格子QCD実践入門

２００４年1月１４，１５日

大阪大学・RCNP

格子QCD

- ユークリッド化（虚時間）経路積分

$$Z = \int DUD\overline{\psi}D\psi\, \mathrm{e}^{-(S_G + \overline{\psi}\Delta\psi)} = \int DU \det\Delta\, \mathrm{e}^{-S_G}$$

  – *U*:グルーオン場、ψ：クォーク場

- ゲージ場（グルーオン場）の量子論的揺らぎ
  ⇨ モンテカルロ計算

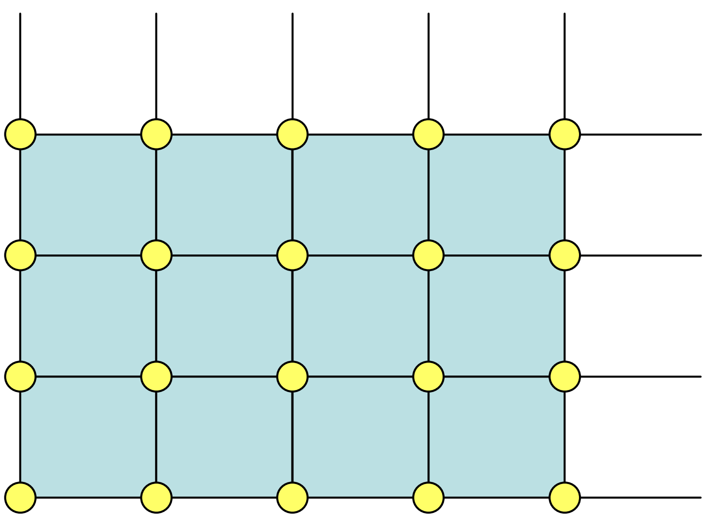
- フェルミオン（クォーク）のプロパゲータ
  ⇨ 線形計算（フェルミオン行列Δの逆行列）

リンク変数 U

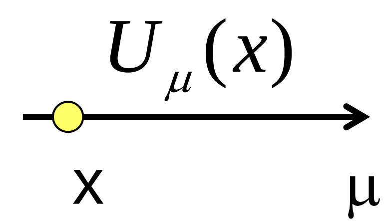
$$Z=\int DUD\overline{\psi}D\psi\, \mathrm{e}^{-(S_G+\overline{\psi}\Delta\psi)}=\int DU \det\Delta\, \mathrm{e}^{-S_G}$$

$$U_\mu(x)=e^{iA_\mu(x)}$$ μ=x,y,z,t or 1,2,3,4

$$x=(x,y,z,t)$$
$$=(x_1,x_2,x_3,x_4)$$



$$x_1=1,2,\cdots,N_x$$
$$x_2=1,2,\cdots,N_y$$
$$x_3=1,2,\cdots,N_z$$
$$x_4=1,2,\cdots,N_t$$

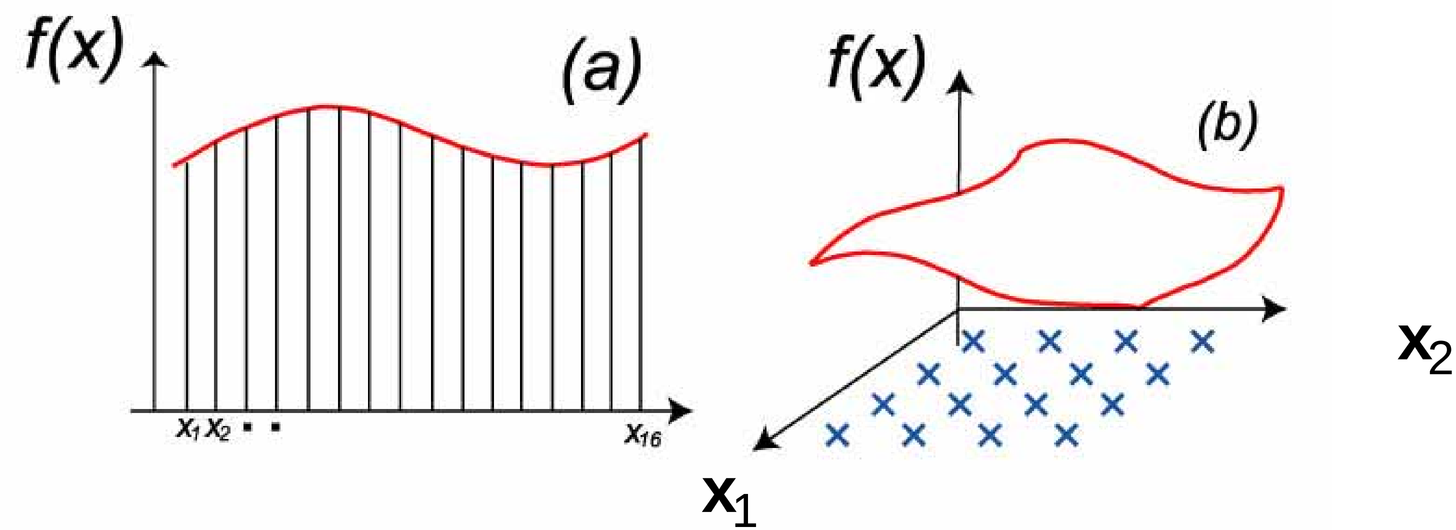


$$\int DU = \int \prod_{\mu=1,2,3,4} \prod_{x_1=1}^{N_x} \prod_{x_2=1}^{N_y} \prod_{x_3=1}^{N_z} \prod_{x_4=1}^{N_t} dU_\mu(x_1, x_2, x_3, x_4)$$

非常に多次元の積分

多次元空間での積分とモンテカルロ

$$I = \int f(x) dx_1 dx_2 dx_3 \cdots dx_n$$



1次元 2次元

数値積分の誤差

$$誤差 \sim \frac{1}{各方向の点の数} = \frac{1}{N^{1/n}}$$

N: **点の (総) 数**

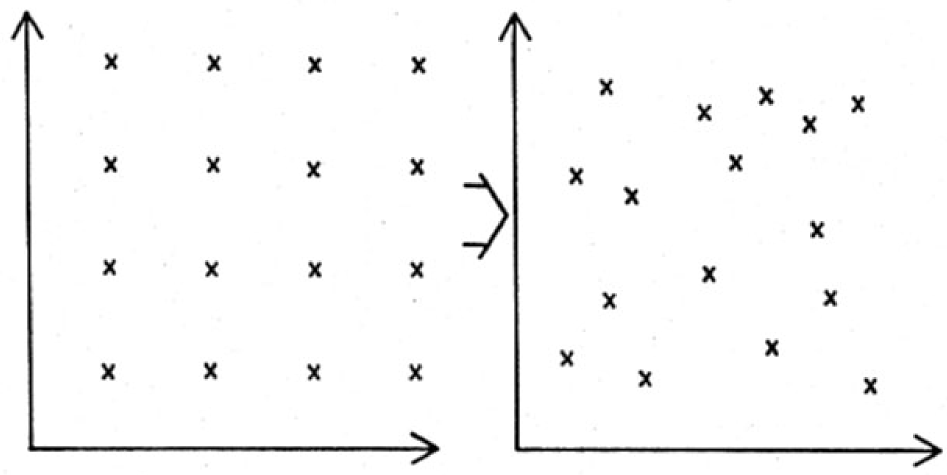
計算時間はN**に比例**

N=1000**でも**n=10**の時** $N^{1/n} = 1.99526$

格子QCD**では** $n = 4N_x N_y N_z N_t \times 8$

通常の数値積分は非現実的

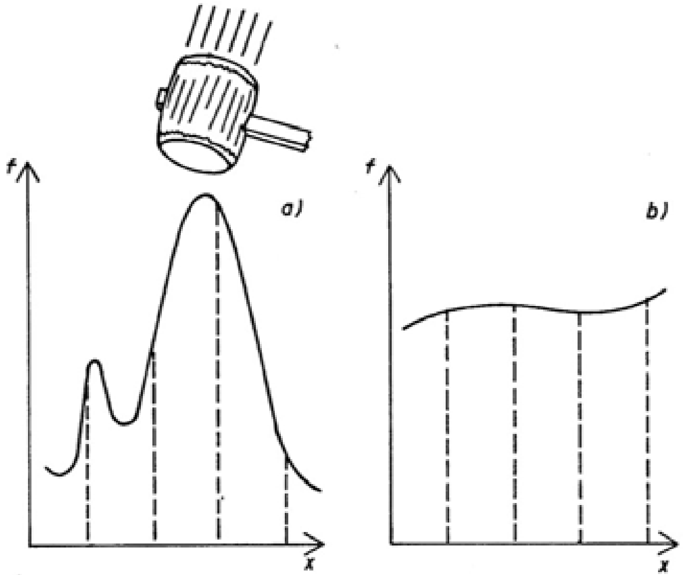
モンテカルロ法での誤差



$$誤差 \sim \frac{1}{\sqrt{N}}$$

次元nによらない！

Importance Sampling



被積分関数が平らなら
数値積分は容易

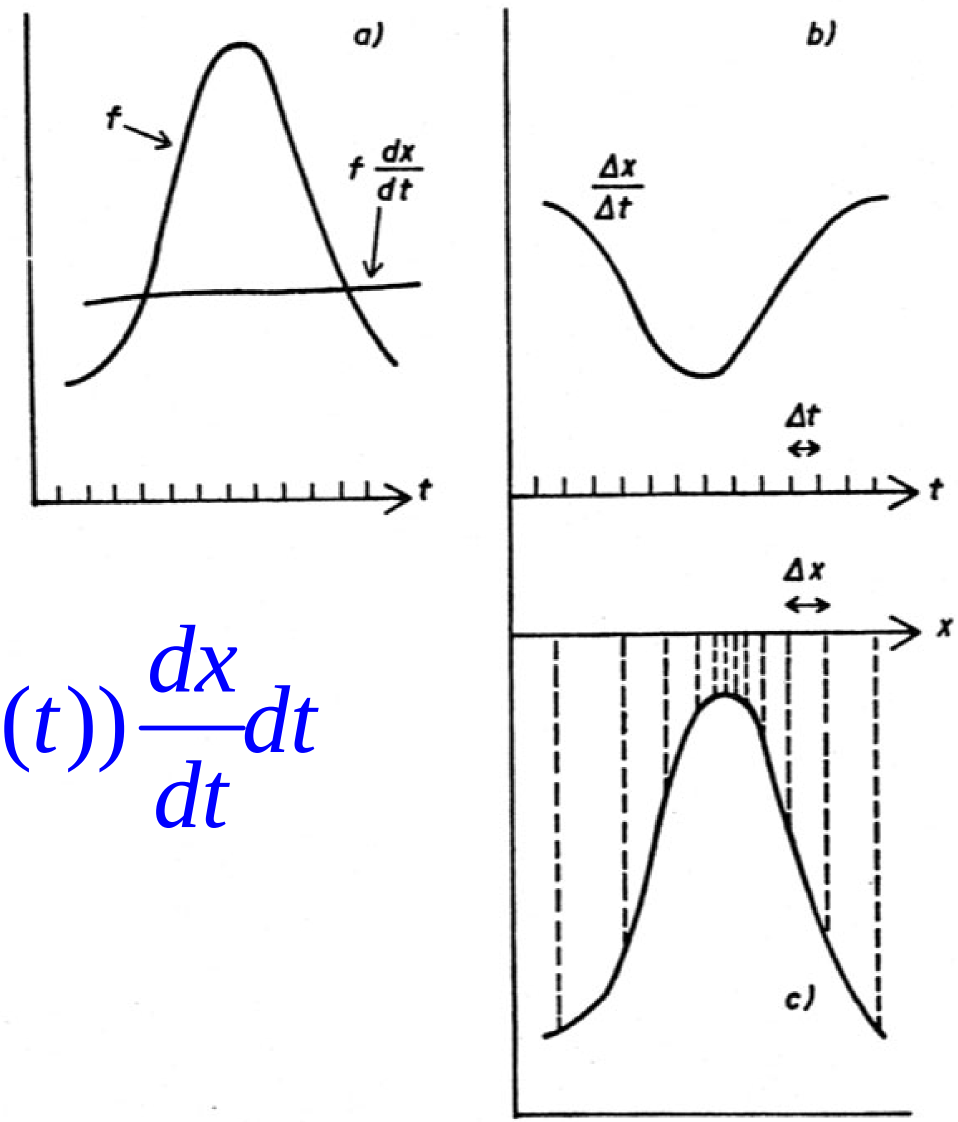
$$\frac{dx}{dt} \sim \frac{1}{f}$$ を満たすような

