 0.9 fm

parity-dependent potential



$\varepsilon < 0$

↕

a smaller V

↕

a higer barrier for even-l

cf. sign(V+ - V-) $= \varepsilon V_0 = - \varepsilon$

Baye's simple rule: ← RGM with two-center HO shell model

D. Baye, J. Deenen, and Y. Salmon, Nucl. Phys. A289('77) 511
D. Baye, Nucl. Phys. A460 ('86)581

$$\mathrm{sign}(V_+ - V_-) = -(-)^{A_<} \prod_{i:valence} \pi_i$$ (nuclear potential)

for $^{12}C+{}^{13}C(p_{1/2})$: $\mathrm{sign}(V_+ - V_-) = -(-)^{12} \cdot (-1) = +1$

Summary

- Fusion oscillations: successive contribution of discrete centrifugal bararaiers

$^{12}C(0^+)$ + $^{12}C(0^+)$
$^{13}C(1/2^-)$ + $^{13}C(1/2^-)$ } symmetrization of relative wave function
^{12}C + ^{13}C --- elastic transfer

cf. ^{14}C + ^{14}C: R.M. Freeman, C. Beck et al., PRC24 (‘81) 2390

- analytic formula for fusion oscillations

← parabolic approximation

Next talk by Rowley: coupled-channels effects