変数変換 x ⇒ t

$$I = \int f(x)dx = \int f(x(t))\frac{dx}{dt}dt$$

ほぼ平ら

Importance Sampling (2)

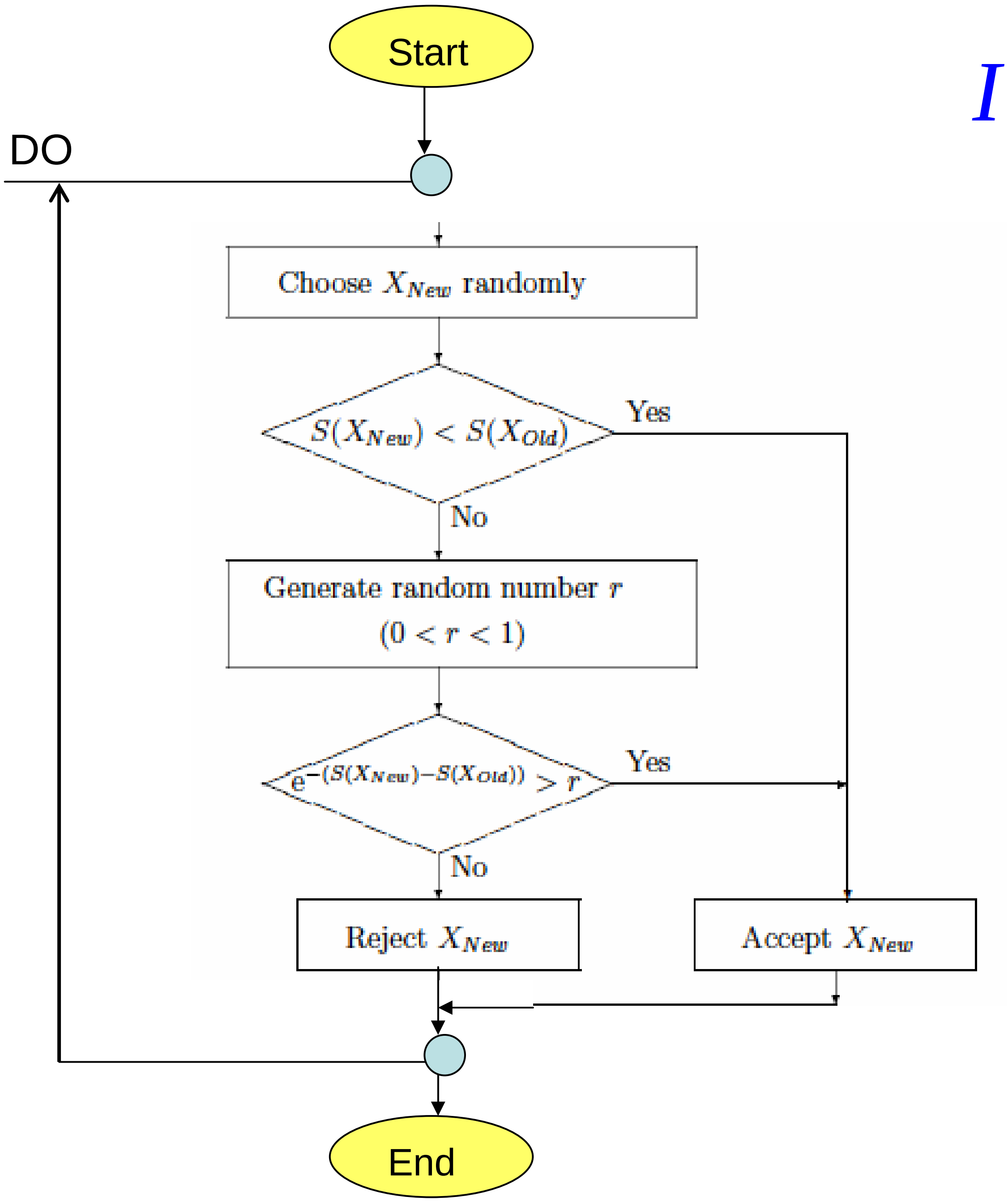


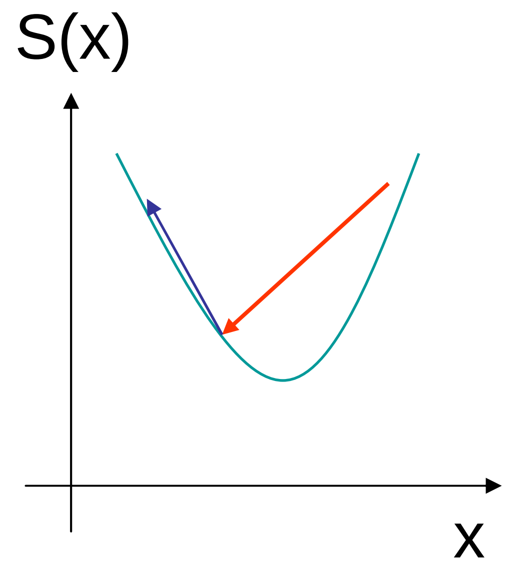
$$I = \int f(x(t)) \frac{dx}{dt} dt$$

Metropolisアルゴリズム

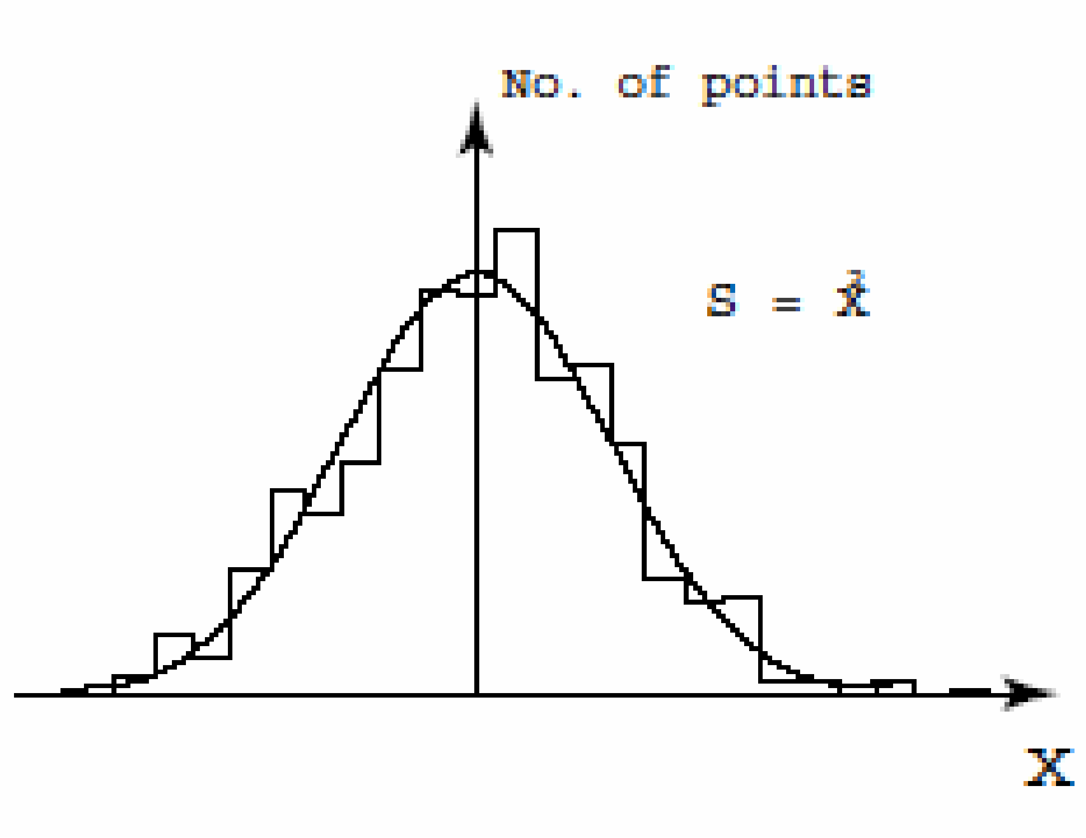
- Importance Sampling + Random Sampling
  ⇒　多次元でも通用するモンテカルロ法
- **そんなことができる？！**
  - N.Metropolis et al.
    J. Chem. Phys. 21, 1087 (1953)

$$I = \int e^{-S(x)} dx$$


Start
DO
Choose X_{New} randomly
$S(X_{New}) < S(X_{Old})$
Yes
No
Generate random number r
$(0 < r < 1)$
$e^{-(S(X_{New})-S(X_{Old}))} > r$
Yes
No
Reject X_{New}
Accept X_{New}
End


S(x)
x

$$I = \int e^{-S(x)} dx = \int e^{-x^2} dx$$

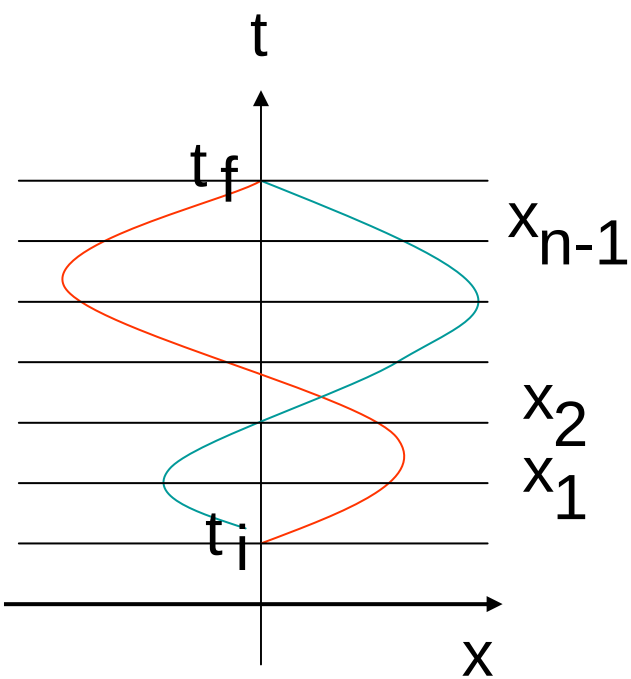


1次元量子力学

$$Z = \int Dx e^{\frac{i}{\hbar}\int dt L},$$

$$L = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 - V(x),$$

$$Dx = \lim_{n\to\infty} dx_1 dx_2 \cdots dx_n$$



$$x_0 \equiv x(t_i), x_1 \equiv x(t_1), x_2 \equiv x(t_2), \ldots, x_{n-1} \equiv x(t_{n-1}), x_n \equiv x(t_f)$$

ユークリッド化（虚時間化）

$$t \to -i\tau,$$

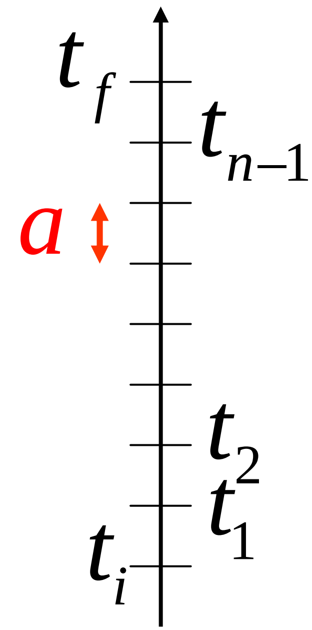
$$L \to -\frac{1}{2}m\left(\frac{dx}{d\tau}\right)^2 - V(x) = -H,$$

$$Z \to \int Dx e^{\frac{i}{\hbar}\int(-id\tau)(-H)} = \int Dx e^{-\frac{1}{\hbar}\int d\tau H} = \int Dx e^{-\frac{1}{\hbar}S}$$

離散化

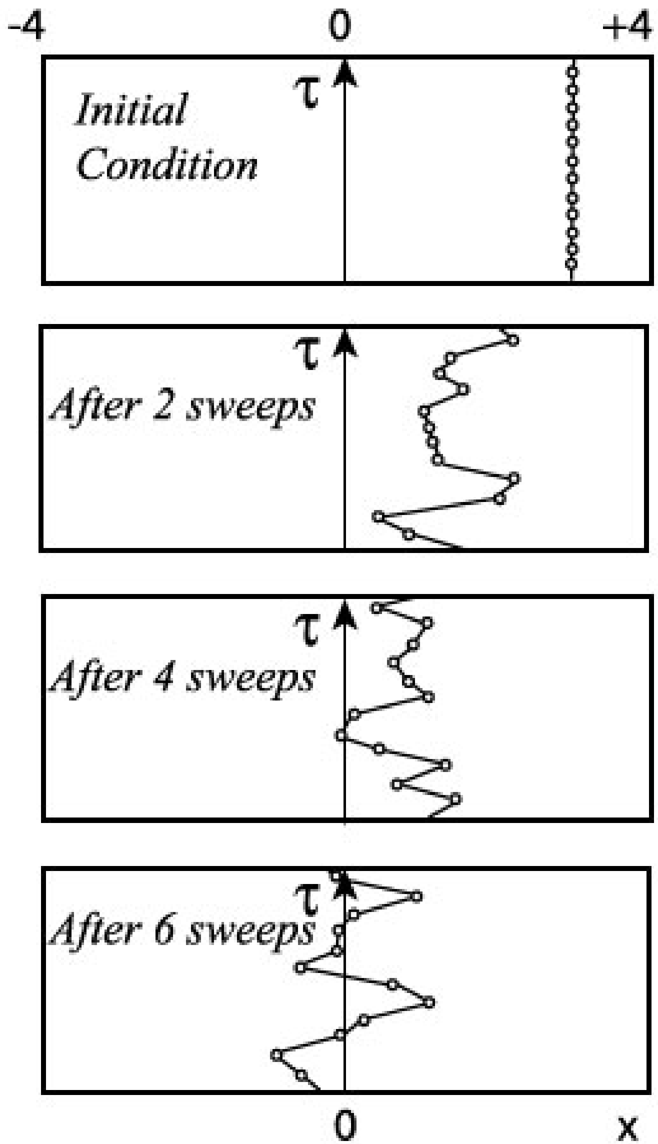
$$Z \simeq \int dx_1 dx_2 \cdots dx_{n-1} e^{-S/\hbar},$$

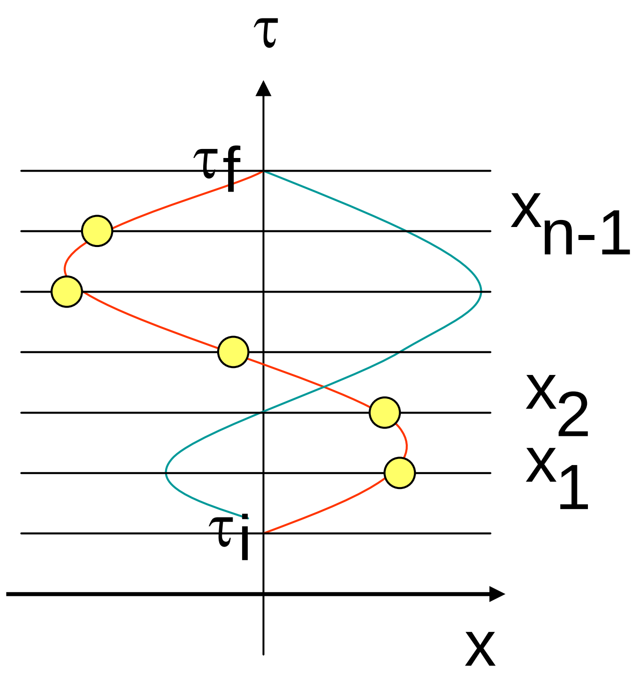
$$S = \sum_j a \left[\frac{m}{2} \left(\frac{x_{j+1} - x_j}{a} \right)^2 + V(x_j) \right]$$

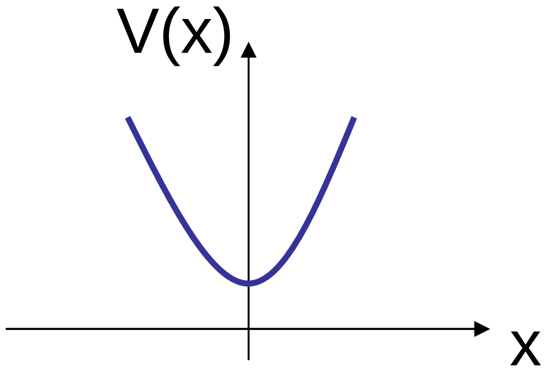


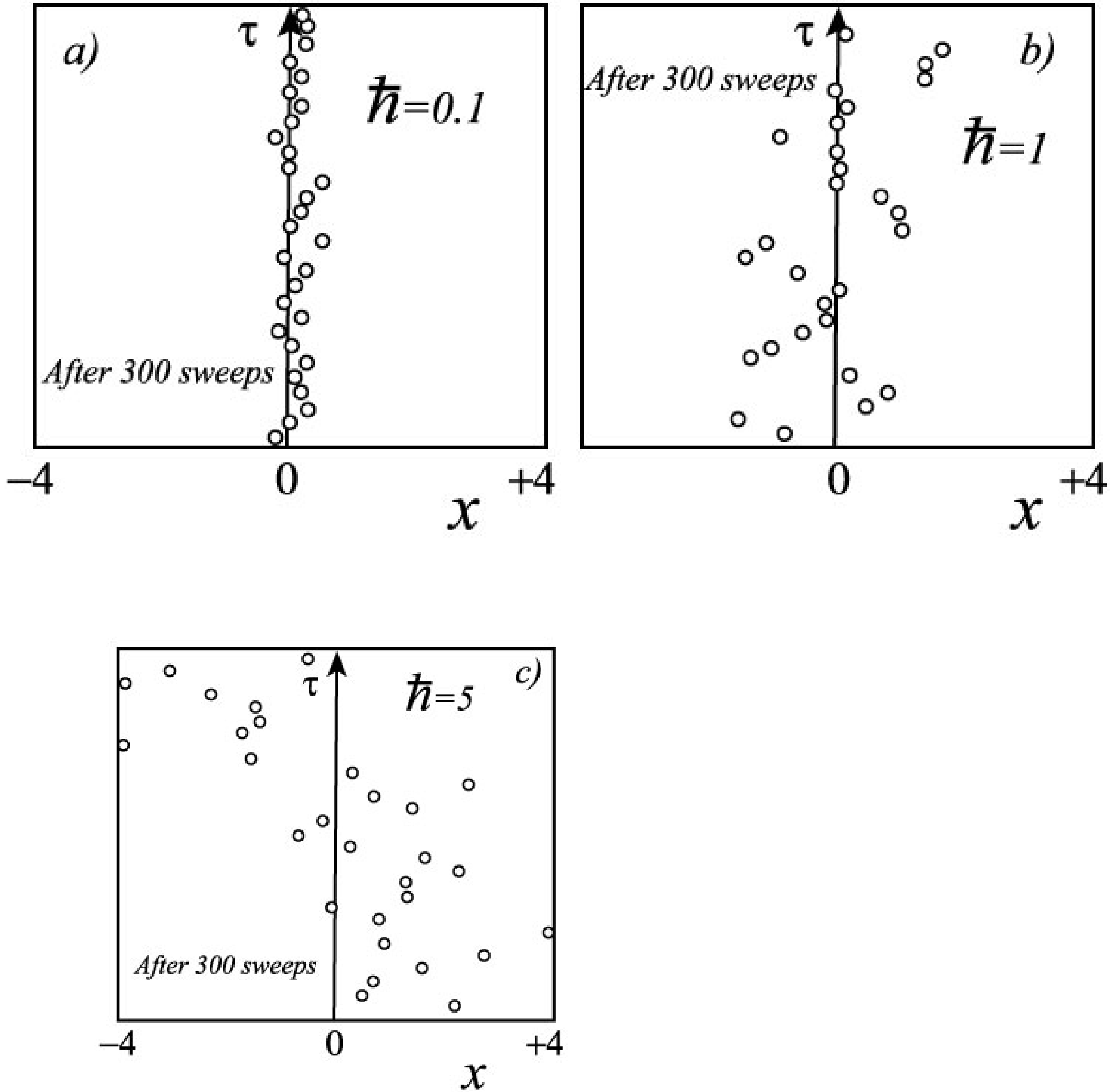
$$x_0 \equiv x(t_i), x_1 \equiv x(t_1), x_2 \equiv x(t_2), \ldots, x_{n-1} \equiv x(t_{n-1}), x_n \equiv x(t_f)$$

シミュレーション結果

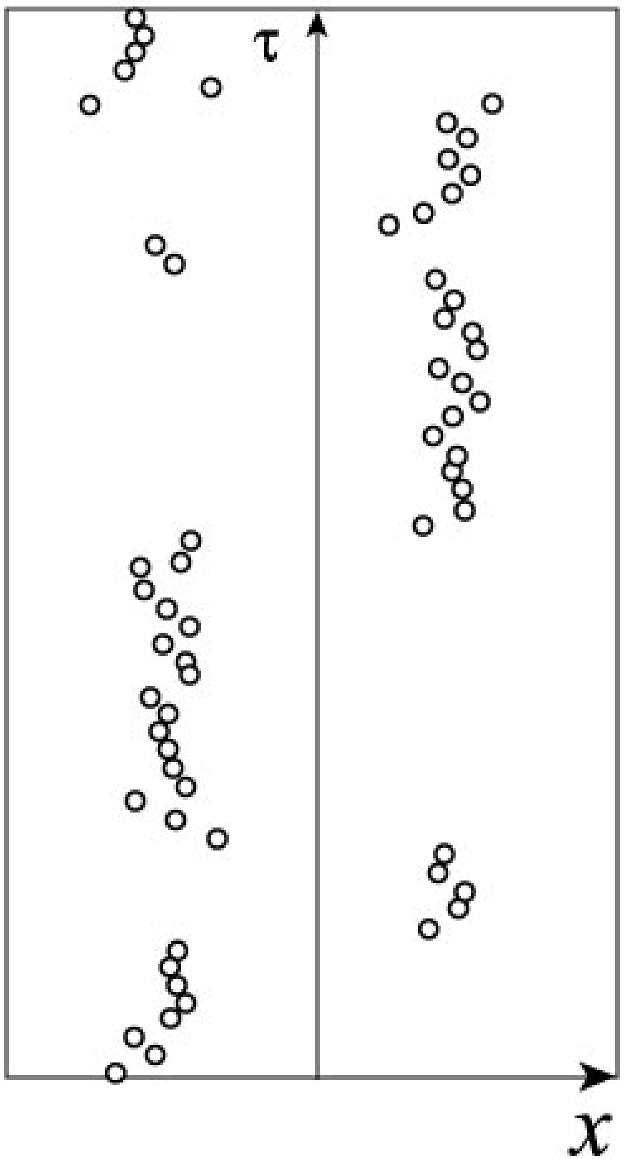

-4
0
+4
τ
Initial Condition
τ
After 2 sweeps
τ
After 4 sweeps
τ
After 6 sweeps
0
x

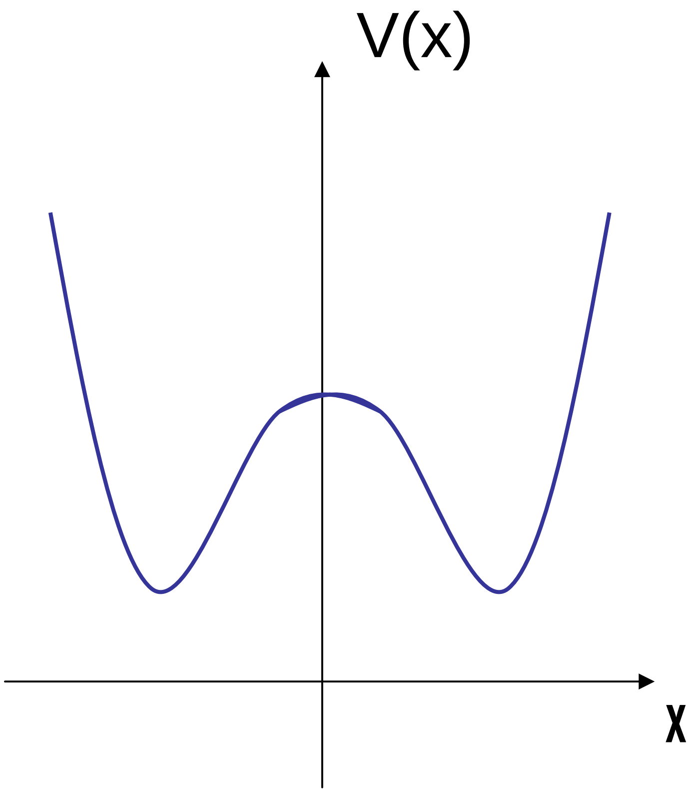

τ
τf
xn-1
x2
x1
τi
x


V(x)
x

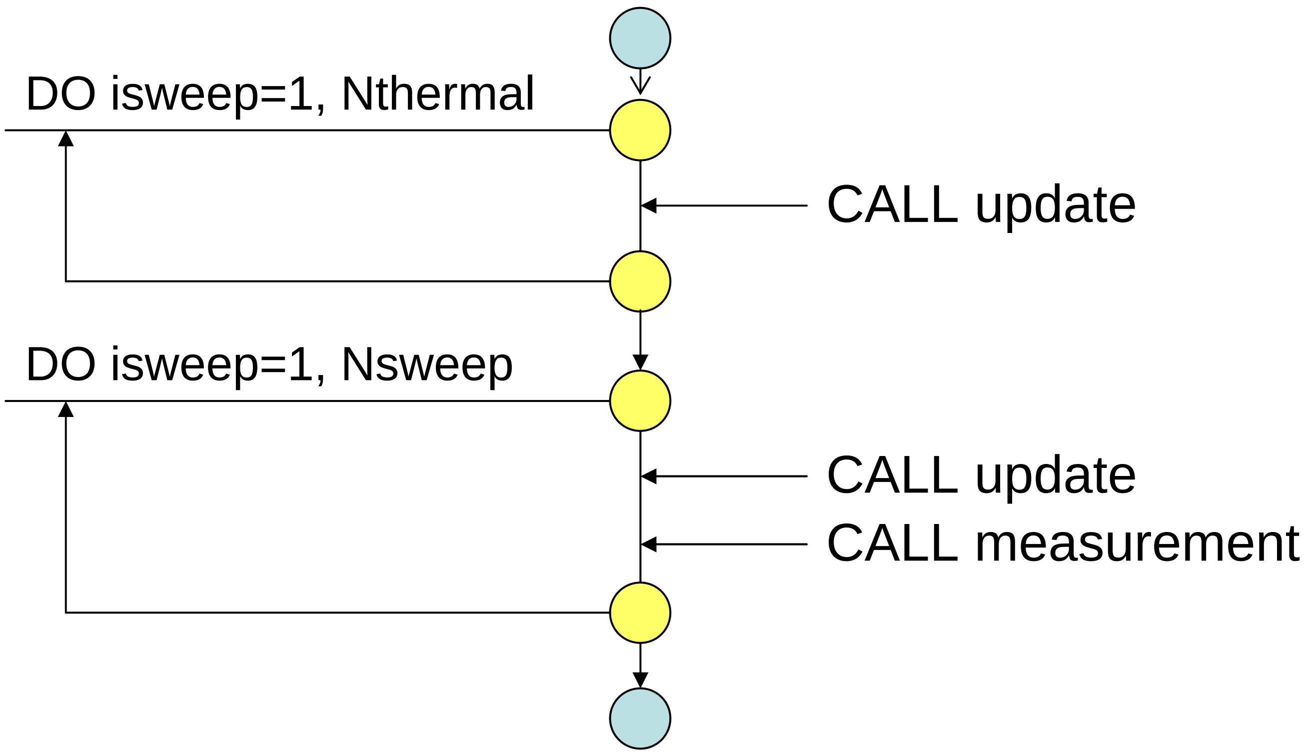

a)
τ
ħ=0.1
After 300 sweeps
−4
0
+4
x
b)
τ
After 300 sweeps
ħ=1
0
+4
x
c)
τ
ħ=5
After 300 sweeps
−4
0
+4
x

非調和振動子

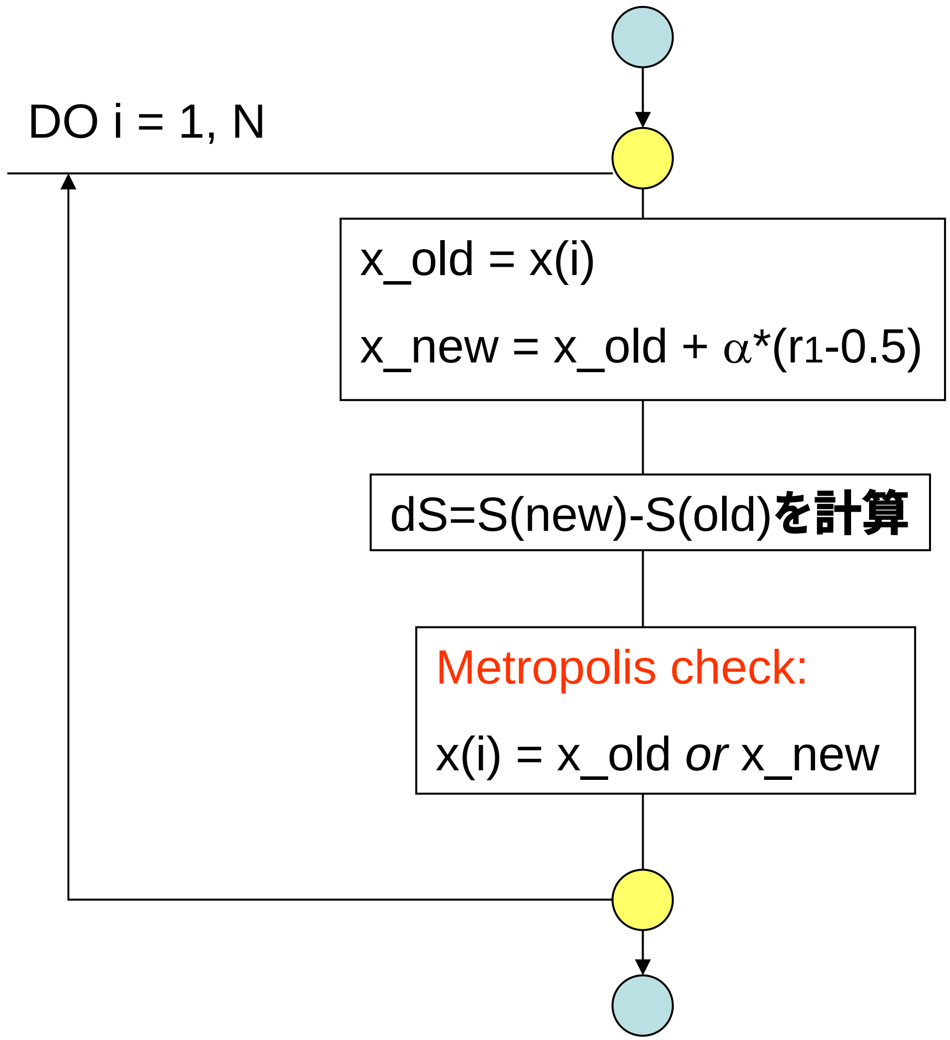




フローチャート
(1) MAIN



フローチャート
(2) update



境界条件の処理

- **周期的境界条件：** x（N+1）=x(1), x(0）＝x（N)
- **（反周期的：** x（N+1）=-x(1), x(0）＝-x（N) ）

```
DO i = 1, N
  ia = i + 1
  ib = i  - 1
  IF( i==N ) ia = 1
  IF( i==1 )  ib = N
  xa = x(ia)
  xb = x(ib)
  . . .
```

```
REAL, DIMENSION(0:N+1) :: x
x(0) = x(N)
x(N+1) = x(1)
DO i = 1, N
  xa = x(i+1)
  xb = x(i -1)
  . . .
```

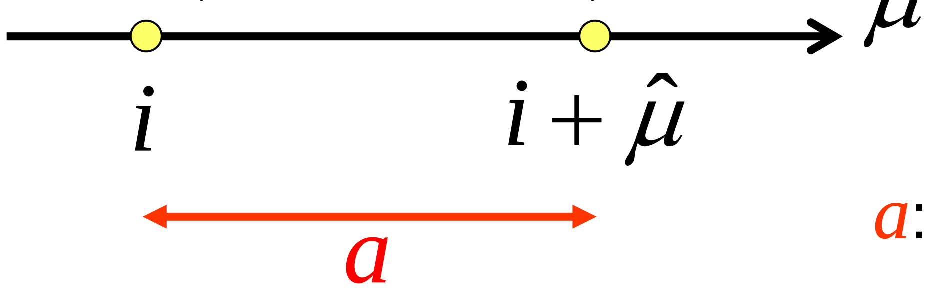
境界条件の処理（2）

```
INTEGER,  DIMENSION(N,2) :: inn

DO i = 1, N
  xa = x(inn(i,1))
  xb = x(inn(i,2))
  . . .
```

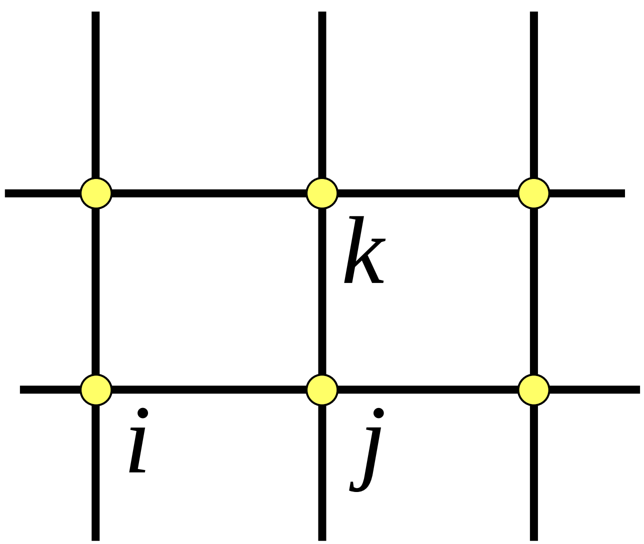
```
SUBROUTINE MakeTable

DO i = 1, N
  ia = i + 1;  ib = i – 1
  IF( i==N ) ia = 1
  IF( i==1  ) ib = N
  inn(i,1) = ia
  inn(I,2) = ib
ENDDO
RETURN
END
```

格子QCDのラグランジアン (準備)

$$U_\mu(i) = U_{i,i+\hat{\mu}}$$



a:格子間隔



$$U_{i,j} U_{j,k}$$

$$\bar{\psi}_i U_{i,j} \psi_j$$

$$U_{j,i} = U_{i,j}{}^\dagger$$

$$U = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix}$$

$$U^{\dagger} = {}^{t}U^{*}$$

$$UU^{\dagger} = I \qquad \det UU^{\dagger} = \det U (\det U)^{*}$$

$$\det U = 1$$

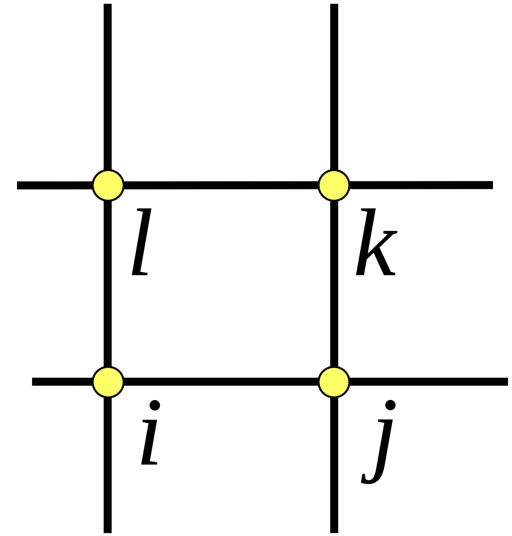
$$U = e^{iA}$$

$$A^{\dagger} = A,$$

$$\det U = e^{Tr \log U} = e^{iTrA} = 1$$

格子QCDのラグランジアン

- K.G.Wilson
  - Phys. Rev. D10, 2445 (1974)
  - Erice Lecture Note 1977



$$S = S_G + S_F$$

$$S_G = \beta \sum_{plaquette} \left\{ 1 - \frac{1}{N_c} Tr(U_{ij} U_{jk} U_{kl} U_{li}) \right\}$$

$$\beta \equiv \frac{2N_c}{g^2}$$

$$U_{i,j} \in SU(N_c)$$

問題：サイズ NxNyNzNt**の格子にプラケットはいくつあるか？**

フェルミオン（クォーク）作用

$$S_F = \sum_{i,j} \bar{\psi}_i W(i,j) \psi_j$$

$$W(i,j) = I - \kappa \sum_{\mu=1}^{4} \left\{ (1-\gamma_\mu) U_{i,j} \delta_{i+\hat{\mu},j} + (1+\gamma_\mu) U_{i,j} \delta_{i-\hat{\mu},j} \right\}$$

$$W_{\alpha\beta}^{ab}(i,j) = \delta_{\alpha\beta}\delta_{ab}\delta_{ij} - \kappa \sum_{\mu=1}^{4} \left\{ (1-\gamma_\mu)_{\alpha\beta} U_{i,j}{}^{ab} \delta_{i+\hat{\mu},j} + (1+\gamma_\mu)_{\alpha\beta} U_{i,j}{}^{ab} \delta_{i-\hat{\mu},j} \right\}$$

κ : hopping parameter

(古典) 連続極限

$$U_{\mu}(n) = e^{igaA_{\mu}(na)}$$

$$\psi_{n} = \sqrt{\frac{a^{3}}{2\kappa}}\psi(na)$$

$$\lim_{a\to 0} S_{G} = \frac{1}{2}\int d^{4}x Tr\left\{F_{\mu\nu}{}^{2}\right\}$$

$$\lim_{a\to 0} S_{F} = -\int d^{4}x\left\{m\bar{\psi}(x)\psi(x) + \bar{\psi}(x)\gamma_{\mu}(\partial_{\mu} + igA_{\mu}(x))\psi(x)\right\}$$

ウォーミングアップ
U(1)の場合

$$P_{\mu\nu}(x) \equiv U_\mu(x)U_\nu(x+\hat{\mu})U_\mu{}^\dagger(x+\hat{\nu})U_\nu{}^\dagger(x)$$

$$= e^{iagA_\mu(x)} e^{iagA_\nu(x+\hat{\mu})} e^{-iagA_\mu(x+\hat{\nu})} e^{-iagA_\nu(x)}$$

$$= e^{ia^2 g\left(\frac{A_\nu(x+\hat{\mu})-A_\nu(x)}{a} - \frac{A_\mu(x+\hat{\nu})-A_\mu(x)}{a}\right)}$$

$$= e^{ia^2 g\tilde{F}_{\mu\nu}(x)} = 1 + ia^2 g\tilde{F}_{\mu\nu} - \frac{1}{2}a^4 g^2 \tilde{F}_{\mu\nu}{}^2 + \ldots.$$

$$\sum_{plaquette} P_{\mu\nu}(x) = \sum_x \left(1 - \frac{1}{2}a^4 g^2 \tilde{F}_{\mu\nu}{}^2 + ..\right)$$

必要な関係式

$$e^X e^Y = e^F$$

$$F = X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}([X,[X,Y]] + [Y,[Y,X]]) + \ldots$$

$$f(x+\hat{\mu}) = f(x) + a\partial_\mu f(x) + O(a^2)$$

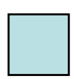
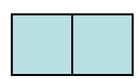
$$\kappa = \frac{1}{8+2ma} \qquad \psi_n = \sqrt{\frac{a^3}{2\kappa}}\psi(na)$$

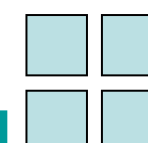
練習問題

- S_G,S_Fがa→0で通常のQCDの作用になることを示せ

作用はユニークではない

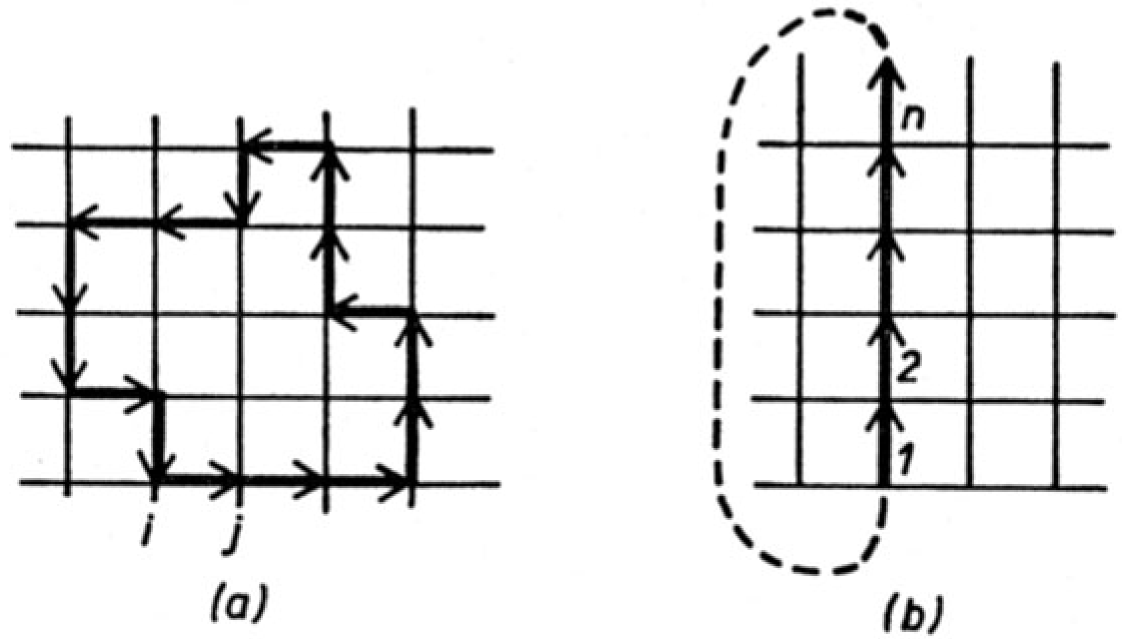
- 古典連続極限(naïve classical limit)がQCD作用になる
- ゲージ不変な $TrU_{ij}U_{jk}U_{kl}\ldots U_{xi},\ \bar{\psi}\ldots\ldots\psi$ はみなOK
  - O(a)の高次項の効果を減少するもの：Improved action
  - よく使われるもの
    - ゲージ：Iwasaki 作用、Syzmanzik作用、(DBW2作用)

$\beta(C_0$  $+C_1$  $)$



- フェルミオン：Wilson fermions、Wilson with Clover作用
  KS(Kogut-Susskind, or staggered) fermions

Wilson LoopとPolyakov Line



$$W = \frac{1}{N_c} Tr(U_{ij} U_{jk} \cdots U_{li})$$

$$L = \frac{1}{N_c} Tr(U_{12} U_{23} \cdots U_{n-1,n})$$

ゲージ場のexternal source $j_\mu = g\delta^3(x_\mu - x_\mu(t))$

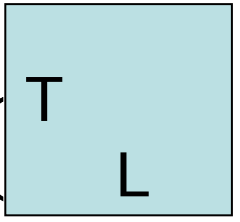
⇒ **系のエネルギーの増加** $i\int d^4x j_\mu A_\mu = ig\int dx_\mu A_\mu$

$$e^{-S_G} \to e^{-ig\int dx_\mu A_\mu - S_G}$$

$$= e^{igaA_n} e^{igaA_{n-1}} \cdots e^{igaA_1} e^{-SG}$$

$$= We^{-SG} \text{ or } Le^{-SG}$$

$$\frac{e^{-(F+\Delta F)}}{e^{-F}} = \frac{\int dU e^{-S_G} W}{Z} = < W >

$< Tr($ 

$) >= e^{-TV(L)}$

Polyakov　Line

- Polyakov line:**クォークラインが1本あるときのエネルギー増加**

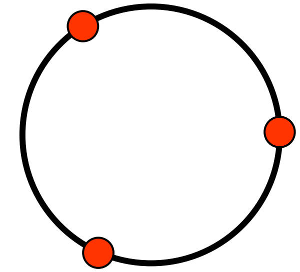
$$< L >= e^{-\Delta F}$$

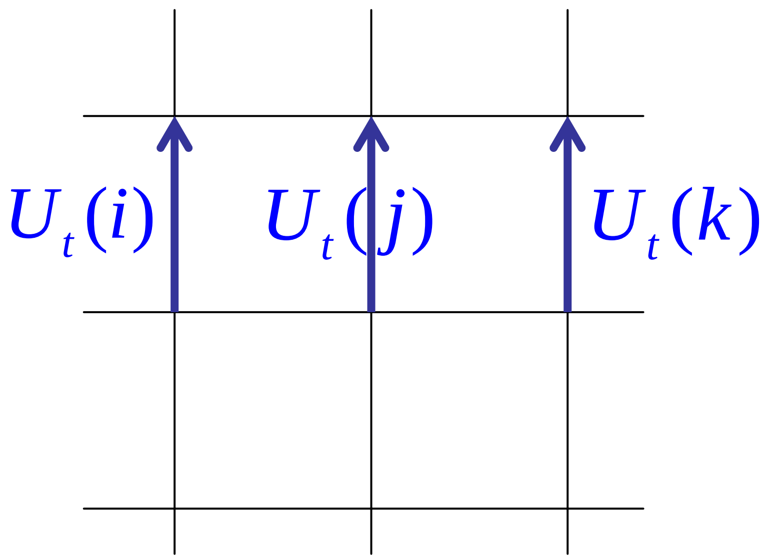
Confinement： $\Delta F = \infty$

⇒ $< L >= 0$

Z3対称性

- SU(3)の要素のうち、1、 $e^{i\frac{2\pi}{3}}, e^{i\frac{4\pi}{3}}$ は他と可換





$$U_t(i), U_t(j), \cdots U_t(k)$$

$$\rightarrow zU_t(i), zU_t(j), \cdots zU_t(k)$$

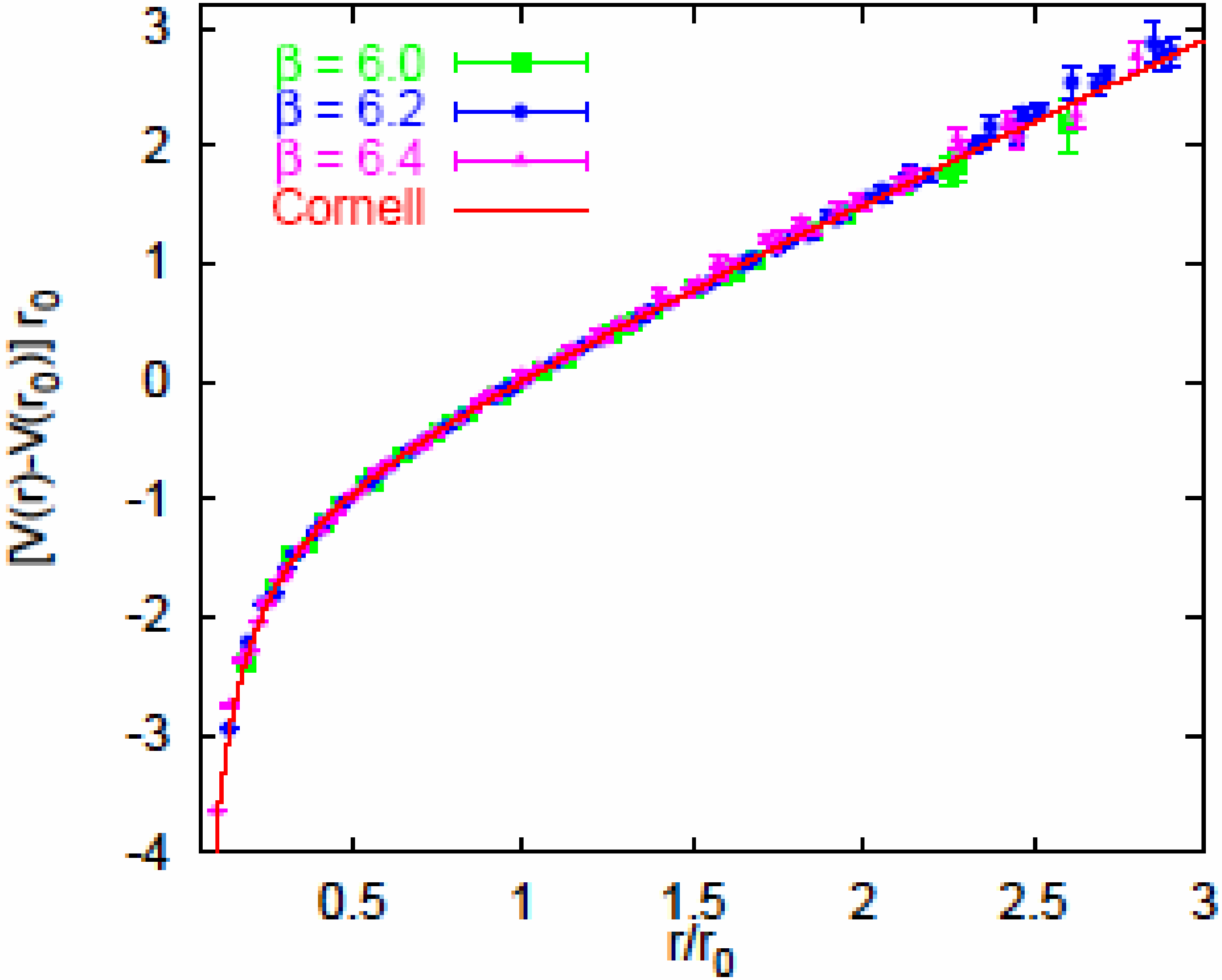
$$z \in Z_3$$

S_G は不変

$$L \rightarrow zL$$

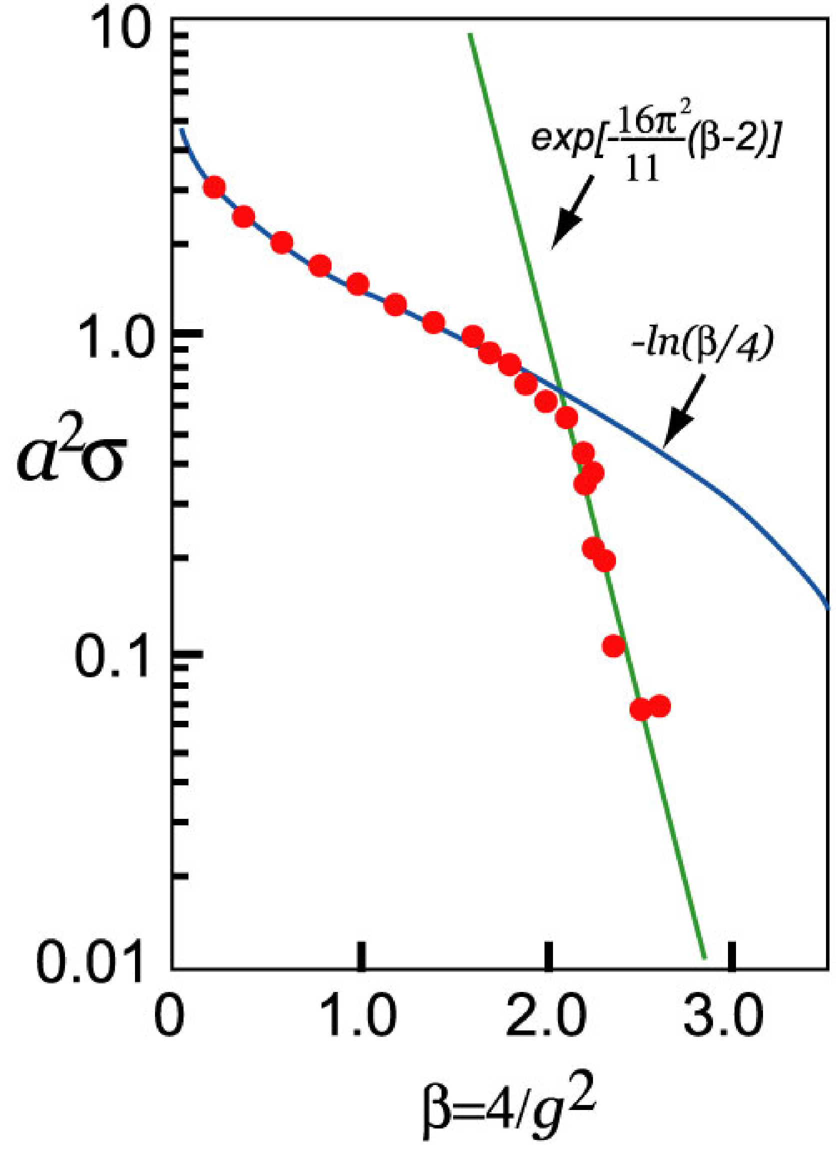
(クエンチ近似では) $< L > \neq 0$ $\Longrightarrow$ Z3対称性の自発的破れ

重クォークポテンシャル

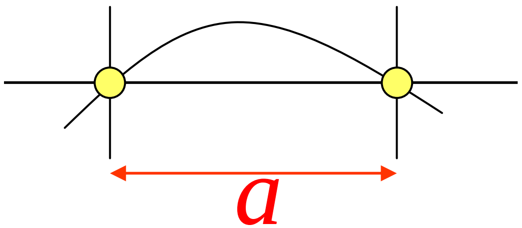


ゲージ結合定数と格子間隔

- M.Creutz,
  - Phys.Rev.D21, 2308 (1980)
  - SU(2)



- **格子**： $(\text{カットオフ}) = \dfrac{\pi}{a}$



$$m = \frac{1}{a} F(g)$$

m：質量次元をもった量

$$\frac{d}{da} m = 0$$

$$\Rightarrow \quad F = a \frac{dF}{da} = a \frac{dg}{da} \frac{dF}{dg} = -\beta(g) \frac{dF}{dg}$$

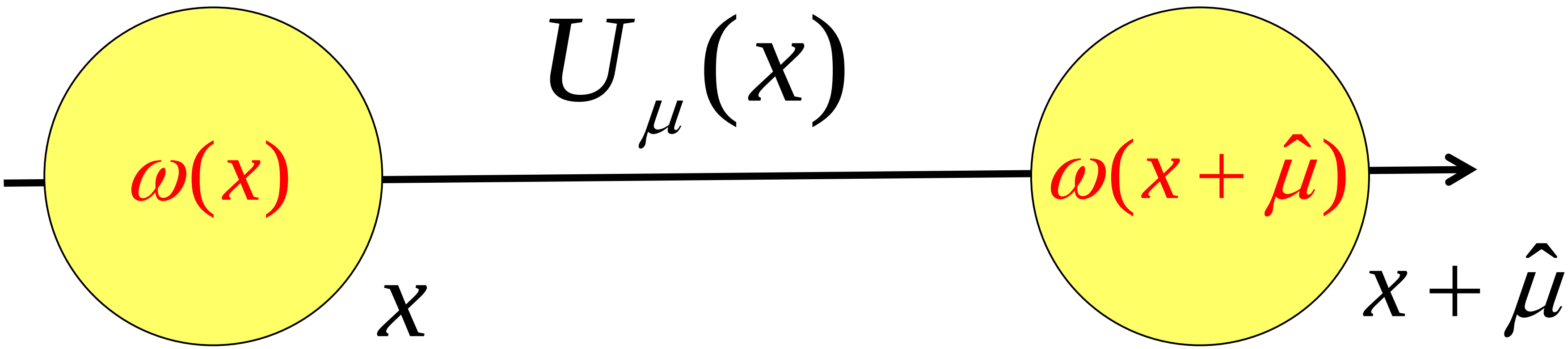
$$\beta(g) = -a \frac{dg}{da}$$

$$\beta(g) = -\beta_0 g^3 - \beta_1 g^5 + \dots$$

$$\int \frac{da}{a} = \int \frac{dg}{-\beta(g)} = \int \frac{dg}{\beta_0 g^3 + \beta_1 g^5}$$

$$a = \frac{1}{\Lambda}\left(\frac{1}{\beta_0 g^2}\right)^{\frac{\beta_1}{2\beta_0{}^2}} e^{-\frac{1}{2\beta_0 g^2}}$$

格子上のゲージ変換



$$U_\mu(x) \to \omega(x)^\dagger U_\mu(x)\omega(x+\hat{\mu})$$

$$\bar{\psi}(x) \to \bar{\psi}(x)\omega(x)$$

$$\psi(x) \to \omega(x)^\dagger \psi(x)$$

$$\bar{\psi}(x)\psi(x)$$

$$\bar{\psi}(x)U_\mu(x)\psi(x+\hat{\mu})$$

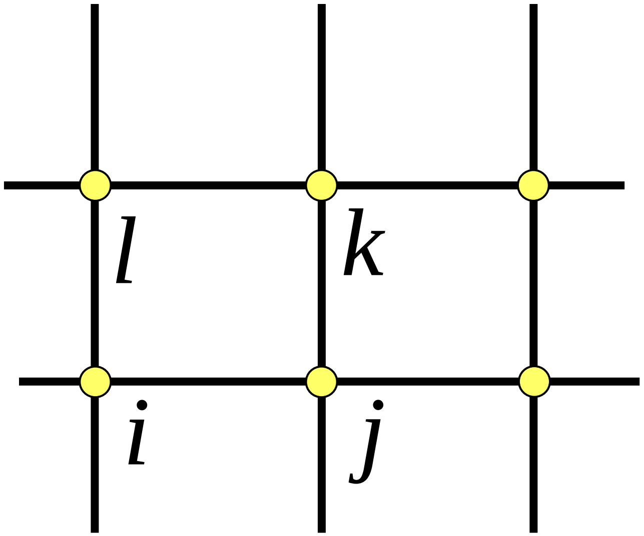
不変

$$U_{ij}U_{jk}U_{kl}U_{li} = U_{ij}U_{jk}U_{lk}{}^{\dagger}U_{il}{}^{\dagger}$$

$$\to (\omega_i{}^{\dagger}U_{ij}\omega_j)(\omega_j{}^{\dagger}U_{jk}\omega_k)(\omega_l{}^{\dagger}U_{lk}\omega_k)^{\dagger}(\omega_i{}^{\dagger}U_{il}\omega_l)^{\dagger}$$

$$= (\omega_i{}^{\dagger}U_{ij}\omega_j)(\omega_j{}^{\dagger}U_{jk}\omega_k)(\omega_k{}^{\dagger}U_{lk}{}^{\dagger}\omega_l)(\omega_l{}^{\dagger}U_{il}{}^{\dagger}\omega_i)$$

$$= \omega_i{}^{\dagger}U_{ij}U_{jk}U_{lk}{}^{\dagger}U_{il}{}^{\dagger}\omega_i$$



$$Tr\,U_{ij}U_{jk}U_{kl}U_{li}$$

不変

ゲージ変換 (連続極限)

$$\omega(x)^\dagger U_\mu(x)\omega(x+\hat{\mu})$$

- U(1)ケース

$$\omega(x) = e^{i\chi(x)} \qquad U_\mu(x) = e^{iaA_\mu(x)}$$

$$U_\mu(x) = e^{iaA_\mu(x)} \to e^{-i\chi(x)} e^{iaA_\mu(x)} e^{i\chi(x+\hat{\mu})}$$

$$A_\mu(x) \to A_\mu(x) + \frac{\chi(x+\hat{\mu}) - \chi(x)}{a}$$

$$= A_\mu(x) + \partial_\mu \chi + O(a)$$

- SU(N)ケース

$$e^{iaA_\mu(x)} \to \omega(x)^\dagger e^{iaA_\mu(x)} \omega(x+\hat{\mu})$$

$$(1+iaA_\mu(x)+..) \to \omega(x)^\dagger (1+iaA_\mu(x)+..)(\omega(x)+a\partial_\mu \omega(x)+..)$$

$$A_\mu(x) \to \omega(x)^\dagger A_\mu(x)\omega(x) - i\omega(x)^\dagger \partial_\mu \omega(x)$$

$$+O(a)$$

クォークプロパゲータ

- クォークプロパゲータ=フェルミオン行列Wの逆
- Gaussの消去法？
  - N^3の演算　（N：行列のランク）
  - Wが疎行列(Sparse行列) であることを利用できない
- 多くの場合 $W\vec{x} = \vec{b}$ を解ければ充分。

$$\vec{b} = \vec{e}_i = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \qquad \vec{x} = \left(W^{-1}\right)\vec{e}_i$$

共役勾配法 (Conjugate Gradient Method, CG法)

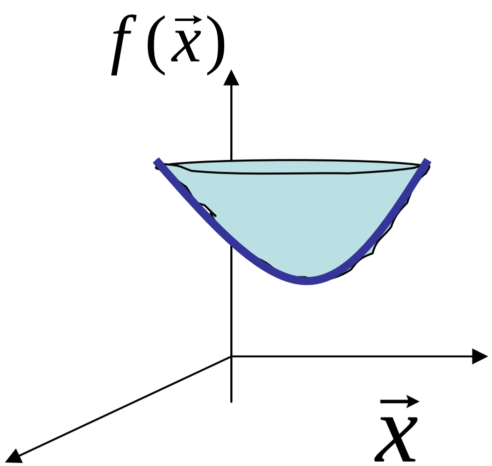
$$A\vec{x} = \vec{b}$$

A:対称、正定値とする。

$(\vec{x}, A\vec{x}) \geq 0, \quad$ for $\forall \vec{x}$

そうでない場合は $^tAA\vec{x} = {}^tA\vec{b}$ とする。

$$f(\vec{x}) = \frac{1}{2}(\vec{x}, A\vec{x}) - (\vec{b}, \vec{x})$$



を最小化

解はもちろん底の所で

$$\nabla f(\vec{x}) = A\vec{x} - \vec{b} = \vec{0}$$

CG法

$$\vec{p}^{(0)} = \vec{r}^{(0)} = \vec{b} - A\vec{x}^{(0)}$$

DO i

$$\alpha^{(i)} = \frac{(\vec{p}^{(i)}, \vec{r}^{(i)})}{(\vec{p}^{(i)}, A\vec{p}^{(i)})}$$

$$\vec{x}^{(i+1)} = \vec{x}^{(i)} + \alpha^{(i)}\vec{p}^{(i)}$$

$$\vec{r}^{(i+1)} = \vec{r}^{(i)} - \alpha^{(i)}A\vec{p}^{(i)}$$

$$\beta^{(i)} = \frac{(\vec{r}^{(i+1)}, A\vec{p}^{(i)})}{(\vec{p}^{(i)}, A\vec{p}^{(i)})}$$

$$\vec{p}^{(i+1)} = \vec{r}^{(i+1)} + \beta^{(i)}\vec{p}^{(i)}$$

$$\vec{r}^{(i)} = \vec{b} - A\vec{x}^{(i)}$$

Residue,残差

$\vec{p}^{(1)}, \vec{p}^{(2)}, \vec{p}^{(3)} \cdots$ 独立

$\vec{r}^{(1)}, \vec{r}^{(2)}, \vec{r}^{(3)} \cdots$ 独立

最大でもN回で収束

(行列) x (ベクトル)、
ベクトルの内積のみ

グラスマン変数

$$\bar{\psi}_i \psi_j + \psi_j \bar{\psi}_i = \delta_{ij},$$
$$\bar{\psi}_i \bar{\psi}_j + \bar{\psi}_j \bar{\psi}_i = 0,$$
$$\psi_i \psi_j + \psi_j \psi_i = 0$$

$$\int d\bar{\psi}_i = \int d\psi_i = 0,$$
$$\int \bar{\psi}_i d\bar{\psi}_i = \int \psi_i d\psi_i = 1$$

Berezin（１９６６）

$$\int D\bar{\psi} D\psi e^{-\bar{\psi} A \psi} = \det A,$$
$$\int D\bar{\psi} D\psi (\bar{\psi}_i \psi_j) e^{-\bar{\psi} A \psi} = \left(A^{-1} \right)_{ji} \det A,$$
$$\int D\bar{\psi} D\psi (\bar{\psi}_i \psi_j \bar{\psi}_k \psi_l) e^{-\bar{\psi} A \psi} = \left\{ \left(A^{-1} \right)_{ji} \left(A^{-1} \right)_{lk} - \left(A^{-1} \right)_{jk} \left(A^{-1} \right)_{li} \right\} \det A$$

Matthews-Salam公式

練習

$$\bar{\psi} A \psi = \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{pmatrix} \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} \psi_1 & \psi_2 \end{pmatrix}$$ の時

$$\int d\bar{\psi}_1 d\psi_1 d\bar{\psi}_2 d\psi_2 e^{-\bar{\psi} A \psi} = \det A$$ を示せ

$$e^{-\bar{\psi} A \psi} = 1 + (\bar{\psi}_1 A_{11} \psi_1 + \bar{\psi}_1 A_{12} \psi_2 + \bar{\psi}_2 A_{21} \psi_2 + \bar{\psi}_2 A_{22} \psi_2)$$
$$+ \frac{1}{2} (\bar{\psi}_1 A_{11} \psi_1 + \bar{\psi}_1 A_{12} \psi_2 + \bar{\psi}_2 A_{21} \psi_2 + \bar{\psi}_2 A_{22} \psi_2)^2 + ..$$

ここしか効かない

メソンのプロパゲータ

- 例1 $\pi(x) = \bar{u}(x)\gamma_5 d(x) = \bar{u}^a_\alpha(x)(\gamma_5)_{\alpha\beta} d^a_\beta(x)$

$$\frac{1}{Z}\int DUD\bar{u}DuD\bar{d}\,Dde^{-S_G-\bar{u}Wu-\bar{d}Wd}\,\pi(x)\pi(y)^\dagger$$

$$\bar{u}^a_\alpha(x)(\gamma_5)_{\alpha\beta} d^a_\beta(x)(-\bar{d}^b_{\alpha'}(y)(\gamma_5)_{\alpha'\beta'} u^b_{\beta'}(y))$$

$$= \frac{1}{Z}\int DUe^{-S_G} \det W^{(u)} \det W^{(d)}$$

$$\times G^{(u)ba}{}_{\beta'\alpha}(y,x)(\gamma_5)_{\alpha\beta} G^{(d)ab}{}_{\beta\alpha'}(x,y)(\gamma_5)_{\alpha'\beta'}$$

$$Tr\left(G^{(u)}(y,x)\gamma_5 G^{(d)}(x,y)\gamma_5\right)$$

$$\frac{1}{Z}\int DU e^{-S_G} \det W^{(u)} \det W^{(d)}$$
$$\times Tr\left(G^{(u)}(y,x)\gamma_5 G^{(d)}(x,y)\gamma_5 \right)$$

$$G^{(u)} \equiv W^{(u)-1}, G^{(d)} \equiv W^{(d)-1}$$

- **例2** $$\sigma(x) = \frac{\bar{u}(x)u(x) + \bar{d}(x)d(x)}{\sqrt{2}}$$

$$= \frac{\bar{u}^a_\alpha(x)u^a_\alpha(x) + \bar{d}^a_\alpha(x)d^a_\alpha(x)}{\sqrt{2}}$$

$$\frac{1}{Z}\int DUD\bar{u}DuD\bar{d}Dde^{-S_G-\bar{u}Wu-\bar{d}Wd}\sigma(x)\sigma(y)^\dagger$$

$$\frac{\bar{u}^a_\alpha(x)u^a_\alpha(x) + \bar{d}^a_\alpha(x)d^a_\alpha(x)}{\sqrt{2}} \times \frac{\bar{u}^b_\beta(y)u^b_\beta(y) + \bar{d}^b_\beta(y)d^b_\beta(y)}{\sqrt{2}}$$

$$\Rightarrow (G^{(u)aa}_{\alpha\alpha}(x,x)G^{(u)bb}_{\beta\beta}(y,y) - G^{(u)ab}_{\alpha\beta}(x,y)G^{(u)ba}_{\beta\alpha}(y,x))$$

$$+G^{(d)aa}_{\alpha\alpha}(x,x)G^{(u)bb}_{\beta\beta}(y,y) +$$

$$\frac{1}{Z}\int DUe^{-S_G}\det W^{(u)}\det W^{(d)}$$

$$Tr\left(G^{(u)}(x,x)\right)Tr\left(G^{(u)}(y,y)\right)-Tr\left(G^{(u)}(x,y)G^{(u)}(y,x)\right)$$

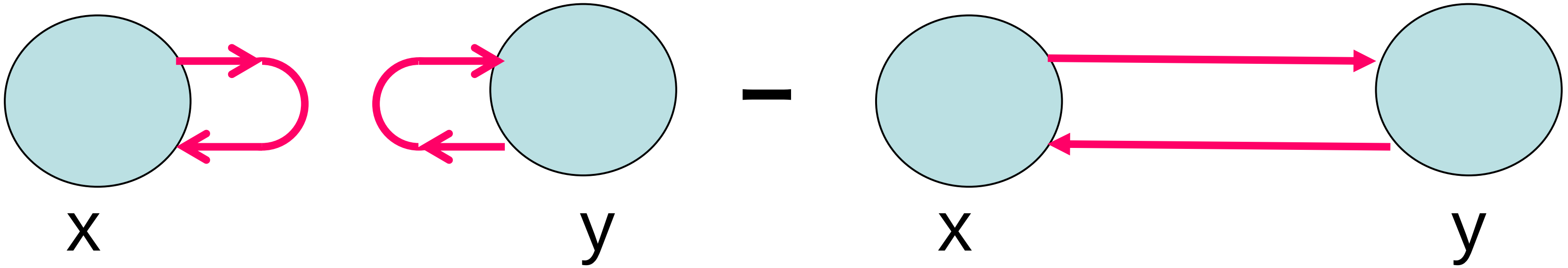
$$+Tr\left(G^{(d)}(x,x)\right)Tr\left(G^{(u)}(y,y)\right)+Tr\left(G^{(u)}(x,x)\right)Tr\left(G^{(d)}(y,y)\right)$$

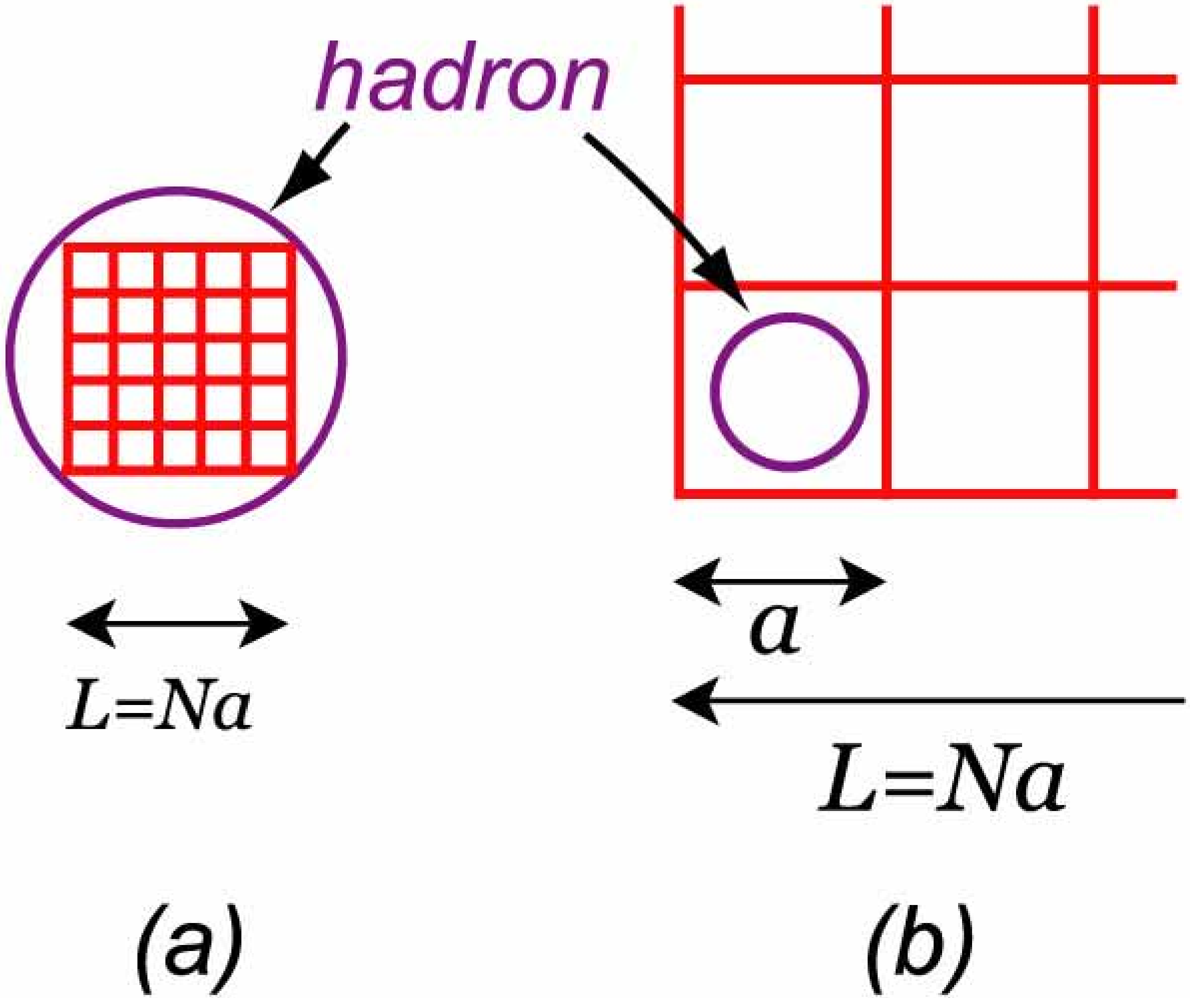
$$+Tr\left(G^{(d)}(x,x)\right)Tr\left(G^{(d)}(y,y)\right)-Tr\left(G^{(d)}(x,y)G^{(d)}(y,x)\right)$$

$G^{(u)}=G^{(d)}$ **の時**

$$2Tr\left(G(x,x)\right)Tr\left(G(y,y)\right)-2Tr\left(G(x,y)G(y,x)\right)$$

$$+2Tr\left(G(x,x)\right)Tr\left(G(y,y)\right)$$




hadron
L=Na
a
L=Na
(a)
(b)